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Dynamic Analysis of a Moored Wave Energy Buoy

by

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Report

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SUMMARY

Time Domain Equations of Motions for a wave energy buoy is presented. The equations are derived for a small body that is moored via mooring buoys.

The equations are derived in a body fixed coordinate system, which was most convenient. The motions are then integrated in an Earth Fixed Frame.

Hydrodynamic forces are derived assuming the body to be small compared to the wave length. Wind and current forces are included, but potential wave drift forces are excluded. Dynamic mooring forces are assumed on the mooring buoys. Wave energy is converted by damping an extra motion degree of freedom in heave.

The main scope of the text is in topics such as hydrodynamic properties and loading. Kinematics of rigid bodies are also discussed, especially coordinate transformations.

LIST OF CONTENTS

PREFACE

The work is the first part of two, about the dynamics of a wave energy convertor. It is dealing with basic hydrodynamics and mechanics, and is to be followed by a text on the numerical simulation.

I wish to thank my tutor, Dr. Lars Bergdahl, and others at the Department of Hydraulics for help throughout the work.

A1 INTRODUCTION

Converting wave energy into electricity using a buoy is a delicate matter. Apart from choosing a properly tuned floater, i.e. with a response that fits the wave climate, a mooring system has to be used to keep it in position. This mooring have to be designed so that it does not introduce any negative effects on the buoy motion. To be able to design such a system, there has to be a tool that is able to consider effects of dynamic mooring forces exerted on the buoy.

The normal procedure when calculating the motions of a floating body, is to assume that the mooring forces act as static forces. In doing this assumption mass and damping forces are neglected. If these are of importance, as they could be for a small buoy, the only way to evaluate this is to use the dynamic forces from the moorings.

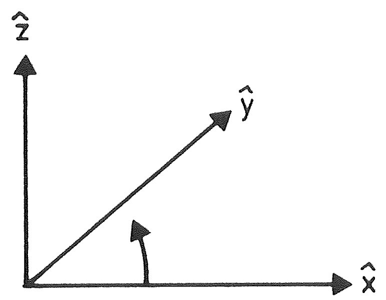
The main equations of motions are first to be established from fundamental hydrodynamics, and then to be specified on an object. In doing this different forces can be distinguished and their significance can be estimated.

A1.1 Coordinate system

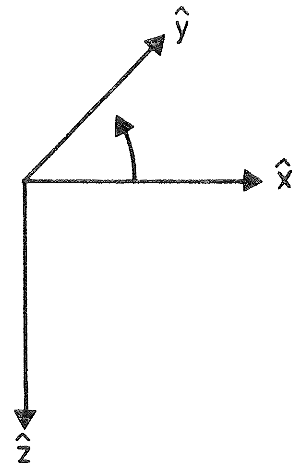
A proper choice of the coordinate system is of interest when deriving the equations of motions. Especially when dealing with the algebraic expressions for physical phenomena, e.g. torque.

There are two types of orthogonal coordinate systems, left-handed systems (LHS) and right-handed systems (RHS). In this report the RHS is used. Properties of both are stated below, to ease the understanding of the difference between the systems.

The space coordinate system consists of one zero point (origin) in space, and one base vector ($\hat{x}$, $\hat{y}$, $\hat{z}$) which is assumed to be orthonormed. Then the base vectors in the following order $\hat{x}$, $\hat{y}$, $\hat{z}$ makes a right-handed system if the smallest rotation that makes $\hat{x}$ parallell to $\hat{y}$ is viewed anti-clockwise from the tip of the $\hat{z}$-axis. If on the contrary the smallest rotation is viewed clock-wise then the system is left-handed.



Right-handed system



Left-handed system

Fig. A1:1 Base vectors in RHS and LHS respectively. The $\hat{x}$- and $\hat{z}$-axis are in the plane of the paper, and the $\hat{y}$-axis is directed into the paper.

It is most important to notice the sequence of the vectors;

If $\hat{x}$, $\hat{y}$, $\hat{z}$ is a RHS, then $\hat{y}$, $\hat{z}$, $\hat{x}$ and $\hat{z}$, $\hat{x}$, $\hat{y}$ are also RHS.

In the following two sections the definitions of the systems are presented using perhaps more illustrative definitions.

A1.1.1 Defined by vector product

The vector product is defined so that positive rotations - between successive axes will give a RHS system and negative rotations will give a LHS.

The vector products of the base vectors are:

RHS
$$\hat{x} = \hat{y} \times \hat{z}$$
$$\hat{y} = \hat{z} \times \hat{x}$$
$$\hat{z} = \hat{x} \times \hat{y}$$

LHS
$$\hat{x} = -(\hat{y} \times \hat{z})$$
$$\hat{y} = -(\hat{z} \times \hat{x})$$
$$\hat{z} = -(\hat{x} \times \hat{y})$$

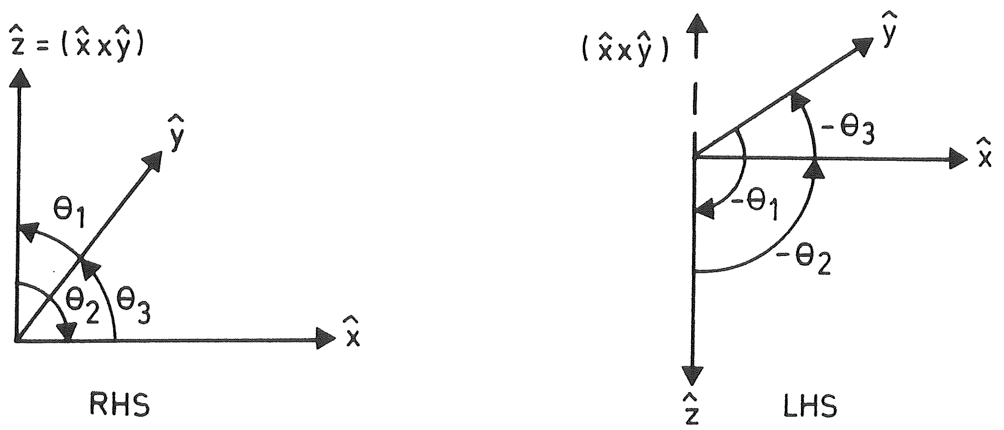


Fig. A1:2 Vector products.

A1.2 The independent coordinates of a rigid body

Before discussing the motion of a rigid body we must first establish how many independent coordinates that are necessary to specify its configuration. The position in space of a rigid body is specified, if the positions of three non-colinear points in the body (or rigidly connected with it) are known in a given frame. Every point requires three coordinates, but the constraint that the distances between all parts of the body are constant gives three relations. This reduces the number of independent coordinates that are needed to specify the position of the body to (3x3 -3 =)6.

Of course, there may be additional constraints on the body besides the constraints of rigidity. For example, the body may be constrained to move non-rotationally. In such a case the independent coordinates will reduce to three.

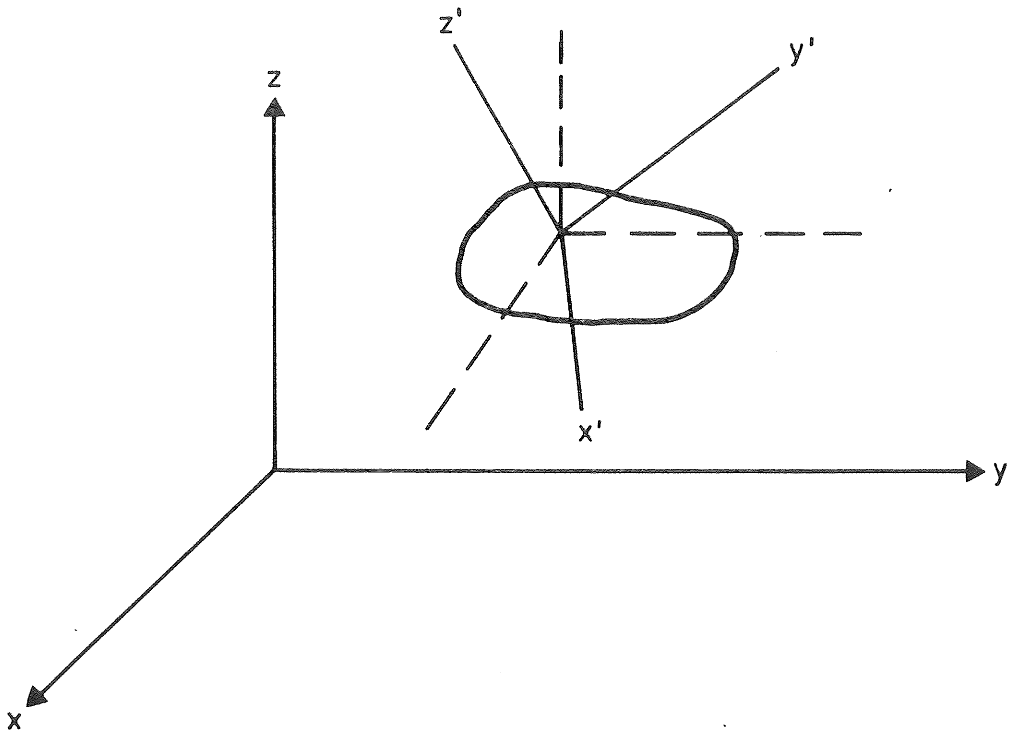


Fig. A1.3 Unprimed axes represent an external reference set of axes, and the primed axes represent the body fixed axes.

There are various ways of assigning these six coordinates. One way is to locate a Cartesian set of coordinates fixed in the rigid body (the primed axes in Fig. A1.3) relative to the coordinate axes of the external space. Clearly three of the coordinates are needed to specify the coordinates of the origin of the body fixed axes. The remaining three coordinates must then specify the orientation of the primed axes in the translated frame.

A2 MOTION RELATIVE TO A MOVING REFERENCE FRAME

In this chapter the relations between two moving systems are discussed.

A2.1 Orthogonal transformation

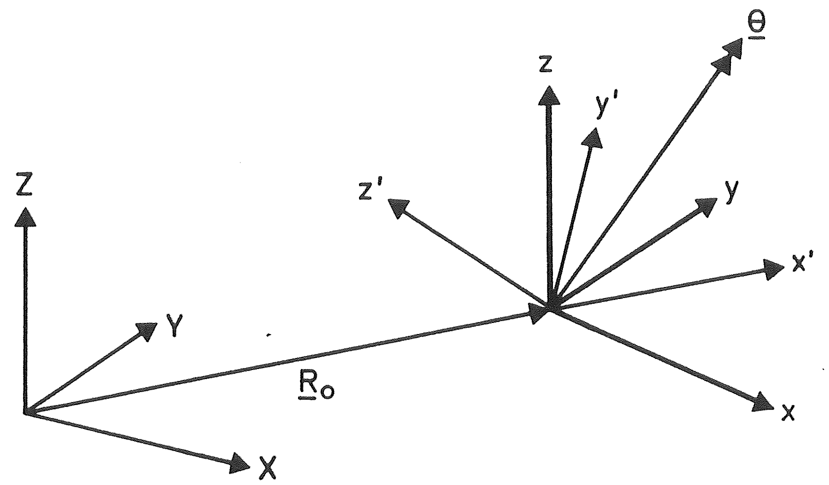


Fig. A2.1 Three sets of coordinate axes.
$\underline{X}$ is space fixed, then translationed to $\underline{x}$,
and finally rotated to $\underline{x}'$.

Different sets of coordinate axes (or frames) are related in some way. The relation is most easily understood if the vector properties are related. Knowing that a vector is defined by its magnitude and direction, and that the direction only is defined in a set of coordinate axes, it is easy to derive the relation.

A2.1.1 Transformation matrix

If one system is regarded as fixed, the other one is displaced, i.e. rotated and translated, relative to the fixed system. With notations from Fig. A2.1 this can be successively written as follows

Relation $\underline{X} = f(\underline{x})$

$$\underline{x} = \underline{X} - \underline{R}_o \qquad \text{...(A2.1)}$$

Here $\underline{R}_o$ is the position vector of the origin of the displaced coordinate system. $\underline{x}$ and $\underline{X}$ are as defined in Fig. A2.1.

Relation $\underline{x} = f(\underline{x}')$

$$\underline{x}' = \underline{T}\ (\underline{\theta}_o)\ .\ \underline{x} \qquad \text{...(A2.2)}$$

Here θ_o is Euler angles, which are the angles between the two systems. $\underline{T}$ is the so called transformation matrix, which is a 3 x 3 matrix containing the nine direction cosines.

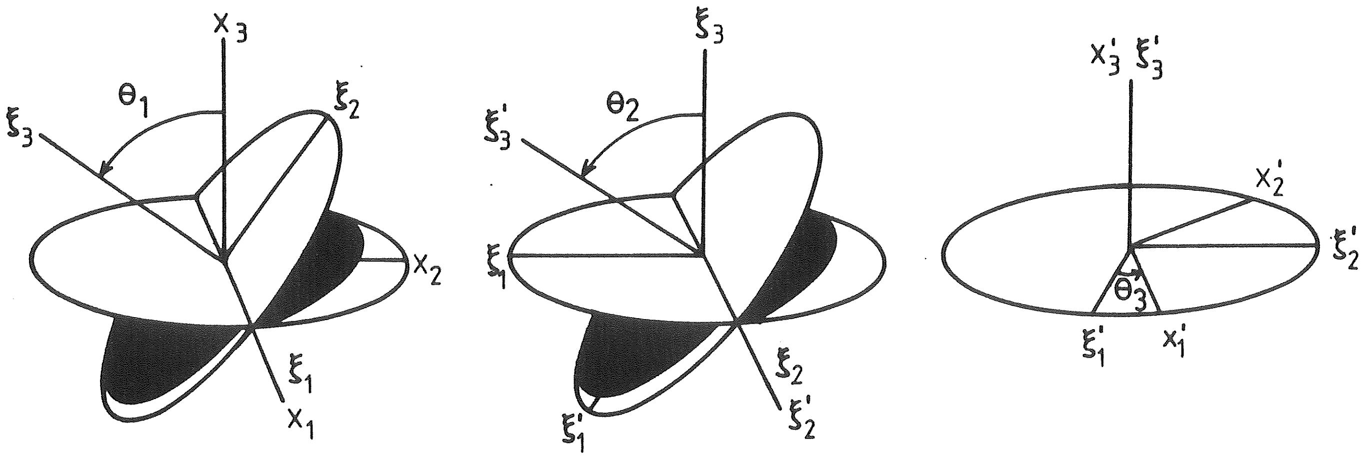


Fig. A2.2 Consequtive rotations about
(A) x_1-axis, angle θ_1
(B) ξ_2-axis, angle θ_2
(C) ξ'_3-axis, angle θ_3

There are twelve different conventions possible when choosing the rotation angles (see Goldstein (1980) App.B.). In this text the xyz-convention will be used which implies that each rotation is about different labelled axis. Furthermore the 1-2-3 sequence of rotation is used. This means that the first rotation is the roll angle θ_1 about the x-axis (x_1-axis), the second is the pitch angle θ_2 about the intermediary y-axis (ξ_2-axis), and the third is the yaw angle θ_3 about the final z-axis (x_3'-axis or ξ_3'-axis), see Fig. A2.2.

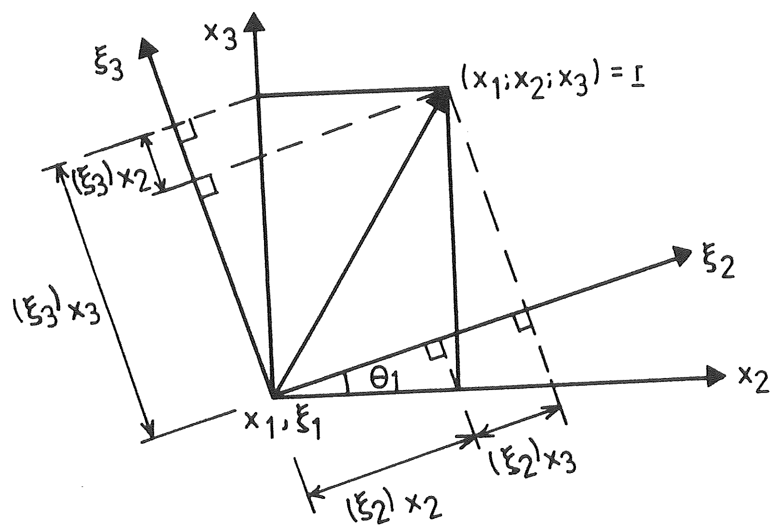


Fig. A2.3 Frame relations of first rotation θ_1 (roll)

From Fig. A2.3 we conclude that the relation between the systems of coordinates $\underline{x}$ and $\underline{\xi}$ is as follows:

$$\begin{aligned} \xi_1 &= x_1 \\ \xi_2 &= x_2 \cos\theta_1 + x_3 \sin\theta_1 \\ \xi_3 &= -x_2 \sin\theta_1 + x_3 \cos\theta_1 \end{aligned} \qquad \text{(A2.3)}$$

which can be written in matrix form as

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \text{(A2.4)}$$

or more compact as

$$\underline{\xi} = \underline{\underline{A}}(\theta_1)\ \underline{x} \tag{A2.5}$$

In a similar fashion as above we can write the relations due to the rotations θ_2 and θ_3

$$\underline{\xi}' = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \underline{\xi} = \underline{\underline{B}}(\theta_2)\ \xi \tag{A2.6}$$

and

$$\underline{x}' = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{\xi}' = \underline{\underline{C}}(\theta_3)\ \underline{\xi}' \tag{A2.7}$$

Combining Eqs. (A2.5) to (A2.7) we can transform the coordinate $\underline{x}$ to $\underline{x}'$ as follows

$$\underline{x}' = \underline{\underline{C}}(\theta_3)\underline{\xi}' = \underline{\underline{C}}(\theta_3)\ \underline{\underline{B}}(\theta_2)\ \underline{\xi} =$$

$$= \underline{\underline{C}}(\theta_3)\ \underline{\underline{B}}(\theta_2)\ \underline{\underline{A}}(\theta_1)\ \underline{x} \tag{A2.8}$$

The transformation matrix is

$$\underline{\underline{T}}(\underline{\theta}) = \underline{\underline{C}}(\theta_3)\ \underline{\underline{B}}(\theta_2)\ \underline{\underline{A}}(\theta_1) \tag{A2.9}$$

which when multiplying gives

$$\underline{\underline{T}}(\theta) = \begin{bmatrix} c\theta_2 c\theta_3 & c\theta_1 s\theta_3 + s\theta_1 s\theta_2 c\theta_3 & s\theta_1 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 \\ -c\theta_2 s\theta_3 & c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3 & s\theta_1 c\theta_3 + c\theta_1 s\theta_2 s\theta_3 \\ s\theta_2 & -s\theta_1\ c\theta_2 & c\theta_1 c\theta_2 \end{bmatrix} \tag{A2.10}$$

with the abbreviations c = cos and s = sin. Now it is quite easy to establish the inverted transformations. That is, from $\underline{x}'$ to $\underline{x}$.

$$\underline{x} = \underline{\underline{T}}^{-1} \, \underline{x}' \tag{A2.11}$$

At this stage it should be quite proper to use the orthogonal properties of the transformation, without missing anything in the analysis. The property to be used is that the inverted matrix $\underline{\underline{T}}^{-1}$ is equal to the transposed matrix $\underline{\underline{T}}^{t}$. (This is shown in the next Chapter A2.1.2).

$$\underline{\underline{T}} \, \underline{\underline{T}}^{-1} = \underline{\underline{T}} \, \underline{\underline{T}}^{t} = \underline{1} \quad => \underline{\underline{T}}^{-1} = \underline{\underline{T}}^{t} \tag{A2.12}$$

Which of course also can be derived similarly as in eqs. (A2.5) to (A2.9)

$$\begin{aligned} \underline{\underline{T}}^{-1} &= \underline{\underline{A}}^{-1}(\theta_1) \, \underline{\underline{B}}^{-1}(\theta_2) \, \underline{\underline{C}}^{-1}(\theta_3) = \\ &= \underline{\underline{A}}^{t} \, \underline{\underline{B}}^{t} \, \underline{\underline{C}}^{t} = (\underline{\underline{A}} \, \underline{\underline{B}} \, \underline{\underline{C}})^{t} = \underline{\underline{T}}^{t} \end{aligned} \tag{A2.13}$$

The transposed (or inverted) transformation matrix becomes,

$$\underline{\underline{T}}^{t} = \begin{bmatrix} c\theta_2 c\theta_3 & -c\theta_2 s\theta_3 & s\theta_2 \\ c\theta_1 s\theta_3 + s\theta_1 s\theta_2 c\theta_3 & c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3 & -s\theta_1 c\theta_2 \\ s\theta_1 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 & s\theta_1 c\theta_3 + c\theta_1 s\theta_2 s\theta_3 & c\theta_1 c\theta_2 \end{bmatrix} \tag{A2.14}$$

With the same abbreviations as in eq. (A2.10).

This gives the inverted transformation using eq. (A2.11) and eq. (A2.13)

$$\underline{x} = \underline{\underline{T}}^{t} \, \underline{x}' \tag{A2.15}$$

A2.1.2 Formal properties of the transformation matrix

The transformation matrix is a very useful mathematical tool in rigid body mechanics. It is therefore of great interest to know a little about its properties, such as its inverse etc.

Inverse

Of great importance is the transformation inverse to $\underline{\underline{T}}$, called $\underline{\underline{T}}^{-1}$. The first property is obvious as

$$\underline{\underline{T}}\ \underline{\underline{T}}^{-1} = \underline{\underline{1}} \tag{A2.16}$$

which indicates why it is called the inverse matrix.

The transformation matrix and its inverse are commutative

$$\underline{\underline{T}}\ \underline{\underline{T}}^{-1} = \underline{\underline{T}}^{-1}\ \underline{\underline{T}} \tag{A2.17}$$

In general the inverse matrix is the Wronskij determinant, $W(\underline{\underline{T}})$, divided by the determinant of the matrix.

$$\underline{\underline{T}}^{-1} = |\underline{\underline{T}}|^{-1}\ W(\underline{\underline{T}}) \tag{A2.18}$$

Further, knowing that for a righthanded orthogonal matrix the determinant is +1 and the Wronskijan determinant is the transpose matrix,

$$|\underline{\underline{T}}| = 1; \quad W(T) = \underline{\underline{T}}^{t} \tag{A2.19}$$

and as this is what we are dealing with. Combining eq. (A2.18) and (A2.19) gives the useful relation

$$\underline{\underline{T}}^{-1} = \underline{\underline{T}}^{t} \tag{A2.20}$$

which is used in the previous chapter A2.1.1.

The product of the transformation matrix and a vector matrix is also commutative.

$$\underline{\underline{T}}\ \underline{A} = \underline{A}^{t}\ \underline{\underline{T}}^{t} \tag{A2.21}$$

Symmetry and antisymmetry

A square matrix that is the same as its transpose

$$B_{ij} = B_{ji}$$

is said to by symmetric. When the transpose is the negative of the matrix

$$B_{ij} = -B_{ji}$$

The matrix is antisymmetric or skew symmetric. It is obvious that in an antisymmetric matrix, the diagonal elements are always zero. For any square matrix $\underline{\underline{B}}$, the matrix $\underline{\underline{B}}_s$ defined as

$$\underline{\underline{B}}_s = 1/2 \; (\underline{\underline{B}} + \underline{\underline{B}}^t) \tag{A2.22}$$

is symmetric, and a corresponding antisymmetric one can be defined as

$$\underline{\underline{B}}_a = 1/2 \; (\underline{\underline{B}} - \underline{\underline{B}}^t) \tag{A2.23}$$

It obviously follows that

$$\underline{\underline{B}} = \underline{\underline{B}}_s + \underline{\underline{B}}_a \tag{A2.24}$$

and

$$\underline{\underline{B}}^t = \underline{\underline{B}}_s - \underline{\underline{B}}_a \tag{A2.25}$$

With the transformation matrix and its inverse derived in eq. (A2.10) and (A2.14) the symmetric transformation matrix is

$$\underline{\underline{T}}_s = \frac{1}{2} \begin{bmatrix} 2c\theta_2 c\theta_3 & (c\theta_1 - c\theta_2)s\theta_3 + s\theta_1 s\theta_2 c\theta_3 & s\theta_1 s\theta_3 + (1 - c\theta_1 c\theta_3)s\theta_2 \\ & 2(c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3) & s\theta_1(c\theta_3 - c\theta_2) + c\theta_1 s\theta_2 s\theta_3 \\ \text{SYM.} & & 2\,c\theta_1\,c\theta_2 \end{bmatrix} \tag{A2.26}$$

and the antisymmetric transformation matrix is

$$\underline{\underline{T}}_a = \frac{1}{2}\begin{bmatrix} 0 & (c\theta_1+c\theta_2)s\theta_3+ +s\theta_1 s\theta_2 c\theta_3 & s\theta_1 s\theta_3-(1+c\theta_1 c\theta_3)s\theta_2 \\ & 0 & s\theta_1(c\theta_3+c\theta_2)+ +c\theta_1 s\theta_2 s\theta_3 \\ \text{ANTISYM.} & & 0 \end{bmatrix} \qquad \text{(A2.27)}$$

which both are very complex, but if assuming small rotations ($c\theta= 1$, $s\theta=\theta$ and $\theta\theta=0$) they will become much simpler.

$$\overset{\Delta}{\underline{\underline{T}}}_s = \begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ \text{S.} & & 1 \end{bmatrix} = \underline{\underline{1}} \qquad \text{(A2.28)}$$

$$\overset{\Delta}{\underline{\underline{T}}}_a = \begin{bmatrix} 0 & \theta_3 & -\theta_2 \\ & 0 & \theta_1 \\ \text{A.S.} & & 0 \end{bmatrix} \qquad \text{(A2.29)}$$

which can be combined to form their corresponding transformation matrix and its inverse, using eq. (A2.24) and (A2.25).

Transformation of operators

The two interpretations of an operator as transforming the vector, or alternatively the coordinate system, are both involved if we seek to find the transformation of an operator. Under a change of coordinates let $\underline{\underline{A}}$ be considered an operator acting upon a vector $\underline{F}$ to produce a vector $\underline{G}$.

$$\underline{G} = \underline{\underline{A}}\ \underline{F} \qquad \text{(A2.30)}$$

The components of the vector $\underline{G}$ in the new system will be given by

$$\underline{\underline{T}}\ \underline{G} = \underline{\underline{T}}\ \underline{\underline{A}}\ \underline{F} \qquad \text{(A2.31)}$$

If the vector $\underline{F}$ is to be in the new system, it is convenient to rewrite eq. (A2.31) as

$$(\underline{\underline{T}}\ \underline{G}) = (\underline{\underline{T}}\ \underline{\underline{A}}\ \underline{\underline{T}}^{-1})(\underline{\underline{T}}\ \underline{F}) \qquad \text{(A2.32)}$$

The expressions enclosed in the brackets are the corresponding elements from eq. (A2.30) in the new coordinate system. If the new system is denoted with a prime eq. (A2.32) becomes

$$\underline{G}' = \underline{\underline{A}}'\ \underline{F}' \qquad \text{(A2.33)}$$

where the operator A' is

$$\underline{\underline{A}}' = \underline{\underline{T}}\ \underline{\underline{A}}\ \underline{\underline{T}}^{-1} \qquad \text{(A2.34)}$$

in the new set of axes. Any transformation of this kind is known as a similarity transformation.

One important property of the operator transformation, and of course of all coordinate transformations, is that the transformed vector or matrix must have the same "magnitude" as the vector or matrix operated upon.

$$|\underline{\underline{A}}|' = |\underline{\underline{A}}| \qquad \text{(A2.35)}$$

This is always the case when using the transformation described in chapter A2.1.1.

A2.2 Rate of change of a vector

We will now discuss the rate of change of some arbitrary vector $\underline{G}$ (as done by Goldstein (1980)). The vector can represent a position vector in a body or the total angular momentum. Clearly the position vector appears constant in the body frame. However, to an observer fixed in space the vector will move.

The vector $\underline{G}$ is defined by its magnitude $G = |\underline{G}|$ and direction. Clearly the rate of change in time of the vector should be expressed in these two quantities as

$$\frac{d\underline{G}}{dt} = \left(\frac{\partial \underline{G}}{\partial t}\right)_{magn.} + \left(\frac{\partial \underline{G}}{\partial t}\right)_{dir} \qquad \text{(A2.36)}$$

the sum of the time rate of change in magnitude and direction. The rate of change in time of the direction can be expressed using the instantaneous angular velocity, ω, as

$$\left(\frac{\partial \underline{G}}{\partial t}\right)_{dir} = \underline{\omega} \times \underline{G} \qquad \text{(A2.37)}$$

which leads to the well known expression for the rate of change in time

$$\frac{d\underline{G}}{dt} = \frac{\underline{G}}{G}\frac{\partial G}{\partial t} + \underline{\omega} \times \underline{G} \qquad \text{(A2.38)}$$

But as stated in the beginning of this chapter, the rate of change is dependent on its reference frame. Obviously, there are three frames that are of special interest. They are:

- Inertial (space fixed) frame
- Rotated (space fixed but changing) frame
- Rotating (body fixed) frame

The first and second frames of reference are fixed, and as such the rate of change in time of a vector $\underline{G}$, in them, will be expressed as in eq. (A2.38). In the third frame the rotations are solidly connected with the body. The only change will then be due to change in magnitude.

To clarify all these changing rates, the following subchapters will be spent on how to derive eq. (A2.38) in the different frames.

A2.2.1 The infinitesimal rotation transform

The existence of a rotation vector, i.e. a vector that is uniquely represented by three axis of rotation, is only possible if the rotations are infinitely small. Otherwise the order of the rotations are important. This is due to the fact that the order of rotations are material. The rotation will then not satisfy vector properties such as, for instance, the addition rule of vectors

$$\underline{A} + \underline{B} = \underline{B} + \underline{A}$$

namely the addition being a cumutative process.

