

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

# Towards Driver Adaptive Active Safety Systems

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*“The future belongs to those who believe in the beauty of their dreams”*  
Eleanor Roosevelt



# Abstract

In recent years, active safety systems have become a standard accessory in new cars and trucks. This thesis addresses one of the major flaws of most such systems: they act the same independently of who is operating the vehicle. Driver modeling has been in focus for research the past 60-70 years, but it is only recently that adapting driver assistance systems to the current driver has been one of its main application areas. The first part of this thesis gives an overview of the historical development of driver models and leads up to one of the most popular modern modeling frameworks, hybrid driver models. In particular, the framework of hybrid ARX models, including probabilistic ARX models, is described. It is shown how these models can be used to predict a driver's steering behavior and to classify the dominating driving style based on vehicle sensor measurements. An algorithm for online steering angle prediction is designed and validated in Paper 1. The driver behavior is also classified as a normal or aggressive. The concept of driver behavior prediction and classification is analyzed in depth for vehicles driving behind another vehicle in Paper 2. It is concluded that for long prediction horizons, there is no gain using a complex model structure. Instead, one-step ahead prediction can be used for classification of the driver's braking behavior. The thesis is summed up with a discussion on how to integrate driver classification models into existing active safety systems.

**Keywords:**

Driver Modeling, Classification, Parameter Estimation, Hybrid Systems, Threat Assessment, Collision Avoidance, Active Safety



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Malin Sundbom  
Göteborg, January 2014





# List of publications

This thesis is based on the following appended papers:

## Paper 1

M. Sundbom, P. Falcone and J. Sjöberg, Online Driver Classification Using PrARX Models, *Intelligent Transportation Systems Conference*, October 2013, The Hague, The Netherlands.

## Paper 2

M. Sundbom and J. Sjöberg, Classification of Longitudinal Driver Behavior, *Edited version submitted to IEEE 2014 Intelligent Vehicles Symposium*, June 2014, Dearborn, Michigan, USA.



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# Part I

## Introductory chapters





# Chapter 1

## Overview

### 1.1 Automotive Safety

Every year, more than one million people die as a result of traffic accidents and for each of them there are 20-50 persons sustaining non-fatal injuries [1]. For middle-income countries the cost of injuries is estimated to be 1-2% of the gross national product each year [2]. Furthermore, the World Health Organization predicts that by 2030, road traffic accidents will be the fifth leading cause of death unless precautions are made. To reduce this huge loss of lives and money, **active safety systems** that take control of a vehicle in emergency situations are becoming increasingly common. In contrast to **passive safety systems**, which aims to mitigate the consequences of a crash by e.g. seatbelts and airbags, active safety systems try to prevent a crash from occurring in the first place. An example of such a system is the **emergency brake system**, which is commonly installed in new cars and by 2015 will be required by law on most types of trucks and coaches in Europe [3]. The purpose of the system is to take control from the driver in case they does not react to a critical situation that would otherwise result in a collision. Another example is the **lane keeping system**, which issues a warning if the driver unintentionally crosses the lane markings.

### 1.2 Driver Adaptive Safety Systems

In order to be minimally intrusive, an active safety system shall only intervene when it is judged that the driver is incapable of avoiding an accident. To accomplish this, most systems assume that the driver is able to perform complicated evasive maneuvers with small time margins before entering a potentially threatening traffic situation. However, regular (i.e., not professional) drivers do not have these skills and would therefore probably accept

earlier interventions of an assisting system. Allowing the thresholds of assisting systems to adapt to the driver would make the range of automatic maneuvers wider and it would be possible to reduce the intervention severity. Another benefit would be the possibility to delay or even turn off certain warnings, in order to avoid annoying a skilled driver to the extent that he or she deactivates the system. This thesis deals with the problem of integrating an estimation of the driver capability into current active safety systems, envisioning to provide adaption to the individual.

### 1.3 Thesis Contributions

The principal objectives of this thesis are to: (1) give a historical orientation on the development of driver models from the 1950's until today, (2) provide a basic understanding of parameter estimation techniques applied to a hybrid framework for driver modeling and (3) discuss the gain of integrating driver models into existing active safety systems. The main contributions are:

- Application of a hybrid driver model to online classification of driving style and prediction of lateral acceleration.
- Investigation of the benefits of hybrid models for prediction of pedal operation behavior in a car-following scenario.
- Application of the probabilistic ARX model framework to longitudinal and lateral behavior respectively.
- Analysis of the predictive power of different dynamic driver models for varying time horizons in the car-following case.
- Classification of a driver's car-following behavior into safe and dangerous modes, and based on this prediction of when the driver will start and stop pressing the brake pedal.
- Comparison of adaptive driver classification methods based on hybrid models and non-adaptive driver model classification.

### 1.4 Thesis Outline

This thesis consists of two parts. Part I, consisting of seven chapters, serves as an introduction, giving a context and some background to the appended papers that constitute the second part of the thesis. Chapter 2 starts with

a historical overview of driver modeling frameworks, leading up the most recent advances in the topic which are discussed in more depth in Chapter 3. Chapter 4 deals with how to identify the driver specific model parameters offline and online respectively. The probabilistic ARX model and its application to driver classification is given in Chapter 5, where also driver model integration into active safety systems is briefly discussed. In Chapter 6, the included papers are summarized, while complete versions of the papers are appended in Part II of the thesis. Chapter 7 closes Part I with some concluding remarks and a look into the future of driver modeling for active safety systems.



# Chapter 2

## Driver Modeling through History

Modeling of driver behavior became popular in the mid 1950's and has been focus for a large number of publications ever since. This chapter gives a historical overview on how the research has progressed from the first linear feedback models and up to today's popular hybrid methods. Figure 2.1 shows an overview of the development from the 1940's until today.

