Effects of the Second Harmonic on the Geodesic Acoustic Modes in Electron Scale Turbulence

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15th EFTC Fusion Theory Conference

September 23-26, 2013, Oxford, England

CHALMERS

Outline

- Goals and motivation
- Electron Temperature Gradient Modes
- Geodesic Acoustic Modes (GAM)
- Non-linearly Driven GAM
- Results and discussion

Goals and motivation

- Goal 1: Derive the corrections to the dispersion relation for the electron branch of the GAM originating from coupling to the m=2 mode.
- Motivation: In simulations growth rates of the GAMs are signicantly influenced by the m=2 interactions.

Motivation

- In the ASDEX-U tokamak, a periodic modulation of flow and turbulence level, with the characteristics of a limit cycle oscillation at the geodesic acoustic mode (GAM) frequency, has been observed preceding the L-H transition in low density plasmas. [1]
- A complex interaction between turbulence driven ExB zonal flow oscillations, i.e., geodesic acoustic modes (GAMs), the turbulence, and mean equilibrium flows is observed during the low to high (L-H) plasma confinement mode transition. [1]
- It was observed that GAMs are only somewhat less effective than the residual zonal flow in providing the non-linear saturation. [2]

1) G. Conway et al PRL 2011
2) Waltz et al PoP 2008

Generation of the GAM

- The basic mechanism is found in the poloidal component of the equation for parallel motion.
- In toroidal geometry, a coupling between the n=0=m electric potential perturbation and the m=1, n=0 density perturbation is established by toroidicity resulting in the GAM.

Electron Temperature Gradient Modes

We use the electron continuity and energy equations adapted from the Braginskii's model

$$\frac{dn_e}{dt} + \nabla\cdot(n_e\vec{v}_E + n_e\vec{v}_{*e}) + \nabla\cdot(n_e\vec{v}_{pe} + n_e\vec{v}_{\pi e}) + \nabla\cdot(n_e\vec{v}_{\parallel e}) = 0$$

$$\frac{3}{2}n_e\frac{dT_e}{dt} + n_eT_e\nabla\cdot\vec{v}_e + \nabla\cdot\vec{q}_e = 0$$

where $\vec{q}_e = -\frac{5}{2}\frac{p_e}{2m_e\Omega_e}\vec{e}_\parallel\times\nabla T_e$ and $\frac{\mathrm{d}}{\mathrm{dt}} = \frac{\partial}{\partial t} + \rho_e c_e \vec{e}\times\nabla\phi\cdot\nabla$

$$\nabla_\perp^2\tilde{A}_\parallel = -\frac{4\pi}{c}\tilde{J}_\parallel$$

$$\tilde{\mathrm{n}}_\mathrm{j} = -\left(\frac{z\tau_j}{1-\omega^2/(k_\perp^2c_j^2)}\right)\tilde{\phi}$$

Ion and impurity non-adiabatic response with

$$\tilde{n}_i = \delta n/n_{i0},\ \tilde{n}_I = \delta n_{I0}/n_I,\ \tilde{\phi} = e\phi/T_e$$

$$z_{eff} \approx (n_i + z^2 n_I)/n_e.$$

Normalized Electron Equations

$$-\frac{\partial \tilde{n}_e}{\partial t} - \nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi} - (1 + (1+\eta_e)\nabla_\perp^2)\frac{\partial \tilde{\phi}}{\partial y} - \nabla_\parallel \nabla_\perp^2 \tilde{A}_\parallel +$$

$$\varepsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial y} + \sin\theta \frac{\partial}{\partial x} \right)(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e) = 0$$

$$\left((\beta_e / 2 - \nabla_\perp^2)\frac{\partial}{\partial t} + (1+\eta_e)(\beta_e / 2)\frac{\partial}{\partial y} \right)\tilde{A}_\parallel + \nabla_\parallel(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e) = 0$$

