Institutionen för Vattenbyggnad
Chalmers Tekniska Högskola

Department of Hydraulics
Chalmers University of Technology

ISSN 0348-1069

MATHEMATICAL MODEL OF SEWAGE DISCHARGE INTO CONFINED, STRATIFIED BASINS - ESPECIALLY FJORDS

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Paper presented at the 17th Congress of the International Association for Hydraulic Research

Baden-Baden, Fed. Rep. of Germany, 15-19 Aug. 1977.

Report
Series B:9 Göteborg 1977

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ERRATA

Page 348, line 6, "dimensional" should be "non-dimensional"

line 9, "accoumt" should be "account"

Page 349, line 9, "Gaussion" should be "Gaussian"

last line" "pp 432-" should be "pp 423-"

Page 350, line 9 from the bottom, "10^{-9}" should be "10^{-8}"

line 14 from the bottom, "(5)" should be "(4)"

Page 352 line 2, "considrably" should be "considerably"

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

MATHEMATICAL MODEL OF SEWAGE DISCHARGE INTO CONFINED, STRATIFIED BASINS - ESPECIALLY FJORDS

(Subject B.c.)

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SYNOPSIS

Eutrophication can in stratified waters often be considerably reduced by discharge of the sewage below the pycnocline. Sewage discharged into the deep water of a fjord or similar confined, stratified basins will be accumulated in the deep water for certain periods of time. This accumulation, the transport of sewage to the surface water by diffusion, and deep water renewals caused by the discharge or by natural causes are predicted by the present model. The model has been tested in a tracer study in the Byfjord, Sweden, and applied for predictional purposes to deep water discharges of sewage in the same fjord and the Oslo Fjord, Norway.

RESUME

L´eutrophication peut, dans les eaux stratifiées, souvent être notablement réduite par les evacuations d´eaux usées sous le pycnocline.
Les eaux usées qui sont déversées en eau profonde dans un fjord ou dans un bassin d´eau stratifiée s´accumulent en eau profonde pendant certaines périodes. Cette accumulation, le transport des eaux usées à l´eau de surface par diffusion, et le renouvellement de l´eau profonde, occasionnés par les émissions ou en raison de processus naturels, sont prévus dans le présent modèle. Le modèle a fait l´objet de tests à l´aide de traceurs dans le fjord "Byfjorden" en Suède, et a été appliqué dans des buts de pronostics pour déversement d´eaux usées en eau profonde dans le même fjord et dans le fjord "Oslofjorden" en Norwège.

INTRODUCTION

Discharge of sewage into surface waters often causes a significant eutrophication in estuaries, fjords, and other partly landlocked coastal waters. In regions with stratified waters this drawback can often be considerably reduced by discharging the sewage below the pycnocline.

A fjord is characterized by steep sides, a deep basin, and a sill at its mouth. A strong stratification often occurs at or below the depth of the sill. The pycnocline thereby acts as a lid on top of the deep water. Complete renewals of the deep water by inflow of denser water over the sill occur infrequently at intervals of several years to a few times a year. Sewage discharged into the deep water of a fjord or similar confined, stratified basins will therefore be accumulated in the deep water for certain periods of time. The model has features that in several cases are similar to those of management models for deep, stratified reservoirs (see for example [1] and [2]).

BASIC PRINCIPLES

When discharged into the deep water, the buoyant sewage jets from the manifold are deflected upwards during a continuous entrainment of the ambient water. The density within the jets increases with height, whereas the density of the ambient stratified water decreases. The vertical rise of the jets finally stops near the level of neutral buoyancy. This normally occurs in the lower part of the halocline. When leaving the buoyant jet phase, the diluted sewage is spread horizontally over the fjord in a layer, here called the trapping layer. This spreading is governed by natural advective motions and gravity forces induced by the sewage. The ambient water entrained into the buoyant jets must for continuity reasons be replaced by water from the trapping layer. This creates a forced vertical circulation in the fjord within the height of the rising jets, i.e. from the discharge level to the trapping level. Above this level, the pynocline is continuously raised by a vertical advection of equal rate as that of the sewage discharge. This advection is, however, orders of magnitude weaker than the induced circulation.