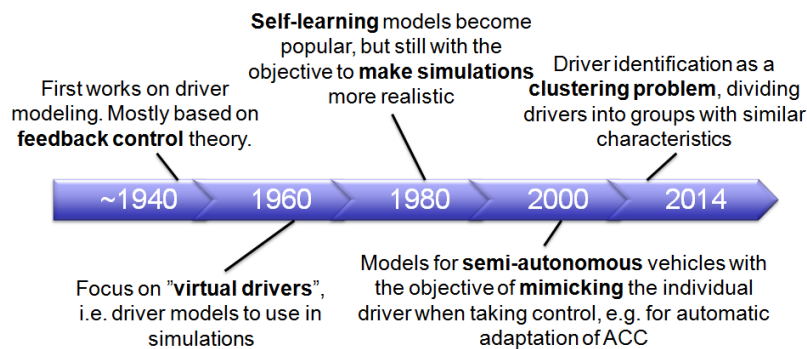


Figure 2.1: Timeline showing the historical development of driver modeling methods.

Most driver models focus on describing specific driving tasks, such as longitudinal acceleration behavior or the characteristics of a lane change maneuver, and are optimized to suit their specific fields of application. The application fields can be divided into four parts:

- *Virtual driver models* – Used to automatically drive in e.g. a simulation environment and perform various tests without the need of a real driver.
- *Anomaly detection models* – Used to detect when the driver is out of their usual behavior, e.g. because of distraction or sleepiness.

- *Cognitive models* – Models based on psychology and behavior analysis, i.e. not necessarily purely mathematical models.
- *Driver behavior recognition/prediction models* – Used to predict driver intent or to identify driver preferences and capabilities in order to adapt a safety system to the individual driving the vehicle.

The application fields should not be seen as absolute partitions of the modeling methods but they overlap and much could be gained from studying models from several application areas.

## 2.1 1940–1960: Feedback Control Models

The first application field of driver models was virtual drivers. Already in 1953, Pipes studied lines of vehicles and concluded that the behavior seems to follow a **law of separation**, meaning that each vehicle strives to keep a specific distance in time to the vehicle ahead [4]. His studies were followed by several others (e.g. [5], [6]), all based on the assumption that each driver has a specific sensitivity to different stimuli, resulting in a certain acceleration. At the same time as Pipes, Kondo developed a model based on **linear feedback** of the deviation of yaw angle, course angle and lateral position [7]. This resulted in simple models that were used mainly for simulating driving behavior.

## 2.2 1960–1980: Virtual Drivers

Around 1960, the modeling of drivers started to gain increased attention. Human-machine and aircraft pilot studies formed the bases for the models. In 1964, Sheridan came up with the first **preview model**, which later became a key property in many models [8]. It builds upon the assumption that a driver looks ahead and acts as a controller, trying to minimize the deviation from the planned path. This was taken up by Kondo *et al.* who in 1968 designed a steering model based on deviations between the driver's sight-point and the desired course [9]. In 1969, a **3-part model** was devised in a theory of manual vehicular control by McRuer and Weir [10]. It consists of three parts: (i) **compensatory** lane position control; (ii) **anticipation** of changes in the upcoming path; (iii) **precognitive control** for executing driver commands.

Control theoretic models dominated the period between 1970-1990. These often relied on the preview model or concept of linear feedback. Weir and McRuer started to apply the well-known **crossover model** for car drivers

in 1973 [11], a model that was earlier used to describe the dynamics of aircraft piloting [12]. The crossover model is based on a describing function of the vehicle and driver **combined**. It describes driving as a compensatory tracking task, where the driver tries to adapt their behavior such that it remains as linear as possible. The behavior can be modeled with a quasi-linear transfer function close to the crossover frequency. The driver behavior is described in terms of control bandwidth and phase margin, using a linear transfer function.

## 2.3 1980–2000: Self-learning Models

In 1981, Gipps proposed a more general model of driving behavior, able to switch between two modes [13]. One mode is free acceleration where the driver accelerates just enough to reach a desired speed and then keeps this speed. The other mode consists in keeping a safe distance and relative speed to the vehicle ahead. The Gipps model has been popular due to its simplicity and its low computational complexity. It was one of the first attempts to describe driver behavior by using different dynamics for different driving situations, i.e. one of the first **hybrid models**.

The same year as Gipps presented his model of longitudinal driver behavior, MacAdam published his work on modeling of lane change maneuvers [14]. Assuming the driver to operate as a controller in a closed loop system of driver/vehicle, he concluded that driver steering control during path following can be considered a time-lagged optimal preview control process.

In the end of the 1980's and during the 1990's, the focus of driver modeling shifted from control theoretical models to self-learning models, in particular models based on **artificial neural networks**. In this way nonlinear models for driver behavior identification were introduced. In 1991, Kreiss and Küttelwesch applied neural networks to model human driver behavior [15]. It was experimentally shown that driving characteristics could be identified by the network and also be used as an information source for driver assistance systems. MacAdam and Johnson later tried to combine the neural network approach with the preview model and apply it to modeling of driver steering behavior [16]. In 1998, MacAdam *et al.* also used the neural networks approach to study longitudinal car-following behavior [17]. During this period, other search-based and machine learning methods became popular as well. For example, Nagai used **genetic algorithms** for optimization of a basic preview model with the aim of analyzing driver behavior in collision avoidance scenarios [18].

Liu and Pentland were among the first to use HMMs, more specifically

**hidden Markov dynamic models**, to recognize driving behavior and to predict the most likely behavior in a short time horizon [19]. They considered the human as a Markov device with a number of mental states, each corresponding to a state in a Markov chain. Each state corresponds to a single behavior and has its own defined probabilities for transition to other states. A driving maneuver can then be described as a sequence of states, e.g. car-following, speed adjustment and changing lanes might be three states together defining a lane change maneuver. This work marked the transition into to a new millennium and a new era of driver modeling methods.



# Chapter 3

## Recent Advances in Driver Modeling

When entering into the 2000s, the areas of driver assistance and active safety systems gained increased attention. With this, the need for predicting driver intention in line with what Liu and Pentland did in 1997 [19] and the need for classification of driver behavior emerged. However, as driving simulators got to be more and more realistic, many vehicle manufacturers realized the usefulness of integrating virtual drivers into the simulator environments in order to perform various tests on their systems. This led to two clear driver modeling tracks, where the virtual driver track still is very much influenced by the preview theory developed in the 1960's and the driver prediction track is dominated by hybrid modeling methods.

