Subset-Optimized Polarization-Multiplexed PSK for Fiber-Optic Communications

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Martin Sjödin, Erik Agrell, *Senior Member, IEEE*, and Magnus Karlsson

***Abstract*—A more power-efficient modulation format than polarization-multiplexed quadrature phase-shift keying (PM-QPSK) can be obtained by optimizing the amplitude ratio between symbols with odd and even parity in the PM-QPSK constellation. The optimal amplitude ratio approaches the golden ratio at high signal-to-noise ratio (SNR), yielding a 0.44 dB increase in asymptotic power efficiency compared to PM-QPSK. Union bound expressions are derived for the bit and symbol error rate of the new format, which together with Monte Carlo simulations give the power efficiency at both low and high SNR. A similar optimization performed on PM-8PSK gains 1.25 dB.**

***Index Terms*—Polarization-multiplexing, quadrature phase-shift keying, power-efficiency, four-dimensional modulation.**

I. Introduction

COHERENT detection has been used for only a short time in optical communications, since it only recently became feasible to implement at high data rates. Compared to traditional noncoherent detection, it offers advantages with respect to required received signal power and spectral efficiency and enables digital equalization of transmission impairments. In addition, it gives access to the full four-dimensional (4-D) signal space of the electromagnetic field (two polarization components and two quadratures) for data transmission. The benefits of coherent receivers are currently being exploited to increase the data rates in future systems. Binary modulation formats such as on-off keying (OOK) and differential phase-shift keying (DPSK) have been used in fiber-optical networks for many years, but there is now an urgent need for more bandwidth-efficient modulation schemes.

The most common use of the 4-D signal space is to transmit independent two-dimensional constellations in the two polarizations, which is known as *polarization-multiplexing*. Quadrature phase-shift keying with polarization-multiplexing (PM-QPSK) has attracted enormous attention in recent years and has, e.g., been used to set the current world record of 112 Pbit/s·km for the product of capacity and transmission distance [1]. On the other hand, there are modulation formats with similar complexity and better performance than PM-QPSK. By choosing one of the subsets with even or odd parity of the PM-QPSK constellation, a format which may be described as QPSK transmitted in one selected polarization-state per symbol is created [2]. It has become known as *polarization-switched* QPSK (PS-QPSK) [3], [4], and is the most power efficient modulation format in signal spaces with up to four dimensions. Unfortunately, PS-QPSK, with 8 different levels, is less spectrally efficient than PM-QPSK, and the 16-point analogy of PS-QPSK is less power-efficient [2]. It would be valuable to find modulation formats with higher power efficiency than PM-QPSK and similar or higher spectral efficiency. The best 4-D 16-level constellation exhibits 1.11 dB gain over PM-QPSK but is far too complex to be implemented at high data rates [5]. The 16-level constellation suggested by Kim *et al*. [6] is nearly optimal with 0.9 dB gain but is also very difficult both to generate and to decode.

In this letter we report on a new modulation format called *subset optimized* PM-QPSK (SO-PM-QPSK), which is the first practically feasible 4-D 16-level constellation with higher power efficiency than PM-QPSK. SO-PM-QPSK is obtained by scaling the relative amplitude between symbols with odd and even parity in the PM-QPSK constellation. We show that the improvement in asymptotic power efficiency over PM-QPSK is 0.44 dB for an amplitude ratio equal to the golden ratio (≈ 1.618). In addition, by using the union bound approximation and Monte Carlo simulations, we investigate the performance of SO-PM-QPSK with different amplitude ratios as a function of the signal-to-noise ratio (SNR) and determine the optimal ratio for each SNR. We also show that a similar optimization can be made for PM-8PSK, with 1.25 dB asymptotic gain.

While the performance benefits of the new modulation formats are smaller at limited SNR, they will be useful for uncoded applications with strong constraints on latency, such as stock market trading and cellular base station coordination. By studying uncoded transmission, we follow a long-standing tradition in communication theory, exemplified by numerous classical papers by, e.g., Forney and Sloane. Most of the papers in our reference, which also focus on uncoded transmission, follow the same tradition.

