Institutionen för vattenbyggnad
Chalmers Tekniska Högskola

Department of Hydraulics
Chalmers University of Technology

ISSN 0348 - 1069

Two papers on
Urban Runoff Modelling:

Base Catchment Modelling in Urban Runoff Simulation

An Improved Rational Method for Urban runoff Application

Sven Lyngfelt

Report
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PREFACE

This report includes two papers:

Base Catchment Modelling
in Urban Runoff Simulation

An Improved Rational Method
for Urban runoff Application

In the first paper
basic theory in urban runoff simulation
application of the kinematic wave in base catchment modelling
is discussed

In the second paper
the Rational Method and its relation to the kinematic wave theory
an improved method
is discussed.

The papers are based on field measurements and numerical experiments performed at the Department of Hydraulics CTH.

The papers have been published in the International Journal Nordic Hydrology, Vol 22 (3) 1991.

Base Catchment Modelling in Urban Runoff Simulation

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BASE CATCHMENT MODELLING IN URBAN RUNOFF SIMULATION

by

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Summary

The selection of numerical model and the transformation of catchment data into 'input data' are two fundamental problems in urban runoff simulation. They are discussed from a general point of view. A numerical solution method for the kinematic wave equations is proposed for base catchment modeling. In connection to this solution a methodology for the generation of input data representing the individual base catchment is presented.

Introduction

In the application of urban runoff models one of the most important problems is the transformation of catchment characteristics into 'input data'. A transformation which is too detailed is ineffective and very costly. On the other hand a coarse description of the catchment will often result in poor model performance. The selection of numerical model is another factor affecting the model performance. The discretization of geometrical input data should then always be discussed with the properties of available numerical models as a base.

An urban runoff model is principally built up of two main sub-models. One treats the collection of storm water on the surface including the transport to the sewer network system. The other describes the transportation of water within the network system. In available runoff models a wide range of approach is used for overland flow routing from simple time offset methods to the kinematic wave approximation (Huber 1977, National Water Council 1981, Lindberg et al 1986).

The model user is in practice not able to describe the catchment geometry with every pavement and roof in detail. In generating the model input he has to simplify and this he does by defining a main sewer network. The upstream ends of the network are connection points to what will here be called base catchments. These will normally contain several different runoff surfaces, gutters and small diameter sewers. The base catchment is represented by a simplified geometry and the runoff from it by a model containing an overland flow routing element. The important geometrical discretization of the catchment in the model input is thus given by the definition of the main network. As the network is usually well specified, the main approximations and difficulties will be in the modeling of runoff from the base catchments.

In an ordinary case of design or analysis we need to know the hydraulic properties in a number of points along the main sewer network, not just the outflow from the catchment. For practical reasons, the base catchments have to be comparatively large. The time of concentration in the base catchment and in the main network systems (greater velocities) is in reality often of the same magnitude. Most of the design points will then significantly be influenced by the base catchment modeling.

In conclusion, a balanced geometrical discretization and sound base catchment modeling is essential for the effective and precise use of urban runoff models. Different methods of catchment aggregation have been discussed in literature, but mostly based on specific properties of the SWMM model (Proctor and Redfern 1977, Zaghloul 1981). The object of this paper is to discuss base catchment modeling in general terms and to present a base catchment model based on the kinematic wave theory.

Basic runoff models

The shallow water equations are the basic equations describing free surface flow on surfaces, in gutters and in sewers (Yevjevich 1975, Sjöberg 1976). Using the Manning friction relation the equations are given by

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad (1a)$$

$$Q = \frac{1}{n} \cdot A \cdot R^{2/3} \left(S_b - \underbrace{\frac{\partial Y}{\partial x}}_{\substack{\text{pressure} \\ \text{force term}}} - \underbrace{\frac{1}{gA} \cdot \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{1}{gA} \cdot \frac{\partial Q}{\partial t}}_{\substack{\text{acceleration} \\ \text{terms}}} \right)^{1/2} \qquad (1b)$$

where Q is flow , A is cross-sectional area of flow, q is lateral inflow, R is hydraulic radius, Y is water depth, S_b is slope and n is the Manning coefficient of roughness. x and t are the space and time variables respectively. The division of the model into overland flow and sewer flow sub models is not intended to reflect the physical difference between water movement above and below ground. The main point is to divide the catchment into regions where different levels of numerical analysis and sophistication in input data can be applied. If the possibility of back water analysis is restricted to the main sewer line, a natural choice of base catchment model is the kinematic wave model, first presented by Lighthill and Whitham (1955). The model has a well documented performance in cases where no significant influence from the downstream boundary condition exist (Morris and Woolhiser 1980). The kinematic wave equations are obtained from the basic equations by neglecting pressure force and acceleration terms

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{2a}$$

$$Q = \frac{1}{n} \cdot A \cdot R \cdot S_b^{1/2} \tag{2b}$$

Further simplification of the above relations is done by assuming that the flow velocity invariant in time or space. Invariance in space is obtained by applying the kinematic wave equations on the base catchment in one single space step. The approach may physically be interpreted as flow over a surface with uniform water depth along the reach. In some urban runoff models this solution is presented as a kinematic wave solution. However if the cross-sectional area is transformed to a reservoir volume ($s = A \cdot L$ where L is the length in flow direction) the corresponding equations are given by

$$ds = (Q_{in} - Q_{ut}) \cdot dt \tag{3a}$$

$$s = C_1 \cdot (Q_{ut})^{C_2} \tag{3b}$$

where C_1 and C_2 can be identified by equation 1b. The equations describe a nonlinear reservoir.

By assuming the flow velocity constant in time but not in space the following relation is obtained

$$Q(t) = \int_{t-t_c}^{t} \frac{dA_c(t-\tau)}{d\tau}\, i(t)\, d\tau \qquad (4)$$

where A_c is contributing area, t_c is the time of concentration (here taken as L/wave velocity) and i is the rain intensity. The equation describes the Time-Area Method in a continuous form.

Both the nonlinear reservoir model and the Time-Area Method give rise to difficulties when applied in practice. In equation 3b C_1 has, despite the relationship with the Manning formula, to be evaluated from rainfall-runoff measurements. Usually such data are not available for the individual base catchment. The Time-area Method parameter, t_c, is principally related to the rain intensity. When a time varying flow is to be simulated, it is not possible to select a value of t_c that gives a resulting simulated flow which fits at all times during the rainfall event. The performance of the Time-Area Method is improved if the time of concentration is allowed to vary with the rain intensity, Johansen (1985). This shows clearly that this method represents a too simplified version of the basic equations.

The kinematic wave model does not suffer from any of above mentioned problems. As will be shown below the model complexity will not result in difficulties in the numerical solution. The kinematic wave model is therefore regarded as the most suitable alternative for base catchment modeling.

The kinematic wave - numerical solution and attenuation

The kinematic wave equations describe a non attenuating wave propagation. This seems to be a drawback as it is in contrast to a real wave emanating from a rainfall event. However, in application the numerical solution introduces an artificial attenuation which is normally greater than the real one. The problem is then rather to keep the artificial attenuation in the numerical solution as small as possible. There are several alternative numerical solutions presented in literature. The most general approach is the weighted box scheme given in figure 1.

