Comparison of two anisotropic creep models at element level

N. Sivasithamparam
Plaxis B.V, Delft, The Netherlands
University of Strathclyde, Glasgow, United Kingdom

M. Karstunen
Chalmers University of Technology, Gothenburg, Sweden
University of Strathclyde, Glasgow, United Kingdom

R.B.J. Brinkgreve
Plaxis B.V, Delft, The Netherlands
Delft University of Technology, Delft, The Netherlands

P. G. Bonnier
Plaxis B.V, Delft, The Netherlands.

ABSTRACT: This paper presents a comparison of two anisotropic creep models, ACM and Creep-SCLAY1, which differ in their formulation of creep strain rate. Creep is formulated in ACM using the concept of contours of constant volumetric creep strain rate, whereas the newly developed Creep-SCLAY1 model uses the concept of a constant rate of visco-plastic multiplier. The two models are identical in the way the initial anisotropy and the evolution of anisotropy are simulated. A key assumption of both models is that there is no purely elastic domain. The models are compared at element level. The numerical simulations show that the Creep-SCLAY1 model is able to give a better representation of natural clay behaviour at element level.

1 INTRODUCTION

Natural soils behave in a highly anisotropic manner due to the deposition process and subsequent loading history, and show time-dependent (creep) behaviour. An accurate description of anisotropy and rate-dependent behaviour of soft soils is necessary for safe and economic design of structures on soft soils deposits. To obtain realistic solutions for geostructures on natural clays, it is essential to use a constitutive model that accounts for anisotropy and time dependency. Many constitutive models for time-dependency and anisotropy have been proposed in the literature. Time-dependent models that represent only inherent anisotropy have been proposed (e.g Sekiguchi & Ohta (1977) and Zhou et al. (2006)) as well as time-dependent models accounting for both inherent and plastic strain induced anisotropy (e.g. Leoni et al. (2008) and Karstunen & Yin (2010)). The constitutive models should be relatively simple and easy to understand . Ideally, it should be possible to determine the values of the model parameters from standard laboratory tests. This would namely enhance the confidence of practicing geotechnical engineers for adopting the models for numerical analysis.

The ACM (Leoni et al. 2008) is an extension of the Soft Soil Creep model (Vermeer et al. 1998) with rotated ellipses (similar to the S-CLAY1S model (Karstunen et al. 2005)) used as contours of volumetric creep strain rates. The formulation for the Creep-SCLAY1 model was proposed recently by Sivasithamparam (2012). Anisotropy in both models is described by introducing a fabric tensor to represent the rotation of the constitutive ellipses in the $p' - q$ plane, similar to the S-CLAY1S model (Karstunen et al. 2005). Moreover, a rotational hardening law describes the evolution of anisotropy due to volumetric and deviatoric creep strain rates. However, the Creep-SCLAY1 model differs considerably from the ACM in the formulation of creep strain rates. Creep is formulated in Creep-SCLAY1 using the concept of rate of the visco-plastic multiplier (Grimstad et al. 2010). Unlike Grimstad et al. (2010) model that used Janbu's time-resistance concept, the present model uses the more familiar creep coefficient, modified creep index $\mu*$ which can be easily derived from standard laboratory tests. This paper shows a direct comparison of both models and their prediction capability at element level.

The first part of this paper gives a short description of the ACM and Creep-SCLAY1 models in triaxial stress space. In further sections the single element simulations results obtained using the finite element code PLAXIS (Brinkgreve et al. 2012) SoilTest facility are presented, followed by brief conclusions.

2 ANISOTROPIC CREEP MODELS

The elastic and creep parts in the both models are combined with an additive law expressing the total strain rate as a combination of elastic and creep component as in classical elasto-plasticity.

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^c \tag{1}$$

where ϵ is strain, a dot over a symbol implies rate (differentiation with respect to time) and superscripts e and c refer to the elastic and creep components respectively.

For the sake of simplicity, the mathematical formulation of the both models is presented in triaxial stress space, which can be used only to model the testing of cross-anisotropic samples (cut vertically from the soil deposit) in oedometer or triaxial apparatus in the laboratory.