II. Preliminaries

We assume a discrete-time memoryless additive white Gaussian noise channel with noise variance $N_0/2$ per dimension. The constellation of a modulation format with M symbols is defined as $C = \{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_M\}$, where the symbols $\boldsymbol{c}_k$ occur with equal probabilities and are formed from the real and imaginary part of the electromagnetic field's x and y polarization components [7]. The Euclidean distance between the symbols $\boldsymbol{c}_k$ and $\boldsymbol{c}_j$ is denoted by $d_{kj} = \|\boldsymbol{c}_k - \boldsymbol{c}_j\|$, and the smallest distance between any pair of symbols in C is then

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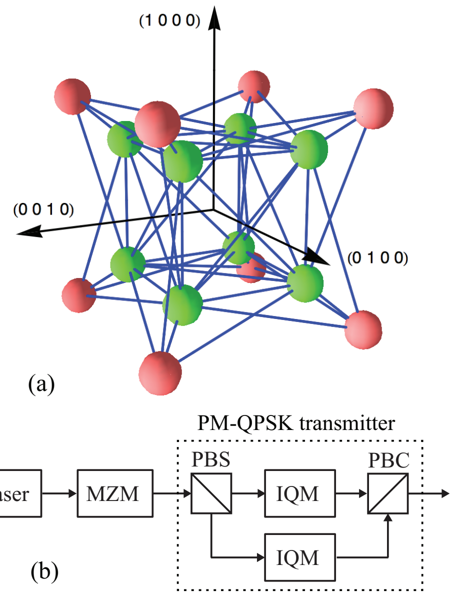

Fig. 1. (a) SO-PM-QPSK with the optimal A_r shown in a three-dimensional projection orthogonal to (0 0 0 1). The lines connect nearest neighbors. (b) The transmitter for SO-PM-QPSK with a Mach-Zehnder amplitude modulator followed by a PM-QPSK transmitter.

$d_{\min} = \min_{k \neq j} d_{kj}$. The asymptotic power efficiency γ of C is [4]

$$\gamma = \frac{d_{\min}^2 \log_2(M)}{4E_s} = \frac{d_{\min}^2}{4E_b}, \tag{1}$$

where the factor of 4 in the denominator is used for normalization to 0 dB for the PM-QPSK constellation, defined by the levels $C_{\text{PM-QPSK}} = \{(\pm1, \pm1, \pm1, \pm1)\}$, E_s is the energy per symbol, and E_b the energy per bit

$$E_b = E_s / \log_2(M) = \frac{1}{M \log_2(M)} \sum_{k=1}^{M} \|\boldsymbol{c}_k\|^2. \tag{2}$$

Finally, the signal-to-noise ratio (SNR) is E_b/N_0, which is varied by changing N_0 while keeping E_b fixed.

III. Subset-Optimized Constellations

To obtain SO-PM-NPSK, we first consider an NPSK constellation in a single polarization. We assume Gray coding, i.e., nearest neighbors have different parity, and the symbols with an even and an odd number of one bits belong to the subset S_e and S_o, respectively. The even and odd parity subsets of PM-NPSK are then $C_{\text{PM-NPSK, even}} = (S_{e,x} \times S_{e,y}) \cup (S_{o,x} \times S_{o,y})$ and $C_{\text{PM-NPSK, odd}} = (S_{e,x} \times S_{o,y}) \cup (S_{o,x} \times S_{e,y})$, where x and y denote the polarization components of the pair of 2-D NPSK symbols used to form a polarization-multiplexed symbol in four dimensions. Scaling the amplitude of the symbols in one of the subsets with a factor A_r gives the SO-PM-NPSK constellation $C_{\text{SO-PM-NPSK}} = C_{\text{PM-NPSK, even}} \cup (A_r \cdot C_{\text{PM-NPSK, odd}})$. Using (2) we find $E_s = 2(A_r^2 + 1)$, regardless of N.

