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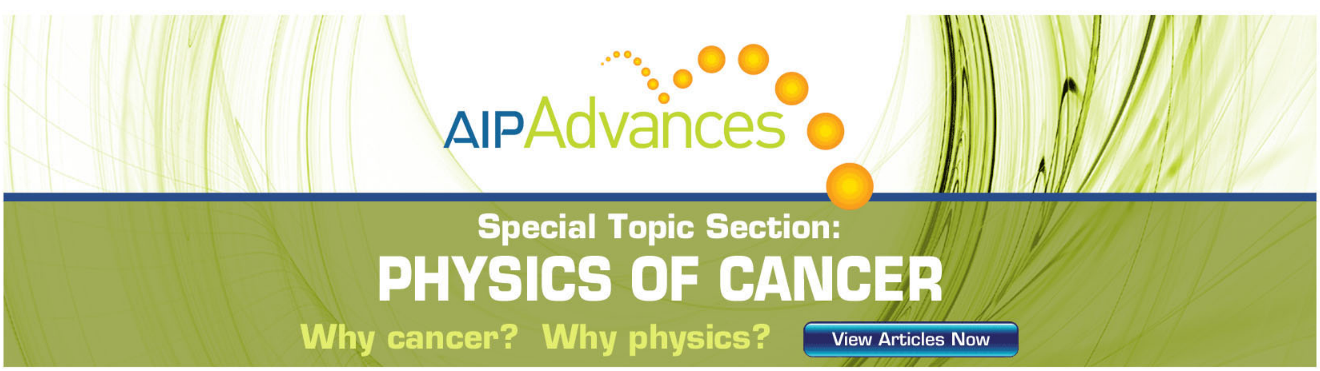

PHYSICS OF PLASMAS **19**, 082307 (2012)

Non-local gyrokinetic model of linear ion-temperature-gradient modes

S. Moradi and J. Anderson
Department of Applied Physics, Nuclear Engineering, Chalmers University of Technology and Euratom-VR Association, Göteborg, Sweden

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The non-local properties of anomalous transport in fusion plasmas are still an elusive topic. In this work, a theory of non-local linear ion-temperature-gradient (ITG) drift modes while retaining non-adiabatic electrons and finite temperature gradients is presented, extending the previous work [S. Moradi *et al*., Phys. Plasmas **18**, 062106 (2011)]. A dispersion relation is derived to quantify the effects on the eigenvalues of the unstable ion temperature gradient modes and non-adiabatic electrons on the order of the fractional velocity operator in the Fokker-Planck equation. By solving this relation for a given eigenvalue, it is shown that as the linear eigenvalues of the modes increase, the order of the fractional velocity derivative deviates from two and the resulting equilibrium probability density distribution of the plasma, i.e., the solution of the Fokker-Planck equation, deviates from a Maxwellian and becomes Lévy distributed. The relative effect of the real frequency of the ITG mode on the deviation of the plasma from Maxwellian is larger than from the growth rate. As was shown previously the resulting Lévy distribution of the plasma may in turn significantly alter the transport as well. [http://dx.doi.org/10.1063/1.4745609]

I. INTRODUCTION

The high level of anomalous transport in magnetically confined fusion plasmas is still an unresolved issue in the quest for controlled fusion. Furthermore, a deterministic description of intermittent events in plasma turbulence is improper due to the stochastic nature of the transport exhibiting non-local interactions as well as non-Gaussian probability density functions (PDFs). Fluctuation measurements by Langmuir probes have provided abundant evidence to support the idea that density and potential fluctuations are distributed according to non-Gaussian PDFs, see Ref. 1 and references there in. By comparing plasma edge density fluctuation PDFs with different types of fusion devices such as the linear device, spherical Tokamak, reversed field pinch, stellarator, and several tokamaks, similar observations were reported. All these PDFs have similar properties and exhibit a clearly skewed, non-Gaussian shape. A large variety of mechanisms can be responsible for these fluctuations such as collisional processes, perturbations in the external electric or magnetic fields imposed on the system, and linear and/or nonlinear interactions between different electromagnetic waves present in fusion plasmas. The PDFs of heat and particle flux also display uni-modal non-Gaussian features which are the signature of intermittent turbulence with patchy spatial structure that is bursty in time. The turbulent behavior in magnetically confined plasmas is the main ingredient in the anomalously high transport of heat, particles, and momentum visible in present day large experiments. One crucial component of the turbulent transport is the so-called ion-temperature-gradient (ITG) driven turbulence. The ITG turbulence is found to be bursty in nature where a significant part of the transport is carried by large avalanche-like events. More specifically, exponential scalings are often observed in the PDF tails in magnetic confinement experiments, and intermittency at the edge strongly influences the overall global particle and heat transport. In particular it may for instance influence the threshold for the high confinement mode (H-mode) in tokamak experiments.[2] In view of these experimental results, theories built on average transport coefficients and Gaussian statistics fall short in predicting vital transport processes. There is a considerable amount of experimental evidence[3–6] and recent numerical gyrokinetic[7–10] and fluid simulations[11] that plasma turbulence in tokamaks is highly non-local. A satisfactorily understanding of the non-local signatures as well as the ever-present non-Gaussian PDFs of transport[12–14] found in experiments and numerical simulations is still lacking.

An attractive candidate for explaining the non-local features of ITG turbulence is by inclusion of a fractional velocity operator in the Fokker-Planck (FP) equation[15] yielding a non-local description that have non-Gaussian PDFs of heat and particle flux. This approach is similar to that of Ref. 16 resulting in a phenomenological description of the non-local effects in plasma turbulence. Moreover, one additional benefit is that anomalous transport features can be described by a purely linear model at the cost of a fractional derivative. The fractional operator introduces an inherently non-local description with strongly non-Maxwellian features of the distribution function resulting in significant modification of the transport process. The non-locality is introduced through the integral description of the fractional derivative.[16,17] There are a number of other phenomenological studies of the effects of fractional derivative models. Using fractional generalizations of the Liouville equation, kinetic descriptions have been developed previously.[18,19] The use of a fractional derivative in velocity space as the source of non-locality in the FP equation allows to link the microscopic stochastic properties of the system to the macroscopic behavior via the solutions of the FP at long times. The underlying physical reasoning is to allow for the non-negligible probability of direction preference and long jumps, i.e., Lévy flights, which therefore allows for asymmetries and long tails in the equilibrium PDFs, respectively. In previous literature, fractional derivatives in real space and time have been reported in various fields. It has been shown that the chaotic dynamics can be described by using the FP equation with coordinate fractional derivatives as a possible tool for the description of anomalous diffusion[20]; however, a fractional derivative in velocity space can be considered as a natural generalization of classical thermodynamics of equilibrium, and much work has been devoted on investigation of the Langevin equation with Lévy white noise, see e.g., Refs. 21 and 22, or related fractional FP equation. We would like to point out that the use of fractional derivatives to model the anomalous transport is still in its infancy that is under development by many authors. Therefore, at the present time one cannot discard any of the developing models, and further experimental tests are needed. Furthermore, fractional derivatives have been introduced into the FP framework in a similar manner[23,24] as the present work; however, a study on ITG modes is still lacking.