### 3.1 Preview Models

In year 2000, Sharp *et al.* formed a mathematical model for steering control using **multipoint preview**. This means that the angular deviation between a set of preview points and corresponding points on a desired driving path is fed back to the system [20]. Eleven years later, Tan *et al.* introduced a **target and control** model which they applied to the double lane change maneuver. Instead of feeding back deviations from a desired path at a preview distance, the angular error between the current heading angle and the heading angle that would be necessary to reach a target point at a preview distance is used as feedback to a system. The system then calculates the resulting steering rate control that minimizes this error [21].

## 3.2 Intent Recognition Models

Three years after Pentland and Liu published their work on driver intent recognition, Pentland published more work in the same application area, this time in collaboration with Oliver [22]. Their work were based on a new test bed platform with a real-time acquisition and playback system. It used machine learning methods to model and recognize driver maneuvers. Particular focus was put on how contextual measures affected driver performance. The authors used both **dynamical graphical models** and an extension to HMMs which they called **coupled HMMs**. With this framework, they managed to recognize 7 different maneuvers in average 1 second before any significant change in vehicle signals occurred. Kuge *et al.* also used Hidden Markov Models to detect driving maneuvers for lane change and lane keeping behavior [23]. They concluded that their HMM-based recognition model had potential to detect lane changes at a very early stage, but also underlined the importance to develop more general and more robust models before integrating the method into a driver assistance system. The driver assistance systems will then have possibility to adapt the behavior to the current driver and the current situation. In 2007, McCall *et al.* integrated a driver intent system based on monitoring the head and feet motions of the driver with a brake assist system based on Bayesian networks [24].

## 3.3 Classification Models

Another way of integrating information about the driver into a driver assistance system is to classify the driving style of the current driver. Raymond *et al.* found that the curve-driving behavior could be used to characterize a driving style as either “normal” or “fast” [25]. It was assumed that the driver controls an individual safety margin of perceived lateral acceleration with respect to the predicted steering deviation.

In 2010, Constantinescu *et al.* investigated how to classify drivers according to driver risk-proneness [26]. Using **cluster and principal component analysis** on statistical cues based on speed, acceleration and mechanical work, they managed to divide the drivers into five groups with different driving characteristics. Also Raksincharoensak *et al.* focused on longitudinal driving situations and classification using statistical machine learning methods [27]. However, the main objective was not to characterize the driver, but to detect driving situations such as car-following and braking.

One of the most recent works in the area was published by Galdepally *et al.* in 2011 [28]. They model the combined driver-vehicle behavior using

a combination of hybrid state systems (HSS) and continuous state systems (CSS), where the driver state is reflected in the HSS and the vehicle dynamics in the CSS. Two methods for driver state estimation are discussed and compared, namely rule-based estimation and HMMs. For the rule-based approach, the relationship between driver state and vehicle dynamics is described through a set of predefined rules, while the HMM based approach consists of a stochastic relationship where the states are not defined in advance but determined by training data. The HMM estimation method showed best results and was further recommended since it would be difficult to have a rule based system that accurately enumerates every possible transition in a system with a large number of states. In the same work, Gadepally *et al.* further developed the HMM approach to predict future driver states by introducing a **hierarchical multilevel HMM structure**, where each hidden state in the HMM consists of another HMM [28].

### 3.4 Hybrid Models

In recent years, one of the most promising approaches for capturing driver capabilities and model driver behavior is to use **hybrid models**. In general, these assume that a driver switches between a number of discrete driving modes while driving, each mode corresponding to a basic driver behavior, e.g., straight driving, lane-change or collision avoidance. Also in this area, HMMs have proved to be a useful tool. Already mentioned work on this was by Liu *et al.* [19], as well as Oliver *et al.* [22], whose work did not only focus on predicting driver behavior, but also clustered it into different modes based on vehicle signals. By estimating the transition probabilities between the modes, it is possible to model the driver’s decision making process. Another advantage is that HMMs also allow to accounting for the integral stochasticity in human behavior, since it is a probabilistic model.

HMMs are often used in combination with other modeling methods. For example, Akita *et al.* used HMMs to cluster the driver behavior and described the behavior in each mode by an **auto regressive with exogenous input (ARX)** model in [29]. This allows for capturing the driver’s behavior in each mode as well as the switching between the modes. Other types of hybrid ARX models for driver behavior have been the focus of several studies, see, e.g., [30], [31]. Akita *et al.* used **stochastic switched-ARX (SS-ARX)** driver models for the first time in [29]. The parameters were estimated using a modified version of the Expectation Maximization algorithm [32]. Driving simulator tests on the SS-ARX model showed good performance in recognizing lateral driving behavior in a car-following scenario.

**probabilistic ARX (PrARX)** models can also be considered a type of switched ARX model, but having a structure that makes it easier to estimate its parameters online. In contrast to the SS-ARX models, where the system output depends only on the current mode, in PrARX models the system output is calculated by weighting the outputs of as many ARX models as modes, with weights corresponding to the probabilities of the system being in each mode. Taguchi *et al.* used a PrARX model in [30] to describe longitudinal driving behavior in a car-following scenario. In [33], it was combined with a model predictive control (MPC) algorithm and integrated into a brake assist system. The MPC was designed to minimize the time spent in modes where the driver showed a collision avoidance behavior.

Hybrid models with a stochastic component is a very promising approach for finding a general driver model which is reliable and robust enough to integrate into a commercial vehicle safety system. In this thesis it is investigated how the hybrid modeling framework can be used to model and classify both longitudinal and lateral driving behavior.

# Chapter 4

## Parameter Estimation

Most driver models contain a finite number of parameters, whose values determine the behavior of the model. By tuning these parameters, it is possible to adapt a driver model to the individual who is operating the vehicle. If the parameter estimation is done on buffered data, allowing to use "future output" in each data point, it is called **offline estimation**. However, real-time driver adaptation demands that the estimated parameter value is continuously re-estimated and updated. No "future data" can be used in the estimation, which is then called **online estimation**. This section describes methods for how this can be achieved and also pinpoints some important aspects on the properties of the dataset on which the estimation is performed.

