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Interference-Free Spectrum Sharing using a Sequential Decoder at the Primary User

Behrooz Makki and Thomas Eriksson
Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden
Email: {behrooz.makki and thomase}@chalmers.se

***Abstract*—Recently, substantial attention has been paid to improve the spectral efficiency of communication setups using different spectrum sharing techniques. This paper studies the ergodic achievable rate of spectrum sharing channels in the case where the primary licensed user is equipped with a sequential decoder, while there is no connection between the transmitters. Assuming Rayleigh block-fading channels, the unlicensed user ergodic achievable rate is obtained under an extremely hard constraint where no interference is tolerated by the licensed user receiver. Simulation results show that using sequential decoders there is considerable potential for data transmission of the unlicensed user with no performance degradation of the licensed user. Moreover, in contrast to previously proposed schemes, the network sum rate increases by implementation of sequential decoders.**

I. Introduction

Spectrum sharing networks are initiated by the apparent lack of spectrum under the current spectrum management policies. Currently, most of frequency bands available for wireless communication are under control of primary license holders that have exclusive right to transmit over their spectral bands. This point has created the perception of spectrum shortage, leading to ever-growing complaints about the available spectral resources. On the other hand, recent studies, e.g., [1], [2], show that at any given time, large portions of the licensed bands remain unused or are under-utilized. Therefore, it is expected that we can improve the data transmission strategies by better utilization of the licensed resources. Spectrum sharing network [3]–[5] is one of the most promising techniques created for this purpose.

Generally, the goal of a spectrum sharing scheme is to better utilize the radio spectrum by allowing the unlicensed secondary users (SUs) to coexist with the licensed primary users (PUs). Along with the standard interference channel [6], where independent transmitters send independent messages to independent receivers, there are other ways such as interference-avoiding and simultaneous transmission approaches to exploit the idea of spectrum sharing. The interference-avoiding paradigm [5], [7], [8] refers to the scheme where, provided that the SU transmitter can sense the temporal, spatial or spectral gaps of the PU resources, it can adjust its transmission parameters to fill these white spaces. Although this scheme theoretically leads to significant spectral efficiency improvement, it suffers from some practical drawbacks mainly related to imperfect gap detection. Also, it is not appropriate for online applications, as the SU data transmission is decided based on the PU activity. In the simultaneous transmission technique, on the other hand, the SU can simultaneously coexist with a PU as long as it operates below a certain interference level imposed by the PU quality-of-service requirements [9], [10].

Assuming different levels of interference at the PU receiver, several results about the performance limits of spectrum sharing networks have been presented recently. For instance, considering different primary or secondary user power constraints, [11]–[14] investigated the SU achievable rates under full channel state information (CSI) assumption. These works were later extended by e.g., [15]–[20] where the secondary channel performance was analyzed under different SU transmitter knowledge imperfection conditions.

Considering, e.g., [3]–[5], [11]–[20], there are three points motivating this paper: 1) in many recently developed spectrum sharing techniques the presence of the SU is at the cost of the PU received interference increment, as using simple decoders the SU signal is treated as an additive noise in the PU receiver. However, this is not acceptable for many license holders, as the interference deteriorates their data transmission efficiency, although it is tried to be kept limited. 2) In the simple-decoder based schemes, interference reduction is done by power allocation at the SU transmitter where, depending on the channel conditions, the SU transmission power changes between $[0, \infty)$. However, this may not be possible in practice where, due to nonlinearity, the power amplifiers work efficiently only in short ranges of transmission power. 3) In the *genie-aided* models [3]–[5], using sequential or simple decoders, a direct noncausal link is considered between the transmitters. However, this is not a practical assumption in many occasions.