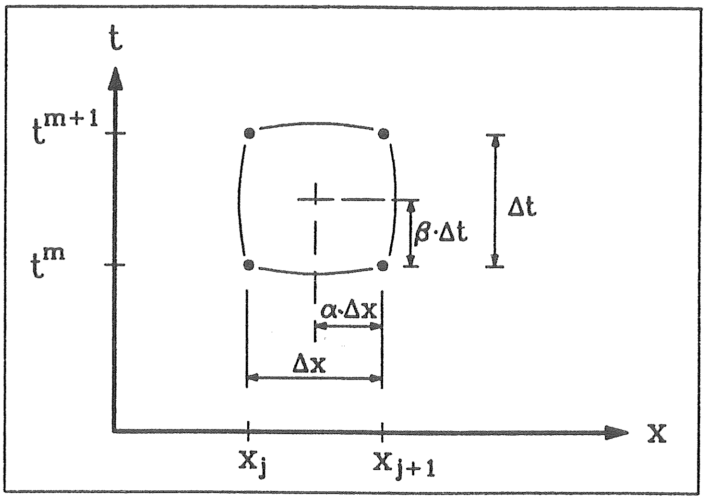

Figure 1 The relation between the center point and weighting factors (from Smith 1980).

In the continuity equation the differentiation of the derivatives by the weighted box scheme takes the form

$$\frac{\partial Q}{\partial x} = \left(\beta(Q^{m+1}_{j+1} - Q^{m+1}_{j}) + (1-\beta)(Q^{m}_{j+1} - Q^{m}_{j})\right)/\Delta x \tag{5a}$$

$$\frac{\partial A}{\partial t} = \left(\alpha(A^{m+1}_{j} - A^{m}_{j+1}) + (1-\alpha)(A^{m+1}_{j+1} - A^{m}_{j+1})\right)/\Delta t \tag{5b}$$

It can be shown by Taylor series expansion that the weighted scheme is a better approximation of the equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + D_n \cdot \frac{\partial^2 Q}{\partial x^2} = q \tag{6}$$

where

$$D_n = (2\alpha - 1)\cdot\frac{\Delta x}{2} + (1-2\beta)c_k\cdot\frac{\Delta t}{2} \tag{7}$$

and c_k is the wave velocity.

A common selection of weights is $\alpha = 0$, $\beta = 0.5$, which are used in the ILLUDAS and the SSARR models (Price 1980). With this set of weights the attenuation will only be influenced by Δx (equation 7). Applied to an ordinary base catchment this model will need a $\Delta x/L$ of approximately 1/20 to provide an attenuation similar to the natural situation, Lyngfelt (1985). It is advantageous to reduce the influence on the attenuation from Δx. According to equation 5 this is done by using $\alpha = 0.5$. In a series of tests it was found that $\Delta x/L$ could be increased to 1/4 using the values of β shown in table 1.

Table 1. Optimal β values ($\Delta x/L=1/4$)

Δt(s)	L(m)	α	β
30	>15	0.5	0.61
30	10	0.5	0.66
30	5	0.5	0.71
60	>15	0.5	0.72
60	10	0.5	0.77
60	5	0.5	0.82

Geometric representation of the base catchment

To use a very detailed description of the base catchment violates the basic idea of the discretization of the runoff system into a main sewer line and base catchments. The geometric representation has to be more generalized in order to reduce the effort of generating input data. During normal application the base catchment includes several separate surfaces and a connecting sewer system. There are several alternative approaches to obtain a simplified geometrical representation of such a catchment. A very simple representation, shown in figure 2, is built up by a rectangular surface feeding a single sewer line. The single sewer represents the main sewer line within the base catchment and is characterized by its length L_m, slope $S_m = \Delta H/L_m$ and diameter D_m. The rectangular surface (parameters; contributing area A_c, length and slope in flow direction L_s and S_s respectively) represents the surface-gutter and sewer branch flows in the real base catchment.

The model is based on the use of the kinematic wave theory, it has 6 parameters and includes a sewer and it is therefore named the KW6S-model. Comparisons with alternative schematic representations show that the model has a well balanced complexity (Lyngfelt 1985).

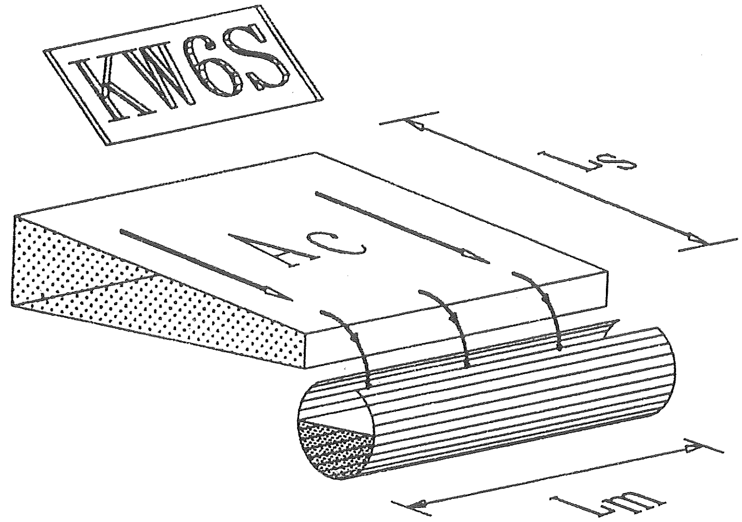

Figure 2 Physical representation of the base catchment by the KW6S-model.

The six parameters in the KW6S-model may be evaluated in three steps:

- Choose a main line sewer in the sewer system of the base catchment and evaluate length L_m and mean slope S_m from the available data.
- Choose a flow path which characterizes the lateral flow into the above defined main sewer line of the base catchment. The path usually includes the elements of runoff surface, gutter and sewer which are evaluated by length, slope etc.
- Estimate the equivalent KW6S-surface length, $(L_s)_{KW6S}$, from evaluated base catchment parameters by the relation

$$(L_s)_{KW6S} = \left(L_s^{3/5} + C_1 \cdot L_g^{3/4} + C_2 \cdot L_p^{3/4}\right)^{5/3} \tag{8}$$

where

$$C_1 = C_3 \cdot i^{3/20} \cdot S_s^{3/10} \cdot S_g^{-3/8} \cdot L_s^{-1/4}$$

$$C_2 = C_4 \cdot i^{3/20} \cdot S_s^{3/10} \cdot S_m^{-3/8} \cdot A_c^{-1/4}$$

and

$$C_3 = n_g^{3/4} \cdot (4(1/z+z))^{1/4}$$

$$C_4 = n_p^{3/4} \cdot (4(1/z+z))^{1/4}$$

In the above equations i is a rain intensity representative for the storms which are to be simulated. z is the side wall slope and s, g, p are indices for surface, gutter and pipe parameters. C_3 and C_4 represent the shape and roughness of gutter and pipe which are normally not varied between different base catchments. In ordinary Swedish catchments relevant values for C_3 and C_4 are 0.058 and 0.036, respectively. Equation (8) is derived from the kinematic wave equations which have an analytical solution in the case of constant rain intensity. It is based on equality between the travel times for waves over the KW6S surface and along the defined characteristic flow path. A similar approach has been discussed for the SWMM-model by Marsalek (1983).