In this paper, based on the non-Gaussian properties of the plasma random fluctuations, a stochastic Langevin equation for the particle's motion is constructed where the stochastic processes are represented by a larger class of statistical distributions, namely, stable distributions. Although this may be a crude assumption that does not represent the full physics, it allows for a natural generalization of the classical example of the motion of a charged Brownian particle with the usual Gaussian statistics. This stochastic process is represented in the Fokker-Planck equation by a fractional velocity derivative operator which as was shown in Ref. 15, resulting in a non-Gaussian (Lévy) equilibrium PDF solution. The modified equilibrium in turn may enhance the unstable fluctuations, i.e., eigenvalues of the unstable modes. Here, we extend the work presented in Ref. 15 to include the effects of finite temperature gradients and non-adiabatic electrons leading to a fractional description of the non-local effects in ITG turbulent transport in a gyrokinetic framework. We quantify the non-local effects in terms of a modified dispersion relation for linear ITG modes. We have considered a case with constant external magnetic field and a shear-less slab geometry. The characteristics of the ITG modes, i.e., the linear eigenvalues, can significantly alter the order of the fractional derivative α. However, the relative effect of the real frequency of the ITG mode on the order of the fractional derivative α deviating from the Maxwellian limit ($\alpha = 2$) is larger than that from the growth rate. This is different from the results obtained in Ref. 15 where the growth rate increased the deviation strongly. We have found that the physics behind this discrepancy is due to the difference in the electron dynamics: adiabatic and non-adiabatic electrons are considered in Ref. 15 and in the present work, respectively. As was shown in Ref. 15 the resulting Lévy distribution of the plasma may in turn significantly alter the transport as well. However, here we limit ourselves to analysis of the impact of the linear phase of ITG instability on modification of the statistical properties of the background equilibrium.

The paper is organized as follows: first we present the mathematical framework of the fractional FP equation (FFPE) which is used to derive a dispersion relation for the ITG modes while retaining the non-local interactions. In the next section, the deviations from a Maxwellian distribution function are investigated, and the dispersion relation is solved. We conclude the paper with a results and discussion section.

II. FRACTIONAL FOKKER-PLANCK EQUATION

Following Ref. 15, the FFPE with fractional velocity derivatives in shear-less slab geometry in the presence of a constant external force can be written as

$$\frac{\partial F_s}{\partial t} + \mathbf{v}\frac{\partial F_s}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m_s}\frac{\partial F_s}{\partial \mathbf{v}} = \nu\frac{\partial}{\partial \mathbf{v}}(\mathbf{v}F_s) + D\frac{\partial^\alpha F_s}{\partial|\mathbf{v}|^\alpha}, \tag{1}$$

where $s(=e,i)$ represents the particle species and $0 \le \alpha \le 2$. Here, the term $\frac{\partial^\alpha F_s}{\partial|\mathbf{v}|^\alpha}$ is the fractional Riesz derivative. The diffusion coefficient, D, is related to the damping term ν, according to a generalized Einstein relation[25]

$$D = \frac{2^{\alpha-1}T_\alpha \nu}{\Gamma(1+\alpha)m_s^{\alpha-1}}. \tag{2}$$

Here, T_α is a generalized temperature, and force $\mathbf{F}$ represents the Lorentz force (due to a constant magnetic field and a zero-averaged electric field) acting on the particles of species s with mass m_s and $\Gamma(1+\alpha)$ is the Euler gamma function. The solution of the Eq. (1), i.e., the generalized equilibrium distribution for a general α can be obtained as[15]

$$F_s(\mathbf{r},\mathbf{v}) = \frac{n_s(\mathbf{r})}{2\pi^{3/2}\sqrt{2\mathcal{D}}}\int\frac{d\mathbf{k}_\perp^v d\mathbf{k}_\parallel^v}{(2\pi)^{3/2}}e^{-i(\mathbf{k}_\perp^v\mathbf{v}_\perp+\mathbf{k}_\parallel^v\mathbf{v}_\parallel)}e^{-\frac{\mathcal{D}}{\alpha}(|\mathbf{k}_\perp^v|^\alpha+|\mathbf{k}_\parallel^v|^\alpha)}, \tag{3}$$

where

$$\mathcal{D} = \frac{V_{T,s}^\alpha}{\Gamma(1+\alpha)}, \tag{4}$$

and we have introduced a generalized thermal velocity as

$$V_{T,s}^\alpha = \frac{2^{\alpha-1}T_\alpha}{m_s^{\alpha-1}}. \tag{5}$$

Using the generalized equilibrium distribution expressed in Eq. (3), we will now quantify the non-local effects on drift waves induced by the fractional differential operator by determining the dispersion relation for ITG driven drift modes. We start by formulating the linearized gyro-kinetic theory where the particle distribution function, averaged over gyro-phase, is of the form (see Ref. 26)

$$f_s(\mathbf{r},\mathbf{v}) = F_s(\mathbf{r},\mathbf{v}) + (2\pi)^{-4} \times \int\int d\mathbf{k}\, d\omega\, \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)\delta f_{\mathbf{k},\omega}^s(\mathbf{v}). \tag{6}$$

We assume that the turbulence is purely electrostatic and neglect magnetic field fluctuations ($\delta\mathbf{B} = 0$). For small deviations from the local equilibrium we find the linearized gyrokinetic equation of the form

$$(\partial_t + ik_\parallel v_\parallel)\delta f^s_{\mathbf{k}}(v_\parallel, v_\perp, t) = i\left[\frac{c}{B}k_y\nabla_x + \frac{e_s}{m_s}k_\parallel\partial_\parallel\right]F_s(x, v_\parallel, v_\perp) \times J_0(|\Omega_s|^{-1}k_\perp v_\perp)\delta\phi_{\mathbf{k}}(t). \quad (7)$$

