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Capacity Bounds for AF Dual-hop Relaying in $\mathcal{G}$ Fading Channels

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Abstract— We investigate the ergodic capacity of amplify-and-forward (AF) dual-hop relaying systems in composite Nakagami-m/Inverse-Gaussian fading channels. This type of fading, which is known in the literature as $\mathcal{G}$ fading, has recently attracted increasing research interest thanks to its ability to better approximate the Nakagami-m/lognormal model compared to the Nakagami-m/gamma model. We study both fixed and variable-gain relaying systems, and present analytical upper and lower bounds for the ergodic capacity of dual-hop relaying systems with not necessarily identical hops; these bounds provide an efficient means to evaluate the ergodic capacity of AF dual-hop relaying systems over $\mathcal{G}$ fading channels. We also establish sufficient conditions for the existence of the bounds depending on the values of the fading parameters. In both cases, our simulation results demonstrate that the proposed upper and lower bounds remain relatively tight for different fading conditions.

Index Terms— Amplify-and-forward relaying, dual-hop transmission, ergodic capacity, Nakagami-m/inverse-Gaussian fading

I. Introduction

Dual-hop relaying, where an intermediate relay node helps forward the signal from the source node to the destination node, is an efficient means to extend the coverage and improve the throughput of communication systems. Among various relaying protocols proposed in the literature, the amplify-and-forward (AF) relaying scheme is of particular interest both in academy and industry due to its low implementation cost [1]. In this case, relays only amplify the source signal and forward it to the destination without performing any decoding. Depending on the availability of instantaneous channel state information (CSI) at the relay node, AF relaying schemes generally fall into two categories: fixed-gain relaying

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(or blind relaying) [2] and variable-gain relaying (or CSI-assisted relaying) [3]. The former configurations do not require knowledge of the instantaneous CSI, but need to know the average fading power of the previous hop, while variable-gain relaying systems amplify the received signal using the instantaneous CSI of the previous hop.

Understanding the ergodic capacity performance of AF dual-hop relaying systems in various practical environments has been an active area of research. The ergodic capacity of AF dual-hop relaying systems in Rayleigh fading channels was investigated in [4–7], while the cases of Nakagami-m fading and Weibull fading channels were studied in [8, 9] and [10], respectively. It is also worth mentioning the seminal work of [11] which established a generic Moment Generating Function-based framework for the performance analysis of relaying systems. Finally, [12] proposed a general framework for analyzing the ergodic capacity of variable-gain multi-hop relaying systems, although the presented results either apply for identically distributed fading distributions (e.g. Nakagami-m) or rely on the classical moment-based approach of [11].

The common characteristic of [4]–[10] is that they consider only small-scale fading and neglect large-scale fading (or shadowing). This can be attributed to the difficulty in averaging the end-to-end Signal-to-Noise ratio (SNR) over the shadowing distribution. In the analysis of composite fading channels, the prevalent model is the Nakagami-m/lognormal model which has been extensively used to approximate the fading fluctuations in radar and RF communication systems [13]. Note that the Nakagami-m distribution has been shown to yield good fit with empirical data in various propagation scenarios [14, 15]. This can, for example, occur for multipath scattering with relatively large delay-time spreads or when two strong paths of comparable power are dominating the other multipath components. Yet, the main disadvantage of the Nakagami-m/lognormal model is that the composite probability density function (p.d.f.) cannot be expressed in closed-form. Consequently, the most important performance measures of interest, such as the ergodic capacity and symbol error rate, cannot be analytically evaluated. For this reason, some alternative models were recently proposed with the most tractable being the Nakagami-m/gamma model, often referred to as the generalized-K model [16–19]. The performance of relaying systems in generalized-K fading channels has been investigated in [12, 20–22] and the references therein.

On the other hand, it was pointed out in [23] that the gamma distribution does not yield a good approximation of the lognormal distribution with large variance. In addition, the

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accuracy of this approximation deteriorates significantly in the tails of the distribution (i.e. low outage region). Motivated by this limitation, the authors in [23] suggested to replace the gamma model with the inverse-Gaussian (IG) model; it was further demonstrated that the composite Rayleigh/IG model gives better characterization of fading channels in comparison with the Rayleigh/gamma distribution. Later in [24], the authors introduced the more general Nakagami-m/IG model (hereafter referred to as the $\mathcal{G}$ distribution) and studied the performance of such fading channels. Despite the importance of this composite fading model, to the best of our knowledge, very little is known about the performance of relaying systems in such fading channels.

On this basis, in this paper, we study the ergodic capacity of AF dual-hop relaying systems in composite $\mathcal{G}$ fading channels. In particular, we provide new analytical upper and lower capacity bounds for both fixed-gain and variable-gain AF dual-hop relaying systems. The derived bounds involve only standard mathematical functions, and therefore can be easily and efficiently evaluated. More importantly, they apply to the case where the first and second hop experience non-identically distributed fading. Our simulation results indicate that these bounds remain relatively tight across the entire SNR range and under different fading conditions. Thus, they can provide an efficient means to evaluate the capacity of AF dual-hop relaying systems in composite $\mathcal{G}$ fading channels. It is also shown that the ergodic capacity of AF dual-hop relaying systems increases when the fading of either hop is less severe.

The remainder of the paper is organized as follows: In Section II, the AF dual-hop relaying system model is presented while Section III introduces some new and fundamental results on the statistics of IG variates. In Section IV, we provide new analytical upper and lower bounds on the ergodic capacity of fixed-gain relaying systems while Section V elaborates on the case of variable-gain relaying systems. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

Let us consider a dual-hop relaying system with one source, relay and destination node as shown in Fig. 1.
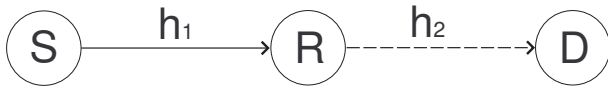

Fig. 1. System model: S, R and D stand for source, relay and destination nodes, respectively, while h_1 and h_2 denote the channel fading coefficients for the source-relay and relay-destination links, respectively.

In this case, the end-to-end input-output relationship can be succinctly expressed as [2, 3]

$$y = h_2 G(h_1 x + n_1) + n_2 \qquad (1)$$

where x denotes the source symbol with $\mathrm{E}\{xx^*\} = P_1$, while $(\cdot)^*$ and $\mathrm{E}\{\cdot\}$ denote complex conjugate and expectation, respectively. The term h_1 represents the random fading coefficient of the source-relay link, while h_2 is the random

fading coefficient of the relay-destination link. Also, G is the power scaling factor which depends on the relay operation mode, and will be defined explicitly in the corresponding sections. In addition, n_1 and n_2 are the additive Gaussian noises at the relay and destination nodes with power N_1 and N_2, respectively.