Performance of the KW6S-model

The performance of the KW6S-model was investigated by comparative studies of a model using the kinematic wave theory with a very detailed description of the catchment. In this model practically every surface and gutter in the catchments are simulated. Three residential areas (figure 3) were used for the investigation. The relevance of the detailed model was shown by comparisons between simulated and measured runoff for 21 different storm events (Lyngfelt 1985). In the comparison between the KW6S and detailed model corresponding real storms have been used.

The Bergsjön and Linköping catchments were subdivided into base catchments according to the above proposed methodology. Three levels of subdivision were investigated:

Level 1 nine base catchments, each $\approx 0.5 \cdot 10^{+4} m^2$ contributing area
Level 2 three base catchments, each $\approx 2 \cdot 10^{+4} m^2$ contributing area
Level 3 one base catchment, $\approx 5 \cdot 10^{+4} m^2$ contributing area

Suitable parameters for comparing simulated runoff are levels and corresponding times for peak flow values and the shape of the hydrographs. The investigation generally showed a good agreement between these parameters- when there was similarity in flow peaks there was also similarity in time and hydrograph shapes. In this paper the emphasis will therefore be on flow peaks. A more general discussion is given in Lyngfelt (1985). In figure 4 runoff simulations using the different models are shown for one of the storm events in Bergsjön.

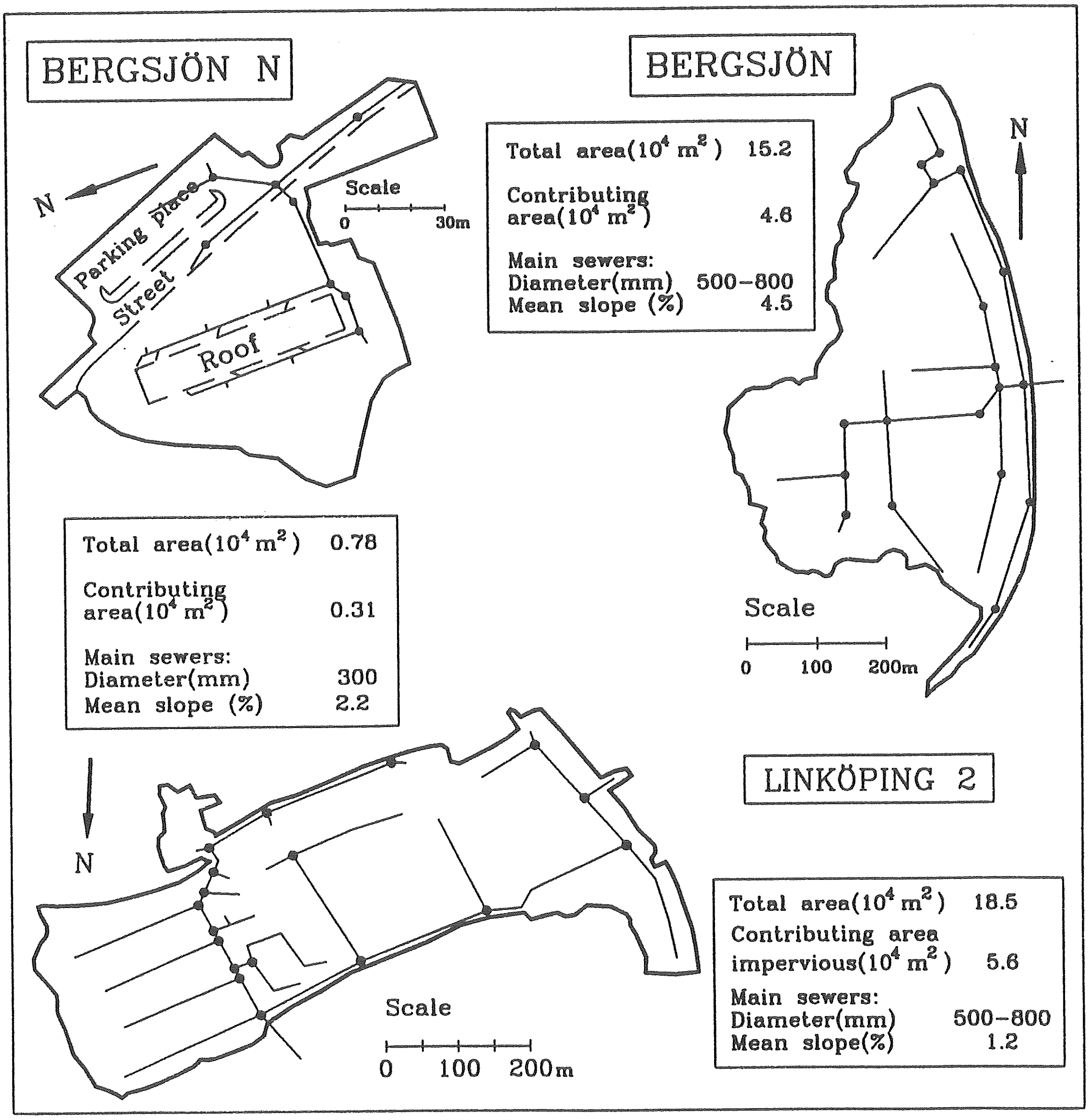

Figure 3 The urban catchments Bergsjön N, Bergsjön and Linköping.

In figure 5 peak flows obtained in the simulations by the detailed and KW6S models have been plotted. It shows that the performance at all levels of discretization is good. This is also evident when the commonly used statistical parameters for the peaks (mean ratio λ_p=0.96, mean standard deviation σ_p=0.13 and absolute error ε_p=11%) are compared with values obtained for corresponding parameters in model performances discussed in the literature (Colyer 1977, Price 1980). Generally the investigation show a weak but notable deterioration of the performance with increasing base catchment area. However, there seems to be no definite upper limit beyond which the base catchment area should be selected using the KW6S-model. For each selection of discretization the performance of

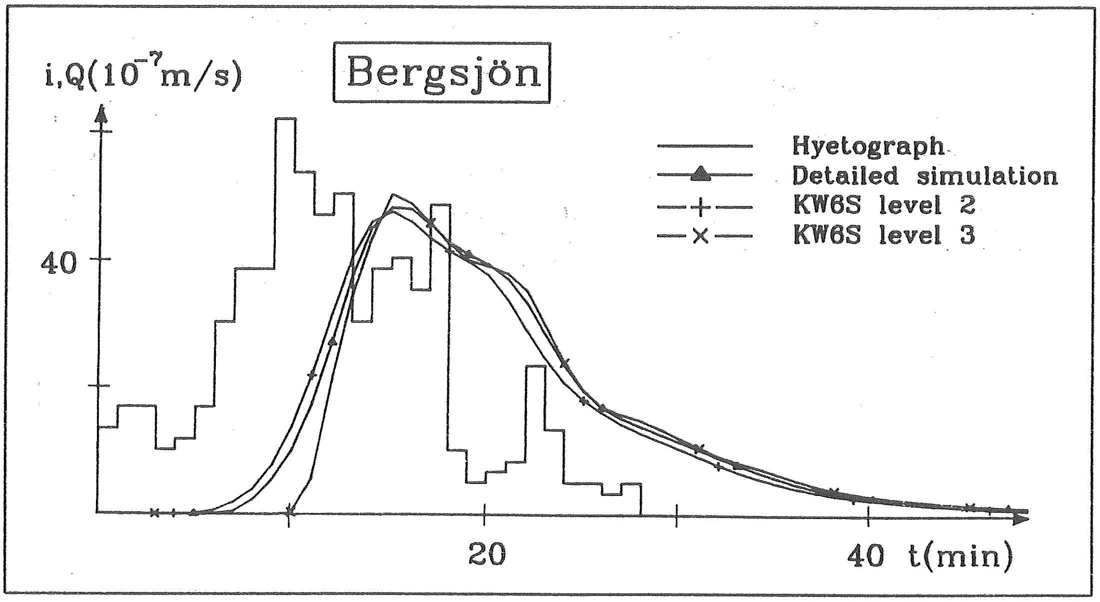

Figure 4 Runoff simulations from a storm event in Bergsjön using different levels of discretization.

