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Adaptive Coded Modulation for Nonlinear Fiber-Optical Channels

Lotfollah Beygi, Erik Agrell, and Magnus Karlsson

***Abstract*—A low complexity hybrid polar low-density parity check (LDPC) coded modulation (CM) scheme is introduced for a single-wavelength fiber-optical channel. In the proposed scheme, we exploit a low complexity probabilistic shaping together with a four-dimensional (4D) mapping to reduce the complexity of a 4D non-binary LDPC CM scheme for fiber-optic channels. The proposed scheme has a flexible structure and it can be used as an adaptive-rate CM scheme. The numerical results show that the forward error correction (FEC) threshold of the introduced CM scheme can be significantly improved by probabilistic shaping with a negligible increase in the system complexity. In particular, the FEC threshold for the uncoded symbol error rate of the introduced CM scheme with 4D 16-ary quadrature amplitude modulation can be improved from 0.058 to 0.072 by exploiting a shaping overhead (redundancy) of 0.016 for an information bit error rate of 10^{-5}.**

***Index Terms*—Coded modulation, low density parity check code (LDPC), polar code, probabilistic shaping, nonlinear Kerr-effect, fiber-optic channel, Chromatic dispersion, adaptive coded modulation, and FEC threshold.**

I. Introduction

The tremendous growth in the demand for high data rates in optical networks motivates exploiting advanced coding and modulation techniques in fiber-optical channels [1]–[4]. The nonlinear Schrödinger equation describes the propagation of light in optical fibers. These channels are nonlinear, and due to the lack of analytical solutions and the complexity of numerical approaches, deriving the statistics of such channels is in general cumbersome.

The recent progress in the modeling of fiber-optical channels has leveraged the design of new coded modulation (CM) techniques based on the accurate channel models [5]–[9]. It has been shown that in quasi-linear [9] regime, a single-wavelength [7], [8] and wavelength-division-multiplexing (WDM) [6] fiber-optic link can be modeled as an additive white Gaussian noise (AWGN) channel with a noise variance nonlinearly proportional to the input power and the channel parameters. This interesting result simplifies the design of CM schemes for these channels. However, the new model is an approximation and exploiting the correlation and the residual signal information in the introduced nonlinear noise is still a challenging open problem.

Using strong coding schemes with 7% redundancy to obtain a forward error correction (FEC) threshold of 3.8×10^{-3} has been well established in fiber-optic channels. To increase this threshold even further, one may need to use modern coding schemes with soft decision decoding [10] or a concatenation of them with conventional hard-decision decoding codes [11]–[14]. Moreover, non-binary low density parity check codes (NB-LDPC) have shown a great potential to replace Reed-Solomon (RS) codes for some applications in data transmission such fiber-optic communications. Therefore, NB-LDPC codes have been widely proposed for fiber-optical channels [15], [16].

The aim of this paper is to introduce a low complexity CM scheme based on the recently introduced channel model for quasi-linear fiber-optic communications [6]–[8]. To this end, we introduce a four dimensional (4D) hybrid polar LDPC CM scheme to reduce the complexity of the NB-LDPC CM [16]. More precisely, we change the bit-to-symbol mapping unit using a new constellation labeling inspired by the polar coding approach [17] to reduce the order of the Gallois field of the NB-LDPC code. Moreover, we combine the introduced CM scheme with a probabilistic shaping based on the shell mapping algorithm [18]–[20] to improve the overall FEC threshold of the system. The simulation results were provided for a 4D NB-LDPC CM with probabilistic shaping and multidimensional mapping over a polarization multiplexed fiber-optic channel. According to the numerical results, one can decrease the complexity of the system for a fixed FEC threshold or increase the FEC threshold of the CM system with a reasonably limited increase in the system complexity.

II. System Model

The system model including the transmitter, the fiber-optical channel, and the receiver has been depicted in Fig. 1. As shown in this figure, the CM unit encodes the information bit vector $\mathbf{u}$ to a matrix $\mathbf{S}$ of coded symbols. Then, the root raised-cosine (RRC) pulse shaping and chromatic dispersion pre-equalization are performed on the coded symbols to compensate for the channel inter-symbol-interference caused by chromatic dispersion. The channel is an uncompensated fiber-optic link with N spans, each consisting of a single mode fiber (SMF) and an erbium doped fiber amplifier (EDFA) with a single-wavelength data transmission scheme. It is assumed that for the output vector $\mathbf{S}[n]$ at time instant n, we have $\mathbb{E}\{\|\mathbf{S}[n]\|^2\} = (P_\text{x} + P_\text{y})/R_\text{s}$, where $P_{\text{x(y)}}$ is the transmitted power in polarization x(y), R_s is the symbol rate, and $\|\cdot\|^2$ denotes the squared Euclidean norm of a complex vector. A linear pre-compensation of the electronic chromatic dispersion [21], as shown in Fig. 1, is used at the transmitter.