In this perspective, this paper investigates the ergodic achievable rates of the secondary user in the case where the PU receiver is equipped with a sequential decoder. Here, there is no connection between the transmitters. The SU transmission rates are selected such that not only the SU data transmission efficiency is optimized but also the SU message is always decoded by the PU receiver. Therefore, removing the SU message from the signal received at the PU receiver, the primary channel remains interference-free. Considering block-fading environments, the ergodic achievable rates are obtained for Rayleigh fading channels. Moreover, we study the network sum rate and compare the results with the case of simple decoders at the PU receiver. As illustrated through simulations, there is considerable potential for data transmission of the unlicensed users with no PU performance degradation. Moreover, there is no need for power allocation where the data transmission is done at fixed power simplifying the power amplifier designing problem. Finally, in contrast to simple decoder-based schemes [11]–[20], spectrum sharing leads to higher network sum rate when sequential decoders are utilized.

II. System model

As illustrated in Fig.1, we consider a standard spectrum sharing network where two primary and secondary users share the same narrow-band frequency with bandwidth W. With no loss of generality we set $W = 1$. Let H_{pp}, H_{ps}, H_{sp} and H_{ss} be the instantaneous channel fading variables of the PU-PU, PU-SU, SU-PU and SU-SU links, respectively. Correspondingly, we define $G_{\mathrm{pp}} \doteq |H_{\mathrm{pp}}|^2$, $G_{\mathrm{ps}} \doteq |H_{\mathrm{ps}}|^2$, $G_{\mathrm{sp}} \doteq |H_{\mathrm{sp}}|^2$ and $G_{\mathrm{ss}} \doteq |H_{\mathrm{ss}}|^2$ which are denoted channel gains in the following. Also the gains probability density functions (pdf:s) are represented by $f_{G_{\mathrm{pp}}}$, $f_{G_{\mathrm{ps}}}$, $f_{G_{\mathrm{sp}}}$ and $f_{G_{\mathrm{ss}}}$, respectively. Although we focus on Rayleigh-fading channels, e.g., $f_{G_{\mathrm{pp}}}(g) = \lambda_{\mathrm{pp}} e^{-\lambda_{\mathrm{pp}} g}, g \geq 0$, the arguments are valid for any combination of independent random variables. The complex white Gaussian noises Z_{p} and Z_{s} added at the PU and SU receivers, are supposed to have distributions $\mathcal{CN}(0, \delta_{\mathrm{p}}^2)$ and $\mathcal{N}(0, \delta_{\mathrm{s}}^2)$, respectively, where with no loss of generality we set the noise variances $\delta_{\mathrm{s}}^2 = \delta_{\mathrm{p}}^2 = 1$. In this way, the channel outputs can be stated as

$$\begin{cases} Y_{\mathrm{p}} = X_{\mathrm{p}} H_{\mathrm{pp}} + X_{\mathrm{s}} H_{\mathrm{sp}} + Z_{\mathrm{p}} \\ Y_{\mathrm{s}} = X_{\mathrm{s}} H_{\mathrm{ss}} + X_{\mathrm{p}} H_{\mathrm{ps}} + Z_{\mathrm{s}} \end{cases}, \mathbf{E}\{|X_{\mathrm{p}}|^2\} = T_{\mathrm{p}}, \mathbf{E}\{|X_{\mathrm{s}}|^2\} = T_{\mathrm{s}} \tag{1}$$

where X_{p} and X_{s} are the PU and SU input messages having powers T_{p} and T_{s}, respectively, and Y_{p} and Y_{s} denote their corresponding outputs. Also, $\mathbf{E}\{.\}$ is the expectation operator.

We focus on block-fading channels where the channels remain fixed for a long time, generally determined by the channel coherence time, and then change independently according to their corresponding distributions. In each block, the channel gains are supposed to be known by the transmitters and receivers which is an acceptable assumption in block-fading channels [11]–[14], [18]. The PU is supposed to be equipped with a sequential decoder [21], also called joint decoder, at the receiver. However, for simplicity, a simple decoder is considered for the SU receiver. In this way, while with proper rates the SU signal is decoded at the PU receiver, the PU signal works as an additive interference at the SU receiver. In Section III, we address the SU ergodic achievable rate under an interference-free PU data transmission constraint.

Remark 1: Rayleigh-fading channels, on which we focus, are good models for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals [22], [23]. Also, it is most applicable when there is no dominant propagation along a line of sight between the transmitters and the receivers.

