

# On the modelling of wheel/rail noise

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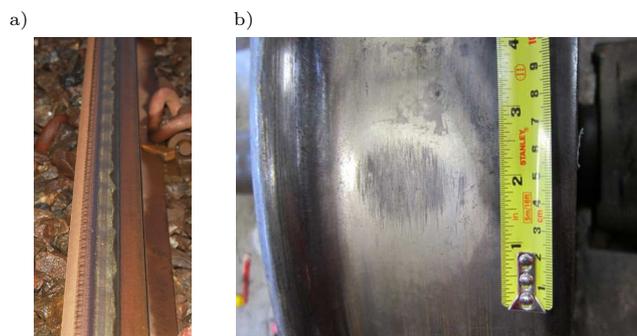
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## Introduction

Noise is one of the main environmental drawbacks of railway operations and is often a critical issue for the expansion of railway infrastructure. Railway noise is caused by diverse sources, whose relative relevance depends on the operating conditions. The importance of traction noise, which includes noise from the power unit and auxiliaries is confined to standstill, acceleration and low speeds below about 60 km/h [1]. For high speeds above approximately 350 km/h, aerodynamic noise, which is generated by unsteady airflow over the train, becomes dominant [2]. In the wide range of train speeds in between, the interaction between wheel and rail induced at the wheel/rail contact is the predominant source of noise emission. This wheel/rail noise consists mainly of rolling noise. Other types of wheel/rail noise are impact noise and curve squeal, with the latter also being relevant at low speeds. While rolling and impact noise are caused by the vertical wheel/rail interaction, curve squeal is induced by a lateral excitation mechanism [2].

Rolling noise is generated by the roughness of the wheel and rail running surfaces, which excites vibrations of track and wheelset in the form of a vertical relative motion. In consequence, the wheelset, the rails and the sleepers radiate noise [2]. The relevant roughness wavelength range is approximately from 5 to 500 mm [2] and typical roughness amplitudes lie in the range 0.1 - 30  $\mu\text{m}$  [3]. In the case of corrugation, which is an extreme form of roughness, greater amplitudes are common. Fig. 1a shows an example of rail corrugation.



**Figure 1:** Excitation at the wheel/rail contact: a) Rail corrugation at a curve in the network of SL, Stockholm. Observe the regular pattern of dark areas in the rolling band. (Photo: P. Torstensson, CHARMEC), b) Wheel flat on the running surface of a railway freight wheel.

Impact noise is caused by discrete irregularities of the wheel and rail running surfaces. The most common such irregularities are wheel flats and rail joints. A wheel flat is a defect of the running surface of a railway

wheel that occurs when the wheel locks and slides along the rail because of malfunction in the brakes or lack of wheel/rail adhesion. The sliding causes severe wear, leading to the wheel being flattened on one side [4], see Fig. 1b. At a rail joint, the rail running surface shows a severe discontinuity characterised by a gap and a height difference between the two sides of the gap [5]. The underlying mechanism of impact noise can be interpreted as an extreme form of roughness excitation [2].

Squeal noise occurring in sharp curves is generated by lateral forces due to frictional instability. It is associated with self-excited vibrations involving stick/slip oscillations at the wheel/rail contact and vibrations of the wheel in one of its resonances [2]. While rolling noise and also impact noise are broad-band phenomena involving a large range of frequencies in the audible range, squeal noise is generally a tonal sound that dominates all other types of noise when it occurs [2].

## Common model assumptions

A model for the prediction of wheel/rail noise needs to cover a large frequency range. Frequencies of up to about 5 kHz should be included. In the case of squeal, even higher frequencies are relevant [2]. Since modelling the dynamics of the complete vehicle/track system in the required frequency range is not possible, it is inevitable to introduce simplifications. First of all, it is assumed that low- and high-frequency dynamics can be decoupled. Programs for classical low-frequency rail vehicle dynamics include typically frequencies up to 20 Hz [6]. Such programs can be used as pre-processing tools for models predicting noise, for instance to obtain the quasi-static curving behaviour of the vehicle as input to squeal models. Furthermore, since the vehicle's primary and secondary suspension isolate the wheelset from the bogie and train body at frequencies of more than a few Hertz [6], the dynamics of the vehicle at higher frequencies is well described by the dynamics of the wheelset. Vehicle components above the primary suspension of the wheelset are commonly introduced as a static preload. While the track is well described by a relatively stiff spring at frequencies below 20 Hz, its inertia becomes increasingly important at higher frequencies [6], and has to be taken into account for the calculation of wheel/rail noise.