2.1 *ACM*

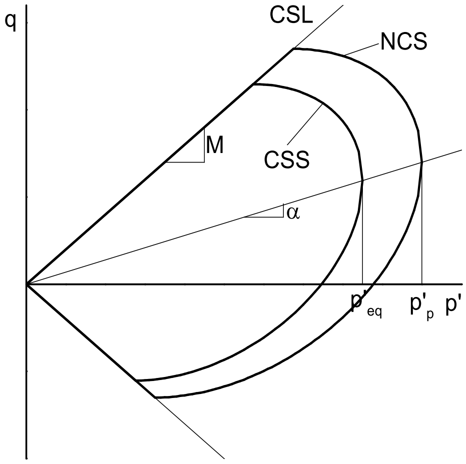
Leoni et al. (2008) proposed the Anisotropic Creep Model (ACM) extending from a previously developed isotropic creep model (Vermeer et al. , Vermeer and Neher) which is based on ellipses of Modified Cam Clay (Roscoe & Burland 1968). An extract of the mathematical formulation from Leoni et al. (2008) is presented below. The outer rotated ellipse defines the normal consolidation surface (NCS) and the size of this ellipse evolves with volumetric creep strains according to the hardening law

$$p'_p = p'_{p0} \exp\left(\frac{\epsilon_v^c}{\lambda^* - \kappa^*}\right) \tag{2}$$

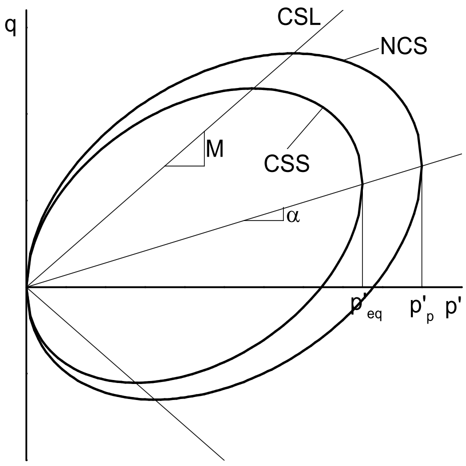
where λ^* and κ^* are the modified compression index and modified swelling index respectively. The intersection of the vertical tangent to the ellipse with p' axis is the isotropic preconsolidation pressure p'_p (see Figure 1(a)).

The inner ellipse passes through the current state of effective stress called the current stress surface (CSS). The intersection of the CSS with the horizontal axis is called the equivalent mean stress p'_{eq}, and it is defined as

$$p'_{eq} = p' + \frac{(q - \alpha p')^2}{(M^2 - \alpha^2)\, p'} \tag{3}$$



(a) ACM



(b) Creep-SCLAY1

Figure 1: Current state surface and normal consolidation surface in triaxial stress space

where M is the stress ratio at critical state and a scalar quantity α is used to describe the orientation of the normal consolidation surface and current stress surface.

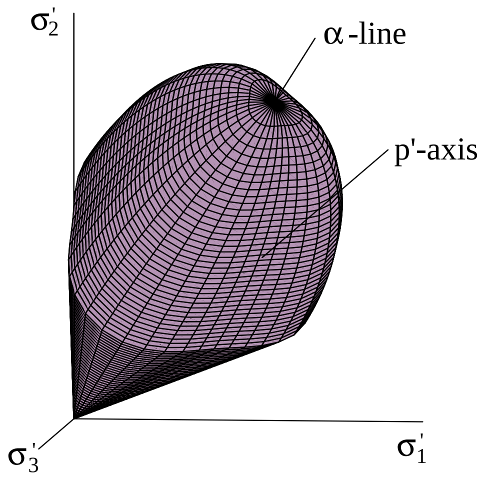
The volumetric creep strain rate is given by a power law as follows:

$$\dot{\epsilon}_v^c = \frac{\mu^*}{\tau}\left(\frac{p'_{eq}}{p'_p}\right)^{\beta} \tag{4}$$

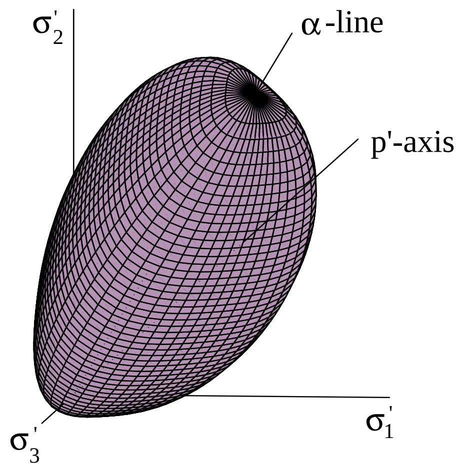
μ^* is referred to as the modified creep index,τ is called the reference time and is set to 1 day if the NCS is found by performing a standard 24h oedometer test, and β is defined as:

$$\beta = \frac{\lambda^* - \kappa^*}{\mu^*} \tag{5}$$

The ACM cannot predict swelling on the ‘dry’ side of critical state line as it does not allow the stress state to cross the failure line represented by Mohr-Coulomb criterion i.e. allowing $d\epsilon_v^c \leq 0$. Because of this, the ACM is limited to the ‘wet’ side of the critical state line only (see Figure 1(a)). In addition, the ACM cannot give a satisfactory response for strain rate changes in undrained tests of normally consolidated clays (Grimstad 2009) as discussed later. For further details of the anisotropy and creep formulation, the interested reader is referred to Leoni et al. (2008), Wheeler et al. (2003) and Karstunen et al. (2005).



(a) ACM



(b) Creep-SCLAY1

Figure 2: Normal Consolidation Surface (NCS) in general stress space

2.2 *Creep-SCLAY1*

In Creep-SCLAY1, Eq. (4) is modified to an expression that gives the rate of the visco-plastic multiplier as follows:

$$\dot{\Lambda} = \frac{\mu^*}{\tau}\left(\frac{p'_{eq}}{p'_p}\right)^{\beta}\left(\frac{M^2-\alpha^2}{M^2-\eta^2}\right) \qquad (6)$$

where $\eta = q/p'$ and the additional term $(M^2-\alpha^2)/(M^2-\eta^2)$ is added to ensure that under oedometer conditions, the resulting creep strain corresponds to Eq. (4). Grimstad (2009) suggested that creep expressed directly on the rate of plastic multiplier gives the “proper” response.

The current stress surface (CSS) and normal consolidation surface (NCS) are defined similar to ACM. However, Creep-SCLAY1 predicts swelling on the ‘dry’ side of the critical state line, unlike ACM (see Figure 1(b)). Figure 2 compares the normal consolidation surface of ACM and Creep-SCLAY1S in general stress space. For further details of the mathematical formulation of the model, the reader is referred to Sivasithamparam (2012).

3 MODEL PARAMETERS

Both models require the same parameters described below.

- Parameters which are similar to the Modified Cam-clay parameters include soils constants ν' (Poisson’s ratio), M (stress ratio at critical state), λ^* (modified compression index) and κ^* (modified swelling index). Furthermore, the initial value for a state variable p'_{m0} (initial size of the yield surface) is required. In the context of finite element analyses, the initial value of p'_{m0} is calculated based on the OCR (vertical overconsolidation ratio) or POP (pre-overburden pressure), normally consolidated K_0^{NC} value (lateral earth pressure at rest, estimated by Jaky’s formula) and the initial vertical effective stress.

- Parameters describing initial anisotropy (α_0) and its evolution, include soil constants ω (rate of rotation of the surfaces) and ω_d (relative rate of surface rotation). The scalar value α_0 and ω_d can be theoretically derived based on M values (see Wheeler et al. (2003) for details) as follows:

$$\alpha_0 = \frac{\eta_0^2 + 3\eta_0 - M^2}{3} \qquad (7)$$

$$\omega_d = \frac{3}{8}\frac{4M^2 - 4\eta_0^2 - 3\eta_0}{\eta_0^2 - M^2 + 2\eta_0} \qquad (8)$$

where $\eta_0 = 3(1-K_0^{NC})/(1+2K_0^{NC})$.

The parameter ω can be estimated based on initial anisotropy (α_0), modified compression index (λ^*), M and ω_d (see Leoni et al. (2008) for details) as follows:

$$\omega = \frac{1}{\lambda^*}\ln\frac{10M^2 - 2\alpha_0\omega_d}{M^2 - 2\alpha_0\omega_d} \qquad (9)$$

In derivation of Eq. (9), a number of assumptions has been made (see Leoni et al. (2008)).

Consequently, with certain parameter combinations Eq. (9) might results with a negative value, which makes no physical sense. As an alternative, an empirical formula suggested by Zentar et al. (2002) to estimate the ω value can be used:

$$\frac{10}{\lambda} \leq \omega \leq \frac{20}{\lambda} \tag{10}$$

- μ^* (modified creep index) can be obtained by measuring the volumetric strain on the long term and plotting it against the logarithmic time. τ (the reference time, which is linked to the definition of vertical preconsolidation stress) can usually be taken to equal one day (see Brinkgreve et al. (2012) for details).