We assume that $|h_1|$ and $|h_2|$ are independent, and follow the $\mathcal{G}$ composite distribution, where $|x|$ denotes the amplitude of a complex number x. This implies that the small-scale fading follows a Nakagami-m distribution, which is characterized by the Nakagami-m parameter, while the large-scale fading follows an IG distribution with parameters λ_i, θ_i. In the following, we are particularly interested in the statistics of the power of each hop, that is $|h_i|^2$, $i = 1, 2$. For the case under consideration, this can be expressed as a product of two independent random variables [24]

$$|h_i|^2 = u_i v_i$$

where u_i follows the gamma distribution with p.d.f.

$$f_{u_i}(x) = \frac{m_i^{m_i}}{\Gamma(m_i)} x^{m_i-1} e^{-m_i x}, \quad x > 0, \ m_i \geq 0.5$$

while v_i follows the inverse Gaussian distribution with parameters λ_i, and θ_i with p.d.f.

$$p_{v_i}(x) = \sqrt{\frac{\lambda_i}{2\pi x^3}} \exp\left(-\frac{\lambda_i(x-\theta_i)^2}{2\theta_i^2 x}\right), \quad x, \theta_i, \lambda_i \geq 0.$$

Following the methodology of [24], it can be shown that the p.d.f. of $|h_i|^2$ reads as

$$f_{|h_i|^2}(x) = \frac{\alpha_i x^{m_i-1}}{(\sqrt{x+\gamma_i})^{m_i+\frac{1}{2}}} K_{m_i+\frac{1}{2}}(\beta_i\sqrt{x+\gamma_i}) \qquad (2)$$

where $K_\nu(\cdot)$ denotes the ν-th order modified Bessel function of the second kind [25, Eq. (8.407.1)], while $\alpha_i \triangleq \frac{m_i^{m_i}}{\Gamma(m_i)}\left(\frac{\lambda_i}{\theta_i^2\beta_i}\right)^{m_i+\frac{1}{2}}\sqrt{\frac{2\lambda_i}{\pi}}\exp\left(\frac{\lambda_i}{\theta_i}\right)$, $\beta_i \triangleq \sqrt{\frac{2\lambda_i m_i}{\theta_i^2}}$ and $\gamma_i \triangleq \frac{\lambda_i}{2m_i}$.

From (1), it is easy to see that the end-to-end SNR equals to

$$\rho = \frac{G^2|h_1|^2|h_2|^2 P_1}{G^2|h_2|^2 N_1 + N_2}.$$

In the following analysis, the hop SNRs defined as $\rho_1 \triangleq \frac{P_1|h_1|^2}{N_1}$, $\rho_2 \triangleq \frac{P_2|h_2|^2}{N_2}$ will be extensively used. Unfortunately, their statistical characterization is not always straightforward, especially when shadowing is taken into account. In this light, we henceforth present some novel results on the statistical properties of ρ_i for the case of composite $\mathcal{G}$ fading.

III. STATISTICAL PROPERTIES OF FUNCTIONS INVOLVING THE HOP SNRS ρ_i

In this section, we provide a set of new statistical results for functions involving $\rho_i \triangleq \frac{P_i}{N_i}|h_i|^2$, $i = 1, 2$. Note that the following results will be handy when studying the ergodic capacity of AF dual-hop systems. Having this in mind, we start with the first positive moment of ρ_i.

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Lemma 1: The first positive moment of ρ_i is given by

$$\mathrm{E}\{\rho_i\} = \frac{P_i}{N_i}\theta_i. \tag{3}$$

Proof: Due to the independence of u_i and v_i, we have that

$$\mathrm{E}\{\rho_i\} = \frac{P_i}{N_i}\mathrm{E}\{u_i\}\mathrm{E}\{v_i\}.$$

Utilizing the fact that $\mathrm{E}\{v_i\} = \theta_i$ [26, Eq. (2.6)] yields the desired result. ∎

Lemma 2: The first negative moment of ρ_i is given by

$$\mathrm{E}\{\rho_i^{-1}\} = \frac{N_i}{P_i}\frac{m_i}{m_i-1}\left(\frac{1}{\lambda_i}+\frac{1}{\theta_i}\right). \tag{4}$$

Proof: See Appendix I-A. ∎

Lemma 3: The expectation of the logarithm of ρ_i is given by

$$\mathrm{E}\{\ln\rho_i\} = \ln\frac{P_i\theta_i}{N_i m_i} + \psi(m_i) + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\mathrm{Ei}\left(-\frac{2\lambda_i}{\theta_i}\right) \tag{5}$$

where $\psi(x)$ is the digamma function [25, Eq. (8.360.1)] and $\mathrm{Ei}(x) = \int_{-\infty}^{x}\frac{e^t}{t}dt$, $x < 0$ is the exponential integral function [25, Eq. (8.211.1)].

Proof: See Appendix I-B. ∎

Lemma 4: The first moment of $\frac{\rho_i}{a+\rho_i}$ can be expressed as

$$\mathrm{E}\left\{\frac{\rho_i}{a+\rho_i}\right\} = \alpha_i\left((-b)^{m_i}R_{m_i}(b,\gamma_i-b,\beta_i)\right.$$
$$\left.+ \sum_{k=1}^{m_i}\binom{m_i}{k}(-b)^{m_i-k}I_{m_i,k}(b,1,\gamma_i-b,\beta_i)\right) \tag{6}$$

where $R_n(u,v,w)$ is defined as

$$R_n(u,v,w) \triangleq \int_1^{\infty}\frac{K_{n+\frac{1}{2}}(w\sqrt{ux+v})}{x(\sqrt{ux+v})^{n+\frac{1}{2}}}dx \tag{7}$$

while $I_{p,q}(u,v,w,z)$ reads as

$$I_{p,q}(u,v,w,z)$$
$$\triangleq (q-1)!\sum_{s=1}^{q}\frac{2^s u^{q-s}}{(vz)^s(q-s)!}\frac{K_{p-s+\frac{1}{2}}(z\sqrt{uv+w})}{(\sqrt{uv+w})^{p-s+\frac{1}{2}}}. \tag{8}$$