When deep water renevals are included in the model, further stages should be considered. The density of the deep water decreases continuously at a rate determined by the sewage discharge and the vertical diffusion. The moment when a deep water renewal actually occurs is determined by the natural density fluctuations at the sill level outside the fjord. The denser inflowing water entrains ambient water along the slope down into the deep basin. The inflowing water may be arrested within the halocline or penetrate to the bottom, all depending on the density relations between the inflowing water and the deep water. The gravity-induced horizontal spreading and the large scale vertical circulation set up by the inflowing water are analogous to the corresponding stages in the spreading of sewage. The time scale is, however, much shorter.

The differences in density, sewage concentration, etc. are usually much greater in the vertical than in the horizontal direction, except when the first wedge of sewage or new deep water is moving horizontally over the basin. The isopycnic surfaces are consequently almost horizontal. In terms of time scales this can be expressed as the time scale for horizontal transport (spreading) being much shorter than that for vertical transport.

As outlined above, the spreading of sewage discharged into the deep water of a confined, stratified basin is basically a three-dimensional problem. One can,

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[1] Huber, W. et.al. (1972) Temperature Prediction in stratified Reservoirs - J.Hydr. Div. ASCE. Vol.98, No. HY4, pp 8839-8845.

[2] Ditmars, J. (1970) Mixing of Density - Stratified Impoundments with Buoyant Jets - Report No. KH-R-22. W.M.Keck Laboratory, California Inst. of Tech. Pasadena, California.

however, transform it into a quasi one-dimensional problem by splitting up the basin into a main region and two subregions. Averaging over the horizontal plane reduces the analysis of the main region to an one-dimensional form. The subregions contain the buoyant jets and the inflowing deep water, respectively (see Fig.1).

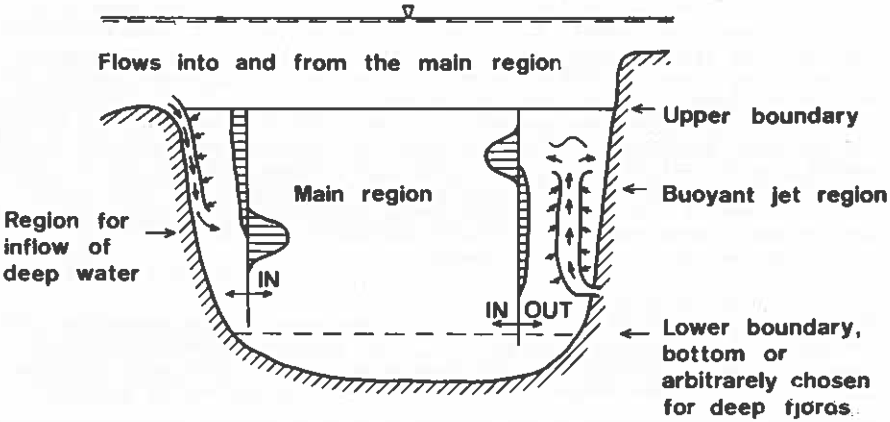

Fig.1 Definition sketch for the model
Figure de définition du modèle.

OUTLINE OF THE MODEL

Governing equations

The governing equations for the main region are based on continuity and conservation of mass for the density-stratifying agent and the sewage, respectively. The equations are written

$$\frac{\delta Q}{\delta y} = q_{in_1} + q_{in_2} - (q_{out_1} + q_{out_2}) \tag{1}$$

$$A\frac{\partial c}{\partial t} + \frac{\partial}{\partial y}(Qc) = \frac{\partial}{\partial y}\left(K_y A \frac{\partial c}{\partial y}\right) + q_{in_1} c_{in_1} + q_{in_2} c_{in_2} -$$

$$(q_{out_1} + q_{out_2})\, c + Ar \tag{2}$$

where, Q = vertical flow rate; q_{in} and q_{out} inflow and outflow rates per unit height; c = concentration of density-stratifying agent or sewage; c_{in} = corresponding concentration in inflow water; A = horizontal area; K_y = turbulent diffusivity; r = rate of internal production or decay.