4 NUMERICAL SIMULATION

This section discusses the performance of both models in a single element simulation. Both models are implemented into the finite element code PLAXIS as user-defined soil models. The Creep-SCLAY1 model has been implemented by the first author and the ACM has been implemented by Leoni et al. (2008). Single element simulations were done using the PLAXIS SoilTest facility to highlight the similarities and the differences in the model predictions. Parameters used for these simulations corresponding to Bothkennar clay parameters (Symposium 1992) are summarized in Table 1.

Table 1: Bothkennar clay parameters.

Parameters	value
λ^*	0.1
ν'	0.2
κ^*	0.00667
M	1.5
OCR	1.5
α_0 (initial anisotropy coefficient)	0.59
ω (anisotropy coefficient)	50.0
ω_d (anisotropy coefficient)	1.0
μ^* (viscosity coefficient)	5.07x10^{-3}
τ (viscosity coefficient)	1.0 day

Firstly, Creep-SCLAY1 and ACM were compared in undrained compression simulations with two strain rates (10% per day and 100% per day). Initial effective stress $\sigma'_3 = 100\ kPa$ and $K_0 = 0.5$ were assumed and 10% maximum strain was applied. Figures 3(a) and 3(b) show the stress paths and deviatoric stress versus axial strain predicted by the two models. Though both models are able to predict dependence on strain rate, the peak undrained strength predicted by ACM is lower than that predicted by the Creep-SCLAY1 model. In contrast to Creep-SCLAY1, ACM predicts stress path aproaches the CSL with reducing p' and q, converging towards the stress origin due to the assumption of constant volumetric creep strains. In the ACM simulations, jumbs were observed as highlighted in Figure 3 due to the transition between current state surface to Mohr-Coulomb failure surface. Furthermore, the ACM cannot reach to a critical state condition with shearing at constant volume and effective stresses.

Several publications (e.g. Graham et al. (1983), Tatsuoka et al. (2002), Tavenas et al. (1978) and Vaid & Campanella (1977)) showed the influence of step changes in strain rate on the stress-strain behaviour of soft soil in undrained triaxial compression. Immediately after an increase in strain rate the stress-strain path is seen to jump upwards and show an initial stiff response. If the strain rate is reduced back to the original strain rate then a downwards stress jump is observed after which the path rejoins the original curve defined by the lower strain rate. The paths in stress-strain curves are indicated to be uniquely defined by the strain rate and the effects of strain rate changes are observed to be persistent, which is a characteristic of isotach behaviour, i.e., there is a unique stress-strain strain-rate relation for a given soil. Most soft clays in both undisturbed and reconstituted states, undisturbed natural stiff clays and cases of soft rock all show isotach viscous behaviour. Figure 4 shows a stepwise change in strain rate undrained compression simulations using Creep-SCLAY1 and ACM to verify the capability of both models to predict the isotach behaviour. Figure 4(d) clearly demonstrates that ACM cannot properly simulate the isotach behaviour observed in natural soft clays under a stepwise change in strain rate. Furthermore, the stress path simulated by ACM cannot overpass the critical state as shown in Figure 4(c). This too is not in agreement with experimental observations for slightly structured or reconstituted clays (Yin et al. 2010).

There is a mathematical difference between the two models to calculate the creep strain components in general stress space. In ACM and Creep-SCLAY1, creep strains are calculated as follows:

ACM:

$$\dot{\epsilon}^c_{ij} = \frac{\dot{\epsilon}^c_v}{\frac{\partial p'_{eq}}{\partial p'}} \frac{\partial p'_{eq}}{\partial \sigma_{ij}} \tag{11}$$

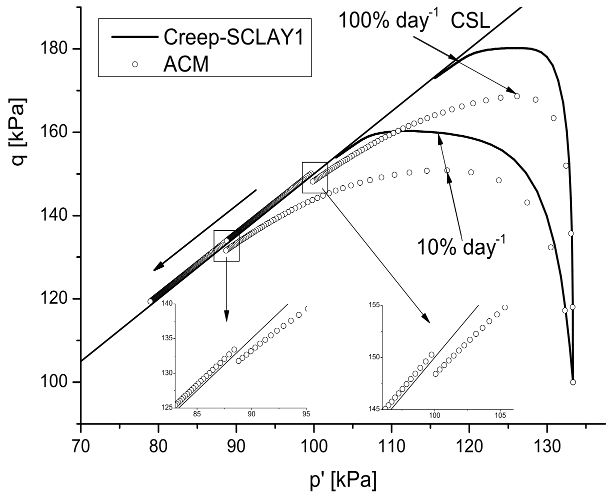
Creep-SCLAY1:

$$\dot{\epsilon}^c_{ij} = \frac{\dot{\epsilon}^c_v}{\left(\frac{\partial p'_{eq}}{\partial p'}\right)_{NC}} \frac{\partial p'_{eq}}{\partial \sigma_{ij}} \tag{12}$$