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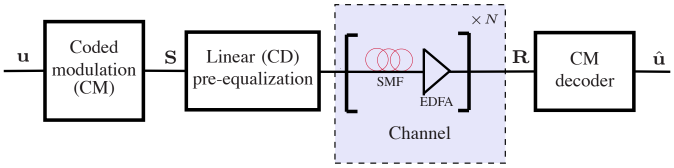

Fig. 1. A fiber link including a coded modulation (CM) unit ($\mathbf{u}$ is the input information-bit sequence) and pre-electronic chromatic dispersion compensation at the transmitter, a fiber-optical channel with N spans, each consisting of an SMF and an EDFA, and the CM decoder at the receiver.

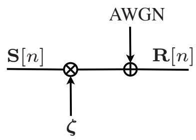

Fig. 2. The discrete-time equivalent model of the fiber-optic link for sufficiently high symbol rates ($\boldsymbol{\zeta}$ is a complex vector).

Moreover, we assume that each EDFA compensates for the attenuation in each fiber span and adds a circular white complex Gaussian amplified spontaneous emission (ASE) noise vector in each span with variance $\sigma^2_{\text{ASE}} = GF_n h\nu_{\text{opt}}/(2T)$ in each polarization [22, eq. 8.1.15], where G is the required gain to compensate for the attenuation in a span, $F_n = 2n_{\text{sp}}(1 - G^{-1})$ is the noise figure, in which n_{sp} is ASE noise factor, and $h\nu_{\text{opt}}$ is the photon energy.

At the receiver, we consider a perfect synchronization consisting clock, carrier, and polarization tracking schemes. We also assume a quasi-linear [9] fiber-optical data transmission with a standard quadrature amplitude modulation (QAM)[1], e.g., dual polarization 16-QAM, therefore we exploit the Gaussian channel model introduced in [6]–[8]. According to this model, the conditional distribution of the received signal for a given transmitted symbol on each polarization is described by a Gaussian distribution. More precisely, a fiber-optical channel with N spans is replaced by an AWGN channel with a complex constant attenuation factor $\boldsymbol{\zeta} = (\zeta_{\text{x}}, \zeta_{\text{y}})$, which attenuates and rotates the transmitted symbol in each polarization by the angle and the amplitude of the constant complex factor $\zeta_{\text{x(y)}}$, respectively. Furthermore, the variance of the zero-mean AWGN in polarization x is given by $\sigma^2_{\text{x}} = N\sigma^2_{\text{ASE}} + \sigma^2_{\text{NL}_\text{x}}$, where

$$\sigma^2_{\text{NL}_\text{x}} = f(\alpha, L_{\text{span}}, \beta_2, R_{\text{s}})N\gamma^2 P^3_{\text{x}}, \tag{1}$$

$f()$ is given in [6] for a WDM link and in [7], [8] for a single-channel link, α is the SMF attenuation coefficient, L_{span} is the span length, β_2 is the SMF dispersion coefficient, and γ is the SMF nonlinear coefficient. This variance is used to compute a posteriori probabilities of coded symbols required for the CM decoder.

III. Encoder of Coded Modulation

We introduce an adaptive CM by exploiting a hybrid polar [17] and NB-LDPC [24, ch. 14] scheme. To this end, we first describe a multidimensional mapping scheme, which performs

[1]This model is not valid for the satellite constellation introduced in [23].