the simulation is in principle better downstream compared to upstream. The main sewer net should therefore be defined at least one or two sewer reaches upstream of the points where the hydraulic properties are analysed.

Conclusions

In runoff simulation the kinematic wave equations are from both a theoretical and practical point of view a suitable base catchment model. There is no reason using very simplified versions of the basic equations such as the nonlinear reservoir model and the Time-Area Method.

In practical application it is usually necessary to describe the catchment in a simplified way. This is preferably done by subdivision into a suitable number of base catchments where each is represented by the KW6S-model using 6 parameters. A good performance implies serious consideration of these parameters. This may be done with maps and simple field observations as a base.

If the proposed method is applied, the study shows that reasonably large catchments may be used - an important property in practice.

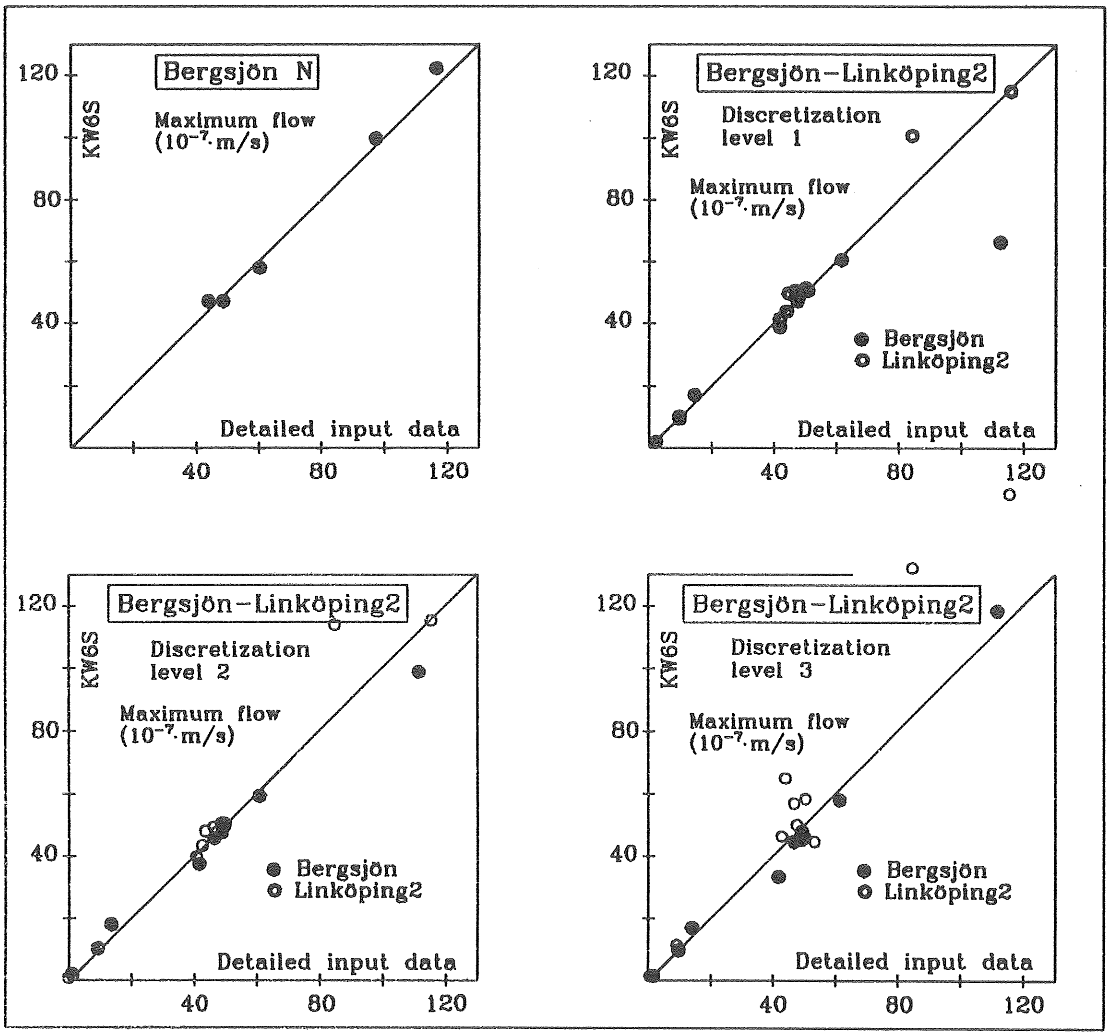

Figure 5 Comparison between simulated peak flow values obtained by a kinematic wave model using detailed input data and the KW6S-model. Three different discretization levels are used.

It should be stressed that a proper evaluation of contributing area is very important for satisfactory model performance, independent of the model used.

Acknowledgements

The work described in this paper has been carried out at the Department of Hydraulics, Chalmers University of Technology as a part of a research program within the Geohydrological Research Group. The program is sponsored by grants from the Swedish Council for Building Research.

List of symbols

A	cross-section of flow
A_c	contributing catchment area
C_1-C_4	parameters
i	rain intensity
j	space step
m	time step
n	Manning's coefficient of roughness
Q	flow rate
R	hydraulic radius
S_b	slope in flow direction
t	time
x	space coordinate
Y	cross-sectional water depth
z	slope factor of side walls
α	numerical parameter in the weighted box scheme
β	numerical parameter in the weighted box scheme
ε_p	mean absolute error in compared peak flow values
λ_p	mean of the ratio between flow peaks
σ_p	standard deviation of the ratio between compared flow peaks

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An Improved Rational Method for Urban runoff Application

CONTENTS sid

AN IMPROVED RATIONAL METHOD FOR URBAN RUNOFF APPLICATIONAPPLICATION

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Summary

In the initial stages of drainage systems planning and for independent tests of advanced runoff model performance there is a need for a simple point flow model. In these cases a method is proposed which is an improved version of the traditional Rational Method. The method, usually regarded as empirical, has a certain relationship with the kinematic wave theory. It is then discussed from both a theoretical and practical point of view based on comparisons with the performance of an advanced continuous model.