In general, the prediction of wheel/rail noise includes the calculation of the wheel/rail interaction and a subsequent calculation of noise radiation and propagation. The model for wheel/rail interaction gives the response of the vehicle/track system to an excitation acting in the

contact zone in the terms of vehicle and track vibrations and dynamic contact forces. This model often consists of three subsystems: a vehicle model, a track model and a contact model. The vehicle and track models describe the global dynamics of the vehicle and the track. They are coupled via the contact model, which comprises the local deformation in and close to the contact zone of wheel and rail. The sound radiation model calculates the noise radiation from wheelset, rails and sleepers, and the total sound pressure at the receiver location is obtained from the model for sound propagation. The presentation is in the following limited to wheel/rail interaction models. An overview of models for sound radiation and propagation from wheel and track is given in Thompson's book [2].

Vehicle, track and contact models of varying complexity have been used in published wheel/rail interaction models and their choice limits the frequency range in which the model is applicable and often implies further assumptions. For instance, the rail can be modelled as a single Timoshenko beam up to about 2.5 kHz if only vertical vibrations are of interest [6]. Above this frequency, cross-sectional deformations of the rail (not modelled by Timoshenko-beam models) have to be taken into account. As another example, the most common normal contact model used in wheel/rail interaction is a Hertzian spring [7]. The use of this model implies the assumption that the excitation in the contact effectively acts at one point. The effect of the finite area of the contact zone on the excitation has then to be taken into account by pre-processing with an adequate contact filter [8]. In the case of roughness excitation, several dB difference are possible in the contact force level depending on the type of contact filter and contact model used. Significant differences are especially obtained if – in situations where the lateral correlation between longitudinal roughness lines is low – measured roughness data from only one longitudinal line is considered instead of the roughness for the complete contact zone [8]. In the case of excitation by wheel flats, calculations based on assumed wheel flat shapes indicate that the three-dimensional description of the wheel flat in comparison to a two-dimensional one is less important [9]. However, when using the Hertzian contact model for wheel flats, it is essential to adequately pre-process the geometrical shape of the wheel flat in order to account for the finite size of the wheel.

## Brief overview of approaches

Wheel/rail interaction models can be formulated either in the frequency or in the time domain. By their nature, frequency-domain models are completely linear models, while time-domain models are suitable to include all kinds of non-linearities. A disadvantage of time-domain models is that they are generally more computationally demanding than are frequency-domain models.

Frequency-domain models work with frequency response functions such as receptances or impedances that represent the dynamic behaviour of the vehicle, the track

and the contact zone. The most well-known frequency-domain model for the calculation of rolling noise is a model going back to Remington [10], which has been generalised and further improved by Thompson [11]. His formulation is implemented in the software package TWINS [12] which is widely used in industry today.

The underlying linearity assumption of frequency domain models is valid for most cases of roughness excitation, but non-linearities in the contact model cannot be neglected in cases of severe roughness and/or a low static contact preload, which can cause loss of contact between wheel and rail [13, 14]. If the response to discrete irregularities such as wheel flats and rail joints is to be calculated, time-domain models are the only option. Only they can capture the discrete nature of the phenomenon and model the loss of contact that is likely to occur [5, 9]. With regard to curve squeal, frequency-domain models can only predict which wheel modes are prone to squeal [15, 16]. As curve squeal is an intrinsically non-linear and transient phenomenon, models aiming to predict squeal amplitudes have to be formulated in the time domain.

Time-domain models essentially solve the system of differential and algebraic equations describing vehicle, track and contact by a time-stepping procedure. Due to the required computational effort of time-domain solutions, it is usually necessary to simplify vehicle, track and contact dynamics or to apply reduction techniques in order to include more detailed submodels. A common approach is to use modal decomposition and reduction for vehicle and track modelled by finite element (FE) procedures [6]. An alternative, computationally efficient approach is to represent vehicle and track by Green's functions. This approach is presented in the following section.