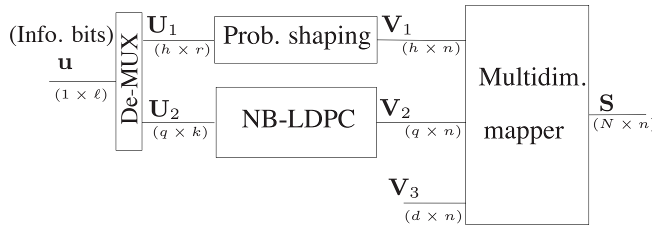

Fig. 3. The encoder of the proposed CM scheme consisting of the 4D (N=4) mapping, probabilistic shaping, and non-binary LDPC units.

the role of a polar code in our scheme. Then, we discuss the combination of this unit with NB-LDPC code and probabilistic shaping units. The multidimensional mapping (constellation labeling) scheme divides bit positions in the binary labeling of constellation symbols into three main groups, namely good, moderate, and bad channels. As described in Section III-A, this technique helps us reduce the CM complexity compared to multilevel coding [25] without performance degradation. Moreover, the proposed approach provides a flexible structure to implement an adaptive rate CM scheme. Before continuing further, we describe the proposed encoder structure. As seen in Fig. 3, a sequence of information bits $\mathbf{u}$ of length ℓ is split into two groups. The first group, denoted by the matrix $\mathbf{U}_1$ of size $h \times r$, is encoded by a probabilistic shaping unit to generate the matrix $\mathbf{V}_1$ of size $h \times n$.

The goal of the probabilistic shaping algorithm is to change the distribution of symbols in the 2D constellation and make them as close as possible to a Gaussian distribution. Clearly, this reduces the average transmitted power. On the other hand, the variance of the introduced nonlinear noise is proportional to the input power, therefore the system performance improves by performing probabilistic shaping as discussed in Section VI. Then, the second group, denoted by matrix $\mathbf{U}_2$ of size $q \times k$, is encoded by an NB-LDPC code. This matrix represents k uncoded symbols from a Galois field of order q denoted by GF(q). The NB-LDPC encoder generates the matrix $\mathbf{V}_2$ of size $q \times n$ consisting n symbols from the GF(q). The matrix $\mathbf{V}_3$ is an all-zero matrix of size $d \times n$.

The multi-dimensional mapper unit maps the input bit vector $V = \{\mathbf{V}_1^T[i], \mathbf{V}_2^T[i], \mathbf{V}_3^T[i]\}$ at the time instant i to an N-dimensional symbol $\mathbf{S}[i]$ selected from a constellation $\mathcal{C}$, where $(\cdot)^T$ is the transpose and $\mathbf{V}_1[i], \mathbf{V}_2[i]$, and $\mathbf{V}_3[i]$ are column i of matrices $\mathbf{V}_1, \mathbf{V}_2$, and $\mathbf{V}_3$, respectively. The matrix $\mathbf{S}$ of size $N \times n$ consisting of n N-dimensional (ND) symbols is pre-distorted by an electronic chromatic dispersion pre-equalization unit and transmitted over the fiber-optic link. Due to four available dimensions in a fiber-optical coherent link, i.e., the possibility of sending two complex signals in each polarization, we consider $N = 4$.

A. Multidimensional mapping (labeling)

We use an example to describe the multidimensional labeling scheme unit. To this end, we assume the 4D constellation $\mathcal{C}$ is a 4D pulse amplitude modulation (PAM), denoted by (4-PAM)^4, with 256 symbols, therefore the input of the

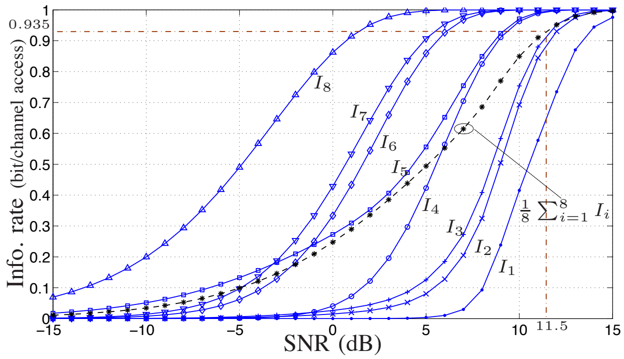

Fig. 4. The bitwise conditional mutual information bits for a 4D mapper based on the Ungerboeck set partitioning with $(4\text{-PAM})^4$ constellation.

multidimensional mapper V, is a binary vector of length $h+q+d=8$. The grouping of the input bits is performed based on the conditional bitwise mutual information of the input bits and the received symbol $\mathbf{R}$. One can represent the channel between the vector V and the received signal $\mathbf{R}$ by some binary channels using the chain rule as [25]

$$I(V;\mathbf{R}) = \sum_{i=0}^{7} I(V[i];\mathbf{R} \mid V[0],\ldots,V[i-1]) = \sum_{i=0}^{7} I_i, \quad (2)$$

where $I_i = I(V[i];\mathbf{R} \mid V[0],\ldots,V[i-1])$; $0 \le i < 8$, is the mutual information or information rate of the binary channel i conditioned on the bits transmitted on channels $0,\ldots,i-1$. The decomposition of a multidimensional signaling to binary channels is determined by the labeling of the exploited constellation; however, the mutual information of the multidimensional scheme, which is the sum of the mutual information of the binary channels, is independent of its labeling. Fig. 4 shows these information rates, $I_0,\ldots,I_7$, of different binary channels for an AWGN channel. In this figure, the Ungerboeck set partitioning has been performed over $(4\text{-PAM})^4$ constellation [26].

