Thesis for the Degree of Licentiate of Engineering

On the Design of Communication Systems for Strong Oscillator Phase Noise

Detection Methods and Constellation Optimization

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Abstract

The problem of designing wireless communication systems to operate in the presence of oscillator phase noise is a classical problem in communication theory. In recent times, there has been a renewed interest in this problem for a multitude of reasons. One of the main factors for this is the unprecedented explosion in the number of wireless and mobile devices that are enabled for communication-intensive and bandwidth hungry applications. This, in turn, is exerting a tremendous pressure on the network infrastructure, where more cost-effective, flexible, high speed connectivity solutions are being sought for. In this regard, wireless backhaul links are an effective solution to transport data by using high order signal constellations, which are extremely prone to hardware impairments like phase noise from imperfect oscillators. Phase noise is also dominant in communication systems that operate over millimeter-wave bands like 60 GHz and higher.

This work is devoted to the classical problem of designing wireless communication systems in the presence of phase noise. First, we address the problem of maximum-likelihood detection of data in the presence of random phase noise due to imperfect oscillators. This is done by designing a low-complexity joint phase-estimator data-detector. We show that the proposed method outperforms existing detectors, especially when high order signal constellations are used.

Then, in order to further improve performance, we consider the problem of designing signal constellations that are optimal in the presence of phase noise. We present two methods for solving this problem; in the first method, constellations are designed such that they minimize the symbol error rate performance of the system impaired by phase noise. In the second method, constellations are designed to maximize the information rate of the system. We observe that these optimal constellations significantly improve the system performance, when compared to conventional constellations and those proposed in the literature.

Keywords: Oscillator, phase noise, maximum likelihood (ML) detection, maximum a posteriori (MAP) estimation, extended Kalman filter (EKF), constellations, symbol error probability, mutual information.

List of Publications

This thesis is based on the following three appended papers[1]:

Paper 1

R. Krishnan, M. Reza Khanzadi, T. Eriksson, T. Svensson, "Soft metrics and their Performance Analysis for Optimal Data Detection in the Presence of Strong Oscillator Phase Noise," *Under Review, IEEE Trans. Commun.*, 2012.

Paper 2

R. Krishnan, H. Mehrpouyan, T. Eriksson, T. Svensson, "Optimal and Approximate Methods for Detection of Uncoded Data with Carrier Phase Noise," *IEEE Global Telecomm. Conf. (GLOBECOM)*, Houston, pp.1-6, 5-9 Dec. 2011.

Paper 3

R. Krishnan, A. Graell i Amat, T. Eriksson, G. Colavolpe, "Optimal Constellations in the Presence of Strong Phase Noise," *Under Review, IEEE Trans. Commun.*, 2013.

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Part I

The Big Picture

Chapter 1

Introduction

Since the landmark paper by Shannon [1], substantial research has been devoted to the design of communications systems that operate close to the ultimate performance limit, i.e., the channel capacity, with an arbitrary small probability of error. Particularly in single-input single-output (SISO) point-to-point wireless systems, much of these efforts have been based on several idealized assumptions like perfect channel state information, perfect synchronization, perfect hardware, untethered implementation complexity, and much more.

As a result of the aforementioned idealized assumptions, there has been a significant gap between the theoretical and practically achieved performance levels in various communication systems, including today's operational 3G and 4G cellular networks. Errors in the channel state information are a major source of performance loss, which is around $2-3$ dB even when the best estimator is used [2]. Nonlinearities in the power amplifier cause distortions in the transmitted signal and its bandwidth expansion, which has to be appropriately modeled and mitigated [3]. Synchronization errors that occur primarily due to phase noise in frequency sources like oscillators, result in significant performance degradation [4].

Oscillators are central to the design of a wireless communication system, and they should be accurate, inexpensive and desirably compact. They provide the carrier and pilot signals required for communication and navigation purposes. They also provide clock signals and reference signals that are used for various purposes like synchronization. All practical oscillators suffer from phase noise, which manifests as a spectrum of noise around its operating frequency. Thus, when information is conveyed from a source (transmitter) to a destination (receiver), a random time-varying phase difference inevitably arises between their respective local oscillators. This is detrimental given that many communication systems are designed to operate synchronously and coherently. If the phase noise is not appropriately addressed, it can result in the distortion of the received signal and undesirably high error rates in phase modulated transmission systems.

1.1 Aim of the Thesis

In this thesis, we will focus on the problem of compensating wireless communication systems impaired by oscillator phase noise by addressing the following questions:

1. How can we systematically derive a low-complexity joint phase-estimator data-detector that is (near) optimal in system performance?

2. How can we design signal constellations that are to be transmitted in a system impaired by phase noise, such that the error rate performance or the information rate of the system is optimized?

In order to comprehensively answer the above questions, it is imperative to understand the phase noise phenomenon and its impact on the communication system performance. Thus, we will discuss about some important results from prior work that are related to the following questions:

- What is the maximum a posteriori (MAP) joint estimator for random phase and data?

- What are the bounds for estimating the random phase noise when data is unknown?

- How can error correcting codes be designed to improve system performance when impaired by phase noise and operate close to channel capacity?

- What is the capacity of channels with phase noise, including those phase noise channels with memory?

