A Plane Wave Approximation in the Computation of Multiscattering Effects in Reflector Systems

7th European Conference on Antennas and Propagation, EuCAP 2013, Gothenburg, Sweden, 8-12 April 2013

Citation for the published paper:
Iupikov, O. ; Maaskant, R. ; Ivashina, M. (2013) "A Plane Wave Approximation in the Computation of Multiscattering Effects in Reflector Systems". 7th European Conference on Antennas and Propagation, EuCAP 2013, Gothenburg, Sweden, 8-12 April 2013 pp. 3828-3832.

A Plane Wave Approximation in the Computation of Multiscattering Effects in Reflector Systems

Oleg Iupikov*, Rob Maaskant*, and Marianna V. Ivashina*

*Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden
Email: oleg.iupikov@chalmers.se, rob.maaskant@chalmers.se, marianna.ivashina@chalmers.se

***Abstract*—A hybrid MoM/PO method for the analysis of multiple scattering effects between reflector and large feeds, such as dense multibeam phased array feeds or multifrequency front-ends (MFFEs) in which higher frequency feeds operate in the vicinity of an extended metal structure, has been presented and studied. This paper evaluates the accuracy and computational efficiency for the MoM/PO method with and without using a uniform plane wave approximation of the reflector scattered field.**

***Index Terms*—multiscattering; standing wave phenomena; phased array feeds; reflector antennas**

I. INTRODUCTION

Prime-focus reflector antennas are widely used for radio astronomy, satellite and radio link communication thanks to their relatively low cost as compared to that of more complex offset- and multi-reflector systems. When designing these antennas, one focuses on the optimization of the antenna feed to realize high gain, low sidelobes, and low spillover loss for the selected reflector, often under stringent dimensional constraints to minimize the aperture blockage and frequency variation of the antenna characteristics due to multiple scattering effects of electromagnetic waves traveling between the feed and reflector antenna.

During the last decades, a number of analytic and numerical techniques have been developed to model feed-reflector interaction effects. For example, in [1] the scattered field of the feed is approximated by a geometric series of fields scattered by the antenna feed due to an incident plane wave at each iteration, where the amplitudes of these plane waves are expressed analytically for a given reflector geometry. This method is very fast and, for the case of a horn feed with an aperture diameter in the order of one wavelength, has been demonstrated to have an accuracy comparable to that of a MoM approach. An alternative to this method is the use of more rigorous (though more time-consuming) hybrid numerical methods combining Physical Optics or Gaussian beams for the analysis of reflectors with the Method of Moments and/or Mode Matching techniques for radiating horns feeds [2], [3]. The recent article [4] has introduced the PO/Generalized-Scattering-Matrix approach for solving multiple domain problems, and has shown its application to a cluster of a few horns. This approach is generic and accurate, but requires the filling of a large scattering matrix that can be time consuming especially for more complex feed systems, such as (i) multifrequency front-ends (MFFEs) in which higher frequency feeds operate in the vicinity of an extended metal structure, or (ii) dense multibeam phased array feeds (PAFs [5], [6]). On the other hand, the above-mentioned analytic method may be inaccurate for these systems, due to a much larger physical area and higher complexity of radiation/scattering mechanisms (the plane wave approximation may not hold). To examine this multiple domain problem with MFFEs and PAFs, we propose to use a hybrid MoM/PO approach as described in [7]. While in [7] the field is computed at each mesh cell of the feed and reflector structure, herein we investigate the approximation of the field scattered by the reflector with a (single) uniform plane wave defined over the area of the feed. As will be shown in this paper, the scattered field computed through integration of the reflector PO currents needs to be known only at a few points in the focal plane region in order to determine the plane wave expansion coefficient in an accurate manner. This significantly reduces the simulation time relative to a direct MoM/PO solution.