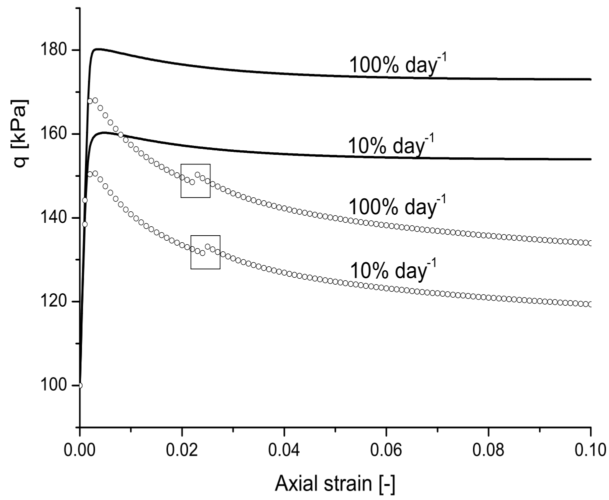
The value of $\partial p'_{eq}/\partial p'$ in the ACM reaches to infinity when η/M becomes to 1, i.e., the stress condition reaches to critical state (see Figure 5). This causes numerical problems.

5 CONCLUSIONS

This paper studies the performance of two anisotropic creep constitutive models at element level. In the