1.2 Thesis Outline

The thesis is organized as follows: In Chapter 2, we explain about the phase noise phenomenon and its sources in an oscillator. Then, we discuss about the model that represents a communication system impaired by phase noise. In Chapter 3, we cover prior work related to designing systems affected by phase noise. We first discuss about phase noise trackers and the different low-complexity algorithms for joint phase-estimation and data-detection. For a theoretical understanding of this topic, the reader is referred to [5].

Then, we examine prior work related to constellation design in the presence of phase noise and the capacity of channels impaired by phase noise. Furthermore, we review results related to the design of error control codes for phase noise channels. Finally, we summarize our papers and contributions in Chapter 4.

Chapter 2

Phase Noise in Communication Systems

Oscillators that are used in communication systems are imperfect, in that their output signals are affected by random phase and frequency instabilities. These instabilities manifest themselves as a spectrum of noise around the oscillators' operating frequency. An oscillator signal can suffer both amplitude and phase perturbations. Amplitude fluctuations are attenuated by an amplitude limiting mechanism present in the oscillator circuitry [6]. For this reason, the amplitude noise originating from an oscillator can be ignored, and phase noise is our focus in this thesis. In this chapter, we will briefly review the various sources and models for the phase noise phenomenon. Then, we will discuss about the model that represents a wireless communication system impaired by phase noise.

2.1 Noise Sources in an Oscillator

An oscillator signal is affected by a number of factors. Broadly speaking, these factors can be categorized as short-term instabilities, deterministic instabilities and long-term instabilities [7]. Short-term instabilities, which typically last for a duration of a few seconds, are mainly caused by the following sources of noise in the oscillator:

- Thermal Noise - This is the white noise caused by random motion of electrons due to thermal excitation, and its level is equal to kTB, where k is the Boltzman constant, T is the absolute temperature in Kelvin, and B is the 3$-$dB noise bandwidth [7]. The instantaneous electron motion is completely independent of its past, i.e., the noise is memoryless, and its power spectral density is regarded as white.

- Colored Noise or $1/f$ Noise - This is the spectral noise dominated by low-frequency components that mixes with frequencies close to the carrier frequency of the oscillator [8]. Its instantaneous fluctuations depend on its past and therefore has memory.

The main deterministic sources of oscillator noise are identified as in [9]:

- Power supply feed-through and other interfering sources - Coupling can happen in an oscillator circuit between the oscillator signal and the other signals in the circuitry. This can amplitude/phase modulate the output signal from the oscillator. Other oscillators and digital frequency dividers in the circuitry can also modulate the local oscillator output.

- Spurious signals - Generally, an oscillator is designed to have just one feedback path for phase correction and to generate the desired output signal. However, several feedback paths may exist, which may in turn result in spurious output signals.

In contrast to the above forms of noise, long term instabilities occur due to aging of the resonator material in the oscillator. Typically, these have very slow variations that occur over hours, days, months, or even years and are therefore less critical.

2.2 Phase Noise in an Oscillator

Consider a noisy oscillator that operates at a center frequency of f_{osc} and is affected by white noise and colored noise processes as described before. Let $\Phi(t)$ represent the sum of all these noise processes. Then the phase noise in the output signal of the oscillator is given as

$$\phi(t) \propto \int_0^t \Phi(t')\,\mathrm{d}t', \tag{2.1}$$

where $\Phi(t')$ is a Gaussian process by the central limit theorem [10]. The phase noise process $\phi(t)$ in (2.1) is also a Gaussian process with a variance that increases with time [10]. In other words, *the phase noise in an oscillator is an accumulative Gaussian process that results from integrating both the white and colored noise perturbations over time.* When the cumulative noise process $\Phi(t)$ in the oscillator is assumed to be *white* and Gaussian, then the phase noise $\phi(t)$ defined in (2.1) is a *Wiener process* [10].

The power spectral density (PSD) of the phase noise process $\phi(t)$ in (2.1) is approximately [11]

$$S_\phi(f) \propto \frac{k_2}{f^2} + \frac{k_3}{f^3}, \tag{2.2}$$

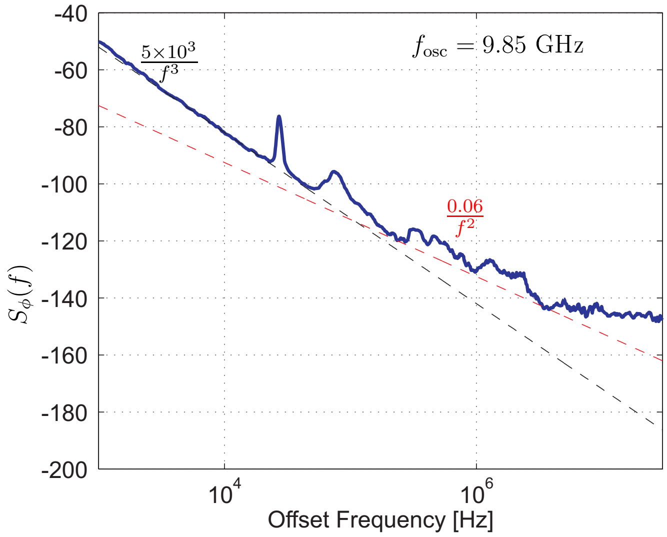


Figure 2.1: Real Oscillator measurement.

where k_2 and k_3 are positive constants that depend on the quality of the oscillator. In Fig. 2.1, we have presented the PSD measurements from a real oscillator operating at a frequency of $f_{\rm osc} = 9.85$ GHz.