II. MODELING PROCEDURE AND NUMERICAL RESULTS

The MoM/PO method [7] consists of the following steps: (i) the antenna feed currents are computed through a method-of-moments (MoM) approach by exciting the antenna port(s) in the absence of the reflector; (ii) these currents generate an EM field which induces PO-currents on the reflector surface; (iii) the PO currents create a scattered field that, in turn, induces currents on the feed structure. The steps (ii) and (iii) are repeated until the sum of the multiply induced currents – which forms the total current – has converged (typically, for low-scattering feeds, 2-3 iterations are enough to achieve an error less than 1% relative to a MoM solution). Afterwards, we can determine the antenna radiation pattern, the input impedance (matrix), and derived antenna parameters affecting the receiving sensitivity.

The third step of this procedure is the most time-consuming since it requires the field computation (integrating of PO currents) at each mesh cell of the feed. To alleviate this computational burden, the field scattered from the reflector can be expanded into a plane wave spectrum, each spectral component of which induces a current on the feed. This approach is much faster since it does not require the integration of the reflector currents at each basis function of the feed; the smoothly-varying field has to be tested at a few points only to find the expansion coefficients of the corresponding plane wave modes. The incident field on the feed is then tested through these plane wave modes.

The model $\mathbf{E}^{\text{mod}}$ of the actual focal field $\mathbf{E}^{\text{ref}}$ of the reflector antenna, due to a radiating PO current on the reflector, can be expanded into a set of plane wave modes $\{\mathbf{E}_n\}$ as

$$\mathbf{E}^{\text{mod}} = \sum_{n=1}^{N} \alpha_n \mathbf{E}_n. \tag{1}$$

The least squares error ϵ between the actual field $\mathbf{E}^{\text{ref}}$ and the modeled field (1) can be expressed as

$$\epsilon(\boldsymbol{\alpha}) = \langle \mathbf{E}^{\text{ref}} - \mathbf{E}^{\text{mod}}(\boldsymbol{\alpha}), \mathbf{E}^{\text{ref}} - \mathbf{E}^{\text{mod}}(\boldsymbol{\alpha}) \rangle \tag{2}$$

where

$$\langle \mathbf{a}, \mathbf{b} \rangle = \iint_{S_a} \mathbf{a}^H \mathbf{b} \, \mathrm{d}S; \tag{3}$$

$(\ldots)^H$ is the Hermitian operator; and S_a is the area constituting the support of the vector function $\mathbf{a}$.

It can be shown that, the solution that minimizes ϵ is obtained through solving the matrix expression

$$\mathbf{A}\boldsymbol{\alpha} = \mathbf{b} \tag{4}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$;

$$\mathbf{A}_{mn} = \langle \mathbf{E}_m, \mathbf{E}_n \rangle \quad \text{and} \quad \mathbf{b}_m = \langle \mathbf{E}_m, \mathbf{E}^{\text{ref}} \rangle \tag{5}$$

for $m, n = 1, 2, ..., N$.

Since the scattered field from large parabolic reflectors resembles a plane wave in the vicinity of the antenna feed, it is sufficient to employ only a single plane wave expansion function [1]. Hence, we can solve Eq. (4) analytically for the coefficient α_1:

$$\alpha_1 = \frac{\langle \mathbf{E}_1, \mathbf{E}^{\text{ref}} \rangle}{\langle \mathbf{E}_1, \mathbf{E}_1 \rangle}. \tag{6}$$

If we choose the plane wave expansion function to have unit amplitude, the coefficient α_1 will be equal to

$$\alpha_1 = \frac{1}{A_{\text{f}}} \iint_{A_{\text{f}}} E_p^{\text{ref}} \, \mathrm{d}S \tag{7}$$

where the subscript p denotes the dominant component of the field $\mathbf{E}^{\text{ref}}$; A_{f} is the area in the focal plane occupied by the feed. Eq. (7) can be evaluated numerically using the midpoint integration rule, i.e.,

$$\alpha_1 \approx \frac{1}{K} \sum_{k=1}^{K} E_p^{\text{ref}}(\mathbf{r}_k), \tag{8}$$

where the set $\{\mathbf{r}_k\}_{k=1}^{K}$ are K sample points, which are assumed to be located on a uniform grid.