III. Theoretical results

Let R_{s} be the SU transmission rate. The SU signal is decoded at the PU receiver if

$$R_{\mathrm{s}} \leq \log(1 + \frac{T_{\mathrm{s}} G_{\mathrm{sp}}}{1 + T_{\mathrm{p}} G_{\mathrm{pp}}}). \tag{2}$$

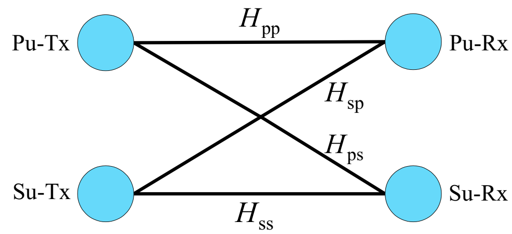

Figure 1. Channel model. The channels share the same narrow-band frequency with bandwidth W.

This is particularly because of the PU sequential decoder that, for decoding the SU message, considers the PU signal is as an additive interference. On the other hand, the signal-to-interference-and-noise ratio (SINR) received at the SU receiver is

$$\mathrm{SINR}_{\mathrm{s}} = T_{\mathrm{s}} U_{\mathrm{s}},\, U_{\mathrm{s}} \doteq \frac{G_{\mathrm{ss}}}{1 + T_{\mathrm{p}} G_{\mathrm{ps}}}. \tag{3}$$

Hence, the maximum achievable rate at the SU receiver is $R_{\mathrm{s}} = \log(1 + \frac{T_{\mathrm{s}} G_{\mathrm{ss}}}{1+T_{\mathrm{p}} G_{\mathrm{ps}}})$. In this way, with a PU interference-free data transmission constraint, the SU transmission rate in each block is obtained as $R_{\mathrm{s}} = \min\{\log(1+\frac{T_{\mathrm{s}} G_{\mathrm{sp}}}{1+T_{\mathrm{p}} G_{\mathrm{pp}}}), \log(1+\frac{T_{\mathrm{s}} G_{\mathrm{ss}}}{1+T_{\mathrm{p}} G_{\mathrm{ps}}})\}$ and the SU ergodic achievable rate, also called the average or the expected rate [18], [24]–[27], is found as

$$\eta_{\mathrm{s}}^{\text{sequential}} = \mathbf{E}\{\min\{\log(1 + \frac{T_{\mathrm{s}} G_{\mathrm{ss}}}{1 + T_{\mathrm{p}} G_{\mathrm{ps}}}), \log(1 + \frac{T_{\mathrm{s}} G_{\mathrm{sp}}}{1 + T_{\mathrm{p}} G_{\mathrm{pp}}})\}\}. \tag{4}$$

Remark 2: According to (4), the PU quality-of-service requirements are satisfied by rate adaptation. This is in contrast to the previously proposed schemes [11]–[20], where the PU received interference constraints are satisfied by power allocation at SU transmitter which, due to power amplifiers nonlinearity, is not practically feasible. Finally, among practical coding schemes providing the rate adaptation requirements, we can mention, e.g., [28]–[31].

Remark 3: In power allocation-based techniques the SU may turn off, due to either PU quality-of-service requirements or the water-filling properties [11]–[20]. However, this may be not acceptable in, e.g., online applications. On the other hand, in our proposed model the SU is always active sending information to the corresponding receiver.

Remark 4: Similar to all sequential decoder-based schemes, there is no privacy for the SU, as its message is decoded by the PU receiver as well. This point may be unacceptable in some applications.

Remark 5: The only cost for our scheme is to replace the simple decoder by a sequential decoder at the PU receiver. On the other hand, there is no extra power cost for the PU transmitter, which is required in the previously developed techniques to compensate the PU interference increment. Finally, unlike the genie-aided models [3]–[5], our scheme requires no connection between the transmitters.

