MICROWAVE CORONA BREAKDOWN AROUND HELIX ANTENNAS IN AIR

J. Rasch[(1)], V. E. Semenov[(2)] , D. Anderson[(1)], M. Lisak[(1)] and J. Puech[(3)]

[(1)]Chalmers University of Technology,
Department of Earth and Space Sciences
SE-412 96, Gothenburg, Sweden
Email: joel.rasch@chalmers.se

[(2)]Institute of Applied Physics,
Nizhny Novgorod, Russia

[(3)]Centre National d'Études Spatiales
Toulouse, France

ABSTRACT

Microwave corona breakdown in gases still presents a severe problem in many applications and there is a great need for reliable predictions of the breakdown threshold in different new device geometries. One important application area where comparatively few investigations have been made is open antenna structures, e.g. quadri-filar helix antennas (QFHA), which consists of four circular conductors that are bent into a helix shape. The engineering method for predicting breakdown levels in this configuration, as recommended by the European Cooperation for Space Standardization (ECSS), is based on the classical picture of a homogeneous field between two parallel plates, where the electric field is taken as the voltage between the helix wires divided by the distance between them. Clearly the accuracy of such an estimate is uncertain and may lead to non-optimized designs and large safety margins or expensive testing. The purpose of the present investigation is to analyze the breakdown strength of a helix antenna in more detail and to compare the result with the prediction of the "homogeneous" approximation.

Typical helix antennas are characterized by small helical pitch angles and long lengths (in terms of wave lengths). This implies that a simplified canonical structure in the form of a two-wire transmission line consisting of two parallel straight infinite cylinders should be a good approximation of the structure. An analysis using numerical methods is made of the problem of microwave breakdown around a helix antenna in air, simplified to a model geometry consisting of two parallel cylindrical wires. The breakdown threshold is determined by a pressure dependent interplay between three different physical processes - ionization, diffusion, and attachment taking place in the inhomogeneous electric field around the wires. We use three techniques to evaluate the breakdown threshold - a full numerical solution of the 2-D problem, a numerical solution of a parallel plates model with an inhomogeneous field profile, and the classical analytical solution for parallel plates with homogeneous electrical field. A comparison between these three models show that the parallel plates approximation gives poor results for small cylinder radii, whereas the 1-D parallel plates model gives a good agreement over the entire relevant pressure interval, implying that it can be used for quick engineering calculations.

INTRODUCTION

A necessary consideration for high power transmission systems that operate in gases, the most common being air, is to determine the power level at which the gas will undergo breakdown and become electrically conducting. This is necessary because the breakdown plasma will disturb the transmission, and may under certain circumstances damage or destroy the equipment [1], [2] and [3]. A typical situation where noise and power reflection is the main issue is during the re-entry into the atmosphere of e.g. a spacecraft or a missile [4], [5], [6] and [7]. During the descent the vehicle will move from near vacuum to atmospheric pressure, consequently moving through the Paschen minimum, where the energy exchange between free electrons and neutral atoms and molecules is the most effective. This typically occurs around the point where the electron-neutral collision frequency equals the field frequency. At this point, breakdown is most likely to occur, since the necessary electrical field strength will be at its lowest - the breakdown voltage threshold is low.

There is a breakdown threshold associated with every pressure region, and to find it one needs to calculate the balance between the rate of creation and the rate of losses of free electrons. A slightly higher rate of electron creation than loss will immediately result in the exponential growth of electron number. This increase in the amount of electrons will continue until stopped by some other mechanism, typically the reflection of the electromagnetic field. Although the full theoretical treatment of this type of gas kinematic processes would have to involve kinetic theory, it is quite well known that one can use more simplified methods, but still get result that will agree well with experiments. Normally, when calculating the breakdown threshold in a system, one needs to consider only one equation, the electron continuity equation, albeit a complicated equation that will not produce analytical solutions.

The continuity equation will be hard to solve when the electrical field strength is not homogeneous in the system, and when the boundary conditions are complicated. Basically, the only system which is easy to solve is the parallel plates setup. This is the classical situation where there are two infinite metal plates that face each other. The electric field will be homogeneous and the boundary condition is zero electron density on the plate surfaces. Because of the simplicity, this is the case which have been most studied experimentally, but it has limited applicability to realistic systems. In any other setup one need to use other methods. Either one simplifies the source and loss terms, and chooses a geometry with certain symmetries, to be able to use some standard set of non-elementary functions, or one uses some approximate method, the most common being variational methods and numerical computer solutions.

In this paper we present a first study of the air breakdown threshold in a helical quadri-filar antenna [8]. The antenna produces a circulary polarized EM-wave by alternating the direction of the electric field successively between pairs of conductors, meaning that at the moments and positions of electric field maximum, the field will be localized between two cylinders. This fact along with the small pitch angle of the wires motivates the simplification of only considering a reduced problem, where the breakdown takes place between two infinite parallel cylinders. The big benefit of this simplification is that the electromagnetic field is known analytically, and the only analysis necessary to do is to solve the electron continuity equation. In addition to this simplification, we will only consider the case of cylinders of equal radii, not because the full analysis would be significantly harder, but to not confuse the reader, and because the equal radii case is the most realistic.

