THE NONLINEAR EVOLUTION OF A SMALL SPHERICAL MICROWAVE BREAKDOWN PLASMA IN AIR

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ABSTRACT

High power microwave systems filled with gas run the risk of corona breakdown. The exponential growth of free electrons in the gas might cause a number of problems, ranging from noise and detuning to the generation of intense heat and severe damage to the system. Normally, when considering the risk for corona breakdown, it is not relevant to investigate the post-breakdown evolution of the plasma. However, when the electric field is inhomogeneous, the initial breakdown region might be very small, and not necessarily harmful in itself. In addition, small areas of field enhancement will suffer breakdown at much lower power levels than what is required to cause breakdown in the whole system. It is therefore of interest to investigate the possible mechanisms for such a small breakdown plasma to expand and lead to a full scale breakdown.

The electron avalanche is generally stopped by the suppression of the field in the plasma, and for a short time a quasi-steady state will persist. On longer time scales, the motion of the electrons will heat the gas, and lower the breakdown threshold. The critical question becomes that of determining the thermal balance, and the connection between electron density and temperature. A general description of the interplay between a breakdown plasma and an incoming EM wave is theoretically intractable, and to gain fundamental knowledge of the different processes, a simplified scenario was conceived and analyzed. We consider the evolution of a small spherical breakdown plasma in a homogeneous microwave electric field in atmospheric pressure air. It is shown that the situation is thermodynamically unstable, and the plasma sphere will either shrink and disappear, or expand indefinitely, depending on the initial radius of the plasma. This allows us to determine a critical size in realistic microwave systems, under which regions of intense field or heating should be kept to avoid full scale breakdown.

INTRODUCTION

When one normally estimates the risk for microwave breakdown in air filled rf equipment, one tries to determine the breakdown threshold in terms of the electric field strength, voltage or power level. The assumption is that breakdown anywhere in the device will lead to large disruptions on the operation of the device, or even complete system failure. At the same time, it is well known that small microwave discharge regions can exist in areas of field intensification without influencing the overall performance of the device. Such regions will on the other hand generate heat, and if the breakdown volume is large enough, this heat will be enough to melt parts of the device. This is typically seen experimentally, where small protrusions in the conducting surfaces will be molten after intense operation. Breakdown will be localized around such small protrusions since there will be an intensification in the electric field strength, and the local field might be above the breakdown threshold at the same time as the ambient field will be well below the threshold, [1] and [2]. If such field intensifications are taking into account when trying to estimate the breakdown threshold one might get a far too restrictive value, it is therefore necessary to examine what the risks are with small local volumes of breakdown discharges. This was done and published in [3], and we shall only present the most important features and the results below.

It is well known that in the high pressure region of the Paschen curve, the homogeneous breakdown field can be approximated with,

$$\frac{E_{bd}}{p} \approx 30 \quad (V/cm/Torr) \tag{1}$$

where p is the effective pressure, meaning that it has a dependence on temperature as $p \propto 1/T$, [4], [5] and [6]. This leads to the following expression for the breakdown field

$$E_{bd}(T) \approx E_{bd,0} \frac{T_0}{T} \tag{2}$$

Therefore, in a homogeneous field with amplitude E_0, we can induce local breakdown at a point by heating the gas to the temperature T_1, given by

$$T_1 \approx T_0 \frac{E_{bd,0}}{E_0} \tag{3}$$

It is also clear that such a discharge region will absorb energy from the field and heat the surrounding gas. If this heating is strong enough, the breakdown region should be able to expand by the continuous heating of the gas on the edge of the discharge region.

We wish to investigate this mechanism of expansion, and what it means for rf systems in general. Of course, the complete treatment of this problem is more or less impossible. The fundamental reason for this is that rf discharges span such a wide range of combinations of pressure, field strength, frequency and geometry. We will have to restrict ourselves to the range which is of highest interest, namely atmospheric pressure Air breakdown conditions. This restriction enables us to neglect diffusion when solving the electron continuity equation, which results in Eq. (1). This simplification is justifiable as long as the dimension of the discharge, or breakdown plasma, L_p, is much larger than the attachment length $L_a \equiv \sqrt{D/\nu_a} \propto 1/p$, where D is the diffusion coefficient, and ν_a is the attachment frequency. Furthermore, we restrict the analysis to a plasma volume which is much smaller than the wavelength of the field, i.e. $L_p << \lambda$. This enables us to use the quasi-static approach for the interplay between the breakdown region and the electric field. In addition to this, we assume that any gradients in the field, which are not caused by the presence of the discharge, are very weak, allowing us to treat the field as homogeneous. We also assume that the initial breakdown is spherical, and has been created by some unspecified instantaneous heating of this spherical region. The general temperature distribution will be

$$\begin{cases} T(r < R) \geq T_1 \\ T(R) = T_1 \\ T(r > R) < T_1 \end{cases} \tag{4}$$