A. Calculating the ergodic achievable rate

Considering Rayleigh-fading channels, the cumulative distribution function (cdf) of the auxiliary variable U_{s}, defined in

(3), is found as

$$\begin{aligned}F_{U_{\rm s}}(u) &= \Pr\{\frac{G_{\rm ss}}{1+T_{\rm p}G_{\rm ps}} \le u\}\\ &= \int_0^\infty \lambda_{\rm ps}e^{-\lambda_{\rm ps}x}\Pr\{G_{\rm ss}\le u(1+T_{\rm p}x)\}{\rm d}x\\ &= \int_0^\infty \lambda_{\rm ps}e^{-\lambda_{\rm ps}x}(1-e^{-\lambda_{\rm ss}u(1+T_{\rm p}x)}){\rm d}x\\ &= 1-\frac{e^{-\lambda_{\rm ss}u}}{1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u}. \end{aligned}\tag{5}$$

Similarly, we have

$$F_{U_{\rm p}}(u) = 1-\frac{e^{-\lambda_{\rm sp}u}}{1+\frac{\lambda_{\rm sp}}{\lambda_{\rm pp}}T_{\rm p}u}\tag{6}$$

where $U_{\rm p} \doteq \frac{G_{\rm sp}}{1+T_{\rm p}G_{\rm pp}}$. In this way, the ergodic achievable rate (4) is obtained by

$$\begin{aligned}\eta_{\rm s}^{\rm sequential} &= \int\limits_{u=0}^{\infty}\int\limits_{v=u}^{\infty}\log(1+T_{\rm s}u)f_{U_{\rm s}}(u)f_{U_{\rm p}}(v){\rm d}u{\rm d}v\\ &+\int\limits_{v=0}^{\infty}\int\limits_{u=v}^{\infty}\log(1+T_{\rm s}v)f_{U_{\rm s}}(u)f_{U_{\rm p}}(v){\rm d}u{\rm d}v\\ &=\int\limits_{u=0}^{\infty}\log(1+T_{\rm s}u)f_{U_{\rm s}}(u)\left(1-F_{U_{\rm p}}(u)\right){\rm d}u\\ &+\int\limits_{v=0}^{\infty}\log(1+T_{\rm s}v)f_{U_{\rm p}}(v)\left(1-F_{U_{\rm s}}(v)\right){\rm d}v.\end{aligned}\tag{7}$$

Here, $f_{U_{\rm s}}$ and $f_{U_{\rm p}}$ respectively are the pdf:s of the auxiliary variables $U_{\rm s}$ and $U_{\rm p}$ obtained by, e.g.,

$$\begin{aligned}&f_{U_{\rm s}}(u)=\frac{{\rm d}F_{U_{\rm s}}(u)}{{\rm d}u}\\ &\Rightarrow f_{U_{\rm s}}(u)=\frac{\lambda_{\rm ss}e^{-\lambda_{\rm ss}u}}{1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u}+\frac{\lambda_{\rm ss}T_{\rm p}e^{-\lambda_{\rm ss}u}}{\lambda_{\rm ps}(1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u)^2}.\end{aligned}\tag{8}$$

To find (7), let us focus on the first integration while the second one is obtained with the same procedure. From (6)-(8), we have

$$\begin{aligned}\Gamma &= \int_{u=0}^\infty \log(1+T_{\rm s}u)f_{U_{\rm s}}(u)\left(1-F_{U_{\rm p}}(u)\right){\rm d}u=\Gamma_1+\Gamma_2,\\ \Gamma_1 &= \lambda_{\rm ss}\int_{u=0}^\infty\frac{\log(1+T_{\rm s}u)e^{-\lambda_{\rm ss}u}}{(1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u)(1+\frac{\lambda_{\rm sp}}{\lambda_{\rm pp}}T_{\rm p}u)}{\rm d}u,\\ \Gamma_2 &= \frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}\int_{u=0}^\infty\frac{\log(1+T_{\rm s}u)e^{-\lambda_{\rm ss}u}}{(1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u)(1+\frac{\lambda_{\rm sp}}{\lambda_{\rm pp}}T_{\rm p}u)^2}{\rm d}u\end{aligned}\tag{9}$$