### 4.1 Offline Parameter Estimation

Offline estimation is performed on data sets, where a whole set is available at the time of estimation. This results in parameter values for which the model best describes the data in an average sense for all data samples. The estimation is commonly done using a **prediction error method (PEM)**, which means that the following problem is solved:

$$\hat{\theta} = \arg \min_{\theta} \epsilon(\theta), \quad (4.1)$$

$$\epsilon(\theta) = \frac{1}{N} \sum_{k=1}^N l(y_k - \hat{y}_k(\theta)), \quad (4.2)$$

where  $y_k$  is the measured output signal from the data set,  $\hat{y}_k(\theta)$  is the estimated output value at time sample  $k$ . The values of the estimated parameters, denoted by  $\hat{\theta}$ , are used to calculate  $\hat{y}$ . A common choice of the

function  $l$  is  $l(y_k - \hat{y}_k(\theta)) = (y_k - \hat{y}_k(\theta))^2$ , in which case the estimation is called **least squares estimation**.

In the case where  $l(\theta)$  is a convex function, e.g. linear or squared, the parameter estimation is rather straight forward and can be done by e.g. resorting to a gradient descent method, such as the method of **steepest descent**. Steepest descent is an iterative method which uses the direction of steepest descent in the search space. In each iteration it takes a step in that direction and updates the function value until convergence is reached or a certain stopping criteria is fulfilled [34]. In the case of non-convex functions, gradient descent methods risk to get stuck in local minima, see Fig. 4.1. This risk implies that the model must always be validated to ensure that the estimates of the parameter values have converged to the global optimum.

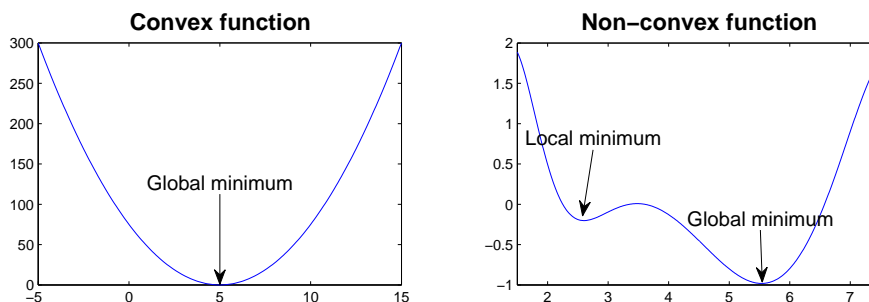


Figure 4.1: The leftmost figure shows the global minimum of a convex function, which will be found using the method of steepest descent. The rightmost figure shows the local and global minimum of a non-convex function, where the steepest descent method risks to get stuck in the local minimum and thus not reach the global optimum.

## 4.2 Online Parameter Estimation

In many cases, the system behavior changes over time and it is desirable to update the model parameter values accordingly while the system is in operation. To obtain this, online estimation is used. It is also called **recursive estimation** since the data are processed recursively as new samples are added to the data set. This can be accomplished in several ways, but the method used in this work is to use a sliding window. It is based on assigning a weighting factor  $\beta(t, k)$  to each measurement  $k$ , where  $t$  is the time and  $k = 1, 2, \dots$ . The sliding window method assumes that  $\beta(t, k) = 1, \forall k < \tau$  and  $\beta(t, k) = 0 \quad \forall k \geq \tau$ , where  $\tau$  is the window

length, see Fig. 4.2. This means that only the  $\tau$  most recent measurements have an impact on the estimation, the rest of the data is disregarded.

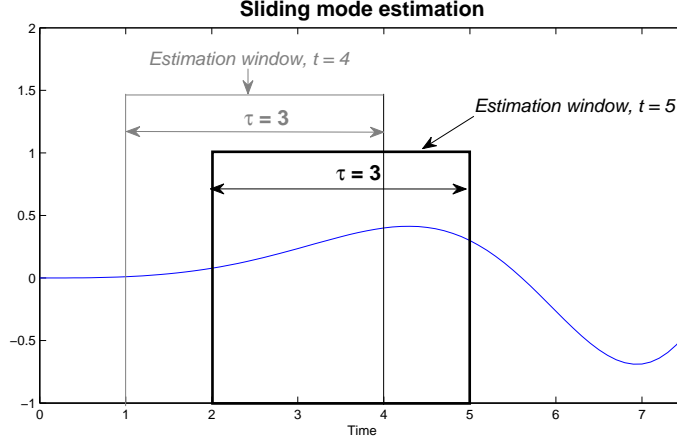


Figure 4.2: Sliding mode estimation with a window of length 3. The boxes show the included time samples for estimation when  $t = 3$  and  $t = 4$  respectively. All samples outside the boxes are disregarded in the estimation algorithm.

In the following equations, the PEM method is applied to formulate the problem of online estimating the parameters  $\theta$  using the sliding window approach. Denote the discrete time index by  $t$  and let  $\tau$  be the length of a sliding estimation time window. At every time instant, the following optimization problem is solved

$$\theta^*(t) = \arg \min_{\theta} \epsilon_t(\theta), \quad (4.3)$$

$$\text{with } \epsilon_t(\theta) = \frac{1}{\tau} \sum_{k=t-\tau}^t l(y_k - \hat{y}_k(\theta)). \quad (4.4)$$

where  $l$  is often chosen as  $l(y_k - \hat{y}_k(\theta)) = (y_k - \hat{y}_k(\theta))^2$ . At time  $t = 0$ , initial values of parameters  $\theta$  are used, which can e.g. be estimated by offline estimation using the first  $\tau$  samples in the dataset. The online estimation is then performed on the remaining dataset, starting from  $t = \tau + 1$ .

## 4.3 Signal Excitation

Reliable identification of the model parameters is possible only if the system is excited with enough input energy [35]. For driver identification this

may be a problem because of the data window containing only small signal variations. During a normal drive, much of the driving is usually performed on highways with relatively small variation in velocity, acceleration etc., i.e. only very small changes in input and output signals. It is possible to overcome the problem by e.g. using different lengths on the estimation windows depending on the traffic condition, or by only including certain data in the estimation data set. The latter approach was used in Paper 1 of this thesis, where all data collected on road segments with a curvature below a certain threshold were excluded.



# Chapter 5

## Driver Classification and System Integration

In this chapter, the dominating modeling framework used in this work is described, namely PrARX-models. It is shown how the framework can be adapted to perform classification of driving style, in particular to distinguish between “normal” and “aggressive” driving behavior, as well as predicting driver braking behavior. The chapter is concluded with a discussion on how to integrate this kind of modeling into existing active safety systems.