Now consider the phase noise caused during an interval τ, and define it as

$$\Delta(\tau) \triangleq \phi(t+\tau) - \phi(t) \propto \int_t^{t+\tau} \Phi(t')\,\mathrm{d}t', \tag{2.3}$$

where $\Delta(\tau)$ refers to the phase noise increment representing the phase noise that has accumulated over the time interval τ. The increment process in (2.3) is also called an *innovation* process. As shown in [12], the increment process is stationary and Gaussian, and its variance is given as

$$\sigma_\Delta^2(\tau) = \int_{-\infty}^{\infty} S_\phi(f) 4 \sin{(\pi f \tau)}^2 \mathrm{d}f, \tag{2.4}$$

When only white noise sources are assumed to be present in the oscillator, the variance of the increment process in (2.3) is obtained by evaluating the integral in (2.4) as

$$\sigma_\Delta^2(\tau) = 4\pi^2 K_{\rm w}\tau, \tag{2.5}$$

where $K_{\rm w}$ is a constant that depends on the cumulative white noise processes in the oscillator. For the remainder of the thesis, we will assume that the oscillator has only white noise sources and $\phi(t)$ is a Wiener process. This is a widely used model for oscillator phase noise [10].

2.3 Communication System Model with Phase Noise

Consider an information signal $m(t)$ that is defined as

$$m(t) = \sum_{l=0}^{L-1} m_k p(t - lT_\mathrm{s}), \tag{2.6}$$

where T_s is the symbol period, $p(\cdot)$ is a bandlimited square root Nyquist pulse [13] and L is the number of information symbols transmitted. The symbols m_k in (2.6) are drawn from the signal constellation $\mathcal{M} = \{s_i, \forall\, i \in \{1, ..., M\}\}$, where M is the size of the constellation. Using the signal from an oscillator at the transmitter, $m(t)$ is up-converted to obtain the pass-band information signal [13] as

$$m_{\mathrm{pb}}(t) = \Re\{\sqrt{2} m(t) \mathrm{e}^{j(2\pi f_{\mathrm{osc}} t + \phi_{\mathrm{tx}}(t))}\}, \tag{2.7}$$

where $\Re\{\cdot\}$ denotes the real part of a complex number, and $\phi_{\mathrm{tx}}(t)$ is the Wiener phase noise process in the oscillator. The pass-band signal $m_{\mathrm{pb}}(t)$ is transmitted from a source to a destination and is affected by phase noise and additive white Gaussian noise (AWGN) processes. Let $\tilde{r}_{\mathrm{pb}}(t)$ denote the pass-band signal received at the destination that is given as

$$\tilde{r}_{\mathrm{pb}}(t) = m_{\mathrm{pb}}(t) + \tilde{n}_{\mathrm{pb}}(t), \tag{2.8}$$

where $\tilde{n}_{\mathrm{pb}}(t)$ is the pass-band AWGN process with double sided noise PSD N_0. The pass-band signal $\tilde{r}_{\mathrm{pb}}(t)$ is down-converted to base-band by first using the signal from an oscillator at the receiver as

$$\tilde{r}'(t) = \Re\{\sqrt{2} \tilde{r}_{\mathrm{pb}}(t) \mathrm{e}^{j(2\pi f_{\mathrm{osc}} t + \phi_{\mathrm{rx}}(t))}\}, \tag{2.9}$$

where $\phi_{\mathrm{rx}}(t)$ is the Wiener phase noise process in the oscillator. The signal $\tilde{r}'(t)$ is then low-pass filtered to obtain $\tilde{r}(t)$ that can be written as

$$\tilde{r}(t) = m(t) e^{j\phi(t)} + \tilde{n}'(t), \tag{2.10}$$

where $\phi(t) = \phi_{\mathrm{tx}}(t) + \phi_{\mathrm{rx}}(t)$, and $\tilde{n}'(t)$ is the complex envelope of $\tilde{n}_{\mathrm{pb}}(t)$ and an additive Gaussian noise process with double sided noise PSD N_0. The noise processes $\phi(t)$, $\tilde{n}'(t)$ are independent of each other and the transmitted information signal $m(t)$.

The received signal (2.10) is passed through a matched filter $p^*(-t)$ and sampled at the Nyquist rate kT_s as

$$\begin{aligned} \tilde{r}(kT_\mathrm{s}) &= \sum_{l=0}^{L-1} m_k \int_{-\infty}^{\infty} p(kT_\mathrm{s} - lT_\mathrm{s} - \tau) p^*(-\tau) e^{j\phi(kT_\mathrm{s})} \mathrm{d}\tau + \int_{-\infty}^{\infty} \tilde{n}'(kT_\mathrm{s} - \tau) p^*(-\tau) \mathrm{d}\tau \\ &\overset{(a)}{=} m_k e^{j\phi(kT_\mathrm{s})} + \tilde{n}(kT_\mathrm{s}), \end{aligned} \tag{2.11}$$