where Γ_1 is simplified to

$$\begin{aligned}\Gamma_1 &= r_1(\int\limits_{u=0}^{\infty}\frac{\log(1+T_{\rm s}u)e^{-qu}}{(1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u)}{\rm d}u-\int\limits_{u=0}^{\infty}\frac{\log(1+T_{\rm s}u)e^{-qu}}{(1+\frac{\lambda_{\rm sp}}{\lambda_{\rm pp}}T_{\rm p}u)}{\rm d}u)\\ &\overset{(a)}{=} r_1\sum_{n=1}^\infty T_{\rm s}^n\frac{(-1)^{n+1}}{n}\int\limits_{u=0}^{\infty}(\frac{u^ne^{-qu}}{1+\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}T_{\rm p}u}-\frac{u^ne^{-qu}}{1+\frac{\lambda_{\rm sp}}{\lambda_{\rm pp}}T_{\rm p}u}){\rm d}u\\ &\overset{(b)}{=} r_1e^{\frac{q\lambda_{\rm ps}}{\lambda_{\rm ss}T_{\rm p}}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm ps}^{n+1}\phi(n,k)}{(\lambda_{\rm ss}T_{\rm p})^{n+1}}\int\limits_{x=1}^{\infty}x^{k-1}e^{-\frac{q\lambda_{\rm ps}}{\lambda_{\rm ss}T_{\rm p}}x}{\rm d}x\\ &-r_1e^{\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm pp}^{n+1}\phi(n,k)}{(\lambda_{\rm sp}T_{\rm p})^{n+1}}\int\limits_{u=0}^{\infty}x^{k-1}e^{-\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}}x}{\rm d}x\\ &\overset{(c)}{=} r_1e^{\frac{q\lambda_{\rm ps}}{\lambda_{\rm ss}T_{\rm p}}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm ps}^{n+1}\phi(n,k)}{(\lambda_{\rm ss}T_{\rm p})^{n+1}}E_{1-k}(\frac{q\lambda_{\rm ps}}{\lambda_{\rm ss}T_{\rm p}})\\ &-r_1e^{\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm pp}^{n+1}\phi(n,k)}{(\lambda_{\rm sp}T_{\rm p})^{n+1}}E_{1-k}(\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}}).\end{aligned}\tag{10}$$

Here, it is defined $r_1=\frac{\lambda_{\rm ss}}{T_{\rm p}(\frac{\lambda_{\rm ss}}{\lambda_{\rm ps}}-\frac{\lambda_{\rm sp}}{\lambda_{\rm pp}})}$, $q=\lambda_{\rm ss}+\lambda_{\rm sp}$ and $\phi(n,k)=\frac{(-1)^{2n+1-k}}{n}\begin{pmatrix}n\\k\end{pmatrix}$ where $\begin{pmatrix}n\\k\end{pmatrix}$ is the "*n choose k*" operator. Then, (a) is obtained by Taylor expansion of the function $h(u)=\log(1+T_{\rm s}u)$, (b) comes from variable transformation and some manipulations and (c) is obtained by the definition of the exponential integral function $E_k(x)\doteq\int_1^\infty\frac{e^{-xt}{\rm d}t}{t^k}$.

Furthermore, with the same procedure as in (10), Γ_2 in (9) is found as

$$\begin{aligned}\Gamma_2 &= \frac{\lambda_{\rm sp}e^{\frac{q\lambda_{\rm ps}}{\lambda_{\rm ss}T_{\rm p}}}A}{\lambda_{\rm pp}T_{\rm p}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm ps}^{n+1}\phi(n,k)}{(\lambda_{\rm ss}T_{\rm p})^{n+1}}E_{1-k}(\frac{q\lambda_{\rm ps}}{\lambda_{\rm ss}T_{\rm p}})\\ &+\frac{\lambda_{\rm sp}e^{\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}}}C}{\lambda_{\rm pp}T_{\rm p}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm pp}^{n+1}\phi(n,k)}{(\lambda_{\rm sp}T_{\rm p})^{n+1}}E_{1-k}(\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}})\\ &+\frac{\lambda_{\rm sp}e^{\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}}}B}{\lambda_{\rm pp}T_{\rm p}}\sum_{n=1}^\infty\sum_{k=0}^n T_{\rm s}^n\frac{\lambda_{\rm pp}^{n+2}(-1)^{2n+2-k}}{(\lambda_{\rm sp}T_{\rm p})^{n+2}}\begin{pmatrix}n+1\\k\end{pmatrix}E_{1-k}(\frac{q\lambda_{\rm pp}}{\lambda_{\rm sp}T_{\rm p}})\end{aligned}\tag{11}$$