We can instantly determine the internal field in the breakdown plasma. Analogous to a dielectric sphere, the internal field will be given by

$$E_i = \left| \frac{3E_0}{\varepsilon + 2} \right| = \frac{3E_0}{\sqrt{(3-n)^2 + \delta^2 n^2}} \tag{5}$$

where $n = Ne^2/(\varepsilon_0 m(\omega^2 + \nu^2))$, N is the electron density, e is the electron charge, m is the electron mass, ε_0 is the vacuum permittivity, ω is the field frequency, ν is the electron-neutral collision frequency, [6] and [7].

The electron density will grow until the internal field is suppressed to the breakdown threshold. This is a very fast process, and the temperature of the discharge will not change significantly under this process. We therefore have the equality

$$E_i = E_{bd}(T) \tag{6}$$

Here we will make the assumption, or approximation, that this rapid saturation of the electron density will lead to a quasi-static state, where we have a spherical discharge region, and the temperature on the edge is T_1. It is unclear how exact this approximation will be, since the polarization of the discharge volume will lead to field intensification on the poles of the sphere (with respect to electric field direction). This polarization field will have a phase difference with respect to the motion of electrons and the external field, and it is unclear how much it will contribute to an increase in the electron energy in the vicinity of the poles. Assuming that this approximation is appropriate we can regard the interplay between electron density and external field as solved, and the remaining problem is to determine the thermodynamical stability properties of the breakdown plasma.

THE THERMODYNAMICAL INSTABILITY

The discharge region will absorb energy from the electric field and undergo heating, and also heat the surrounding gas. If the heating inside the discharge is larger than the amount of heat that can be transported over the edge the breakdown gas will start to heat up, and the temperature on the edge of the discharge will heat up. Consequently, the breakdown sphere will expand. To solve the problem of thermal balance we employ the heat conduction equation

$$\rho c_v \frac{\partial T}{\partial t} = \nabla(\kappa \nabla T) + q \tag{7}$$

where ρ is the gas density, c_v the heat capacity of air, κ the heat conductivity and q the heating caused by the motion of the free electrons, i.e. Joule heating. In the above equation we have neglected convection due to the smallness of the discharge region, and radiation due to the relatively low temperatures we are interested in.

Since we know the internal field in the sphere, we can solve for the electron motion, and the heat generated by the motion. We find an approximation for the heating term, viz.

$$q(T) \approx q_E \frac{T_1}{T} \sqrt{1 - \left(\frac{T_1}{T}\right)^2} \tag{8}$$

where $q_E \equiv 3\varepsilon_0 \omega E_0^2$. The heat conductivity also has a dependence on temperature as

$$\kappa(T) \approx \kappa_0 \left(\frac{T}{T_0}\right)^{3/4} \tag{9}$$

where T_0 is the temperature in the far distance $r >> R$, [8].

Since the temperature is highest in the center of the sphere, and heating goes down with temperature, we can get an upper limit on the heating in the sphere by using the edge temperature, T_1, throughout the volume. If we integrate Eq. (7) over the volume $4\pi R^3/3$, and use the expressions for Joule heating and heat conduction, we get

$$\frac{\partial W_{Total}}{\partial T} = -Q_{Loss} + Q_{Joule} \tag{10}$$

where W_{Total} is the total heat energy content in the sphere. The heat loss over the edge is given by

$$Q_{Loss} = \frac{16\pi}{7}\left[T_0\left(\frac{T_0}{T_1}\right)^{3/4} - T_1\right] R \propto R \tag{11}$$

The maximum heating in the volume of the sphere is

$$Q_{Joule} = \frac{4\pi}{6} R^3 q_E \propto R^3 \tag{12}$$

As pointed out above, the heating scales as the radius cubed, and the heat loss is proportional to the radius. We illustrate this fact in Fig. 1.