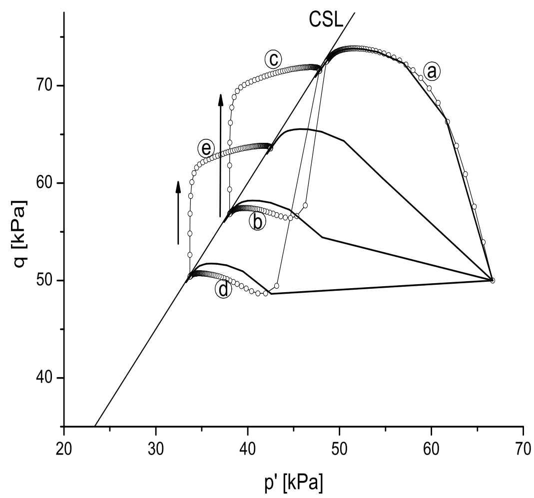


(a) stress path

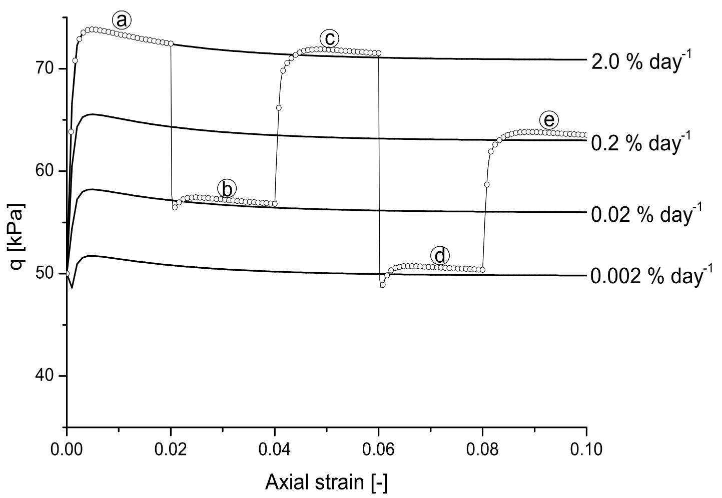


(b) stress-strain path

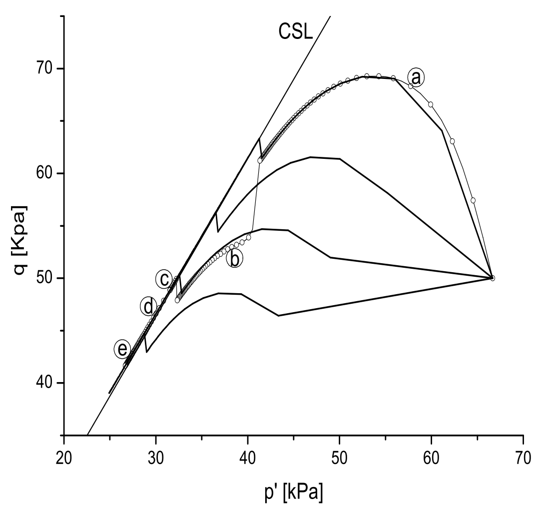
Figure 3: Simulation of undrained triaxial compression with varying strain rate



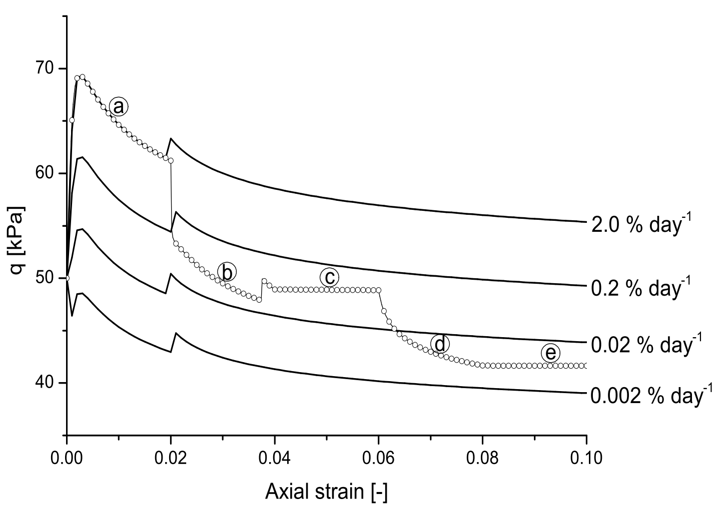
(a) stress path of Creep-SCLAY1



(b) stress-strain path of Creep-SCLAY1



(c) stress path of ACM



(d) stress-strain path of ACM

Figure 4: Simulation of undrained triaxial compression with varying strain rate

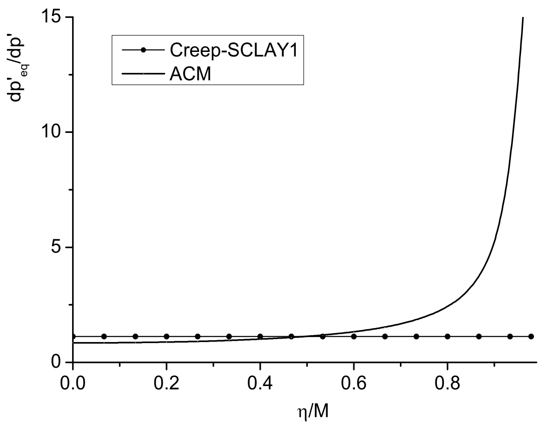


Figure 5: dp'_{eq}/dp' versus η/M plot

first model ACM (Leoni et al. 2008), the creep strain rate is formulated using contours of volumetric creep strain rates whereas in the newly developed model, Creep-SCLAY1 (Sivasithamparam 2012), creep strain rate is formulated using the concept of rate of visco-plastic multiplier. The model simulations demonstrate that the new formulation (Creep-SCLAY1) results in a better prediction of natural soft soil behaviour. The following observations are made from the comparison:

- Though both models are able to predict rate effect dependence in undrained compression simulation, in contrast to the Creep-SCLAY1 model, the ACM predicts stress paths which approach the CSL with reducing p' and q, converging towards the stress origin.
- ACM cannot reach a critical state condition with shearing at constant volume and effective stresses.
- Undrained compression using stepwise change in strain rate simulations demonstrate that the ACM cannot reproduce the isotach behaviour observed in natural soft soils. Furthermore, ACM cannot overpass the CSL; this may not be in agreement with experimental observations for slightly structured or reconstituted clays.
- There is a mathematical difficulty in ACM. When calculating creep strain rates, the value of $\partial p'_{eq}/\partial p'$ can reach infinity when η/M becomes to 1, i.e., the stress condition reaches a critical state.

Further work will involve comparing the performance of the models against experimental data and instrumented test structures.