Introduction

The analysis and design of urban drainage systems was traditionally, and still often is, executed using the Rational Method. That method has for a long time been regarded as too approximate for many applications. With the introduction of computers several advanced runoff models have been developed and brought into common use. In the initial stages of drainage systems planning and for independent tests of advanced runoff model performance there is a need for a simple point flow model. For these applications the Rational Method, properly used, is a very useful tool despite the extended use of advanced runoff models. In this paper the basic concept of the Rational Method is discussed and compared with the kinematic wave theory. An improved method is proposed and verified in three urban areas.

Basic deterministic relationship

The Rational Method is usually referred to as an empirical method. However the basic concept has a certain relationship with the kinematic wave theory (Newton and Painter 1974). This theory has been proved valid in numerous studies of urban runoff (provided there is no significant backwater). The kinematic wave equations are given by

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{1a}$$

$$Q = a \cdot (A)^b \tag{1b}$$

where t and x are time and space variables respectively, Q is discharge, A is cross- sectional area, q is lateral inflow and a and b are constants.

Applied to surface flow (length L, width B) with the boundary condition A(0,t) = 0, according to Figure 1, integration of Q along a characteristic line gives equation (2) (Lyngfelt 1985)

$$Q(L,t) = a \cdot B \cdot \left(\int_{t-t_c}^{t} q(\sigma)\, d\sigma \right)^b \tag{2}$$

where the time of concentration t_c is the time for a wave to travel from upstream end at t-t_c to downstream end (x=L) at t. In the linear case, b = 1, the constant a appears to be the wave velocity defined as $dQ/dA = L/t_c$, giving

$$Q(t) = L \cdot B \cdot \frac{1}{t_c} \cdot \int_{t-t_c}^{t} i(\sigma)\, d\sigma \tag{3}$$

where i is the rainfall intensity. Equation (3) corresponds to an averaging of the rainfall intensities over the time t_c. For each storm event a maximum value of the average intensity can be found

$$i_{max} = \left(\frac{1}{t_c} \cdot \int_{t-t_c}^{t} i(\sigma) \cdot d\sigma \right)_{max} \tag{4}$$

The maximum flow is obtained as

$$Q_{max} = L \cdot B \cdot i_{max} \tag{5}$$

The model (equation 5) expresses the deterministic relation underlaying the Rational Method. Evidently the relation despite its simplicity accounts for not only a transient wave velocity but also detention storage within the flow section.

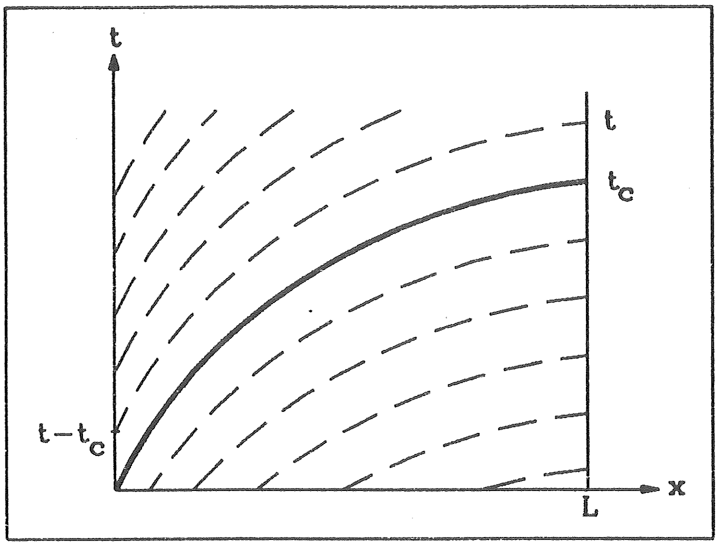

Figure 1 A system of kinematic characteristics in the case of lateral inflow only (from Lyngfelt 1985)

In the case of surface flow (and b = 1) the model provides the analytical kinematic wave solution. The relevance of maximum flow values obtained by this model mainly depends on the accuracy of the estimated time t_c and the error introduced by assuming a linear friction relation. In consequence Hager (1985) has shown by numerical experiments that the variation of rain intensities in an interval equal to the time of concentration has little influence on the peak flow value for a rectangular surface (effects of delay were not significant).

The conclusions from this section cannot automatically be generalized to an arbitrary urban drainage system. However, such a system in many cases is only somewhat more nonlinear than a rectangular surface and in addition the inflow pattern to the system may generally be regarded as lateral. Consequently, from a theoretical point of view there is a great potential in the basic model (equation 5).

Statistical Relationship

The design of a network system is basically a statistical problem. In principle, one possible way to balance pipe size against risk is to design the system for each rainfall event in a long series (perhaps 30 years). The return period for the different flows is calculated and a choice between risk levels with corresponding pipe sizes can be made. A more practical but also more approximate approach is based on storms generated by statistical parameters, i. e. design storms. By using such a storm a design flow is evaluated which is assumed to have the same return period as the storm.

The traditionally used statistical storm is the Maximum Average Intensity storm (MAI-storm) which is defined by its average intensity i_{max} corresponding to average time (duration time t_d): equation (4) defines i_{max}. Each historical storm can be described by a series of MAI-storms with different durations. From a series of historical storms, frequencies of MAI-storms can be evaluated. For each duration a distribution function for the intensities can be plotted. Examples of such functions obtained from a two year series are given in Figure 2 (Arnell and Lyngfelt 1975). The three rain distributions correspond to the durations t_d = 6, 9 and 12 minutes. The frequency is here given as the return period in years (T). The distribution of maximum discharges from a residential area (0.15 km^2) during the same period is also shown in the figure.

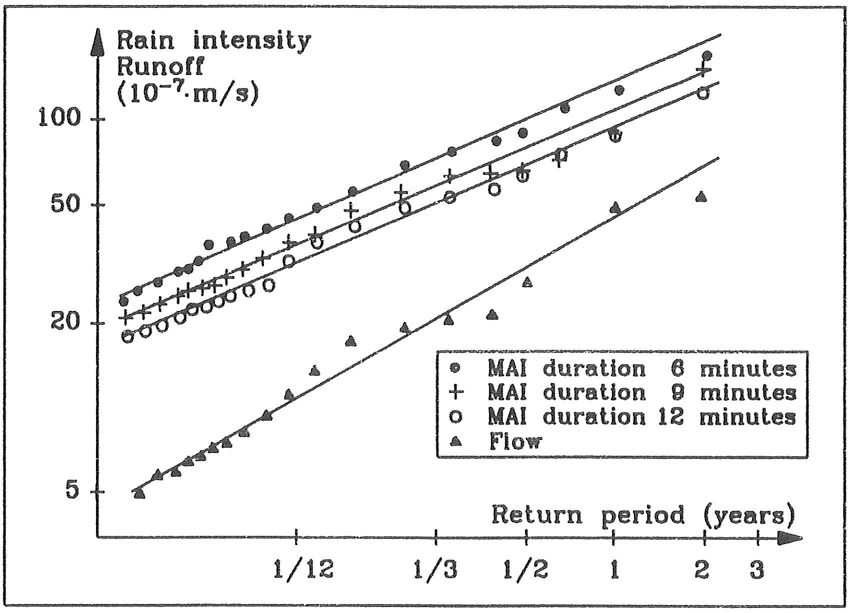

Figure 2 Distribution functions for MAI-storms and maximum discharge (from Arnell and Lyngfelt 1975).