Important information about the basic scheme of vertical distribution of sewage is gained by a dimensional analysis. The main parameters in Eq. (2) can in normalized form be expressed as

$$\frac{\partial c^*}{\partial t^*} + R^{1/2}\ \frac{Q^*}{A^*}\frac{\partial c^*}{\partial y^*} = \frac{1}{R^{1/2}}\ \frac{1}{A^*}\ \frac{\partial}{\partial y^*}\left(K_y^*\ A^*\ \frac{\partial c^*}{\partial y^*}\right) \tag{3}$$

where * denotes nondimensional variables. For the normalizing procedure the following variables are used: Q_o = discharge rate of sewage, Y_o and S_o height and mean dilution of the jets during the initial stage and A_o and K_{yo}, representative values for horizontal area and diffusivity within this height. Identification in Eq.(2) yields

$$R = \frac{Q_o \cdot S_o}{A_o} \Big/ \frac{K_{yo}}{Y_o} = \frac{Y_o^2}{K_{yo}} \Big/ \frac{A_o\ Y_o}{Q_o \cdot S_o} \tag{4}$$

i.e. the dimensional number, R, can be expressed as the ratio between advective and diffusive velocities or as the ratio between characteristic time scales for diffusion and advection within the main storage volume for sewage. Inflow and outflow of sewage has been taken into accoumt, implicitly, by the choice of Q_o, Y_o and S_o in the normalizing procedure. A more reliable expression for characterization of the spreading pattern is gained by introduction of the actual time needed for vertical spread of sewage within the rising height of jets by advection and diffusion, respectively, as time scales (see [3]).Deep water inflow rates are usually orders of magnitude larger than those of the sewage discharge, and, without dimensional considerations, advection can easily be judged to be totally predominant for vertical transport.

Calculation of inflow and outflow

The buoyant phase for determination of q_{out_1} has been computed according to the theories of Cederwall [4] and Fan. A computer application of Fan's theory is given by Ditmars [5] . The results obtained differed only slightly, and therefore the much simpler method of Cederwall is now used in the model.

Analytical works dealing with the vertical flow distribution for trapped sewage or inflowing deep water are not available. The physical behaviour of the horizontal spreading of sewage in an initially motionless recipient has, however, many features common with a wake collapse in stratified waters. Contrary to the wake, the diluted sewage water is certainly not homogeneous when reaching the trapping level, but the degree of stratification can be assumed to be less pronounced compared to that of the ambient water, thus imposing a gravity-induced horizontal transport. For the principal stage of the wake collapse, when the velocity of the wedge, U, does not change so much with time, the velocity according to Long [6] is given by

$$U = \frac{N \cdot h}{2} = \frac{h}{2} \left(\frac{d\rho}{dy} \cdot \frac{g}{\rho} \right)^{1/2} \tag{5}$$

where N = the Brunt-Väisälä frequency; h = the layer thickness and ρ the density of the ambient water. Continuity yields

$$h = \gamma \left(\frac{q}{N} \right)^{1/2} = \gamma \left(q^2 / \frac{d\rho}{dy} \frac{g}{\rho} \right)^{1/4} \tag{6}$$

[3] Göransson, C-G. Mathematical Model of Sewage Discharge into Confined, Stratified Basins - Especially Fjords. Dept. of Hydr., Chalmers Univ. of Tech., Gothenburg, Sweden - In Progress.

[4] Cederwall, K. (1968) Hydraulics of Marine Waste Water Disposal - Report No. 42, Dept.of Hydr., Chalmers Univ. of Tech., Gothenburg, Sweden.

[5] Ditmars, J. (1969) Computer Program for Round Bouyant Jet into Stratified Ambient Environments - Tech. Mem. 69-1, W.M.Keck Laboratory, California Inst. of Tech. Pasadena, California.

[6] Long. R.R. (1974) Turbulence and Mixing Processes in Stratified Fluids - Tech.Rep. No.6 (Series C). The Johns Hopkins Univ., Baltimore,Maryland.

where q = flow rate per unit of width, taken as the ratio between the total inflow rate and the mean width of the fjord. γ has to be determined by experiments. The thickness of a withdrawal layer, which is a gravity-driven mechanism, is given by fundamentally the same expression (see [1]). This similarity seems also reasonable. Eq.(6) has therefore been used for determination of the thickness of trapping layers for sewage and inflowing deep water, respectively. For the vertical distribution of sewage within the trapping layer the same approach as used in [1] is applied, i.e. the inflow distribution is assumed to be of a Gaussion form, and 95% of the inflow should be included in the thickness of the trapping layer.