A. Minimum distance decoding

The squared minimum distance in SO-PM-QPSK is

$$d_{\min}^2 = \min\{8, 8A_r^2, (1 + A_r)^2 + 3(1 - A_r)^2\}, \tag{3}$$

and using (3) in (1) we find a maximum improvement in power efficiency over PM-QPSK of 0.44 dB for $A_r = \varphi$ and $1/\varphi$, where $\varphi = (\sqrt{5} + 1)/2 \approx 1.618$ is the *golden ratio*. The SO-PM-QPSK constellation is shown in a three-dimensional projection in Fig. 1(a), where the green and red spheres represent symbols in the subset with even and odd parity, respectively. The outer spheres have 4 nearest neighbours, just like the symbols in conventional PM-QPSK, while the inner spheres have 10. Although there are 4-D 16-level constellations with higher asymptotic gain over PM-QPSK [5], [6], these have irregular constellations and are harder to generate and to decode. SO-PM-QPSK has, to the best of our knowledge, the best power efficiency of all published 16-level formats with similar implementation complexity. Since it consists of two subsets of PM-QPSK with different amplitudes, it can be generated by placing a single Mach-Zehnder amplitude modulator (MZM) before a conventional PM-QPSK transmitter, as shown in Fig. 1(b). Depending on the parity of the SO-PM-QPSK symbol to be transmitted, the MZM sets the light amplitude to one of two possible values, which have a mutual ratio of A_r.

A maximum-likelihood decoding algorithm for SO-PM-QPSK can be obtained by modifying the standard algorithm for decoding the D_4 lattice [8] as follows.

1) The received symbol $\boldsymbol{r}$ is first decoded as for PM-QPSK. We denote the resulting vector as $\boldsymbol{c}_1$.
2) The most uncertain bit in $\boldsymbol{c}_1$ is inverted to create the vector $\boldsymbol{c}_2$, which will have different parity than $\boldsymbol{c}_1$.
3) In the final step, we find $\min\{\|\boldsymbol{r} - \boldsymbol{c}_1\|, \|\boldsymbol{r} - \boldsymbol{c}_2\|\}$, which gives the decoded codeword.

For SO-PM-8PSK the squared minimum distance is

$$d_{\min}^2 = \min\{b, A_r^2 b, 4 + 4A_r^2 - A_r(4 + 2\sqrt{2})\}, \tag{4}$$

where $b = 8 - 4\sqrt{2}$. Using (4) in (1) we find that the maximum power efficiency improvement over PM-8PSK is 1.25 dB for $A_r = \sqrt{2}$ or $1/\sqrt{2}$. Among modulation formats with the same number of levels suitable for practical implementation, star 8-QAM with polarization multiplexing exhibits an additional 0.35 dB gain over PM-8PSK [9, p. 197]. Fig. 2 shows the dependence of γ on A_r for SO-PM-QPSK and SO-PM-8PSK. There are two optimal ratios, since it does not matter which subset is scaled in amplitude. The conventional formats are obtained when $A_r = 0$ dB, and the gain for optimal A_r is indicated for each case. The subset optimization can be performed for higher-order NPSK formats as well, but since the power efficiencies of these are very low, we focus on QPSK and 8PSK.

B. Symbol and bit error rates

We investigated the performance of the subset optimized formats as a function of the SNR by using both the union bound approximation for the BER and the SER and Monte Carlo simulations. The methods complement each other, since Monte Carlo simulations are very time consuming for simulating low BER, while the union bound is asymptotically correct for high SNR but inaccurate for low SNR. In the Monte Carlo

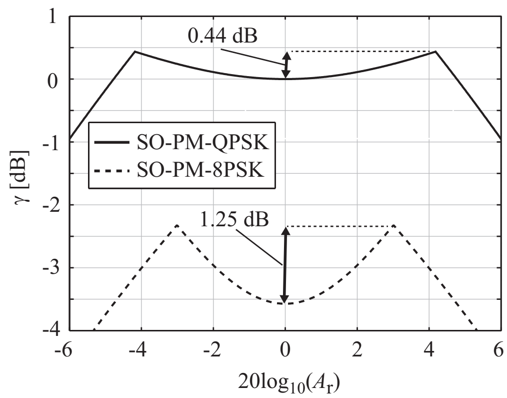

Fig. 2. The improvement in asymptotic power efficiency over PM-QPSK and PM-8PSK as a function of A_r. Maximum gains of 0.44 dB and 1.25 dB are achieved with the subset optimization.