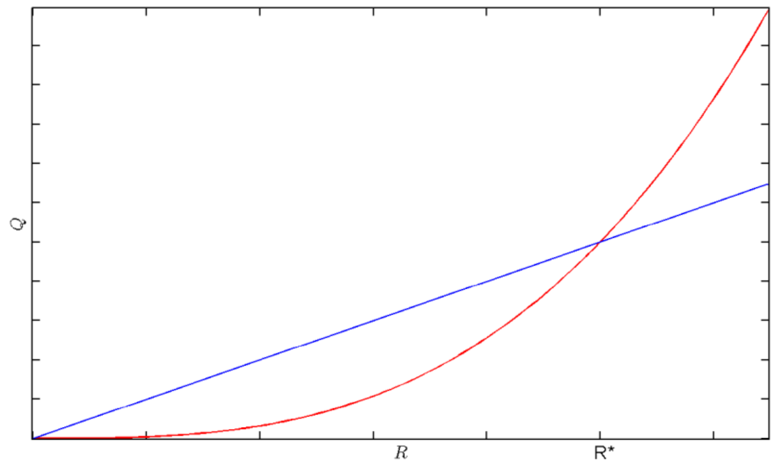

Fig 1. The radial dependence of heating and heat loss for the spherical discharge volume. Red curve is heating and blue curve is heat loss.

There is obviously a point of equilibrium radius, $R*$, where the heat loss balances the heat generation. If the radius increases slightly above this value, the heating will be stronger than the heat loss, and the discharge region will start to expand. In the same way, if the radius decreases slightly from this value, the sphere will start to shrink. In this way, we can make a rough estimate of how large regions of heating or field intensification above the breakdown threshold can be without leading to an expanding breakdown region.

The dimension of the critical radius can be calculated using this formula, see [3],

$$R^* \approx \sqrt{\frac{8}{7}\frac{\kappa_0 T_0}{\varepsilon_0}\frac{(\eta^{7/4}-1)\eta^2}{\omega E_{bd,0}{}^2}} \tag{13}$$

where $\eta = E_{bd,0} / E_0$. This radius can be evaluated numerically for any combination of parameters using empirical approximations for the heat conduction etc., [3]. If we assume room temperature far away, and a ratio between the breakdown field and the ambient field $\eta = 4.5$, we get $R^*\sqrt{\omega} \approx 7$. If we further assume an operating frequency $\omega = 1$ GHz we get $R^* \approx 0.02$ cm. This means that a plasma sphere with a radius smaller than roughly 0.02 cm will shrink and vanish, whereas a larger sphere will expand indefinitely (within this model). Keeping η constant one can find a formula for the dependence of the radius on pressure and frequency.

$$R^*\sqrt{\omega} = \frac{p_0}{p} R_0{}^*\sqrt{\omega_0} \tag{14}$$

This means that the previous result can be rescaled quite easily, and for example, if one desires a critical radius of 1 cm, and to keep the frequency at 1 GHz, one has to decrease the pressure to $0.01 p_0$. However, it is very important to keep in mind the assumptions of the model, so as not to end up with a discharge region larger than the wavelength, or smaller than the attachment length, for in that case, the model does not apply. But, the model does say something about the case when we are at high frequencies and high pressures and the critical radius is smaller than the attachment length. In this case the model does not incorporate the mechanisms that are important when the discharge sphere is smaller than the attachment length, but it does say that any sphere which can be considered as much larger than the attachment length will generate more heat than it can transport over the edge, and consequently expand.

CONCLUSIONS

We have made a qualitative analysis of the mechanisms which determine if a small microwave discharge region will expand and cause a full scale breakdown. We were able to determine a critical radius, above which any breakdown region of spheroid shape will generate more heat than what can be transported over the discharge edge. This will heat the surrounding gas and consequently lower the surrounding breakdown threshold, allowing the initially small breakdown region to expand. On the other hand, any discharge spheroid smaller than this critical radius will cool down and disappear. This model should be able to give order of magnitude estimates for the maximum size of regions of heating or intensified field in rf equipment, so that they will be safe from this type of thermally initiated breakdown.

This model highlights the most important physical mechanisms for the thermal evolution of a discharge volume under the conditions of high pressure and quasi-static electric field. But this clarity in exposition comes at the prize of severe limitations. New and interesting effects would most likely be found by the inclusion of diffusion or external heat sources into the model. This might allow the discharge to become stabilized at a radius below the critical one.

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