Microwave Breakdown due to Heating of an Irradiated Metal Sphere

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Abstract

We investigate the effect on the microwave breakdown threshold by the heating of a metal ball irradiated by a plane electromagnetic continuous wave. It is found that the effect can be substantial for a given combination of parameters, in a certain range of ball radii, but that the time it takes to reach breakdown temperatures increases rapidly with ball radii.

Introduction

Microwave breakdown of the gas in RF equipment is a dangerous failure mechanism [1]. The deleterious effects of the plasma generated by the electron avalanche ranges from noise to the melting and destruction of the system. In order to avoid breakdown one typically calculates the breakdown threshold for the electrical field. The exact value for the threshold depends heavily on the configuration of the electric field, and hence the system geometry. But the general dependence is due to the effective gas pressure. The effective pressure is a measure of the gas density, and there exists a region around 1 Torr where the threshold is at its lowest, the so called Paschen minimum. The gas density depends upon the temperature, and above the Paschen minimum, the breakdown threshold decreases with increasing temperature, provided the absolute pressure is constant. Consequently, it is possible to generate local breakdown in a field which is below the threshold simply by raising the local temperature to the critical level [2]. Since protruding metal elements in RF systems may be heated by microwave absorption, there exists the possibility of breakdown in a system which is operating below the room temperature threshold. It is hard to provide general predictions for when this is a risk, since the thermal balance of an absorbing metal part will depend so much on the geometry of the system. In order to make a worst case analysis, we choose to investigate the situation of a free floating metal sphere in air being irradiated by a plane wave. This setup has the benefit of minimum heat loss, a minimal local field enhancement (which leads to increased heating), and there exists good expressions for the heating power, as well as the heat loss through convection.

Breakdown thresholds

When a metal sphere is irradiated by a plane wave, the local field around the sphere will be changed. Local regions of enhanced field will appear around the ball. This leads to the existence of three separate breakdown thresholds. The first one corresponds to breakdown of the air in the undisturbed field, and is given by the equality of the applied rms field, E_0, and the well-known value for breakdown in a homogeneous field

$$E_I \approx 30 p_{eff} \quad \text{(V/cm)} \tag{1}$$

where the effective pressure is given by

$$p_{eff} = p \frac{T_0}{T} \quad \text{(Torr)} \tag{2}$$

where p is the absolute pressure, and T is the gas temperature.

The field close to the sphere is enhanced with a factor β, where $\beta = 3$ for the quasistatic case ($a << c/\omega$, where a is the sphere radius, c the light velocity, and ω the field frequency), and $\beta = 3$ for the limit of geometrical optics ($a >> c/\omega$), which means that the second breakdown threshold is

$$E_{II} = \frac{E_I}{\beta} \quad \text{(V/cm)} \tag{3}$$

Breakdown due to heating is possible when the temperature on the sphere surface, T_S, is high enough to lower the second threshold to the value of the applied field. We call this temperature the breakdown temperature, T_b,

$$T_b = T_0 \frac{E_{II}}{E_0} \quad \text{(K)} \tag{4}$$

Then, the third threshold is defined by

$$T_S(E_{III}) = T_b \quad \text{(K)} \tag{5}$$

When the electric field is exactly equal to the third threshold, the temperature rise takes an infinite time, and to reach breakdown in a finite time, the field must be between the third and second threshold.

Ball heating, heat loss, and thermal breakdown time

The nature of the heating of the ball depends heavily on the relative size of the ball with respect to the field wavelength. When the ball is smaller than the wavelength (the quasistatic regime), but larger than the nanometre scale, the heating is mainly of the magnetic type, and can be expressed as [3]

$$W_Q = \frac{3\pi}{4} a^2 \delta^3 \sigma \frac{\omega^2}{c^2} E_0^2 \quad \text{(W)} \tag{6}$$

And when the ball is large (geometrical optics limit), the heating is given by

$$W_G = \frac{3\pi}{8} a^2 \delta^3 \sigma \frac{\omega^2}{c^2} E_0^2 \quad \text{(W)} \tag{7}$$

In the range where $a \sim c/\omega$, the heating power will fluctuate and go through a sequence of maxima and minima. In our analysis we shall neglect the detailed behaviour, and simple use the quasistatic approximation for $a < c/\omega$, and the geometrical optics approximation for $a \geq c/\omega$. This goes for the field enhancement factor as well.