where $\tilde{r}(kT_\mathrm{s})$ is the received signal sample, $\tilde{n}(kT_\mathrm{s})$ is the complex Gaussian noise sample with $\mathbb{E}\{\tilde{n}(kT_\mathrm{s})\} = 0$ and $\mathbb{E}\{\tilde{n}(kT_\mathrm{s})\tilde{n}^*(kT_\mathrm{s})\} = N_0$, and $\phi(kT_\mathrm{s})$ is the phase noise sample in the kth time instant. The simplification in step (a) in (2.11) results because $p(t)$ is a square root Nyquist pulse and it is assumed that *the phase noise variation is a constant within* T_s. The discrete (sampled) phase noise process $\phi(kT_\mathrm{s})$ can be expressed, using (2.1) and (2.3), as

$$\begin{aligned} \phi(kT_\mathrm{s}) &= \sum_{i=1}^{k} \int_{(i-1)T_\mathrm{s}}^{iT_s} \Phi(t)\mathrm{d}t = \sum_{i=1}^{k} \Delta(iT_s) \\ &= \phi((k-1)T_\mathrm{s}) + \Delta(kT_\mathrm{s}). \end{aligned} \tag{2.12}$$

With a slight change in notation, we rewrite the discrete phase noise process in (2.12) as

$$\phi_k = \phi_{k-1} + \Delta_k, \tag{2.13}$$

where ϕ_k at $k = 0$ is a uniform random variable (r.v.), and $\Delta_k \sim \mathcal{N}(0, \sigma_\Delta^2)$ is the innovation of the Wiener process. Since only white noise sources are considered in the oscillator, the discrete innovation process is white and distributed as $\mathcal{N}(0, \sigma_\Delta^2)$, where σ_Δ^2 is defined in (2.5) as $\sigma_\Delta^2 = 4\pi^2 K_\mathrm{w} T_\mathrm{s}$. We rewrite the discrete system model in (2.11) as

$$\tilde{r}_k = m_k e^{j\phi_k} + \tilde{n}_k. \tag{2.14}$$

The discrete signal $\tilde{r}_k$ in (2.11) forms a sufficient statistics for the continuous time model in (2.10) [14]. Implicit from (2.11) is that the spectral broadening of the transmitted signal $m(t)$ caused by phase noise is moderate, and there is no inter-channel interference. For the remainder of the thesis, we will use the discrete model in (2.14) to represent an information signal that is affected by phase noise and AWGN.

Chapter 3

Designing Communication Systems for Phase Noise

Oscillators may be carefully designed so that they have low levels of phase noise. However, such accurate oscillators can be expensive and cannot be employed ubiquitously. With an explosion in the number of wireless devices in use/demand in the recent times, their design has to be optimized in several ways, particularly in terms of cost. The use of inexpensive, noisy oscillators in such systems is therefore inevitable, and systems have to be appropriately designed and compensated by accounting for phase noise. In this chapter, we will present a review of prior work related to designing systems in the presence of oscillator phase noise.

3.1 Design Approaches

The problem of designing wireless communication systems in the presence of phase noise such that they achieve near coherent performance has been investigated for decades. The main design approaches to this problem can be summarized as follows:

1. The traditional approach is to design phase noise trackers that would track or estimate the phase noise in the received signals, followed by coherent detection of the transmitted symbols. This can be used in combination with standard error correcting codes like low-density parity-check (LDPC) codes or turbo codes and conventional signal constellations like phase shift keying (PSK) or quadrature amplitude modulation (QAM).

2. One may design low-complexity joint phase-estimation data-detection algorithms for compensating Wiener phase noise. Similar to the tra-

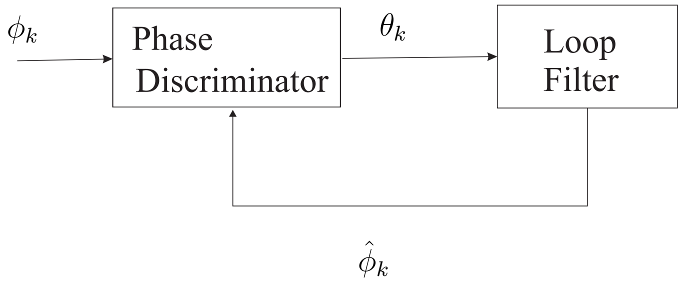


Figure 3.1: PLL Tracking

ditional approach, these joint algorithms can be used along with standard error correcting codes and conventional constellations.

3. In order to improve the system performance, one may design constellations that are optimized for the phase noise channel.

4. Another approach is to design error correcting codes that incorporate the effect of phase noise. This can be used along with conventional constellations for transmission.

3.1.1 Phase Noise Trackers

We will first briefly review some methods for phase noise tracking used in communication receivers by considering the following question: How can phase noise trackers be designed such that near-coherent error rate performance can be achieved in the presence of oscillator phase noise?

Trackers are used to track or estimate the phase noise in the received signal. That is, after matched filtering and sampling of the received signal $\tilde{r}(t)$, the phase noise in the discrete signal $\tilde{r}_k$ is tracked and compensated as

$$\begin{aligned} r_k &\triangleq \tilde{r}_k e^{-j\hat{\phi}_k} = m_k e^{j\theta_k} + n_k \\ \theta_k &\triangleq \phi_k - \hat{\phi}_k,\ n_k \triangleq \tilde{n_k} e^{-j\hat{\phi}_k}, \end{aligned} \tag{3.1}$$

where $\hat{\phi}_k$ is the phase noise estimate, and θ_k is the remaining phase error. Following this compensation, coherent detection of the transmitted symbols is performed by effectively treating the phase error θ_k to be zero.