### 5.1 PrARX-models for Driver Modeling and Classification

PrARX models have previously been used for modeling and predicting continuous driver behavior, mainly in car-following scenarios. In this work, it is used to cluster drivers into two groups depending on their driving style or their braking behavior.

A PrARX model is constituted by a set of different ARX-models. Consider a system  $S$  as shown in Fig. 5.1. An ARX model calculates the output  $y_t$  of the system  $S$  at time  $t$  as:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_n y_{t-n_a} + b_0 u_t + b_1 u_{t-1} + \dots + b_m u_{t-n_b} + e_t, \quad (5.1)$$

where  $e_t$  is an error term. The orders of the ARX-model are denoted  $n_a$  and  $n_b$ , while  $a_1, a_2, \dots, a_{n_a}$  and  $b_0, b_1, \dots, b_{n_b}$  are the model parameters. The ARX model can also be expressed in a more compact form as

$$y_t = \theta^T \phi_t + e_t, \quad (5.2)$$


 Figure 5.1: System with inputs  $u_t$  and outputs  $y_t$ 

where the parameter vector  $\theta$  and the regression vector  $\phi_t$  are defined as

$$\theta = [a_1, a_2, \dots, a_{n_a}, b_0, b_1, \dots, b_{n_b}]^T, \quad (5.3)$$

$$\phi_t = [y_{t-1}, y_{t-2}, \dots, y_{t-n_a}, u_t, u_{t-1}, \dots, u_{t-n_b}]^T, \quad (5.4)$$

respectively.

A PrARX model describes the behavior of the system  $S$  by letting it operate in  $s$  modes  $S_1, \dots, S_s$ . Each mode  $S_i$  is described by a separate ARX-model, and a probability function  $P_i$ . The probabilistic weighting functions  $P_i(\phi_t)$ ,  $i = 1, \dots, s$  express the probability that the system  $S$  is operating in mode  $i$  and are defined by the following softmax function

$$P_i(\phi_t) = \frac{e^{\eta_i^T \phi_t}}{\sum_{j=1}^s e^{\eta_j^T \phi_t}}, \quad (5.5)$$

where the parameter vectors  $\eta_i$ ,  $i = 1, \dots, s - 1$  determine the impact each operating mode has on the model output. Moreover, the system output  $y_t$  is calculated as a linear combination of the ARX model outputs with the corresponding probability functions  $P_i$ , i.e.

$$y_t = \sum_{i=1}^s P_i(\phi_t) \theta_i^T \phi_t. \quad (5.6)$$

The PrARX model can be transformed into a piecewise ARX model (PWARX) described by a Mixed Linear Dynamical (MLD) representation. This is done by introducing an auxiliary binary variable  $z_{i,t}$  with  $z_{i,t} = 1$  for  $i = \arg \max_i P_i(\phi_t)$  and  $z_{j,t} = 0$  for all  $j \neq i$ . The model output  $y_t$  in (9) can be rewritten as

$$y_t = \sum_{i=1}^s z_{i,t} \theta_i^T \phi_t. \quad (5.7)$$

In short, the driver behavior is assumed to be described only by the ARX-model corresponding to the mode with highest value of the probability function. Driver classification can then be carried out by letting one class correspond to one operating mode, i.e. at all times where mode 1 has the highest probability function value, the system belongs to class 1, and at all times where mode 2 has the highest probability function value, the system belongs to class 2 etc. Figure 5.2 shows the different model structures for PrARX and PWARX models.

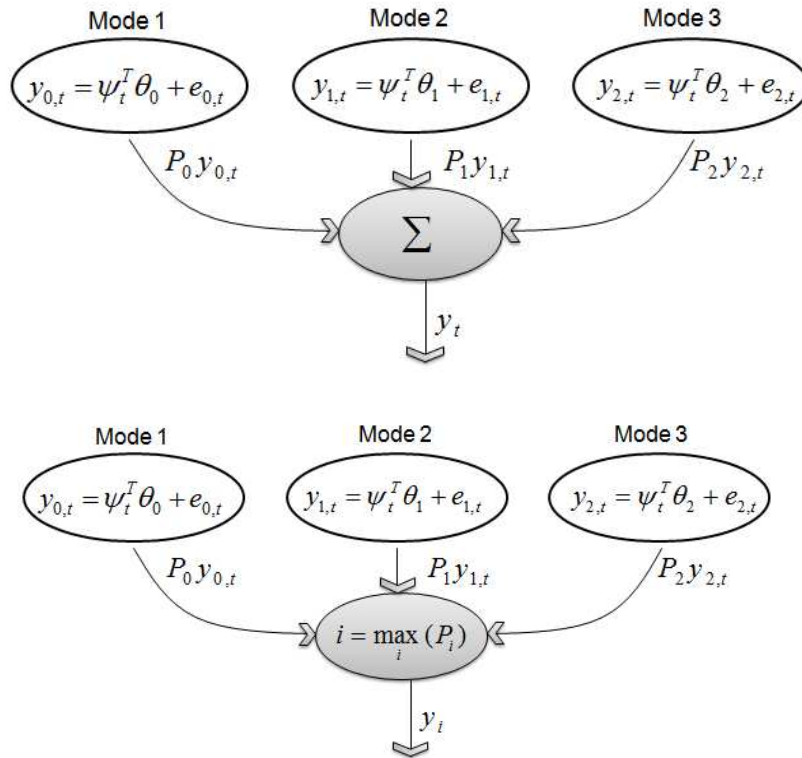


Figure 5.2: Structure of a PrARX (upper) and a PWARX model (lower).

### 5.1.1 Driving Style Classification

Driver classification using PrARX models can be performed online to detect variations in the driver behavior, as described in Paper 1. Based on how the driver conducts the vehicle in curvy sections of the road, it is possible to determine whether they are driving relatively “normal”, or in an unusually aggressive way. This is done by dividing the driver behavior into two modes: (i) one mode corresponding to aggressive driving, typically represented by higher acceleration in curves and more lateral movement compared to a

standard driver, and (ii) one mode representing normal driving, i.e. having a moderate curve acceleration and lateral movement in the lane. This distinction can then be used to tune the behavior of an active safety system. It can for example be assumed that a driver with an aggressive driving style would accept fewer and later warnings than a driver operating in normal mode.