Here $\partial_\parallel = \partial/\partial v_\parallel$. Evaluating explicitly the derivatives of the distribution function in Eq. (3), we obtain the following relations:

$$\begin{aligned}\frac{c}{B}k_y\nabla_x F_s(x,\mathbf{v}) &= \frac{e_s}{T_{s,\alpha}}\omega^s_{*\mathbf{k}}\left[\frac{d\ln n_s(x)}{dx} - \frac{1}{2}\frac{d\ln T_{s,\alpha}(x)}{dx}\right]F_s(x,\mathbf{v}) \\ &+ \frac{e_s}{T_{s,\alpha}}\omega^s_{*\mathbf{k}} \times \left\{\frac{n_s(x)}{2\pi^{3/2}\sqrt{2\mathcal{D}}}\int\frac{d\mathbf{k}^v_\perp d\mathbf{k}^v_\parallel}{(2\pi)^{3/2}}\right. \\ &\times\left[-\frac{\mathcal{D}}{\alpha}(|\mathbf{k}^v_\perp|^\alpha + |\mathbf{k}^v_\parallel|^\alpha)\frac{d\ln T_{s,\alpha}(x)}{dx}\right] \\ &\left.\times e^{-i(\mathbf{k}^v_\perp\mathbf{v}_\perp + \mathbf{k}^v_\parallel\mathbf{v}_\parallel)}e^{-\frac{\mathcal{D}}{\alpha}(|\mathbf{k}^v_\perp|^\alpha + |\mathbf{k}^v_\parallel|^\alpha)}\right\}, \end{aligned} \quad (8)$$

where $\frac{d\ln A(x)}{dx} = \frac{1}{A(x)}\frac{dA(x)}{dx}$, and

$$\frac{e_s}{m_s}k_\parallel\partial_\parallel F_s(x,\mathbf{v}) = \frac{e_s}{m_s}k_\parallel\frac{n_s(x)}{2\pi^{3/2}\sqrt{2\mathcal{D}}}\int\frac{d\mathbf{k}^v_\perp d\mathbf{k}^v_\parallel}{(2\pi)^{3/2}}(-i\mathbf{k}^v_\parallel) \times e^{-i(\mathbf{k}^v_\perp\mathbf{v}_\perp + \mathbf{k}^v_\parallel\mathbf{v}_\parallel)}e^{-\frac{\mathcal{D}}{\alpha}(|\mathbf{k}^v_\perp|^\alpha + |\mathbf{k}^v_\parallel|^\alpha)}, \quad (9)$$

where $\omega^s_{*\mathbf{k}} = \frac{cT_s}{e_s B}k_y$, and we assumed that the space dependence of F_s is only in the x direction perpendicular to the magnetic field as well as for the density gradient. In the equation above, J_0 is the Bessel function of order zero, $v_\parallel$ is the parallel velocity, $v_\perp \equiv (v_x^2 + v_y^2)^{1/2}$ is the perpendicular velocity, and hence we write the total speed as $v = (v_\perp^2 + v_\parallel^2)^{1/2}$. The linearized gyro-kinetic equation could be further Laplace transformed. The Fourier-Laplace transform of the fluctuating electrostatic potential is

$$\delta\phi_{\mathbf{k},\omega} = \int_0^\infty dt e^{i\omega t}\delta\phi_{\mathbf{k}}(t). \quad (10)$$

Similar formula defines the Fourier-Laplace transform of $\delta f_{\mathbf{k},\omega}$. Therefore the Fourier-Laplace transformed gyro-kinetic Eq. (7) is

$$-i(\omega - k_\parallel v_\parallel)\delta f^s_{\mathbf{k}}(v_\parallel, v_\perp, t) = -\Delta^s_{\mathbf{k},\omega}(v_\parallel, v_\perp)\delta\phi_{\mathbf{k},\omega} + \delta f^s_{\mathbf{k}}(v_\parallel, v_\perp, 0). \quad (11)$$

Its solution is

$$\delta f^s_{\mathbf{k},\omega}(v_\parallel, v_\perp) = \mathcal{G}^s_{\mathbf{k},\omega}(v_\parallel, v_\perp)\{-\Delta^s_{\mathbf{k},\omega}(v_\parallel, v_\perp)\delta\phi_{\mathbf{k},\omega} + \delta f^s_{\mathbf{k}}(v_\parallel, v_\perp, 0)\}, \quad (12)$$

where the operator

$$\mathcal{G}^s_{\mathbf{k},\omega}(v_\parallel, v_\perp) = \frac{1}{-i(\omega - k_\parallel v_\parallel)} \quad (13)$$

is the unperturbed propagator of the gyro-kinetic equation, and we have introduced the function $\Delta^s_{\mathbf{k},\omega}(v_\parallel, v_\perp)$ as

$$\begin{aligned}\Delta^s_{\mathbf{k},\omega}(v_\parallel, v_\perp) &= -i\frac{e_s}{T_{s,\alpha}}\omega^s_{*\mathbf{k}}\left[\frac{d\ln n_s(x)}{dx} - \frac{1}{2}\frac{d\ln T_{s,\alpha}(x)}{dx}\right] \\ &\times F_s(x,\mathbf{v})J_0(|\Omega_s|^{-1}k_\perp v_\perp) + i\frac{e_s}{T_{s,\alpha}}\omega^s_{*\mathbf{k}} \\ &\times\left\{\frac{n_s(x)}{2\pi^{3/2}\sqrt{2\mathcal{D}}}\int\frac{d\mathbf{k}^v_\perp d\mathbf{k}^v_\parallel}{(2\pi)^{3/2}}\right. \\ &\times\left[\frac{\mathcal{D}}{\alpha}(|\mathbf{k}^v_\perp|^\alpha + |\mathbf{k}^v_\parallel|^\alpha)\frac{d\ln T_{s,\alpha}(x)}{dx}\right]e^{-i(\mathbf{k}^v_\perp\mathbf{v}_\perp + \mathbf{k}^v_\parallel\mathbf{v}_\parallel)} \\ &\left.\times e^{-\frac{\mathcal{D}}{\alpha}(|\mathbf{k}^v_\perp|^\alpha + |\mathbf{k}^v_\parallel|^\alpha)}\right\}J_0\left(|\Omega_s|^{-1}k_\perp v_\perp\right) \\ &+ i\frac{e_s}{T_{s,\alpha}}\left[\frac{T_{s,\alpha}}{m_s}k_\parallel\frac{n_s(x)}{2\pi^{3/2}\sqrt{2\mathcal{D}}}\int\frac{d\mathbf{k}^v_\perp d\mathbf{k}^v_\parallel}{(2\pi)^{3/2}}(i\mathbf{k}^v_\parallel)\right. \\ &\left.\times e^{-i(\mathbf{k}^v_\perp\mathbf{v}_\perp + \mathbf{k}^v_\parallel\mathbf{v}_\parallel)}e^{-\frac{\mathcal{D}}{\alpha}(|\mathbf{k}^v_\perp|^\alpha + |\mathbf{k}^v_\parallel|^\alpha)}\right]J_0(|\Omega_s|^{-1}k_\perp v_\perp). \end{aligned} \quad (14)$$

