Sensitivity Optimization for Electromagnetic Measurement Systems by Sensor Selection

Oskar Talcoth*, Gustav Risting*, Thomas Rylander*,

* Department of Signals and systems, Chalmers University of Technology, SE-41296 Göteborg, Sweden
Email: oskar.talcoth@chalmers.se, risting@student.chalmers.se, rylander@chalmers.se

***Abstract*—We attempt to maximize the sensitivity of a sensor system, where the system sensitivity is expressed by a goal function formulated in terms of the Fisher information matrix. Given a large set of sensor candidates, we formulate the problem as a sensor selection problem by means of introducing a weight for each sensor candidate. Such a weight corresponds to the fraction of measurements that is performed by a specific sensor and it is allowed to vary continuously, which yields a convex optimization problem.**

***Index Terms*—antenna system, convex optimization, Cramér-Rao inequality, Fisher information, optimal measurements**

I. Introduction

Tracking of the position of an object equipped with an electromagnetic transmitter has many applications. For quasi-magnetostatic sensor systems, this approach is referred to as magnetic tracking and it is exploited for a number of biomedical engineering problems such as tracking of the human eye [1], catheter tracking [2] and real-time organ positioning during radiotherapy [3]. We have applied optimization techniques to such quasi-magnetostatic sensor systems [4], where we optimize a sensor arrangement intended for the positioning of a source coil. In particular, we attempt to find the sensor arrangement that yields the maximum sensitivity with respect to displacements of the source coil.

Here, we present an extension of our optimization technique for electrodynamic problems that involve antennas. Thus, the objective is to determine the position of a radiating antenna by means of a set of receiving antennas, i.e. the sensor antennas. Given a set of sensor candidates, we formulate an optimization problem that maximizes the sensitivity with respect to the position of the radiating antenna. The original sensor selection problem is relaxed with respect to its integer constraints and this yields a convex optimization problem [5], which is attractive from a computational perspective.

II. Optimization problem

The objective is to estimate the position $\vec{r}_q^{\,\mathrm{t}} = (x_q^{\mathrm{t}}, y_q^{\mathrm{t}}, z_q^{\mathrm{t}})$ of a transmitting antenna, where the position is described by the parameter vector $\mathbf{d} = [x_q^{\mathrm{t}}, y_q^{\mathrm{t}}, z_q^{\mathrm{t}}]^T$. (The parameter vector $\mathbf{d}$ could involve other parameters such as the orientation of the antenna but, in this abstract, we limit the discussion to the position as we assume that the orientation is known and fixed.) Let $\boldsymbol{u}_p^{(-)}(\mathbf{d})$ be the received voltage for sensor antenna p and $\nabla_{\mathbf{d}}\boldsymbol{u}_p^{(-)}(\mathbf{d}_0)$ denote the gradient of $\boldsymbol{u}_p^{(-)}(\mathbf{d})$ with respect to the parameter vector $\mathbf{d}$ at the position $\mathbf{d}_0$. We assume additive complex Gaussian noise $\boldsymbol{n}_p \in \mathcal{CN}(0, 2\sigma^2)$, where the noise term for the sensor antenna p is independent with respect to all the other sensor antennas.

The Fisher information matrix $\mathbf{F}$ can be used as a performance metric for the parameter estimation problem. For example, the Cramér-Rao inequality [6], $\operatorname{cov}\hat{\mathbf{d}} \succeq \mathbf{F}^{-1}$, provides a lower bound for the covariance of the estimated $\hat{\mathbf{d}}$ for all unbiased estimators. For our problem, the Fisher information matrix can be expressed as

$$\mathbf{F} = \sum_{p=1}^{N} \mathbf{F}_p = \frac{1}{\sigma^2}\sum_{p=1}^{N} \mathrm{Re}\left\{\nabla_{\mathbf{d}}\boldsymbol{u}_p^{(-)}(\mathbf{d}_0)\nabla_{\mathbf{d}}\boldsymbol{u}_p^{(-)}(\mathbf{d}_0)^H\right\}.$$

In the following, we wish to maximize $\log\det(\mathbf{F})$ in order to find a sensor arrangement that yields a large sensitivity.