One can exploit a multidimensional labeling to obtain more flexibility in categorizing the binary channels into three groups, namely bad, moderate, and good channels. Bad and good channels have information rates 0 and 1, respectively, while the rate of a moderate channel is between 0 and 1. As an example, for SNRs larger than 5 dB in Fig. 4, we solely have moderate and good channels, while for SNRs smaller than 5 dB, the binary channel are categorized in two groups of moderate and bad channels. Since the information rate of bad channels is zero, the input of these channels is frozen to zero or one. Therefore, as mentioned before, the matrix $\mathbf{V}_3$ is selected as a $d \times n$ all-zero matrix, where d may be zero if no bad channels exist.

B. NB-LDPC coding and decoding

The computational complexity required per iteration of the fast Fourier transform 2^q-ary sum product algorithm (FFT-QSPA) in decoding of an NB-LDPC code designed over GF(q) is in the order of $O(J\rho q 2^q \log 2)$ [27], [28], where J and ρ are the number and weight of the rows of the parity-check matrix of the NB-LDPC code, respectively. Therefore, decreasing the order of the GF or the number of bits for representing the NB-LDPC code symbols, without degrading the performance, can significantly decrease the CM complexity. To this end, we used the grouping property of the multidimensional mapper in Section III-A. More precisely, the error protection using NB-LDPC coding is performed over moderate channels. Thus, good channels are left uncoded and no information is transmitted on bad channels. (The inputs of bad channels are fixed to zero which are known to the receiver.) However, the bad channels with rate 0 solely appear in systems with average rates smaller than 0.5 (as seen in Fig. 4). Since the NB-LDPC code is performed on moderate channel, we can use a smaller order for the GF and consequently a CM scheme with lower complexity can be obtained. As seen in Fig. 4, for an average information rate of 0.935 (7% redundancy), we have 3 good channels (i.e., with information rate 1) and five moderate channels, that need to be protected. For this case, one may get a performance very close to the Shannon constrained capacity by exploiting a capacity-approaching 32-ary NB-LDPC code ($q=5$).

It is worth mentioning that the NB-LDPC code performs on a vector of input bits of moderate channels. Since these channels are dependent, they need to be decoded jointly (symbol-wise). Independent bit-wise decoding gives rise to performance degradation of the CM scheme. We notice that the decoding of the binary channels with rate 1 is performed after the detection of moderate bits (i.e., coded by the NB-LDPC code). This is because the information rate of these channels are conditioned on the input bits of moderate channels in (2).

The introduced channel model, shown in Fig. 2 for a fiber-optical channel with large accumulated dispersion and linear chromatic dispersion, simplifies the computation of a posteriori information required for LDPC decoder. In other words, according to this model the residual memory can be neglected. Therefore, the a posteriori of received symbols is computed using (1) and no need for turbo equalization [29, ch. 7].

C. Probabilistic Shaping

Probabilistic shaping is an algorithm that changes the distribution of the constellation symbols to a distribution close to the Gaussian distribution. In other words, instead of uniform distribution of input symbols, the symbols close to origin (with small amplitudes) are sent more often than symbols far from the origin of the constellation (with large amplitudes). Shell mapping [18]–[20] and trellis shaping [30] are two well-known algorithms for performing probabilistic shaping over a constellation with uniformly distributed symbols. Since the joint shell mapping and NB-LDPC codes can be implemented with lower complexity than trellis shaping, we exploited shell mapping in our CM scheme. Indeed, the shell mapping algorithm can be more easily combined with block codes, such as LDPC codes, than with trellis-based codes. The exploited shell mapping is simply described as a mapping from $h \times r$ bits to $h \times n$ bits as shown in Fig. 3, where the output matrices with

TABLE I
THE CHANGE OF THE CM PARAMETERS VS. SNR FOR AN AWGN CHANNEL ((4-PAM)^4 CONSTELLATION)

SNR (dB)	-3	0	5	9	12
CM rate	0.15	0.25	0.5	0.77	0.95
Num. of shaping bits: h	0	0	1	3	5
Num. of coding bits: q	4	5	6	5	3
Num. of dropped bits: d	4	3	1	0	0

Hamming weight (the sum of the Hamming weights of the matrix rows) larger than certain value are removed. Therefore, it is readily seen that $2^r \leq 2^n$ and we define the shaping rate as $R_{\text{shaping}} = r/n$.

