Thesis for the Degree of Licentiate of Engineering

On Nonlinear Compensation Techniques for Coherent Fiber-Optical Channel

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To my family and friends...

"When you really want something to happen, the whole world conspires to help you achieve it."

-Paulo Coelho

Abstract

Fiber-optical communication systems form the backbone of the internet, enabling global broadband data services. Over the past decades, the demand for high-speed communications has grown exponentially. One of the key techniques for the efficient use of existing bandwidth is the use of higher order modulation formats along with coherent detection. However, moving to high-order constellations requires higher input power, and thus leads to increased nonlinear effects in the fiber. In long-haul optical communications (distances spanning from a hundred to a few thousands of kilometers), amplification of the signal is typically needed as the fibers exhibit power losses. Amplifiers add noise and the signal and noise interact, leading to nonlinear signal–noise interactions, which degrade the system performance.

The propagation of light in an optical fiber is described by the nonlinear Schrödinger equation (NLSE). Due to the lack of analytical solutions for the NLSE, deriving statistics of this nonlinear channel is in general cumbersome. The state-of-the-art receiver for combating the impairments existing in a fiber-optical link is digital backpropagation (DBP), which inverts the NLSE, and is widely believed to be optimal. Following this optimality, DBP has enabled system designers to determine optimal transmission parameters and provides a benchmark against which other detectors are compared. However, a number of open questions remain: How is DBP affected by noise? With respect to which criterion is DBP optimal? Can we estimate the optimal transmit power for a system when DBP is used?

In paper A, starting from basic principles in Bayesian decision theory, we consider the well-known maximum a posteriori (MAP) decision rule, a natural optimality criterion which minimizes the error probability. As the closed-form expressions required for MAP detection are not tractable for coherent optical transmission, we employ the framework of factor graphs and the sum-product algorithm, which allow a numerical evaluation of the MAP detector. The detector turns out to have similarities with DBP (which can be interpreted as a special case) and is termed stochastic digital backpropagation, as it accounts for noise, as well as nonlinear and dispersive effects. Through Monte Carlo simulations of a single-channel communication system, we see significant performance gains with respect to DBP for dispersion-managed links.

In paper B, we investigate the performance limits of DBP for a non dispersion-managed fiber-optical link. An analytical expression is derived that can be used to find the optimal transmit power for a system when DBP is used. We found that a first-order approximation is reasonably tight for different symbol rates and it can be used to approximately compute the optimum transmit power in terms of minimizing the symbol error rate. Moreover, the first-order approximation results show that the variance of the nonlinear noise grows quadratically with transmitted power, which limits the performance of a system with DBP.

Keywords: Digital backpropagation, fiber-optical communications, factor graphs, near-MAP detector, nonlinear compensation, performance limits.

Publications

This thesis includes the following papers:

[A] N. V. Irukulapati, H. Wymeersch, P. Johannisson, E. Agrell, "Stochastic Digital Backpropagation", submitted to *IEEE Transactions on Communications*, Jan. 2014.

Part of this paper is also presented in
N. V. Irukulapati, H. Wymeersch, P. Johannisson, E. Agrell, "Extending Digital Backpropagation to Account for Noise", in *Proceedings of European Conference and Exhibition on Optical Communication (ECOC)*, London, UK, Sep. 2013, p. We.3.C.4.

[B] L. Beygi, N. V. Irukulapati, E. Agrell, P. Johannisson, M. Karlsson, H. Wymeersch, P. Serena, and A. Bononi, "On nonlinearly-induced noise in single-channel optical links with digital backpropagation", *Optics Express*, vol. 21, no. 22, pp. 26376–26386, Oct. 2013.

Other contributions by the author (not included in this thesis):

[C] T. Ahmad, Y. Ai, P. Muralidharan, N. V. Irukulapati, P. Johannisson, H. Wymeersch, E. Agrell, P. Larsson-Edefors, and M. Karlsson, "Methodology for Power-Aware Coherent Receiver Design", in *Signal Processing in Photonics Communications (SPPCom) Topical Meeting*, Puerto Rico, Jul. 2013.

[D] D. Sen, H. Wymeersch, N. V. Irukulapati, E. Agrell, P. Johannisson, M. Karlsson, P. A. Andrekson, "MCRB for Timing and Phase Offset for Low-Rate Optical Communication with Self-Phase Modulation", in *IEEE Communication Letters*, vol. 17, no. 5, pp. 1004–1007, May 2013.

Contents

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I owe a debt of gratitude to my main supervisor, Henk, who was very encouraging and supportive right from the start of the PhD. I learnt a lot of things from you, be it attacking problems right at that moment instead of postponing, focusing and multitasking, giving feedbacks for the papers very quickly, prioritizing, answering emails promptly at any time (even at 1 in midnight or on a different time zone) and many more. Thank you Pontus for explaining me even small (maybe stupid sometimes) questions and answering to all those optical communications related questions patiently. You are a person with background in optics but also have good knowledge in DSP, which is a very big plus during my research. My gratitude is also extended to Erik for his time, careful attention to detail, for giving feedback that would help me professionally and as a human.

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Gothenburg,
January, 2014.

Acronyms

ASE	amplified spontaneous emission
FBG	fiber Bragg gratings
CD	chromatic dispersion
DBP	digital backpropagation
DCF	dispersion-compensating fiber
DCM	dispersion-compensating module
DM	dispersion-managed
DSP	digital signal processing
EDC	electronic dispersion compensation
EDFA	erbium-doped fiber amplifier
FG	factor graphs
ISI	inter-symbol interference
MAP	maximum a posteriori
MLSD	maximum likelihood sequence detection
NDM	non dispersion-managed
NLPN	nonlinear phase noise
NLSE	nonlinear Schrödinger equation
PDF	probability density function
PMD	polarization-mode dispersion
PMF	probability mass function
PR	particle representation
QAM	quadrature amplitude modulation
QPSK	quadrature phase-shift keying
SDBP	stochastic digital backpropagation
SER	symbol error rate
SMF	single-mode fibre
SPA	sum-product algorithm
SPM	self-phase modulation
SSFM	split-step Fourier method

Part I

Overview

Chapter 1

Introduction

Today's information society relies to a large extent on solutions based on broadband communications, with applications such as mobile voice and data services, high-speed internet access, and multimedia broadcast systems [1]. Each of these applications brings its own set of challenges, which can be addressed using electronic, radio-frequency, or optical communication systems. Among the different communication technologies, optical communications generally has the edge over baseband electronic or radio-frequency transmission systems whenever high aggregate bit rates and/or long transmission distances are involved. Both advantages are deeply rooted in physics: first, the high optical carrier frequencies allow for high-capacity systems at small relative bandwidths. For example, a mere 2.5% bandwidth at a carrier frequency of 193 THz (1.55 μm wavelength) opens up a 5-THz chunk of contiguous communication bandwidth. Second, fibers exhibit losses of around 0.2 dB/km, which is very little compared to the losses in typical coaxial cables supporting a bandwidth of 1 GHz. The latter generally exhibits losses of 2 to 3 orders of magnitude higher than that for the fiber. Other advantages of using fiber-optical communications are the unregulated spectrum in the optical regime and the absence of electromagnetic interference.

The data rates of the optical communication links has grown exponentially since their introduction in the late 1970s. Until very recently, lightwave systems used binary modulation formats such as on-off keying at the transmitter and power detection at the receiver using a fixed-threshold detection. This receiver does not require any complex hardware; however, the data rates and spectral efficiency that one can achieve is not high. To increase the data rates of the fiber-optical communication systems, modulation formats with higher spectral efficiency is of great interest along with coherent detection at the receiver. However these higher-order modulation formats are less tolerant towards some of the channel impairments [2].