where $A=\frac{1}{(1-\frac{\lambda_{\rm sp}\lambda_{\rm ps}}{\lambda_{\rm ss}\lambda_{\rm pp}})^2}$, $B=-\frac{\lambda_{\rm ss}T_{\rm p}}{\lambda_{\rm ps}(\frac{\lambda_{\rm ss}\lambda_{\rm pp}}{\lambda_{\rm sp}\lambda_{\rm ps}}-1)^2}$ and $C=1-A$. Finally, the second integral in (7) is obtained with the same procedure as in (9)-(11) where $\lambda_{\rm ss}$, $\lambda_{\rm sp}$, $\lambda_{\rm ps}$ and $\lambda_{\rm pp}$ are replaced by $\lambda_{\rm sp}$, $\lambda_{\rm ss}$, $\lambda_{\rm pp}$ and $\lambda_{\rm ps}$, respectively.

Remark 6: Although there are infinite terms in the summations, the results converge very fast when truncating the summations.

B. Evaluating the effect of sequential decoder

With no sequential decoder, the SU signal plays the role of additive interference at the PU receiver, reducing its data transmission efficiency. For instance, with a fixed SU transmission power $T_{\rm s}$ the average interference power received at the PU receiver is $\phi_{\rm p}=T_{\rm s}\mathbf{E}\{G_{\rm sp}\}$. On the other hand, the SU instantaneous transmission rate would be $R_{\rm s}=\log(1+\frac{T_{\rm s}G_{\rm ss}}{1+T_{\rm p}G_{\rm ps}})$. Therefore, the SU ergodic achievable rate is found as

$$\begin{aligned}\eta_{\rm s}^{\rm simple} &= \mathbf{E}\{\log(1+\frac{T_{\rm s}G_{\rm ss}}{1+T_{\rm p}G_{\rm ps}})\}\\ &= \int_0^\infty f_{U_{\rm s}}(u)\log(1+T_{\rm s}u){\rm d}u \overset{(d)}{=} T_{\rm s}\int_0^\infty\frac{1-F_{U_{\rm s}}(u)}{1+T_{\rm s}u}{\rm d}u\\ &\overset{(e)}{=}\frac{e^{\frac{\lambda_{\rm ss}}{T_{\rm s}}}{\rm E}_1(\frac{\lambda_{\rm ss}}{T_{\rm s}})-e^{\frac{\lambda_{\rm ps}}{T_{\rm p}}}{\rm E}_1(\frac{\lambda_{\rm ps}}{T_{\rm p}})}{1-\frac{T_{\rm p}\lambda_{\rm ss}}{T_{\rm s}\lambda_{\rm ps}}}.\end{aligned}\tag{12}$$

Here, (d) is obtained by partial integration and (e) follows from the definition of the exponential integral function and some manipulations. On the other hand, the PU ergodic achievable rate reduces from

$$\begin{aligned}\eta_{\rm p}^{\rm sequential} &= \mathbf{E}\{\log(1+T_{\rm p}G_{\rm pp})\}\\ &= \int_0^\infty\lambda_{\rm pp}e^{-\lambda_{\rm pp}x}\log(1+T_{\rm p}x){\rm d}x = e^{\frac{\lambda_{\rm pp}}{T_{\rm p}}}E_1(\frac{\lambda_{\rm pp}}{T_{\rm p}}),\end{aligned}\tag{13}$$

which is obtained in the presence of sequential decoder, to

$$\eta_{\text{p}}^{\text{simple}} = \mathbf{E}\{\log(1+\frac{T_{\text{p}}G_{\text{pp}}}{1+T_{\text{s}}G_{\text{sp}}})\} = \frac{e^{\frac{\lambda_{\text{pp}}}{T_{\text{p}}}}E_1(\frac{\lambda_{\text{pp}}}{T_{\text{p}}}) - e^{\frac{\lambda_{\text{sp}}}{T_{\text{s}}}}E_1(\frac{\lambda_{\text{sp}}}{T_{\text{s}}})}{1-\frac{T_{\text{s}}\lambda_{\text{pp}}}{T_{\text{p}}\lambda_{\text{sp}}}} \quad (14)$$

when the SU signal is treated as noise at the PU receiver.