Proof: See Appendix I-C. ∎

IV. FIXED-GAIN AF DUAL-HOP SYSTEMS

In this section, we elaborate on the ergodic capacity of fixed-gain AF dual-hop relaying systems. As was previously mentioned, fixed-gain relying schemes do not require instantaneous CSI at the relay node and as such they are more attractive from a practical point of view. For fixed-gain relaying systems, the relaying gain is given by [2]

$$G_f = \sqrt{\frac{P_2}{\mathrm{E}\{|h_1|^2\}P_1 + N_1}}$$

where P_2 is the power of the transmitted signal at the output of the relay. Hence, the end-to-end SNR for fixed-gain AF dual-hop relaying systems is equal to

$$\rho_f = \frac{\rho_1\rho_2}{c+\rho_2}$$

where $c \triangleq \frac{P_2}{G_f^2 N_1}$. With this definition in hand, we can obtain the ergodic capacity of the system under consideration as follows

$$C_f = \frac{1}{2}\mathrm{E}\left\{\log_2\left(1+\frac{\rho_1\rho_2}{c+\rho_2}\right)\right\} \tag{9}$$

where the factor $1/2$ accounts for the fact that the entire communication takes place in two time slots. Unfortunately, an exact evaluation of the ergodic capacity in (9) is in general intractable, due to the presence of the nonlinear log function. Motivated by this, we hereafter seek to deduce upper and lower bounds on C_f. We start with the following Jensen's upper bound:

Theorem 1: The ergodic capacity of fixed-gain AF dual-hop relaying systems in $\mathcal{G}$ fading channels is upper bounded by

$$C_f^{u1} = \frac{1}{2}\log_2\left(1+\frac{P_1\theta_1\alpha_2(-d_2)^{m_2}}{N_1}\left(R_{m_2}(d_2,\gamma_2-d_2,\beta_2)\right.\right.$$
$$\left.\left.+ \sum_{k=1}^{m_2}\binom{m_2}{k}(-d_2)^{-k}I_{m_2,k}(d_2,1,\gamma_2-d_2,\beta_2)\right)\right) \tag{10}$$

where $d_2 \triangleq \frac{cN_2}{P_2}$, $R_n(u,v,w)$ is defined in (7) while $I_{p,q}(u,v,w,z)$ in (8).

Proof: Exploiting the concavity of the log function and applying Jensen's inequality, the ergodic capacity of fixed-gain AF dual-hop systems can be upper bounded by

$$C_f^{u1} = \frac{1}{2}\log_2\left(1+\mathrm{E}\{\rho_1\}\mathrm{E}\left\{\frac{\rho_2}{c+\rho_2}\right\}\right).$$

The proof concludes after invoking Lemma 1, Lemma 4 and factorization. ∎

Corollary 1: When $m_2 = 1$, the capacity upper bound (10) reduces to

$$C_f^{u1} = \frac{1}{2}\log_2\left(1-\frac{P_1\theta_1\alpha_2}{N_1}\left(\frac{\sqrt{2\pi}e^{-\beta_2\sqrt{\gamma_2}}}{\beta_2^{3/2}\sqrt{\gamma_2}}-d_2A_1\right)\right) \tag{11}$$

where $A_1 \triangleq R_1(d_2,\gamma_2-d_2,\beta_2)$.

Proof: When $m_2 = 1$, we notice that $I_{1,1}(u,v,w,z) = \frac{2K_{1/2}(z\sqrt{uv+w})}{vz\sqrt{uv+w}^{1/2}}$. The desired result can then be obtained after some mathematical simplifications. ∎

The main limitation with the Jensen's upper bound C_f^{u1} is that it is not sufficiently tight in the high SNR regime. Motivated by this, we propose the following upper bound which is asymptotically exact in the high SNR regime.

Theorem 2: The ergodic capacity of fixed-gain AF dual-hop systems in $\mathcal{G}$ fading channels is upper bounded by C_f^{u2} shown in (12) at the bottom of next page.

Proof: We know from [21] that the ergodic capacity of fixed-gain AF dual-hop systems is upper bounded by

$$C_f^{u2} = \frac{1}{2}\log_2\left(1+c\mathrm{E}\left\{\rho_1^{-1}\right\}\mathrm{E}\left\{\rho_2^{-1}\right\}+\mathrm{E}\left\{\rho_1^{-1}\right\}\right)$$
$$+ \frac{1}{2}\mathrm{E}\left\{\log_2\rho_1 + \log_2\rho_2 - \log_2(c+\rho_2)\right\}. \tag{13}$$

To this end, the desired result can be obtained by combining Lemma 2, Lemma 3 and a result from [24, Eq. (21)]. ∎

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Note that taking into account the condition for the existence of the first negative moment in (13) (see Lemma 2), the upper bound C_f^{u2} exists only for $m_i > 1$. This implies, that the second upper bound does not exist if small-scale fading in one of the hops is Rayleigh distributed. Now, we turn our attention to the ergodic capacity lower bound, and present the following new result:

Theorem 3: The ergodic capacity of fixed-gain AF dual-hop systems in $\mathcal{G}$ fading channels is lower bounded by C_f^l shown in (14) at the bottom of the page.

Proof: From [21], the ergodic capacity of fixed-gain dual-hop relaying systems is lower bounded by

$$C_f^l = \frac{1}{2}\log_2(1 + \exp(\mathrm{E}\{\ln\rho_1\} + \mathrm{E}\{\ln\rho_2\} - \mathrm{E}\{\ln(c+\rho_2)\})).$$

As a next step, we invoke Lemma 3 and a result from [24, Eq. (21)], and the desired result can be obtained after some basic algebraic manipulations. ∎

We note that all bounds require m_2 to be a positive integer while for C_f^{u2}, i.e., $m_2 = 2, 3, \ldots$. When both the first and second hop channels are subjected to Rayleigh/IG fading, we have the following simplified lower bound:

Corollary 2: When $m_1 = m_2 = 1$, the ergodic capacity lower bound in (14) reduces to C_f^l shown in (15) at the top of next page, where $\gamma_0 = 0.57721$ is the Euler-Mascheroni constant.

Proof: When $m_i = 1$, we have

$$\mathrm{E}\{\log_2(c+\rho_2)\} = \ln c + \frac{2\alpha_2}{\beta_2}R_0\left(b_2, \gamma_2 - b_2, \beta_2\right) \quad (16)$$

and the desired result follows immediately. ∎

It is worth pointing out that the ergodic capacity upper and lower bounds presented in the above theorems only involve standard functions, and hence, can be very fast and efficiently evaluated in popular software packages such as Matlab or

Mathematica. More importantly, C_f^{u2} and C_f^l become exact at high SNRs, as shown in the following corollary:

Corollary 3: At high SNRs, the upper bound C_f^{u2} and lower bound C_f^l become exact as

$$C_f^{u2} = C_f^l = C_f^e, \quad (17)$$

where C_f^e is shown in (18) at the top of next page.