To understand these impairments, recall that an optical optical fiber is a waveguide consisting of a cylindrical core surrounded by a cladding. The refractive index of the core is higher than the cladding so that the light is guided in the optical fiber. The dependence of the refractive index on the frequency and the power gives rise to two dominant impairments in fiber-optical links, namely chromatic dispersion and the nonlinear Kerr effect. Due to chromatic dispersion, the signal that is sent at the input of the fiber is broadened in time, causing inter-symbol interference. The signal phase is changed in proportion to the signal power due to the nonlinearities in the fiber and this power-dependent phase shift causes spectral broadening. In addition to these two impairments, when the signal propagates in the fiber-optical medium, the signal power reduces exponentially due to the fiber loss. Therefore, amplification of the signal is needed for long-haul communications (distances spanning from a hundred to a few thousands of kilometers). Amplifiers add noise and typically the signal and noise interact leading to nonlinear signal–noise interactions, which is seen as one of the major factors for limiting the fiber capacity [3].

To compensate for the dispersion and nonlinearity, solitons were proposed as they preserve their shape in spite of the dispersive and nonlinear effects occurring inside fibers. The existence of solitons was found as a solution obtained by the inverse scattering method [4]. However, solitons were limited to low-order modulation formats such as on-off keying and suffer from pulse interactions. Other techniques for jointly compensating for nonlinearities and dispersion in the optical domain include techniques such as phase-conjugation [5] and twin waves [6]. Due to the advancements in digital signal processing, it has been suggested to combat these impairments in the digital domain using techniques based on Volterra kernels [7, 8], channel inversion using digital backpropagation (DBP) [9, 10], and maximum likelihood sequence detection [11–13].

Among the techniques, DBP is often considered to be a universal technique for jointly compensating the linear and the nonlinear impairments [14]. However, DBP does not account for the noise from the optical amplifiers while compensating for impairments. Maximum likelihood sequence techniques account for nonlinear signal–noise interactions, but the methods currently available in the literature are only applicable for low dispersion where the memory is short and thereby handles only low inter-symbol interference scenarios [12].

Our work aims to answer the following questions:

- Is DBP truly optimal, or can we derive methods that outperform DBP?
- Is it possible to analytically predict the performance of DBP?

Partial answers to these questions are provided in papers A and B, respectively. In paper A, starting from the maximum a posteriori principle, a detector that outperforms DBP is developed. In paper B, a closed-form expression to calculate the optimal power which minimizes the symbol error rate is found. This was done for an optical link without periodic dispersion compensation and the result is found to be a good approximation for low to moderate input powers.

1.1 Organization of the Thesis

In Sweden, the Licentiate is a degree half way through the doctoral studies and writing this thesis is part of the process for getting the degree. Thesis in Sweden can be written either as a monograph or as a collection of papers and the latter is followed for this thesis. The intended audience of the thesis are graduate students and researchers currently working or planning to work in optical communications, who have some background in digital communications.

The second part of the thesis contains the research papers with a modified layout. In the first part of the thesis, an introduction material needed to understand the concepts behind the papers of part II will be presented. Specifically, in chapter 2, we introduce the fiber-optical channel, starting with the signal propagation model and the impairments existing in this channel. Different approximate existing channel models will be presented later followed by the numerical approach used to simulate the signal propagation in a fiber. In chapter 3, nonlinear compensation techniques will be described mainly emphasizing DBP, followed by some basic principles needed for our proposed detector. Chapter 4 summarizes the contributions of the papers and highlights possible future directions of the current research.

Chapter 2

Fiber-Optical Transmission Systems

An optical fiber is a waveguide consisting of a cylindrical core surrounded by a cladding. The refractive index of the core is higher than the cladding so that the light is guided in the optical fiber. A waveguide mode is a configuration of the electric field that propagates without changing its spatial distribution, apart from an amplitude change and a phase shift. A single-mode fiber (SMF), commonly used for transmission in long-haul communications, supports only one propagating mode. In fiber-optical communications, the physical dimensions that can be used for modulation and multiplexing are time, quadrature (amplitude/phase), frequency, polarization, and space (for example by using multiple modes in a multi-mode fiber) [15]. In polarization-multiplexed signals, the spectral efficiency is increased by transmitting two different signals at the same wavelength but in two orthogonal polarizations. For example, for a symbol rate of 28 Gbaud using polarization-multiplexed 16-QAM modulation format, a raw data rate of $28 \times 4 \times 2 = 224$ Gb/s per wavelength can be achieved.

A single-mode optical fiber is an exceptionally transparent medium. Unlike typical coaxial cables, where losses are on the order of several tens of dB/km for a bandwidth of around 1 GHz, a modern telecom fiber features attenuation coefficients below 0.2 dB/km across a bandwidth of many THz. Nevertheless, as the signal propagates in the fiber, the signal power is reduced due to the fiber loss and for long-haul communications, this attenuation calls for amplification of the signal.

This chapter is organized as follows. In Sec. 2.1, starting with the equations governing signal propagation in a fiber, impairments arising in the fiber are described. The system model will be described along with the assumptions in Sec. 2.2, detailing different blocks of the model. Existing analytical channel models will be presented in Sec. 2.3, followed by numerical methods for describing the signal propagation in the fiber in Sec. 2.4.

2.1 Signal Propagation in the Fiber

The propagation of light in an optical fiber is modeled using the Manakov equation with loss included [16]

$$\frac{\partial \mathbf{A}}{\partial z} = i\gamma \|\mathbf{A}\|^2 \mathbf{A} - i\frac{\beta_2}{2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{\alpha}{2}\mathbf{A}, \tag{2.1}$$

where $\mathbf{A} \triangleq [A_x\ A_y]^T$ is the complex envelope of the two polarization components of the optical field, γ is the nonlinear coefficient, $\|\mathbf{A}\|^2 = \mathbf{A}^H\mathbf{A}$ represents the optical power, where H is the hermitian conjugate, β_2 is the group velocity dispersion coefficient, α is the power attenuation factor, z is the distance of propagation, and t is the time coordinate in a reference frame moving with the signal group velocity. The nonlinear Schrödinger equation (NLSE) is the corresponding modeling equation for the single-polarization case.

2.1.1 Chromatic Dispersion

If the group velocity[1] is different for different frequency components of the wave, the medium is said to be dispersive and the effect is known as group velocity dispersion or chromatic dispersion (CD). The CD broadens the pulse in the time domain leading to inter-symbol interference (ISI) as depicted in Fig. 2.1. An important parameter is the dispersion length, L_D, which is the propagation distance after which the dispersive effects become important and is given by $L_\mathrm{D} = 1/(|\beta_2|W^2)$ [17, p. 55], where W is the bandwidth of the transmitted signal.

When $\gamma = 0$ and $\alpha = 0$ in (2.1), a closed-form solution is given by $\tilde{\mathbf{A}}(z,\omega) = \tilde{\mathbf{A}}(0,\omega)\exp(i\beta_2\omega^2 z/2)$, where $\tilde{\mathbf{A}}(z,\omega)$ is the spectrum of $\mathbf{A}(z,t)$. Hence CD can be modeled as an all-pass filter. CD does not change the amplitude of the spectrum but causes a frequency-dependent phase shift in the frequency domain [16, 18] as shown in Fig. 2.2.

Dispersion can be compensated for in the optical domain either using dispersion-compensating fibers (DCFs) or fiber Bragg gratings (FBG) [19]. The DCFs have the opposite sign of β_2 compared to the SMF and also have higher nonlinear coefficient than the SMF [16, ch. 9]. An FBG has no nonlinearities and low insertion loss. When the dispersion is compensated optically within the fiber-optical transmission link, the system is known as a dispersion-managed (DM) link. Otherwise the link is said to be non-dispersion-managed (NDM). The CD can also be compensated through digital signal processing (DSP) in the receiver using an electronic dispersion compensation (EDC) block. This EDC is a filter with a frequency response equal to $\exp(-i\beta_2\omega^2 z/2)$.