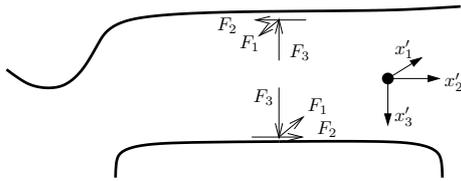
## Green's function approach in the time domain

Under the assumption that vehicle and track are linear and time-invariant, their description in the frequency domain by frequency response functions is completely equivalent to a description by impulse response functions – also called Green's functions – in the time domain. These Green's functions give the response of vehicle or track to an impulse excitation at the contact point. The response to a time series of contact forces is then obtained by convoluting the contact forces with the Green's functions. Since the Green's functions can be pre-calculated before carrying out the time-stepping procedure, this approach leads to relatively short calculation times and makes it consequently possible to include computationally intensive non-linear, non-Hertzian and transient contact models in the calculation [17]. Additionally, the approach is very versatile because any vehicle or track model represented by Green's functions can be used without changing the mathematical formulation of the interaction model.

The method has been introduced in the railway community in 1989 by Manfred Heckl with his proposal for

a railway simulation program [18] and has subsequently been used by several researchers in the area [19, 14, 20, 8, 17]. Already in 1979, McIntyre and Woodhouse [21] proposed a time-domain model for the dynamics of bowed strings based on Green's functions and the approach is also used in models for tyre/road interaction, see e.g. [22].

The Green's function approach is in the following exemplified on the basis of the wheel/rail interaction model published in [17]. The reference frame of the model is depicted in Fig. 2. It is moving with the (pre-calculated) nominal contact point along the rail in rolling direction  $x'_1$ . The forces  $F_1$ ,  $F_2$  and  $F_3$  are respectively the longitudinal, lateral and vertical contact force.



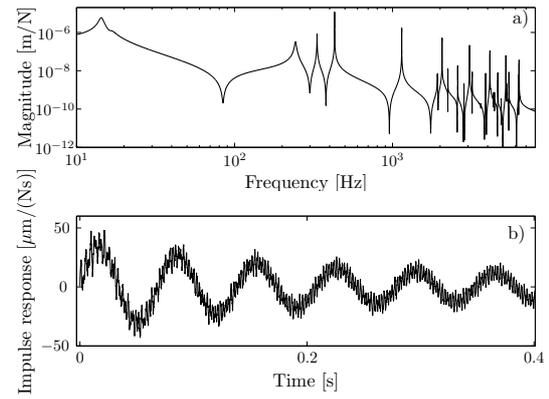
**Figure 2:** Moving reference frame of the wheel/rail interaction model from [17].

The dynamics of the vehicle at the contact point is represented by its Green's functions,  $g_{ij}^W(t)$ , which are obtained from the corresponding receptances  $G_{ij}^W(f)$  by an inverse Fourier transform. The indices  $i$  and  $j$  indicate the direction of the exciting force and of the displacement response, respectively. In the interaction model, the displacements of the vehicle in direction  $j$ ,  $\xi_j^W(t)$ , are then obtained by convoluting the contact forces with the Green's functions of the vehicle:

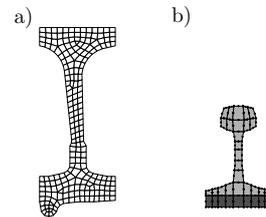
$$\xi_j^W(t) = - \int_0^t \sum_{i=1}^3 F_i(\tau) g_{ij}^W(t - \tau) d\tau, \quad j = 1, 2, 3. \quad (1)$$

Fig. 3 shows as an example the lateral point receptance  $G_{22}^W$  and Green's function  $g_{22}^W$  which have been calculated for the vehicle model depicted in Fig. 4a at an off-centre node located towards the field side of the wheel tread. The vehicle has been represented by one flexible wheel with a rigid constraint applied at the inner edge of the hub, where the wheel would be connected to the axle. The wheel has been modelled by axi-symmetric finite elements with a commercial FE software. In addition to the eigenmodes of the wheel, the receptance from Fig. 3 contains also the rigid body modes of the complete wheelset. Since the wheel is only lightly damped, long Green's functions have to be considered. The total length of the Green's functions taken into account is 20 s. The influence of wheel rotation has been neglected.