To illustrate the tightness of the proposed ergodic capacity upper and lower bounds, we compare the analytical results against Monte-Carlo simulation results. For all simulations, the parameters of the IG distributions are obtained by matching the first and second moment with those of the lognormal distributions as per [24]

$$\lambda = \frac{\exp\mu}{2\sinh(\sigma^2/2)}, \qquad \theta = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

where $\sinh(x)$ is the sine hyperbolic function, while μ and σ are the mean and the standard deviation of a lognormal distribution.

Figure 2 shows the impact of small-scale fading on the ergodic capacity of fixed-gain AF dual-hop systems. In the simulations, we set $\mu_i = 0.115$ and $\sigma_i = 0.115$, $i = 1, 2$, which corresponds to an infrequent light shadowing scenario [27]. As can be readily seen, the second upper bound C_f^{u2} is much tighter than the first upper bound C_f^{u1} in the high SNR regime; in fact, C_f^{u2} almost overlaps with C_f^l when $P_1/N_1 \geq 20$dB, thereby demonstrating that C_f^{u2} and C_f^l are asymptotically exact at high SNRs. We also observe the intuitive result that the ergodic capacity of the system improves when the fading severity level is reduced, i.e., when m_i increases from 2 to 20. Moreover, the tightness of all three bounds improves when m_i becomes larger. To further examine the impact of m_i, in Fig. 3, we set $\frac{P_1}{N_1} = 10$dB, $\frac{P_2}{N_2} = 2\frac{P_1}{N_1}$, $\mu_1 = \mu_2 = 0.115$, $\sigma_1 = \sigma_2 = 0.115$. As can be seen, when m_i increases, the ergodic capacity of the system improves;

$$C_f^{u2} = \frac{1}{2}\log_2\left(1 + \frac{N_1 m_1}{P_1(m_1-1)}\left(\frac{1}{\lambda_1} + \frac{1}{\theta_1}\right)\left(\frac{cN_2 m_2}{P_2(m_2-1)}\left(\frac{1}{\lambda_2} + \frac{1}{\theta_2}\right) + 1\right)\right)$$
$$+ \frac{\log_2 e}{2}\left(\ln\frac{P_1 P_2 \theta_1 \theta_2}{N_1 N_2 m_1 m_2} + \psi(m_1) + \psi(m_2) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathrm{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right) + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathrm{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)$$
$$- \frac{\log_2 c}{2} - \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}(-d_2)^{m_2-k}R_{m_2-k}(d_2, \gamma_2 - d_2, \beta_2)$$
$$- \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}(-d_2)^{m_2-k-j}I_{m_2-k,j}(d_2, 1, \gamma_2 - d_2, \beta_2). \quad (12)$$

$$C_f^l = \frac{1}{2}\log_2\left(1 + \frac{P_1 P_2 \theta_1 \theta_2}{N_1 N_2 m_1 m_2}\exp\left(\psi(m_1) + \psi(m_2) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathrm{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right)\right.$$
$$\times\exp\left(\exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathrm{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right) - \ln c - \sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}(-d_2)^{m_2-k}R_{m_2-k}(d_2, \gamma_2 - d_2, \beta_2)\right.$$
$$\left.\left.- \sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}(-d_2)^{m_2-k-j}I_{m_2-k,j}(d_2, 1, \gamma_2 - d_2, \beta_2)\right)\right). \quad (14)$$

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$$C_{\mathsf{f}}^l = \frac{1}{2}\log_2\left(1 + \frac{P_1 P_2}{N_1 N_2 c}\exp\left(-2\gamma_0 + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right) + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)\right.$$
$$\left. \times \exp\left(-\frac{2\alpha_2}{\beta_2}R_0\left(d_2, \gamma_2 - d_2, \beta_2\right)\right)\right). \tag{15}$$

$$C_{\mathsf{f}}^e = \frac{\log_2 e}{2}\left(\ln\frac{P_1 P_2}{N_1 N_2 m_1 m_2} + \psi(m_1) + \psi(m_2) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right) + \right) - \frac{\log_2 c}{2}$$
$$+ \frac{\log_2 e}{2}\left(\exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right) - \sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\left(-d_2\right)^{m_2-k}R_{m_2-k}\left(d_2, \gamma_2 - d_2, \beta_2\right)\right)$$
$$- \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}\left(-d_2\right)^{m_2-k-j}I_{m_2-k,j}\left(d_2, 1, \gamma_2 - d_2, \beta_2\right). \tag{18}$$
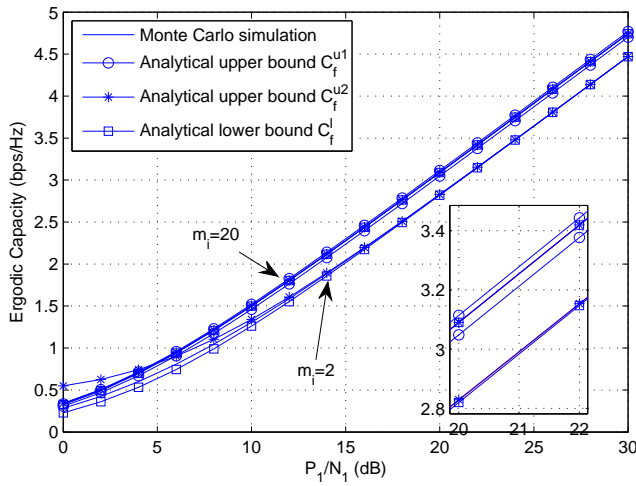

Fig. 2. Ergodic capacity of fixed-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results vs. Analytical upper bounds, $\frac{P_2}{N_2} = 3\frac{P_1}{N_1}$.
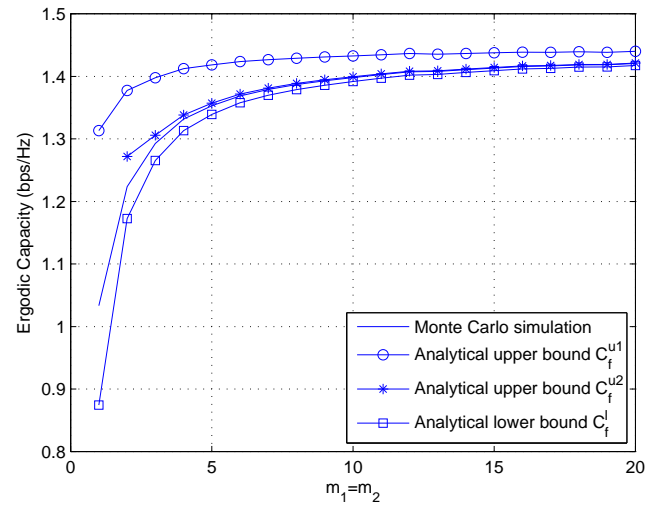

Fig. 3. Ergodic capacity of fixed-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels with $\frac{P_2}{N_2} = 2\frac{P_1}{N_1}$ and $\frac{P_1}{N_1} = 10$dB: Impact of m_i.

however, this improvement gradually diminishes when m_i becomes sufficiently large, i.e., $m_i > 10$.