Assuming parallel intensity and discharge distributions we obtain

$$Q_{max}(T) = c_1 \cdot i(T,t_d) \tag{6}$$

where $Q_{max}(T)$ and $i(T,t_d)$ are flow and MAI-storm distributions respectively, T is the return period, t_d is the duration time and c_1 is a constant. As we can see, all the chosen MAI-storm distributions diverge slightly from this assumption. The storm distributions get closer to the flow distribution with increasing return period. The same tendency can be found in other catchments analyzed in a similar way (Shaake et al. 1967).

If the time of concentration is used as the duration of the MAI-storm, the corresponding intensity distribution will have a 'steeper' slope. In a study of five catchments it was found that the distribution $i(T,t_c)$ was always in better accordance with the flow distribution than any distribution $i(T,t_d)$ using constant duration. It was also found that the constant c_1 (equation 6) was close to the estimated contributing area A_c (Lyngfelt 1981). The relation becomes

$$Q_{max}(T) = A_c \cdot i(T,t_c) \qquad (7)$$

where t_c is a function of i.

Time of concentration

The traditional way of presenting MAI-storm distributions for a series of historical storms is the Intensity Duration Frequency diagram (IDF-diagram). In Sweden IDF-curves have been established at six locations. In Figure 3 an IDF-diagram from Göteborg is shown.

The curves are characterized by having steep gradients for the durations of most interest in urban drainage design (5-20 minutes). Overestimating the time of concentration by, for example, five minutes may result in an underestimation of the discharge by more than 20%. The time of concentration is thus a significant parameter and the estimation of the parameter is of great importance in the application of the method.

From the kinematic wave equations it is possible to derive analytical expressions relating the time of concentration to geometric parameters, provided constant rain intensity is assumed. The expressions for surface, gutter and sewer flow may be summarized in equation (8) (Lyngfelt 1985)

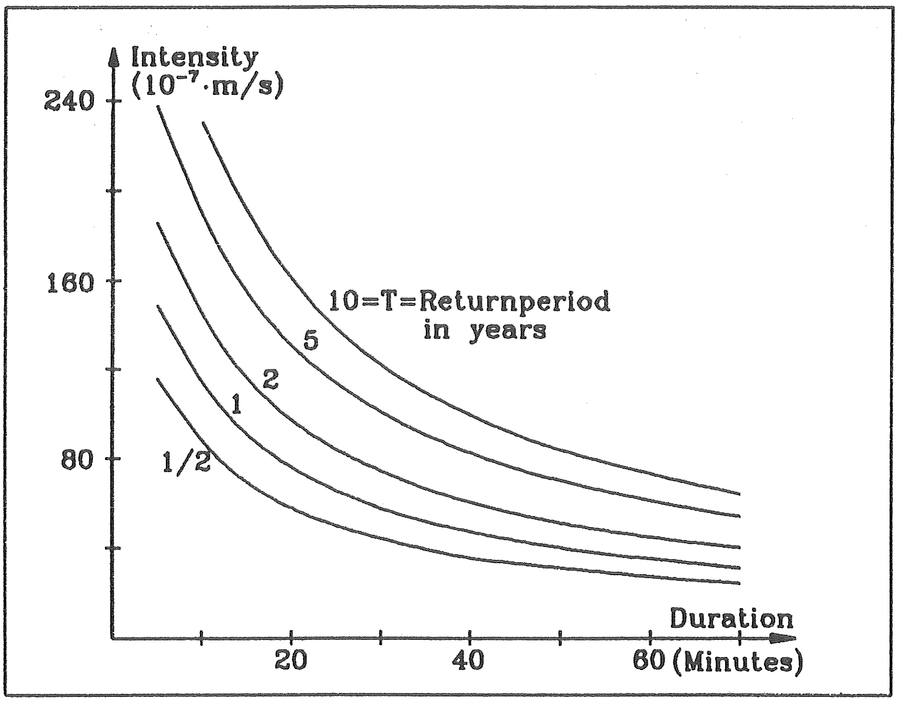

Figure 3 IDF-diagrams used in Göteborg (from VAV 1976).

$$t_c = K_1/\, i^{2/5} + K_2/\, i^{1/4} \tag{8}$$

where

$$K_1 = (n_s \cdot L_s)^{3/5} \,/\, S_s^{3/10} \tag{9}$$

representing the surface flow part (index s) and

$$K_2 = \left(\frac{n_g^2 \cdot L_g^2}{S_g}\right)^{3/8} \cdot \left(\frac{4(1/z_g+z_g)}{L_s}\right)^{1/4}$$

$$+\left(\frac{(n_p)^2}{S_p}\right)^{3/8} \cdot \left(\frac{4(1/z_p+z_p)}{A_c}\right)^{1/4} \cdot L_p \tag{10}$$

representing the gutter flow (index g) and pipe flow (index p) respectively. For simplicity,the sewer cross section is assumed here to have a V-shape. n is Manning's coefficient of roughness, L and S are length and slope in flow direction, z is the slope factor of side walls. Similar relations also based on the kinematic wave theory has been presented by Singh (1975) and Akan (1984).

Evaluation of the maximum flow

The IDF-curves may be expressed by the relation

$$i = \frac{a}{t_d+b} + c \tag{11}$$

where a, b and c are parameters which vary with location and return period. Using the Rational Method we are looking for the rain intensity corresponding to the time of concentration estimated by equation (8) which is a function of the rain intensity.The intensity is obtained, together with the time of concentration, by solving equations (12a) and (12b)

$$i(T,t_c) = \frac{a}{t_c+b} + c \tag{12a}$$

$$t_c = K_1/(i(T,t_c))^{2/5} + K_2/(i(T,t_c))^{1/4} \tag{12b}$$

This may be done by iteration in the equation (13)

$$(t_c)_{n+1} = K_1/\left(\frac{a}{(t_c)_n+b} + c\right)^{2/5} + K_2/\left(\frac{a}{(t_c)_n+b} + c\right)^{1/4} \tag{13}$$

The intensity $i(T,t_c)$ is obtained from Equation (12a) for a given t_c and the maximum flow by equation (7).

Verification and application

The improved Rational Method relates the distribution functions of rain intensity and flow in a catchment. It is therefore only possible to investigate the relevance of the model in catchments where such functions have been established. Arnell (1982) used an advanced continuous runoff model and a series of historical storms to evaluate distribution functions for discharge and rain intensity in 21 catchments and sub catchments in three different urban areas. The continuous model used was calibrated and verified by measurements of the outflow from the three areas. The study was basically done in order to compare different design storms, used for the design of sewer nets, by detailed runoff models. In Figure 4 the main sewer network of the three areas Bergsjön, Linköping1 and Linköping2 are shown with calculation points.