Fortunately, the rate of change of a vector can be satisfied by infinitesimal rotations without putting any restrictions on the motion. This can easily be obtained by using small enough time steps.

Before starting discussing the rate of change of vectors, we shall recall the orthogonal transformations from chapter A2.1. In that chapter a vector was transformed from one frame to another.

The transformation can also be thought of as an operator that rotates the vector and expresses the resulting vector in the same frame. Then the rotation of the vector will be opposite to the equivalent rotation of a frame. This is shown in Fig. A2.4 below.

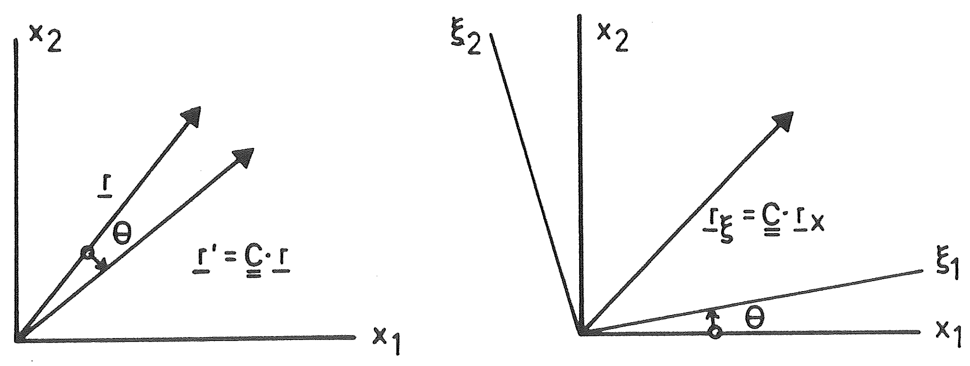


Fig. A2.4 The effect of transforming the 2-D vector $\underline{r}$; firstly relative the same frame, secondly from one frame to another.

In fig. A2.4 the transformation matrix $\underline{\underline{C}}$ is the one derived in eq. (A2.7) in chapter A2.1.

Using the rotational effect of the transformation it is quite easy to derive the rate of change of the vector in direction. Furthermore infinitesimal rotations will make the rates of change in direction and magnitude orthogonal, which simplifies a lot.

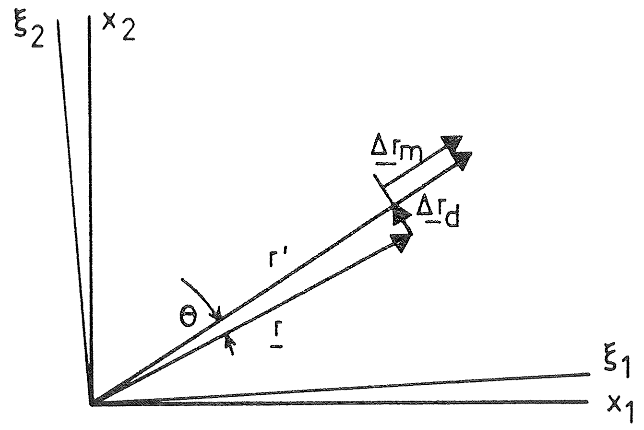


Fig. A2.5 Rate of change of a vector, $\underline{r}$.

With the use of the rotations in fig. A2.4, the rate of change $\Delta\underline{r}$ can be expressed as

$$\Delta\underline{r} = \underline{r}' - \underline{r} = \Delta\underline{r}_m + \Delta\underline{r}_d \qquad \text{(A2.39)}$$

This difference $\Delta\underline{r}$ is of course independent of the frame, but it is only quantified in a certain frame. If the rotated frame (ξ) is choosen and the vectors are fixed in direction in their respective frames that is $\underline{r}$ in x-frame and $\underline{r}'$ in ξ-frame. Then it is convenient to express $\underline{r}$ in the rotated frame,

$$\underline{r}_\xi = \underline{\underline{T}}\ \underline{r}_x \qquad \text{(A2.40)}$$

as done previously in eq. (A2.8), where $\underline{\underline{T}}$ is the 3-D transformation matrix. The rate of change due to change of magnitude is the easiest to express. Because if the vector's direction is fixed in its frame (as stated above), then the change is just the difference between them in their respective frames.

$$(\Delta \underline{r}_m)_\xi = (\Delta \underline{r}_m)_x = \underline{r}_\xi - \underline{r}'_x \tag{A2.41}$$

which expresses the independency of the rotation.

The rate of change due to rotation is only experienced if seen from one of the frames. Using Eqs. (A2.40) and (A2.41) the rate of change due to the rotation is (in ξ-frame)

$$(\Delta \underline{r}_d)_\xi = (\underline{r}' - \Delta \underline{r}_m - \underline{r})_\xi \tag{A2.42}$$

Now, knowing that

$$\underline{r}_x = (\underline{r}' - \Delta \underline{r}_m)_\xi \tag{A2.43}$$

and eq. (A2.40) gives

$$(\Delta r_d)_\xi = \underline{r}_x - (\underline{\underline{T}}\underline{r}_x)_\xi \tag{A2.44}$$

This rate can of course also be expressed in the x-frame as

$$(\Delta r_d)_x = (\underline{\underline{T}}^t (\underline{r}' - \Delta \underline{r}_m)_\xi)_x - \underline{r}_x \tag{A2.45}$$

and using eq. (A2.43)

$$(\Delta r_d)_x = (\underline{\underline{T}}^t \underline{r}_x)_x - \underline{r}_x \tag{A2.46}$$

It is easy to prove that rates satisfy the wanted change in the vector by deriving vector $\underline{r}'$ in the following way

$$\underline{r}' = \underline{r} + \Delta \underline{r}_m + \Delta \underline{r}_d \tag{A2.47}$$

which in the ξ-frame is

$$\underline{r}'_\xi = (\underline{\underline{T}}\,\underline{r}_x)_\xi + \underline{r}'_\xi - \underline{r}_x + \underline{r}_x - (\underline{\underline{T}}\,\underline{r}_x)_\xi = \underline{r}'_\xi \tag{A2.48}$$

This far the properties of the infinitesimal transformation matrix $\underline{\underline{T}}_{\Delta}$ has not been clarified. But in chapter A2.1.2 there are expressions with which it is possible to derive both $\underline{\underline{T}}_{\Delta}$ and its transpose $\underline{\underline{T}}_{\Delta}^{t}$. The infinitesimal transformation matrix is dervied by inserting eqs. (A2.28) and (A2.29) into eq. (A2.24) which gives,

$$\underline{\underline{T}}_{\Delta} = \begin{bmatrix} 1 & \Delta\theta_3 & -\Delta\theta_2 \\ -\Delta\theta_3 & 1 & \Delta\theta_1 \\ \Delta\theta_2 & -\Delta\theta_1 & 1 \end{bmatrix} \qquad \text{(A2.49)}$$

and its transpose by inserting eqs. (A2.28) and (A2.29) into eq. (A2.25)

$$\underline{\underline{T}}_{\Delta}^{t} = \begin{bmatrix} 1 & -\Delta\theta_3 & \Delta\theta_2 \\ \Delta\theta_3 & 1 & -\Delta\theta_1 \\ -\Delta\theta_2 & \Delta\theta_1 & 1 \end{bmatrix} \qquad \text{(A2.50)}$$

In both eqs. (A2.49) and (A2.50) second and higher order terms are neglected.

With the use of $\underline{\underline{T}}_{\Delta}$ as in eq. (A2.49), it is possible rewrite the expression for the rate of change in rotation, eq. (A2.44), as

$$(\Delta r_d)_{\xi} = \underline{r}_x - (\underline{\underline{1}} \cdot \underline{r}_x - \Delta\underline{\theta} \times \underline{r}_x)_{\xi} = (\Delta\underline{\theta} \times \underline{r}_x)_{\xi} \qquad \text{(A2.51)}$$

the vector product of the infinite rotation vector and the position vector. In the x-frame the expression is derived similarily, but using eq. (A2.50) for the transpose transformation.

$$(\Delta r_d)_x = (\underline{\underline{1}} \cdot \underline{r}_x + \Delta\underline{\theta} \times \underline{r}_x) - \underline{r}_x = (\Delta\underline{\theta} \times \underline{r}_x)_x \qquad \text{(A2.52)}$$

Not surprisingly the results are identical, regardless of frame.

But, as earlier stated in chapter A2.1.1 a transformed vector only changes in direction and not in magnitude. This will give a second order error in the magnitude, while the magnitude of the transform is

$$|\underline{\underline{T}}_{\Delta}| = \begin{vmatrix} 1 & \Delta\theta_3 & -\Delta\theta_2 \\ -\Delta\theta_3 & 1 & \Delta\theta_1 \\ \Delta\theta_2 & -\Delta\theta_1 & 1 \end{vmatrix} =$$

$$= 1 + (\Delta\theta_1)^2 + (\Delta\theta_2)^2 + (\Delta\theta_3)^3 \qquad \text{(A2.53)}$$

However, previously made approximations are of the same order, why it is proper also to neglect these terms.

A2.2.2 Rate of change in time

The rates of change in time will be experienced (and expressed) differently in a fixed frame than in a rotating frame. In the following text two types of frames will be used, those fixed (but of course they can be displaced) in space and those rotating with the vector ("body"). To distinguish them, the vector will be denoted with a prime, ', when related to the rotating frame.

Vector notation

$\underline{r}$ is in a fixed frame, and

$\underline{r}'$ is in a rotating frame

For comparison, we will derive the first and second time derivatives for both a vector that is constant in the rotating frame and a vector varying in the rotating frame. But first of all we will derive the general expression for the time derivative.

To obtain the time derivative we first recall from the previous chapter A2.2.1, that the incremental change of the vector $\underline{r}$ can be separated into two perpendicular components. One of the components is in the direction of $\underline{r}$ and is called $\Delta\underline{r}_m$, and the other is perpendicular to $\underline{r}$ and is called $\Delta\underline{r}_d$. Obviously $\Delta\underline{r}_m$ is the change in magnitude and $\Delta\underline{r}_d$ is the change in direction. The sum of these changes is the infinitesimal change of the vector. Recalling eq. (A2.47) it can be expressed as

$$\Delta\underline{r} = \Delta\underline{r}_m + \Delta\underline{r}_d \qquad \text{(A2.54)}$$

This can be rewritten using eq. (A2.51), and using that $\underline{r}$ and $\underline{r}'$ represent the same vector (see also fig. A2.5)

$$\Delta\underline{r} = \Delta\underline{r}' + \Delta\underline{\theta} \times \underline{r} = \Delta\underline{r}' + \Delta\underline{\theta} \times \underline{r}' \qquad \text{(A2.55)}$$

Dividing the increment in space, $\Delta\underline{r}$, with its associated increment in time, Δt, and letting them approach zero will give the time derivative of $\underline{r}$.

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\underline{r}}{\Delta t} = \lim_{\Delta t \to 0} \left(\frac{\Delta\underline{r}'}{\Delta t} + \frac{\Delta\underline{\theta}}{\Delta t} \times \underline{r} \right) = \dot{\underline{r}}' + \underline{\omega} \times \underline{r} \qquad \text{(A2.56)}$$

Clearly the time derivative is the sum of a radial component $\dot{\underline{r}}'$, (which is the only time derivative experienced in the rotating frame, i.e. the only "motion" relative the rotating frame), and a tangential component $\underline{\omega} \times \underline{r}$. But, as a consequence of the approximations used when deriving the incremental change in space, eq. (A2.56) only is a first order approximation of the time derivative. If the time derivatives has a high absolute value another expression should be used.

The time derivative equations (A2.56) can be expressed in a more general way as an operator equation acting upon any vector.

$$\left(\frac{d}{dt} \right)_{fixed} = \left(\frac{d}{dt} \right)_{rotating} + \underline{\omega} \times \qquad \text{(A2.57)}$$

The subscripts indicates that the time derivatives are to be observed in the fixed (inertial) and rotating (body fixed) systems of axes, respectively. But, it must be emphasized that the time rate of change is only given relative to the specified coordinate system, and components of the rate may be taken along another set of axes only after the differentiation has been carried out.

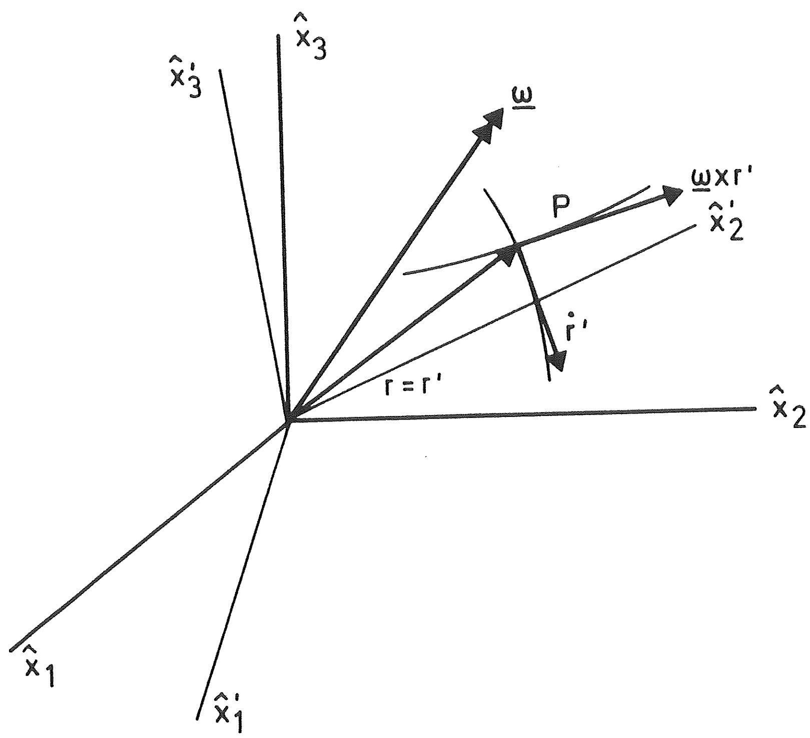


Fig. A2.6 A fixed frame (x) and a frame rotating (x') with the velocity $\underline{\omega}$.

This far an operator equation for time derivatives has been derived. Using this eq. (A2.57) it is easy to derive the expressions for; at first a vector constant in the rotating frame and secondly for a vector changing in the rotating frame.

Refer to Fig. A2.6 and denote vector OP with $\underline{r}$ expressed in frame $\underline{x}$, and $\underline{r}'$ expressed in frame $\underline{x}'$, see eq. (A2.55). They represent the same vector why

$$\underline{r}_x = \underline{r}'_x = \underline{\underline{K}}\,\underline{r}'_\xi \tag{A2.58}$$

This means that expressing them in the same frame gives identical vectors but in different frames they only will have the same absolute value. Note also that matrix $\underline{\underline{K}}$ is dependent on frame, see also chapter A2.2.4.

Expressing the motion in the rotating system, and differentiating in time gives the changing vector's (eq. (A2.56)) first time derivative.

$$\underline{\dot{r}} = \underline{\dot{r}}' + \underline{\omega} \times \underline{r} \qquad \text{(A2.59a)}$$

or using eq. (A2.58),

$$\underline{\dot{r}} = \underline{\dot{r}}' + \underline{\omega} \times \underline{r}' \qquad \text{(A2.59b)}$$

and the constant (in magnitude) vector

$$\dot{r} = \underline{\omega} \times \underline{r} = \underline{\omega} \times \underline{r}' \qquad \text{(A2.60)}$$

The second time derivative of $\underline{r}$ is for the changing vector

$$\underline{\ddot{r}} = \frac{d}{dt}(\underline{\dot{r}}') + \frac{d}{dt}(\underline{\omega}) \times \underline{r} + \underline{\omega} \times \frac{d}{dt}(\underline{r}) =$$
$$= \underline{\ddot{r}}' + \underline{\omega} \times \underline{\dot{r}}' + \underline{\dot{\omega}} \times \underline{r} + (\underline{\omega} \times \underline{\omega}) \times \underline{r} +$$
$$+ \underline{\omega} \times \underline{\dot{r}}' + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \qquad \text{(A2.61)}$$

Using that

$$\underline{\omega} \times \underline{\omega} = 0 \qquad \text{(A2.62)}$$

gives the obvious result that

$$\underline{\dot{\omega}} = \underline{\dot{\omega}} + \underline{\omega} \times \underline{\omega} = \underline{\dot{\omega}} \qquad \text{(A2.63)}$$

Inserting these two relations in eq. (A2.61) gives the second derivative as

$$\underline{\ddot{r}} = \underline{\ddot{r}}' + 2\underline{\omega} \times \underline{\dot{r}}' + \underline{\dot{\omega}} \times r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) =$$

$$= \underline{\ddot{r}}' + 2\underline{\omega} \times \underline{\dot{r}}' + \underline{\dot{\omega}} \times \underline{r}' + \underline{\omega} \times (\underline{\omega} \times \underline{r}') \qquad \text{(A2.64)}$$

The vector that is constant in the rotating frame will have the expression

$$\underline{\ddot{r}} = \frac{d}{dt}(\underline{\omega}) \times \underline{r} + \underline{\omega} \times \frac{d}{dt}(\underline{r}) =$$

$$= \underline{\dot{\omega}}' \times \underline{r} + (\underline{\omega} \times \underline{\omega}) \times \underline{r} + \underline{\omega} \times \dot{r}' + \underline{\omega} \times (\underline{\omega} \times \underline{r}) =$$

$$= \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\dot{\omega}} \times \underline{r}' + \underline{\omega} \times (\underline{\omega} \times \underline{r}') \qquad \text{(A2.65)}$$

As is seen in eq.(A2.65) in a position,p, fixed in the rotating frame one will only experience normal (radial) and tangential accelerations. When using these equations one should be aware of what ω is and how it is derived, see next chapter A2.2.3.

In terms of accelerations,

$\underline{\ddot{r}}$	is the acceleration of p in relation to the fixed space
$\underline{\ddot{r}}'$	is the acceleration of p in relation the rotating frame
$2\ \underline{\omega} \times \underline{\dot{r}}'$	is the Coriolis acceleration
$\underline{\dot{\omega}} \times \underline{r}'$	is the tangential acceleration
$\underline{\omega} \times (\underline{\omega} \times \underline{r}')$	is the centripetal or normal acceleration

This far the origins of the system has been thought of as coinciding , if this is not the case there will be some more terms to regard. If the origin, O, of the frame translates with velocity $\underline{v}_o$ and acceleration $\underline{a}_o$, relative the inertial space, the absolute motion of point P are

$$\underline{v} = \underline{v}_o + \underline{\dot{r}} \qquad \text{(A2.66)}$$

and

$$\underline{a} = \underline{a}_o + \underline{\ddot{r}} \qquad \text{(A2.67)}$$

respectively, with $\underline{\dot{r}}$ from eq. (A2.59) or (A2.60) and $\underline{\ddot{r}}$ from eq. (A2.64) or (A2.65).

A2.2.3 Vector transformation of the time rates of rotation

If the rotations and their time rates are to be expressed in terms of the Euler angles and their time rates, then one must be aware that the Euler angles are not perpendicular. But, instead they are directed along successively changing axes. We will now see how this affects the angular velocity.

This far we have expressed the angular velocity along its instantaneous axis of rotation, that is along the rotated (body fixed) frame. It has the components

$$\underline{\omega} = (\omega_1, \omega_2, \omega_3) \tag{A2.68}$$

as in eq. (A2.56)

Along the inertial (space fixed) frame it will have the components

$$\underline{\omega}^I = \underline{\underline{T}}^t \underline{\omega} \tag{A2.69}$$

and here $\underline{\underline{T}}^t$ is the transposed transformation matrix from eq. (A2.14).

From now on it is necessary to distinguish the Euler angle derivatives. Its components will here be denoted by

$$\underline{\dot{\theta}}^E = (\dot{\theta}_1^E, \dot{\theta}_2^E, \dot{\theta}_3^E) \tag{A2.70}$$

Where $\dot{\theta}_1^E$ is along the space x-axis,

$\dot{\theta}_2^E$ is along the instantaneous η-axis,

$\dot{\theta}_3^E$ is along the ζ'-axis,

with axis notation as in fig. A2.7. Their associated Euler angles are denoted by

$$\underline{\theta} = (\theta_1, \theta_2, \theta_3) \tag{A2.71}$$

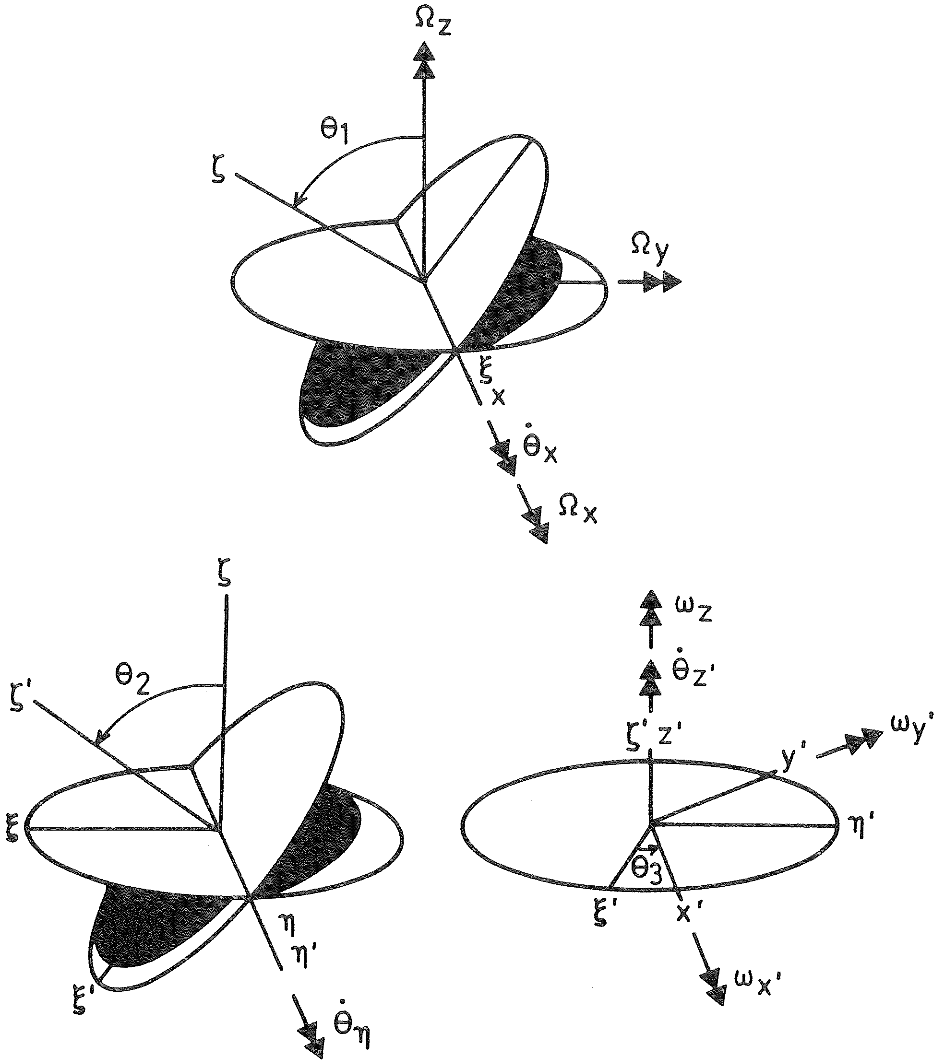


Fig. A2.7 Euler angle derivatives and angle velocities in their respective frames

The angular velocities along the body axes can be expressed using the Euler angle derivatives and the transformation matrices $\underline{A}$, $\underline{B}$ and $\underline{C}$. Since $\dot{\theta}_1$ is parallell to the space x-axis, and also the ξ-axis, its components are transformed using the transform (compare eq. (A2.9)).

$$\underline{T}_1 = (\underline{\underline{C}} \;\; \underline{\underline{B}} \;\; \underline{\underline{A}})_{x1} = (\underline{\underline{C}} \;\; \underline{\underline{B}})_{x1} \qquad \text{(A2.72)}$$

Where the index x1 means the first column vector of $\underline{\underline{C}}\underline{\underline{B}}\underline{\underline{A}}$ or $\underline{\underline{C}}\underline{\underline{B}}$.

The second rotation velocity $\dot{\theta}_2$ is parallell to the η and η'-axis, and consequently transformed by

$$\underline{T}_2 = (\underline{\underline{C}} \;\; \underline{\underline{B}})_{x2} = (\underline{\underline{C}})_{x2} \qquad \text{(A2.73)}$$

where the index x2 means the second column vector of $\underline{\underline{C}}\underline{\underline{B}}$ or $\underline{\underline{C}}$.

Finally the third component Θ_3 is parallell to the ζ' and z'-axis and transformed by

$$\underline{T}_3 = (\underline{\underline{C}})_{x3} = (0,\ 0,\ 1)^t \tag{A2.74}$$

where the index x3 means the third column vector of $\underline{\underline{C}}$.
Adding those separate components will give the angular velocities along the body set of axes as

$$\underline{\omega}_{x'} = \underline{\underline{R}}'\ \underline{\dot{\Theta}}^E = \begin{bmatrix} (C\ B\ A)_{11} & (C\ B)_{12} & (C)_{13} \\ (C\ B\ A)_{21} & (C\ B)_{22} & (C)_{23} \\ (C\ B\ A)_{31} & (C\ B)_{32} & (C)_{33} \end{bmatrix} \begin{bmatrix} \dot{\Theta}_x \\ \dot{\Theta}_\eta \\ \dot{\Theta}_{\zeta'} \end{bmatrix} =$$

$$= \begin{bmatrix} (C\ B)_{11} & (C)_{12} & 0 \\ (C\ B)_{21} & (C)_{22} & 0 \\ (C\ B)_{31} & (C)_{32} & 1 \end{bmatrix} \begin{bmatrix} \dot{\Theta}_\xi \\ \dot{\Theta}_{\eta'} \\ \dot{\Theta}_{z'} \end{bmatrix} \tag{A2.75}$$

or written out using the abbreviations $c\Theta = \cos\Theta$ and $s\Theta = \sin\Theta$.

$$\underline{\omega}_{x'} = \begin{bmatrix} c\Theta_2 c\Theta_3 & s\Theta_3 & 0 \\ -c\Theta_2 s\Theta_3 & c\Theta_3 & 0 \\ s\Theta_2 & 0 & 1 \end{bmatrix} \underline{\dot{\Theta}}^E \tag{A2.76}$$

The angular velocities expressed in the inertial frame in terms of Euler angle derivatives, $\dot{\Theta}^E$, is, inserting eq. (A2.76) into eq. (A2.69), with abbreviations as above

$$\underline{\omega}_x = \underline{\underline{R}}\ \dot{\underline{\theta}}^E = \underline{\underline{T}}^t\ \underline{\underline{R}}'\ \dot{\underline{\theta}}^E =$$

$$= \begin{bmatrix} A^t_{11} & (AB)^t_{12} & (ABC)^t_{13} \\ A^t_{21} & (AB)^t_{22} & (ABC)^t_{23} \\ A^t_{31} & (AB)^t_{32} & (ABC)^t_{33} \end{bmatrix} \dot{\underline{\theta}}^E = \begin{bmatrix} 1 & A^t_{12} & (AB)^t_{13} \\ 0 & A^t_{22} & (AB)^t_{23} \\ 0 & A^t_{32} & (AB)^t_{33} \end{bmatrix} \dot{\underline{\theta}}^E = \begin{bmatrix} 1 & 0 & s\theta_2 \\ 0 & c\theta_1 & -s\theta_1 c\theta_2 \\ 0 & s\theta_1 & c\theta_1 C\theta_2 \end{bmatrix} \dot{\underline{\theta}}^E$$

(A2.77)

These derived transformations, $\underline{\underline{R}}$ and $\underline{\underline{R}}'$ are not orthogonal. Therefore the inverse transformation matrix is not equal to the transpose transformation matrix, and the inverse relation is

$$\dot{\underline{\theta}}^E = (\underline{\underline{R}}')^{-1} \underline{\omega}_{x'} = \frac{1}{c\theta_2} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ c\theta_2 s\theta_3 & c\theta_2 c\theta_3 & 0 \\ s\theta_2 c\theta_3 & -s\theta_2 s\theta_3 & c\theta_2 \end{bmatrix} \underline{\omega}_{x'} \qquad (A2.78)$$

Obviously this inverse matrix does not exist if

$$\cos\theta_2 = 0 \quad \Rightarrow \quad \theta_2 = 90^o + n \cdot 180^o \omega \quad n=1,\ 2.....$$

This restriction is a property of the choosen set of Euler angles. Although this will not affect anything in the kinematics of a floating body, which hopefully will have moderate values of both θ_1 and θ_2. This can, however, be of importance in other problems, and shall therefore be noticed.

In terms of inertial frame rotation, the Euler angle time derivative is

$$\dot{\underline{\theta}}^E = (\underline{\underline{R}})^{-1} \, \underline{\omega}_x = (\underline{\underline{R}}')^{-1} \, \underline{\underline{T}} \, \underline{\omega}_x =$$

$$= \frac{1}{c\theta_2} \begin{bmatrix} c\theta_2 & s\theta_1 s\theta_2 & -c\theta_1 s\theta_2 \\ 0 & c\theta_1 c\theta_2 & s\theta_1 c\theta_2 \\ 0 & s\theta_1 & c\theta_1 \end{bmatrix} \underline{\omega}_x \qquad \text{(A2.79)}$$

Of course, the inverse transformation $(\underline{\underline{R}})^{-1}$ has the same restriction as $(\underline{\underline{R}}')^{-1}$.