2.1.2 Nonlinear Kerr Effect

The term with the nonlinear parameter γ in the Manakov equation (2.1) represents the effect due to Kerr nonlinearity. This arises due to the power-dependent

[1] The group velocity is the velocity with which the complex envelope of the wave propagates through the fiber.

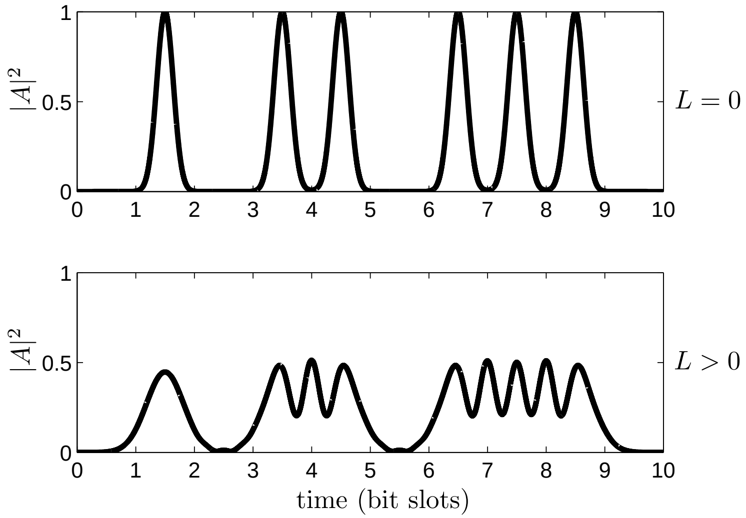

Figure 2.1: Effect of the CD in the time domain. In the top (resp., bottom) figure, a pulse at the input (resp., output) of a fiber can be seen. One can see that pulses broaden in the time domain and start to interfere.

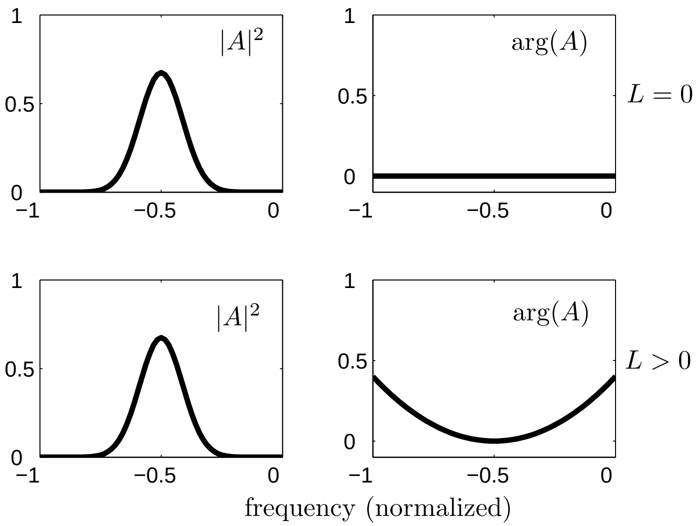

Figure 2.2: Spectrum of a single pulse affected by the CD. In the top (resp., bottom) row of the figures, a pulse at the input (resp., output) of a fiber is shown. One can see that the amplitude is not changed and only a quadratic phase modulation occurs. In the time domain, this corresponds to a broadening of the pulses, as in Fig. 2.1.

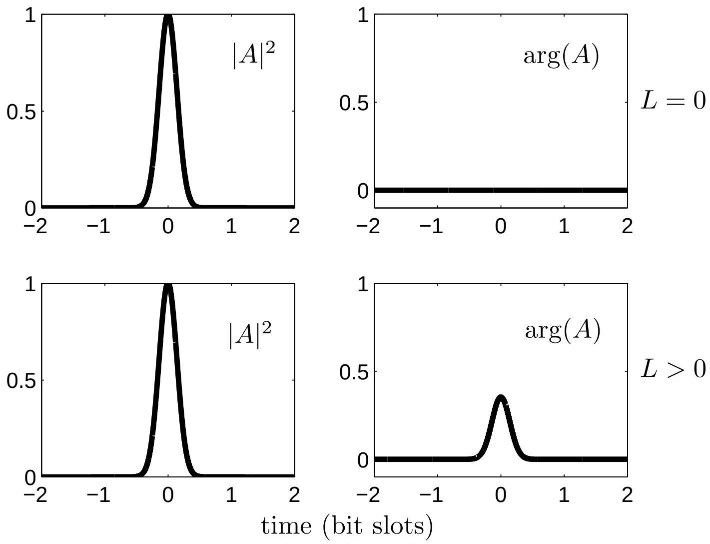

Figure 2.3: Effect of the Kerr nonlinearity in the time domain. In the top (resp., bottom) row of the figure, a pulse at the input (resp., output) of a fiber can be seen. The amplitude of the pulse is not changed but a phase shift is introduced.

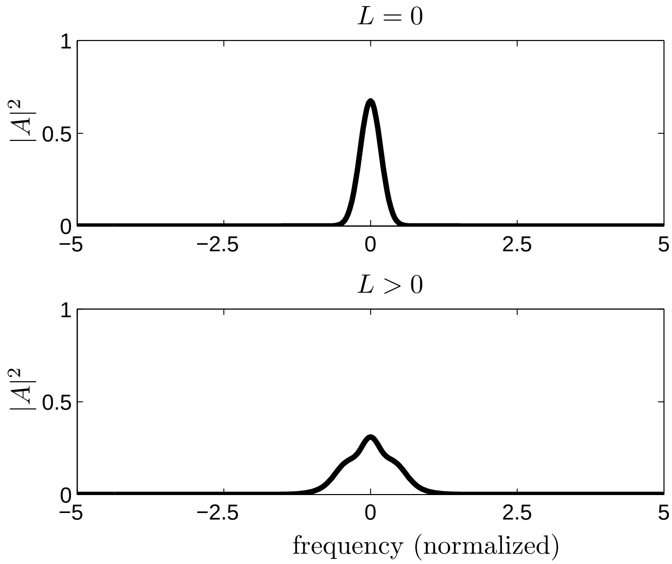

Figure 2.4: Spectrum of a single pulse affected by the Kerr nonlinearity. In the top (resp., bottom) figure, a pulse at the input (resp., output) of a fiber can be seen. Due to the phase shift in the time domain (Fig. 2.3), the spectrum is broadened.

refractive index of the fiber. The solution of the Manakov equation (2.1) setting $\alpha = 0$ and $\beta_2 = 0$ is

$$\mathbf{A}(L,t) = \mathbf{A}(0,t)\exp[i\gamma\|\mathbf{A}(0,t)\|^2 L_{\text{eff}}], \tag{2.2}$$

where the signal phase is changed in proportion to the signal power and this effect is called self-phase modulation (SPM).[2] Here L is the length of the fiber. The nonlinear phase shift is denoted by $\phi_{\text{NL}} \triangleq \gamma\|\mathbf{A}(0,t)\|^2 L_{\text{eff}}$. The effective fiber length, $L_{\text{eff}} = [1-\exp(-\alpha L)]/\alpha$, is an indication of the fiber length along which the nonlinearities are effective. The amplitude of the time-domain signal is not changed but a power-dependent phase shift is introduced due to the SPM as shown in Fig. 2.3. As a result in the frequency domain, the spectrum is broadened as depicted in Fig. 2.4.