A proper description of the deep water renewals requires that the vertical density distribution above the sill outside the fjord is known for a given period of simulation. Corresponding density distribution inside the fjord is determined by the renewal flow itself, due to the lifting of the deep water. A method for calculation of the flow over the sill where the continuous variation in density difference with depth is considered, is described in [7] . By applying the theory of Ellison and Turner [8] , an expression for the entrainment of water along the slope into the deep basin has been derived.

$$\frac{dQ}{ds} = a\,(\Delta\rho \cdot Q)^{1/3} \tag{7}$$

where s = coordinate along the slope, a = coefficient which is dependent on the inclination of the slope and $\Delta\rho$ and Q are density difference and flow rate at the actual depth, respectively. The a-value must often be found by calibration from field studies.

The sewage discharge often changes the vertical density distribution considerably and this may in turn affect the turbulent diffusivity. In deep basins diffusivity is in several cases found to be closely related to the stability of the density stratification, [7] . For the two fjords dealt with in this paper the relation is of the form

$$K_y = \beta\,(N^2)^{-\alpha} \tag{8}$$

where the coefficients α and β are found from field studies.

<u>Boundary conditions and solution of the basic equations</u>

Boundary conditions are given as

$$\left(\frac{\partial c}{\partial y}\right)_{y=y_b} = 0 \qquad c_{y=y_s} = c_r\,(t) \tag{9}$$

where y_b and y_s denote the elevation for the bottom and the sill, respectively; c_r = density or concentration at the sill level outside the fjord, or a given fixed reference value. During deep water renewals, c_r is calculated with respect payed to the lifting of the deep water.

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[7] Göransson, C-G, and Svensson,T. (1975) The Byfjord: Studies of Water Exchange and Mixing Processes - SNV PM 594, Swedish Environmental Protection Board, Stockholm, Sweden. (In Swedish with summary and captions in English).

[8] Ellison, T.H. and Turner, J.S. (1959) Turbulent Entrainment in Stratified Flows. - J.Fluid.Mech. Vol.6, part 3, pp 432-449.

The governing equation, (2), has been solved by both explicit and implicit finite difference schemes. The former impose for stability reasons that

$$K_y \frac{\Delta t}{(\Delta y)^2} \leq \frac{1}{2} \tag{10}$$

and

$$\frac{Q}{A} \cdot \frac{\Delta t}{\Delta y} < 1 \tag{11}$$

where Δy and Δt are the time and space increments, respectively. Unconditionally stable implicit schemes can be used but, for reasons of accuracy, they ought to fulfil the relation given in Eq.(11). This relation has in most applications been the most restrictive one. Therefore, an explicit scheme has commonly been used.

APPLICATION TO A FIELD STUDY

The author has participated in extensive multidisciplinary research on an eutrophicated fjord, the Byfjord, situated on the West Coast of Sweden. The hydrodynamical studies [7] included an extensive tracer simulation of a proposed deep water discharge of sewage from the adjacent town of Uddevalla. The fjord is approximately 4 km long, 1.5 km wide and has a maximum depth of 50 m. The fjord is highly stratified below the sill level. The sill is situated at the narrow entrance, 100 m wide, of the fjord at a depth of 11 m. The mean rate of flow of sewage is expexted to increase to 500 l/s in the future.

Brackish surface water was continuously pumped into the deep water at a depth of 25 m to simulate the sewage discharge. Pumping was performed for a period of 2.5 months at a flow rate of 200 l/s. A fluorescent dye, Rhodamine-B, was used as a tracer. The first, gravity driven, wedge of the tracer was spread over the entire fjord in less than a week. The horizontal gradients within the trapping layer became relatively small after 2-3 weeks. The thickness of the tracer-marked layer was found to be 3.5 m during the progress of the first wedge. By using the mean width of the fjord, which is almost of a rectangular form, and the buoyant jet flow reaching the level of neutral buoyancy, one gets a value of $\gamma = 7$ in Eq.(6).