The above derived transformations are the same for the rotation and acceleration vectors.

A2.2.4 General expressions of rate of change in time

Bodies that rotates with high speeds will make violence on the first order approximation of the angular velocity, and therefore also on the kinematic relations derived in chapter A2.2.2. The general expression of the time derivatives is, using previously used notations (the primed-values related to the rotating frame).

The position relation is

$$\underline{r} = \underline{r}_o + \underline{\underline{T}}^t \, \underline{r}' \qquad \text{(A2.80)}$$

The velocity relation is

$$\dot{\underline{r}} = \dot{\underline{r}}_o + \dot{\underline{\underline{T}}}^t \, \underline{r}' + \underline{\underline{T}}^t \, \dot{r}' \qquad \text{(A2.81)}$$

The acceleration relation is

$$\ddot{\underline{r}} = \ddot{\underline{r}}_o + \ddot{\underline{\underline{T}}}^t \, \underline{r}' + 2\dot{\underline{\underline{T}}}^t \, \dot{r}' + \underline{\underline{T}}^t \, \ddot{r}' \qquad \text{(A2.82)}$$

The inverse relations are

Position relation

$$\underline{r}' = \underline{\underline{T}}(\underline{r} - \underline{r}_o) \tag{A2.83}$$

Velocity relation

$$\dot{\underline{r}}' = \dot{\underline{\underline{T}}}(\underline{r}-\underline{r}_o) + \underline{\underline{T}}(\dot{\underline{r}} - \dot{\underline{r}}_o) \tag{A2.84}$$

Acceleration relation

$$\ddot{r}' = \ddot{\underline{\underline{T}}}(\underline{r}-\underline{r}_o) + 2\dot{\underline{\underline{T}}}(\dot{\underline{r}}-\dot{\underline{r}}_o) + \underline{\underline{T}}(\ddot{\underline{r}}-\ddot{\underline{r}}_o) \tag{A2.85}$$

The time derivatives of the transformation matrix are quite complex, but still possible to derive.

As the previous mentioned results are the first order relations, it can be of interests to identify the first order appproximation. This is done by comparing Eqs. (A2.55), (A2.59) and (A2.64) with Eqs. (A2.80) to (A2.82).

The first order approximations are on operator form

$$\underline{\underline{T}}^{t(1)} = \Delta\Theta x \tag{A2.86}$$

$$\dot{\underline{\underline{T}}}^{t(1)} = \underline{\omega} x \tag{A2.87}$$

$$\ddot{\underline{\underline{T}}}^{t(1)} = \underline{\omega} x + \underline{\omega} x(\underline{\omega} x) \tag{A2.88}$$

A3. DYNAMICS OF A MOORED FLOATING RIGID BODY

A3.1 Equation of motion

The equation of motion, Newton´s second law, is derived in a lot of literature on fundamental mechanics, such as Goldstein (1980) and Meirovitch (1970). These equations will therefore be assumed to be known.

The force equation of motion is as follows

$$\underline{F} = \frac{d}{dt} (\underline{\underline{m}}\, \underline{\dot{r}}_c) \qquad \text{(A3.1)}$$

where

$\underline{\underline{m}}\, \underline{\dot{r}}_c$	is	linear momentum of the body,
$\underline{\underline{m}}$	is	total mass,
$\underline{\dot{r}}_c$	is	velocity of centre of mass of body and
$\underline{F}$	is	excitation force on body

Similarly, the torque equation of motion about the centre of mass G, is

$$\underline{M}_c = \frac{d}{dt} (\underline{\underline{I}}_c \underline{\omega}) \qquad \text{(A3.2)}$$

where

$\underline{\underline{I}}_c \underline{\omega}$	is angular momentum of body,
$\underline{\underline{I}}_c$	is inertia matrix, with moments and products of inertia with respect to a body set of axes with the origin at the mass center,
$\underline{\omega}$	is angular velocity of the body set of axes relative to an inertia space and
$\underline{M}_c$	is excitation moment on body

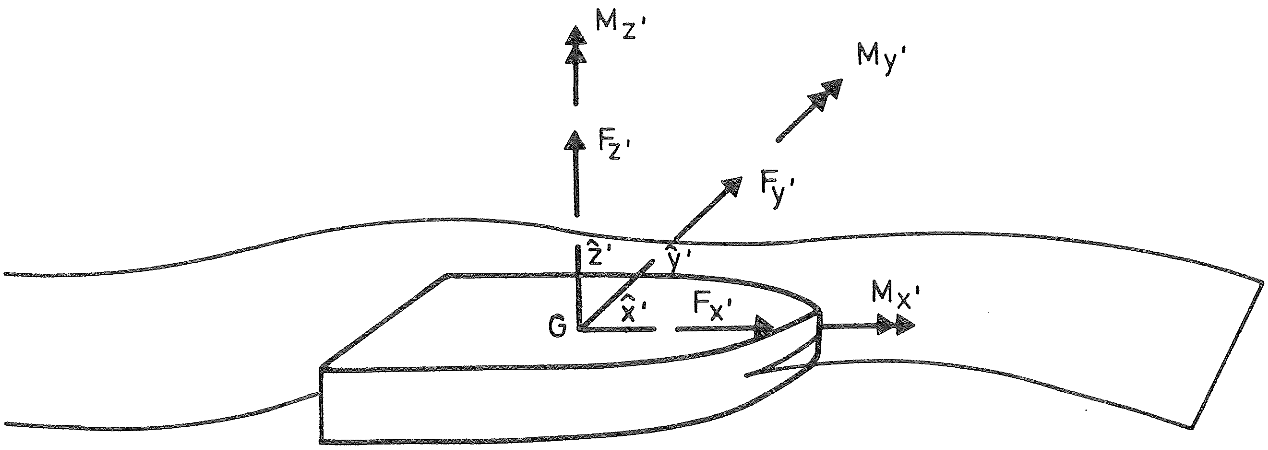


Figure A3.0 Force components acting in the centre of gravity. F_x = surge force, F_y = sway force, F_z = heave force, M_x = roll moment M_y = pitch moment and M_z = yaw moment.

In some cases it may be desirable to refer the motion of a rigid body to a system of body axes x', y', z', see Fig. 3.0. Moreover, it is convenient to place the origin of the body axes in the center of mass G.

Recalling from chapter A2.2.2, the time derivative of a vector expressed in terms of components along a moving system, Eq. (A3.1) then will assume the form

$$\underline{F} = \underline{\underline{m}}\ \ddot{\underline{r}}_c + \underline{\omega} \times (\underline{\underline{m}}\dot{\underline{r}}_c) \qquad \text{(A3.3)}$$

where, except notations from eq. (A3.1)

$\ddot{\underline{r}}_c$ is acceleration of center of mass of body

$\underline{\omega}$ is the angular velocity of the body (axes).

Eq. (A3.2) will assume the form

$$\underline{M}_C = \underline{\underline{I}}_C \dot{\underline{\omega}} + \underline{\omega} \times (\underline{\underline{I}}_C \ \underline{\omega}) \qquad \text{(A3.4)}$$

where

$\dot{\underline{\omega}}$ is angular acceleration of body axes.

The moment equation (A3.4) is independent of eq. (A3.3) unless the forces and moments are coupled. Eq. (A3.3) though depends on (A3.4) through ω.

Now we know the equation of motion, but we have not specified the exciting forces and moments. For a moored floating body these can be divided in three parts;

forces caused by the surrounding fluid (wind, wave and current),

forces caused by the moorings and

gravity forces.

A3.2 Fluid loading

The fluid forces are of two major types, namely those that vary in time and those that are constant or slowly varying in time. Forces of the first type are first order wave forces.

The slowly varying (or constant) forces are second order wave forces, wind forces and current forces. This is no strict definition but a common approach.

These fluid forces can be divided in three parts. Firstly fluid forces that are due to the disturbances in the fluid generated in the far, F_e. There are also fluid forces on a moving structure in a fluid that initially is at rest, F_S. Finally forces due to nonlinear coupling effects between the motion of the undisturbed flow and the motion of the structure, F_n. The total force on the structure can be written as a sum of the above mentioned forces, as

$$\underline{F} = \underline{F}_s + \underline{F}_e + \underline{F}_n \qquad \text{(A3.5)}$$

The force caused by the structure in the fluid is considered as a property of the structure, and consequently is called the hydrodynamic properties of the body.

A3.2.1 Hydrodynamic properties

For the evaluation of hydrodynamic properties one usually limits the characteristics of the motion of the structure to three quantities only

$$\underline{\underline{F}} = \underline{F}_s \; (\underline{r}_c, \dot{\underline{r}}_c, \ddot{\underline{r}}_c) \qquad \text{(A3.6)}$$

where

$\underline{r}_c$ is position of the structure
$\dot{\underline{r}}_c$ is velocity of the structure
$\ddot{\underline{r}}_c$ is acceleration of the structure

In a linearized form equation (A3.5) is often described by

$$\underline{F}_s(\underline{r}_c) = -\underline{\underline{a}}\ddot{\underline{r}}_c - \underline{\underline{b}}\dot{\underline{r}}_c - \underline{\underline{c}}\underline{r}_c \qquad \text{(A3.7)}$$

where

$\underline{\underline{a}}$ is added mass
$\underline{\underline{b}}$ is damping coefficient
$\underline{\underline{c}}$ is spring constant

A structure of arbitrary shape will possess hydrodynamic properties a_{mn}, b_{mn} and c_{mn} in each direction m due to any motion in the direction n. As a consequence one finds that the six components of motion are described by six coupled equations of motion. But usually only a few properties will couple the equations. As for instance couplings between surge-pitch and sway-roll.

The main interest within this scope of work is in structures with constant hydrodynamic properties. Bodies that penetrates the surface always has time varying hydrodynamic properties.

These properties varies with the wave frequency, and must be averaged in some way. This and other things will be discussed in the following subchapters on the hydrodynamic properties.

A3.2.1a Buoyancy

The buoyant force is the static reaction force that brings the floating body into static equilibrium after an excursion. These buoyant forces are due to the structures under-water shape and effected by the mass distribution.

For submerged bodies the only stiffness is for roll and pitch, as long as the fluid is uniform. When the body is floating it also has heave stiffness.

The forces in the static equilibrium equation is gravity forces and buoyant forces. Gravity forces act in the centre of gravity, G. Buoyant forces act in the centre of buoyancy.

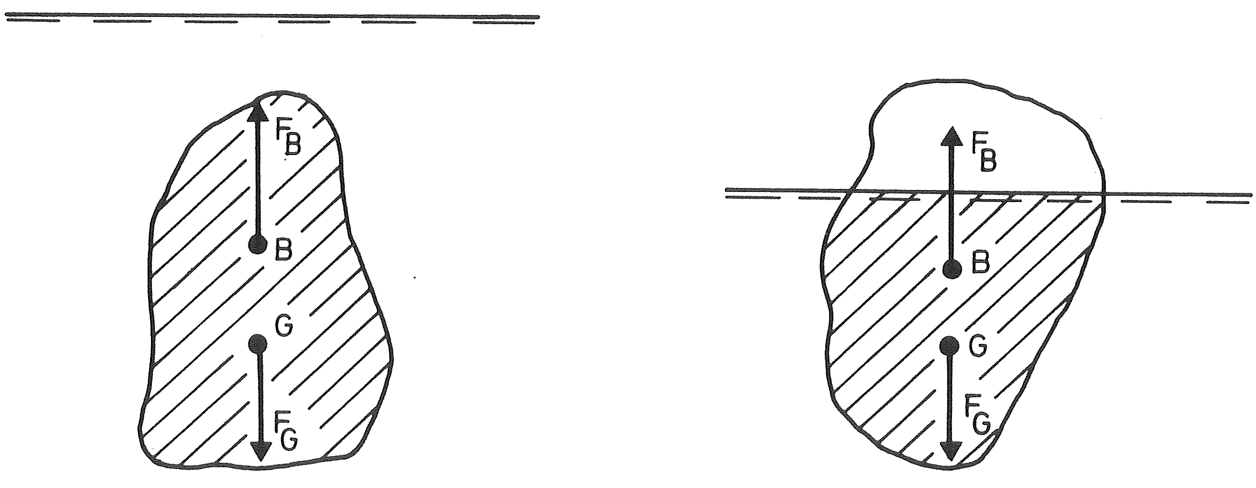


Fig. A3.1 Schematic indication of resultant forces and their reduction point for a submerged and a floating body.

The gravity force is a body force. Its force components are

$$\underline{F}_G = \underline{g} \int_m dm = \underline{g} \int_V \rho dV = m\underline{g} \tag{A3.7}$$

and its moment components are

$$\underline{M}_G = \underline{r}_G \times \underline{F}_G = \int (\underline{r} \times d\underline{F}_G) = \int_V (\underline{r} \times \underline{g}\ \rho) dV \tag{A3.8}$$

where

$\underline{g}$	is	acceleration of gravity,
m	is	mass,
ρ	is	density of structure,
V	is	volume,
$\underline{r}$	is	position vector of dV and
$\underline{r}_G$	is	position vector of center of gravity.

To determine the location of the center of gravity of any body, we apply the principle of moments to the parallell system to determine its resultant. The moment of the resultant gravitational force F_G about any axis equals the sum of the moments about the same axis of gravitational forces dF_G acting on all particles considered as infinitesimal elements of the body.

$$\underline{r}_G = \frac{\int_m \underline{r}\ dm}{m} = \frac{\int_V \underline{r}\ \rho dV}{\int_V \rho dV} \tag{A3.9}$$

The buoyancy force is a boundary force on the body. The force on the wetted surface is caused by the surrounding pressure. This force is the pressure, p, in the normal direction of the surface, $\hat{n}$, times the surface area, A.

$$\underline{F}_B = - \int_A \hat{n} p\ dA \tag{A3.10}$$

where $\hat{n}$ is directed outwards from the surface.

The surrounding pressure is assumed to be constant and equal to the atmospheric pressure above the sea surface. Below the sea surface the pressure is the sum of air and hydrostatic pressure. Using the coordinate system at the sea surface with z-direction upwards, the pressure is

$$p_{tot} = p_{atm} \qquad z > 0 \qquad \text{(A3.11)}$$

$$p_{tot} = p_{atm} - \rho g z \qquad z < 0$$

or with the atmospheric pressure as reference level.

$$p = 0 \qquad z > 0 \qquad \text{(A3.12)}$$
$$p = -\rho g z \qquad z < 0$$

The resultant buoyancy force is directed upwards as long as hydrostatic pressure is assumed.

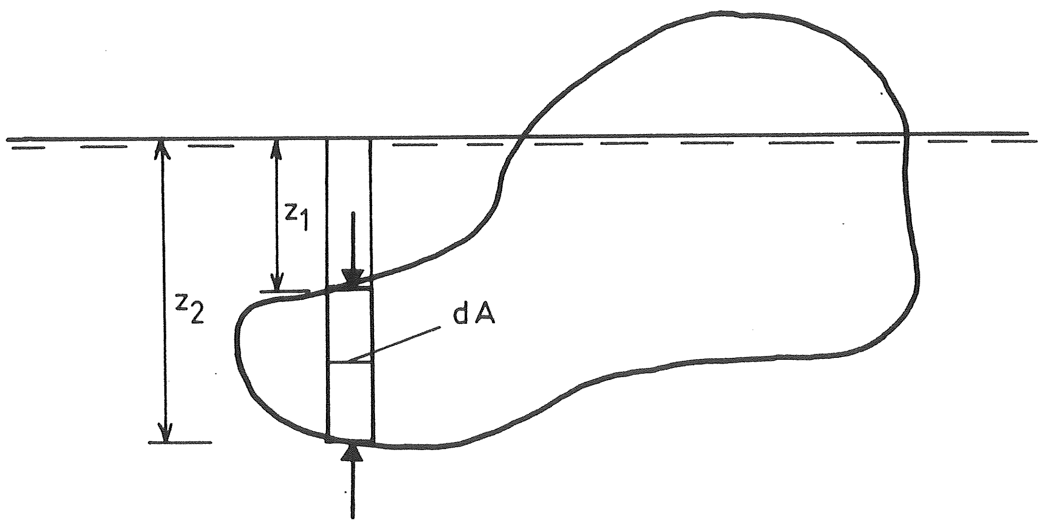


Fig. A3.2 Buoyancy force on the body.

Inserting eq. (A3.12) in eq. (A3.10) it is now possible to calculate the resultant buoyancy force using the principle of buoyancy, the discovery of which is credited Archimedes.

Using the notations from fig. A3.2 the force is

$$F_{Bz} = \rho g \int_A z \; dA = \rho g \int_{A_z} (z_2 - z_1) \; dA_z \qquad \text{(A3.13)}$$

$$= \rho g V_s \quad (=F_G \text{ Equilibrium})$$

where

V_s is the submerged volume.
A is the overall wetted surface.
A_z is the horizontal (z) projecting of A.

The buoyancy force is equal to the force on the water displaced by the body. This force is vertical and acts through the center of buoyancy. Given the same assumptions as eq. (A3.9), but assuming a constant density of the water, the buoyancy center $\underline{r}_B$, is

$$\underline{r}_B = \frac{\int_{V_s} \underline{r}\rho \; dV}{\int_{V_s} \rho \; dV} = \frac{\int_{V_s} \underline{r} \; dV}{V_s} \qquad \text{(A3.14)}$$

with notations as earlier. If the body is in equilibrium then B and G is on the same vertical line ($x_B = x_G$, $y_B = y_G$).

Knowing that the gravity force, F_G, is constant, it is obvious that the restoring force on the body must be caused by a change in the buoyancy force.

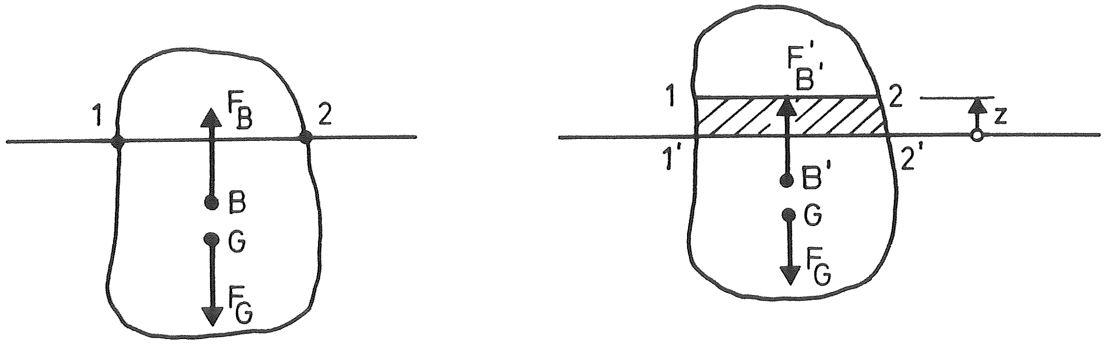


Fig. A3.3 Schematic indication of the heave restoring force ΔF_H on a floating structure caused by a heave z from the equilibrium position.

Translations will not cause any change in $\underline{F}_B$, except for the heave of a floating body. Which will cause a change in both $\underline{F}_B$ and $\underline{r}_B$, although the change in $\underline{r}_B$ does not influence the restoring force in heave for a floating body.

$$\Delta F_H = F'_{B'} - F_G = \rho g (V_{os} - V_s) =$$

$$= -\rho g \Delta V_s = - \rho g \int_o^z A_w(z) dz \qquad \text{(A3.15)}$$

where

V_{os}	is	displaced volume in equilibrium,
V_s	is	displaced volume and
$A_w(z)$	is	water line area, as a function of z.

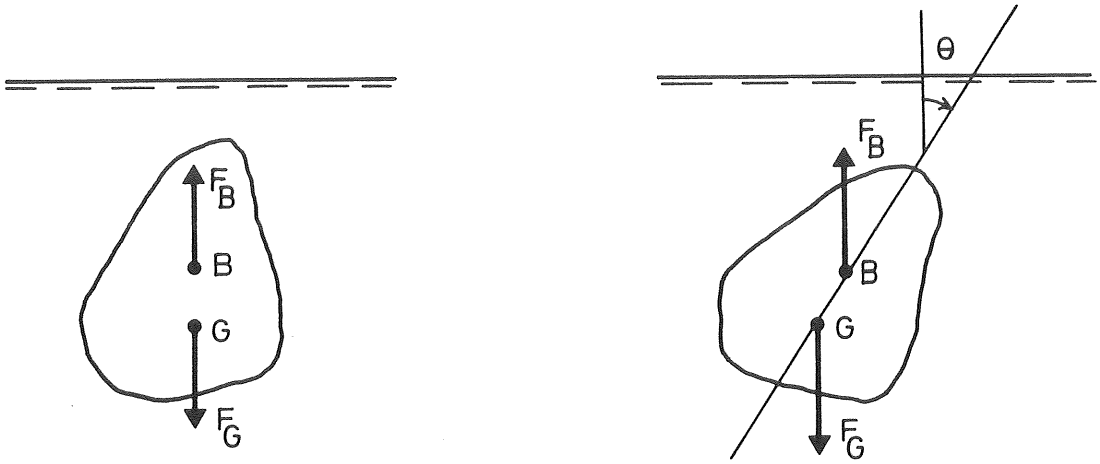
If the water line area is constant the restoring force is

$$\Delta F_H = -\rho g \, A_w \, z \qquad \text{(A3.16)}$$

which gives the linear heave stiffness, C_H, as

$$C_H = \frac{\Delta F_H}{z} = -\rho g \, A_w \qquad \text{(A3.17)}$$

Rotations will, on the contrary to the translations, cause restoring forces for all motions except the yaw motion. There is a fundamental difference between submerged bodies and those floating. Therefore, we will first discuss submerged bodies and then floating bodies.



Equilibrium position   Inclined position

Fig A3.4   Schematic indication of the pitch or roll restoring moment on a submerged body, caused by an inclination from the equilibrium position

The restoring moment on a submerged body will be

$$\Delta\underline{M} = \underline{GB} \times \underline{F}_B = (\underline{r}_B - \underline{r}_G) \times \underline{F}_B \qquad \text{(A3.18)}$$

where $\underline{GB}$ is the vector from G to B,
$\underline{r}_B$ and $\underline{r}_G$ are the location vectors of B and G.

Eq. (A3.18) is for a pure pitch or roll motion

$$\Delta M = \overline{GB} \; mg \sin\theta \qquad \text{(A3.19)}$$

where $\overline{GB}$ is the magnitude of $\underline{GB}$ and
θ is inclination angle.

This gives the vertical rotation stiffnesses as

$$c_{P-R} = \overline{GB} \; mg \; \frac{\sin\theta}{\theta} \qquad \text{(A3.20)}$$

which for small rotations* gives the linear stiffness

$$C_{P-R} = \overline{GB}\ mg \qquad (A3.21)$$

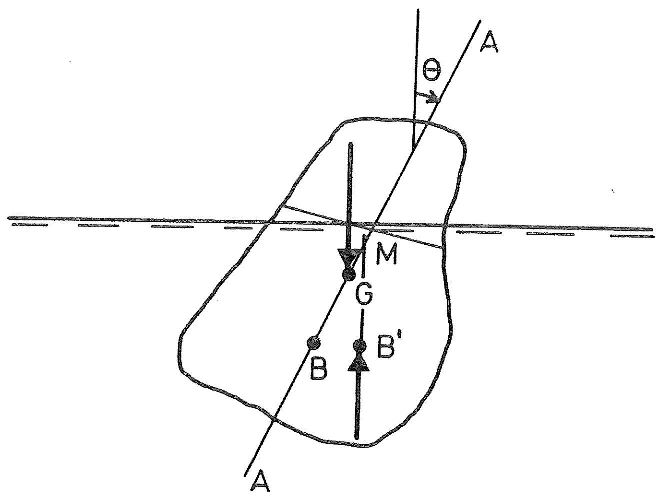


Fig. A3.5 Schematic indication of the pitch and roll restoring moment on a floating body, caused by an inclination θ around the center of gravity.

The restoring moments on a floating body is more complicated to derive. Eq. (A3.18) gives the moment, but the problem is that both $\underline{GB}$ and $\underline{F}_B$ are dependent on the inclination angle θ. Using eqs. (A3.13) and (A3.14) the problem is solved.

Before starting the derivation of the restoring moment we have to define the metacentre M (see fig. A3.5). As noted in fig. A3.5 the centre of buoyancy will shift. The intersection of the vertical through B' with the inclined axis AA is called the metacentre M. Consequently the distance between the gravity centre G and metacentre is called the metacentric height, GM. This metacentric height is a measure of how stable a body is floating. The consequences of GM are

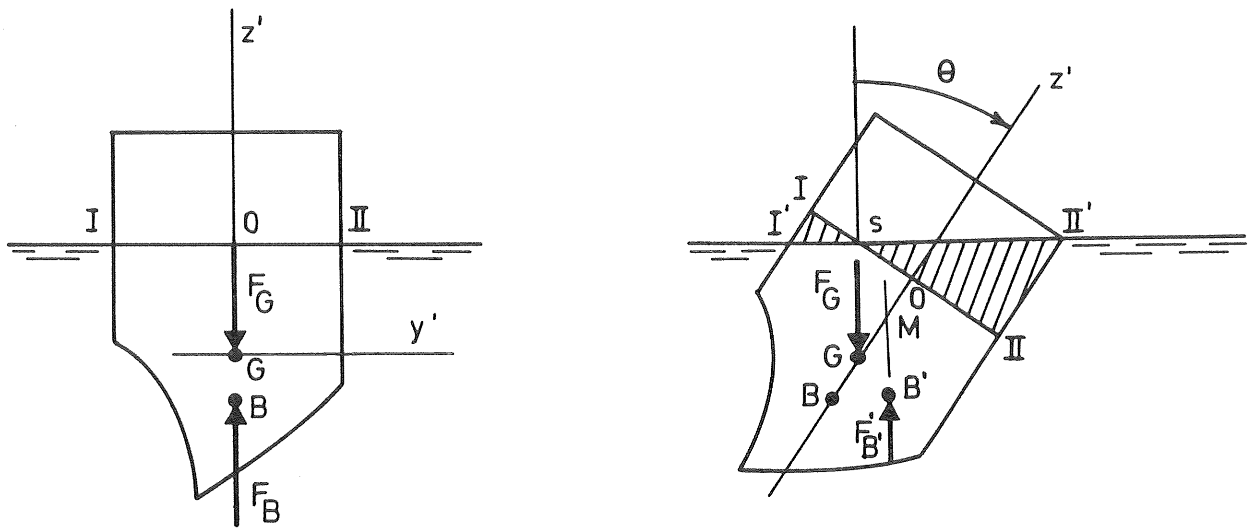
GM > 0	stable equilibrium
GM = 0	indifferent -"-
GM < 0	unstable -"-

---

* Note

$$1 - \frac{\sin\theta}{\theta} \leq 1\% \qquad \Rightarrow \theta \leq 14^{\circ}$$

This means that the linear assumption has a fault less than 1% for an inclination angle of less than 14°.



A3.6 Schematic presentation of the determination of the restoring moment on a floating body with vertical sides

As an example of the determination of the roll restoring moment using eqs. (A3.13) and (A3.14), consider a floating body with vertical sides as in fig. A3.6, as done in Hooft (1982). The restoring moment is determined as the sum of the moment due to equilibrium buoyancy force acting through B and the moment due to change in immersion (lined area in fig. A3.6). For the condition presented in fig. A3.6 the restoring moment is, assuming a homogenous fluid,

$$-M_x = (\underline{GB}' \times \underline{F}'_{B'})_x = (\underline{GB} \times \underline{F}_B)_x + \Delta M_I \qquad \text{(A3.22)}$$

where ΔM_I is the moment due to change in immersions.

This moment, ΔM_I, is given by the difference between the up-ending moment of the submerged part S-II-II' and the over-turning moment of the part raised from the water S-I-I'. After some calculations

$$\Delta M_I = \rho g \sin\theta \left(I_w\left(1 + \tfrac{2}{3}\tan^2\theta\right) + \tfrac{1}{2}\,\overline{OG}^2 \tan^2\theta \sin\theta\, A_w\right) \qquad \text{(A3.23)}$$

assuming that the water line has a constant transsection over the inclination. Here

I_w is the area moment of inertia of the water plane

$$I_w = \int_0^L \int_{-Y_I}^{Y_{II}} y^2 \, dxdy \tag{A3.24}$$

A_w is the water plane area

$$A_w = \int_0^L B(x)\, dx \tag{A3.25}$$

while

$B(x)$ is the width of the water line at the cross section x

$$B(x) = \overline{I\ II}(x) = Y_{II} + Y_I \tag{A3.26}$$

L is the water line length

Y_I and Y_{II} are the y coordinates of the sides at the water line (fig A3.6).

For a circular water line cross section the area moment of inertia is

$$I_{circ} = \frac{\pi}{4} r^4 \tag{A3.27}$$

where r is the radius of the circle.

The additional moment ΔM_I in equation (A3.22) is often written

$$\Delta M_I = \rho g V \overline{MB} \sin\theta \tag{A3.28}$$

Substituting this result and eq. (A3.18), also using eq. (A3.19), into eq. (A3.22) leads to the total roll restoring moment

$$M_x = -\rho g V \, \overline{GM} \sin\theta \tag{A3.29}$$

in which

$$\overline{GM} = \overline{GB} + \overline{BM} \tag{A3.30}$$

$$\overline{BM} = \frac{I_w}{V} \left(1 + \frac{2}{3} \tan^2\theta\right) + + \frac{1}{2} \frac{A_w \, \overline{OG}^2}{V} \tan^2\theta \sin\theta \tag{A3.31}$$

where V is displaced volume

$$V = V_o + \Delta V = V_o + A_w \, \overline{OG} \, (\sec\theta - 1) \tag{A3.32}$$

V_o is displaced volume in equilibrium and ΔV is the change in immersion due to the inclination.