Now, we wish to find N sensors among a given set of sensor candidates, where the number of sensor candidates is denoted K and these are located at $\vec{r}_1^{\,\mathrm{r}}, \ldots, \vec{r}_K^{\,\mathrm{r}}$. In order to formulate this as a computationally attractive optimization problem, we allow the p-th sensor to perform m_p measurements and we denote the total number of measurements by M. The fraction of the total number of measurements for sensor p is denoted $w_p = m_p/M$ and, for large M, we can relax the constraint $w_p \in \mathbb{Q}$ to be $w_p \in \mathbb{R}$. This relaxation allows us to solve the convex optimization problem [7]

$$\begin{aligned} \underset{w_p}{\text{minimize}} \quad & -\log\det\left(\sum_{p=1}^{K} w_p\mathbf{F}_p(\mathbf{d})\right) \\ \text{subject to} \quad & \sum_{p=1}^{K} w_p = 1 \\ & w_p \geq 0, \quad p = 1, \ldots, K \end{aligned} \tag{1}$$

We use CVX, a package for specifying and solving convex programs [8], [9], to solve the optimization problem.

III. Sensor system model

In this section, we describe the physical model of the sensor system and its transmitting antenna, which we wish to track. We denote the receiving antennas (i.e. the sensor antenna candidates) with the index $p = 1, \ldots, K$ and the transmitting antenna with the index $q = K + 1$. In the model that we use in this abstract, each antenna is connected to a coaxial cable, which allows us to also incorporate the mismatch of the antenna in relation to its transmission line. The received voltage $\boldsymbol{u}_p^{(-)}\big|_{\xi=\xi_p}$ on port p due to the field transmitted by antenna q, with excitation $\boldsymbol{u}_q^{(+)}\big|_{\xi=\xi_q}$, is [10]

$$\boldsymbol{u}_p^{(-)}\big|_{\xi=\xi_p} = -\frac{2j\lambda_0}{\eta}\frac{Z^{\mathrm{c}}}{\boldsymbol{Z}_p^{\mathrm{a}}+Z^{\mathrm{c}}} \frac{2}{\boldsymbol{Z}_q^{\mathrm{a}}+Z^{\mathrm{c}}}e^{-jk^{\mathrm{c}}(\Delta\xi_p+\Delta\xi_q)}\boldsymbol{a}_{pq}\boldsymbol{u}_q^{(+)}\big|_{\xi=\xi_q} \quad (2)$$

where

$$\boldsymbol{a}_{pq} = \frac{e^{-jk_0 r_{pq}}}{r_{pq}}\vec{\boldsymbol{G}}_p(-\hat{r}_{pq})\cdot\vec{\boldsymbol{G}}_q(\hat{r}_{pq}). \quad (3)$$

where $\boldsymbol{Z}_q^{\mathrm{a}}$ denotes the impedance of antenna q and $\Delta\xi_q$ is the distance between the antenna port and the reference plane associated with the scattering parameters for port q. For the cable connected to the antennas, we have the characteristic impedance Z^{c} and the propagation constant k^{c}. In free space, we denote the wavelength by λ_0 and the wave impedance η. Further, $\vec{r}_{pq} = \vec{r}_p - \vec{r}_q$ is the vector from the antenna located at $\vec{r}_q$ to the antenna located at $\vec{r}_p$, which gives the unit vector $\hat{r}_{pq} = \vec{r}_{pq}/|\vec{r}_{pq}|$. Equation (3) can be used when antenna p and q are sufficiently separated, such that the far-field approximation is valid. For closer distances, we exploit the method of moments to compute a database of values for $\boldsymbol{a}_{pq}$ and $\nabla_{\mathbf{d}}\boldsymbol{a}_{pq}$.

IV. Preliminary results

In this abstract, we present some preliminary results that only depend on $\exp(-jk_0 r_{pq})/r_{pq}$. Our ambition is to show results that are valid for physical antennas at the conference. In the present text, we assume that $\vec{\boldsymbol{G}}_p(-\hat{r}_{pq})\cdot\vec{\boldsymbol{G}}_q(\hat{r}_{pq}) = \eta^2$ and, consequently, we deal with omnidirectional antennas that do not have any losses associated with the polarization. Furthermore, we assume that $\boldsymbol{Z}_p^{\mathrm{a}} = \boldsymbol{Z}_q^{\mathrm{a}} = Z^{\mathrm{c}}$ for all antennas, i.e. that the antennas are perfectly matched to the transmission lines. Finally, we put the port very close to the antenna for all antennas, i.e. $k^{\mathrm{c}}(\Delta\xi_p+\Delta\xi_q)\ll 1$ which gives $e^{-jk^{\mathrm{c}}(\Delta\xi_p+\Delta\xi_q)}\simeq 1$. Thus, we have

$$\boldsymbol{u}_p^{(-)}\big|_{\xi=\xi_p} = -\frac{j\eta\lambda_0}{Z^{\mathrm{c}}}\frac{e^{-jk_0 r_{pq}}}{r_{pq}}\boldsymbol{u}_q^{(+)}\big|_{\xi=\xi_q}. \quad (4)$$