The most widely used tracker is the phase locked loop (PLL) [4,15] that is shown in Fig. 3.1, whose operation can be summarized as follows: Let $\hat{\phi}_k$ be the tracked phase from a loop filter and ϕ_k be the phase noise in the received signal. They are the inputs to the phase discriminator. Let $\theta_k \triangleq \phi_k - \hat{\phi}_k$ denote the phase error process. This error signal is then fed to the loop filter, which produces an estimate $\hat{\phi}_k$ of the phase noise in the received signal. The estimate $\hat{\phi}_k$ is generated such that it decreases the phase error θ_k. When a PLL initially seeks to track the phase of the incoming signal, the phase error is large, and the error steadily decreases with time. This transient operating mode is called the acquisition mode of the PLL. When the phase error becomes very small, the PLL is said to be locked to the incoming signal. Another tracker that is commonly used is the extended Kalman Filter (EKF) [16, 17]. It has been shown in prior work [18] that an EKF has a structure and performance similar to that of a PLL.

The performance of the trackers can be evaluated by comparing their mean square error (MSE) with a lower bound on the phase estimation MSE. One way of characterizing the MSE lower bound is to evaluate the Bayesian Cramer-Rao bound (CRB) [19] for the phase noise model in (2.13). Particle filters [20], extended Kalman fitlers or smoothers, and the MAP estimation algorithm in [21] have been shown to reach the CRB. Note that the bounds for the MSE of these algorithms are known and characterized only when the data is known. They are generally harder to derive when the transmitted data is unknown, or when the estimator has only limited prior information about the transmitted data [19].

In recent times, there has been a lot of effort towards improving the performance of coded systems (like turbo codes) in the presence of random phase noise. To address this problem, the per-survivor processing (PSP) algorithm proposed in [22] has been widely used, where phase estimation is first performed using an estimator like the PLL followed by (Viterbi or BCJR) sequence detection. Another widely used technique for this problem is called turbo synchronization [23]. In this technique, phase estimation is performed using the expectation-maximization (EM) algorithm. The phase estimates are then used to compute the a posteriori bit and symbol probabilities using algorithms like the BCJR [24–26]. In both PSP and turbo synchronization, the phase noise estimates rendered by the estimation algorithm are treated as the true value of phase noise.

There are other numerous algorithms that have been proposed for phase noise tracking, and we refer the readers to [4, 15] for a fairly exhaustive review. Note that the traditional approach can be viewed as a special case of the approach where algorithms for phase-estimation data-detection are jointly designed for compensating Wiener phase noise.

Phase Error Models

In the context of phase noise tracking, it is important to study models for the phase error process θ_k. Recall that the phase error θ_k in the traditional

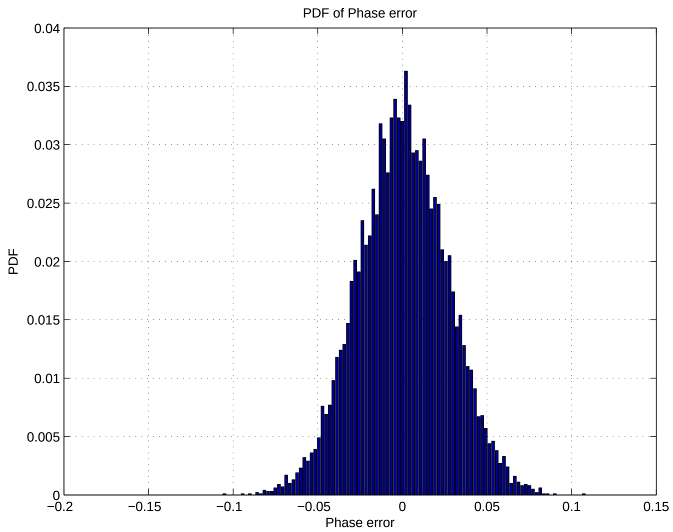


Figure 3.2: PDF of phase error resulting from the compensation of the received signal with an EKF for $\sigma_{\Delta}^2 = 10^{-2}\text{rad}^2$.

approach is treated as zero. However, as we shall see in the sequel, its statistics can be used for designing joint phase-estimation data-detection algorithms, which can help to achieve significant gains in the system error rate performance [27]. A common assumption for the phase error process θ_k resulting from the PLL, for a given symbol amplitude, is that it is Tikhonov [28]. The Tikhonov or Von Mises PDF with circular mean 0 and variance $1/\rho$ is given as