IV. Simulation Results and Discussions

Considering different SU and PU transmission powers, Fig. 2 shows the secondary channel ergodic achievable rate (7) in the presence of sequential decoder. Here, we set $\lambda_{\text{ss}} = \lambda_{\text{pp}} = 1$ and $\lambda_{\text{ps}} = \lambda_{\text{sp}} = 0.1$. Moreover, Figs. 3 and 4 investigate the effect of SU transmission power and the fading parameter λ_{sp} on the network sum ergodic achievable rate when different kinds of decoders are implemented at the PU receiver. Note that for the case of sequential decoder the network sum ergodic achievable rate is obtained by summation of (7) and (13). On the other hand, (12) and (14) give the network sum ergodic achievable rate in the presence of simple decoder at the PU receiver. Here, the simulations show that:

- Using sequential decoders, there is considerable potential for the SU for data transmission with no problem for the PU. However, the achievable rates decrease when the PU transmission power increases (Fig. 2).
- With a sequential decoder at the PU receiver, the network sum ergodic achievable rate increases with both the SU and the PU transmission powers (sequential decoder-based curves in Fig. 3). However, this is not generally valid when simple decoders are utilized by the PU receiver (simple decoder-based curves in Fig. 3). There is an interesting intuition behind this point; With sequential decoder, increasing, e.g., the SU transmission power does not affect the PU data transmission efficiency as, with proper rate allocation, the SU message in always decoded at the PU receiver. However, the SU achievable rate increases with the SU transmission power leading to higher network sum rate. On the other hand, when simple decoders are considered for the PU receiver, the SU data transmission reduces the PU ergodic achievable rate as the PU received interference increases. Therefore, although increasing the SU transmission power increases the SU ergodic achievable rate, the network sum rate may increase or decrease, depending on the channel conditions. In other words, the gain due to spectrum sharing in the secondary channel may be less than the loss in the primary channel, reducing the network sum rate. Finally, it is interesting to remind that the PU interference increment in the case of simple decoders (power allocation-based schemes [11]–[20]) may be not acceptable by the PU, as its performance is deteriorated.
- Increasing the fading parameter λ_{sp} increases (decreases) the network sum rate in the case of simple decoders (sequential decoders). Moreover, the network sum ergodic achievable rate is less sensitive to this fading parameter variations when sequential decoders are utilized. Again, the reason behind this is that the parameter variation does not affect the PU ergodic achievable rate when implementing sequential decoder at the PU receiver.

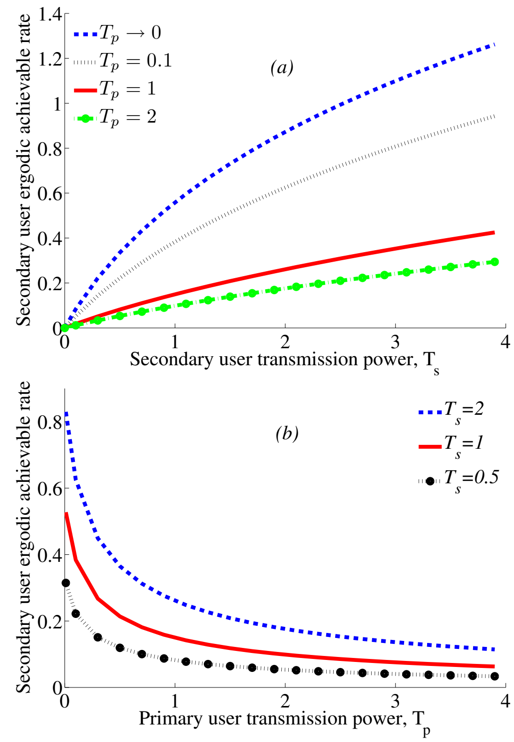

Figure 2. Secondary user ergodic achievable rate vs (a): the SU transmission power T_{s} and (b): the PU transmission power T_{p}. Sequential decoder is implemented at the PU receiver, $\lambda_{\text{ss}} = \lambda_{\text{pp}} = 1$ and $\lambda_{\text{ps}} = \lambda_{\text{sp}} = 0.1$.