The present model has been applied to the tracer test. Calculated and measured values averaged in the horizontal plane are shown in Fig.2. The averaging procedure smooths out the measured profiles somewhat. The upper part of the pycnocline was disturbed during the study by smaller inflows over the sill. Measured and predicted values show a fairly good agreement, especially in view of the complexity of the field measurements. The vertical distribution of the tracer reveals that both advection and diffusion are important spreading mechanisms in this case. From Eq.(5), R = 12 is found.

Measurable concentrations of the tracer remained in the deep water for more than two years until a complete renewal occurred. This implied excellent conditions for studies of vertical diffusion and the effect of minor renewals within the pycnocline. From these studies the coefficients in Eq.(8) have been determined to be $\alpha = 0{,}6$ and $\beta = 1{,}2 \cdot 10^{-9}$ (K_y in m^2/s).

APPLICATIONS FOR PREDICTIONAL PURPOSES

The Byfjord.

For predictional purposes the present model is applied to the previously mentioned sewage discharge in the deep water, with a flow rate of 500 l/s and a discharge depth of 30 m. The result for a period without any deep water renewals is shown in Fig.3.

The stronger vertical circulation induced by the sewage discharge compared with that of the tracer study results in a pronounced advection-dominated

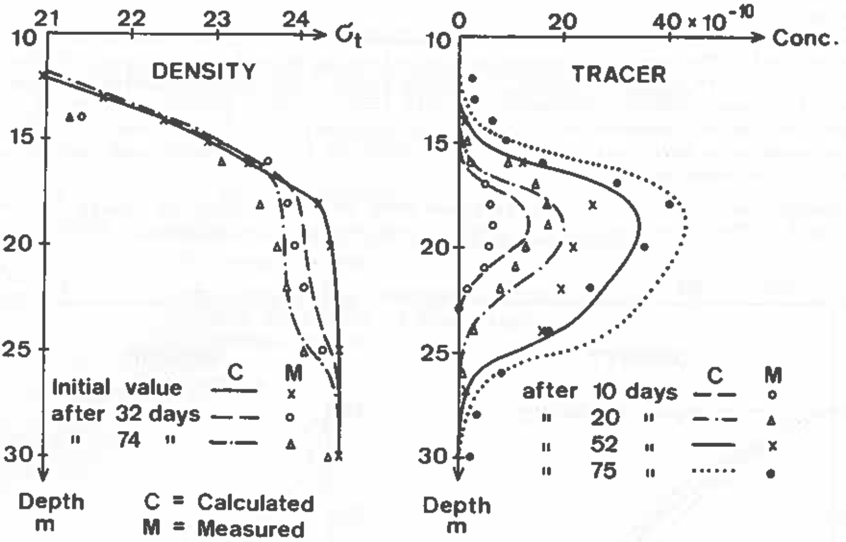

Fig. 2 Calculated and measured concentrations for the tracer test.
Concentrations calculéet mesurés de l´expérience de traceur.

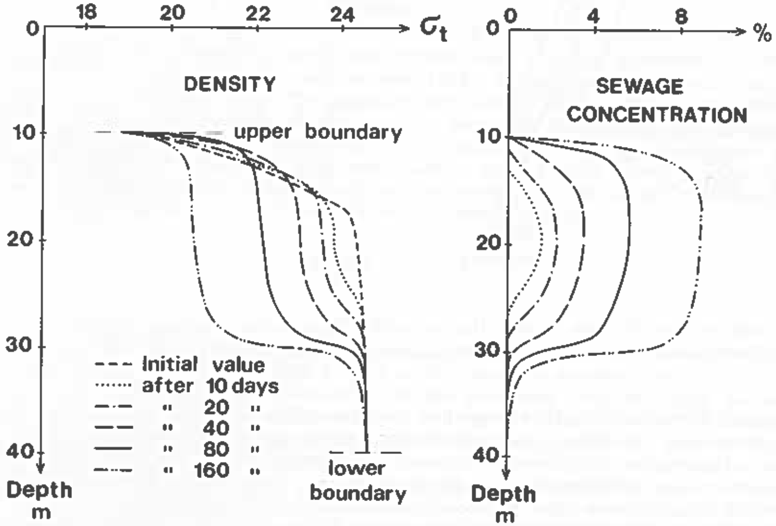

Fig. 3 Simulation of deep water discharge of sewage in the Byfjord.
Simulation de la décharge d´eaux usées dans Byfjorden.

spreading pattern. Consequently, a much higher value, R=80, is found for the ratio between advective and diffusive velocities. Density and sewage concentration within the storage volume,i.e. the deep water volume within the height of the buoyant jets, are almost homogeneous. Thus, a simple box model with a moving upper boundary can be used to give a good simulation of a deep water discharge[3].