Similarly, Fig. 4 investigates the impact of shadowing on the ergodic capacity of fixed-gain AF dual-hop systems when $m_i = 20$. As we can observe, when the shadowing effect becomes more frequent and heavy, the ergodic capacity of the system is reduced.

V. Variable-Gain AF Dual-Hop Systems

In this section, we study the ergodic capacity of variable-gain AF dual-hop relaying systems. The variable-gain relaying scheme exploits the instantaneous CSI at the relay node, such that corresponding relay amplification factor is given by [3]

$$G_v = \sqrt{\frac{P_2}{|h_1|^2 P_1 + N_1}}.$$

Hence, the end-to-end SNR for variable-gain AF dual-hop relaying systems equals

$$\rho_v = \frac{\rho_1\rho_2}{\rho_1 + \rho_2 + 1}. \tag{19}$$

From (19), the ergodic capacity of the system can be expressed as

$$C_v = \frac{1}{2}\mathtt{E}\left\{\log_2\left(1 + \frac{\rho_1\rho_2}{\rho_1 + \rho_2 + 1}\right)\right\}.$$

Similar to the fixed-gain relaying case, an exact expression for C_v is in general very difficult to derive. Hence, we focus on deriving closed-form upper and lower bounds on C_v.

Theorem 4: The ergodic capacity of variable-gain AF dual-hop relaying systems in $\mathcal{G}$ fading channels is upper bounded by C_v^u shown in (20) at the bottom of next page, where $f_i \triangleq \frac{P_i}{N_i}$, $i = 1, 2$.

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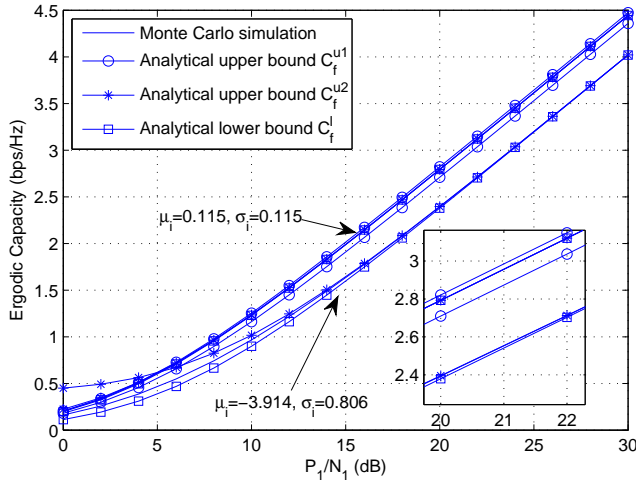
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Fig. 4. Ergodic capacity of fixed-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results vs. Analytical upper and lower bounds, $\frac{P_1}{N_1} = \frac{P_2}{N_2}$.

Proof: From [7], the ergodic capacity of variable-gain AF dual-hop systems can be alternatively expressed as

$$C_{\mathsf{v}} = \frac{1}{2}\mathrm{E}\left\{\log_2(1+\rho_1)\right\} + \frac{1}{2}\mathrm{E}\left\{\log_2(1+\rho_2)\right\}$$
$$- \frac{1}{2}\mathrm{E}\left\{\log_2(1+\rho_1+\rho_2)\right\}. \tag{21}$$

Since the first two terms can be expressed in closed-form expressions, the key task is to bound the third term. It is easy to show that $f(x,y) = \log_2(1+e^x+e^y)$ is a convex function with respect to x and y, hence, we have

$$\mathrm{E}\left\{\log_2(1+\rho_1+\rho_2)\right\} \geq \log_2(1+e^{\mathrm{E}\{\ln\rho_1\}}+e^{\mathrm{E}\{\ln\rho_2\}}).$$

The desired result is finally obtained by using Lemma 3. ∎

When both the first and second hop channels are subjected to the Rayleigh/IG fading, we have the following simple capacity upper bound:

Corollary 4: When $m_1 = m_2 = 1$, the ergodic capacity

upper bound in (20) reduces to

$$C_{\mathsf{v}}^u = \frac{\alpha_1 \log_2 e}{\beta_1} R_0\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right)$$
$$+ \frac{\alpha_2 \log_2 e}{\beta_2} R_0\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right)$$
$$- \frac{1}{2}\log_2\left(1 + f_1 \exp\left(-\gamma_0 + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right)\right.$$
$$\left.+ f_2\exp\left(-\gamma_0 + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)\right). \tag{22}$$

Proof: The result follows immediately by invoking (16). ∎

Now, we turn our attention to the ergodic capacity lower bound, and we have the following key result.

Theorem 5: The ergodic capacity of variable-gain AF dual-hop relaying systems in $\mathcal{G}$ fading channels is lower bounded by C_{v}^l shown in(23) at the top of next page.