6 ACKNOWLEDGMENTS

The research was carried out as part of GEO-INSTALL (Modelling Installation Effects in Geotechnical Engineering, PIAP-GA-2009-230638) and CREEP (Creep of Geomaterials, PIAP-GA-2011-286397) supported by the European Community through the programme Marie Curie Industry-Academia Partnerships and Pathways (IAPP).

REFERENCES

Brinkgreve, R., E. Engin, & W. M. Swolfs (2012). *PLAXIS Finite Element Code for Soil and Rock Analyses*. The Netherlands: 2D-Version 2011.

Graham, J., J. Crooks, & A. Bell (1983). Time effects on the stress-strain behaviour of natural soft clays. *Geotechnique 33(3)*, 327–340.

Grimstad, G. (2009). *Development of effective stress based anisotropic models for soft clays*. Ph. D. thesis, Norwegian University of Science and Technology (NTNU), Norway.

Grimstad, G., S. Abate, S. Nordal, & M. Karstunen (2010). Modeling creep and rate effects in structured anisotropic soft clays. *Acta Geotechnica 5*, 69–81.

Karstunen, M., H. Krenn, S. Wheeler, M. Koskinen, & R. Zentar (2005). The effect of anisotropy and destructuration on the behaviour of murro test embankment. *International Journal of Geomechanics (ASCE) 5(2)*, 87 – 97.

Karstunen, M. & Z. Y. Yin (2010). Modelling time-dependent behaviour of murro test embankment. *Geotechnique 29*, 1 – 34.

Leoni, M., M. Karstunen, & P. Vermeer (2008). Anisotropic creep model for soft soils. *Gotechnique 58 (3)*, 215–226.

Roscoe, K. & J. Burland (1968). On the generalised stress-strain behaviour of wet clay. *Engineering Plasticirv*, 535 – 609.

Sekiguchi, H. & H. Ohta (1977). Induced anisotropy and time dependency in clays. *9th ICSMFE, Tokyo, Constitutive equations of Soils 17*, 229 – 238.

Sivasithamparam, N. (2012). Modelling creep behaviour of soft soils. *Internal report Plaxis B. V.*

Symposium (1992). Bothkennar soft clay test site: Characterization and lessons learned (géotechnique symposium in print). *Géotechnique 42(2)*, 161–380.

Tatsuoka, F., M. Ishihara, H. Di Benedetto, & R. Kuwano (2002). Time-dependent shear deformation characteristics of geomaterials and their simulation. *Soils & Foundations 42(2)*, 103–138.

Tavenas, F., S. Leroueil, P. La Rochelle, & M. Roy (1978). Creep behaviour of an undisturbed lightly overconsolidated clay. *Canadian Geotechnical Journal 15(3)*, 402–423.

Vaid, Y. & R. Campanella (1977). Time-dependent behaviour of undisturbed clay. *ASCE J Geotech Eng Div 103(7)*, 693–709.

Vermeer, P. A. & H. Neher. A soft soil model that accounts for creep. *Beyond 2000 in Computational Geotechnics, R.B.J. Brinkgreve(eds), Balkema, Rotterdam.*.

Vermeer, P. A., D. F. E. Stolle, & P. G. Bonnier. From classical theory of secondary compression to modern creep analysis. *Proc. 9th Int. Conf. Comp. Meth. and Adv. Geomech., Yuan(eds)*.

Wheeler, S., A. Näätänen, M. Karstunen, & M. Lojander (2003). An anisotropic elasto-plastic model for soft clays. *Canadian Geotechnical Journal 40*, 403 – 418.

Yin, Z. Y., C. S. Chang, M. Karstunen, & P. Y. Hicher (2010). An anisotropic elastic-viscoplastic model for soft clays. *Int. J. of Solids and Structures 47*, 665 – 677.

Zentar, R., M. Karstunen, C. Wiltafafsky, H. F. Schweiger, & M. Koskinen (2002). Comparison of two approaches for modelling anisotropy of soft clays. *Proc. 8th Int. Symp. on Numerical Models in Geomechanics (NUMOG VIII)*, 115–121. In G. N. Pande & S. Pietruszczak (eds.), Rome.

Zhou, C., J.-H. Yin, J.-G. Zhu, & C.-M. Cheng (2006). Elastic anisotropic viscoplastic modeling of the strain-rate dependent stressstrain behaviour of k0-consolidated natural marine clays in triaxial shear test. *Int. J. Geomech 5(3)*, 218–232.