Here, the wave vector perpendicular to magnetic field is $k_\perp = (k_x^2 + k_y^2)^{1/2}$. The gyro-kinetic Eq. (6) is complemented with Poisson equation for the electric potential. For fluctuations with wave vectors much smaller than the Debye wave vector, the Poisson equation becomes the quasi-neutrality condition

$$\sum_s e_s\delta n^s_{\mathbf{k},\omega} = 0, \quad (15)$$

where the density fluctuation is related to the distribution function through

$$\delta n^s_{\mathbf{k},\omega} = -\frac{e_s}{T_s}n_s\delta\phi_{\mathbf{k},\omega} + \int d\mathbf{v}J_0(|\Omega_s|^{-1}k_\perp v_\perp)\delta f^s_{\mathbf{k},\omega}(v_\parallel, v_\perp). \quad (16)$$

In the above equation we have separated the adiabatic response (first term on the right hand side) from the non-adiabatic response (second term on the right hand side). We have to keep in mind that the density n_s coming from the $F_s(x,\mathbf{v})$ in the adiabatic response is also given by Eq. (3), and for a general $0 \le \alpha \le 2$ the adiabatic response can be different than that calculated by Maxwellian distribution. Using the quasi-neutrality condition (9) we find the dispersion equation which determines the eigenfrequencies as a function of the wave vector, $\omega = \omega(\mathbf{k}) = \omega_r(\mathbf{k}) + i\gamma(\mathbf{k})$. In the simplest case we consider a plasma consisting of electrons and a single species of singly charged ions with equal temperatures. For the density fluctuation therefore we have

$$\delta n^s_{\mathbf{k},\omega} = -n_s(x)\frac{e_s}{T_s}\delta\phi_{\mathbf{k},\omega}[M^{ad,s} + M^s_{\mathbf{k},\omega}]. \quad (17)$$

Therefore, the dispersion equation as in the Ref. 26 is

$$M^{ad,e} + M^{e}_{\mathbf{k},\omega} = -M^{ad,i} - M^{i}_{\mathbf{k},\omega}, \tag{18}$$

where

$$M^{ad,s} = \int d\mathbf{v} \frac{1}{2\pi^{3/2}(\Gamma(1+\alpha))^{-1/2}\sqrt{2V^{\alpha}_{T,s}}} \int \frac{d\mathbf{k}^{v}_{\perp} d\mathbf{k}^{v}_{\parallel}}{(2\pi)^{3/2}} \times e^{-i(\mathbf{k}^{v}_{\perp}\mathbf{v}_{\perp}+\mathbf{k}^{v}_{\parallel}\mathbf{v}_{\parallel})} e^{-\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})}, \tag{19}$$

gives the adiabatic contribution, and

$$\begin{aligned} M^{s}_{\mathbf{k},\omega} &= \frac{1}{n_s(x)} \int d\mathbf{v} \mathcal{G}^{s}_{\mathbf{k},\omega}(v_{\parallel}, v_{\perp}) \Delta^{s}_{\mathbf{k},\omega}(v_{\parallel}, v_{\perp}) J_0(|\Omega_s|^{-1} k_{\perp} v_{\perp}) = -\omega^{s}_{*\mathbf{k}} \left[\frac{d\ln n_s(x)}{dx} - \frac{1}{2}\frac{d\ln T_{s,\alpha}(x)}{dx}\right] \int d\mathbf{v} \frac{J_0^2(|\Omega_s|^{-1}k_{\perp}v_{\perp})}{\omega - k_{\parallel}v_{\parallel}} \\ &\times \left\{\frac{1}{2\pi^{3/2}(\Gamma(1+\alpha))^{-1/2}\sqrt{2V^{\alpha}_{T,s}}} \times \int \frac{d\mathbf{k}^{v}_{\perp} d\mathbf{k}^{v}_{\parallel}}{(2\pi)^{3/2}} e^{-i(\mathbf{k}^{v}_{\perp}\mathbf{v}_{\perp}+\mathbf{k}^{v}_{\parallel}\mathbf{v}_{\parallel})} e^{-\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})}\right\} + \omega^{s}_{*\mathbf{k}} \int d\mathbf{v} \frac{J_0^2(|\Omega_s|^{-1}k_{\perp}v_{\perp})}{\omega - k_{\parallel}v_{\parallel}} \\ &\times \left\{\frac{1}{2\pi^{3/2}(\Gamma(1+\alpha))^{-1/2}\sqrt{2V^{\alpha}_{T,s}}} \times \int \frac{d\mathbf{k}^{v}_{\perp} d\mathbf{k}^{v}_{\parallel}}{(2\pi)^{3/2}} \left[\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})\frac{d\ln T_{s,\alpha}(x)}{dx}\right] e^{-i(\mathbf{k}^{v}_{\perp}\mathbf{v}_{\perp}+\mathbf{k}^{v}_{\parallel}\mathbf{v}_{\parallel})} e^{-\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})}\right\} \\ &+ \frac{T_{s,\alpha}}{m_s} k_{\parallel} \int d\mathbf{v} \frac{J_0^2(|\Omega_s|^{-1}k_{\perp}v_{\perp})}{\omega - k_{\parallel}v_{\parallel}} \left\{\frac{1}{2\pi^{3/2}(\Gamma(1+\alpha))^{-1/2}\sqrt{2V^{\alpha}_{T,s}}} \times \int \frac{d\mathbf{k}^{v}_{\perp} d\mathbf{k}^{v}_{\parallel}}{(2\pi)^{3/2}} [i\mathbf{k}^{v}_{\parallel}] e^{-i(\mathbf{k}^{v}_{\perp}\mathbf{v}_{\perp}+\mathbf{k}^{v}_{\parallel}\mathbf{v}_{\parallel})} e^{-\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})}\right\} \end{aligned} \tag{20}$$

gives the non-adiabatic contribution.