Numerical Example

Similar to the dispersion length, the nonlinear length is defined as $L_{\text{NL}} = 1/(\gamma P_0)$, where P_0 is the initial peak power. For a particular system, comparing L_{D} and L_{NL} for a fiber of length L helps in determining whether the dispersion or the nonlinearity will be the dominant effect. If $L \gtrsim L_{\text{D}}$ and $L \ll L_{\text{NL}}$, then the CD dominates over the nonlinearities. As an example, consider a standard telecommunication fiber at wavelength 1550 nm, dispersion parameter[3] $D = 17$ ps/(nm km), and bandwidth $W = 28$ Gbaud, then $L_{\text{D}} \approx 60$ km. If the nonlinear parameter $\gamma = 1.3$ 1/(W km) and $P_0 = 0$ dBm, then $L_{\text{NL}} \approx 750$ km. If the fiber length is $L = 80$ km, then CD is the dominant effect.

2.1.3 Power Losses

By setting $\gamma = 0$ and $\beta_2 = 0$ in the Manakov equation (2.1), a closed-form solution is given by $\mathbf{A}(z,t) = \mathbf{A}(0,t)\exp(-\alpha z/2)$. That is, when the signal propagates in the fiber, the signal power reduces exponentially due to the fiber loss. Over sufficiently long distances z, the signal-to-noise ratio of the detected signal will be too low, leading to a high bit-error rate. Therefore amplification of the signal is needed for long-haul communications. Optical amplification can be done in a distributed manner using Raman amplification or in lumped components using erbium-doped fiber amplifiers (EDFAs). Unlike radio frequency amplifiers, most of the optical amplifiers exhibit constant gain across the spectrum and do not distort the optical signals. Instead, the main degrading effect of the optical amplification is the generation of amplified spontaneous emission (ASE) noise.[4] The ASE noise

[2] The Kerr effect also causes cross-phase modulation and four-wave mixing, but these are not considered in this work. The reason is that these effects are only present in wavelength-division multiplexed systems and in this thesis, we consider a single-wavelength scenario.

[3] The dispersion parameter, D, is related to β_2 as $D = -2\pi c\beta_2/\lambda^2$, where c is the speed of light and λ is the wavelength.

[4] Light that is coupled into the erbium-doped fiber is amplified through stimulated emission: incident photons stimulate the excited ions to return to the ground state and emit a photon of identical frequency, phase and polarization. However, ions also return to their ground state

can be modeled as additive white Gaussian noise [16]. The one-sided power spectral density per polarization is given by $S_{\mathrm{sp}}(\nu) = (G-1)n_{\mathrm{sp}}h\nu$, where $G = \exp(\alpha L)$ is the required gain needed to compensate for the attenuation in the fiber of length L, ν is the optical frequency with $h\nu$ being the photon energy, and n_{sp} is the spontaneous-emission factor. In this thesis, EDFAs are used for optical amplification.

2.1.4 Other Impairments

There are number of impairments not modeled by the Manakov equation (2.1). This includes polarization-mode dispersion (PMD), polarization-dependent loss, third order dispersion, and laser phase noise [16]. Different polarizations of light travel at slightly different speeds, leading to random spreading of the optical pulses, and this effect is known as PMD. A related effect is polarization-dependent loss, in which two polarizations suffer different rates of loss in the optical components of the fiber-optical link such as couplers.

2.2 System Model

The system model is shown in Fig. 2.5 and consists of a data sequence of K symbols,[5] $\underline{\mathbf{s}} = [\underline{\mathbf{s}}[1], \underline{\mathbf{s}}[2], \cdots, \underline{\mathbf{s}}[K]] \in \Omega^K$, where Ω is the set of symbols in the constellation, a pulse shaper, a fiber-optical link with N spans, and a receiver with a compensation algorithm followed by a decision unit. Each span of the fiber-optic link consists of an SMF followed by an optional dispersion compensating module (DCM) for the DM links. In this work, we have considered either DCF or FBG as DCM. In between the fiber spans, there are EDFAs compensating for the losses in the preceding fiber. At the receiver, the signal is sent into a compensating unit, where the impairments are compensated for. In this work, either digital backpropagation, stochastic digital backpropagation algorithm (SDBP), or an electronic dispersion compensation algorithm is applied for compensation of impairments. DBP and SDBP will be explained in Ch. 3. This signal is then sent to the decision unit where the symbols are decoded. The problem of interest is to estimate the data sequence $\underline{\mathbf{s}}$ given the received signal $\mathbf{r}(t)$. As a closed-form expression for the input-output relationship of the fiber-optical link is not present, many approximate channel models have been proposed in literature, which will be described next. In this thesis, a "channel" refers to the fiber-optical link of

spontaneously, thereby emitting photons of random phase and polarization; this spontaneous emission becomes amplified upon propagation along the fiber link, a process that is known as amplified spontaneous emission.

[5] Lower case bold letters (e.g., $\mathbf{r}$) are used for vector representation of the continuous-time signals and underlined lower case bold letters (e.g., $\underline{\mathbf{s}}$) for a vector of discrete-time symbols. Note that even though both these classes of signals are represented by vectors, they are quite distinct. The sequence $\underline{\mathbf{s}}$ denotes the symbol-spaced data and $\mathbf{r}$ is the oversampled continuous-time optical signal $\mathbf{r}(t)$. With a slight abuse of notation, $\mathbf{r}(t)$ is used to represent a vector of dual-polarization single-wavelength continuous-time signal and $\mathbf{r}$ is used to represent samples of $\mathbf{r}(t)$, where the data for each polarization is combined.

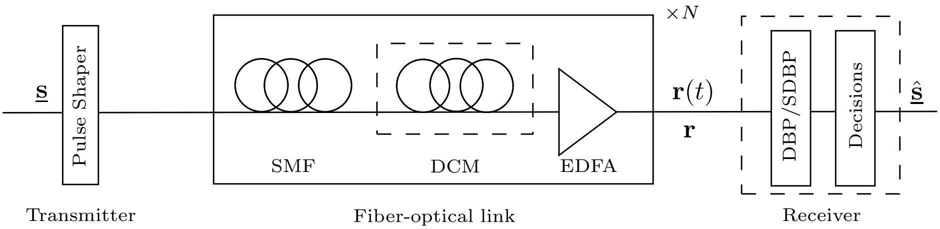

Figure 2.5: A fiber-optical link with N spans where each span consists of an SMF, a DCM (for DM links), and an EDFA. The transmitter consists of a pulse shaper and the receiver consists of a compensation algorithm (DBP/SDBP) and a decision block. The transmitted data is denoted by $\underline{\mathbf{s}}$, decoded data by $\hat{\underline{\mathbf{s}}}$, and the received signal by $\mathbf{r}(t)$ (or $\mathbf{r}$).

Fig. 2.5 for a single-wavelength system with a dual-polarization complex envelope signal.

Coherent detection with perfect timing, phase, and frequency synchronization, as well as perfect polarization tracking at the receiver, is assumed. A long-haul single-channel system is considered in the theory and the simulations. However, the theory can be extended to wavelength-division multiplexed systems.

2.3 Existing Channel Models

As described earlier, the signal propagation in a fiber is modeled through the Manakov equation (2.1) and the NLSE for a single-polarization signal. The Manakov equation does not lend itself to an analytic solution except for some specific cases, but approximate analytical solutions exist for the fiber-optical link of Fig. 2.5, including dispersion, nonlinearity, losses in the fiber, and noise from the amplifiers.

2.3.1 Perturbation Analysis

Linearization of the Manakov equation (2.1) is often used to find approximate analytical solutions of the equation. Often in these techniques, noise-less amplifiers are assumed and the total noise from all the inline amplifiers is added either at the receiver or at the transmitter instead. Most linearization techniques can be classified broadly into two categories: perturbation-based techniques [20–23] and techniques based on a Volterra series transfer function [7, 8].