For small rotations the restoring moment is

$$M_x = -\rho g V_o \, \overline{GM} \, \theta \tag{A3.33}$$

where

$$\overline{GM} = \overline{GB} + \frac{I_w}{V} \tag{A3.34}$$

This gives the linear rotational stiffness as

$$C_{R-P} = - \rho g \, V_o \, \left(\overline{GB} + \frac{I_w}{V}\right) \tag{A3.35}$$

In doing this linearization we, besides neglecting higher order terms in roll and pitch, also neglect a roll (pitch) induced translation. This induced translation is due to the change in immersion caused by the inclination. This force is using eq. (A3.32)

$$F(\theta) = \rho g \, A_w \, \overline{OG} \, (\text{see } \theta - 1) \tag{A3.36}$$

By this we conclude the discussion on the hydrostatic stiffness of (partly or fully) submerged bodies. There are of course many more subjects in this topic, but they will not be discussed.

A3.2.1b Hydrodynamic damping

When a body is moved in water there will be a loss of kinetic energy. This will make the equations of motions non conservative. Moreover it can be noticed that this energy loss usually is written as an equivalent force times body velocity. This force is called the hydrodynamic damping force, and can be divided into two parts; viscid and inviscid damping.

Inviscid damping

Inviscid damping only exists for a body which affects the free surface, i.e. a body that generates waves when it moves. This wave making is of two kinds. The first kind is due to the oscillation of the body and the second is the waves caused by a body moving at a constant speed at or near the surface. The second kind of damping is called wave resistance and will not be further discussed, as it is a second order effect. The first is the radiation damping, which causes a resistance force that is a linear function of the body velocity,

$$\underline{F}_r = \underline{\underline{b}}\,\dot{\underline{r}}_c \tag{A3.37}$$

where $\underline{\underline{b}}$ is the radiation damping which is a function of the frequency ω of the oscillation.

To determine radiation damping (and added mass), the problem is described with potential theory and solved with diffraction theory approach. This will be discussed somewhat in chapter A3.2.1c, in connection with the added mass problem.

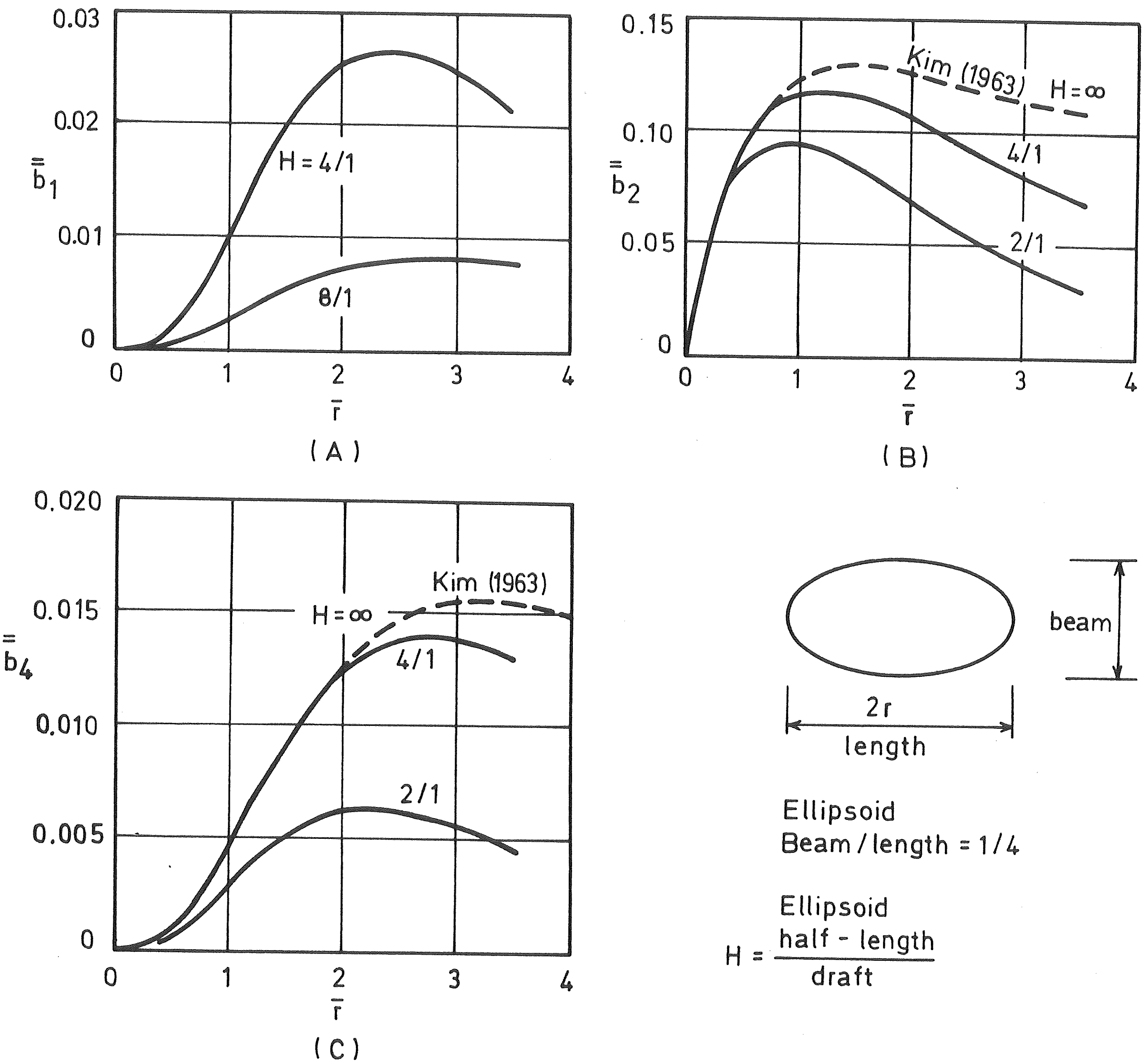


Fig. A3.7 Damping coefficients of ellipsoids for

(A) Surge (or sway) $\bar{\bar{b}}_1 = b_1/\rho\omega r^3$

(B) Heave $\bar{\bar{b}}_3 = b_3/\rho\omega r^3$

(C) Roll (or pitch) $\bar{\bar{b}}_4 = b_4/\rho\omega r^4$

which all are functions of $\bar{\bar{r}} = r\omega^2/g$. (From Kim (1965)).

In order to clarify radiation damping, we will first discuss a floating body that is given a unit velocity impulse. The body will after this experience a decreasing and oscillating retardation force, as shown in Figure A3.8, due to the waves made by the body.

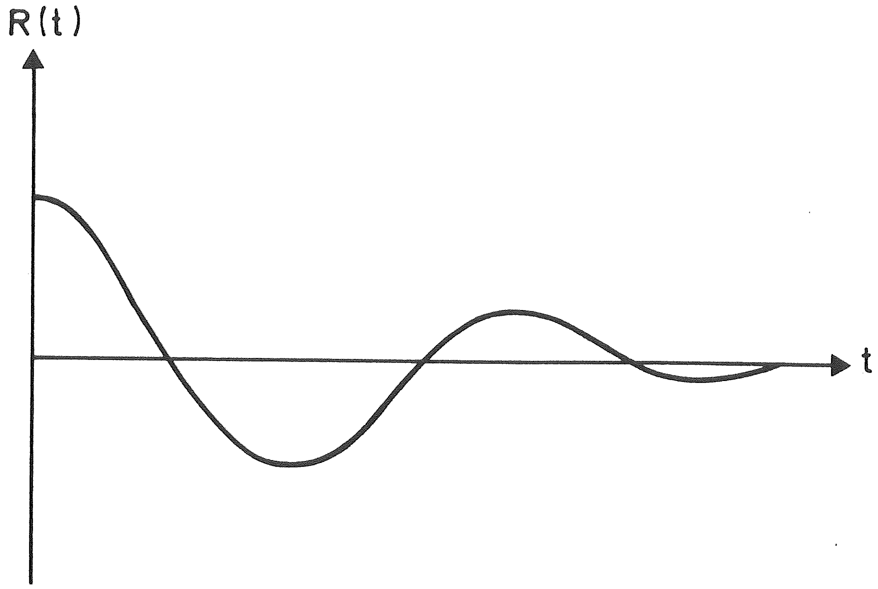


Fig. A3.8 Retardation force on a floating body.

When the damping is known as a function of frequency ϕ, then the retardation force of the structure is known. This force will then be a linear function of the relative velocity between the fluid and the structure, as supposed in Eq. (A3.37). From this it follows, that the fluid reaction force on the body can be deduced from that force, which is due to a unit velocity impulse, as shown in Figure A3.8.

$$R(t) = \frac{2}{\pi} \int_0^\infty b(\omega) \cos(\omega t) \, d\omega \tag{A3.38}$$

Why it is as simple as this, can be explained knowing that a unit velocity impulse is the fourier transform of unity,

$$\dot{x}(t) = \delta(t) \qquad \hat{\dot{x}}(\omega) = 1,$$

which means, that a unit velocity impulse contains all frequencies. Hence, it will excite the whole frequency band of damping coefficients.

The physical meaning of a unit velocity impulse is not easily understood. If instead the displacement is dicussed. The displacement being the integral of the velocity. So, the displacement will be a step function, with a unit step at time zero.

$$x(t) = \int \dot{x}(t)\,dt = \int \delta(t)\,dt = \theta(t) = \begin{matrix} 0 & t<0 \\ 1 & t>0 \end{matrix}$$

All of which is a boundary condition on which Eq. (A3.38) is based.

This requires that the damping is known. We will therefore continue with the derivation of it.

The radiation damping coefficient $\underline{\underline{b}}(\omega)$ is calculated assuming that the loss of kinetic energy of the structures motion that corresponds to the energy of the outgoing waves, which has led to the idea that the damping due to the outgoing waves should be related to the excitation force on the structure due to the incoming waves, as discussed in Newman (1980) and Hooft (1982). According to these it is found that

$$b_{nn}(\omega) = \frac{k}{16\pi} \int_0^{2\pi} \frac{|F_{an}(\phi,\omega)|^2}{P(\omega)}\,d\phi = c_f \int_0^{2\pi} \frac{F_{an}(\phi\ ,\omega)}{\zeta_a}^2 d\phi \qquad \text{(A3.39)}$$

where

$b_{mn}(\omega)$ is the damping coefficient in direction m due to a velocity in direction n.

$P(\omega)$ is the wave energy flux in the outgoing wave, which is

$$P(\omega) = F\,c_g = \frac{1}{2}\,\rho g \zeta_a^2\,c_g \qquad \text{(A3.40)}$$

$F_{am}(\phi,\omega)$ is the exciting force amplitude in direction m due to waves in the ϕ-direction relative to the x-axis,

ζ_a is the wave amplitude

ϕ is the wave direction,

c_f is the incoming normalized wave energy flux coefficient, which is

$$c_f = \frac{k}{8\pi\rho g\ c_g} \quad \text{(A3.41)}$$

k is the wave number, which is

$$k = \frac{2\pi}{L} \quad \text{(A3.42)}$$

L is the wave length,

c_g is the group velocity of the wave, which is

$$c_g = \frac{1}{2}\, c\ \left(1 + \frac{2kd}{\sinh\ (2kd)}\right) \quad \text{(A3.43)}$$

c is the phase velocity of the wave, which is

$$c = \frac{\omega}{k} \quad \text{(A3.44)}$$

ω is the angular frequency of the wave, which is

$$\omega = \frac{2\pi}{T} \quad \text{(A3.45)}$$

T is the wave period,

ω is related to k through the wave dispersion relation, which is

$$\omega^2 = gk\ \tanh\ (kd) \quad \text{(A3.46)}$$

d is the water depth.

Eqs. (A3.43) and (A3.46) are asymptotic for both shallow water (kd -> 0) and deep water (kd -> ∞) which simplifies these equations, see Mårtensson (1983).

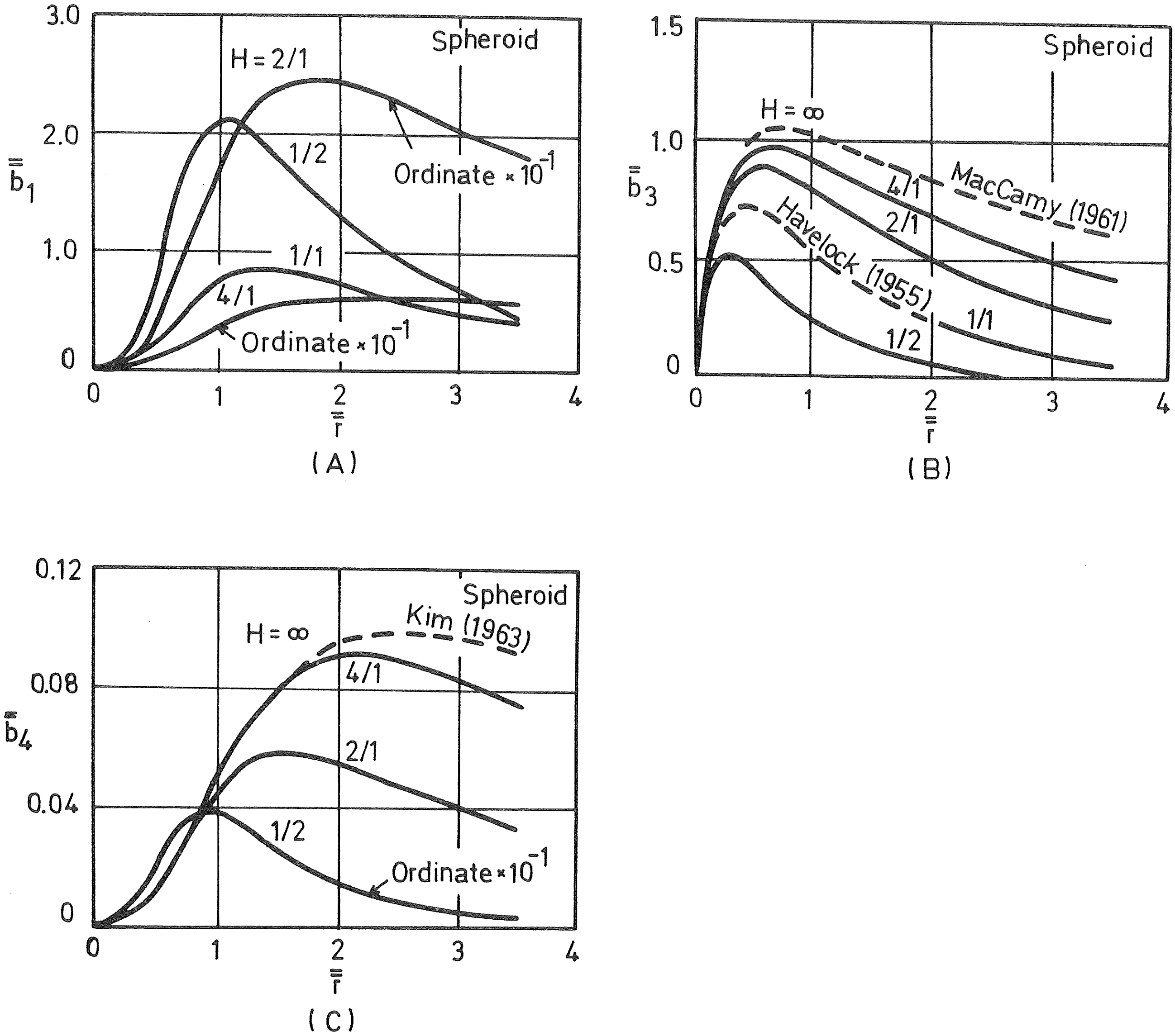


Fig. A3.9 Damping coefficients of spheroids for;

(A) Surge (or sway) $\bar{\bar{b}}_1 = b_1/\rho\omega r^3$;

(B) Heave $\bar{\bar{b}}_3 = b_3/\rho\omega r^3$ and

(C) roll (or pitch) $\bar{\bar{b}}_4 = b_4/\rho\omega r^4$

which all are functions of $\bar{\bar{r}} = r\omega^2/g$.

(From Kim (1965)).

(r = radius of spheroid)

Special results of equation (A3.39) are obtained for structures that are rotary symmetric around a vertical axis such as floating spheroids, see fig. A3.9. For such structures the integrated equations are

$$b_{11}(\omega) = b_{22}(\omega) = \pi c_f \left| \frac{F_{a1}(\phi=0,\omega)}{\zeta_a} \right|^2 \tag{A3.47}$$

$$b_{33}(\omega) = 2\pi\ c_f \left| \frac{F_{a3}(\phi=0,\ \omega)}{\zeta_a} \right|^2 \tag{A3.48}$$

$$b_{44}(\omega) = b_{55}(\omega) = \pi c_f \left| \frac{M_{a4}(\phi=0,\omega)}{\zeta_a} \right|^2 \tag{A3.49}$$

$$b_{66}(\omega) = 0 \tag{A3.50}$$

Similarly, in two dimensions, for a body symmetric about the plane x=0, it can be shown that

$$b_{nn}(\omega) = \frac{1}{4} \frac{|F_{an}(\omega)|^2}{P(\omega)} = \frac{1}{2\rho g\ c_g} \left| \frac{F_{an}(\omega)}{\zeta_a} \right|^2 \tag{A3.51}$$

for the 2-D motions, surge, heave and pitch. This is visualized for the heave motion in fig. A3.10 and A.3.11.

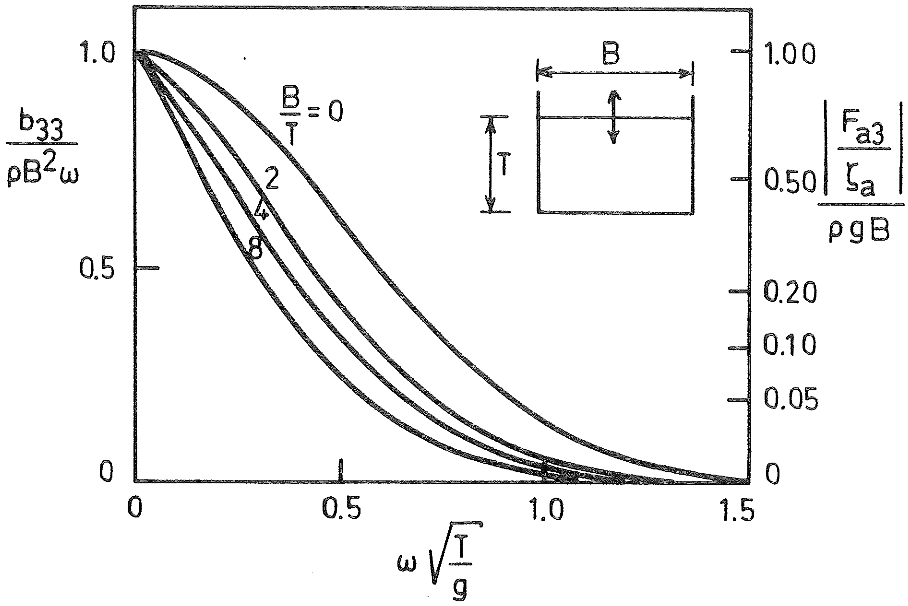


Fig. A3.10 Damping coefficient for a family of two-dimensional rectangular parallelepipeds (boxes) heaving in deep water. Included is the thin-ship approximation labeled B/T=0. Also shown here by the scale on the right is the heave exciting force coefficient, obtained in accordance with eq. (A3.51). (From Newman (1980)).

The major difference between 2-D and 3-D bodies are in the limiting values as $\omega \to 0$. It is seen in both fig. A3.10 and A3.11 that for 2-D bodies the heave damping does not approach zero as fast as it does for 3-D bodies. The box has a damping that approaches zero just as fast as ω does, therefore the non-dimensional damping equals unity as $\omega \to 0$.

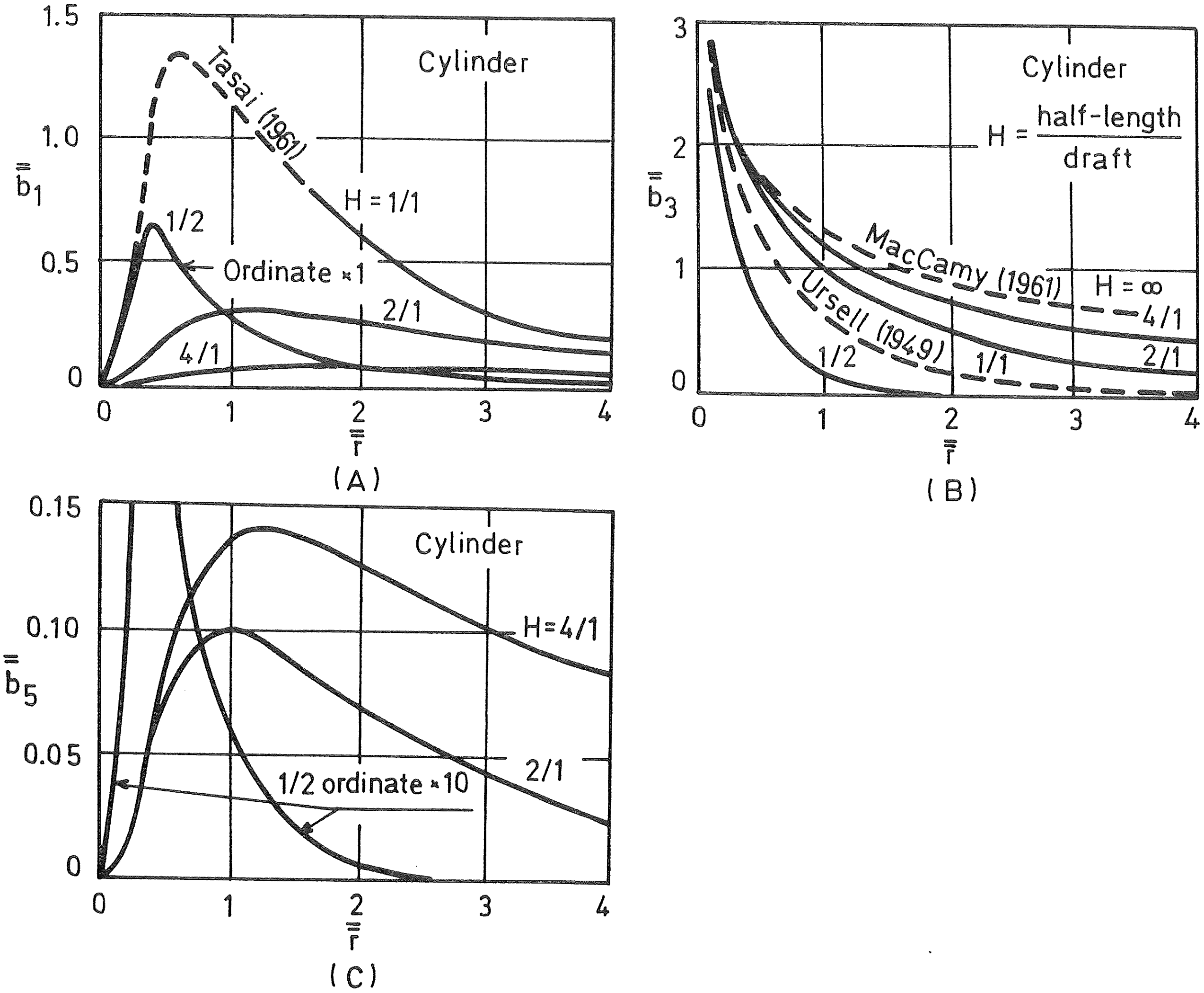


Fig. A3.11 Damping coefficients of horizontal cylinders (2D);

(A) surge $\bar{\bar{b}}_1 = b_1/\rho\omega r^2$;

(B) heave $\bar{\bar{b}}_3 = b_3/\rho\omega r^2$ and

(C) pitch $\bar{\bar{b}}_5 = b_5/\rho\omega r^3$

which all are functions of $\bar{\bar{r}} = r_\omega{}^2/\rho$.
(From Kim (1965)).

The radiation damping problem is coupled to the added mass (see chapter A3.2.1c) in the way that if either of them are known the other is also known. Usually the radiation damping is computed using some type of wave diffraction program, which are of some finite element method or boundary element method type, using sink-sources.

Viscid damping

Far below the surface the body will not make any waves. Then the radiation damping will be constant and equal to zero. Nevertheless energy will be dissipated when oscillating the body. This energy loss is caused by the viscosity of the water.

Viscid damping is the only damping that exists for a body which does not affect the free water surface. But, of course, it exists for a body anywhere in a viscous fluid. This viscid damping is often called drag.

When the velocity of the body is large enough, the influence of viscosity will start to affect the motions of the body. This while the viscous reaction force will become of the same order of magnitude as the inviscid forces on the body.

In the absence of wave resistance the total drag is the profile drag, which in turn is due to pressure and viscous shear. The drag may be due to pressure or viscous shear only, or a combination of both.

The drag is expressed as the product of the dynamic pressure of the free stream, some characteristic area, and a drag coefficient. The drag as coefficient is a function of a number of parameters which influence the boundary layer and its separation. These influencing parameters include the body shape, Reynolds number, Mach number, Froude number, surface roughness, and free stream turbulence.

All these influencies can be found in some books on basic fluid mechanics, such as Olson (1980). In any instance, the drag force is expressed as

$$\underline{F}_d = \frac{1}{2}\rho \underline{u}_s \, u_s A \, C_D \qquad \text{(A3.52)}$$

where

$\frac{1}{2}\rho u_s^2$ is dynamic pressure of the free stream,

A is area being for pure skin friction drag, the chord area for litting varies, or the projected frontal area for other shapes

C_D is the drag coefficient

C_f is skin friction drag coefficient

C_p is pressure drag coefficient.

The vector formulation of the drag force on a axisymmetric slender body may be found in Lindahl and Sjöberg (1983). Using this approach it is assumed that the drag force may be divided in two components, one normal and one tangential component. The force per unit length is

$$\underline{f} = \underline{f}_{DT} + \underline{f}_{DN} = \tfrac{1}{2}\rho d_o (C_{DT} u_{ST} \underline{u}_{ST} + C_{DN} u_{SN} \underline{u}_{SN}) \qquad \text{(A3.53)}$$

Where

d_o is the drag diameter,

$\underline{u}_{ST}$ is the velocity in the tangential direction or the body, that is

$$\underline{u}_{ST} = u_{ST}\hat{t} = \underline{u}_S \cdot \hat{t}\hat{t}$$

where

$\hat{t}$ is the tangential direction

$\underline{u}_{SN}$ is the normal velocity, that is

$$\underline{u}_{SN} = \underline{u}_S - \underline{u}_{ST}$$

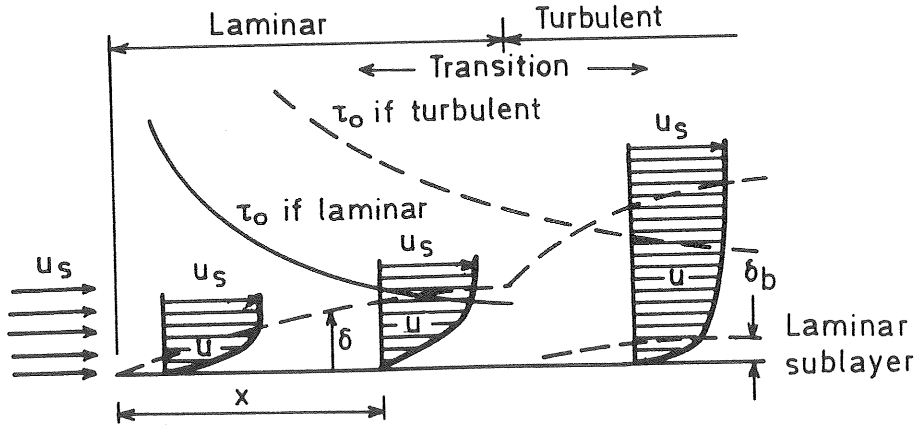


Fig. A3.12 Flow of viscous fluid along a flat plate. (τ_o is the shear stress). (From Olson (1980).)

Purely, viscous shear drag for parallel flow past a smooth flat plate depends on the rate of change of the velocity in the boundary layer. The drag coefficient C_f depends on whether the boundary layer is laminar or turbulent. If the boundary layer is laminar, C_f depends on the Reynolds number, Re, of the flow based on the free stream velocity u_s and the length of the plate x. If the boundary layer is turbulent, C_f depends on the Reynolds number, the roughness of the plate, and on the location of the transition from a laminar to a turbulent boundary layer.

It is noted that the skin-friction, C_f is dependent on the boundary layer. Integrating the shear stresses in the boundary layer will give the viscous shear force on the plate. Resulting curves of drag coefficient as a function of Reynolds number for smooth flat surfaces are given in fig. A3.13 below, including a typical ship hull with a rough surface. As can be seen in the figure the transition is at higher Re_x for smoother surfaces or lower free-stream turbulence.