In summary, the plane-wave-enhanced MoM/PO method consists of the following steps: (i) the antenna feed currents are computed through a method-of-moments (MoM) approach by exciting the antenna port(s) in the absence of the reflector; (ii) these currents generate an EM field which induces PO-currents on the reflector surface; (iii) the PO currents create a scattered field that is tested at only a few points in the focal plane; (iv) the field intensity at the sample points is averaged in accordence with (8), and the obtained value is used as the expansion coefficient for the plane wave traveling from the reflector towards the feed; (v) this incident plane wave induces a new current distribution on the feed structure. The steps (ii)–(v) are repeated until a convergence condition is met.

The following three types of feeds were used to illuminate a reflector antenna: (i) a pyramidal horn with aperture diameter in the order of one wavelength, (ii) a pyramidal horn with extended ground plane, and (iii) an 121-element dual-polarized dipole array [see Fig. 1(a)]. All antennas are impedance power-matched, so that antenna component [8] of their corresponding radar cross-section (RCS) is minimized. However, the residual component of the RCS of the horn with ground plane is still high due to the extended metal structure surrounding it, so that this feed is a high scattering antenna and strong feed-reflector coupling can be expected.

The corresponding E- and H-plane focal field distribution cuts at the 1st and 2nd iterations are shown in Figs. 2(b) and (c), respectively, each for the reflector antenna with the semi-subtended angle of 70 deg and a respective diameter of 38λ and 118λ. This result clearly demonstrates that the field scattered by the reflector differs slightly from a uniform plane wave, where the largest variation in amplitude is about 0.8–1.5 dB (and 4–6 degrees in phase) over the area occupied by the feed (the vertical dashed line). The ripples in the focal plane field at the 1st iteration are due to diffraction effects from the reflector edges when it is illuminated by the primary feed pattern and, as expected, are more pronounced for the electrically smaller reflector, regardless of the type of the feed. It is also observed that, at the 2nd iteration, when the scattered field component of the feed is incident on the reflector, the focal field distribution due to the horn feed remains rather uniform, but becomes more tapered for the case of the electrically larger feeds (the PAF and horn with the extended ground plane) because of the much narrower scattered patterns of these feeds [see Fig. 1(b)]. Thus, larger errors due to the plane wave approximation can be expected for these feed structures. Another important observation is that the shapes of the scattered patterns and the corresponding focal fields at the 2nd iteration are rather similar in case of the PAF and horn with the extended ground plane, as the result of the equal aperture areas. This similarity, however, does not imply that modeling errors due to the plane wave approximation will be close as well. This can be readily seen from Table I, where the errors in the total focal field and several antenna characteristics such as the gain, the gain at -3 dB level, and the antenna input impedance (in case of an array – the input impedance of the most excited antenna element) are summarized.

The errors in focal field and scalars antenna characteristics are computed as

TABLE I

ERRORS DUE TO A PLANE WAVE APPROXIMATION

	Focal field		**Gain (on-axis)**		**Gain (@-3 dB)**		**Impedance**	
Reflector diameter D	38λ	118λ	38λ	118λ	38λ	118λ	38λ	118λ
Feed: Pyramidal horn								
Parameter variation, %	3.91	1.23	1.98	0.62	3.99	2.16	15.05	4.66
Method:	**Error, %**							
Method 1	0.3	0.05	0.28	0.05	0.36	0.14	1.37	0.18
Method 2	0.1	0.04	0.16	0.04	0.3	0.13	0.09	0.03
Feed: Pyramidal horn with extended ground plane								
Parameter variation, %	139.3	39.1	19.2	3.4	29.4	3.56	43.4	6.1
Method:	**Error, %**							
Method 1	37.7	1.29	12.7	0.1	10.1	0.17	18.5	0.2
Method 2	11.9	0.48	2.23	0.07	4.71	0.15	12.46	0.11
Feed: 121-element dual-polarized dipole array								
Parameter variation, %	8.45	3.28	1.84	0.28	3.68	0.73	5.8	1.7
Method:	**Error, %**							
Method 1	0.61	0.11	0.21	0.03	0.15	0.02	0.34	0.08
Method 2	0.44	0.1	0.12	0.03	0.08	0.03	0.58	0.05