Proof: Applying Jensen's inequality on the third term of (21), we have

$$\mathrm{E}\left\{\log_2(1+\rho_1+\rho_2)\right\} \leq \log_2(1+\mathrm{E}\{\rho_1\}+\mathrm{E}\{\rho_2\}).$$

Hence, the desired result can be obtained by invoking Lemma 1. ∎

When both the first and second hop channels are subjected to Rayleigh/IG fading, we have the following simple capacity lower bound:

Corollary 5: When $m_1 = m_2 = 1$, the ergodic capacity lower bound in (23) reduces to

$$C_{\mathsf{v}}^l = \frac{\alpha_1 \log_2 e}{\beta_1} R_0\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right) + \tag{24}$$
$$\frac{\alpha_2 \log_2 e}{\beta_2} R_0\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right) - \frac{1}{2}\log_2\left(1 + f_1\theta_1 + f_2\theta_2\right).$$

Proof: The result follows immediately by invoking (16). ∎

We note that both the upper bound in (20) and the lower bound in (23) require that both m_1 and m_2 are positive integers. We can now examine the tightness of the above upper and lower capacity bounds. Figure 5 shows the impact of small-scale fading on the tightness of the capacity bounds when $\mu_i = 0.115$, $\sigma_1 = 0.115$, $i = 1, 2$. Generally speaking,

---

$$C_{\mathsf{v}}^u = \frac{\log_2 e}{2}\sum_{k=1}^{m_1}\frac{\alpha_1\Gamma(m_1)2^k}{\beta_1^k(m_1-k)!}\left(-\frac{1}{f_1}\right)^{m_1-k}R_{m_1-k}\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right)$$
$$+ \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\left(-\frac{1}{f_2}\right)^{m_2-k}R_{m_2-k}\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right)$$
$$+ \frac{\log_2 e}{2}\sum_{k=1}^{m_1}\frac{\alpha_1\Gamma(m_1)2^k}{\beta_1^k(m_1-k)!}\sum_{j=1}^{m_1-k}\binom{m_1-k}{j}\left(-\frac{1}{f_1}\right)^{m_1-k-j}I_{m_1-k,j}\left(\frac{1}{f_1}, 1, \gamma_1 - \frac{1}{f_1}, \beta_1\right)$$
$$+ \frac{\log_2 e}{2}\sum_{k=1}^{m_2}\frac{\alpha_2\Gamma(m_2)2^k}{\beta_2^k(m_2-k)!}\sum_{j=1}^{m_2-k}\binom{m_2-k}{j}\left(-\frac{1}{f_2}\right)^{m_2-k-j}I_{m_2-k,j}\left(\frac{1}{f_2}, 1, \gamma_2 - \frac{1}{f_2}, \beta_2\right)$$
$$- \frac{1}{2}\log_2\left(1 + \frac{f_1}{m_1}\exp\left(\psi(m_1) + \exp\left(\frac{2\lambda_1}{\theta_1}\right)\mathtt{Ei}\left(-\frac{2\lambda_1}{\theta_1}\right)\right)\right.$$
$$\left.+ \frac{f_2}{m_2}\exp\left(\psi(m_2) + \exp\left(\frac{2\lambda_2}{\theta_2}\right)\mathtt{Ei}\left(-\frac{2\lambda_2}{\theta_2}\right)\right)\right). \tag{20}$$

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$$C_{\mathsf{v}}^l = \frac{\log_2 e}{2} \sum_{k=1}^{m_1} \frac{\alpha_1 \Gamma(m_1) 2^k}{\beta_1^k (m_1-k)!} \left(-\frac{1}{f_1}\right)^{m_1-k} R_{m_1-k}\left(\frac{1}{f_1}, \gamma_1 - \frac{1}{f_1}, \beta_1\right)$$
$$+ \frac{\log_2 e}{2} \sum_{k=1}^{m_2} \frac{\alpha_2 \Gamma(m_2) 2^k}{\beta_2^k (m_2-k)!} \left(-\frac{1}{f_2}\right)^{m_2-k} R_{m_2-k}\left(\frac{1}{f_2}, \gamma_2 - \frac{1}{f_2}, \beta_2\right)$$
$$+ \frac{\log_2 e}{2} \sum_{k=1}^{m_1} \frac{\alpha_1 \Gamma(m_1) 2^k}{\beta_1^k (m_1-k)!} \sum_{j=1}^{m_1-k} \binom{m_1-k}{j} \left(-\frac{1}{f_1}\right)^{m_1-k-j} I_{m_1-k,j}\left(\frac{1}{f_1}, 1, \gamma_1 - \frac{1}{f_1}, \beta_1\right)$$
$$+ \frac{\log_2 e}{2} \sum_{k=1}^{m_2} \frac{\alpha_2 \Gamma(m_2) 2^k}{\beta_2^k (m_2-k)!} \sum_{j=1}^{m_2-k} \binom{m_2-k}{j} \left(-\frac{1}{f_2}\right)^{m_2-k-j} I_{m_2-k,j}\left(\frac{1}{f_2}, 1, \gamma_2 - \frac{1}{f_2}, \beta_2\right)$$
$$- \frac{1}{2} \log_2 \left(1 + f_1 \theta_1 + f_2 \theta_2\right). \tag{23}$$

the effect of m_i on the capacity becomes less pronounced as m_i gets larger (i.e. the relatively difference between the capacity curves gets smaller).
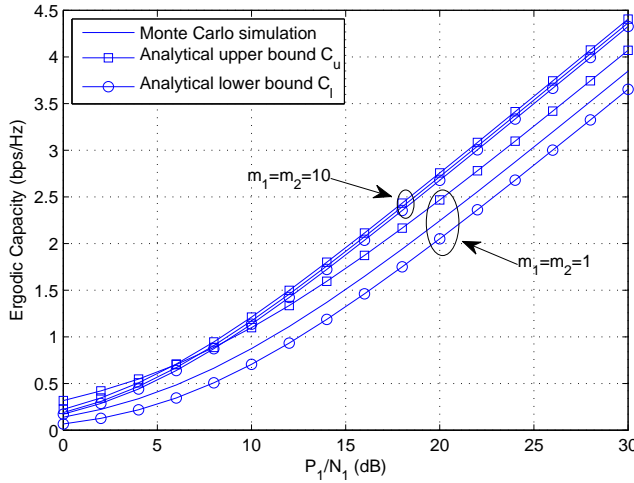

Fig. 5. Ergodic capacity of variable-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results vs. Analytical upper and lower bounds, $\frac{P_1}{N_1} = \frac{P_2}{N_2}$.

In addition, Fig. 6 illustrates how the large scale fading affects the performance of the capacity bounds when $m_i = 10$, $i = 1, 2$. For all cases under consideration, it can be readily observed that the capacity bounds perform much better for the less severe fading scenarios.

Finally, Fig. 7 investigates the impact of asymmetric fading channels on the system capacity. The curves indicate that the capacity bounds become tighter when the fading level of either hops improves.