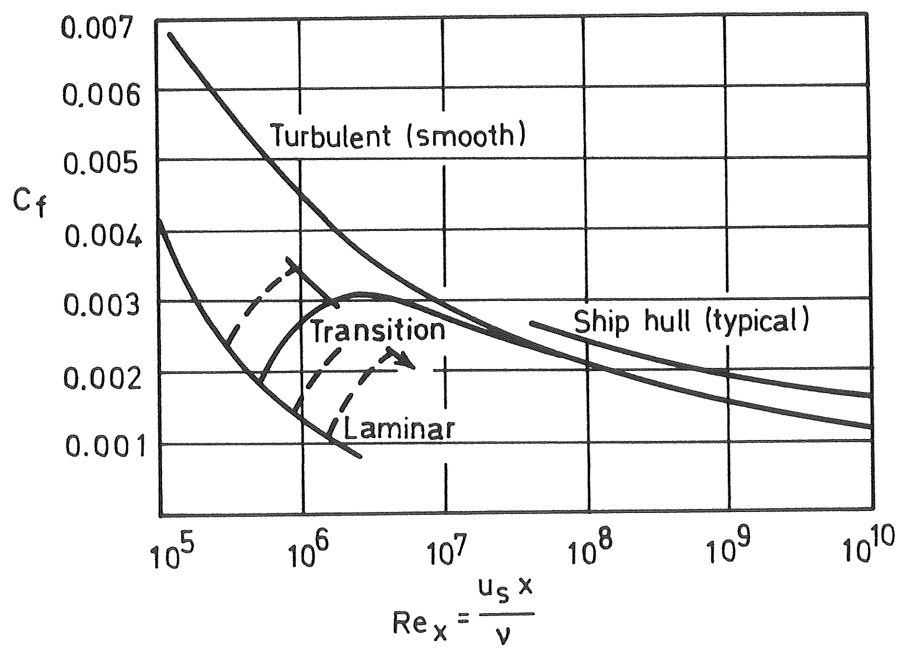


Fig. A3.13 Drag coefficients for plane surfaces parallel to flow. (Source: H.E.Saunders, Hydrodynamics in Ship Design, Vol.2 (1957), p.100).

The Reynolds number is the product of the free stream velocity, u_s, and the exposed length, x, divided by the dynamic viscosity, ν, of the fluid.

$$Re_x = \frac{u_s \cdot x}{\nu}$$

It is common to divide the Reynolds number range into three regions (see fig. A3.13 for the boundary layer). They are

laminar boundary layer region	$Re < 1.1\ 10^5$,
transition boundary layer region	$1.1\ 10^5 < Re < 2.\ 10^7$
turbulent boundary layer region	$2.\ 10^7 < Re$

In these three regions the velocity profile over the surface will be fundamentally different.

The skin-friction drag is derived assuming a certain rate of change of velocity in the boundary layer. This may not be the actual velocity distribution, but nevertheless the values of fig. A3.13 are good approximations.

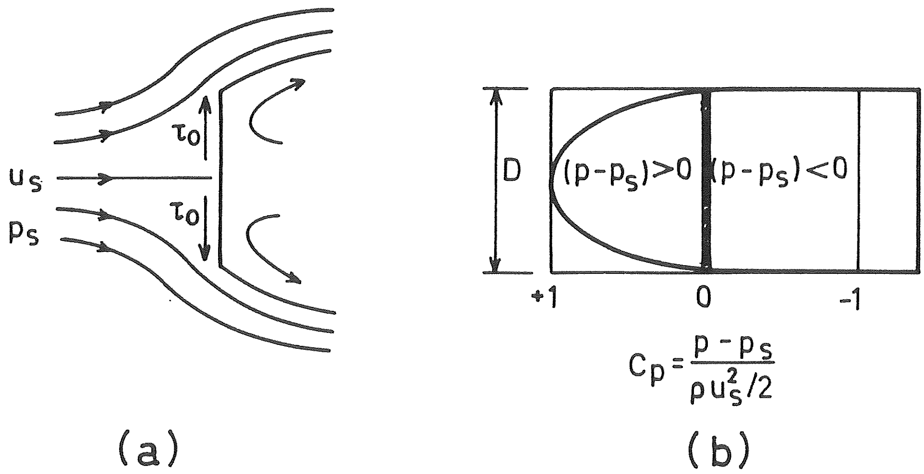


Fig. A3.14 Two-dimensional flow past a flat plate normal to the free stream (a) Flow pattern, with shear forces normal to the free stream, (b) Pressure distribution (From Olson (1980)).

Pressure drag is caused by the drop in pressure due to the separation of the boundary layer from the body surface. Pure pressure drag exists only for flow past a flat plate normal to the stream. As seen in fig. A3.14 the shear will act normal to the approaching stream and thus do not contribute to the drag force directly. The shear affects the growth of the boundary layer and therefore has a minor effect on the pressure distribution.

The drag coefficient for pure pressure drag depends on the shape of the surface and the Reynolds number of the flow based on some characteristric length D.

In Lighthill (1979) the pressure drag is derived on basis that the energy of the vortex wake is, rather roughly, proportional to 1/2 ρAu^2, where ρ is the rate of lengthening of the vortex wake and A is the frontal area. This is because the mass of a unit length of wake should vary as ρA, while the motions generated by the vorticing, which has just been shed should vary primarily with the body's instantaneous speed U. Then the rate of increase of kinetic energy per unit wake length is

$$\frac{1}{2} \rho A U^3 C_p = F_p U \qquad \text{(A3.54)}$$

which is equal to the rate of work, F_pU. The corresponding vortex-flow component of drag (pressure drag) is

$$F_p = \frac{1}{2} \rho A U^2 C_p \qquad \text{(A3.55)}$$

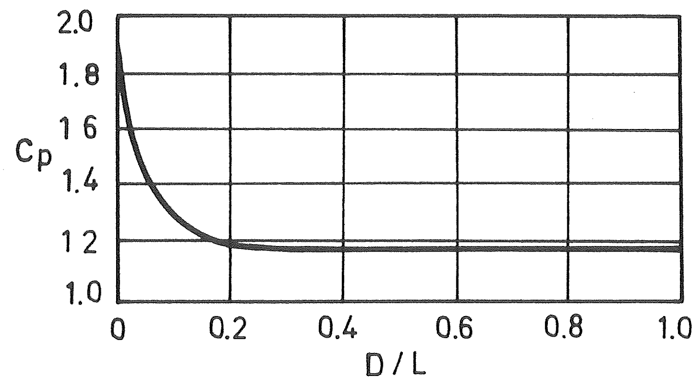


Fig. A3.15 Effect of aspect ratio on drag coefficient for rectangular plate normal to the flow (from measurements by C.Wieselsberger and O.Flachsbart). (From Olson (1980)).

The drag coefficient for a finite plate depends on the ratio D/L (see Fig. A3.15) as well as Re, because of the end effects. For a plate of infinite length $C_p = 1.9$ if $Re > 10^3$. Combined skin-friction and pressure drag, profile drag, is the most common situation of viscous drag on a body. This is the type of drag for which the drag coefficient is C_D, which in each limit can equal either C_f or C_p. Although most often it is the sum of them both.

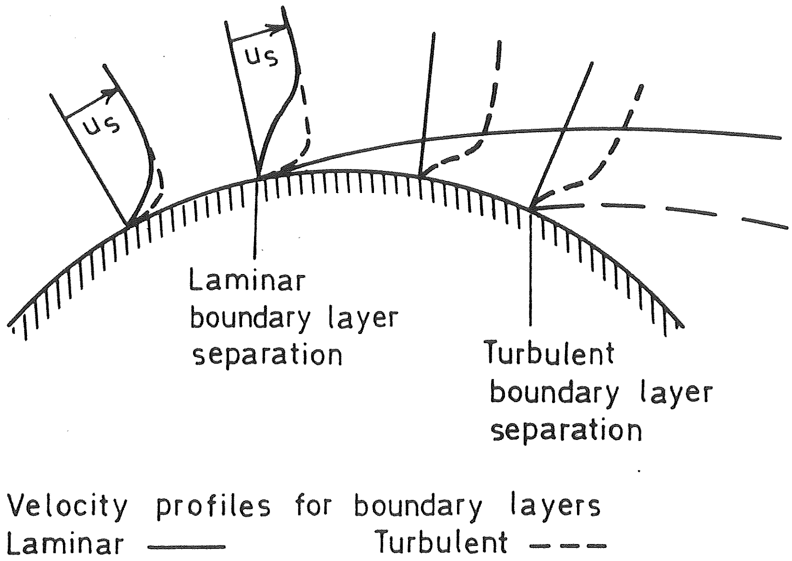


Fig. A3.16 Schematic presentation of a profile drag situation.

The drag coefficient depends on background turbulence, type of boundary layer (Reynolds number Re), body shape, surface roughness (k/D), and, if the flow is oscillating, on the Keulegan-Carpenter number (KC).

The classical C_D values are based on measurements in stationary flow on smooth bodies.

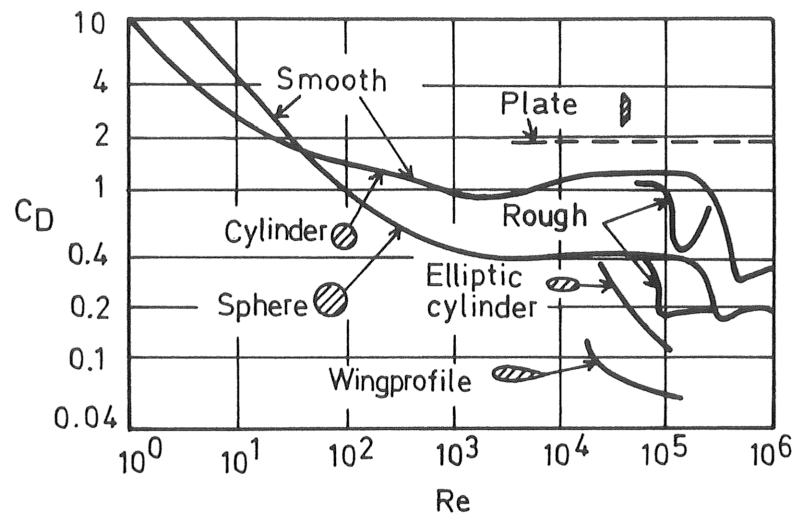


Fig. A3.17 Drag coefficient versus Re, for a family of 2-D transections and a spere.

The nature of the curves for both the typical profile drag bodies; the sphere, and the cylinder, can be explained in terms of boundary layer type. Below $Re \cong 10^5$ the boundary layer is laminar and the change in C_D is due to the change of the separation point. Above $Re \cong 5.10^5$ the boundary layer is turbulent and then the drag will decrease.

Furthermore it is of interest to know that all sharp-edged profiles are independent of scale (Re effects) due to the fact that they only experience nearly pure pressure drag. Semi-aerodynamic shapes have drag coefficients varying with wind speed.

In table A3.1, C_D values of various smooth 2-D body shapes are listed. If a body is infinitely long, there is the same pattern of flow around it at every cross section. More commonly, the body has one or two ends, and fluid then escapes around these ends and reduces the average drag per unit length. This reduction is a function of length/diameter or aspect ratio ε which is therefore defined

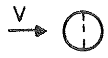
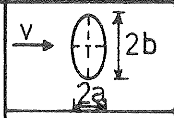
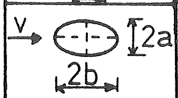
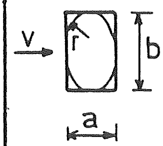
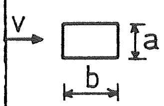
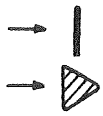
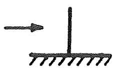
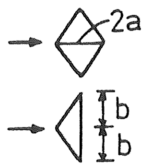
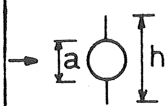
$$\varepsilon = L/D \tag{A3.56}$$

where one end of the member is sitting on a flat surface, in a uniform fluid stream, the surface has a mirror effect, so that the aspect ratio is defined as

$$\varepsilon = 2L/D \tag{A3.57}$$

Correction factors are common to most body shapes, and are given in table A3.2. Correction factors multiply the appropriate 2-D shape factor C_D(2-D). For instance as C_D=2.0 for an 2-D body, C_D(3-D) = 0.6·2.0 = 1.2 for ε = 3.

Table A3.1 Viscous drag coefficients for various 2-D bodies.

2-D Body shape		Boundary layer L=laminar T=turbulent	Dimension ratio	Drag Coefficient C_D		
Description	Sketch					
Circle or half-circle		L		1.2		
		T		0.7		
Ellipse		L	½<(b/a)<2	$1.2(b/a)^{½}$		
		T		$0.7(b/a)^{½}$		
		L	½<(b/a)<2	$1.2(a/b)^{½}$		
		T		$0.7((a/b)^{½}$		
Rectangle with or without corner radius				(b/a)<2	2≦(b/a)≦5	5<(b/a)
		L,T	(r/b)≦0.10	2.0	2.0 k_2	1.0
		L,T	0.1<(r/b)<0.15	k_1	k_1k_2	$½k_1$
		L	0.25<(r/b)	1.2	1.2 k_2	0.6
		T		0.7	0.7 k_2	0.35
		L,T		$k_2 = (8-(b/a))/6$		
		L		$k_1 = (7.6-16(r/b))/3$		
		T		$k_1 = (8.6-26(r/b))/3$		
				As above but b/a → a/b r/b → r/a		
Plate or triangle		L,T		2.0		
Plate on flat surface		L,T		1.2		
Parallelogram or triangle with edge to wind with radius		L,T	0.5<(a/b)<1.0 0≦(r/b)<0.25	$1.5(b/a)^{½}$		
Circle with fins		L		1.2(h/a)		
		T		0.7(h/a)		
Circular flat plate		L,T		1.12		

Values are from Sachs (1972), Vedeld (1983) and Hullam et al (1978).

Table A3.2 Correction factor for aspect ratio

Aspect ratio ε	Correction factor
0 - 4	0.6
4 - 8	0.7
8 - 40	0.8
Above 40	1.0

(From Sachs (1978)).

As we are interested in oscillating fluid around bodies both the Keulegan-Carpenter number (KC = $u_{max}T/D$), and the surface roughness (k/D) are of interest. In Sarpkaya (1976) model tests are carried out for various KC, Re, k/D on a vertical cylinder. The model test are based on undisturbed rectilinear flow, and are suggested by Sarpkaya to be considered as an upper limit for the wave motion.

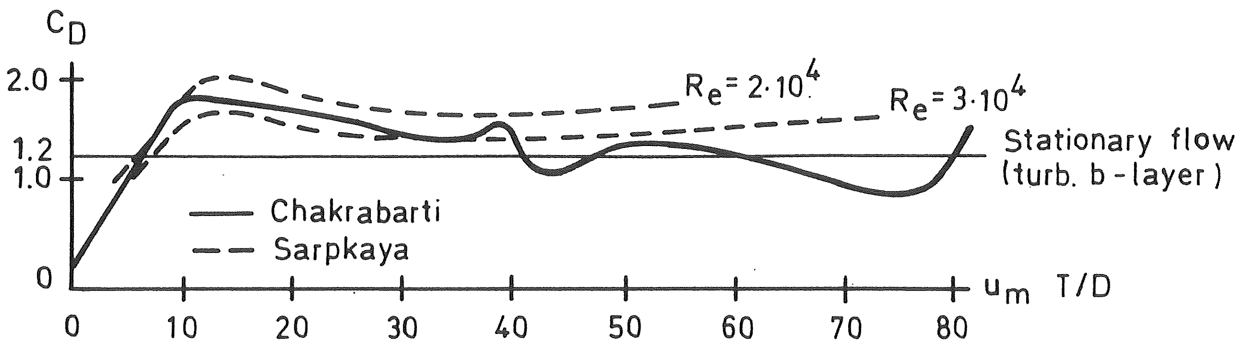


Fig. A3.18 Comparison of Sarpkaya's 2-D flow test results with Chakrabarti's wave tank test result. (From Chakrabarti (1980)).

In Chakrabarti (1980) test results from a wave tank are referred. The used cylinder and the range of Reynolds number was between 2×10^4 and 3×10^4. The comparison between the two tests are shown in fig. A3.18. It is seen that the correlation is good except for higher values. Chakrabarti´s tests are sparse in this region, why it does not necessarily indicate fundamental differencies.

The values of the tests show a decreasing tendency, but they are significantly different from the value of C_D in stationary flow.

The increase in C_D due to roughness is perhaps more significant than the decrease with KC number. Values of twice them for smooth cylinders in stationary flow are frequent. It is, therefore hard to choose a good value, knowing that marine growth can increase roughness significantly over a year only.

A rough engineering approximation, for tubular sections, seems to be to use the C_D values for laminar boundary layers, (see table A3.1), where the turbulent values should have been used.

A3.2.1c Added mass

A floating body that is dropped into the water will start to oscillate with an eigenperiod that is much longer than that corresponding to the body mass solely. This is the case because the body will start to push around the water, see figure A3.19. Therefore the water will cause pressure changes on the body surface. Which in turn will cause this unexpectedly long eigenperiod.

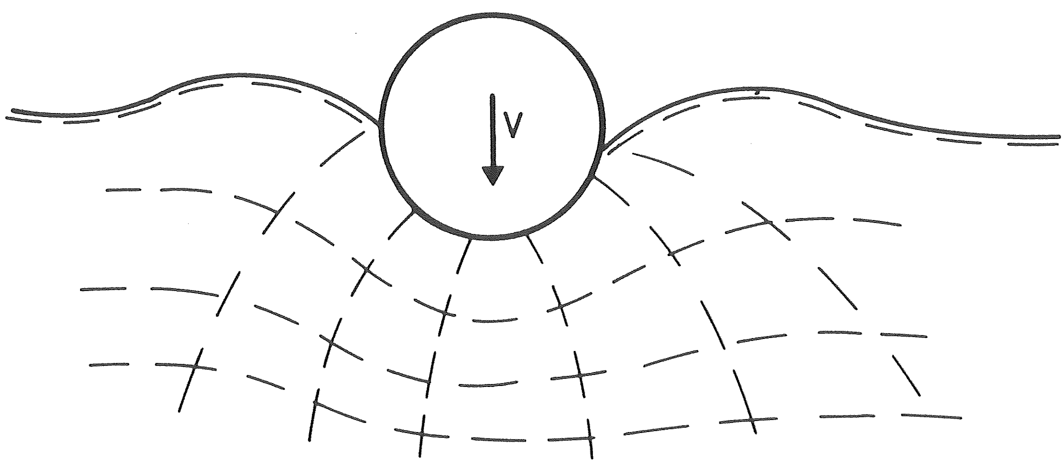


Fig. A3.19 Flow net around a dropped body.

In the far distance the water will remain unaffected. Integrating over the affected water volume will then give the total affected mass, or as it is usually called the added mass.

The added mass is usually thought of as the added mass in an inviscid fluid, with or without the presence of a free boundary. Potential theory is then used to solve the problem. For the structure affecting the free boundary, the special branch of potential theory called diffraction theory is used.

Diffraction theory:

Deriving the expressions for the added mass, is really a matter of solving the hydrodynamic reaction force caused by the body. Therefore the force will be solved for a floating body using diffraction theory. The problem is solved in all fundamental literature on hydrodynamics, such as Newman (1980). Using Newman, but also Hooft (1982) and Olsson (1982), the added mass will be derived in a classical way.

The diffraction theory states that the waves around a floating body can be described by the sum of three wave velocity potentials.

$$\Phi = \Phi_I + \Phi_S + \Phi_R \qquad \text{(A3.58)}$$

These are the potentials:

Φ_I, of the incoming wave,
Φ_S, of wave scattered by the body and
Φ_R, of wave radiated by the body.

The incoming wave is coming from the far distance unaffected by the structure. The scattered wave is the one which is reflected on the body surface. The radiated wave is caused by the motions of the body. Both the scattered and the radiated wave must satisfy the Sommerfeldt condition, i.e. vanish in the far distance.

Moreover the incoming wave is independent of the structure, while the others both depend on the body shape, although the scattered wave also depends on the incoming wave. The velocity potentials can be divided into two groups. The first is potentials that exist around a fixed structure. These are of course Φ_I and Φ_S. On the contrary we have a floating body oscillating in an otherwise calm water. Then $\Phi_I = \Phi_S = 0$, and Φ_R is the only to exist. We will now continue by deriving the hydrodynamic forces on the body, in this last case when only the body is moving.

Moving body in calm water

The motion of the body is based on Newtons equation of motion. This equation can be written in the following way, dividing the restoring forces into a hydrostatic and a hydrodynamic part.

$$\underline{\underline{m}}\,\underline{\ddot{x}} = -\underline{\underline{c}}\,\underline{x} - \rho \int_{S_o} \frac{d\Phi}{dt}\,\underline{n}\,dS \qquad \text{(A3.59)}$$

Here

$\underline{\underline{m}}$ is the mass,

$\underline{\underline{c}}$ is the hydrostatic "spring",

$\underline{n}$ is the local normal of the wet body surface,

S_o is the wet body surface,

$\underline{x}$ is the excursion from the equilibrium position.

We will continue with the hydrodynamic part of eq. (A3.59).

$$\underline{F} = -\rho \int_{S_o} \frac{d\Phi}{dt}\,\underline{n}\,dS \qquad \text{(A3.60)}$$

or really with the velocity potential which here is the radiated potential only

$$\Phi = \Phi_R \qquad \text{(A3.61)}$$

If a floating body is forced to perform harmonic motion x_k in one of its motion degrees of freedom, k, and the fluid is at rest initially, then the velocity potential Φ_k of the fluid can be written as

$$\Phi_k = \phi_k \dot{x}_{ak} e^{j\omega t} \tag{A3.62}$$

Here

$\dot{x}_{ak}$ is the velocity amplitude for motion k

$\phi_k \dot{x}_{ak}$ is the velocity potential amplitude for motion k.

This harmonic behaviour is due to the structure being linear. Its motion is then described by

$$x_k(t) = x_{ak} e^{j\omega t} \tag{A3.63}$$

Its velocity is

$$\dot{x}_k (t) = j\omega x_{ak} e^{j\omega t} = \dot{x}_{ak} e^{j\omega t} \tag{A3.64}$$

Its acceleration is

$$\ddot{x}_k (t) = - \omega^2 x_{ak} e^{j\omega t} = j\omega \dot{x}_{ak} e^{j\omega t} = \ddot{x}_{ak} e^{j\omega t} \tag{A3.65}$$

The wet surface boundary condition says, that the fluid motion at a surface is perpendicular to that surface.

$$\frac{\partial \Phi_k}{\partial n} = \dot{x}_k \frac{\partial k}{\partial n} = \dot{x}_{ak} e^{j\omega t} \frac{\partial k}{\partial n} \tag{A3.66}$$

From the combination of eqs. (A3.62) and (A3.66) one finds

$$\frac{\partial \phi_k}{\partial n} = \frac{\partial k}{\partial n} \tag{A3.67}$$

The time derivative of the velocity potential (eq. (A3.62)) is in direction 1

$$\frac{\partial \Phi_1}{\partial t} = \phi_1 j\omega \, \dot{x}_{a1} \, e^{j\omega t} \qquad \text{(A3.68)}$$

Inserting eqs. (A3.66) and (A3.67) in eq. (A3.60) gives the reaction force in the k degree of freedom, caused by a motion in direction 1, as follows

$$F_k = -\rho \, j\omega \, \dot{x}_{a1} \, e^{j\omega t} \int_{S_o} \phi_1 \frac{\partial \phi_k}{\partial n} \, dS \qquad \text{(A3.69)}$$

To get beyond this we have to apply more boundary conditions. This will not be done, instead the reader is advised to study the references mentioned earlier.

The reaction force, F_k, is split up into two parts, one including the acceleration and the other including the velocity.

$$F_k = a_{k1} \, \ddot{x}_k + b_{k1} \, \dot{x}_k \qquad \text{(A3.70)}$$

Then one finds from the identification of the terms in eq. (A3.70) with the use of eqs. (A3.64) and (A3.65), and a further combination of the result with eq. (A3.69) that

$$a_{k1} = \text{Re} \left(\rho \int_{S_o} \phi_1 \frac{\partial \phi_k}{\partial n} \, dS \right) \qquad \text{(A3.71)}$$

$$b_{k1} = \text{Im} \left(\rho\omega \int_{S_o} \phi_1 \frac{\partial \phi_k}{\partial n} \, dS \right) \qquad \text{(A3.72)}$$

where

a_{k1} is the added mass, which causes a force in direction k due to an acceleration in direction 1.

b_{k1} is the radiation damping, which causes a force in direction k due to velocity in direction 1. It was discussed further in the foregoing chapter A3.2.1b.

We have now derived general expressions for both added mass and radiation damping in an inviscid fluid. Some applications of the added mass will be shown.

We will in the following primarily discuss bodies in inviscid fluid, both floating and submerged. Thus we will discuss the radiation problem for the floating body. But, it is of course also of interest that the viscous effects are discussed.

Surface effects

We will start by discussing a floating body. If the radiation damping is derived, as done in the previous chapter, then it can be shown that the complete hydrodynamic properties of the body are known. This will be discussed in the following assuming that the requirements of the Kramers-Kronig relations are fulfilled. (These relations are discussed in Van Wijngaard (1963).)

When the retardation function R(t) is known, eq. (A3.38), and derived from a known radiation damping, $b(\omega)$, then it is possible to derive the added mass $a(\omega)$, based on the hydrodynamic equation, eq. (A3.8). For the added mass one deduces from R(t) that

$$a(\omega) = a_\infty - \frac{1}{\omega} \int_0^\infty R(t)\, \sin(\omega t)\, dt \qquad \text{(A3.73)}$$

where a_∞ is the added mass at infinite frequency.

The radiation damping will in a similar way be expressed as

$$b(\omega) = \int_0^\infty R(t)\, \cos(\omega t)\, dt \qquad \text{(A3.74)}$$

which is the Fourier inversion of eq. (A3.38).

We now have the hydrodynamic coefficients in terms of velocity potentials, eqs. (A3.71) and (A3.72), and in terms of the retardation function, eqs. (A3.73) and (A3.74). Out of these two sets of equations, the latter could be used to establish the coefficients. For calculations of the added mass and radiation damping the first set is used, however.

Hooft (1982) has in chapter 3.5 done a very thorough examination of the influence of the free surface, which is recommended to read if a more complete understanding is wanted.

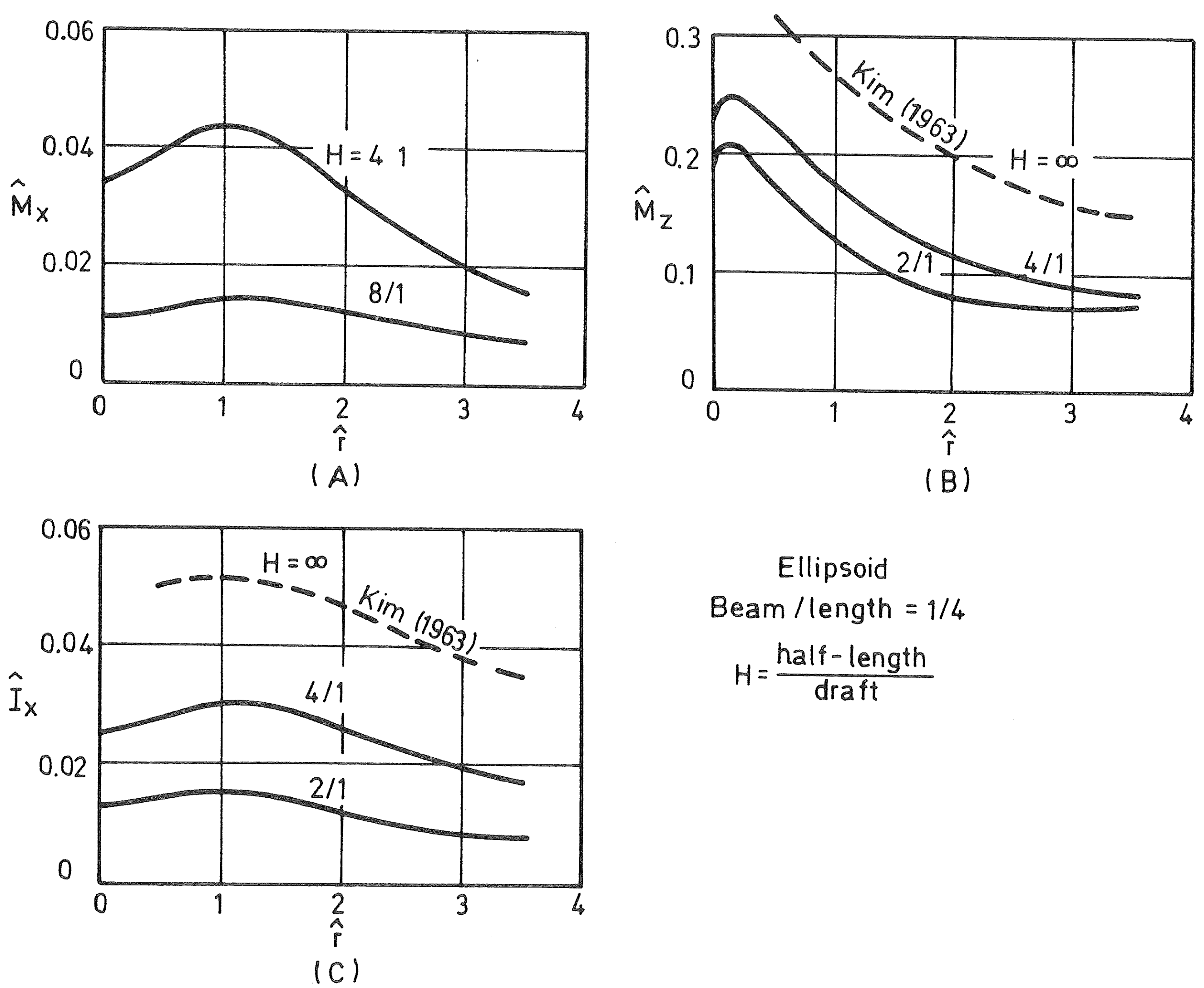


Fig. A3.20 Added mass of ellipsoids for;
(A) surge (or sway) $\hat{M}_x = M_x/\rho r^3$
(B) heave $\hat{M}_z = M_z/\rho r^3$ and
(C) roll (or pitch) $\hat{I}_x = I_x/\rho r^4$
which all are functions of $\hat{r} = r\omega^2/g$.
(From Kim (1965))

It is now possible to derive the added mass for floating structures. This can be done for 3-D structures with the use of Eq. (A3.38) inserted in Eq. (A3.73) and some knowledge of the asymptotic behaviour of $a(\omega)$, see Figure A3.20. If the 3-D body is rotary symmetric about the horisontal axes, then it is possible to derive $a(\omega)$. This is done with the use of Eqs. (A3.47) to (A3.50) inserted in Eq. (A3.73) and knowing the asymptotic values.