TABLE II

TOTAL SIMULATION TIME

	Horn	**Horn with ground plane**	**Dipole array**	**Vivaldi array**
MoM-PO, no approximations	9 min 05 sec (100%)	59 min 21 sec (100%)	71 min 09 sec (100%)	197 min 04 sec (100%)
Method 1	0 min 39 sec (7%)	1 min 12 sec (2%)	4 min 49 sec (7%)	33 min 58 sec (17%)
Method 2	2 min 32 sec (28%)	13 min 28 sec (23%)	19 min 19 sec (27%)	67 min 06 sec (34%)

$$\epsilon_1 = \frac{\sqrt{\sum_k |E^{\text{ref}}_{p;k} - E^{\text{mod}}_{p;k}|^2}}{\sqrt{\sum_k |E^{\text{ref}}_{p;k}|^2}} \times 100\% \tag{9}$$

$$\epsilon_2 = \frac{|f^{\text{ref}} - f^{\text{mod}}|}{|f^{\text{ref}}|} \times 100\%, \tag{10}$$

where $E^{\text{ref}}_{p;k}$ and $E^{\text{mod}}_{p;k}$ are the k-th sample of the discretized p-components of the focal E-field $\mathbf{E}^{\text{ref}}$ and $\mathbf{E}^{\text{mod}}$ respectively; f^{ref} and f^{mod} is the gain or antenna input impedance, reference and modeled values respectively. The MoM/PO results without the plane wave approximation are used as the reference solution.

The above values were also computed using the method described in [1], where the plane wave coefficient α_1 is computed analytically from the field intensity in the on-axis direction of both the original and the scattered feed pattern due to an incident plane wave. We will refer to this method as “Method 1” while the herein proposed approach is denoted as “Method 2”.

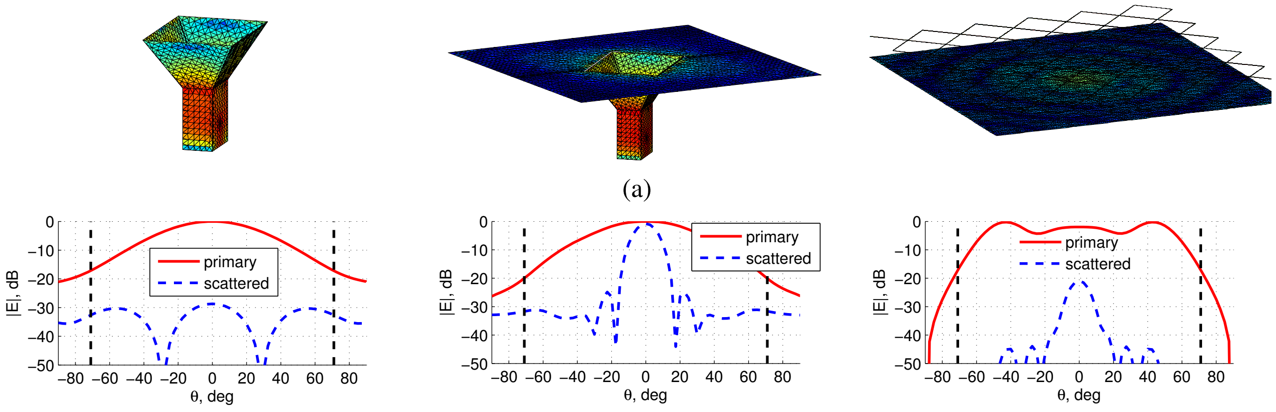
The total simulation time (10 frequency points) for the 38λ reflector fed by the considered feeds is shown in table II. Virtually all simulation time is consumed by the field computation on the reflector surface for obtaining its PO currents, while the computation of the currents on the feed due to the currents on the reflector is more than 1000 times faster when a plane wave approximation is used.