The main idea behind the perturbation techniques is to decompose $\mathbf{A}(z,t)$ into a linear term and a small perturbative term. An analytic expression is derived using the decomposition [21, eq. (37)]

$$\mathbf{A}(z,t) \approx \mathbf{A}_0(z,t) + \mathbf{A}_1(z,t), \tag{2.3}$$

where $\mathbf{A}_0$ is the linear solution, obtained by setting $\gamma = 0$ in (2.1), and $\mathbf{A}_1$ is the first-order nonlinear perturbation. As the regular perturbation technique is applicable for low to moderate launched peak powers, an enhanced regular perturbation technique has been suggested that is fairly accurate up to launched peak powers of 10 dBm [21].

A Volterra series is a polynomial expansion of the input-output relationship for a nonlinear system with memory. In the Volterra series-based approach, the NLSE is expressed as a polynomial expansion in the frequency domain and the most significant terms are retained in the transfer function. When the nonlinearity is due to the Kerr effect alone, then the order n regular perturbation solution coincides with the order $2n + 1$ Volterra series solution [21]. This is a coincidence as both these methods have different approaches: namely, the regular perturbation method seeks a solution in the form of a power series of γ, whereas the Volterra series is a generalized Taylor power series of the frequency domain input field [21]. All the above methods are applicable for special cases. They either deal with small input powers [21], or zero or very low fiber loss [22], or consider noiseless amplifiers [21], or the complexity is high with high-order kernels as in the case for Volterra series transfer function. No method considers all practical parameters and get an approximated analytical solution for (2.1).

2.3.2 Statistics of a Non-Dispersive Channel

In parallel, research has been conducted on finding the statistics of a simplified channel. The model, often referred to as a memory-less channel model, is to neglect CD and to consider the interactions of the signal and the ASE noise from optical amplifiers due to the Kerr effect, causing nonlinear phase noise (NLPN). This phenomenon was first reported by Gordon and Mollenauer [24] and is sometimes referred to as the Gordon-Mollenauer effect. By studying the correlation of the NLPN with the received signal, analytical expressions for the probability density function (PDF) of the NLPN have been derived [25–28]. The effect of the CD on the variance of the NLPN has also been studied for example using linearization techniques [27, 29, 30]. A comprehensive survey of different available techniques for the NLPN and the impact on the system performance is found in [27].

2.3.3 Statistics of a Highly Dispersive Channel

The methods in Sec. 2.3.2 assume that the dispersion is either completely compensated for through the DCMs or assuming zero dispersion fiber for transmission. Even though it is not a valid assumption in long-haul transmission, the research has clarified a lot of concepts and has been qualitatively useful in understanding the actual problem of how the signal is affected in nonlinear dispersive media.

On the other hand, there is a quite active research going on in finding an analytical expression for the system without inline CD compensation, i.e., NDM links. These models are for highly dispersive channels, where the dispersion length, L_{D}, is much shorter than the nonlinear length, L_{NL}. It was reported through simulations that for NDM links at high symbol rates, there is a residual signal distortion due to nonlinear effects even after CD is compensated for at the receiver. This distortion has a statistical distribution that typically is very close to Gaussian, even in the absence of ASE noise [31]. This means that for high-enough symbol rates, the channel can be modeled as an AWGN channel. This finding has attracted a lot of interest and analytical expressions were proposed in the literature [32–37]. A time-domain analytical model for NDM links has been derived based on the split-step Fourier method (SSFM) and also accounting for the noise from the inline EDFAs [34]. Based on this result[6], an NDM link can be viewed as a linear dispersive channel with AWGN noise and a complex scaling.

2.4 Numerical Methods for Signal Propagation

The Manakov equation (or the NLSE for the single-polarization case) does not generally have an exact analytical solution and approximate analytical solutions are not available for all cases. Numerical approaches such as the SSFM [38] are often used to describe, understand, and solve for the signal propagation in dispersive and nonlinear media [16]. In this section, the main idea behind the SSFM will be discussed.

The Manakov equation (2.1) can be re-written as

$$\frac{\partial \mathbf{A}}{\partial z} = (\hat{D} + \hat{N})\mathbf{A} \tag{2.4}$$

where $\hat{D}$ is a linear differential operator accounting for dispersion and losses in the fiber and $\hat{N}$ is a nonlinear operator. Even though nonlinearity and dispersion act together in the fiber, the SSFM assumes that when the optical field is propagated over a small distance h, the dispersion and the nonlinear effects act independently. Using this assumption, an approximate solution is obtained by propagating the signal from z to $z+h$ in a two-step process. In the first step, by setting $\hat{D} = 0$ in (2.4), one can account for the nonlinear effects. In the second step, by setting $\hat{N} = 0$ in (2.4), dispersion and losses are accounted for. Mathematically, given the field at z, the field at $z + h$ can be approximated as $\mathbf{A}(z + h, t) \approx \exp(h\hat{D}) \exp(h\hat{N})\mathbf{A}(z, t)$, where the exponential operator $\exp(h\hat{D})$ can be evaluated in Fourier domain [17, eq. 2.4.5]. The interpretation and implementation of this equation is as follows. In the first step, the phase change due to the Kerr effect is applied in the time domain using $\mathbf{A}(z + h, t) = \mathbf{A}(z, t) \exp(j\gamma h\|\mathbf{A}(z, t)\|^2)$. In the second step, phase changes due to CD and power losses in the fiber are introduced in the frequency domain as $\tilde{\mathbf{A}}(z + h, \omega) = \tilde{\mathbf{A}}(z, \omega) \exp((j\beta_2\omega^2 - \alpha)h/2)$.

The SSFM explained above is known in the literature as asymmetric SSFM. To model the fiber with better accuracy, symmetric SSFM can be used. In symmetric SSFM,[7] the signal propagation over a segment from z to $z + h$ is performed such that the nonlinearity is placed in the middle of the segment rather than at the

[6] In paper B, we extended [34] to get the performance of the system with digital backpropagation, which will be explained in detail in Ch. 3.

[7] For the analysis in paper A, asymmetric SSFM is used but in the simulations, symmetric SSFM is used. However, the principles suggested in paper A do not change even if we had used symmetric SSFM. For paper B, we used asymmetric SSFM for both theory and simulations.

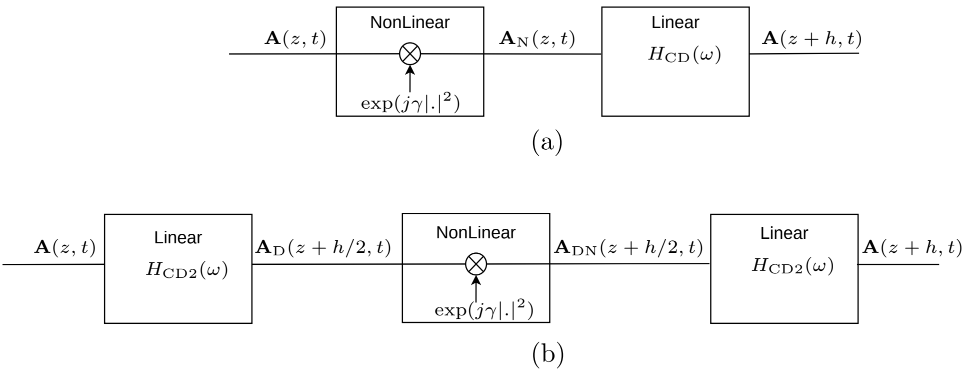

Figure 2.6: Model of the fiber as a concatenation of nonlinear and linear blocks using (a) Asymmetric (b) Symmetric SSFM. The frequency response of CD filter is $H_{\mathrm{CD}}(\omega) = \exp(i\beta_2\omega^2 z/2)$ and $H_{\mathrm{CD2}}(\omega) = \exp(i\beta_2\omega^2 z/4)$ as defined in Sec. 2.1.1.

segment boundary as

$$\mathbf{A}(z+h,t) \approx \exp\left(\frac{h}{2}\hat{D}\right)\exp(h\hat{N})\exp\left(\frac{h}{2}\hat{D}\right)\mathbf{A}(z,t). \tag{2.5}$$

This equation is then applied repeatedly over the length of the fiber L, divided into M segments each of length h, i.e., $L = Mh$. Even though these methods are straightforward to implement, the selection of the step size h is crucial and involves a complexity-accuracy tradeoff [39].