$$\frac{\partial \tilde{T}_e}{\partial t} + \frac{5}{3}\varepsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial y} + \sin\theta \frac{\partial}{\partial x} \right)\frac{1}{r}\frac{\partial}{\partial y}\tilde{T}_e + (\eta_e - \frac{2}{3})\frac{1}{r}\frac{\partial}{\partial y}\tilde{\phi} - \frac{2}{3}\frac{\partial \tilde{n}_e}{\partial t} = 0$$

$$(\tilde{\phi}, \tilde{n}_e, \tilde{T}_e) = (L_n / \rho_e)(e\delta\phi / T_{e0}, \delta n_e / n_0, \delta T_e / T_{e0})$$

$$\tilde{A}_\parallel = (2 c_e L_n / \beta_e c \rho_e) e A_\parallel / T_{e0}$$

$$\beta_e = 8\pi n T_e / B_0^2$$

Anderson et al PoP 2012 and IAEA 2012

The m=0, m=1 and m=2 Equations

$$-\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi}^{(0)} - \varepsilon_n \sin\theta \frac{\partial}{\partial x}(\tilde{n}_e^{(1)} + \tilde{T}_e^{(0)}) = 0$$

m=0

$$-\frac{\partial \tilde{n}_e^{(1)}}{\partial t} - \nabla_\parallel \nabla_\perp^2 \tilde{A}_\parallel^{(1)} - \varepsilon_n \sin\theta \frac{\partial}{\partial x}(-\tilde{\phi}^{(0)} + \tilde{n}_e^{(2)} + \tilde{T}_e^{(2)}) = 0$$

$$\left((\beta_e/2 - \nabla_\perp^2)\frac{\partial}{\partial t}\right)\tilde{A}_\parallel^{(1)} - \nabla_\parallel(\tilde{n}_e^{(1)} + \tilde{T}_e^{(1)}) = 0$$

m=1

$$\frac{\partial \tilde{T}_e^{(1)}}{\partial t} - \frac{2}{3}\frac{\partial \tilde{n}_e^{(1)}}{\partial t} = 0$$

$$-\frac{\partial \tilde{n}_e^{(2)}}{\partial t} - \nabla_\parallel \nabla_\perp^2 \tilde{A}_\parallel^{(2)} - \varepsilon_n \sin\theta \frac{\partial}{\partial x}(\tilde{n}_e^{(1)} + \tilde{T}_e^{(1)}) = 0$$

m=2

$$\left((\beta_e/2 - \nabla_\perp^2)\frac{\partial}{\partial t}\right)\tilde{A}_\parallel^{(2)} - \nabla_\parallel(\tilde{n}_e^{(2)} + \tilde{T}_e^{(2)}) = 0$$

$$\frac{\partial \tilde{T}_e^{(2)}}{\partial t} + \frac{5}{3}\varepsilon_n \sin\theta \frac{\partial}{\partial r} T_e^{(1)} - \frac{2}{3}\frac{\partial \tilde{n}_e^{(2)}}{\partial t} = 0$$

Solving for the el-GAM dispersion relation starting with the m=2 equations

There is a simple relation between the m=2 density perturbation and the m=1 density and temperature perturbation

$$\tilde{n}_e^{(2)} = -\frac{\varepsilon_n q_r}{\Omega_q} \sin\theta \left(\frac{1}{C_1}\tilde{n}_e^{(1)} + \tilde{T}_e^{(1)}\right)$$

$$C_1 = \frac{5}{3}\frac{q_\parallel^2 q_\perp^2}{\Omega_q^2\left(\frac{\beta_e}{2} + q_\perp^2\right)}$$

We can use this to compute a relation between the m=1 density and temperature Perturbations, where C_1=1 and C_0 = 2/3 if the m=2 contributions are neglected,

$$\tilde{T}_e^{(1)} = \frac{2}{3}\frac{1 - \frac{5}{3}\frac{\varepsilon_n^2 q_r^2}{\Omega_q^2 C_1}\sin^2\theta}{1 - \frac{5}{3}\frac{\varepsilon_n^2 q_r^2}{\Omega_q^2}\sin^2\theta}\tilde{n}_e^{(1)}$$

$$C_0 = \frac{2}{3}\frac{1 - \frac{5}{3}\frac{\varepsilon_n^2 q_r^2}{\Omega_q^2 C_1}\sin^2\theta}{1 - \frac{5}{3}\frac{\varepsilon_n^2 q_r^2}{\Omega_q^2}\sin^2\theta}$$