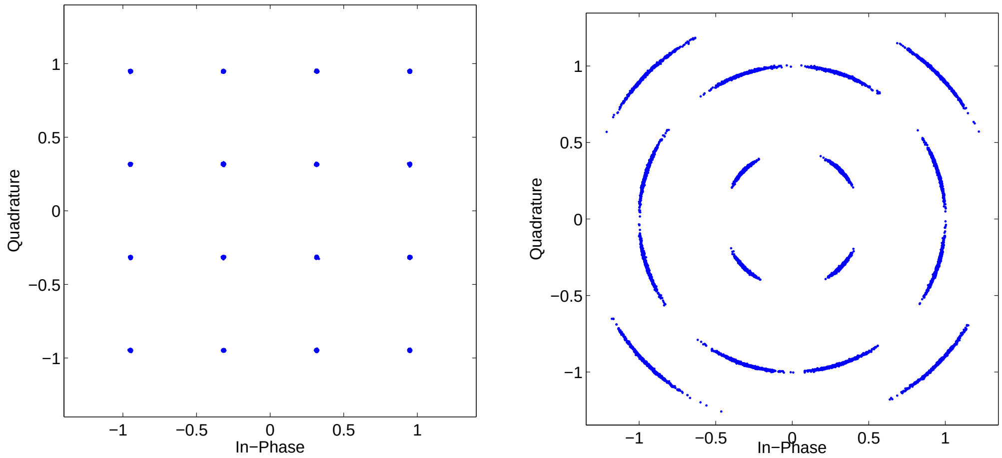
$$p(\theta_k) = \frac{e^{\rho\cos(\theta_k)}}{2\pi I_0(\rho)},\ \theta_k \in [-\pi, \pi], \tag{3.2}$$

This PDF is approximately Gaussian for large values of ρ, and is used to model the phase error after compensation with the PLL or a tracker. Another PDF model that is used to describe the phase error process is the wrapped Gaussian distribution [29]

$$p(\theta_k) = \frac{1}{\sqrt{2\pi\sigma_{\text{p}}^2}} \sum_{l\in\mathbb{Z}} e^{\frac{-(\theta_k - 2l\pi)^2}{2\sigma_{\text{p}}^2}},\ \theta_k \in [-\pi, \pi] \tag{3.3}$$

where σ_{p}^2 denotes the variance of θ_k. After compensation by the EKF, the phase error is approximately Gaussian or Tikhonov for a given symbol amplitude. The PDF of the phase error for this case is presented in Fig. 3.2. However, the error PDF becomes complicated when the transmitted symbols have different amplitudes.

Let us now try to visualize in Fig. 3.3 the effect of the Gaussian phase error on a simple constellation, where we observe the rotational effect of



(a) (b)

Figure 3.3: 16-QAM constellation at SNR per bit of 40 dB, when (a) no phase noise is present, (b) affected by a Gaussian distributed phase error of variance $\sigma_{\mathrm{p}}^2 = 1 \times 10^{-2}$ rad^2.

phase error on the transmitted symbols. Here, the symbols are drawn from a 16-QAM signal constellation and transmitted over a wireless link at a signal-to-noise (SNR) per bit of 40 dB and phase error variance $\sigma_{\mathrm{p}}^2 = 10^{-2}$ rad^2.

3.1.2 Joint Phase-Estimation Data-Detection Algorithms

We will now review prior work that has attempted to address the following question: When the transmitted information signal is affected by AWGN and phase noise, how can a low-complexity joint phase-estimation data-detection algorithm be designed such that (near) optimal system performance is achieved?

The problem of receiver design for joint phase estimation and data detection in SISO point-to-point links has been extensively studied, e.g., refer to [4, 15] and references therein. One of the earlier approaches adopted to solve this problem was reported in [30, 31], which proposed simultaneous maximum-likelihood (ML) estimation of the data symbols, the carrier phase and the timing offset. In [32], MAP estimation based on the Viterbi algorithm was proposed for joint estimation of phase and data. The phase noise model considered was similar to the random walk model in (2.13), but the innovations Δ_k were restricted to be discrete binary jumps. This shortcoming was addressed in [33], where the discrete Wiener process (2.13) was used. Specifically, the phase random variable was assumed to be discrete in the range $[-\pi, \pi]$, and the Viterbi algorithm was employed to find the MAP phase and symbol estimates. A similar approach using the BCJR algorithm was proposed in [34]. The algorithms in [33, 34] are regarded as the MAP phase-estimation data-detection algorithms. However, they are extremely complex and are used as a benchmark to compare with the performance of other low complexity joint phase-estimation data-detection algorithms.

In [35], an optimum symbol-by-symbol (SBS) receiver was derived, where it was illustrated that this receiver has a separable estimator-detector structure. The received signals were first used to compute the a posteriori PDF of phase noise. This PDF was then used to perform SBS detection. The problem of computing the a posteriori PDF of phase noise given the received signals has been demonstrated to be intractable in general. However, it was observed that the optimum receiver structure can be analytically obtained only for some cases of the phase noise a posteriori PDF, e.g., the phase noise PDF is uniform. On a related note, it is possible to restrict the a posteriori phase noise PDF to a canonical family of distributions and then derive the ML symbol detector. This approach was reported in a much earlier work by Foschini *et al.* [27]. In their work, it was assumed that the phase of the received signal is tracked and compensated using a PLL. Then the posteriori phase error PDF (2.14), was approximated as a Tikhonov PDF [28] and used to derive the ML detector. In a more recent effort, a similar detector was derived in [36] for the phase noise channel in (2.14).

When the transmitted symbols are affected by random phase noise, methods based on the sum-product algorithm (SPA) [37] on factor graphs have also been used for designing joint phase-estimation data-detection algorithms. The SPA does not employ an explicit estimator and approximates the a posteriori symbol probabilities based on a marginalization of the phase noise, which is treated as a nuisance parameter. A joint phase-estimator data-detector that is similar to an extended Kalman smoother was proposed in [16,38]. In [39], the messages used in the SPA were restricted to a canonical set of distribution, namely the Tikhonov distribution. An extension of the approach in [39] was proposed in [40] to handle both time varying phase noise and a constant frequency offset.