To combine the probabilistic shaping with the hybrid polar LDPC CM scheme proposed in Section III-B, we exploited the 4D set partitioning introduced in [26]. This labeling helps us control the average energy of the 4D constellation by changing the probability of 1s and 0s in the good channels. The input distribution of moderate channels has no effect on the average energy of the 4D constellation. The nonuniform distribution of the inputs bits of the good binary channels control the average energy of the 4D constellation. Therefore, one may solely need to change the distribution of the input bits to a nonuniform distribution in these channels, where we exploited the shell mapping algorithm for this purpose. An advantage of applying probabilistic shaping in good channels is the possibility of using hard decision de-shaping without performance degradation at the receiver.

IV. Adaptive rate CM scheme

The introduced CM scheme can be adjusted to support the information rate introduced by the Shannon constrained capacity for different SNRs. As discussed in Section III-A, the grouping of good, bad and moderate binary channels changes by SNR. Therefore, one may need to change the parameters of the NB-LDPC code and probabilistic shaping units, while, the structure remains unchanged for different SNRs. For example, the required parameters of the system with the multidimensional labeling given in Fig. 4 have been listed in Table I for different SNRs. The total rate of the CM scheme is given by CM rate = $h/(h+q+d)R_{\text{shaping}} + q/(h+q+d)R_{\text{coding}}$, where $R_{\text{coding}} = k/n$ is the coding rate. The number of shaping, coding, and dropped bits are computed using Fig. 5. For example at the CM rate of 0.5, one may find the SNR that gives rise to this rate using the average rate, $\sum_{i=1}^{8} I_i/8$, curve. Then, a vertical line can be drawn to cross the information rate curves of the binary channels. The number of channels with rate 0 and 1 determine d and h, respectively and q is the number of moderate channels (i.e., channels with rates between 0 and 1). As shown in Table I, the number of coded bits is smaller than 8 and the complexity can be reduced without any degradation in the performance.

V. Simulation Results

In this section, we evaluate the performance of the proposed CM scheme for a single-channel fiber-optical system with (4-PAM)^4 constellation, 32-ary (1024,928) NB-LDPC code designed using the method introduced in [24, sec. 14.4.2] ($k = 928$, $n = 1024$, $q = 5$, $h = 3$, and $d = 0$), and three different probabilistic shaping schemes: no probabilistic shaping ($r = 1024$) and probabilistic shaping of rates 0.9375 ($r = 1003$) and 0.875 ($r = 981$). The split-step Fourier method [31, eq. 2.4.10] (SSFM) is used to simulate a fiber-optic channel based on the Manakov equation with an adaptive segment length [32] of $\Delta_i = (\epsilon L_{\text{N}} L_{\text{D}}^2)^{\frac{1}{3}}$, where i is the segment index, $\epsilon = 10^{-4}$, L_{D} is the dispersion length, and L_{N} is the nonlinear length of segment $i-1$ [31, p. 55]. In the simulations, the receiver is assumed to have perfect knowledge of the polarization state. The EDFA filters are assumed to be unity gain with double-sided bandwidth equal to the exploited sampling frequency, which is usually greater than the signal bandwidth. The following channel parameters are used for the numerical simulations: the nonlinear coefficients $\gamma_{\text{SMF}} = 1.4$ W^{-1}km^{-1}, the optical frequency $\nu_{\text{opt}} = 193.55$ THz, the attenuation coefficients $\alpha_{\text{SMF}} = 0.2$ dB/km, $L = 100$ km, $N = 15$, the symbol rate of 28 Gbaud, a dispersion coefficient of $D = 17$ ps/(nm·km), and $F_n = 6$ dB. Moreover, we consider two pulse shapes: An RRC pulse [33, p. 675] with an excess bandwidth of 0.25 and a truncation length of 8 symbols. The chromatic dispersion is compensated by an electronic chromatic dispersion filter at the transmitter.