GAM Dispersion Relation

To find the dispersion relation, we must determine the relation between the m=1 components and the potential

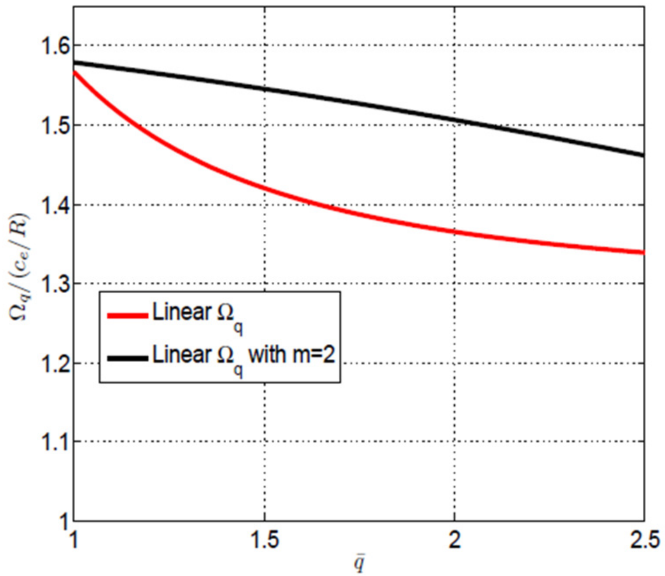
$$\tilde{n}_e^{(1)} = \frac{\dfrac{\varepsilon_n q_r}{\Omega_q}\sin\theta}{1 - \dfrac{q_{\parallel}^2 q_{\perp}^2}{\Omega_q^2}\dfrac{1+C_0}{\dfrac{\beta_e}{2}+q_{\perp}^2} + \dfrac{5}{3}\dfrac{\varepsilon_n^2 q_r^2}{\Omega_q^2 C_1}\sin^2\theta}\tilde{\phi}^{(0)}$$

The dispersion relation is now:

$$1 - \frac{q_{\parallel}^2 q_{\perp}^2}{\Omega_q^2}\frac{1+C_0}{\dfrac{\beta_e}{2}+q_{\perp}^2} + \frac{5}{3}\frac{\varepsilon_n^2 q_r^2}{\Omega_q^2 C_1}\left\langle\sin^2\theta\right\rangle = \frac{q_r^2\varepsilon_n^2}{q_{\perp}^2\Omega_q^2}(1+C_0)\left\langle\sin^2\theta\right\rangle$$

Note that this dispersion relation reduces to the previous relation (see page 11) for C_1=1 and C_0 = 2/3.

Solutions to the linear dispersion relation including m=2 effects



The linear dispersion relation w/o m=2 contributions is

$$\Omega_q^2 = \frac{5}{3}\frac{c_e^2}{R^2}\left(2 + \frac{1}{q^2}\frac{1}{1+\beta_e/(2q_r^2)}\right)$$

Figure 1. (color online) The linear el-GAM real frequency (with $m = 2$ harmonics included in black line and without represented by the red line) normalized to (c_e/R) as a function of the safety factor $\bar{q}$ is shown for the parameter $\eta_e = 4.0$ whereas the remaining parameters are $\epsilon_n = 0.909$, $\beta = 0.01$, $q_x\rho_e = 0.3$ in the strong ballooning limit $g(\theta) = 1$.

Summary

- A first derivation of the effects of higher harmonics on the electron branch of the Geodesic Acoustic Mode (el-GAM) is presented.
- An analytical dispersion relation for the el-GAM growth rate was derived.
- Allowing for interactions with the higher harmonics (m = 2) components moderates the decrease in the frequency. This effect is due to the third term on the left hand side arising from the m = 2 higher harmonics. Note that, the C0 term describes the effect of including temperature perturbations in the system and would vanish if these could be neglected.