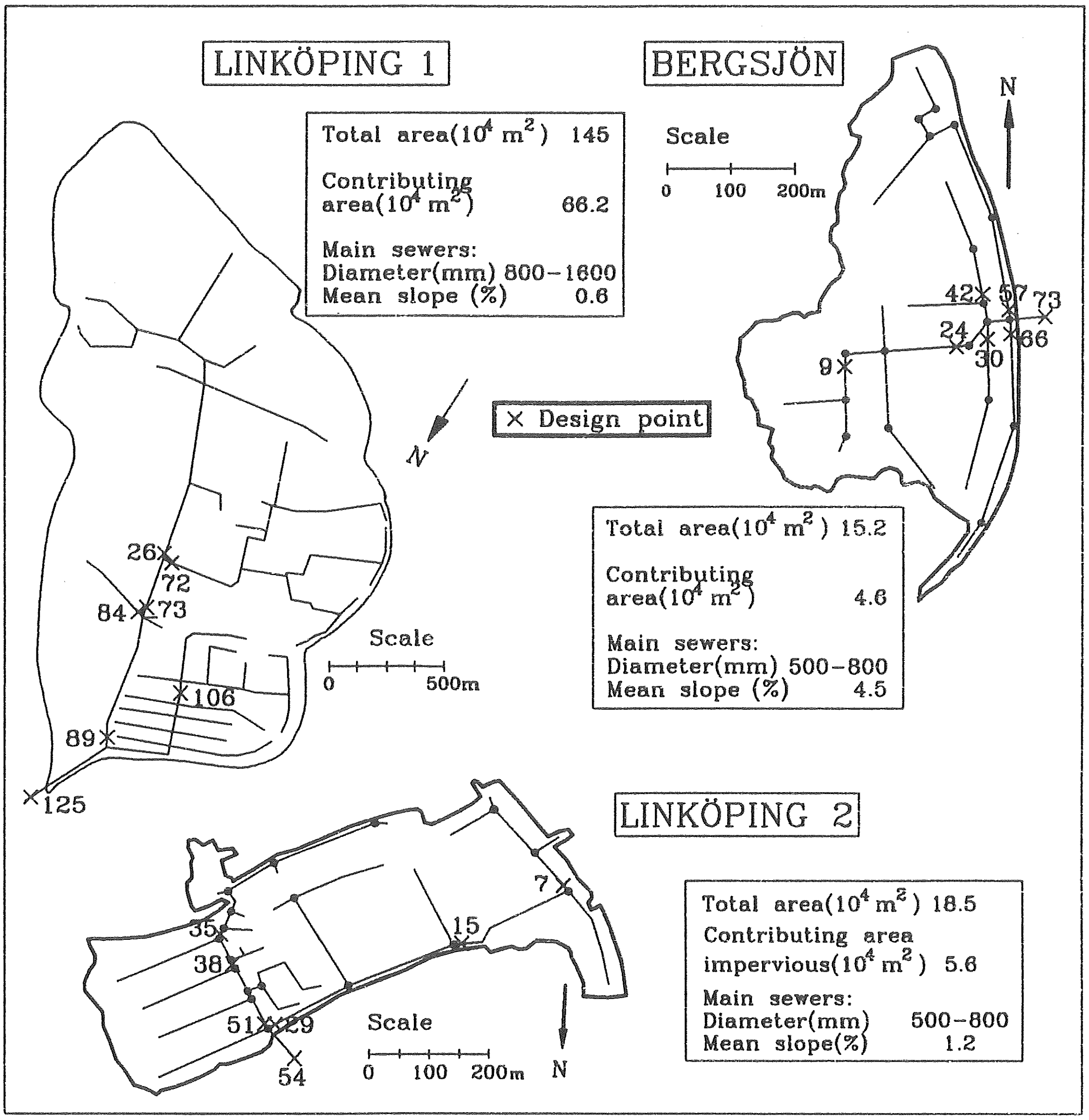

Figure 4 The three urban areas used in the study with calculation points (sub catchments.

In this study Arnell's distribution functions were used for comparisons with statistical peak flows evaluated by the Rational Method as described in the last two sections. The intensity-duration-frequency relation (equation 11) used in the Rational Method was evaluated from the above mentioned set of historical storms. In figure 5 the distribution functions for six of the 21 catchments or calculation points are shown together with corresponding peak flows evaluated by the improved Rational Method. The performance appears by visual inspection of the

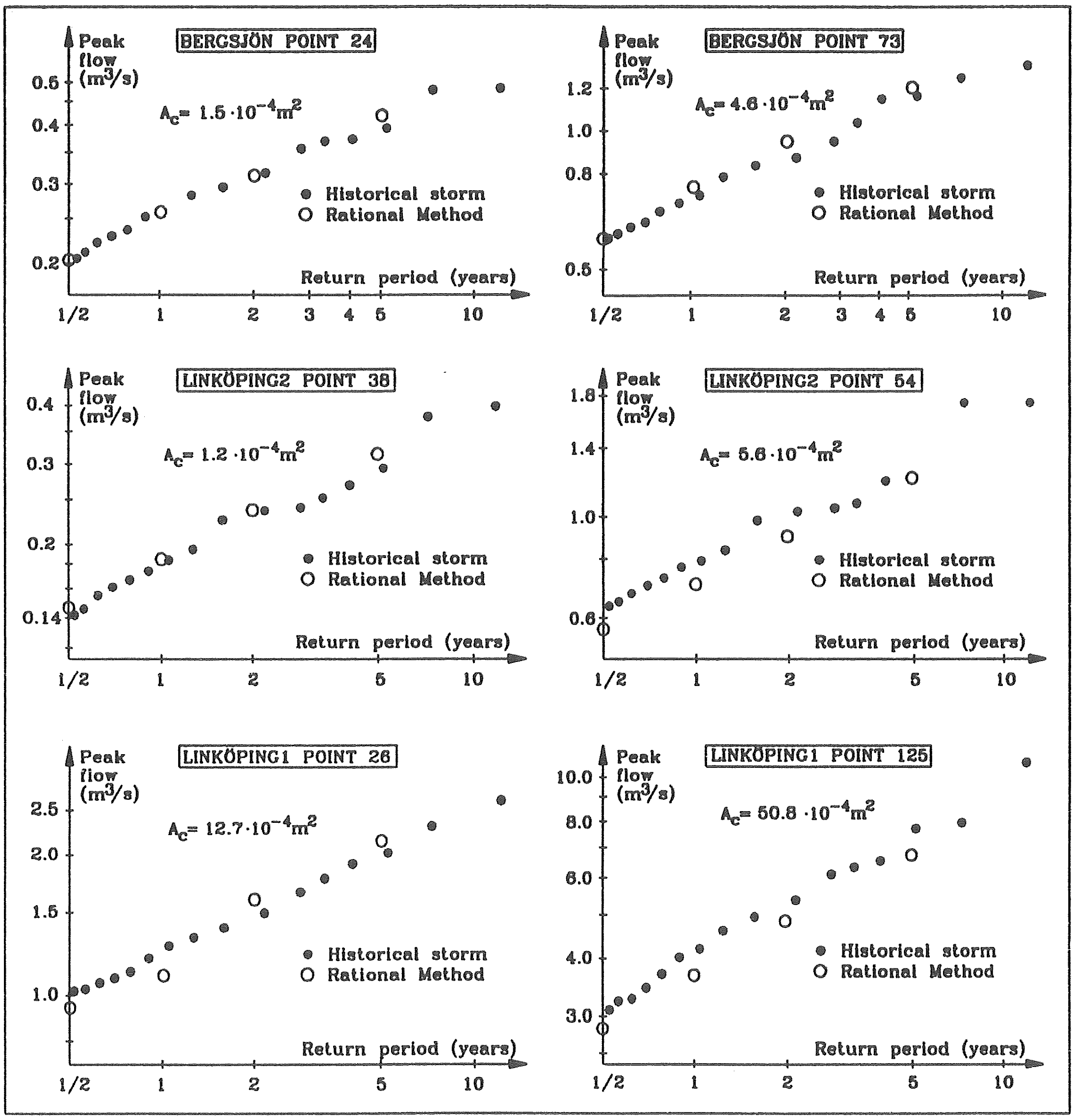

Figure 5 Distribution functions for peak flows in 6 of the 21 analyzed catchments with corresponding statistical peak flows obtained by the Rational Method.

plots to be satisfactory. This is also evident when commonly used statistical parameters for the peaks, namely the mean ratio λ_p, the mean standard deviation σ_p and the absolute error ε_p, are compared with values which in the literature generally are taken as evidence for a good model performance (Colyer 1977, Price 1980), see table 1.