In Fig. 5, the information rates of the binary channels of the described fiber-optical link have been plotted versus the transmit power (the same transmit power is used in both polarizations, i.e., $P_{\text{x}} = P_{\text{y}}$). As shown in this figure, a CM scheme with total rate of 0.941 was selected for numerical simulations. At this rate, $q = 5$ bits need to be protected by NB-LDPC code and the rest of the bits ($h = 3$) can be left uncoded. The bit, symbol, and frame error rates (BER, SER, and FER) of this system are evaluated for three different scenarios: The CM scheme with a total rate of 0.941 but no probabilistic shaping and the CM scheme with probabilistic shaping and total rates of 0.926 and 0.934. As seen in Fig. 6, exploiting a probabilistic shaping with an overhead (added redundancy) of 0.016 can improve the system performance around 0.4 dB at BER $= 10^{-5}$. This improves the uncoded 4D FEC threshold of the system from a SER of 0.058 to 0.072 at the information BER of 10^{-5}.

VI. Conclusion

This paper introduced a new hybrid polar LDPC coded modulation (CM) scheme for nonlinear fiber-optic channels. The proposed multidimensional labeling inspired by polar coding reduced the computational complexity of the CM scheme without any degradation in the system performance. The new scheme provides a flexible structure that can be used as an adaptive-rate CM scheme for fiber-optic networks. Furthermore, exploiting a probabilistic shaping based on the shell mapping algorithm, the system FEC threshold can be improved with a reasonable increase in the system complexity.

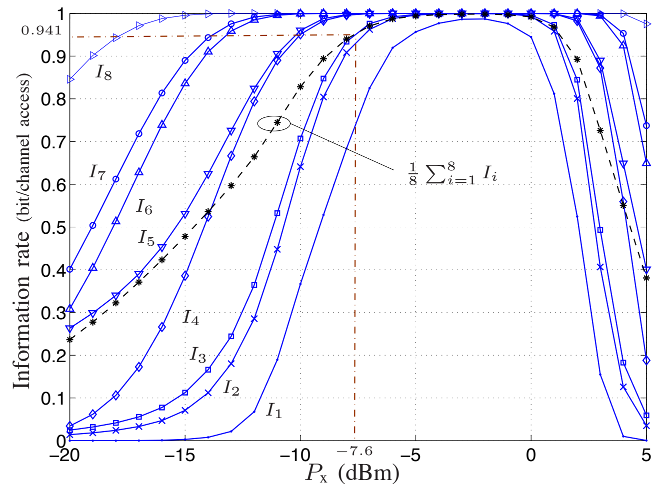

Fig. 5. The information rate of binary channels for a 4D mapper with $(4\text{-PAM})^4$ constellation and a fiber-optical link with 15 spans of length 100 km at 28 Gbaud.

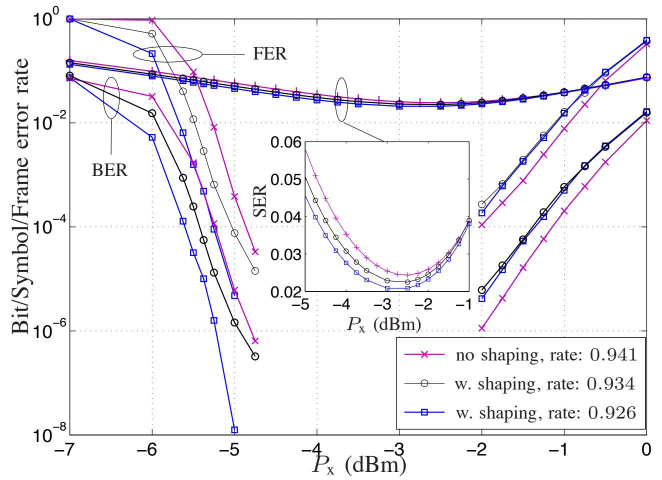

Fig. 6. The performance of CM with $(4\text{-PAM})^4$ (or DP-16QAM) constellation, 32-ary (1024,928) NB-LDPC code, and three different probabilistic shaping scenarios: no probabilistic shaping (no shaping) with total rate of 0.941, probabilistic shaping with total rate of 0.934 (shaping rate: 0.008), and probabilistic shaping with total rate of 0.926 (shaping rate: 0.016). The fiber-optical link is assumed with 15 spans of length 100 km at 28 Gbaud.

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