The analytical solutions for integrals over $\mathbf{k}^{v}$ with an arbitrary α in the Eqs. (19) and (20) requires rather tedious calculations. Instead we consider an infinitesimal deviation of the form $\alpha = 2 - \epsilon$, where $0 \le \epsilon \ll 2$ and expand the terms depending on α in the Eq. (19) around $\epsilon = 0$ as follows:

$$\begin{aligned} &\frac{1}{2\pi^{3/2}(\Gamma(1+\alpha))^{-1/2}\sqrt{2V^{\alpha}_{T,s}}} e^{-\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})} \\ &= \frac{e^{-\frac{V^{2}_{T,s}}{4}(|\mathbf{k}^{v}_{\perp}|^{2}+|\mathbf{k}^{v}_{\parallel}|^{2})}}{2\pi^{3/2}V_{T,s}} + \Lambda(\mathbf{k}^{v}_{\perp}, \mathbf{k}^{v}_{\parallel})\epsilon + \mathcal{O}[\epsilon^2], \end{aligned} \tag{21}$$

where

$$\begin{aligned} \Lambda(\mathbf{k}^{v}_{\perp}, \mathbf{k}^{v}_{\parallel}) = \frac{e^{-\frac{V^{2}_{T,s}}{4}(|\mathbf{k}^{v}_{\perp}|^{2}+|\mathbf{k}^{v}_{\parallel}|^{2})}}{8\pi^{3/2}V_{T,s}} \Bigg\{ &-3 + 2\gamma_E + 2\log[V_{T,s}] \\ &- 2V^2_{T,s}\left[|\mathbf{k}^{v}_{\perp}|^2 + |\mathbf{k}^{v}_{\parallel}|^2\right] + \gamma_E V^2_{T,s}\left(|\mathbf{k}^{v}_{\perp}|^2 + |\mathbf{k}^{v}_{\parallel}|^2\right) \\ &+ V^2_{T,s}\left(|\mathbf{k}^{v}_{\perp}|^2\log\left[|\mathbf{k}^{v}_{\perp}|^2\right] + |\mathbf{k}^{v}_{\parallel}|^2\log\left[|\mathbf{k}^{v}_{\parallel}|^2\right]\right) \\ &+ V^2_{T,s}\log[V_{T,s}]\left(|\mathbf{k}^{v}_{\perp}|^2 + |\mathbf{k}^{v}_{\parallel}|^2\right)\Bigg\} \end{aligned} \tag{22}$$

and in Eq. (20) the expansion for the second term on the RHS gives

$$\begin{aligned} &\frac{\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})}{2\pi^{3/2}(\Gamma(1+\alpha))^{-1/2}\sqrt{2V^{\alpha}_{T,s}}} e^{-\frac{V^{\alpha}_{T,s}}{\Gamma(1+\alpha)\alpha}(|\mathbf{k}^{v}_{\perp}|^{\alpha}+|\mathbf{k}^{v}_{\parallel}|^{\alpha})} \\ &= \frac{e^{-\frac{V^{2}_{T,s}}{4}(|\mathbf{k}^{v}_{\perp}|^{2}+|\mathbf{k}^{v}_{\parallel}|^{2})} V_{T,s}\left(|\mathbf{k}^{v}_{\perp}|^2 + |\mathbf{k}^{v}_{\parallel}|^2\right)}{8\pi^{3/2}} + \Sigma(\mathbf{k}^{v}_{\perp}, \mathbf{k}^{v}_{\parallel})\epsilon \\ &+ \mathcal{O}[\epsilon^2], \end{aligned} \tag{23}$$

where

$$\begin{aligned} \Sigma(\mathbf{k}^{v}_{\perp}, \mathbf{k}^{v}_{\parallel}) = \frac{e^{-\frac{V^{2}_{T,s}}{4}(|\mathbf{k}^{v}_{\perp}|^{2}+|\mathbf{k}^{v}_{\parallel}|^{2})} V_{T,s}}{32\pi^{3/2}} \Bigg\{ &5(|\mathbf{k}^{v}_{\perp}|^2 + |\mathbf{k}^{v}_{\parallel}|^2) \\ &- (2\gamma_E + 2\log[V_{T,s}])\left(|\mathbf{k}^{v}_{\perp}|^2 + |\mathbf{k}^{v}_{\parallel}|^2\right) \\ &- 4\left(|\mathbf{k}^{v}_{\perp}|^2\log[|\mathbf{k}^{v}_{\perp}|] + |\mathbf{k}^{v}_{\parallel}|^2\log[|\mathbf{k}^{v}_{\parallel}|]\right) \\ &+ \left(-2V_{T,s} + \gamma_E V^2_{T,s} + \log[V_{T,s}]V^2_{T,s}\right) \\ &\times \left(|\mathbf{k}^{v}_{\perp}|^4 + |\mathbf{k}^{v}_{\parallel}|^4\right)\Big(-4V_{T,s} + 2\gamma_E V^2_{T,s} \\ &+ 2\log[V_{T,s}]V^2_{T,s})(|\mathbf{k}^{v}_{\perp}|^2|\mathbf{k}^{v}_{\parallel}|^2\Big) \\ &+ V^2_{T,s}\left(|\mathbf{k}^{v}_{\perp}|^4\log[|\mathbf{k}^{v}_{\perp}|] + |\mathbf{k}^{v}_{\parallel}|^4\log[|\mathbf{k}^{v}_{\parallel}|]\right) \\ &+ V^2_{T,s}\left(|\mathbf{k}^{v}_{\perp}|^2|\mathbf{k}^{v}_{\parallel}|^2\right)(\log[|\mathbf{k}^{v}_{\perp}|] + \log[|\mathbf{k}^{v}_{\parallel}|])\Bigg\}. \end{aligned} \tag{24}$$

Here, we have used the Euler-Mascheroni constant $\gamma_E \approx 0.57721$.