In the study a wide range of catchment sizes have been used from the three smallest, $\approx 0.6 \cdot 10^{+4} m^2$, to the greatest $50 \cdot 10^{+4} m^2$ contributing area. The deviation in flow values between the catchments is shown in figure 6 where the peak flows

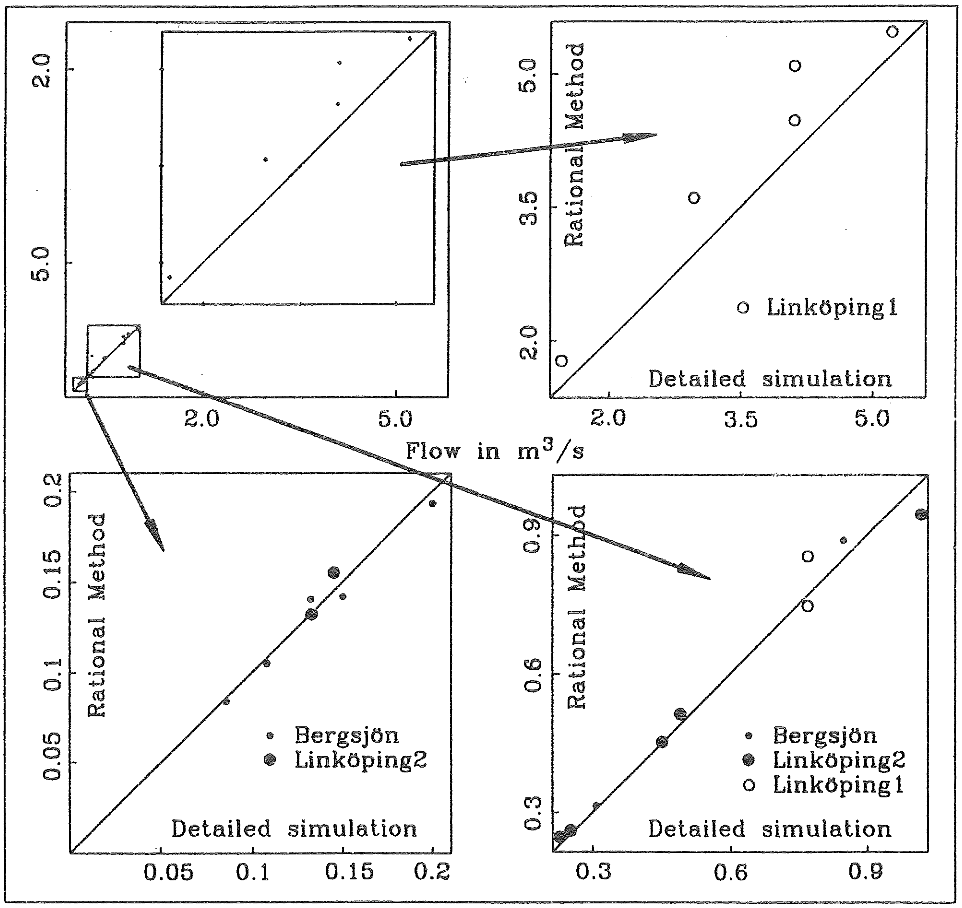

Figure 6 Statistical flow peaks estimated by the advanced continuous model and the improved Rational Method plotted together.

Table 1. Mean ratio $_p$,standard deviation $_p$ and absolute error $_p$ for peak flows evaluated by the detailed model and the Rational Method.

Selected catchments	Mean ratio λ_p	Standard dev. σ_p	Abs. error ε_p %	Number of peaks
All	1.05	0.04	5	21
$<1 \cdot 10^{-4} m^2$	1.00	0.05	2	9
$1\text{-}5 \cdot 10^{-4} m^2$	1.01	0.06	3	7
$12\text{-}50 \cdot 10^{-4} m^2$	1.16	0.08	15	5

from distributions and the Rational Method are plotted together (return period 2 years). In table 1 the statistical parameters evaluated for three classes (sizes) of catchments are given. Because of the few values in each class there should not be drawn to many conclusions from the material. It is, however, clearly shown in the table that the deviation increases with increasing catchment area or time of concentration.

Conclusions and recommendations

The Rational Method, usually referred to as an empirical method, has a sound theoretical basis with a close relationship to the kinematic wave theory. It appears to be capable of estimating statistical design flows. The accuracy of estimated peak flows decreases with increasing contributing area but the method may still be used in quite large catchments.

It should be stressed that the method, as presented here, requires much the same amount of input data as a kinematic wave model. In addition it is usually advantageous to have the entire hydrograph and not only the design flow as a basis in the design situation. Runoff systems with retention storages or overflows are examples where routing methods are preferred.

The improved Rational Method is a very suitable method for calculating peak flows in the preliminary design stage of a network system, in small or simple systems and also for checking advanced continuous models. Basic in the method is the careful estimation of the time of concentration with relations based on the kinematic wave theory.

From a practical point of view the proposed method with careful estimation of the time of concentration seems rather complicated. However, when the method has been used for a while the user will get a better feeling for variations in time of concentration and flow with slope, shape, area and so on. Because of this, there is an important educational aspect of the method; the practicing engineer will improve his skill to a level which is hardly reached when using the traditional method.

Acknowledgements

The work described in this paper has been carried out at the Department of Hydraulics, Chalmers University of Technology as a part of a research program within the Geohydrological Research Group. The program is financially supported by the Swedish Council for Building Research. The author wishes to thank Dr Arnell who evaluated the distribution functions for the rain intensities and peak flows that have been used in the study.

List of symbols

A	cross-section of flow
A_c	contributing catchment area
a	parameter in the nonlinear friction relation
	parameter in the IDF-relation
B	width of channel
b	parameter in the nonlinear friction relation
i	rain intensity
i_{max}	maximum average rain intensity
L	length in flow direction
n	Manning's coefficient of roughness
Q	flow rate
Q_{max}	maximum flow rates obtained by the Rational Method
S_b	slope in flow direction
T	return period
t	time
t_c	time of concentration
t_d	duration of the rain
x	space coordinate
z	slope factor of side walls
s, g, p	index denoting surfaces gutters and pipes respectively
ε_p	meanabsolute error in compared peak flow values
λ_p	mean of the ratio between flow peaks
σ_p	standard deviation of the ratio between compared flow peaks

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