2.5 Summary and Our Contribution

In summary, starting from the Manakov equation that governs the signal propagation in the nonlinear dispersive medium, impairments existing in the fiber were described. Then the system model used in the papers was described along with the assumptions assumed in this work. Existing channel models are explained and it can be seen that there exists no channel model that account for the dispersion, nonlinearity, and noise from amplifiers for a DM link, and thereby the PDF of the signal is still unknown. Hence, existing techniques for compensating these impairments are not optimal. For uncompensated links, analytical models exist only for a highly dispersive regime and were discussed in Sec. 2.3.3. The SSFM used to numerically model the fiber is described in Sec. 2.4.

In paper A, we proposed a detector that takes β_2, γ, α, and also noise into account. We showed that significant improvements in the performance can be achieved compared to the existing algorithms. This proposed detector along with the existing state-of-the-art detector, DBP, will be introduced in the next chapter.

Chapter 3

Nonlinear Compensation Techniques

The most widely studied strategies for compensating nonlinearities and dispersion jointly are nonlinear equalization based on Volterra kernels [7,8], channel inversion using digital backpropagation [9,10], and maximum likelihood sequence detection (MLSD). Several MLSD methods have been proposed, which solve the detection problem by training sequence approach and using a look-up table at the receiver [11–13]. However, this approach is limited to a very low dispersion fiber and as a result applicable only when the ISI and the nonlinear memory is low [12]. For practical parameters, better results can be obtained with the DBP approach. Even though the original proposed approach has high complexity, many low complexity versions of DBP have been proposed in the literature.

3.1 Digital Backpropagation

In the absence of noise, the transmitted signal can be found by solving the inverse of the Manakov equation (2.1) by propagating the output signal with inverse parameters ($-\beta_2$, $-\gamma$, $-\alpha$), and this technique is therefore called back-propagation. This approach is optimal when noise from the optical amplifiers of the fiber-optical link (Fig. 2.5) is ignored. This idea was considered already in 1979 by Yariv et al. [36], who suggested the use of phase-conjugation for the dispersion compensation [40]. Fisher et al. extended this notion in 1983 to compensate for both dispersion and Kerr nonlinearity [5]. The use of a medium with negative nonlinear index (e.g., semiconductors) to reverse the effects of transmission without phase conjugation was suggested by Paré in 1996 [41].

DBP [9,10] is done in DSP using the SSFM introduced in Sec. 2.4 with inverse parameters of β_2, γ, α to invert the channel effects and get $\mathbf{A}(0,t)$. To backpropagate the signal through a section of fiber which extends from $z+h$ to z, methods such as noniterative asymmetric SSFM [9] and iterative symmetric SSFM [10, 42] have been used. In spite of the high computational complexity, DBP has been proposed as a universal technique for jointly compensating the linear and nonlinear impairments and its performance is often used to benchmark schemes proposed in the literature [42–47]. The assumed optimality of the DBP approach has spurred intense research in low-complexity variations, including weighted DBP [43], perturbation DBP [44, 45], and filtered DBP [46, 48].

3.2 Stochastic Digital Backpropagation

Unlike DBP, which finds the inverse of the channel by ignoring noise, our aim is to build a detector from basic principles of digital communications. In this section, the principles and concepts behind the proposed detector will be explained.

3.2.1 Bayesian Inference

Given the received signal $\mathbf{r}(t)$ of Fig. 2.5, the aim of any detector is to estimate the transmitted data $\underline{\mathbf{s}}$. Optimal detectors in terms of minimizing the symbol error rate can be built based on the maximum a posteriori (MAP) principle, which is used in this work. Mathematically, MAP detection involves the optimization

$$\hat{\underline{\mathbf{s}}} = \arg\max_{\underline{\mathbf{s}} \in \Omega^K} p(\underline{\mathbf{s}}|\mathbf{r}), \tag{3.1}$$

where $p(\underline{\mathbf{s}}|\mathbf{r})$ is a shorthand notation for $p_{\underline{\mathbf{S}}|\mathbf{R}}(\underline{\mathbf{s}}|\mathbf{r})$.

In the case of coherent optical communications, finding a closed form expression for $p(\underline{\mathbf{s}}|\mathbf{r})$ is difficult except in some simplified cases. However, the joint distribution of the input and all intermediates states of the system is generally available, and the determination of $p(\underline{\mathbf{s}}|\mathbf{r})$ can be seen as a marginalization of this joint distribution.

For instance, assume we have a joint distribution with four random variables $\underline{\mathbf{S}}$, $\mathbf{Y}$, $\mathbf{Z}$, $\mathbf{R}$ and we would like to find $p(\underline{\mathbf{s}}|\mathbf{r})$, assuming that the underlying structure behind these variables is governed by the Markov property.[1] In particular, let the random variables $\underline{\mathbf{S}}, \mathbf{Y}, \mathbf{Z}, \mathbf{R}$ form a Markov chain as in Fig. 3.1, i.e., $p(\mathbf{r}|\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}) = p(\mathbf{r}|\mathbf{z})$, $p(\mathbf{z}|\underline{\mathbf{s}}, \mathbf{y}) = p(\mathbf{z}|\mathbf{y})$. Using Bayes' rule and the Markov property, the joint distribution can be factorized as

$$\begin{aligned} p(\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}, \mathbf{r}) &= p(\mathbf{r}|\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}) p(\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}) \\ &= p(\mathbf{r}|\mathbf{z}) p(\mathbf{z}|\underline{\mathbf{s}}, \mathbf{y}) p(\underline{\mathbf{s}}, \mathbf{y}) \\ &= p(\mathbf{r}|\mathbf{z}) p(\mathbf{z}|\mathbf{y}) p(\mathbf{y}|\underline{\mathbf{s}}) p(\underline{\mathbf{s}}). \end{aligned} \tag{3.2}$$

Now for a given observation $\mathbf{r}$, $p(\underline{\mathbf{s}}|\mathbf{r})$ can be written as a marginal of $p(\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}|\mathbf{r})$

$$p(\underline{\mathbf{s}}|\mathbf{r}) = \sum_{\mathbf{y},\mathbf{z}} p(\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}|\mathbf{r}), \tag{3.3}$$

[1] A stochastic process has the Markov property if the conditional probability distribution of future states of the process (conditional on both past and present values) depends only upon the present state, not on the sequence of events that preceded it.

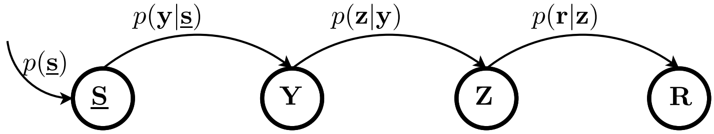

Figure 3.1: The random variables $\underline{\mathbf{S}}, \mathbf{Y}, \mathbf{Z}, \mathbf{R}$ forming a Markov chain.

where $p(\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}|\mathbf{r}) = p(\underline{\mathbf{s}}, \mathbf{y}, \mathbf{z}, \mathbf{r})/p(\mathbf{r})$. To find this marginal, the factorization (3.2) can be used.

It is clear that the marginal posteriors play an important role in inference problems. Factor graphs (FGs) and the sum-product algorithm (SPA) are tools to compute these marginal posteriors rigorously and efficiently.