Inserting the zeroth order terms in ϵ from the expansion (21) into Eq. (19) will produce the Maxwellian adiabatic response

$$M^{ad,s} = 1, \tag{25}$$

and by inserting the zeroth order terms in ϵ from the expansion (22) into Eq. (20) will produce the Maxwellian non-adiabatic response

$$M^{s}_{\mathbf{k},\omega} = \frac{2}{\sqrt{\pi}V^3_{T,s}} \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} dv_{\perp} v_{\perp} \frac{k_{\parallel}v_{\parallel} - \omega^{s,T}_{*k}(v_{\parallel}, v_{\perp})}{-\omega + k_{\parallel}v_{\parallel}} \times J_0^2(|\Omega_s|^{-1}k_{\perp}v_{\perp}) e^{-\frac{v^2_{\parallel}+v^2_{\perp}}{V^2_{T,s}}}, \tag{26}$$

where

$$\omega_{*k}^{s,T}(v_\parallel, v_\perp) = \omega_{*\mathbf{k}}^{s}\left[\frac{d\ln n_s(x)}{dx} + \left(\frac{v_\parallel^2 + v_\perp^2}{V_{T,s}^2} - \frac{3}{2}\right)\frac{d\ln T_s(x)}{dx}\right]. \tag{27}$$

By using the expansion defined by the expressions (21) and (23) to first order in ϵ from Eqs. (19) and (20), the adiabatic and non-adiabatic parts of the dispersion relation $M^{ad,s}$ and $M_{\mathbf{k},\omega}^{s}$ are as follows:

$$M^{ad,s} = 1 + \left(2\pi\int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp \times \int \frac{d\mathbf{k}_\perp^v d\mathbf{k}_\parallel^v}{(2\pi)^{3/2}} \times e^{-i(\mathbf{k}_\perp^v \mathbf{v}_\perp + \mathbf{k}_\parallel^v \mathbf{v}_\parallel)}\Lambda(\mathbf{k}_\parallel^v, \mathbf{k}_\perp^v)\right)\epsilon = 1 + \epsilon W^{ad,s} \tag{28}$$

and

$$\begin{aligned} M_{\mathbf{k},\omega}^{s} = {} & \frac{2}{\sqrt{\pi}V_{T,s}^3}\int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp \frac{k_\parallel v_\parallel - \omega_{*k}^{s,T}(v_\parallel, v_\perp)}{-\omega + k_\parallel v_\parallel} J_0^2\left(|\Omega_s|^{-1}k_\perp v_\perp\right)e^{-\frac{v_\parallel^2+v_\perp^2}{V_{T,s}^2}} \\ & + \epsilon\Bigg\{-2\pi\omega_{*\mathbf{k}}^{s}\left[\frac{d\ln n_s(x)}{dx} - \frac{1}{2}\frac{d\ln T_s(x)}{dx}\right]\int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp \frac{J_0^2\left(|\Omega_s|^{-1}k_\perp v_\perp\right)}{\omega - k_\parallel v_\parallel} \times \int \frac{d\mathbf{k}_\perp^v d\mathbf{k}_\parallel^v}{(2\pi)^{3/2}} e^{-i(\mathbf{k}_\perp^v \mathbf{v}_\perp + \mathbf{k}_\parallel^v \mathbf{v}_\parallel)}\Lambda(\mathbf{k}_\perp^v, \mathbf{k}_\parallel^v) \\ & + 2\pi\omega_{*\mathbf{k}}^{s}\frac{d\ln T_s(x)}{dx}\int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp \frac{J_0^2\left(|\Omega_s|^{-1}k_\perp v_\perp\right)}{\omega - k_\parallel v_\parallel} \times \int \frac{d\mathbf{k}_\perp^v d\mathbf{k}_\parallel^v}{(2\pi)^{3/2}} e^{-i(\mathbf{k}_\perp^v \mathbf{v}_\perp + \mathbf{k}_\parallel^v \mathbf{v}_\parallel)}\Sigma(\mathbf{k}_\perp^v, \mathbf{k}_\parallel^v) \\ & + 2\pi\frac{T_s}{m_s}k_\parallel \int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp v_\perp \frac{J_0^2\left(|\Omega_s|^{-1}k_\perp v_\perp\right)}{\omega - k_\parallel v_\parallel} \times \int \frac{d\mathbf{k}_\perp^v d\mathbf{k}_\parallel^v}{(2\pi)^{3/2}} e^{-i(\mathbf{k}_\perp^v \mathbf{v}_\perp + \mathbf{k}_\parallel^v \mathbf{v}_\parallel)}(i\mathbf{k}_\parallel)\Lambda(\mathbf{k}_\perp^v, \mathbf{k}_\parallel^v)\Bigg\} = N_{\mathbf{k},\omega}^{s} + \epsilon W_{\mathbf{k},\omega}^{s}. \end{aligned} \tag{29}$$

III. DISPERSION EQUATION

We will now turn our attention to the problem of solving the dispersion relation described by Eq. (18). In order to solve this dispersion equation we use the method described in Ref. 15, where the dispersion relation is in the form

$$(1 + N_{\mathbf{k},\omega}^{e}) + \epsilon_e(W^{ad,e} + W_{\mathbf{k},\omega}^{e}) = -(1 + N_{\mathbf{k},\omega}^{i}) - \epsilon_i(W^{ad,i} + W_{\mathbf{k},\omega}^{i}). \tag{30}$$

Note that we have expanded in ϵ_e and ϵ_i for electrons and ions, respectively, and that there exist a relation between the two see Ref. 15. The first terms on the right and left hand sides generate the usual contributions to the dispersion equation as in Ref. 26, and the terms proportional to ϵ generate the non-Maxwellian contributions. For the non-adiabatic Maxwellian response, we have

$$N_{\mathbf{k},\omega}^{s} = \frac{2}{\sqrt{\pi}}\int_{-\infty}^{\infty} dw \int_0^\infty duu\left[\frac{w - \bar{\omega}_{*k}^{s,T}(u,w)}{w - \bar{\omega}}\right]J_0^2(b_s u)e^{-(u^2+w^2)}, \tag{31}$$

with

$$\bar{\omega}_{*k}^{sT}(u,w) = \bar{\omega}_{*\mathbf{k}}^{s}\left[1 + \left(u^2 + w^2 - \frac{3}{2}\right)\eta_s\right]. \tag{32}$$