As a low complexity alternative to SPA, the work in [17] proposed joint phase estimation and detection based on the Variational-Bayesian (VB) framework, which was found to be efficient in the presence of random phase noise. However, variational methods may not be as effective compared to the SPA algorithm since the a posteriori symbol probabilities are computed based on a marginalization of the phase noise, for which the SPA is more effective. In [41], an algorithm similar to the SPA based on forward-backward recursions for phase estimation and data detection was proposed for the discrete Wiener phase noise model. Application of Monte Carlo sampling

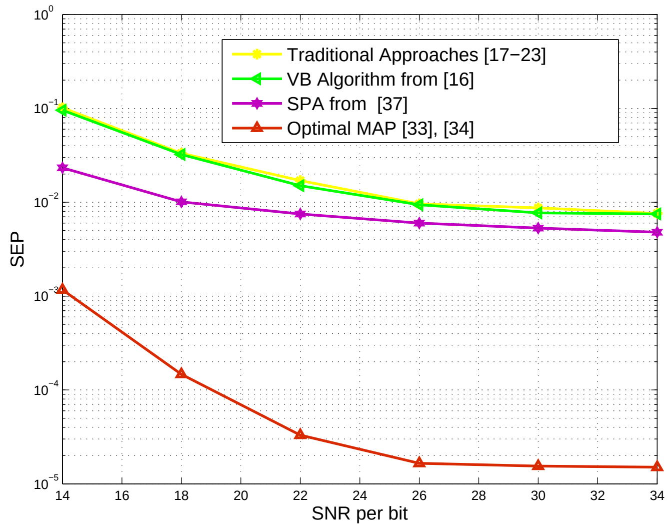


Figure 3.4: Comparison of the SEP performance between the various detectors and optimal MAP for 16-QAM, $\sigma_\Delta^2 = 10^{-2}\text{rad}^2$.

methods for joint phase estimation and data detection was investigated in [42] for both coded and uncoded systems.

Let us now see how the various low-complexity estimator-detectors proposed in prior work perform in terms of SEP with respect to the optimal MAP algorithm [33, 34]. We consider uncoded data transmission, and use symbols from the 16-QAM constellation. The phase noise model used is the discrete Wiener phase noise model in (2.13) with $\sigma_\Delta^2 = 10^{-2}$ rad^2. The comparison is shown in Fig. 3.4, where we observe that the gap in performance between the various proposed algorithms and the MAP is significantly large. This emphasizes that the problem of computing accurate a posteriori symbol probabilities and a posteriori phase PDF for deriving joint phase-estimation data-detection algorithms is challenging. The gap in performance motivates the need to design new low-complexity algorithms for performing joint phase estimation and data detection for severely strong phase noise scenarios, particularly considering high order constellations. This is investigated in our works in [43, 44] that are appended to this thesis, where we propose a low-complexity phase-estimator data-detector that is demonstrated to outperform all the algorithms existing in the literature.

3.1.3 Constellation Design

Another approach for improving system performance when affected by phase noise is to design optimal constellations that are to be transmitted over the wireless link. In this regard, we summarize prior work that have sought to address the following: How can two-dimensional signal constellations be designed for channels with phase noise, such that a target objective function like error rate performance or the mutual information (MI) is optimized?

The problem of arranging M points in a two-dimensional plane such that a target objective function is optimized is a classical problem in communication theory [45]. For decades, this problem has been studied for different channel conditions and communication models [46–49]. SEP and MI are some of the performance measures that have been used as the target objective function. SEP optimized constellations enhance asymptotic performance of coded systems. Furthermore, the effects of phase noise impairments can be quite complicated to be incorporated in the design of long capacity-achieving codes. Then it might be a reasonable approach to first design a constellation that is optimized in terms of SEP, and then combine it with a standard error-correcting code like turbo or LDPC codes. SEP minimizing constellations do not necessarily maximize MI, which is a more relevant figure of merit in coded systems [50]. For coded systems, it is interesting to optimize constellations such that they maximize MI for the phase noise channel.

The design of constellations for wireless systems impaired by phase noise was first addressed by Foschini *et al.* in [27]. In their work, an approximate ML detector and its SEP were derived for the phase noise channel in (2.14). The SEP derived was seen to be an upper bound, and the constellations that minimized it were obtained using a heuristic algorithm developed in [45]. In [51], constellations robust to phase noise were constructed heuristically such that they have low decoding complexity or simple decision regions (thus enabling quadrant or threshold-based decoding). In [52], the approximate SEP for a given phase offset in (2.14) was derived, and it was minimized for designing constellations. In [53], a simple method for constructing spiral-shaped constellations was presented, and their performances were compared to that of the conventional constellations in the presence of memoryless phase noise. In a more recent effort [54], the problem of designing constellations that maximize the MI of the memoryless phase noise channel was addressed. In their work, first the (approximate) MI for the channel was derived, and optimal constellations were obtained by maximizing it using the simulated annealing algorithm.