In order to classify the driving behavior, a PrARX-model with two modes is used and its parameters are estimated to correctly determine the one-step ahead prediction of the steering angle. The parameters, including the probability weights of each mode, are continuously re-estimated using online parameter estimation with a sliding widow, as described in Chapter 4. In Paper 1, the following four input signals were selected, with the intent of capturing the above differences between normal and aggressive driving:

- Lateral acceleration  $u_1 = a_{lat}$  [m/s]
- Yaw rate  $u_2 = \dot{\psi}$  [ $s^{-1}$ ]
- Absolute value of movement in lane  $u_3 = |d\dot{\psi}|$ , [%/s], where  $\dot{\psi}$  is the yaw rate and  $d$  is defined as

$$d = \frac{d_l}{w_{lane}}. \quad (5.8)$$

with  $w_{lane}$  being the lane width and  $d_l$  the lateral displacement in the lane.

- Absolute value of the heading error  $u_4 = |e_\psi|$

The heading error  $e_\psi$  is defined as the difference between the angle of the tangent to the road center line at a point 0.7 s ahead of the vehicle,  $\psi_{road,lp}(t)$ , and the vehicle's yaw angle  $\psi(t)$  at time  $t$ :

$$e_\psi(t) = \psi_{road,lp}(t) - \psi(t). \quad (5.9)$$

Since the chosen input signals reflect the driver's response to the vehicle's lateral and yaw behavior only, in the straight parts of the test track the regressor does not contain enough information to predict the driver's steering. Hence, only samples corresponding to a curvature higher than a certain threshold are included in the parameter estimation. Schematics of the model can be found in Fig. 5.3.

Fig. 5.4 shows the result obtained in Paper 1. It was obtained using online estimation performed on data collected by real drivers on a test track. It can be seen that the model gives very accurate results in the curvy parts on the road.

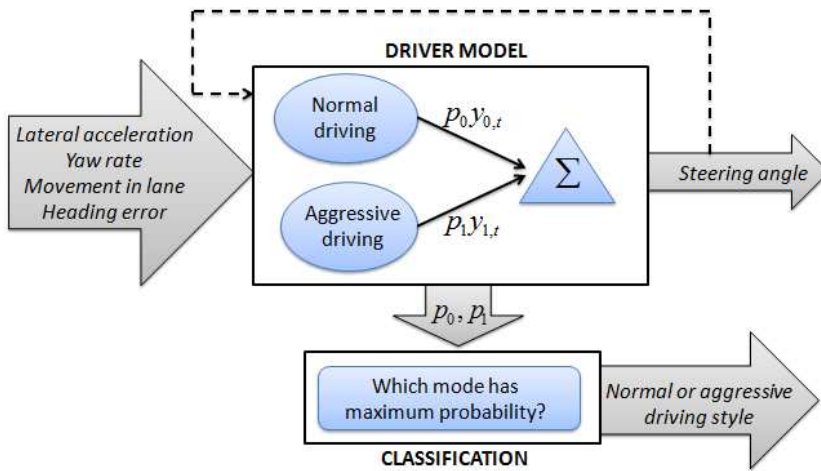


Figure 5.3: Schematics on driver style classification using PrARX models

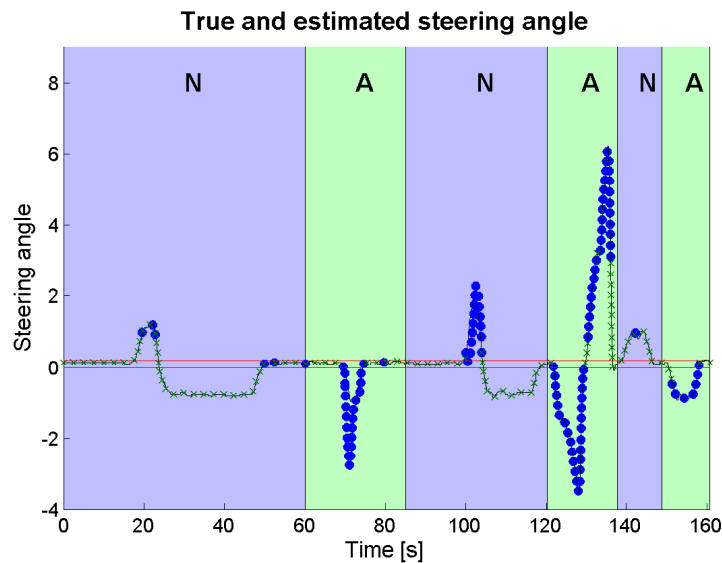


Figure 5.4: One step ahead prediction of the steering angle: estimated aggressive driving (blue circles) and estimated normal driving (green crosses). The shaded areas correspond to the true driver behavior: normal driving (N) and aggressive driving (A). The black line is the measured steering angle. The red lines indicate the curvature limits, i.e. samples outside the red lines should be correctly estimated, while samples between the red lines do not have enough information for a reliable estimate.

### 5.1.2 Driving Braking Behavior Classification

It is also possible to formulate a classification model which estimates whether the driver is aware of a dangerous situation or not, in the sense that they are going to brake, based on PrARX modeling techniques. In Paper 2, this is done by simplifying the PrARX model by removing its switching parameters and create a model switching modes according to whether the value of the pedal operation signal exceeds a certain threshold or not. In the paper, a **PrARX model with simplified mode transition (PrARX-SMT)** is used. The transition mode in this model is not completely sharp, but made smooth in a small region by using a sigmoid function with two parameters,  $\alpha$  and  $\beta$ :

$$\sigma(x) = \frac{1}{1 + \beta e^{x-\alpha}} \quad (5.10)$$

where  $x$  is the pedal operation signal. Mode  $\sigma(x) = 1$  corresponds to the driver operation in a safe mode and  $\sigma(x) = 0$  to the driver operating in a dangerous mode. The parameters of the model were estimated offline using data from a real driver, assumed to act in a correct fashion when approaching a threatening situation. The model can be used to compare the pedal operation of a real driver to that of a correct driver. If the difference is large, the real driver can be assumed to drive in a way that is not safe and it might be suitable to issue a warning. See Fig. 5.5 for a schematics of the comparison.