We consider a transmitting antenna at $(0,0,z_0)$ above the xy-plane containing the receiving antennas. Due to the rotational symmetry of the transmitted field around the z-axis, the optimized sensor configuration will be circularly symmetric around the origin. Therefore, we arrange the candidate sensors in a polar grid with $\varphi\in\{0,2\pi/3,4\pi/3\}$ and $\rho\in[0,z_0]$ where the radial resolution is $z_0/1000$. Due to the symmetry of the problem, all sensors with a specific ρ will take identical weights. Results show that only one sensor circle with radius ρ_{opt} is present in the optimized solution. The radius ρ_{opt} is not influenced by the number of uniformly distributed sensors over the circle circumference N_s, as long as $N_s\geq 3$.

Figure 1 shows ρ_{opt} as function of frequency. The optimized radius is $\rho_{\mathrm{opt}} = 0.535z_0$ when the transmitter is close to the sensor plane in terms of wavelengths, i.e. $z_0 < 10^{-2}\lambda_0$, and $\rho_{\mathrm{opt}} = 0.707z_0$ when the transmitter is far away from the sensor plane, i.e. $z_0 > \lambda_0$. For the latter case, the angle between the z-axis and $\vec{r}_{pq}$ for the sensors at ρ_{opt} is $45°$.

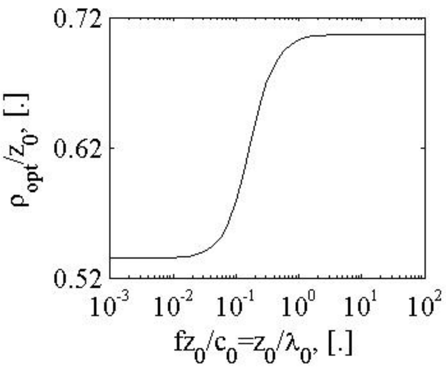

Fig. 1. Optimized radius ρ_{opt} of the circular sensor configuration as a function of scaled frequency. Note that ρ_{opt} has been normalized with z_0, that the scaled frequency is expressed as z_0/λ_0, and that c_0 denotes the speed of light in free space.

V. Conclusion

In this abstract, we have presented a sensor selection method for maximizing the sensitivity of electromagnetic measurement systems that yields a convex optimization problem. Preliminary results were given for a simplified sensor system model.

Acknowledgment

This work has been supported by the Swedish research council VINNOVA via a project within the VINN Excellence center CHASE, and by the Swedish National Graduate School in Scientific Computing.

References

[1] A. Plotkin, O. Shafrir, E. Paperno, and D. M. Kaplan, "Magnetic eye tracking: A new approach employing a planar transmitter," *IEEE Trans. Biomed. Eng.*, vol. 57, no. 5, pp. 1209–1215, 2010.

[2] Webster Biosense, "Carto 3 System." [Online]. Available: http://www.biosensewebster.com/products/navigation/carto3.aspx

[3] R. Iustin, J. Linder, E. Isberg, T. Gustafsson, and B. Lennernäs, "A Model Based Positioning System," Patent. WO 2008/079071 A1, World Intellectual Property Organization, 2008.

[4] O. Talcoth and T. Rylander, "Sensor selection in magnetic tracking based on convex optimization," *IET Electronics Letters*, vol. 49, no. 1, Jan. 2013.

[5] S. Joshi and S. Boyd, "Sensor Selection via Convex Optimization," *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 451–462, Feb. 2009.

[6] E. Walter and L. Pronzato, *Identification of parametric models from experimental data*, ser. Communications and control engineering. Springer, 1997.

[7] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, UK, 2004.

[8] Grant, M., and Boyd, S.: 'CVX: Matlab Software for Disciplined Convex Programming', version 1.21, April 2011, http://cvxr.com/cvx

[9] Grant, M., and Boyd, S.: 'Graph implementations for nonsmooth convex programs', in V. Blondel, S. Boyd, and H. Kimura (Eds.): 'Recent Advances in Learning and Control', (Springer-Verlag, 2008), pp. 95-110

[10] P.-S. Kildal, "Equivalent circuits of receive antennas in signal processing arrays," *Microwave and Optical Technology Letters*, vol. 21(4), pp. 244–246, 1999.