The inclusion of the track dynamics in the wheel/rail interaction model is very similar to the inclusion of the vehicle dynamics. However, in order to take into account the motion of the nominal contact point on the rail, a special type of Green's functions is needed. These moving Green's functions [14],  $g_{ij,v}^{R,x_1^0}(t)$ , describe, for excitation of the track in  $i$ -direction at the position  $x_1^0$  at time  $t^0 = 0$ , the displacement response of the track in  $j$ -direction at a point moving at train speed  $v$  away from



**Figure 3:** Example of representation of the wheel in frequency and time domain : a) Lateral point receptance for excitation at the wheel tread at an off-centre node located towards the field side of the tread b) Corresponding lateral Green's function.



**Figure 4:** Flexible wheel and rail models: a) FE mesh of the wheel cross section b) Waveguide FE mesh of the rail.

the excitation, thus at the nominal contact point on the rail. The discrete version of  $g_{ij,v}^{R,x_1^0}(t)$  is constructed from a series of ordinary Green's functions,  $g_{ij}^{R,x_1^0,x_1^0+\chi}(t)$ , where the superscripts specify the excitation point,  $x_1^0$ , and the response point,  $x_1^0 + \chi$ , along the rail. These Green's functions are obtained from the corresponding track transfer receptances,  $G_{ij}^{R,x_1^0,x_1^0+\chi}(f)$  by inverse Fourier transform. In the interaction model, the displacement of the track at the nominal contact point in direction  $j$ ,  $\xi_j^R(t)$ ,  $j = 1, 2, 3$ , is calculated by convoluting the contact forces with the moving Green's functions of the track

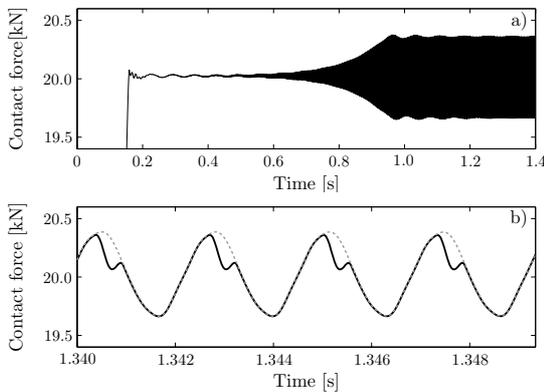
$$\xi_j^R(t) = \int_0^t \sum_{i=1}^3 F_i(\tau) g_{ij,v}^{R,x_1^0}(t - \tau) d\tau, \quad j = 1, 2, 3. \quad (2)$$

The track has been represented by one continuously supported rail modelled with the waveguide FE method using the software WANDS [23], see Fig. 4b. The total length of the moving Green's functions taken into account is 0.25 s.

In the wheel/rail interaction model from [17], vehicle and track model are coupled via a three-dimensional, non-linear and transient contact model, which is an implementation of Kalker's variational theory [24].

A result from the interaction model is given in Fig. 5. The shown simulation corresponds to quasi-static curving of the inner wheel of the leading wheelset in a bogie with the wheelset in an underradial position. The main parameters are: Imposed lateral creepage  $\eta = -1 \cdot 10^{-2}$ , constant friction coefficient  $\mu = 0.3$ , train speed  $v = 50$  km/h and static preload  $P = 65$  kN. Fig. 5 shows that

a stick/slip oscillation develops in the wheel/rail contact. This oscillation is associated with curve squeal, and its occurrence is attributed to the coupling between vertical and tangential dynamics of the vehicle/track system. The main frequency of the oscillation is very close to the eigenfrequency 430 Hz of the axial mode of the wheel with two nodal diameters and zero nodal circles, which is depicted in Fig. 6.



**Figure 5:** Stick/slip oscillations in the wheel/rail contact in the case of a constant friction coefficient  $\mu$ : a) Time series of the lateral contact force  $F_2$ . b) Zoom on the time series of the lateral contact force  $F_2$  (solid line) in comparison to the traction bound  $\mu F_3$  (dashed line).



**Figure 6:** Axial mode of the wheel with two nodal diameters and zero nodal circles, which has an eigenfrequency of 430 Hz.

## Conclusions

Both frequency- and time-domain models have been presented in the literature to calculate the noise caused by the interaction between wheel and rail induced at the wheel/rail contact. In this paper, a time-domain approach has been pointed out, which is based on the representation of vehicle and track by Green's functions. This approach allows the inclusion of any linear and time-invariant vehicle and track model. As the Green's functions are pre-calculated before starting the dynamic simulations, the modelling approach is computationally efficient and makes it possible to consider detailed non-linear contact models.

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