VI. CONCLUSION

The capacity characterization of AF dual-hop relaying systems has not so far properly addressed the effects of shadowing. On this basis, we have considered the generic $\mathcal{G}$ distribution which is the combination of small-scale Nakagami-m fading and large-scale IG fading. We note that the $\mathcal{G}$

Fig. 6. Ergodic capacity of variable-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results vs. Analytical upper and lower bounds, $\frac{P_1}{N_1} = \frac{P_2}{N_2}$.
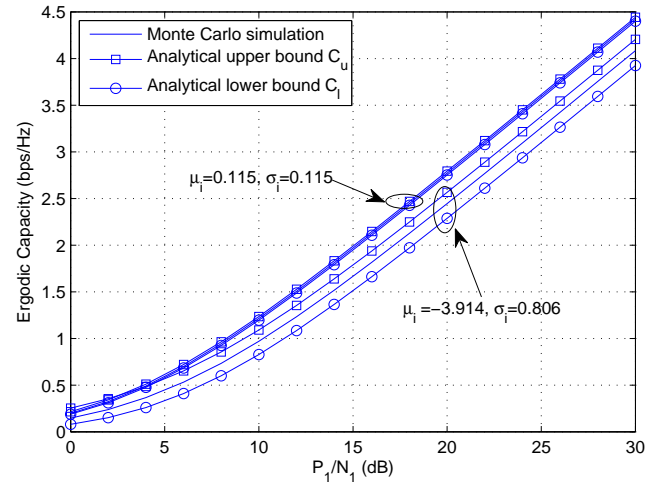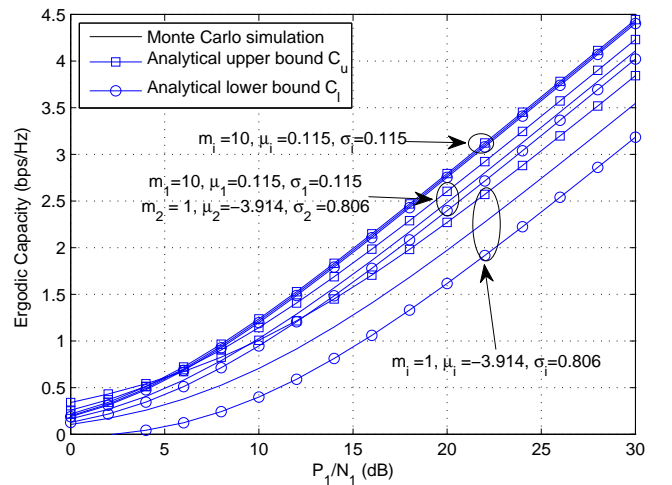

Fig. 7. Ergodic capacity of variable-gain AF dual-hop systems in composite $\mathcal{G}$ fading channels: Simulation results vs. Analytical upper and lower bounds, $\frac{P_1}{N_1} = \frac{P_2}{N_2}$.

fading model can more effectively approximate the Nakagami-m/lognormal model than the widely used Nakagami-m/gamma model. On the other hand, the mathematical analysis becomes more challenging.

In this paper, we investigated the ergodic capacity of AF dual-hop relaying systems in $\mathcal{G}$ fading channels. More specifically, some new analytical upper and lower bounds were derived for the ergodic capacity of both fixed-gain and variable-gain AF dual-hop relaying systems. It was demonstrated that the performance of the capacity upper and lower bounds remain good across the entire SNR range and under different fading conditions; therefore they can be used to efficiently assess the ergodic capacity performance of AF dual-hop systems over composite channels. Moreover, our numerical results suggested that the ergodic capacity of the system is degraded when the small-scale fading becomes more severe or when the shadowing becomes more frequent and heavy.

APPENDIX I
MAIN PROOFS

A. Proof of Lemma 2

Due to the independence of u_i and v_i, the first negative moment of ρ_i simplifies to

$$E\{\rho_i^{-1}\} = \frac{N_i}{P_i}E\{u_i^{-1}\}E\{v_i^{-1}\}.$$

The first negative moment of u_i can be computed as

$$E\{u_i^{-1}\} = \frac{m_i^{m_i}}{\Gamma(m_i)}\int_0^\infty x^{m_i-2}e^{-m_i x}dx = \frac{m_i}{m_i-1}$$

where we have used [25, Eq. (3.381.5)] to solve the corresponding integral. It is noteworthy that $E\{u_i^{-1}\}$ requires $m_i > 1$ to exist.[1] Now, the first negative moment of v_i can be computed as

$$E\{v_i^{-1}\} = \sqrt{\frac{\lambda_i}{2\pi}}\int_0^\infty x^{-\frac{5}{2}}\exp\left(-\frac{\lambda_i(x-\theta_i)^2}{2\theta_i^2 x}\right)dx$$

$$= \sqrt{\frac{\lambda_i}{2\pi}}\exp\left(\frac{\lambda_i}{\theta_i}\right)\int_0^\infty x^{-\frac{5}{2}}\exp\left(-\frac{\lambda_i x}{2\theta_i^2}-\frac{\lambda_i}{2x}\right)dx \quad (25)$$

$$= \sqrt{\frac{\lambda_i}{2\pi}}\exp\left(\frac{\lambda_i}{\theta_i}\right)2\theta_i^{-\frac{3}{2}}K_{\frac{3}{2}}\left(\frac{\lambda_i}{\theta_i}\right) \quad (26)$$

$$= \frac{1}{\lambda_i}+\frac{1}{\theta_i} \quad (27)$$

where from (25) to (26) we have used [25, Eq. (3.471.12)] while from (26) to (27) a property of Bessel functions of order equal to an integer plus one-half [25, Eq. (8.468)].

B. Proof of Lemma 3

The expectation of the logarithm of ρ_i can be trivially expressed as

$$E\{\ln\rho_i\} = \ln\frac{P_i}{N_i} + E\{\ln u_i\} + E\{\ln v_i\}.$$

[1]For a detailed discussion of this condition, the interested readers are referred to [21].

With the help of the integral identity [25, Eq. (4.352.1)], the expectation of $\ln u_i$ becomes

$$E\{\ln u_i\} = \psi(m_i) - \ln m_i.$$

To compute the expectation of $\ln v_i$, we first work out the general moment of v_i as

$$E\{v_i^n\} = \sqrt{\frac{2\lambda_i}{\pi}}\exp\left(\frac{\lambda_i}{\theta_i}\right)\theta_i^{n-\frac{1}{2}}K_{n-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right).$$