V. Conclusion

This paper studies the ergodic achievable rate of the spectrum sharing networks when a sequential decoder is implemented at the PU receiver and there is no connection between the transmitters. Here, the SU transmission rates are selected such that, while the SU data transmission efficiency is maximized, no interference is added at the PU receiver. Theoretical and simulation results show that there is considerable data transmission potential for the SU with no PU rate decrement when sequential decoders are utilized at the PU receiver. Moreover, the network sum rate is substantially increased by sequential encoders, when compared with the case of simple decoders. Also, in comparison to the case of simple decoders, the network performance is less sensitive to fading parameters variations when using sequential decoders. Finally, implementation of sequential decoders leads to much simpler power amplifier designing problem as no adaptive power allocation is required for spectrum sharing.

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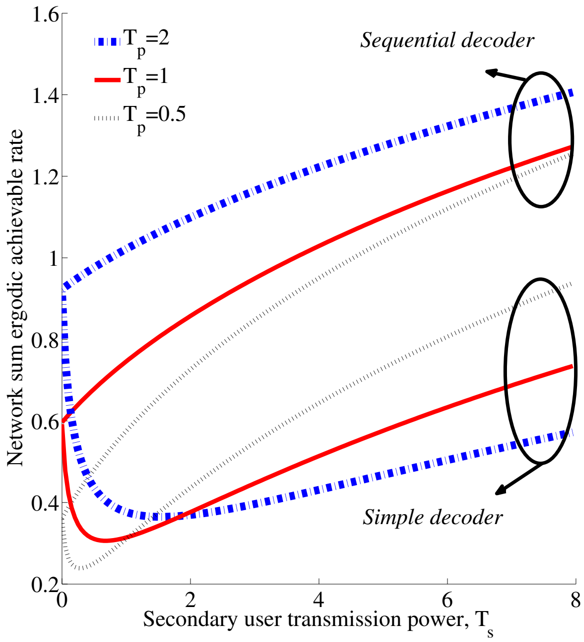

Figure 3. Network sum ergodic achievable rate vs the secondary user transmission power T_s and different kinds of decoders at the PU receiver, $\lambda_{ss} = \lambda_{pp} = 1$ and $\lambda_{ps} = \lambda_{sp} = 0.1$.

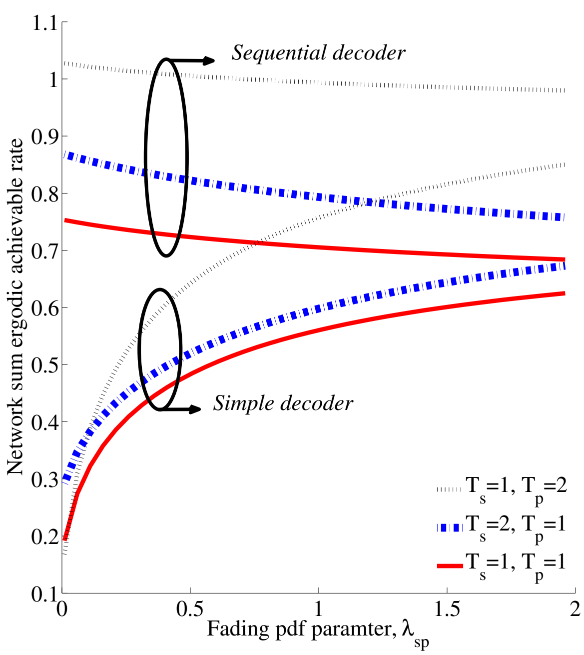

Figure 4. Network sum ergodic achievable rate vs the SU-PU fading parameter λ_{sp} and different kinds of decoders at the PU receiver, $\lambda_{ss} = \lambda_{pp} = 1$ and $\lambda_{ps} = 0.1$.

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