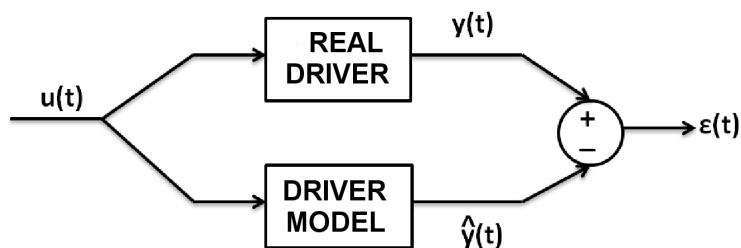


Figure 5.5: Schematics on how the driver model is compared to a real driver. As long as the model error  $\epsilon$  is kept small, the driver can be assumed to drive safely.

The data used in Paper 2 was collected by the Suzuki lab at Nagoya University, Japan. It was acquired from five persons driving in an advanced driving simulator with a stereoscopic immersive environment and focused on car-following on expressways [30]. The following signals were used as inputs:

- Range  $r$  [m], which is the relative distance between the subject vehicle and that in front of it.
- Range rate  $\dot{r}$  [m/s], which is the relative velocity with respect to the vehicle in front.
- KdB index, which is a risk index which can be understood as a measure similar to the logarithm of the inverse time-to-collision signal.

The KdB index can be calculated from the range and range rate as follows [36]:

$$\text{KdB} = \begin{cases} -10 \log | -\frac{\dot{r}}{r^3} 4 \cdot 10^7 |, & \dot{r} > 0 \\ 10 \log | -\frac{\dot{r}}{r^3} 4 \cdot 10^7 |, & \dot{r} \leq 0. \end{cases} \quad (5.11)$$

The continuous model output describes the use of brake and gas pedal. It is represented by a signal called pedal operation, defined as a positive value between 0 and 1 when the gas pedal is pressed and a negative value between 0 and -1 when the brake pedal is pressed. 1 and -1 corresponds to maximal pressure on the pedal, and 0 to no pressure at all.

This model can be used to predict when the driver will start and stop braking, by monitoring whether the current dominant mode in the model is the safe mode or the dangerous mode. The model was validated on several drivers and compared to other classification models, among which a model which is independent on the driver. This fixed model only uses a threshold on the time-to-collision (TTC) value to determine whether it is appropriate to brake or not. TTC is a measure on how close a collision is, assuming the current velocity is kept. It is defined as:

$$\text{TTC} = -\frac{-r}{\dot{r}}, \quad (5.12)$$

where  $r$  is the range, and  $\dot{r}$  the range rate. The comparison between the TTC-model and the PrARX-SMT model was based on sensitivity and specificity measures, as well as on a brake start/stop measure based on the amount of correctly detected, incorrectly detected and missed occurrences of when the brake pedal should be pressed and loosened. See Paper 2 for details on this measure. The PrARX-SMT model showed much better performance than the TTC model, which can be seen in Fig. 5.6. This implies that much can be gained from using the driver adapted PrARX-SMT model instead of the driver independent TTC-model, given that the model output is used to make a decision of whether an active safety system should intervene or not, e.g. if an automatic emergency brake system should start braking.

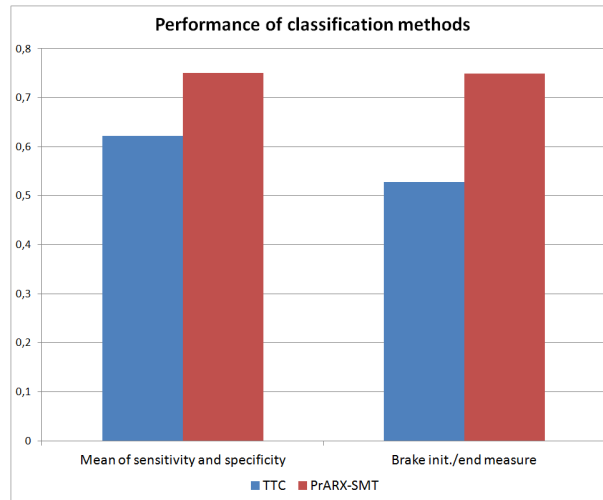


Figure 5.6: Classification Performances of the TTC-based model and the PrARX-SMT model.

## 5.2 Integration Into Active Safety Systems

In the previous section it was shown that it is possible to divide the driver operation into two modes using a PrARX-SMT model based on the gas and brake behavior on the driver. By continuously collecting data from the driver and adapt the parameters of this mode in real time, it would be possible to integrate this into a brake assist system according to the schematics in Fig. 5.7.

If the driver is estimated to be in the dangerous mode, it is assumed that they is prepared to take care of a potentially dangerous situation and would thus not desire a warning. However, if they is still in the non-braking mode while the threat assessment system detects a potentially dangerous situation, the driver is not likely to have recognized the threat and a warning would be considered as appropriate. Moreover, if the threat is severe to the point that the driver has little chance to avoid it, the system should intervene even if the driver operates in the braking mode. This would also reduce the risk of collisions caused by a false classification.

The benefit of having a system as described above is that the drivers will have fewer warnings they think are “false” in the sense that the situation is under control. This minimizes the risk that the driver turns the system off because they finds it annoying. Also, an unskilled driver may have more and earlier warnings, given that the threshold of what is perceived by the system as a dangerous situation is set at a suitable level.



## 5.2. INTEGRATION INTO ACTIVE SAFETY SYSTEMS

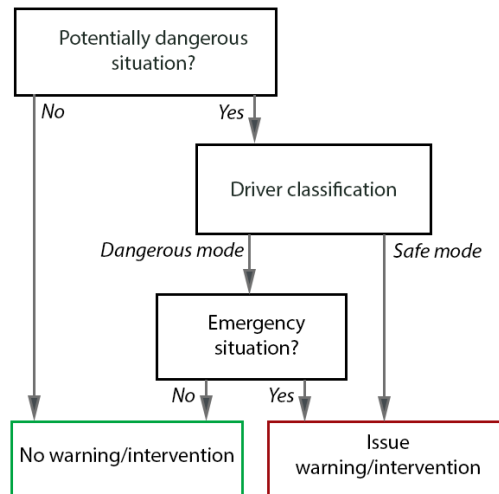


Figure 5.7: Schematics on how to integrate driver classification into a brake assist system.