To this end, the expectation of $\ln v_i$ can be derived as

$$E\{\ln v_i\} = \frac{dE\{v_i^n\}}{dn}\bigg|_{n=0}$$

where we have used the following derivative property

$$\frac{dx^n}{dn} = x^n\ln x.$$

Hence, we get

$$E\{\ln v_i\} = \sqrt{\frac{2\lambda_i}{\pi\theta_i}}\ln\theta_i\exp\left(\frac{\lambda_i}{\theta_i}\right)K_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)$$
$$+ \sqrt{\frac{2\lambda_i}{\pi\theta_i}}\exp\left(\frac{\lambda_i}{\theta_i}\right)K_{n-\frac{1}{2}}^{\{1,0\}}\left(\frac{\lambda_i}{\theta_i}\right)\bigg|_{n=0}$$

where $K_\nu^{\{1,0\}}(x)$ denotes the derivative of the Bessel-K function with respect to the order ν. Using [28], $K_{n-\frac{1}{2}}^{\{1,0\}}\left(\frac{\lambda_i}{\theta_i}\right)\big|_{n=0}$ can be explicitly expressed as

$$K_{n-\frac{1}{2}}^{\{1,0\}}\left(\frac{\lambda_i}{\theta_i}\right)\bigg|_{n=0} =$$
$$\frac{\pi}{2}\left(I_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+I_{-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)\right)\left(\mathsf{Chi}\left(\frac{2\lambda_i}{\theta_i}\right)-\mathsf{Shi}\left(\frac{2\lambda_i}{\theta_i}\right)\right)-$$
$$\frac{\theta_i}{\lambda_i}K_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+\sqrt{\frac{\pi\theta_i}{\lambda_i}}\left(I_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+I_{-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)\right)K_{\frac{1}{2}}\left(\frac{2\lambda_i}{\theta_i}\right)$$

where $I_\nu(\cdot)$ denoting the ν-th order modified Bessel function of the first kind [25, Eq. (8.406.1)], while $\mathsf{Chi}(x)$ and $\mathsf{Shi}(x)$ are the hyperbolic cosine integral and hyperbolic sine integral, respectively. After some algebraic manipulations, we arrive at

$$E\{\ln v_i\} =$$
$$\left(\ln\theta_i-\frac{\theta_i}{\lambda_i}\right)+\sqrt{\frac{\pi\lambda_i}{2\theta_i}}\exp\left(\frac{\lambda_i}{\theta_i}\right)\left(I_{\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)+I_{-\frac{1}{2}}\left(\frac{\lambda_i}{\theta_i}\right)\right)$$
$$\times\left(\mathsf{Chi}\left(\frac{2\lambda_i}{\theta_i}\right)-\mathsf{Shi}\left(\frac{2\lambda_i}{\theta_i}\right)+\frac{\theta_i}{\lambda_i}\exp\left(-\frac{2\lambda_i}{\theta_i}\right)\right). \quad (28)$$

We now invoke the following properties of Bessel functions [25, Eq. (8.467)]

$$I_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}}\sinh(z)$$

$$I_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}}\cosh(z)$$

and combine them with the relationship $\sinh(z)+\cosh(z) = \exp(z)$ to reformulate (28) as

$$E\{\ln v_i\} = \ln\theta_i + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\left(\mathsf{Chi}\left(\frac{2\lambda_i}{\theta_i}\right)-\mathsf{Shi}\left(\frac{2\lambda_i}{\theta_i}\right)\right).$$

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We now recall the integral definitions of $\mathsf{Chi}(x)$ and $\mathsf{Shi}(x)$ [25, Eq. (8.221)]

$$\mathsf{Chi}(x) = \gamma_0 + \ln x + \int_0^x \frac{\cosh(t) - 1}{t} dt \qquad (29)$$

$$\mathsf{Shi}(x) = \int_0^x \frac{\sinh(t)}{t} dt. \qquad (30)$$

We can now combine (29)–(30) with the trigonometric relationship $\cosh(z) - \sinh(z) = \exp(-z)$, to get

$$\mathsf{E}\{\ln v_i\}$$

$$= \ln \theta_i + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\left(\gamma_0 + \ln\left(\frac{2\lambda_i}{\theta_i}\right) + \int_0^{\frac{2\lambda_i}{\theta_i}} \frac{e^{-t} - 1}{t} dt\right)$$

$$= \ln \theta_i + \exp\left(\frac{2\lambda_i}{\theta_i}\right)\mathsf{Ei}\left(-\frac{2\lambda_i}{\theta_i}\right).$$

Note that for the evaluation of the integral we have used the integral identity [25, Eq. (8.212.1)]. This concludes the proof.

C. Proof of Lemma 4

With the p.d.f. given in (2), the considered first moment can be re-expressed as

$$\mathsf{E}\left\{\frac{\rho_i}{a + \rho_i}\right\} = \mathsf{E}\left\{\frac{|h_i|^2}{\frac{aN_i}{P_i} + |h_i|^2}\right\}$$

$$= \alpha_i \int_0^\infty \frac{x^{m_i}}{(x + b)(\sqrt{x + \gamma_i})^{m_i + \frac{1}{2}}} K_{m_i + \frac{1}{2}}(\beta_i \sqrt{x + \gamma_i}) dx$$

where $b \triangleq \frac{aN_i}{P_i}$. We now make a change of variables, $t = x + b$, and apply the binomial theorem, to get

$$\mathsf{E}\left\{\frac{\rho_i}{a + \rho_i}\right\} = \alpha_i \sum_{k=0}^{m_i} \binom{m_i}{k}(-b)^{m_i - k}$$

$$\times \int_b^\infty \frac{t^{k-1}}{(\sqrt{t - b + \gamma_i})^{m_i + \frac{1}{2}}} K_{m_i + \frac{1}{2}}(\beta_i \sqrt{t - b + \gamma_i}) dt. \qquad (31)$$

For the first term $k = 0$, we get

$$\mathsf{term}_1 = \alpha_i(-b)^{m_i} \int_1^\infty \frac{K_{m_i + \frac{1}{2}}(\beta_i \sqrt{bx - b + \gamma_i})}{x(\sqrt{bx - b + \gamma_i})^{m_i + \frac{1}{2}}} dx$$

$$= \alpha_i(-b)^{m_i} R_{m_i}(b, \gamma_i - b, \beta_i). \qquad (32)$$

Note that an explicit expression for $R_n(u, v, w)$ is provided in [24, Appendix C]. With the help of [24, Appendix A], the rest of the terms $k \geq 1$ can be computed as

$$\mathsf{term}_2 = \alpha_i \sum_{k=1}^{m_i} \binom{m_i}{k}(-b)^{m_i - k} I_{m_i, k}(b, 1, \gamma_i - b, \beta_i). \qquad (33)$$

Substituting (32) and (33) into (31) yields the desired result after appropriate factorization.

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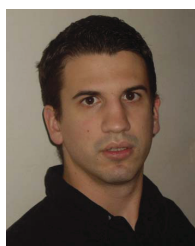
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