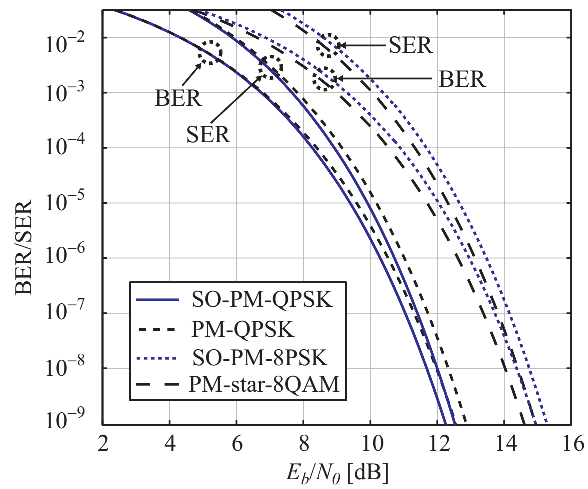

Fig. 3. The BER and SER as a function of E_b/N_0 for the different formats. For SO-PM-QPSK and SO-PM-8PSK we used the optimal A_r for each SNR.

simulations we used 2^{20} symbols of random independent data and counted at least 25000 bit errors for each SNR value. For SO-PM-QPSK the decoding scheme in III was used for symbol decisions while full search over the symbol alphabet was employed for SO-PM-8PSK.

The SER is approximated by the union bound [9, p. 184]

$$\text{SER} \leq \frac{1}{M}\sum_{k=1}^{M}\sum_{\substack{j=1\\j\neq k}}^{M} Q\left(\frac{d_{kj}}{\sqrt{2N_0}}\right), \tag{5}$$

where $Q(x) = \text{erfc}(x/\sqrt{2})/2$. Due to the symmetry of SO-PM-QPSK the SER expression in (5) can be expanded as

$$\text{SER} \leq 3Q\left(\sqrt{\frac{1}{A_r^2+1}\frac{8E_b}{N_0}}\right) + 3Q\left(\sqrt{\frac{A_r^2}{A_r^2+1}\frac{8E_b}{N_0}}\right) + 4Q\left(\sqrt{\frac{A_r^2-A_r+1}{2A_r^2+2}\frac{8E_b}{N_0}}\right). \tag{6}$$

By considering all pairwise symbol errors independently, we obtain an expression for the BER of SO-PM-QPSK

$$\text{BER} \leq \frac{3}{2}Q\left(\sqrt{\frac{1}{A_r^2+1}\frac{8E_b}{N_0}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{2}{A_r^2+1}\frac{8E_b}{N_0}}\right) + \frac{3}{2}Q\left(\sqrt{\frac{A_r^2}{A_r^2+1}\frac{8E_b}{N_0}}\right) + 3Q\left(\sqrt{\frac{A_r^2+A_r+1}{2A_r^2+2}\frac{8E_b}{N_0}}\right) + Q\left(\sqrt{\frac{A_r^2-A_r+1}{2A_r^2+2}\frac{8E_b}{N_0}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{2A_r^2}{A_r^2+1}\frac{8E_b}{N_0}}\right). \tag{7}$$

IV. Numerical Results and Conclusions

Fig. 3 shows the BER and the SER for SO-PM-QPSK and SO-PM-8PSK with A_r optimized for each SNR value. The Monte Carlo results and the union bound expressions are used below and above E_b/N_0 corresponding to a BER of less than 5.0×10^{-4}, respectively. For comparison we show the BER and SER for PM-QPSK (using the exact expressions in [9, pp. 192–193]) and PM-star-8QAM (using Monte Carlo simulations and the Union bound). In the low SNR regime, PM-QPSK and SO-PM-QPSK perform similarly since the optimal $A_r \approx 1$. The difference gets noticeable when increasing the SNR and at a BER of 10^{-3} $A_r = 1.275$ gives SO-PM-QPSK a 0.05 dB reduction in required SNR compared to PM-QPSK. The optimal A_r converges slowly to φ and a reduction in required SNR of 0.31 dB is gained at BER $= 10^{-9}$ for $A_r = 1.525$. The amplitude scaling gives negligible BER reduction for E_b/N_0 of less than approximately 5 dB, but the SER is decreased noticeably already from $E_b/N_0 \approx 2$ dB. When comparing SO-PM-8PSK and PM-star-8QAM we find the expected 0.35 dB difference in the high SNR regime, which decreases to 0.30 dB at a BER of 10^{-3}. For SO-PM-8PSK we note that $A_r = 1$ (conventional PM-8PSK) and $A_r = \sqrt{2}$ yield similar performance at low SNR.

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