3.2.2 Factor Graphs

As seen in the previous section, the global function of several variables is factorized into several local functions, each involving a small subset of variables. In many applications, as in our case, the global function represents the joint probability density and the corresponding local functions are various conditional distributions. FGs visualize this factorization and the interaction of the various random variables that are involved in a particular problem. One of the key features of FGs is that they support a variety of summary propagation or message-passing algorithms (e.g., the SPA, the min-sum algorithm, and other variations) that can be used for Bayesian inference.

Factor graphs are a generalization of other graphs proposed in the literature. FGs are strongly connected with coding theory, and the foundations of graphical models usage in coding dates back to Gallager, who in his PhD thesis in 1963 visualized a code as a graph [49]. Forney in 1973 introduced a trellis diagram as a way to show the time evolution of a finite-state machine [50]. Tanner graphs were introduced in 1981 as a way to describe a family of codes [51]. The work of Pearl in 1988 on probability propagation (or belief propagation) in Bayesian networks has attracted much attention in artificial intelligence and statistics. Applications of these graph-theoretical models beyond coding were described by Wiberg in his PhD thesis in 1996 [52]. Wiberg also introduced SPA as a message-passing algorithm over a graph [53]. A large number of existing algorithms in the fields of coding, signal processing, and computer science can be viewed as instances of the SPA. The algorithms derived in this way often are viewed as special cases or as obvious approximations of existing well-known algorithms. For example, the decoding algorithm for low-density parity check codes, the Viterbi algorithm, Kalman filtering, and the fast Fourier transform can be seen as an instance of the SPA over an appropriately chosen FG. New algorithms for complex detection and estimation problems can also be derived as instances of the SPA [54, 55].

A Forney-style FG (FFG) will be used in this work and it generally contains

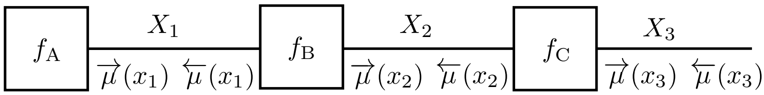

Figure 3.2: A simple factor graph for variables X_1, X_2, X_3 where every factor has at most two variables. The leftward and rightward messages are also shown in the figure.

nodes, edges and half-edges (if connected to only one node) and is drawn according to following rules[2]:

- a node is created for every factor,
- an edge (or half-edge) is drawn for every variable,
- node f_A is connected to edge X_i iff variable X_i appears in factor f_A.

Let us now take a simple example to show how an FG is drawn. Consider a joint distribution $f : \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3 \to \mathbb{R}$, which is factorized into 3 factors,

$$f(x_1, x_2, x_3) = f_\mathrm{A}(x_1) f_\mathrm{B}(x_1, x_2) f_\mathrm{C}(x_2, x_3), \tag{3.4}$$

where an instance (a realization) of a random variable, say x_i, belongs to the set $\mathcal{X}_i$. Here f is the global function and $f_\mathrm{A}, f_\mathrm{B}, f_\mathrm{C}$ are the non-negative valued local functions. From this factorization, an FG can be drawn according to the rules stated above and is shown in Fig. 3.2.

To find the marginals, messages, which are functions of the corresponding variables, are to be exchanged over the edges of the FG through a message-passing algorithm: the SPA. The SPA for our FFG is summarized as follows. The message out of a factor node $h(X_m, X_n)$ along the edge X_m is the product of $h(X_m, X_n)$ and the message towards h from edge X_n summed over all possible values of X_n. Here h represents f_A, f_B, f_C of Fig. 3.2. We denote messages associated with x_m by $\mu_{X_m}(x_m)$ (or $\mu(x_m)$ when the variable is clear from the context), and the direction of the message will be represented by arrows: $\overleftarrow{\mu}_{X_m}(x_m)$ and $\overrightarrow{\mu}_{X_m}(x_m)$. In Fig. 3.2, the leftward message $\overleftarrow{\mu}_{X_2}(x_2)$ is from the node f_C towards f_B and $\overrightarrow{\mu}_{X_2}(x_2)$ is other way around. The detailed SPA is given in [53, p. 39] but in our case, where a factor has at most two variables and a variable can be in at most two factors, the FG is linear (a FG without any branches and loops) as shown in Fig. 3.2. We note that messages can be normalized without affecting the normalized marginal. These normalized messages can be interpreted as distributions: PMFs (resp., PDFs) when the variables are discrete (resp., continuous).

- *Initialization*: All edges X_k connected to a single node, such as X_3 in Fig. 3.2, transmit message $\mu_{X_k}(x_k) = 1$, $\forall x_k \in \mathcal{X}_k$. Nodes h connected to a single

[2] If a variable appears in more than two factors, special measures have to be taken and the above rules have to be modified slightly. This will not be described here as the problem of our interest deals with variables appearing in at most two factors.

edge X_m, such as f_{A} of Fig. 3.2, transmit message $\mu_{X_m}(x_m) = h(x_m)$, $\forall x_m \in \mathcal{X}_m$.

- *Message computation*: When a node h has received the incoming message, the outgoing message on the remaining edge, say X_m, is computed as

$$\mu_{X_m}(x_m) = \sum_{x_n} h(x_m, x_n)\mu_{X_n}(x_n), \forall x_m \in \mathcal{X}_m. \tag{3.5}$$

  Note that the summation is over all possible values of x_n.

- *Termination*: The marginal for variable x_k can be obtained once the two messages on the corresponding edge are available using[3]

$$g_{X_k}(x_k) = \overleftarrow{\mu}_{X_k}(x_k)\overrightarrow{\mu}_{X_k}(x_k), \forall x_k \in \mathcal{X}_k. \tag{3.6}$$

Summation should be replaced by integration for continuous variables.

For example in Fig. 3.2, suppose the aim is to find the marginal distribution of variable X_1. The first step is to start with message $\overleftarrow{\mu}_{X_3}(x_3)$, which is a constant. Then the leftward messages for variables X_2 and X_1 are computed using (3.5) to get $\overleftarrow{\mu}_{X_1}(x_1)$. During the initialization stage, the rightward message for X_1, $\overrightarrow{\mu}_{X_1}(x_1)$ can also be found. In the last step, the leftward and rightward messages for variable X_1 are multiplied to get the marginal of X_1.

When the variables X_i are continuous (resp., discrete), the messages are scaled probability density (resp., mass) functions. For such variables, SPA rules often lead to intractable integrals and therefore the representation of messages is an important issue in such works. Many different approaches exist how to solve this problem such as the following.

- Considering a grid and evaluating the message at each of the grid points, which leads to a vector that represents the message.
- Approximating the message with parameterized distributions such as a mixture of Gaussians.
- Approximating the message by a list of samples or particles.

In the next section, the interpretation of the messages and the message passing rules will be detailed assuming that the variables are discrete.

Interpretation of Messages for Discrete Variables

Consider two discrete random variables q_1 and q_2 related by a joint distribution function $f(q_1, q_2)$. The messages, $\overleftarrow{\mu}_{Q_1}(q_1)$ and $\overleftarrow{\mu}_{Q_2}(q_2)$ in Fig. 3.3, are then probability mass functions (PMFs) of the corresponding random variables and thus can be interpreted as vectors. The message passing rules can be determined (and even interpreted) using a matrix-vector multiplication.

[3] The details behind getting marginals when two messages are multiplied is not explained here but is described in the literature [54, 55].

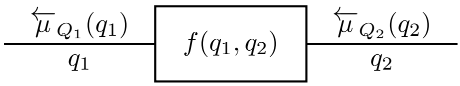

Figure 3.3: A simple factor graph with random variables q_1 and q_2 related by the factor $f(q_1, q_2)$. Also shown are the messages corresponding to these random variables.