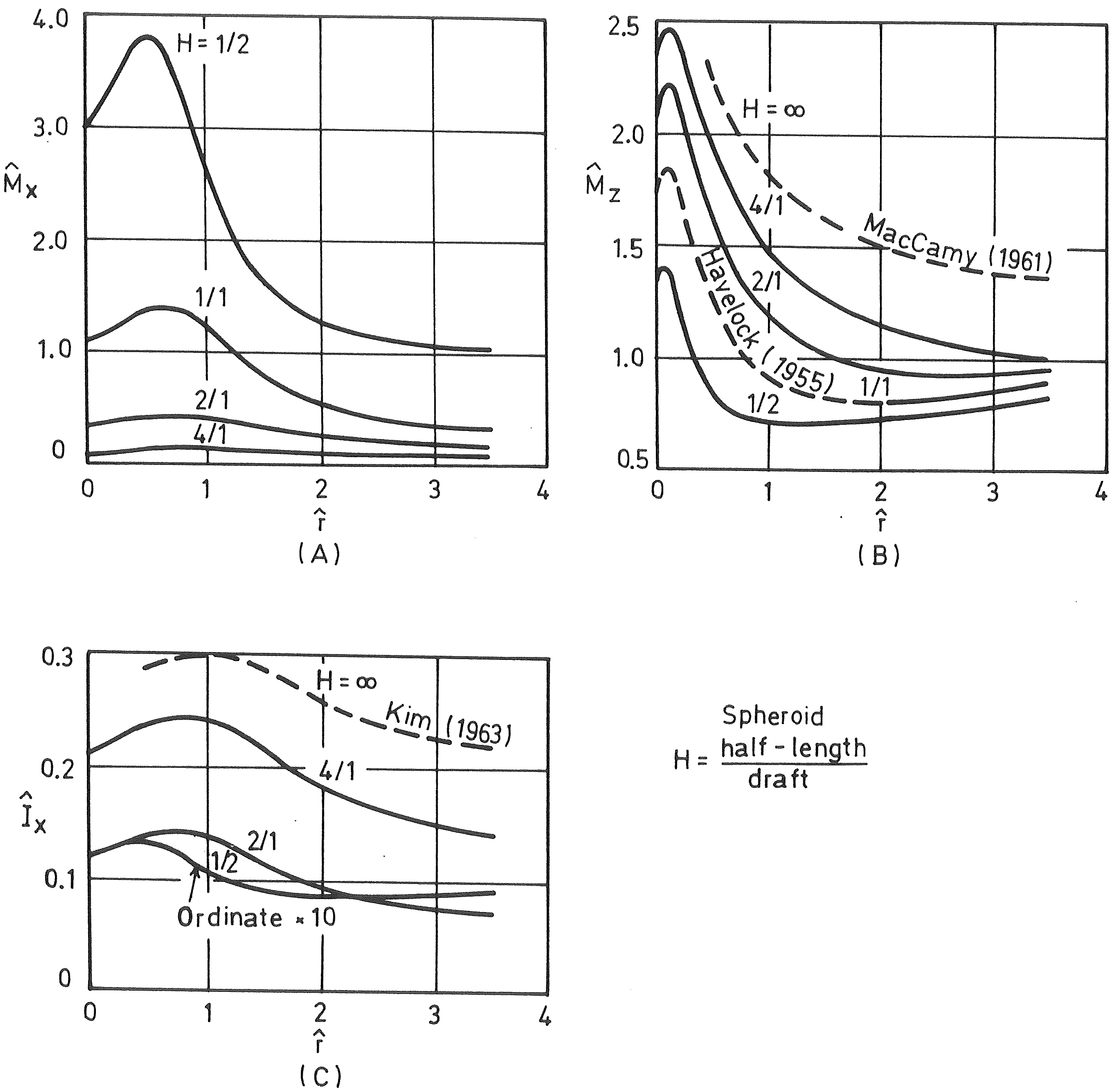


Fig. A3.21 Added mass of spheroids for;
(A) surge (or sway) $\hat{M}_x = M_x/\rho r^3$
(B) heave $\hat{M}_z = M_z/\rho r^3$ and
(C) roll (or pitch) $\hat{I}_x = I_x/\rho r^4$
which all are functions of $\hat{r} = r\omega^2/g$.
(From Kim (1965))

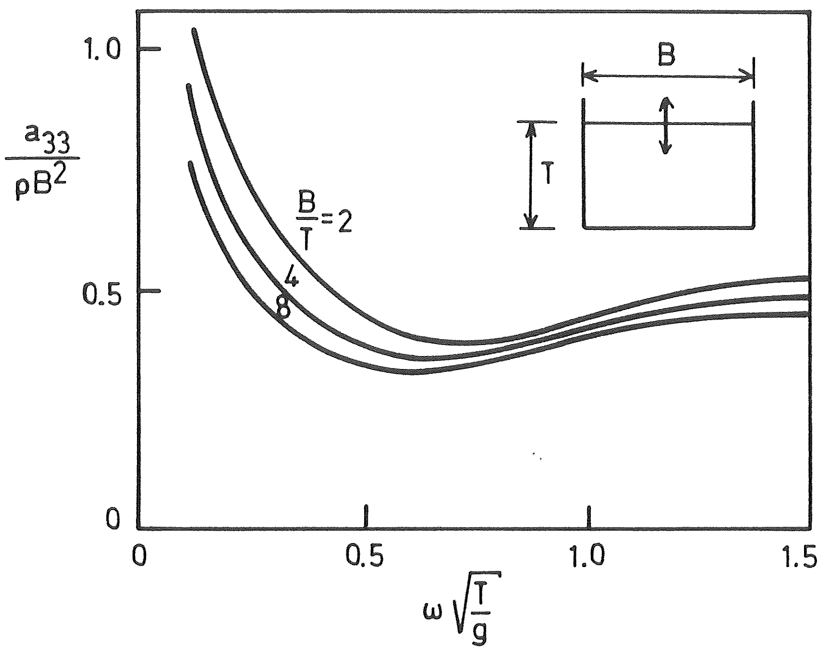


Fig. A3.22 Added mass coefficient for a family of 2-D boxes, heaving in deep water. (From Newman (1980))

Another family of bodies that were mentioned in chapter A3.2.1b are the 2-D bodies which are symmetric about x=0. Here are shown 2 types of shapes, namely boxes, in figure A3.22, and horizontal cylinders, in figure A3.23. These curves can be derived using eq. (A3.51) in eq. (A3.73) and knowing the asymptotic values.

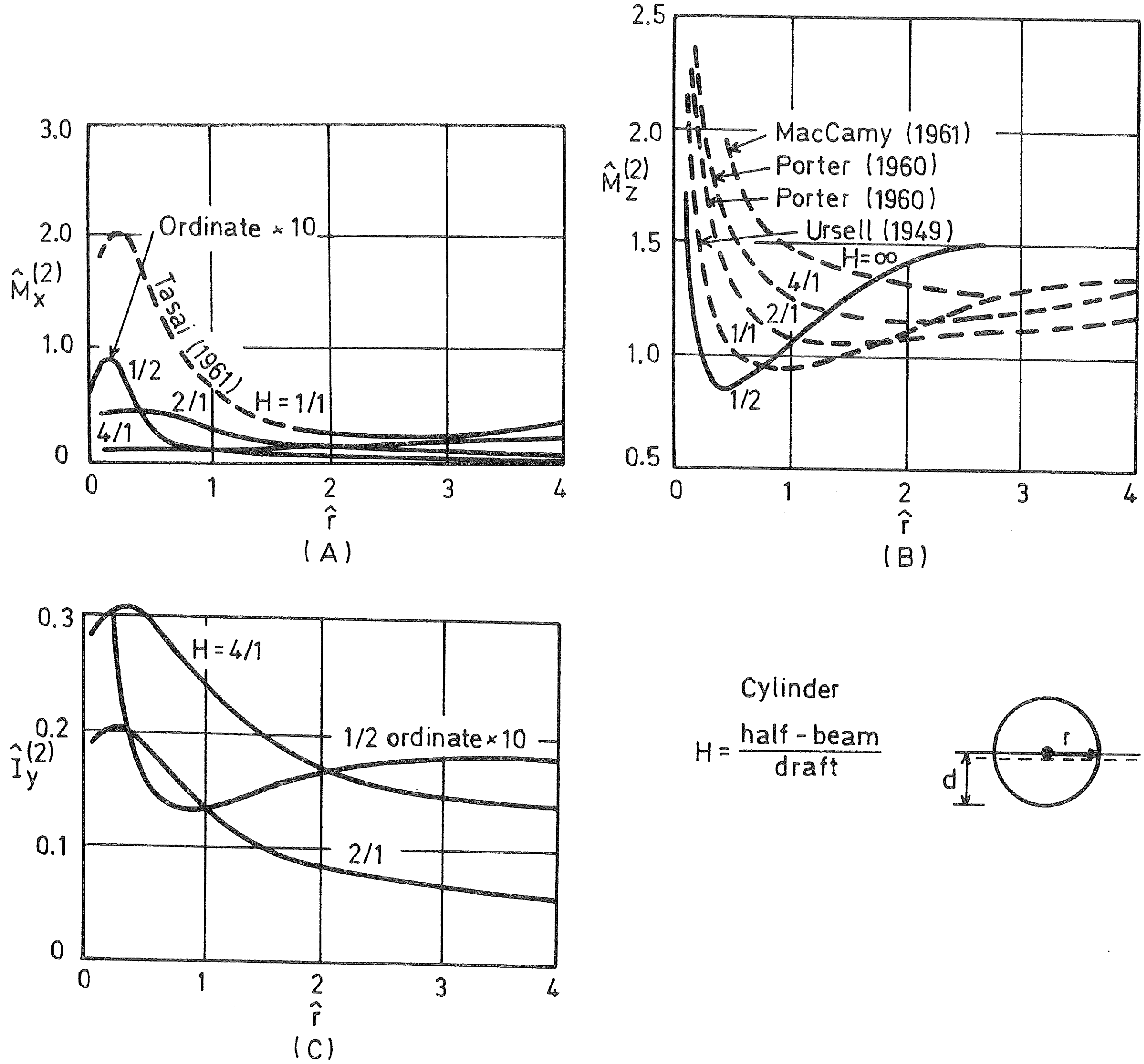


Fig. A3.23 Added mass of horizontal cylinders (2D);
(A) surge $\hat{M}_x^{(2)} = M_x^{(2)}/\rho r^2$
(B) heave $\hat{M}_z^{(2)} = M_z^{(2)}/\rho r^2$ and
(C) pitch $\hat{I}_y^{(2)} = I_y^{(2)}/\rho r^3$
which all are functions of $\hat{r} = r\omega^2/g$.
(From Kim (1965))

Immersed structures

The bodies that will be discussed here, are positioned so far from any boundary that they will not be affected by them. Being far away from the free surface means that the added mass will be constant. Furthermore, being far away from any other boundary (such as the bottom) means that the fluid moved by the body will not affect the fluid in the far distance boundaries.

We will not derive any other expressions for the added mass than that of eq. (A3.71). If wanted, this expression can be further developed using the Laplace equation on the surrounding fluid.

$$\Delta\Phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi = 0 \qquad (A3.75)$$

The added mass has been derived for various bodies. Values are presented by for instance Korvin-Kroukovsky (1961).

It is common to normalize the added mass by the body mass (mass of the displaced volume), and calling it the added mass coefficient

$$C_m = a_T / (\rho V) \qquad (A3.76)$$

or

$$C_m = a_R / (\rho V \, r_m^2) \qquad (A3.77)$$

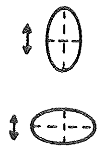
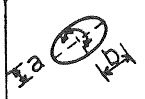
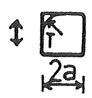
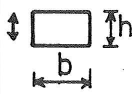
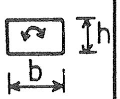
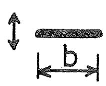
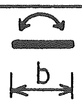
where a_T is added mass (translation)
a_R is added inertia (rotation)
ρ is density of surrounding fluid
V is volume of the body
r_m is radius of inertia.

Note that in the literature C_M sometimes denotes the added mass plus the mass of the displaced fluid, thus

$$C_M = 1 + C_m \qquad (A3.78)$$

Values of C_m are presented in table A3.3, which is from Korvin-Kroukovsky (1961). But, occurring approximations are proposed by the author.

Table A3.3 Added mass coefficients for various 2-D bodies. Translation a_T, Rotation a_R.

Body shape: Description	Body shape: Sketch	Dimension ratio	Added mass coefficient $C_m = a_T/\rho V$; $C_m = a_r/\rho V r^2$
Circle			1
Ellipse			a/b
			$(a^2-b^2)^2/(ab(a^2+b^2))$
Square with or without corner radius		$0 \leq r/a \leq 1$	$1.2 - 0.12(r/a) - 0.08(r/a)^2$ or approximation $1.2-0.2(r/a)$
			0.26
Rectangle		$1/10 \leq b/h \leq 10$	b/h: 0.1, 0.2, 0.5, 1, 2, 5, 10 C_m: 0.18, 0.31, 0.67, 1.19, 2.14, 4.75, 8.95 or approximately $1.02\ (b/h)^{0.82} + 0.19(b/h)$
		 $0.1 \leq b/h \leq 0.5$ $2. \leq b/h \leq 10.$	b/h: 0.1, 0.2, 0.5, 1.0, 2, 5, 10 C_m: 3.43, 1.70, 0.57, 0.26, 0.57, 1.70, 3.43 approximately $0.353((b/h)^3\ (1+(h/b)^2)^{-1}$ $0.353((h/b)^3(1+b/h)^2))^{-1}$
Plate			$a_T = (\rho\pi/4)b^2$
			$a_r = (\rho\pi/64)b^4$

Source: Korvin-Kroukovsky (1961) and the author's approximations.

Effects of oscillation

As we are interested in fluid oscillating around the bodies both the Keulegan-Carpenter number (KC), and the surface roughness (k/D) are of interest. In Sarpkaya (1976) model tests are carried out for various KC, Re, k/D, on a vertical cylinder. The model tests are based on undisturbed rectilinear flow, and are suggested by Sarpkaya to be considered as an upper limit for the wave motion. These tests are referred earlier, in chapter A3.2.1b, where they also are explained.

It is seen in Figure A3.24 that the value of table A3.3 is an underestimate of the added mass in an oscillating viscid fluid.

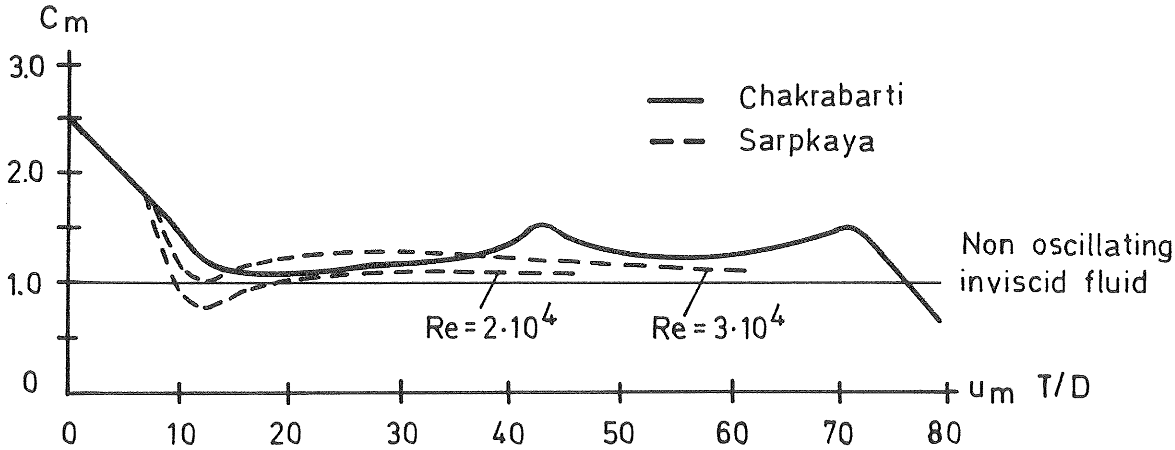


Fig. A3.24 Comparison of Chakrabarti's wave tank tests and Sarpkaya's 2-D flow test. (From Chakrabarti (1980))

A3.2.2 Wave force

A body that is fixed will experience hydrodynamic forces from the surrounding fluid, when that fluid is moving. These forces are of two kinds, namely shear forces and pressure forces.

Pure shear force are excerted on the body by flow parallel to the body surface. This is commonly called skin friction, and it only exists in a viscid fluid.

Pressure forces are excerted on the body perpendicular to the body surface. These forces exist both in viscid and inviscid fluid.

In the following the forces are presented divided into linear forces and higher order forces. Linear wave forces are caused by fluid pressure. Higher order forces can be caused by both shear force, pressure forces, and the combination of them.

A3.2.2a Linear wave forces

The only first order wave forces that are excerted on the body are those caused by the linear wave pressure. The linearised wave pressure is usually expressed as follows

$$p = -\rho \frac{\partial \Phi}{\partial t} \qquad z<0; \quad z<\zeta$$

$$p = +\rho g \xi \qquad 0<z<\zeta; \; \zeta<z<0 \tag{A3.79}$$

where Φ is the wave velocity potential
ξ is the wave surface and
z is the vertical coordinate, positive upwards and zero at mean sea surface.

When discussing wave forces, the structure is supposed to be fixed (as mentioned earlier). The linear potential around the structure is then the sum of that from the incoming wave, Φ_I, and that from the scattered wave

$$\Phi = \Phi_I + \Phi_S \tag{A3.80}$$

On the surface of the body the velocity of the incoming wave and the scattered wave are equal in values but opposite in direction.

$$\left(\frac{\partial\Phi_I}{\partial \underline{n}}\right)_S = -\left(\frac{\partial\Phi_s}{\partial \underline{n}}\right)_S \qquad \text{(A3.81)}$$

where $\underline{n}$ is the local normal vector of the surface of the body, directed outwards. In this way we also satisfy the boundary condition, that the velocity normal to the body surface is zero at the body surface.

$$\left(\frac{\partial\Phi}{\partial \underline{n}}\right)_S = 0 \qquad \text{(A3.82)}$$

The linear wave force on a fixed structure is obtained by integrating the linear wave pressure (Eq. A3.79) over the mean wet surface, S_o.

$$F = \iint_{S_o} \rho g \zeta \underline{n}_o \, dS + \iint_{S_o} \rho \frac{\partial\Phi}{\partial t} \underline{n}_o \, dS =$$

$$= R(0,0,\rho g \Delta V_\zeta) + \iint_{S_o} \rho \frac{\partial\Phi}{\partial t} \underline{n}_o \, dS \qquad \text{(A3.83a)}$$

Here the first term is the hydrostatic restoring force (chapter A3.2.1.a), acting only on that part of the body that is between the wave surface and the mean sea level. This means that ΔV_ζ is

$$\Delta V_\zeta = V(z=\zeta) - V(z=0) - \zeta \cdot A(z=0) \qquad \text{(A.83b)}$$

where A is the horizontal projection of the body. Then clearly the body will only experience a force if the water plane area changes.

As noted earlier the problem is to solve the scattered wave potential problem. This can be done by the Green's function method (see Hooft (1982) and Chakrabarti (1980)). The Green's function, G, is then chosen as a singular potential which satisfies the Laplace equation (A3.75) in the flud. On the boundaries the same boundary conditions as the scattered wave potential, Φ_s, are satisfied. If Green's theorem is applied to Φ_s and the spatial part of Green's function, G_X, and if the body surface boundary condition is satisfied, then the two-dimensional Fredholm integral equation, solving the scattered wave problem, is obtained.

$$\Phi_s = \frac{1}{2\pi} \iint (\Phi_s \frac{\partial G_X}{\partial n_o} + G_X \frac{\partial \phi_I}{\partial n_o})\ dS \qquad (A3.84)$$

The equation is solved numerically for Φ_s on the surface of the object.

The potential of the incoming wave is

$$\Phi_I = \frac{g\xi}{\omega} \frac{\cosh(k(d+z))}{\cosh(kd)} \cos(\omega t - kx) \qquad (A3.85)$$

Here ω is the angular frequency of the wave
k is the wave number,

$$k = \frac{2\pi}{\lambda} \qquad (A3.86)$$

λ is the length of the wave and
d is the water depth.

The first order wave exciting force can be written in terms of the two wave potentials as follows

$$\underline{F} = \iint_{S_o} \rho \frac{\partial \Phi_I}{\partial t} \underline{n}_o\ dS + \iint_{S_o} \rho \frac{\partial \Phi_s}{\partial t} \underline{n}_o\ dS \qquad (A3.87)$$

The first term is called the Froude-Krylov force and the second term is the diffraction force. In general, this force can only be derived in a numerical way, as mentioned earlier.

Eq. (A3.87) can be approximated assuming small body theory. This assumption is valid if the wave potential is constant over the body. If this is the case then the Froude-Krylov force is

$$\underline{F}_{fk} = \iint_{S_o} \rho \frac{\partial \Phi_I}{\partial t} \underline{n}_o\ dS \qquad (A3.88a)$$

which in all directions, except in the vertical direction of a floating body, can be linearized as

$$\underline{F}_{fk} = \rho V \dot{\underline{u}} \qquad (A3.88b)$$

where $\dot{\underline{u}}$ is the wave acceleration.

In the vertical direction the force is expressed differently due to that the force only acts on the wetted surface. In deep waterwaves the force is

$$(F_{fk})_z = \rho g \zeta \int_{-d}^{0} e^{kz} \, dA_z(z)$$

where ζ is the wave elevation
A_z is the horizontal projection of the body surface, S,
z is the vertical coordinate of the body surface, S.

The diffraction force, on the other hand, can be approximated in all degrees of freedom as

$$\underline{F}_D = \iint_{S_o} \rho \frac{\partial \Phi_s}{\partial t} \underline{n}_o \, dS = \underline{\underline{a}} \, \underline{\dot{u}} + \underline{\underline{b}} \, \underline{u} \qquad \text{(A3.89)}$$

where $\underline{\underline{a}}$ and $\underline{\underline{b}}$ are the added mass and the radiation damping respectively.

The linear wave force on small bodies is inserting Eqs. (A3.88) and (A3.89) in Eq. (A3.83), as follows

$$\underline{F} = R(0,0,\rho g \Delta V_\zeta) + \rho V \underline{\dot{u}} + \underline{\underline{a}} \, \underline{\dot{u}} + \underline{\underline{b}} \, \underline{u} \qquad \text{(A3.90)}$$

The hydrodynamic coefficients to be used are derived in chapters A3.2.1b and c.

This concludes the chapter on linear wave forces, and leads us further to the higher order wave exciting forces. These higher order forces will in most cases give a significant contribution to the wave exciting force, but more about this in the next chapter.

A3.2.2.b Second and higher order wave forces

Wave forces of second and higher order can be divided into two families.

- Those caused by the fluid viscosity, the damping force
- Those caused by the fluid pressure.

These are of varying magnitude depending on both the fluid motion and the body.

Viscous damping

The damping force due to the fluid viscosity is of major interest only when the fluid velocity is large enough. This force acts in the same direction as the fluid velocity and can be expressed as follows

$$F_D = \frac{1}{2} \rho \underline{u} u \quad A C_D \qquad \text{(A3.91)}$$

where u is the fluid velocity,
A is the projected area in the fluid direction and
C_D is the drag coefficient.

All of this viscous damping problem is already analysed in chapter A3.2.1b. The second order viscous force is therefore not further discussed.

Pressure force

The second and higher order pressure forces can be divided in one fluid pressure part, p, and one body surface location part $\underline{n}$.

The fluid pressure is to the second order

$$P = - \rho g z - \rho \frac{\partial}{\partial t} (\Phi_1 + \Phi_2) - \frac{1}{2} \rho (\nabla \Phi_1)^2$$

$$- \rho (\underline{s}_1 \cdot \nabla \frac{\partial \Phi_1}{\partial t}) + 0(3) \qquad \text{(A3.92)}$$

or divided into zeroth order pressure

$$p_o = -\rho g z \qquad \text{(A3.93)}$$

first order pressure, Eq. A.79,

$$p_1 = - \rho \frac{\partial \Phi_1}{\partial t} \qquad \text{(A3.94)}$$

and second order pressure

$$p_2 = - \frac{1}{2} \rho (\nabla \Phi_1)^2 - \rho \frac{\partial \Phi_2}{\partial t} - \rho (\underline{s}_1 \cdot \nabla \frac{\partial \Phi_1}{\partial t}) \qquad \text{(A3.95)}$$

The velocity potential is also divided in first and second order terms as

$$\Phi = \Phi_1 + \Phi_2 \tag{A3.95}$$

where ϕ_1 is found in Eq. (A3.85), and ϕ_2 is the second order wave potential, as defined by Stokes.

$$\Phi_2 = \frac{3}{8} k\ c\ \zeta^2 \frac{\cosh(2k(z+d))}{\sinh^4(kd)} \cos(2(kx-\omega t) \tag{A3.96}$$

as presented in Wiegel (1964).

Inserting the abbreviations from the zeroth to the second order pressure terms, the pressure can be written as follows

$$p = p_o + p_1 + p_2 + O(3) \tag{A3.97}$$

When the body is rotating then the local normal vector, $\underline{n}$, of the surface changes. From chapter A2.1 we know that the application of the transformation matrix on the normal vector will give the change due to the rotation. In the same way as the pressure, this can be written as different orders of rotation.

$$\underline{n} = \underline{n}_o + \underline{n}_1 + \underline{n}_2 + O(3) \tag{A3.98}$$

The pressure force can be deduced by integrating the pressure, directed normal into the body, over the total wet surface

$$\underline{F} = - \iint_S p\ \underline{n}\ dS \tag{A3.99}$$

Inserting Eqs. (A3.97) and (A3.98) into Eq. (A3.99) gives

$$\underline{F} = - \iint_S (p_o + p_1 + p_2 + O(3))(\underline{n}_o + \underline{n}_1 + \underline{n}_2 + O(3))dS \tag{A3.100}$$

out of which we can get the second order pressure force by integrating all products of the pressure, p, and normal vector, $\underline{n}$, which give second order contributions over the mean wet surface, S_o. Then the first order wave pressure is integrated over the oscillating surface, S_1.

$$\underline{F}_2 = - \iint_{S_o} (p_o \underline{n}_2 + p_1 \underline{n}_1 + p_2 \underline{n}_o) dS - \iint_{S_1} (p_1 \underline{n}_o) \, dS \qquad \text{(A3.101)}$$

After some algebra (Hooft (1982)), and assuming small enough rotations to neglect $p_o \underline{n}_2$, then it is possible to rewrite Eq. (A3.101) with the use of Eqs. (A3.94) and (A3.95) as follows.

$$\underline{F}_2 = - \int_{l_{wl}} \frac{1}{2}\rho g \zeta_{1r}^2 \underline{n}_o \, dl + \Delta \underline{\underline{T}} \; \underline{\underline{m}} \; \ddot{\underline{s}}_1 + \iint_{S_o} \frac{1}{2} \rho |\nabla \Phi_1|^2 \underline{n}_o dS + \iint_{S_o} \rho \frac{\partial \Phi_2}{\partial t} \underline{n}_o \, dS + \iint_{S_o} \rho (\underline{s}_1 \nabla \frac{\partial \Phi_1}{\partial t}) \underline{n}_o \, dS \qquad \text{(A3.102)}$$

where

ζ_{1r}	is the relative wave height $\zeta_{1r} = \zeta_1 - z_{1wL}$
z_{1wL}	is the z-coordinate of the static equilibrium water line,
ζ_1	is the first order wave elevation,
l_{wl}	is the length of the water line
$\Delta \underline{\underline{T}}$	is the first order transformation matrix (see chap. A2.1.2)
$\underline{\underline{m}}$	is the mass matrix of the structure
$\ddot{\underline{s}}_1$	is the first order acceleration of the structure

and the rest of the terms are explained earlier in this chapter. Usually the first term in Eq. (A3.102) is the dominant term for surface piercing bodies, but if the body rotates a lot then second term also will be significant in magnitude.

A3.2.3 Current force

The current forces are the forces caused by a stationary flow around the body. These are equivalent to the viscous damping force and the wave making resistance, which are presented in chapter A3.2.1. Normally the current speed is not of that magnitude that it will give a significant wave making resistance. The wave making resistance will therefore not be further discussed.

The current force is expressed in the same way as in Eq. (A3.91).

$$F_C = \frac{1}{2} \rho \underline{u} u A C_D \qquad \text{(A3.103)}$$

where ρ is the surrounding fluids density

u is the free stream velocity,

A is the area projected normal to the free stream velocity and

C_D is the drag coefficient

A3.2.4 Wind force

The wind, being a fluid in motion, is of course gouverned by the same equations as the water. It is even assumed to be incompressible in the subsonic region. The only difference is the free surface boundary conditions, and therefore we will not have any diffraction forces caused by the wind. But, it will exist a fluid inertia force and a viscous damping force. Due to the very low density of air the inertia force will be very small, and is neglected here .

The wind force is considered to be caused only by the viscous drag force on the structure, and is expressed in the same way as the current force.

$$F_W = \frac{1}{2} \rho \underline{u} u A C_D \qquad \text{(A3.104)}$$

where ρ is the air density,
u is the free stream velocity,
A is the area projected normal to the free stream velocity and
C_D is the drag coefficient.