The heat loss is mainly due to natural convection, and the temperature of the ball can be found by solving

$$\frac{4\pi}{3} a^3 c_p \rho \frac{\partial T_S}{\partial t} = W - 4\pi a^2 h (T_S - T_0) \tag{8}$$

where c_p is the heat capacity (W/kg*K), ρ the density (kg/m^3), T_0 the room temperature, and h the average heat transfer coefficient (W/m^{2}K) [4]. The exact form of h depends on the temperature and ball size, but for small temperature gradients it can be approximated with $h \approx k_0 / a$, where $k_0 \approx 0.025$ (W/m*K). For small temperature gradients, the solution to Eq. (8) becomes

$$T_S(t) = T_0 + \frac{W}{4\pi a k_0}(1 - \exp(-\frac{3k_0}{a^2 \rho c_p} t)) \tag{9}$$

The breakdown temperature, T_b, is reached after a time t_b, where $t_b \to \infty$ corresponds to the third threshold.

When the electric field is close to the second threshold, heating is very rapid, we can neglect the heat loss term from Eq. (8), and find the solution

$$T_S(t) = T_0 + \frac{3W}{4\pi a^3 \rho c_p} t \tag{10}$$

Equations (9) and (10) together form the asymptotic solutions which bound the more exact solution. In figure 1, the third threshold is drawn for silver at 1 GHz, by using Eq. (9) and $t_b \to \infty$, and the field corresponding to four different values of $t_b = 100$, 10, 1 and 0.1 seconds are drawn by using Eq. (10).

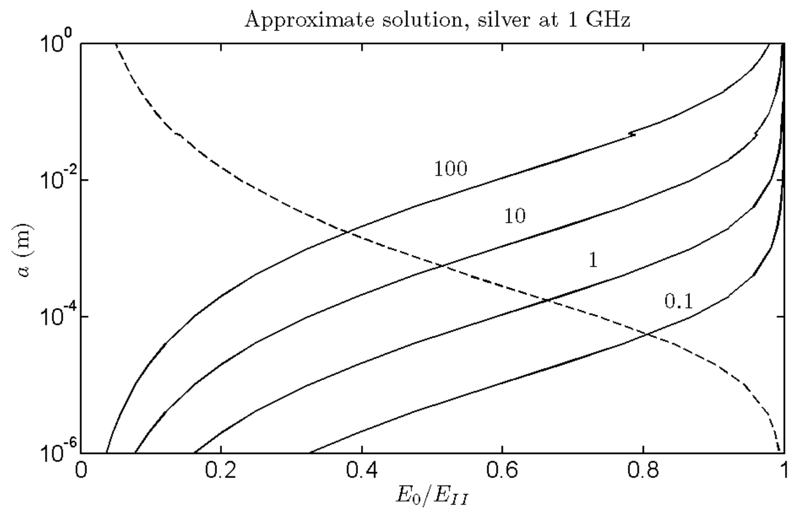

Figure 1. The approximate solution for silver ($\sigma = 6.3\times10^{7}$ (A/Vm), $c_p = 236$ (J/KgK), and $\rho = 10.5\times10^{3}$ (kg/m^3)) at 1 GHz, and four values of breakdown times, $t_b = 100$, 10, 1 and 0.1 seconds (solid lines). The dashed line corresponds to the third threshold, and exactly on the dashed line, the breakdown time is infinite. A small increase in the field leads to breakdown in a finite time, and the wedge-like regions to the left of the intersections of the dashed and solid lines corresponds to the parameter regions where breakdown is possible in a certain time. The kink in the lines around $a \sim 0.1$(m) is due to the discontinuous form used for the heating term, and the field enhancement.

Conclusions

We have qualitatively shown that there exists a certain ball radius which will lead to breakdown at the lowest field strength, and that on both sides of this minimum there is a range of radii which will lead to breakdown in finite time for a certain electrical field above the third threshold. The possibility of verifying the existence of the thermal effect in experiments depend heavily on the maximum CW operation time of the microwave generator in question, and for large spheres, a long time is needed to achieve breakdown, whereas for small spheres, the necessary electric field is close to the room temperature threshold.

References

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