The Oslo Fjord

The Oslo Fjord, Norway, is a water area of considrably larger dimensions than the Byfjord. The area just below the halocline is 90 km^2 for the part of the fjord dealt with in this example, the Vest Fjord. Vertical diffusion is a much more effective transport mechanism in this larger fjord, almost an order of magnitude larger compared with that of the Byfjord, α =0.8 and β =$6.2 \cdot 10^{-8}$. This leads to a more rapid decrease in density of the deep water and yearly renewals of the deep water.

An example of a simulation of a proposed deep water discharge of sewage is shown in Fig.4. The flow rate, Q_o, is 3 m^3/s, and the depth of discharge is 42 m.

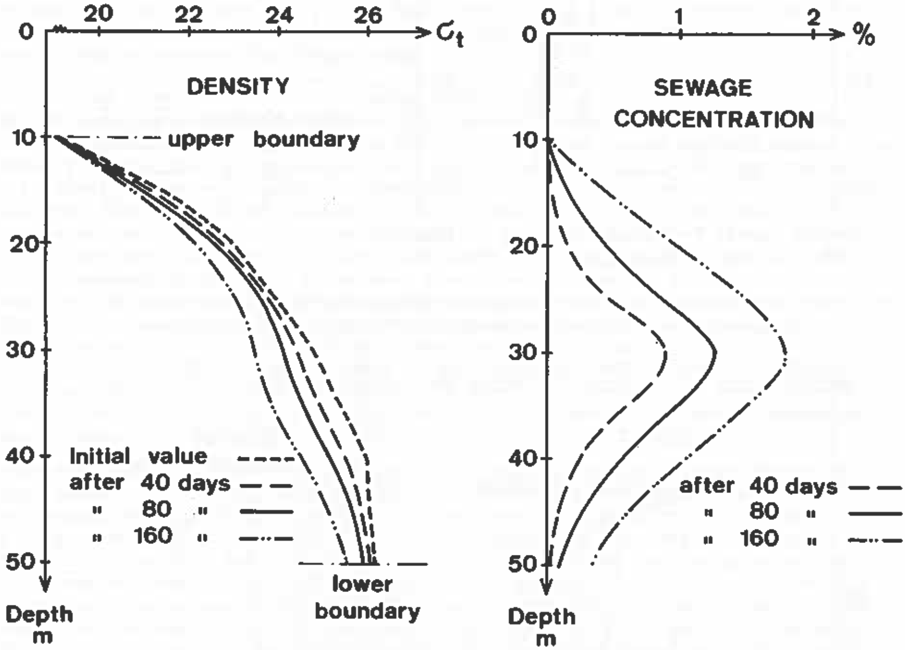

Fig. 4 Simulation of deep water discharge of sewage in the Oslo Fjord.
Simulation de la décharge d'eaux usées dans Oslofjorden.

This vertical distribution of sewage has a quite different pattern showing that vertical spreading is almost exclusively determined by diffusion. Consequently, R=1.1 has a low value. In this case a very good estimate of the vertical sewage distribution can be achieved by simply applying the analytical solution for a continuous point source into a basin of constant cross-sectional area. The common features of the density profiles are to a very little extent influenced by the sewage discharge, which also reveals the predominance of diffusion. This means that only small horizontal density gradients will be established. The horizontal gravity-induced spreading of sewage within the trapping layer will consequently be weaker here, and natural currents may play a more important role. Greater horizontal variations in sewage concentration can be expected. Furthermore, it is doubtful whether the whole fjord area really participates in the spreading of the sewage. Nevertheless, the resulting vertical transport of sewage will be well simulated by the model.

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