Prior work has demonstrated that constellations designed for phase noise substantially outperform conventional constellations in terms of SEP and MI. However, in most prior work (except [27] and [54]) ad-hoc methods have been used. There has been very limited effort to address this problem based on rigorous optimization formulations. These factors motivate the need to revisit the problem of constellation design based on optimization formulations that use target objective functions like SEP or MI. This is the theme of our work in [55] that is appended to this thesis, where we design constellations based on different objective functions like SEP and MI and demonstrate that the optimal constellations obtained outperform the conventional constellations and those proposed in the literature.

Capacity of Phase Noise Channels

In the context of optimizing finite sized constellations such that the MI is maximized, it is relevant to discuss about some important results related to capacity of channels with phase noise and AWGN, which is an active area of research. In [56], bounds on the capacity for channels with uniform phase noise were derived. It was also shown that the capacity achieving PDF is discrete with infinite mass points. A similar conjecture was presented for the case of partially coherent channels. In [57], the capacity achieving input PDF for a partially coherent channel was found to be circularly symmetric and not Gaussian. In [58], upper bounds on the capacity for phase noise channels with and without memory were derived. The capacity of channels with memory is an open problem [59]. Specifically for phase noise channels with memory, as is the case for Wiener phase noise, the capacity is not analytically characterized, but can be numerically computed using a technique in [59].

3.1.4 Coding

Designing error correcting codes that achieve the ultimate performance limit, i.e., the channel capacity, with arbitrarily small probability of error at affordable complexity is the holy grail for researchers in communication theory and information theory [60]. For systems that are affected by phase noise, codes that would incorporate the nature of phase noise impairments have to be designed. We now review prior work that has addressed the following question: How can capacity-achieving error correcting codes be designed for systems impaired by phase noise?

Designing error correcting codes so that they are amenable for phase noise scenarios is a challenging problem and is an active area of research [61–64]. In [36], the impact of phase noise on the error rate performance of standard error correcting codes was investigated. In particular, it was concluded that standard LDPC and turbo codes were effective in reducing the performance degradation incurred by phase noise. It was also noted that trellis coded modulation schemes experienced significant degradation in their performance in the presence of strong phase noise. The design of codes for the phase noise channel have been explored in [61,63,64]. These designs, aided by phase estimation, have been demonstrated to achieve near-coherent performance for channels with strong phase noise in many scenarios. In [62], LDPC codes were designed by dividing the codewords into sub-blocks such that the variation of phase noise over each sub-block was small. Phase estimates were used to correct each sub-block and phase ambiguity checks were applied using local check nodes. In [65], repeat-accumulate (RA) codes were designed where the phase ambiguity was resolved through differential encoding.

Chapter 4

Contributions and Future Directions

This thesis addresses two approaches for compensating oscillator phase noise in SISO point-to-point systems. In the first approach, we seek to design joint phase-estimator data-detector in the presence of oscillator phase noise in (2.13). In the second approach, we seek to design constellations that are optimized for the phase noise channel in (2.14). We will now briefly discuss about the papers that have been appended to this thesis and our main contributions:

1. **Paper A: Soft Metrics and their Performance Analysis for Optimal Data Detection in the Presence of Strong Oscillator Phase Noise**

   In this work, we derive an approximate ML detector for detecting data in the presence of a phase error that is Gaussian distributed. This detector is used along with an EKF for joint phase-estimation data-detection in a system impaired by Wiener phase noise. We compare the SEP performance of the proposed technique with that of other algorithms from prior work. We observe that our technique outperforms all other algorithms from prior work for a wide range of SNR values and phase noise variances. We also derive an upper bound on the SEP of this ML detector, and it is shown to be a tight upper bound for interesting SNR values and phase noise scenarios.

2. **Paper B: Optimal and Approximate Methods for Detection of Uncoded Data with Carrier Phase Noise**

   We show that the ML data detector for SBS detection in the presence of phase noise can be formulated as a weighted sum of central moments of the conditional PDF of phase noise. Furthermore, we present a simple approximation of this ML detection rule, and observe that the approximate rule renders SEP performance close to the optimum for low-to-high phase noise variance and low-to-medium SNR.

3. **Paper C: Optimal Constellations in the Presence of Strong Phase Noise**

   In this work, we study the problem of designing constellations for a channel with strong phase noise that is memoryless using three optimization formulations. In the first formulation, constellations that minimize the SEP of the system are obtained. In the next two formulations, constellations that maximize the MI of the phase noise channel are obtained. To this end, we analytically characterize the MI of the phase noise channels under consideration. It is observed that the optimal constellations obtained significantly outperform conventional constellations and those proposed in the literature for a wide range of SNR values and phase noise variances.

4.1 Future Work

Ongoing research and some possible topics for future research are described in the following:

- Currently, we are working on a joint phase-estimation data-detection algorithm based on the SPA and Gaussian mixture reduction that extends our work in [43] by allowing the phase error PDF to be multimodal.

- We are also investigating the performance of the proposed constellations in the presence of oscillator phase noise with memory. Specifically, we are exploring if our analysis and findings for the memoryless phase noise channel can be extended to the channel with Wiener phase noise. Also of interest is to investigate the constrained capacity of phase noise channels with/without memory, where the input is restricted to be discrete and have finite mass points.

- Another area that we intend to investigate in this doctoral thesis is the design of error correcting codes that incorporate the effect of phase noise impairments. As observed before, this is a challenging problem and is an active area of research.

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