# Chapter 6

## Summary of Included Papers

This chapter provides a brief summary of the papers that constitute the base for this thesis. Full versions of the papers are included in Part II. The papers have been reformatted to increase readability and to comply with the layout of the rest of the thesis.

### Paper 1

M. Sundbom, P. Falcone and J. Sjöberg, Online Driver Classification Using PrARX Models, *Intelligent Transportation Systems Conference*, October 2013, The Hague, The Netherlands.

In Chapter 5, probabilistic ARX models were described. Paper 1 shows how this framework can be used to predict a driver's steering behavior. It is also described how to classify the current driving style, based on measurements from the vehicle sensors. Two modes are assigned to the model. One mode represents normal driving, i.e. the behavior of a driver using e.g. moderate acceleration in curves and a small range of lateral acceleration. The second mode describes aggressive driving, which typically contains higher curve accelerations and more lateral movement of the vehicle. An algorithm, online classifying the driving style and predicting the steering behavior, is designed and validated on data recorded on a test track. The algorithm is designed to distinguish between two driving styles corresponding to normal and aggressive driving and shows good performance.

### Paper 2

M. Sundbom and J. Sjöberg, Classification of Longitudinal Driver Behavior, *Edited version submitted to IEEE 2014 Intelligent Vehicles Symposium* June 2014, Dearborn, Michigan, USA.

Paper 2 analyses the PrARX modeling framework for describing a driver's longitudinal behavior in a car-following scenario. In the first part of the paper, the predictive performance of the PrARX model is compared to other dynamical models with a simpler structure. Range, range rate and the KdB risk index are used to predict the driver's pedal operation. It is found that for longer prediction horizons than one step ahead, there is no advantage having a more complex model structure than a linear ARX model. Moreover, in order to assess the severity of a given situation, it is enough to predict whether there is a need for braking or not. Thus, the second part of the paper deals with classification of driver behavior into a "safe" and "dangerous" mode. The one-step ahead prediction from the dynamical models are used for classification of whether the driver is braking or not. These are also compared to a simpler classification method and a fixed model, i.e. a model having a predetermined set of parameter values, independent on the current driver. It is found that the best classification performance is obtained using a PrARX model with simplified mode transition, where the mode partitioning is based on the drivers pedal operation in the previous time step. This driver adaptive classification model performs significantly better than the fixed model, in particular for detecting when to start and stop braking.

# Chapter 7

## Concluding Remarks And Future Challenges

### 7.1 Conclusions

One of the main challenges in today's active safety systems is how these can be adapted to the driver. Modeling the human being is a complex task, much because of the stochastic component that is integral to most human behavior. It is not likely that the same person will perform a task *exactly* the same way two or more times in a row, no matter if the surrounding conditions are identical.

This thesis investigates the history of driver modeling and the different areas of application. Hybrid driver models are judged to be the most promising framework for integration of driver models into active safety systems, because of their ability to use different dynamics in different traffic situations and/or for drivers showing different driving characteristics. In particular, the PrARX framework is described and applied to modeling and classification of both longitudinal and lateral driving behavior. One of the strongest features of these kinds of models is that they divide the driver behavior into a certain number of modes and also comprise a stochastic part in the mode switching logic. In addition, it is possible to estimate both the switching parameters and the ARX-parameters simultaneously both offline and online.

The lateral driver behavior is modeled by predicting the steering angle based on vehicle sensor data which was collected in a curvy environment. It is shown possible to distinguish between a driver expressing a normal curve taking behavior and a driver using a more aggressive behavior when driving through curves. Also the longitudinal driver behavior in a car-following scenario can be modeled by PrARX models. Range and range rate seems to be among the most important input signals to such models.

In Paper 2, a comparison of the predictive power of different dynamical models show that it is very difficult to get reliable long-term predictions. However, for one-step ahead prediction of the pedal operation signal, much can be gained by using a multi-mode hybrid ARX-model instead of a simple ARX-model. This implies that different dynamics dominate for different parts of the data. The driver seems to follow one kind of dynamics when pressing the gas pedal, but another when braking. Since only one-step ahead prediction is to trust, it may be advantageous to instead use the one-step ahead prediction for classification of the driver behavior. In Paper 2, it is shown that classification based on parametric models, where the parameters are adapted to the driver, gives better results in predicting the braking behavior of a driver than the more traditional fixed model, based on TTC-value. The most promising model structure is however a PrARX-model with simplified mode transition, having a somewhat smooth transition between its modes.

In Chapter 5 it is discussed how driver models and classification may be integrated to existing active safety systems in order to make them adapt to the individual driving the vehicle. By adding a driver model to the safety system functionality, it is possible to take into consideration the driver's ability to handle threatening situations. This contributes to a proper timing of system warnings and interventions and facilitates the driver acceptance of active safety systems that may take over control from the driver. For example, it can be assumed that a driver with an aggressive driving style accepts fewer and later warnings than a driver operating in a more cautious mode.

## 7.2 Future Work

Future challenges in the driver modeling area involve combination of lateral and longitudinal driver behavior modeling, in order to obtain a more generic model of driver skills. Such a combination would allow for implementing only one driver model in an active safety system, which could be used by many different functions, e.g. both lane keeping support and automatic emergency brake systems. Also, a hierarchical model could be advantageous in order to let different sub models of driver behavior be available in different traffic environments, such as city and highway driving. This has already been studied in some publications, but can be further enhanced.

Moreover, it would be interesting to develop an automatic tuning of the optimal number of modes to use in a hybrid model. In this thesis, only two modes have been used. It is possible that other driving scenarios would gain in having three, four or an even higher number of modes to

## 7.2. FUTURE WORK

accurately discriminate different kinds of driving behavior. Much could probably also be gained from estimating the number of modes, as well as the parameters, both online and offline. This combination would permit the parameter identification to reflect both long term and short term driver behavior. All modeling techniques should also be tested on several drivers in real vehicles and realistic driving environments with the purpose of proper model validation.





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