Here, $b_s = k_\perp V_{T,s}/\Omega_s$, $\{w, u\} = \{v_\parallel/V_{T,s}, v_\perp/V_{T,s}\}$, we have introduced the following notation $L_A = \frac{d\ln A(x)}{dx}$, $\eta_s = L_T/L_n$, and $\omega_{*\mathbf{k}}^{s} = \frac{cT_s}{e_s B}k_y/L_n$. Bar denotes normalization to $\mathbf{k}_\parallel V_{T,s}$. The effects of the fractional velocity derivative can result in the non-Maxwellian contribution of the form

$$W_{\mathbf{k},\omega}^{s} = \frac{2}{\sqrt{\pi}}\int_{-\infty}^{\infty} dw \int_0^\infty duu\left[\frac{w\Upsilon(u,w) - \bar{\Omega}_{*k}^{sT}(u,w)}{w - \bar{\omega}}\right] \times J_0^2(b_s u)e^{-(u^2+w^2)}, \tag{33}$$

where

$$\bar{\Omega}_{*k}^{sT}(u,w) = \bar{\omega}_{*\mathbf{k}}^{s}\left[1 - \frac{1}{2}\eta_s\right]\Phi(u,w) - \bar{\omega}_{*\mathbf{k}}^{s}\eta_s\Psi(u,w). \tag{34}$$

The functions $\Phi(u,w)$, $\Psi(u,w)$, and $\Upsilon(u,w)$ are given in Appendix.

IV. RESULTS AND DISCUSSION

In this section we present the solutions of the dispersion Eq. (30) using Eqs. (31) and (33). We can find an expression for ϵ_e as

$$\epsilon_e = -\frac{2 + N_{\mathbf{k},\omega}^{e} + N_{\mathbf{k},\omega}^{i}}{67.32 + W_{\mathbf{k},\omega}^{e} + 1.42W_{\mathbf{k},\omega}^{i}}. \tag{35}$$

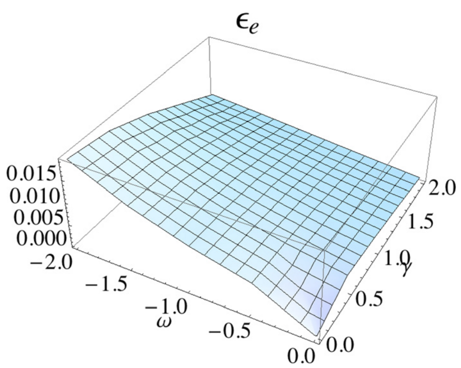

FIG. 1. ϵ_e as function of ω and γ, where the frequencies are normalized to $|k_\parallel|V_{Te}$.

Here, we have used the results shown in Ref. 15: $\epsilon_i = 1.42\epsilon_e$, $W_e^{ad} = 33.724$, and $W_i^{ad} = 23.6591$, $V_{T,e} = 5.93\,10^9[\mathrm{cm/s}]$, $V_{T,i} = 1.38\,10^8[\mathrm{cm/s}]$, and $b_i = 0.42$. We normalize all the frequencies to $|k_\parallel|V_{Te}$ where $k_\parallel = 10^{-3}$, and we solve Eq. (35) for given values of γ and ω where $\bar{\omega} = \omega_r + i\gamma$ corresponding to the real and imaginary (also called growth rate) parts of the eigenvalue.

Figure 1 shows the deviation factor ϵ_e, as defined in Eq. (35) calculated for given values of γ and ω with $\eta_i = 5$ and $\eta_e = 0$. As seen in this figure, the deviation factor increases as the frequency and growth rate of the ITG mode increase. These results are in agreement with results given in Ref. 15, where it was shown that as the growth rate increases, e.g., the plasma becomes more linearly unstable, the plasma starts to deviate from a Maxwellian, and becomes Lévy distributed. This qualitative behavior is observed for all relevant values of the temperature gradient through the parameter η_i. Note that the relative effect of the real frequency is larger compared to the effect of the growth rate. This behavior is different from the results shown in Ref. 15 where the main effect on the deviation of the plasma from Maxwellian was due to the growth rate of the density gradient mode. From our findings we expect that the basis of this discrepancy is the result of the different assumptions on the electron dynamics: adiabatic or non-adiabatic electrons were assumed in Ref. 15 and here, respectively.

In summary, the impact of the plasma background fluctuations are introduced into the Langevin equation for the particle motion by a stochastic process obeying the statistical properties of a larger class of distributions, namely, stable distributions. Such a stochastic process is represented by a fractional velocity derivative in the corresponding Fokker-Planck equation. The solution of the fractional Fokker-Planck equation represents the equilibrium PDF of the system and, due to the non-Gaussian assumption on the background fluctuations, is no longer the classical Maxwellian distribution but a Lévy distribution. Through the plasma quasi-neutrality condition one can find an expression for the exponent of the Lévy distribution, i.e., the order of the fractional derivative in the Fokker-Planck equation as a function of the linear eigenvalues of the unstable modes. By solving this expression for given eigenvalues, it is shown that as the linear eigenvalues of the modes increase, the order of the fractional velocity derivative deviates from 2 and therefore, plasma becomes Lévy distributed. In Ref. 15 by solving the dispersion equation for eigenvalues with a given deviation order ϵ_e, it was also shown that the modified equilibrium in turn may strongly enhance the unstable fluctuations, i.e., eigenvalues of the unstable modes, cf., Ref. 27. Therefore, when analyzing the turbulence driven transport one has to take into account that if the statistical properties of the underlying plasma fluctuations are non-Gaussian, the resulting transport due to the unstable fluctuations may be modified significantly. The present work is a step on the way to establish the connection between the microscopic physics of turbulence and fractional derivative models.