By analyzing Table I and Table II the following observations can be made:

- Method 1 is numerically efficient and accurate for small feeds (whose size is in the order of one wavelength) and for low-scattering feeds, but fails in case of large high-scattering feeds, such as MFFEs, because the focal field produced by the feed scattering pattern has a high level and a highly tapered shape;
- Method 2 provides a better prediction of all the system parameters, since it accounts for the actual shape of the scattered pattern when fitting the plane wave to it; however, it is slower than Method 1;
- Both methods are accurate in case of large reflectors because (i) the multiscattering effects are less pronounced (see “Parameter variation” in Table I), and (ii) the field scattered from the reflector is close to a plane wave at all iterations.

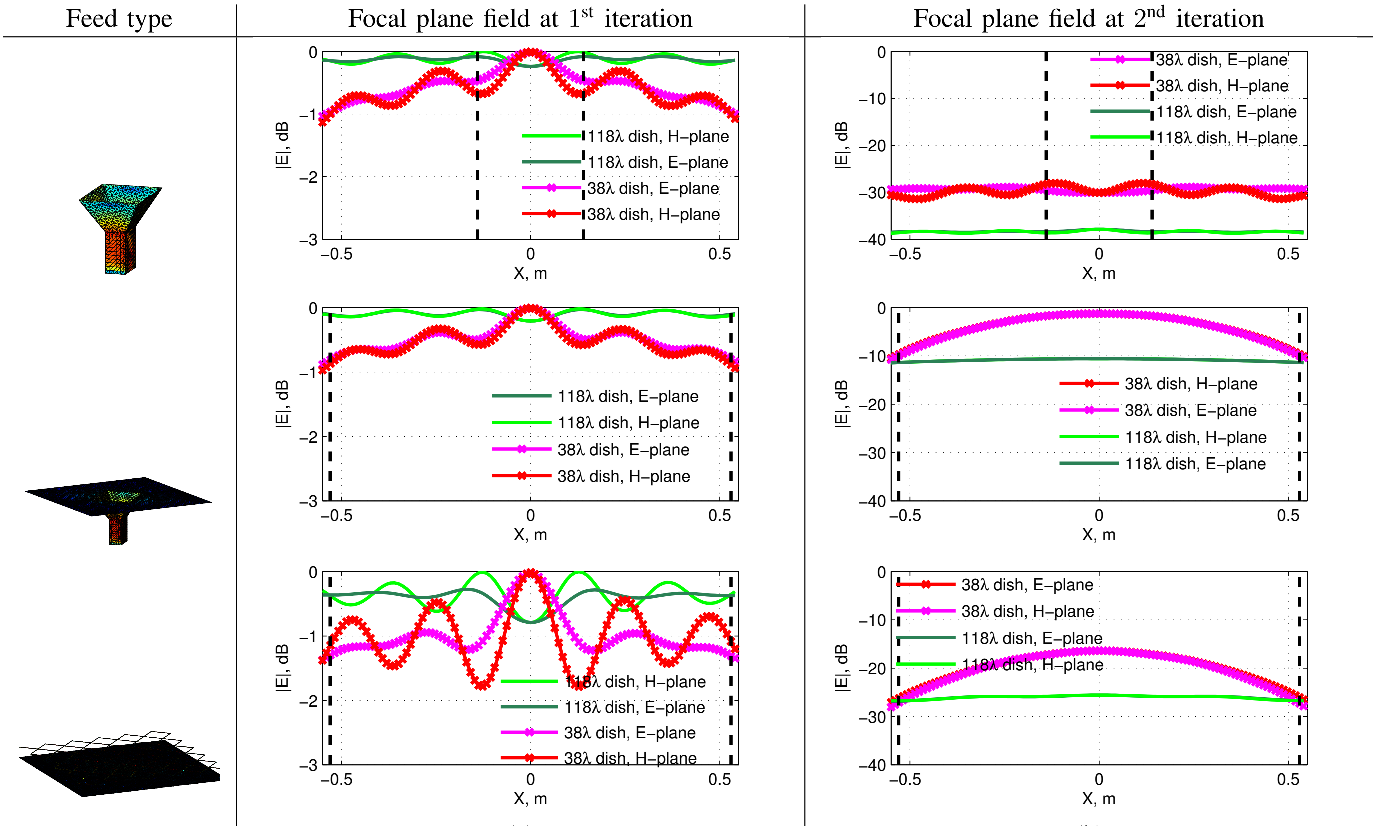
III. CONCLUSIONS

A hybrid MoM/PO method for the analysis of multiple scattering effects between the reflector and large feeds, such as PAFs and MFFEs, has been presented and studied. It has been shown that, although the field scattered by the parabolic reflector differs slightly from that of a uniform plane wave, the plane wave approximation can be used to predict the main antenna parameters with an error less than a few percent relative to a direct MoM/PO approach, while reducing the computational time significantly. It has also been shown that, for electrically large high-scattering feeds (exceeding 2–3 λ in diameter), the plane wave approximation gives rise to an increased error, since the scattered field from a reflector at the 2nd iteration is tapered and has a large amplitude. In the latter



(b)

Fig. 1. (a) EM models of the reflector antenna feeds, including (when viewing from left to right) the pyramidal horn feed (with the aperture diameter of one wavelength) without and with the extended ground plane and the phased array feed of 121 half wavelength dipole antenna elements; and (b) the corresponding primary field patterns of the feeds and their scattered field patterns due to the field incident from the reflector at the 1st iteration.



(a) (b)