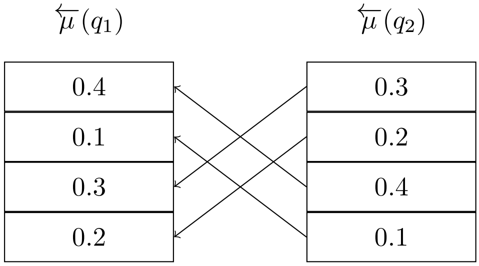

Figure 3.4: The messages, which are PMFs of variables q_1 and q_2, are shown. The arrows represent the permutations of the bijective function ϕ.

For example, assume q_1 and q_2 to be two quaternary random variables and suppose $q_2 \in \{\kappa, \psi, \chi, \zeta\}$ is an instance of the random variable Q_2 that has a PMF: $(0.3, 0.2, 0.4, 0.1)$ and the message, $\overleftarrow{\mu}_{Q_2}(q_2)$, is this PMF represented as a vector. Assume $q_1 \in \{a, b, c, d\}$ and q_2 are related by a bijective function ϕ as $q_2 = \phi(q_1)$ with $\phi^{-1}(q_2) = [c, d, a, b]$ as shown in Fig. 3.4. According to the SPA, the message $\overleftarrow{\mu}_{Q_1}(q_1)$ can be computed as

$$\begin{aligned} \overleftarrow{\mu}_{Q_1}(q_1) &= \sum f(q_1, q_2)\overleftarrow{\mu}_{Q_2}(q_2) = \sum \delta(q_2 - \phi(q_1))\overleftarrow{\mu}_{Q_2}(q_2) \\ &= \overleftarrow{\mu}_{Q_2}(\phi(q_1)), \end{aligned} \tag{3.7}$$

or $\overleftarrow{\mu}_{Q_1}(\phi^{-1}(q_2)) = \overleftarrow{\mu}_{Q_2}(q_2)$ i.e., $\overleftarrow{\mu}_{Q_1}([c, d, a, b]) = \overleftarrow{\mu}_{Q_2}([\kappa, \psi, \chi, \zeta])$. This means that the output message, $\overleftarrow{\mu}_{Q_1}(q_1)$, is a permutation of the input message. Thus the message $\overleftarrow{\mu}_{Q_1}(q_1) \propto (0.4, 0.1, 0.3, 0.2)$. In this example, the message computation can be described as $\overleftarrow{\mu}_{Q_1}(q_1) = A^T\overleftarrow{\mu}_{Q_2}(q_2)$, where A is a permutation matrix based on the relation $f(q_1, q_2) = \delta(q_2 - \phi(q_1))$ and Fig. 3.4.

When the variable is discrete, the extension to non-bijective mappings is straightforward. In this case, the matrix A will not be invertible, but messages can still be found using a matrix-vector multiplication. When q_1 and q_2 are continuous random variables and can take on real values, then instead of the PMFs, we will have the PDFs as messages. In particular, an expression of the form $\int \delta(y - x)\mu_X(x)\mathrm{d}x$ can be simplified immediately to $\mu_X(y)$. However, when the message is of the form $\int \delta(y - \phi(x))\mu_X(x)\mathrm{d}x$, then the random variables need to be transformed (for example by introducing $z = \phi(x)$) before getting the final message.

3.2.3 Particle Representation

As mentioned at the end of Sec. 3.2.2, the SPA rules often lead to intractable integrals. In this section, a brief introduction to the particle representation (PR), used in paper A, will be given.

Given a probability density function p_X, a PR, denoted by PR$\{p_X\}$, is a list of values[4] $\{x^{(k)}\}_{k=1}^{N_p}$, with the property that for any integrable function f

$$\frac{1}{N_p}\sum_{k=1}^{N_p} f(x^{(k)}) \rightarrow \int f(x)p_X(x)\mathrm{d}x, \quad N_p \rightarrow \infty. \tag{3.8}$$

One way to obtain a PR is to draw N_p i.i.d. samples from $p_X(x)$, though many other methods exist [53, ch. 3]. Note that a PR can easily be extended to high-dimensional variables. PR can be interpreted as follows: in the context of (3.8), $p_X(x)$ can be approximated as $p_X(x) \approx 1/N_p \sum_{k=1}^{N_p} \delta(x - x^{(k)})$. In other words PR$\{p_X\}$ can be considered as a uniform probability mass function, so that X is considered to be a uniform discrete random variable that can take on values in the set $\{x^{(k)}\}_{k=1}^{N_p}$.

3.3 Summary and Our Contribution

Starting with a high-level overview of the existing nonlinear compensation algorithms, DBP was presented, which is seen as a universal technique for jointly compensating linear and nonlinear impairments. Then we moved on to describe the principles behind our proposed detector of paper A, starting with Bayesian inference, factor graphs, the sum-product algorithm, and particle representation of the messages. We introduced the notion of global and local functions and how the factorization can be pictorially represented using an FG. Then SPA was introduced, which can be applied to the FG, and a marginal can be found. We noted that the messages represent scaled probability density functions and since a closed-form expression of the messages is difficult, we used a particle representation for the messages. In particular, messages are approximated with a list of samples in paper A.

In paper A, we applied the FG framework for the fiber-optical link of Fig. 2.5. The joint distribution f introduced in Sec. 3.2.2 is the joint distribution of the variables involved in each segment and span of the SSFM representation of the fiber and the EDFAs. The local functions correspond to the CD and nonlinear blocks within each segment, an EDFA within each span, and a pulse shaper at the transmitter. Each of these correspond to one local function. In paper B, we investigate the performance limits of the EDC and DBP for a single-channel NDM fiber-optical link. An analytical expression is derived that can be used to find the optimal power for a system when DBP is used.

[4] Variations exist where the values are weighted by using importance sampling. We do not apply weighting in this context.

Chapter 4

Contributions and Future Work

4.1 Paper A

In this work, we extend the MAP-based detector for a single channel [56] to account for dispersive effects. The proposed detector is based on the MAP criterion and compensates not only for linear and nonlinear effects but also takes the noise from the amplifiers into account. As a consequence, nonlinear signal-noise interactions can be handled using the proposed detector. This allows us to (i) get closer to the fundamental performance limits of the fiber-optical channel and (ii) identify regimes where DBP is not optimal. Our proposed near-MAP detector turns out to be a generalization of DBP, and hence we call the method *stochastic digital backpropagation* .

My Contributions: I derived the SDBP detector, wrote the code, performed the simulations and wrote the paper.

4.2 Paper B

In this work, we investigate the performance limits of EDC and DBP for a single-channel NDM fiber-optical link. An analytical expression is derived that can be used to find the optimal transmit power for a system when DBP is used. This analytical method is extended from [34] to derive a first-order approximation for the nonlinear noise of the system with DBP.

In this work, we found that the first-order approximation is reasonably tight for different symbol rates and it can be used to approximately compute the optimum transmit power in terms of minimizing the symbol error rate. Moreover, the first-order approximation results show the quadratic growth of the nonlinear noise with transmitted power, which is a limit for the performance of a system with DBP.

My Contributions: I derived the equations when the DBP is used as a pre-compensating unit rather than post-compensation unit. I verified that the existing plots are correct and created new plots for the paper. I modified the initial draft of the paper.

4.3 Future Work

Possible extensions on SDBP work include

- Quantifying the loss by approximating a) sequence to symbol-based detector and b) using bi-variate Gaussian approximation for the particle clouds.
- Comparing SDBP with other MLSD approaches. To start with, we can compare SDBP for a scenario as in [12].
- A similar study on the impact of PMD on DBP [57] can be done for SDBP.
- Finding an analytical expression to find the optimum transmit power in terms of minimizing the symbol error rate when SDBP is used for compensation.
- Performance of DBP and SDBP in the presence of a mismatch in the system parameters.
- Performing a similar study as in paper B to know if the quadratic nonlinear noise growth using DBP has decreased using SDBP.

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