As long as the air can be regarded as incompressible, then the drag coefficients are the same as the ones valid in water.

A3.3 Mooring forces

A3.3.1 General aspects

The main problem dealing with catenary moorings and mooring forces, is whether they should be thought of as quasi-static or not. This of course depends on what is demanded from the analysis. It is therefore relevant to start with a little bit of when, where and how the analysis shall be performed.

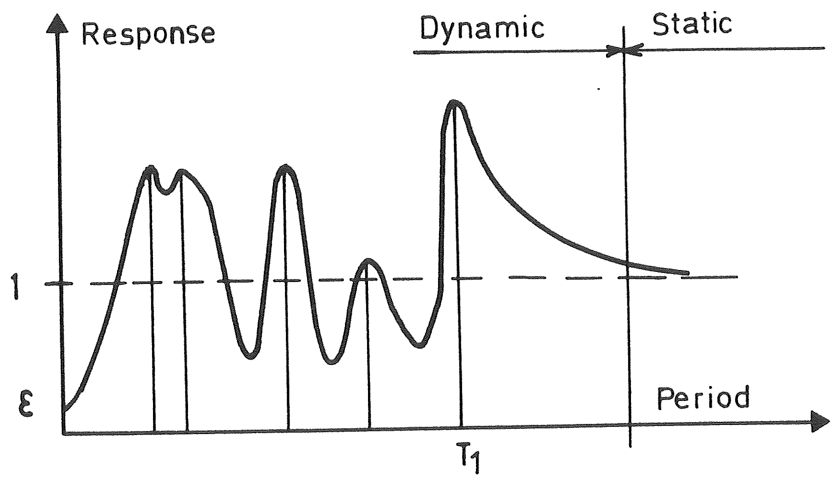


Fig. A3.25 Typical motion amplitude response curve for a mooring line

It is appropriate to use dynamic analysis on a mooring cable, when the excitation period is short enough to approach the highest eigenperiod of the cable, T_1, see Fig. A3.25. Still slower excitations will be more gouverned by the stiffness and then the deformations will follow the excitation. Note that an oscillation that appears to be slow, can be composed of several faster oscillations and should therefore be dynamically analysed.

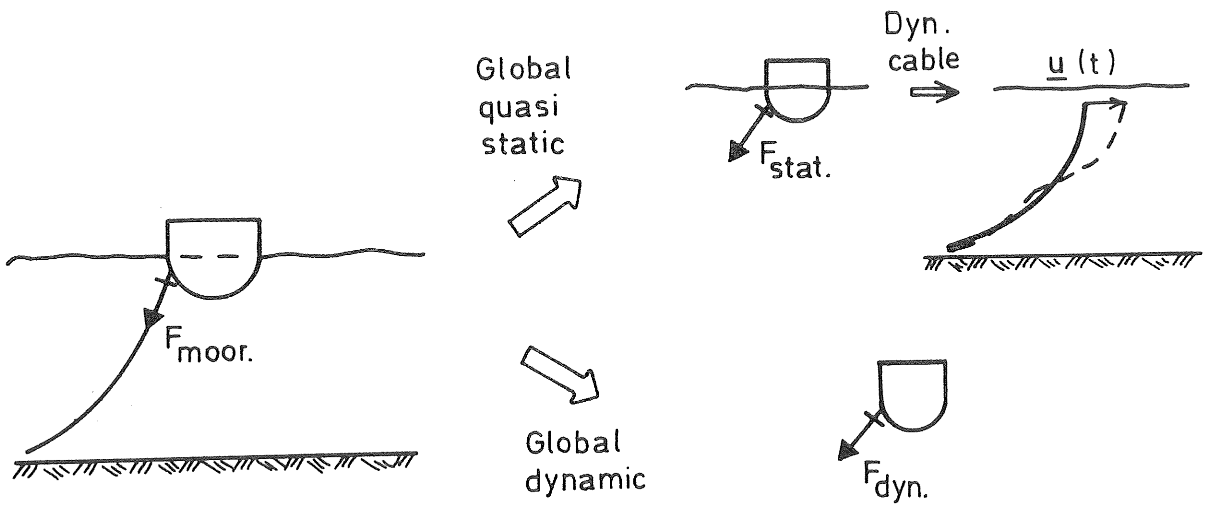


Fig. A3.26 Schematic interpretations of where to use dynamic mooring analysis

Normally it is assumed that dynamics of the cable does not influence the moored body. The mooring force is then considered as static. This is true if the structure is large enough, but for smaller structures the cable dynamics may influence the motion of the moored body.

There are two ways of analysing the cable:

- (Quasi-) Static analysis
- Dynamic analysis

Which one to use is decided by the oscillation period. If the excitation is in the region where the cable acts dynamic, then the dynamic analysis should be used.

The dynamic analysis will be described in the following chapter. Some static solutions of the equation of motion of the cable will also be shown .

A3.3.2 Cable equation of motion

Mooring cables are considered to be so long and slender that any bending resistance can be neglected. They will therefore respond to external loads, in the normal direction, by a significant change of shape. This property may give rise to major geometrical non-linearities. There are also other non-linearities to be considered, such as the dragforce and the sea bed contact.

The fluid loading on the cable is based on the slender (small) body approximation of the fluid loading equations derived in chapter A3.2, i.e. the Morison equation.

Lindahl (1981) presents a very comprehensive way of deriving the equations of motion, which largely will be shown here.

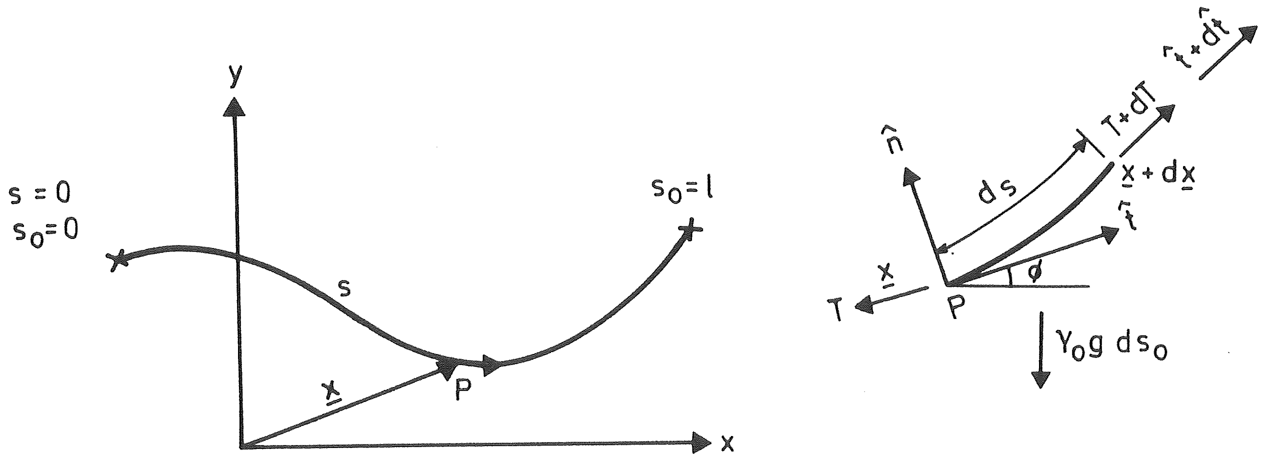


Fig. A3.27 An axially elastic cable

Consider an elastic cable moving in the x-y plane. A position P on the cable has the position $\underline{x} = (x(s_o,t);\ y(s_o,t))$ (see Fig. A3.27), where s_o is the unstretched cable length from $s_o = 0$ to P, and t is time. These two variables, s_o and t, are the two independent variables in which we want to have our expressions.

In a cable element ds the mass is assumed to be constant

$$m\, ds = m_o\, ds_o \qquad (A3.105)$$

where m is cable mass per stretched unit length
m_o is cable mass per unstretched unit length

As mentioned earlier, the bending resistance is neglected. The stress will therefore be constant over the cable cross section and the cable tension T(s,t) is expressed as

$$T = K\varepsilon \qquad (A3.106)$$

where K is the cable stiffness

$$K = EA \qquad (A3.107)$$

E is the modulus of elasticity
A is the cross sectional area
ε is the tangential elongation.

The stretch is defined as

$$\varepsilon = \frac{ds - ds_o}{ds_o} \qquad (A3.108)$$

Eqs. (A3.106) and (A3.108) implies that the cable force T is proportional to the elongation of the cable, which is Hooke´s law. Using Hooke´s law puts practical limitations on the deformations, because the equations will be strongly non-linear. It is therefore useful to derive another expression for the elongation.

Eq. (A3.108) can be rewritten as

$$\varepsilon + 1 = \frac{ds}{ds_o} \qquad (A3.109)$$

Square both sides and neglect higher order terms of ε ($\varepsilon^2 << \varepsilon$ if $\varepsilon << 1$), then another expression of the elongation is obtained

$$\tilde{\varepsilon} = \frac{1}{2}\left(\frac{ds^2}{ds_o^2} - 1\right) \qquad (A3.110)$$

This Eq. (A3.110) is Green's equation of elongation.

Approximating ds to a straight line and using Pythagora's proposition on the arch element in Fig. A3.27, the following relation is obtained

$$ds^2 = d\underline{x} \cdot d\underline{x} \qquad \text{(A3.111)}$$

Using this and denoting

$$\frac{d\underline{x}}{ds_o} = \underline{x}' \qquad \text{(A3.112)}$$

then the stretch expressions of Eqs. (A3.108) and (A3.110) can be rewritten as

$$\varepsilon = (\underline{x}' \cdot \underline{x}')^{1/2} - 1 \qquad \text{(A3.113)}$$

and

$$\tilde{\varepsilon} = \frac{1}{2} (\underline{x}' \cdot \underline{x}' - 1) \qquad \text{(A3.114)}$$

The work dW that is made by the force T on the element ds is equal to the elongation energy of the element ds. Integrating dW over the entire cable length, l, then gives the total elongation energy.

$$V_t = \int_0^l dW = \int_0^l (\frac{1}{2} K\varepsilon)\varepsilon \, ds_o = \frac{1}{2} K \int_0^l \varepsilon^2 ds_o \qquad \text{(A3.115)}$$

This is for a uniform cable, assuming constant stiffness K over l. If the stretch is small then $\varepsilon \cong \tilde{\varepsilon}$ and Eq. (A3.115) can be used in combination with both Eqs. (A3.113) and (A3.114).

The kinetic energy of the cable is, using Eq. (A3.105)

$$U = \int_0^{l+\varepsilon_l} \frac{1}{2} m \, ds \, \dot{\underline{x}} \cdot \dot{\underline{x}} = \frac{1}{2} m_o \int_0^l \dot{\underline{x}} \cdot \dot{\underline{x}} \, ds_o \qquad \text{(A3.116)}$$

Other forces $\underline{f}$ per unit length will give the potential energy

$$V_1 = \int \underline{f} \cdot \underline{x} \, ds_o \qquad \text{(A3.117)}$$

where the other forces are body forces, gravity, surface forces, and hydrodynamic forces.

The total potential energy is the sum of stretch work and the potential energy from other forces.

$$V = V_t + V_1 \tag{A3.118}$$

Knowing the kinetic and potential energy of the cable, it is possible to derive the dynamic equilibrium equations (Euler Lagrange equations). This is done using Hamilton´s principe (see Lindahl (1981)), which results in the Euler Lagrange equation.

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \underline{\dot{x}}}\right) + \frac{\partial}{\partial s_o}\left(\frac{\partial L}{\partial \underline{x}'}\right) - \frac{\partial L}{\partial \underline{x}} = 0 \tag{A3.119}$$

where the Lagrange density is

$$L = \tfrac{1}{2} m_o \underline{\dot{x}} \cdot \underline{\dot{x}} + \underline{f} \cdot \underline{x} - \tfrac{1}{2} k\varepsilon^2 \tag{A3.120}$$

Making the derivatives in Eq. (A3.119) on the Lagrangian of Eq. (A3.120) give two types of equations of motions.

$$m_o \underline{\ddot{x}} - \frac{\partial}{\partial s_o}\left(K\varepsilon \frac{\underline{x}'}{1+\varepsilon}\right) - \underline{f} = 0 \tag{A3.121}$$

with ε from Eq. (A3.113)

$$m_o \underline{\ddot{x}} - \frac{\partial}{\partial s_o}\left(K\tilde{\varepsilon}\underline{x}'\right) - \underline{f} = 0 \tag{A3.122}$$

with $\tilde{\varepsilon}$ from Eq. (A3.114).

In Eqs. (A3.121) and (A3.122) the partial differential equations of an axially elastic cable are presented. The difference between them is the direction in which the tensional force, $k\varepsilon$ or $k\tilde{\varepsilon}$, acts. In Eq. (A3.121) the force acts in the tangential direction of the stretched cable

$$\hat{t} = \frac{\partial \underline{x}}{\partial s} = \frac{\partial \underline{x}}{(1+\varepsilon)\partial s_o} = \frac{\underline{x}'}{1+\varepsilon} \tag{A.123}$$

while in Eq. (A3.122) the force acts in the tangential direction of the unstretched cable.

$$\hat{t}_o = \frac{\partial \underline{x}}{\partial s_o} = \underline{x}' \tag{A3.124}$$

If the tensional force is to act in the accurate stretched direction (Eq. (A2.123)), then the tensional force in Eq. (A3.122) must be

$$T = K\tilde{\varepsilon}(1+\varepsilon) \tag{A3.125}$$

If the tensional force follows this relation, then the force is exactly independent of the elongation size. If the elongations are small enough, then the force is independent of stretch type.

$$T = K\tilde{\varepsilon}(1+\varepsilon) \cong K\tilde{\varepsilon} \cong K\varepsilon \tag{A3.126}$$

Assuming that the tensional force follows the relation of Eq. (A3.125), it is possible to use Eq. (A3.122) as the cable equation of motion. Usually the interest is in studying oscillations of a cable about some known configuration, for example the static equilibrium. A reference configuration is then $x_o(s_o)$, and the strain $\tilde{\varepsilon}_o(s_o)$. If $\underline{u}$ is the displacement and $\Delta\tilde{\varepsilon}$ is the incremental strain between the actual configuration and the reference configuration.

$$\underline{x} = \underline{x}_o + \underline{u} \tag{A3.127}$$

$$\tilde{\varepsilon} = \tilde{\varepsilon}_o + \Delta\tilde{\varepsilon} \tag{A3.128}$$

Using these relations in Eq. (A3.122) it is possible to rewrite the equation of motion of a cable in a fluid of stationary motion (see Lindahl (1983)).

$$m_o \underline{\ddot{u}} + C_4(1+\varepsilon)\underline{\ddot{u}} - \frac{C_4}{1+\varepsilon} (\underline{\ddot{u}} \cdot \underline{x}') x' - - \frac{\partial}{\partial s_o} (k\varepsilon \underline{x}') - \underline{f}_r = 0 \tag{A3.129}$$

where $$C_4 = C_{mN} \frac{\pi d_o^2}{4} \rho_v \tag{A3.130}$$

C_{mN} is the added mass coefficient

and

$\underline{f}_r$ is the resultant force acting on the cable

$$\underline{f}_r = \underline{f}_g + \underline{f}_{DT} + \underline{f}_{DN} \qquad \text{(A.131)}$$

$\underline{f}_g$ is the gravity force

$$\underline{f}_g = |0,0, - m_r g|^T$$

m_r is the efficient mass

$$m_r = \frac{\rho_k - \rho_v}{\rho_k} m_o \qquad \text{(A3.132)}$$

ρ_k is the cable density,

ρ_v is the water density.

$\underline{f}_{DT}$ is the tangential drag.

$$\underline{f}_{DT} = C_2 \, |\underline{v}\cdot\underline{x}'| \, (\underline{v}\cdot\underline{x}')\underline{x}' / (1+\varepsilon)^2 \qquad \text{(A3.133)}$$

$$C_2 = \frac{1}{2} C_{DT} \, d_o \, \rho_v \qquad \text{(A3.134)}$$

C_{DT} is the tangential drag coefficient and

d_o is the line diameter.

$\underline{f}_{DN}$ is the normal drag

$$\underline{f}_{DN} = C_3\{\underline{v}\;\underline{v} - \frac{(\underline{v}\cdot\underline{x}')^2}{(1+\varepsilon)^2})^{\frac{1}{2}} \; (\underline{v} - \frac{(\underline{v}\cdot\underline{x}')\underline{x}'}{(1+\varepsilon)^2})\}(1+\varepsilon) \qquad \text{(A3.135)}$$

$$C_3 = \frac{1}{2} \; C_{DN} \, d_o \, \rho_v \qquad \text{(A3.136)}$$

C_{DN} is the normal drag coefficient.

Furthermore

$$\underline{x}' = \underline{x}' + \underline{u}' \qquad \text{(A3.137)}$$

$$\underline{v} = \underline{v}_c - \dot{\underline{u}} \qquad \text{(A3.138)}$$

where $\underline{v}_c$ is the current velocity.

$$\tilde{\varepsilon} = \tilde{\varepsilon}_o + \Delta\tilde{\varepsilon} \qquad \text{(A3.139)}$$

$$\tilde{\varepsilon}_o = \frac{1}{2}(\underline{x}_o'\cdot\underline{x}_o' - 1) \qquad \text{(A3.140)}$$

$$\Delta\tilde{\varepsilon} = \frac{1}{2}\underline{u}'\cdot\underline{u}' + \underline{x}_o'\cdot\underline{u}' \qquad \text{(A3.141)}$$

The force that acts on the moored body is obtained by integrating Eq. (A3.129) over the entire length of the cable, and this gives the tensional force at the attachment point on the body.

$$\underline{T}(s_o=1,t) = \int_0^1 \{m_o\ddot{\underline{u}} + C_4(1+\varepsilon)\ddot{\underline{u}} - \frac{C_4}{1+\varepsilon}(\ddot{\underline{u}}\cdot\underline{x}')\underline{x}' - \underline{f}_r\}ds_o +$$

$$+ \underline{T}(s_o=0,t) \qquad \text{(A3.142)}$$

with $\underline{T}$ from Eq. (A3.125) directed in the stretched tangential direction, Eq. (A3.123).

Static solution

The static solution with the gravity as the only acting force, is obtained from Eq. (A3.142). The horizontal and vertical components are

$$H_o = K\,\varepsilon_o \frac{x_o'}{1+\varepsilon_o} - m_r g a \qquad \text{(A3.143)}$$

$$V_o = K\varepsilon_o \frac{y_o'}{1+\varepsilon_o} - m_r g\, s_o \qquad \text{(A3.144)}$$

The line force is

$$T_o = (H_o^2 + V_o^2)^{\frac{1}{2}} = m_o g(a^2 + s_o^2)^{\frac{1}{2}} \qquad \text{(A3.145)}$$

Furthermore, the line force is obtained from Hooke´s law.

$$T_o = K\varepsilon_o \qquad \text{(A3.146)}$$

The geometry of the curve is obtained by integrating Eqs.(A3.143) and (A3.144) and using the relations in Eqs. (A3-145) and (A3.146) in the following way

$$x_o = \int (a(a^2+s_o^2)^{-\frac{1}{2}} + \frac{m_r ga}{K}) ds_o =$$

$$= a \operatorname{arcsinh}(\frac{s_o}{a}) + \frac{m_r ga}{K} s_o \qquad \text{(A3.147)}$$

$$y_o = \int (s_o(a^2+s_o^2)^{-\frac{1}{2}} + \frac{m_r g s_o}{K}) ds_o =$$

$$= (a^2+s_o^2)^{\frac{1}{2}} + \frac{m_r g s_o^2}{2K} - a \qquad \text{(A3.148)}$$

The integration parameter a is most easily obtained from Eq. (A3.143), if the force H_o is known.

$$a = \frac{H_o}{m_r g} \qquad \text{(A3.149)}$$

In the case of an axially stiff cable, K -> ∞, then the expressions of the geometry become

$$x_o = a \text{ arc sin h } (\frac{s_o}{a}) \qquad \text{(A3.150)}$$

$$y_o = (a^2+s_o^2)^{\frac{1}{2}} - a \qquad \text{(A3.151)}$$

The unstretched line force is obtained from Eqs. (A3.145) and (A3.151).

$$T_o = m_r g(y_o+a) = m_r g y_o + H_o \qquad \text{(A3.152)}$$

These are the static equilibrium equations, stretched and unstretched, for a uniform cable in a fluid at rest.

A3.4 Gravity force

The earth will affect a moving body with two forces, namely; the gravity force and the Coriolis' force. The Coriolis' force from the rotation of the earth (see chapter A2.2.2) will be neglected.

The gravity forces acting on the floating body, and how it affects it, is already discussed. Hence, the discussion will concentrate on how the gravity force acts in the different sets of frame that exists.

In the earth fixed frame from chapter A1, the gravity force is

$$\underline{F}_G = m(0,0,-g)^t \tag{A3.153}$$

In the body fixed frame, the gravity force is

$$\underline{F}_G' = \underline{\underline{T}}\,\underline{F}_G = mg\,\underline{f}_G' \tag{A3.154}$$

where

$$\underline{f}_G' = -\underline{T}_{x3} = \begin{matrix} s\theta_1 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 \\ s\theta_1 c\theta_3 + c\theta_1 s\theta_2 s\theta_3 \\ c\theta_1 c\theta_2 \end{matrix}$$

A3.5 Hydrodynamic equation of motion

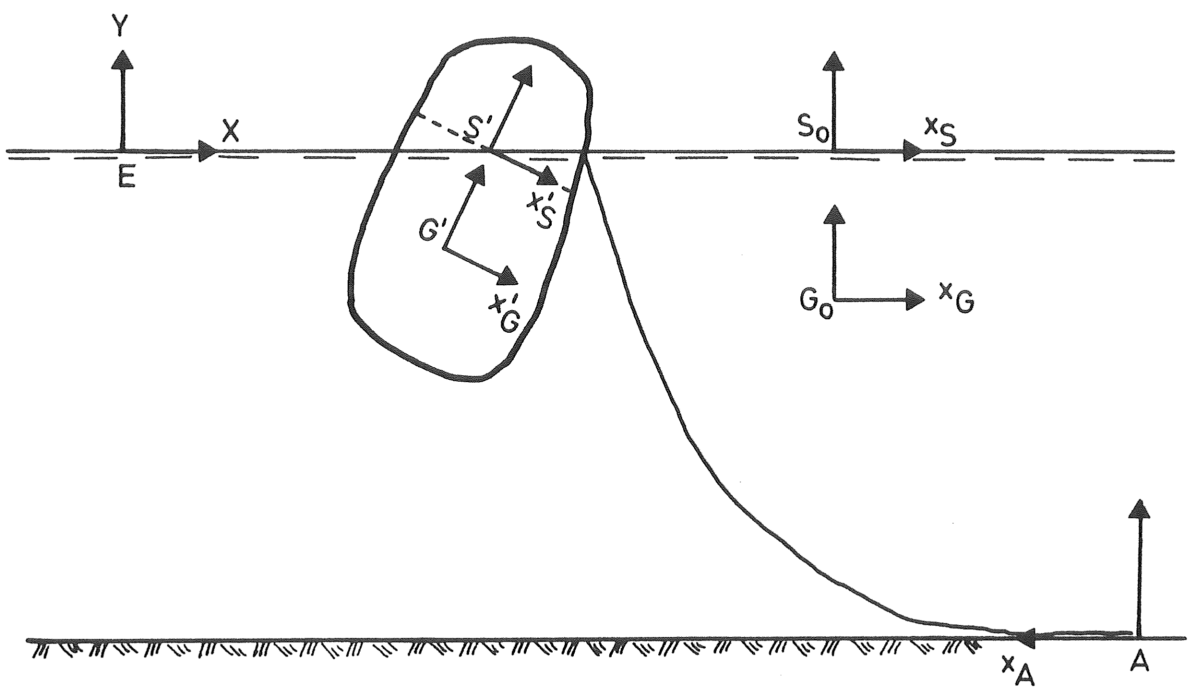


Fig. A3.28 Sets of rotating and fixed frames.

The equations of motion of the floating system are to be expressed in terms of one frame. As can be seen in Figure A3.28 there are various frames to choose. These frames are of two types, one type rotating (body fixed) and the other one fixed.

There are four convenient types of earth fixed systems. Among these the Earth System X is the main reference frame and the others are merely subframes, but the main frame can of course be chosen to coincide with one of the subframes. This is to be preferred if special knowledge is wanted about that substructure. Apart from the main (Earth) frame, there are two frames representing the body and one for each mooring line.

The mooring line frame X_A is located at the lower attachment point of the line, see Figure A3.28. It is directed with its x-axis towards the floating object, the y-axis upwards and the z-axis in the resulting direction.

The fixed body equilibrium frames are located at the centre of the equilibrium water surface x_S and at the equilibrium centre of gravity x_G. They are directed in the way that is presented in Chapter A1. These frames coincide with the rotating, body fixed, frames, x'_S and x'_G, when the system is in the reference equilibrium position.

The reason for having two frames in the body is that different properties are expressed in terms of different frames. So is the frequency dependent hydrodynamic properties mostly given in terms of rotations about the surface, while the body moment of inertia is given about the centre of gravity. The buoyancy rotation stiffness is here derived about the center of gravity frames, though it is perhaps more common to express it in the surface frame.

The above mentioned properties, and their corresponding translational properties, are dependent on the body orientation. This is valid without any restrictions for the body properties of inertia, the buoyancy and the surface independent hydrodynamic properties. On the other hand, the surface (frequency) dependent hydrodynamic properties are derived assuming the rotations about the surface to be linear, and the body shape to be constant at the surface. Then, strictly speaking, only the yaw can be assumed to be large, but if the pitch and roll rotations are small enough ($<10^o$) it is possible to assume all hydrodynamic properties to be body fixed.

For matters of comprehensiveness the equations of motion of the system will be expressed in the following chapters. First they will be expressed in terms of the main frame. Secondly they will be expressed in terms of their substructure frames. In doing this it will be easy to decide which frame is most easy to use.

A3.5.1 In a fixed frame

The force and torque equations of motion for a moored floating body, based in Eqs. (A3.1) and (A3.2), can be written using results from Chapters A3.2 to A3.4. These equations will be expressed in a general form.

The force equation can be written as follows

$$\underline{\underline{m}}\,\ddot{\underline{r}}_c = \underline{F} \qquad \text{(A3.154)}$$

where $\underline{\underline{m}}$ is total mass,
$\ddot{\underline{r}}_c$ is acceleration of center of mass and
$\underline{F}$ is body boundary force.

The body boundary force is

$$\underline{F} = \underline{F}_s + \underline{F}_d + \underline{F}_e + \underline{F}_c + \underline{F}_w + \underline{F}_M + \underline{F}_G \qquad \text{(A3.155)}$$

where $\underline{F}_s$ is the linear force caused by body motion (Eq. (A3.6))

$\underline{F}_d$ is the drag force, 0(2), caused by relative body motion (Eq. (A3.51))

$\underline{F}_e$ is the wave induced forces on the body (Eq. (A3.90))

$\underline{F}_c$ is the current force (Eq. (A3.103))

$\underline{F}_w$ is the wind force (Eq. (A3.104))

$\underline{F}_M$ is the mooring force (Eq. (A3.142)) and

$\underline{F}_G$ is the gravity force (Eq. (A3.154))

Inserting the body boundary forces listed above into Eq. (A3.154) gives the following force Equation of Motion

$$(\underline{\underline{m}}+a)\,\ddot{\underline{r}}_c = -\,\underline{\underline{b}}\,\dot{\underline{r}}_c - \underline{\underline{c}}\,\underline{r}_c + \{\underline{\underline{b}}_1((\underline{u}+\underline{u}_{cu})-\dot{\underline{r}})\,|(\underline{u}+\underline{u}_{cu})-\dot{\underline{r}}|\}|_L$$

$$+\{\underline{\underline{b}}\,\underline{u} + (\underline{\underline{a}}+\rho V)\,\dot{\underline{u}}\}|_L + \{\underline{\underline{b}}_2(\underline{w}-\dot{\underline{r}})\,|\underline{w}-\dot{\underline{r}}|\}|_L +$$

$$+\{\underline{T}\}|_M + \underline{F}_G \qquad \text{(A3.156)}$$

where { }| denotes the integrated value of the term over either the projected length, L, or sum over the mooring lines, M.

Furthermore the notations are

$\dot{r}_c$	velocity of center of gravity
r_c	displacement of center of gravity
$\underline{\underline{a}}$	added mass
$\underline{\underline{b}}$	radiation damping
$\underline{\underline{c}}$	buoyancy stiffness
$\underline{\underline{b}}_1$	water drag damping
$\underline{\underline{b}}_2$	wind drag damping
V	body volume
$\underline{\dot{u}}$	water wave acceleration
$\underline{u}$	water wave velocity
$\underline{u}_{cu}$	current velocity
$\underline{w}$	wind velocity
$\underline{T}$	mooring force
$\underline{F}_G$	gravity force.

All interaction between current and waves are excluded. This restriction is not necessary, but it simplifies the calculations.

In a similar way the torque equation of motion, about the center of gravity, is derived

$$\underline{\underline{I}}_c\underline{\dot{\omega}} = \underline{M}_c \qquad \text{(A3.157)}$$

where

$\underline{\underline{I}}_c$	is body inertia about center of gravity,
$\dot{\omega}$	is angular acceleration
M_c	is excitation torque on the body

The body excitation torque is (same notation as for forces)

$$\underline{M} = \underline{M}_s + \underline{M}_d + \underline{M}_e + \underline{M}_c + \underline{M}_w + \underline{M}_M \qquad \text{(A3.158)}$$

where

$\underline{M}_s$ is the linear moment caused by body motion (from Eq. (A3.6)),

$\underline{M}_d$ is the drag moment, 0(2), caused by relative body motion (from Eq. (A3.51)),

$\underline{M}_e$ is the wave induced moments on the body (from Eq. (A3.90)),

$\underline{M}_c$ is the current induced moment (from Eq. (A3.103)),

$\underline{M}_w$ is the wind induced moment (from Eq. (A3.104) and

$\underline{M}_M$ is the mooring induced moment (from Eq. (A3.142)).