Fig. 2. (a)-(b) The focal plane fields of the reflector antenna on transmit for the feeds shown in Fig. 1(a). The plots in Fig. 2(a) are for the fields computed at the 1st iteration, when the reflector is illuminated by the primary field of each of the considered feeds, and the results in Fig. 2(b) are for the fields obtained at the 2nd iteration, when the illumination source is the scattering field component of the feed due to the scattered field from the reflector at the 1st iteration.

a spectrum of plane waves can be considered, which is planned as future work. Among the antenna characteristics, the input impedance is found to be the most sensitive to errors.

REFERENCES

[1] A. Moldsvor and P.-S. Kildal, "Systematic approach to control feed scattering and multiple reflections in symmetrical primary-fed reflector antennas," *IEEE Trans. Antennas Propag.*, vol. 139, no. 1, pp. 65–71, Sep. 1992.

[2] P. Bolli, G. Gentili, L. Lucci, R. Nesti, G. Pelosi, and G. Toso, "A hybrid perturbative technique to characterize the coupling between a corrugated horn and a reflector dish," *IEEE Trans. Antennas Propag.*, vol. 54, no. 2, pp. 595–603, Sep. 2006.

[3] M. Bandinelli, F. Milani, G. Guida, M. Bercigli, P. Frandsen, S. Sorensen, B. Bencivenga, and M. Sabbadini, "Feed-array design in presence of

strong scattering from reflectors," in *Proc. European Conference on Antennas and Propag. (EuCAP)*, Rome, Italy, Apr. 2011, pp. 3844–3848.

[4] C. D. Giovampaola, E. Martini, A. Toccafondi, and S. Maci, "A hybrid po/generalized-scattering-matrix approach for estimating the reflector induced mismatch," *IEEE Trans. Antennas Propag.*, vol. 60, no. 9, pp. 4316–4325, Sep. 2012.

[5] S. Hay, R. Mittra, and N. Huang. (2010) Analysis of reflector and feed scattering and coupling effects on the sensitivity of phased array feeds. [Online]. Available: http://csas.ee.byu.edu/docs/Workshop/BYU SGH.pdf

[6] M. V. Ivashina, O. Iupikov, R. Maaskant, W. A. van Cappellen, and T. Oosterloo, "An optimal beamforming strategy for wide-field surveys with phased-array-fed reflector antennas," *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 1864–1875, Jun. 2011.

[7] O. Iupikov, R. Maaskant, and M. Ivashina, "Towards the understanding of the interaction effects between reflector antennas and phased array feeds," in *Proc. Int. Conf. on Electromagn. in Adv. Applicat. (ICEAA)*, Cape Town, Sep. 2012, pp. 792–795.

[8] B. Munk, Ed., *Finite Antenna Arrays and FSS*. New York: John Wiley and Sons, 2003.