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APPENDIX: FRACTIONAL VELOCITY FUNCTIONS

The functions $\Phi(u,w)$, $\Psi(u,w)$, and $\Upsilon(u,w)$ are defined as follows:

$$\begin{aligned}\Phi(u,w) = -\frac{i}{8|u|}\Bigg\{& u\mathbf{Erfi}[|u|]\left(\left(-1+3w^2\right)(-2\gamma_E + 2\log[V_{T,s}]) + e^{w^2}\,{}_1F_1(1,0,0)\left[\frac{3}{2},\frac{1}{2},-w^2\right]\right)\\ &- i|u|(14 - 8\gamma_E - 8u^2 + 4\gamma_E u^2 - 8w^2 + 4\gamma_E w^2 - 4\log[V_{T,s}] - 2e^{u^2}\,{}_1F_1(1,0,0)\left[\frac{3}{2},\frac{1}{2},-u^2\right]\\ &- 2e^{w^2}\,{}_1F_1(1,0,0)\left[\frac{3}{2},\frac{1}{2},-w^2\right] - i\mathbf{Erfi}[|u|]\left((-1+2w^2)(-2+\gamma_E+2\log[V_{T,s}]) + e^{w^2}\,{}_1F_1(1,0,0)\left[\frac{3}{2},\frac{1}{2},-w^2\right]\right)\Bigg)\Bigg\},\end{aligned} \tag{A1}$$

and

$$
\begin{aligned}
\Psi(u,w) = \frac{1}{48|u|}\Bigg\{ & iu\mathbf{Erfi}[|u|]\Bigg(\Big(3-12w^2+4w^4\Big)(-8+3\gamma_E+6\log[V_{T,s}]) - 9e^{w^2}\,{}_1F_1(1,0,0)\left[\frac{5}{2},\frac{1}{2},-w^2\right]\Bigg)\\
& + |u|\Bigg(-84+48\gamma_E+180u^2-96\gamma_E u^2-32u^4+24\gamma_E u^4+180w^2-96\gamma_E w^2-192u^2w^2+48\gamma_E u^2 w^2-32w^4\\
& +24\gamma_E w^4+24i\mathbf{Erfi}[u]-9i\gamma_E\mathbf{Erfi}[u]-96iw^2\mathbf{Erfi}[u]+36i\gamma_E w^2\mathbf{Erfi}[u]+32iw^4\mathbf{Erfi}[u]-12i\gamma_E w^4\mathbf{Erfi}[u]\\
& +24\log[V_{T,s}]-24u^2\log[V_{T,s}]-24w^2\log[V_{T,s}]-18i\mathbf{Erfi}[u]\log[V_{T,s}]+72iw^2\mathbf{Erfi}[u]\log[V_{T,s}]\\
& -24iw^4\mathbf{Erfi}[u]\log[V_{T,s}]-6(-1+2u^2)(-1+2w^2)\Big({}_1F_1{}^{(1,0,0)}\left[0,\frac{1}{2},u^2\right]+{}_1F_1{}^{(1,0,0)}\left[0,\frac{1}{2},w^2\right]\Big)\\
& -12e^{u^2}\,{}_1F_1{}^{(1,0,0)}\left[\frac{3}{2},\frac{1}{2},-u^2\right]-12e^{w^2}\,{}_1F_1{}^{(1,0,0)}\left[\frac{3}{2},\frac{1}{2},-w^2\right]+18e^{u^2}\,{}_1F_1{}^{(1,0,0)}\left[\frac{5}{2},\frac{1}{2},-u^2\right]\\
& +18e^{w^2}\,{}_1F_1{}^{(1,0,0)}\left[\frac{5}{2},\frac{1}{2},-w^2\right]+9ie^{w^2}\mathbf{Erfi}[u]{}_1F_1{}^{(1,0,0)}\left[\frac{5}{2},\frac{1}{2},-w^2\right]+6{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},u^2\right]\\
& -24u^2\,{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},u^2\right]-12w^2\,{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},u^2\right]+48u^2w^2\,{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},u^2\right]+6{}_1F_1\left[0,\frac{1}{2},w^2\right]\\
& -12u^2\,{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},w^2\right]-24w^2\,{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},w^2\right]+48u^2w^2\,{}_1F_1{}^{(1,0,1)}\left[0,\frac{1}{2},w^2\right]\\
& +12u^2\,{}_1F_1{}^{(1,0,2)}\left[0,\frac{1}{2},u^2\right]-24u^2w^2\,{}_1F_1{}^{(1,0,2)}\left[0,\frac{1}{2},u^2\right]+12w^2\,{}_1F_1{}^{(1,0,2)}\left[0,\frac{1}{2},w^2\right]\\
& -24u^2w^2\,{}_1F_1{}^{(1,0,2)}\left[0,\frac{1}{2},w^2\right]\Bigg)\Bigg\},
\end{aligned}
\tag{A2}
$$

and

$$
\begin{aligned}
\Upsilon(u,w) = \frac{-1}{8|u|}\Bigg\{ & iu\mathbf{Erfi}[|u|]\Bigg(\Big(-3+2w^2\Big)(-2+\gamma_E+2\log[V_{T,s}])+e^{w^2}\,{}_1F_1{}^{(1,0,1)}\left[\frac{3}{2},\frac{1}{2},-w^2\right]\Bigg)+|u|\Bigg(22-12\gamma_E-8u^2\\
& +4\gamma_E u^2-8w^2+4\gamma_E w^2-6i\mathbf{Erfi}[u]+3i\gamma_E\mathbf{Erfi}[u]+4iw^2\mathbf{Erfi}[u]-2i\gamma_E w^2\mathbf{Erfi}[u]-4\log[V_{T,s}]+6i\mathbf{Erfi}[u]\log[V_{T,s}]\\
& -4iw^2\mathbf{Erfi}[u]\log[V_{T,s}]-2e^{u^2}\,{}_1F_1{}^{(1,0,0)}\left[\frac{3}{2},\frac{1}{2},-u^2\right]-ie^{w^2}(-2i+\mathbf{Erfi}[u]){}_1F_1{}^{(1,0,0)}\left[\frac{3}{2},\frac{1}{2},-w^2\right]\Bigg)\Bigg\}.
\end{aligned}
\tag{A3}
$$

Here, ${}_1F_1[a;b;z]$ denoting Kummer's confluent hypergeometric function and the superscripts represent the derivative of the hypergeometric function with respect to its parameters; for example, ${}_1F_1{}^{(1,0,0)}[a;b;z]$ represents the derivative with respect to the first parameter, i.e., a, and $\mathbf{Erf}[u]$ gives the imaginary error function $\mathbf{Erf}[iu]/i$.

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