Inserting the body boundary moments, which are derived from the above listed force equations, gives the torque Equation of Motion

$$(\underline{\underline{I}}_c+a)\,\dot{\underline{\omega}} = -\,\underline{\underline{b\omega}} - \underline{\underline{c}}\underline{\theta}^E + \{\underline{r}\text{x}(\underline{\underline{b}}_1((\underline{u}+\underline{u}_{cu}) - \dot{r})\,|(u+u_{cu}) - \dot{r}|\}\big|_L$$

$$+ \{\underline{r}\ \text{x}\ \underline{\underline{b}}_T\underline{u} + \underline{r}\ \text{x}(\underline{\underline{a}}_T+\rho V)\dot{\underline{u}}\}\big|_L +$$

$$+ \{\underline{r}\ \text{x}(\underline{\underline{b}}_2(\underline{w} - \dot{r})\,|\underline{w}-\dot{r}|)\}\big|_L + \{(\underline{r}\underline{\text{x}}T)\}\big|_M \qquad \text{(A3.159)}$$

using the same notations as in Eq. (A3.156) plus the following notations

$\dot{\underline{\omega}}$ is the body angular acceleration
$\underline{\omega}$ is the body angular velocity
$\underline{\theta}^E$ is the body angular displacement
$\underline{r}$ is the force location vector

and index T denotes translation.

This gives us the expressions for both force and moment, but a problem is that the body has significant displacements, and therefore it is hard to give the body properties in a simple way. Hence, using the body frame will simplify the problem.

A3.5.2 In a moving frame

The use of a body fixed frame and its related quantities simplifies the problem. Using the body motions and solving the problem in the body fixed frame gives Equations of Motions as below.

The force equation is:

$$\begin{aligned}(\underline{\underline{m}} + \underline{\underline{a}})\underline{\ddot{r}}'_c = & - \underline{\omega}' \mathrm{x} (\underline{\underline{m}}+\underline{\underline{a}})\underline{\dot{r}}'_c - \underline{\underline{b}}\,\underline{\dot{r}}'_c - \underline{\underline{c}}\underline{r}'_c + \\ & + \{\underline{\underline{b}}_1((\underline{u}'+\underline{u}'_{cu}) - \underline{\dot{r}}')\,|(\underline{u}'+\underline{u}'_{cn}) - \underline{\dot{r}}'|\}|_L + \\ & + \{\underline{\underline{b}}\underline{u}' + (\underline{\underline{a}} + \rho V)\underline{\dot{u}}'\}|_L + \\ & + \{\underline{\underline{b}}_2((\underline{u}'+\underline{u}'_{cu})-\underline{\dot{r}}')\,|(\underline{u}'+u'_{cn}) - \dot{r}'|\}|_L + \\ & + \{\underline{T}'\}\,|_M + \underline{F}'_G \end{aligned} \qquad \text{(A3.160)}$$

with notations from Eq. (A3.156), and where the prime denotes that they are expressed in terms of body coordinates.

The torque equation is:

$$\begin{aligned}(\underline{\underline{I}}+\underline{\underline{a}})\underline{\dot{\omega}}' = & - \underline{\omega}' \mathrm{x} (\underline{\underline{I}}+\underline{\underline{a}})\underline{\omega}' - \underline{\underline{b}}\underline{\omega}' - \underline{\underline{c}}\underline{\theta}^E + \\ & + \{\underline{r}' \mathrm{x} (\underline{\underline{b}}_1((\underline{u}+\underline{u}_{cn}) - \underline{\dot{r}})\,|(\underline{u}+\underline{u}_{cn}) - \underline{\dot{r}}'|\}|_L + \\ & + \{\underline{r}' \mathrm{x} (\underline{\underline{b}}_T\underline{u}') + \underline{r}' \mathrm{x} ((\underline{\underline{a}}+\rho V)\underline{\dot{u}}')\}|_L + \\ & + \{\underline{r}' \mathrm{x} (b_2(\underline{w}'-\underline{\dot{r}}')\,|\underline{w}'-\underline{\dot{r}}'|)\}|_L + \\ & + \{\underline{r}' \ \mathrm{x}\ \underline{T}'\}|_M \end{aligned} \qquad \text{(A3.161)}$$

The use of the body frame has the advantage that you do not need to transform the property matrices. Instead you have to transform the motion vectors of the fluid and the body. But this is of course simpler.

A3.5.3 Solution principles

A set of equations that are expressed in quasi coordinates, such as the body frame Equations of Motions, Eqs. (A3.160) and (A3.161), may not be integrated. This is due to the fact that these coordinates are time dependent. It is therefore necessary to relate these body motions to a fixed frame. Hence, it will be possible to integrate them.

The integration scheme is as below.

- Solve E o M (Eq. of Motion): $\underline{\ddot{r}}'$
  (Eq. (A3.160))

- Transform $\qquad \underline{\ddot{r}}=\underline{\underline{T}}^{t}(\underline{\ddot{r}}'+\underline{\omega}\times\underline{\dot{r}}') \qquad$ (A3.162)
  (Eq. (A2.15) and Eq. (A2.67))

- Integrate: $\qquad \underline{\dot{r}} = \int \underline{\ddot{r}}\, dt$

$$\underline{r} = \int \underline{\dot{r}}\, dt \qquad \text{(A3.163)}$$

Rotations:

- Solve E o M: $\qquad \underline{\dot{\omega}}'$
  (Eq. (A3.160))

- Transform: $\qquad \ddot{\theta}^{E}=(\underline{\underline{R}}')^{-1}(\underline{\dot{\omega}}'+\underline{\omega}\times\underline{\omega}') \qquad$ (A3.164)
  (Eq. (A2.79) and Eq. (A2.67))

- Integrate:

$$\dot{\theta}^{E} = \int \ddot{\theta}^{E}\, dt$$
$$\theta^{E} = \int \dot{\theta}^{E}\, dt \qquad \text{(A3.165)}$$

This entire integration scheme is solved using some numerical method, such as a central difference scheme for instance.

A4 DYNAMICS OF A WAVE ENERGY BUOY

A4.1 Description of buoy

Wave energy devices can be divided into three groups, attenuators, terminators and point absorbers, where the denominations hint at the extension and direction of the convertors. Attenuator extend in the wave direction, and terminators are perpendicular to the wave direction.

This work deals with a special point absorber, see Figure A4.1. A point absorber will act as a wave energy sink which will affect the wave situation around the buoy, an effect that is out of scope of this work.

The model chosen will only take into account the effects that are caused by the damped motion of the buoy, i.e. that the dissipated and radiated energy will be changed due to the energy conversion.

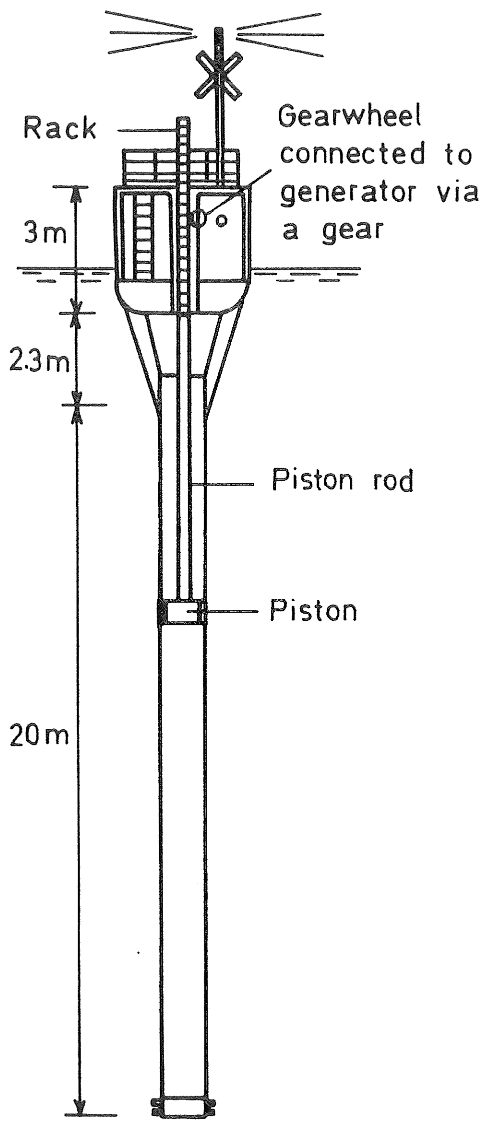


Fig. A4.1 Point absorber.

Choosing a point absorber such as the one in Figure A4.1, will give seven degrees of freedom of the buoy. That is the ordinary six rigid body motions, plus one heave motion for the rod piston.

Energy is extracted from the relative motion between the two heave degrees of freedom.

A4.2 Externally damped buoy

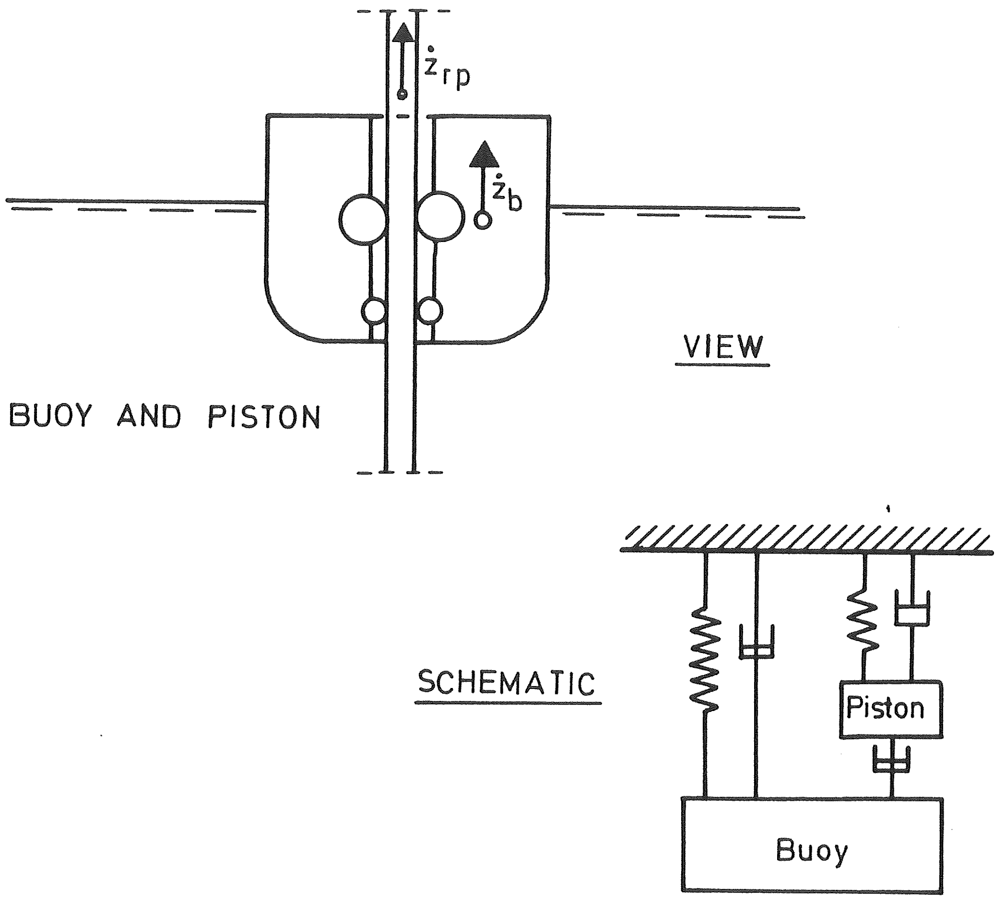


Fig. A4.2 Buoy and piston, and a schematic presentation of the system.

In the wave energy buoy, which is used here, the power take off is from the linear damper between the heave motions of the bodies, see Figure A4.2. All other degrees of freedom are the same for the buoy and the rod.

The power extracted from the linear damper is

$$P = F_c \; u_{rel} = b_1 u_{rel}^2 \qquad \text{(A4.1)}$$

where

b_1 is the linear (or linearized) damping constant of the conversion equipment and

u_{rel} is the relative heave velocity between piston and buoy.

$$u_{rel} = \dot{z}'_{rp} - \dot{z}'_b \qquad \text{(A4.2)}$$

where

$\dot{z}'_b$ is buoy velocity and
$\dot{z}'_{rp}$ is rod piston velocity

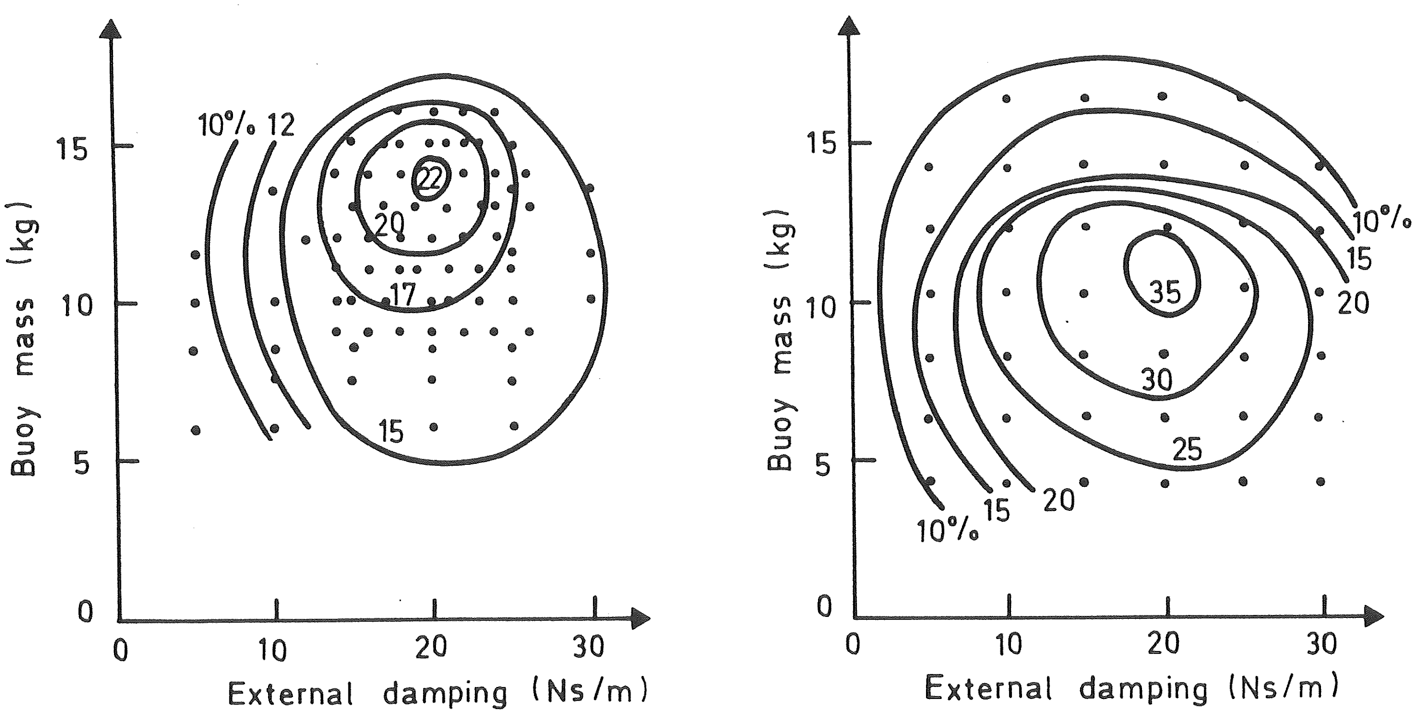


Fig. A4.3 a Contours of total capture width ratio as a function of mass and damping. Wave Spectrum G0. (harder)
b Contours of total capture width ratio as a function of mass and damping. Wave Spectrum G4. (milder)
(From Bergdahl and Mårtensson, 1984)

A proper choice of mass/damping ratio is of vital interest. As seen in Figures A4.3a and b, there is an optimal ratio. This ratio is as expected to depend on the waves, the bigger the waves are, the heavier the buoy must be.

Damping forces, from energy conversion, are as shown in Eq. (A4.1) linear functions of relative velocity. The force on the buoy is

$$(F_c)_{buoy} = b_1 \ (\dot{z}'_{rp} - \dot{z}'_b) \qquad (A4.3)$$

and the force on the rod piston is

$$(F_c)_{rp} = b_1(\dot{z}'_b - \dot{z}'_{rp}) \qquad (A4.4)$$

A4.3 Equations of motions

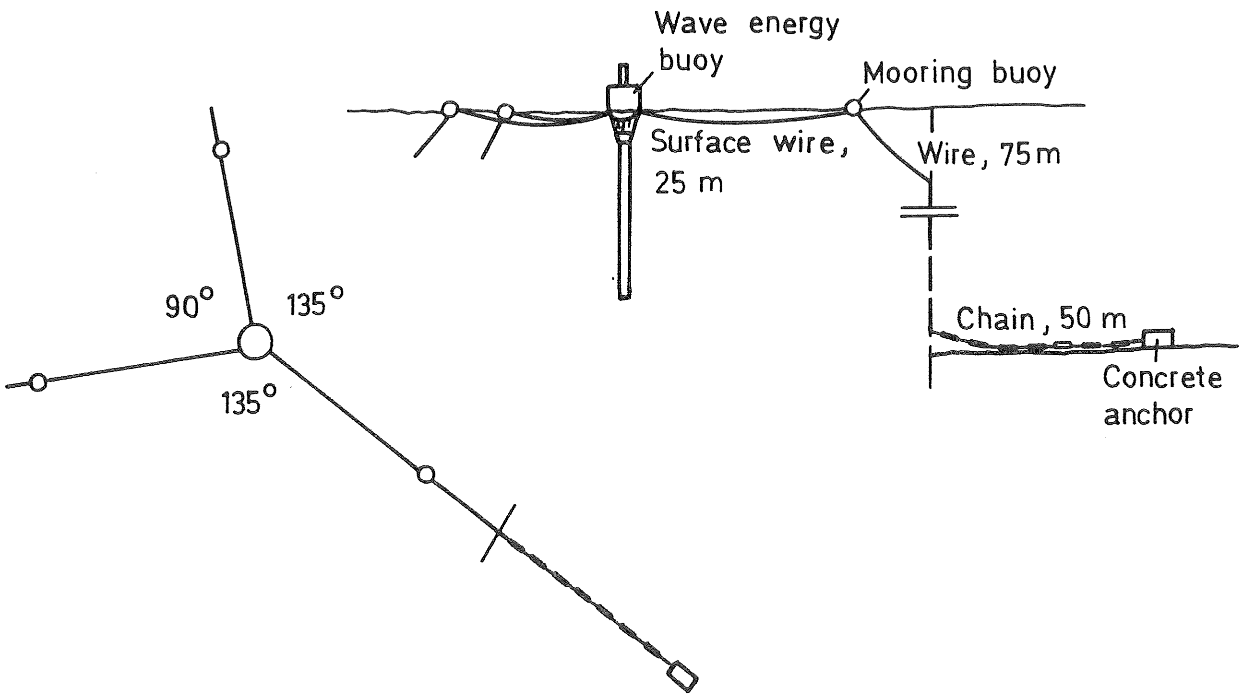


Fig. A4.4 Moored wave energy buoy. Top view and side view.

The moored wave energy buoy consists of one main buoy, a number of achoring buoys, lines connecting the main buoy to the anchoring buoys and finally mooring lines connecting the anchoring buoys with the anchors.

This system has

$$7 + n \cdot 3$$

degrees of freedom, where n is the number of anchoring buoys, assuming that the anchoring buoys only translate, and that the lines between the main and the mooring buoys are weight less.

Schematically the system of equations of motions is for the main buoy

$$\underline{\underline{M}}_B \, \underline{\ddot{x}}_B = \underline{F}_s + \underline{F}_e + \underline{F}_n + \underline{F}_c + \underline{F}_L \qquad \text{(A4.5)}$$

and for the mooring buoy

$$\underline{\underline{M}}_{MB} \, \underline{\ddot{x}}_{MB} = \underline{F}_s + \underline{F}_c + \underline{F}_n + \underline{F}_L + \underline{F}_M \qquad \text{(A4.6)}$$

Here

$\underline{F}_s$ force from moving structure in environment

$\underline{F}_e$ force from environment

$\underline{F}_n$ coupled force from structure motion and environment

$\underline{F}_c$ force from energy conversion

$\underline{F}_L$ force from line connecting main and mooring buoy

$\underline{F}_M$ force from mooring line

$\underline{\ddot{x}}_B$ 7-dimensional acceleration vector consisting of 4 translational and 3 rotational accelerations

$\underline{\ddot{x}}_{MB}$ 3-dimensional translational acceleration vector.

These equations are solved in the way that is described in Chapter 3.5.

The line force is preferably calculated assuming some kind of damping, external or internal. This can be done in the same way as for the mooring force, see Chapter 3.5. Choosing a too small damping will cause numerical instabilities in the equations. It is also possible to assume the connecting line to be rigid. The mooring buoy will then only be adjusted in direction in the system.

BIBLIOGRAPHY

Brebbia and Walker (1979): Dynamic Analysis of Offshore Structures. Newnes-Butterworths, London, Great Britain.

Bergdahl and Mårtensson (1984): Wave-Power Buoys. Model Experiments; Efficiency and Hydrodynamic Coefficients. Group for Wave Energy Research, Rep. no. GR:55, Chalmers Univ. of Technology, Göteborg, Sweden.

Bergdahl and Rylander (1981): Mooring Force Measurements on a Wave Energy Converter at Sea in 1980 (Swedish). Group for Wave Energy Research, Rep. no. GR:38, Chalmers Univ. of Technology, Göteborg, Sweden.

Bergdahl et al (1978): Linear Theory for Energy Absorption by an Oscillating Wave Energy Converter of Buoy Type (Swedish). Group for Wave Energy Research, Rep. no. GR:10, Chalmers Univ. of Technology, Göteborg, Sweden.

Cederwall and Larsen (1976): Hydraulics for Civil Engineers (Swedish). Liber Läromedel, Lund, Sweden.

Chakrabarti (1980): Impact of Analytical, Model and Field Studies on the Design of Offshore Structures. Int. Symp. Ocean Eng. and Ship Handl., SSPA, Göteborg, Sweden.

Chakrabarti (1984): Steady Drift Force on Vertical Cylinder - Viscous vs. Potential. Appl. Ocean Research, Vol 6:2, pp 73-82.

Choo and Casarella (1973): A Survey of Analytical Methods for Dynamic Simulation of Cable-Body Systems. J. Hydronautics, Vol 7:4, pp 137-144.

Claeson (Ed.) (1980): Wave Energy in Sweden (Swedish). Group for Wave Energy Research, Rep. no. GR:16, Chalmers Univ. of Technology, Göteborg, Sweden.

Claeson (Ed.) (1980): Introductory Test with the Inter Project Buoy (Swedish). Group for Wave Energy Research, Rep. no. GR:34, Chalmers Univ. of Technology, Göteborg, Sweden.

Claeson (Ed.) (1982): Wave Energy Research in Sweden. Final report Phase 5. Group for Wave Energy Research, Rep. no. GR:50, Chalmers Univ. of Technology, Göteborg, Sweden.

Clough and Penzien (1975): Dynamics of Structures. McGraw-Hill Kogakusha, Tokyo, Japan.

Craig (1981): Structural Dynamics. An Introduction to Computer Methods. John Wiley & Sons, New York, USA.

van Dao and Penzien (1982): Comparison of Treatments of Non-linear Drag Forces Acting on Fixed Offshore Platforms. Appl. Ocean Research, Vol. 4:2, pp 66-72.

Davis and Snider (): Introduction to Vector Analysis. (4:th ed.)

Goldstein (1980): Classical Mechanics (2:nd ed.). Addison-Wesley Publ. Comp., Reading, Mass., USA.

Greenwood (1977): Classical Dynamics. Prentice-Hall, Englewood Clifts, N.J., USA.

Handa (1984): Compendium in Building Aerodynamics, (rev.) (Swedish). Dep. of Structural Design, No. 1982:21, Chalmers Univ. of Technology, Göteborg, Sweden.

Honkanen (1976): On the Wave Induced Motions of Ships. The Swedish Academy of Engineering Sciences in Finland, Rep. no. 30, Helsinki, Finland.

Hooft, J.P. (1982): Advanced Dynamics of Marine Structures. John Wiley & Sons, New York, USA.

Kim, W. (1965): On the Harmonic Oscillations of a Rigid Body on a Free Surface. J. Fluid Mech., Vol. 21:3, pp 427-451.

Korvin-Kroukovsky, B.V. (1961): Theory of Seakeeping. Soc. of Nav. Arc. and Mar. Eng., New York, N.Y., USA.

Lamb (1932): Hydrodynamics (6:th ed.). University Press, Cambridge, Great Britain.

Larsen, I. (1984): The Concept of Acceleration Net for Defining Added Mass. Prog. Rep. 66, pp 15-22, Inst. Hydrodyn. and Hydraulic Eng., Techn. Univ. Denmark, Lyngby, Denmark.

Lighthill, J. (1979): Waves and Hydrodynamic Loading. Behaviour of Offshore Structures 1979 (2:nd Int. Conf.) Opening Address-1, London, England.

Lindahl (1981): Equation of motion of a cable. (Swedish) Dep. of Hydraulics, Chalmers Univ. of Technology, Rep. Ser. B:24, Göteborg, Sweden.

Lindahl and Sjöberg (1983): Dynamic Analysis of Mooring Cables. Dep. of Hydraulics, Chalmers Univ. of Technology, Rep. Ser. A:9, Göteborg, Sweden.

Lindsey (1940): Simple Shapes. NACA Tech. Rep 619.

Meirovitch, L. (1970): Methods of Analytical Dynamics. McGraw-Hill, New York, USA.

Meriam (1980): Statics and Dynamics. John Wiley & Sons, New York, USA.

Mårtensson (1983): Wind Generated Waves (2:nd ed.). Group for Wave Energy Research, Rep. no. GR:52, Chalmers Univ. of Technology, Göteborg, Sweden.

Nayfeh et al (1973): Nonlinear Coupling of Pitch and Roll Modes in Ship Motions. J. Hydronautics, Vol. 7:4, pp 145-152.

Nayfeh et al (1974): Perturbation-Energy Approach for the Development of the Nonlinear Equations of Ship Motion. J. Hydronautics, Vol. 8:4, pp 130-136.

Newman, J. (1978): The Theory of Ship Motions. Adv. Applied Mechanics, Vol. 18, pp 221-283.

Newman, J. (1980): Marine Hydrodynamics. The MIT Press, Cambridge, Mass., USA.

Olson, R. (1980): Essentials of Engineering Fluid Mechanics (4:th ed.). Harper & Row, New York, USA.

Olsson, G., (1982): Hydrodynamics of a Floating Cylinder (Swedish). Educational Litt. Nr. 1982:3, Dep. of Hydraulics, Chalmers Univ. of Technology, Göteborg, Sweden.

Pio, R. (1964): Symbolic Representation of Coordinate Transformations. IEEE Trans. on Aerospace and Navigational Electronics, Vol. ANE-11, pp 128-134.

Pio, R. (1966): Euler Angle Transformations. IEE Trans. on Automatic Control, Vol. AC-11, No. 4, pp 707-715.

Rodenbusch and Källström (1986): Forces on a Large Cylinder in Random Two-Dimensional Flows. Offshore Technology Conference, Paper no. OTC-5096, Houston, Texas, USA.

Sachs (1972): Wind Forces in Engineering. Pergamon Press, Oxford, Great Britain.

Sarpkaya (1979): Impact of Analytical, Model and Field Studies on the Design of Offshore Structures. Int. Symp. Ocean Eng. and Ship Handl., SSPA, Göteborg, Sweden.

Sjöberg and Bergdahl (1981): Moorings and Mooring Forces (Swedish). Dep. of Hydraulics, Rep. Ser. B:30, Chalmers Univ. of Technology, Göteborg, Sweden.

Smith and Dominguez (1972): Oscillations of Buoy-Cable Mooring Systems. Symp. on Flow-Induced Structural Vibrations, Aug. 14-16, Karlsruhe, Germany.

Standing (1979): Use of Wave Diffraction Theory with Morison's Equation to Compute Wave Loads and Motions of Offshore Structures. National Maritime Institute, Rep. no. NMI R74, Feltham, Great Britain.

Stiansen and Chen (1981): Computational Methods for Predicting Motion and Dynamic Loads of Tension-Leg Platforms. Integrity of Offshore Structures, 2:nd Int. Symp., 1-3 July, Glasgow, Great Britain.

Vedeld (1983): WAMLOS:NV1461: Wave Motions and Loads for Offshore Structures. User's Manual. Technical Report no. 83-6101, Computas A/S, Oslo, Norway.

Wiegel (1964): Oceanographical Engineering. Prentice-Hall Inc, New Jersey, USA.

van Wijngaarden, L. (1963): On the Kramers-Kronig Relations with Special Reference to Gravity Waves. Proc. Kroninklijke Nederlandse Akad. Wet. Ser. B 66(5).

Yeung, R. (1981): Added Mass and Damping of a Vertical Cylinder in Finite-Depth Waters. Appl. Ocean Res., Vol. 3, No. 3.

Zarnick and Casarella (1972): The Dynamics of a Ship Moored by a Multi-Legged Cable System in Waves. 8th Annual Conference and Exposition, Marine Technology Society Sep 11-13, Washington DC.

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