THE ELECTRIC FIELD

The geometry of the two-cylinder system is shown in Fig. (1). Neglecting any self-induction it is possible to find the electromagnetic field using logarithmic potential theory [9].

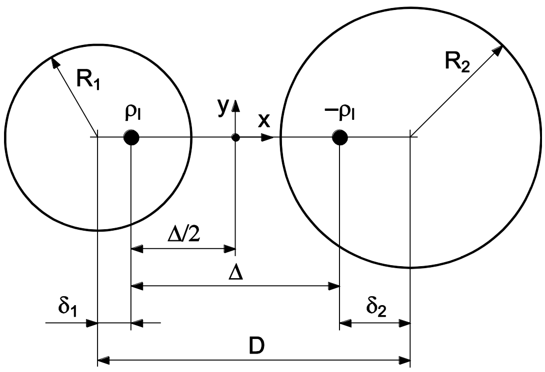

Fig. 1: The geometrical setup of the two cylinder system.

The electric field can be expressed as

$$\overline{E} = \frac{E_0}{4}\frac{(v^2 - u^2 + 1/4)\hat{x} - 2uv\hat{y}}{(v^2 + u^2 - 1/4)^2 + v^2} \tag{1}$$

where $u = x/\Delta$, $v = y/\Delta$, E_0 is the minimum electric field between the cylinders, and when the radii are equal, ($R_1 = R_2 = R$) we have

$$\Delta = \sqrt{D^2 - 4R^2} \tag{2}$$

The maximum field is located on the point on the two surfaces where they are closest to each other, and can be expressed as

$$E_{\max} = \frac{V}{R}\sqrt{\frac{D+2R}{D-2R}}\frac{1}{\cosh^{-1}(\frac{D^2}{2R^2} - 1)} \tag{3}$$

where V is the voltage between the cylinders, and $\cosh^{-1}$ is the inverse of $\cosh$. Another useful formula connects the maximum field with the minimum field, viz.

$$E_{\max} = \frac{E_0}{2}(1+\frac{D}{2R}) \tag{4}$$

CALCULATING THE BREAKDOWN THRESHOLD

To calculate the breakdown threshold one needs to consider the balance of production and loss of electrons. The number density of free electrons in the beginning stage of the breakdown can be described with the electron continuity equation containing only attachment and diffusion as effective loss mechanisms for electrons, viz

$$\frac{\partial n}{\partial t} = \nabla^2(Dn) + n(\nu_i - \nu_a) \tag{5}$$

Here n is the density of electrons, D_e is the free electron diffusion coefficient, ν_i the ionization frequency and ν_a the attachment frequency. The breakdown threshold for CW operation is defined as the point where $\frac{\partial n}{\partial t} = 0$

The ionization and attachment frequencies, as well as the diffusion coefficient all depend on pressure, and they all depend on the electric field strength. But the dependence of the ionization frequency on the electric field strength is so strong compared to the others, that we can regard the attachment and diffusion parameters as constants for a given pressure. A typical approximation for the field dependence of the ionization frequency is

$$\nu_i(E) \approx \nu_{i,\max}\left(\frac{E}{E_{\max}}\right)^{\beta} \tag{6}$$

The exact value of the exponential β is not well defined. Different authors have used different values, typically around 5.3, e.g. [3], [10] and [11]. In our previous publications we have used 5.33, e.g. [12] and [13], and admittedly, such an exact value is impossible to extract from the measurements made and published during the years. However, the aim is to find an order of magnitude estimate, which this expression has been found to accomplish.

Taking in mind that we use constant expressions for the attachment and diffusion rates, and the fact that the geometry in the two-cylinder problem has an infinite extension in the z-direction, Eq. (6) becomes

$$\nabla_{\perp}^2 n + n(\lambda s(x,y) - q) = 0 \tag{7}$$

where $\lambda \equiv \nu_{i,\max}/D_e$, $q \equiv \nu_a / D_e$, and $s(x,y) = (E(x,y)/E_{\max})^{5.33}$. To solve the problem, one now needs to find the eigenvalue λ as a function of q and R. This eigenvalue can then be used to evaluate the breakdown electric field strength, voltage or power, as a function of pressure and system dimensions. Due to the complicated form of the electric field, an analytical solution will not be feasible, and the solution is typically shown in the form of curves. A full numerical solution is not particularly hard to achieve, and there are several different schemes possible. But the actual running of codes can be a lengthy procedure. Therefore, we are looking for an easier way of evaluating the threshold. We will calculate the breakdown threshold using the parallel plates approximation, a parallel plates approximation incorporating the inhomogeneity of the electric field, and the full geometry, and then compare the results to see if any of the simpler routines gives good predictions.

PARALLEL PLATES SOLUTION

The very simplest way of trying to approximate the threshold is by taking the electric field as constant along a line drawn between the cylinder centres. Then one approximates the surfaces of the cylinders with parallel metal plates, on which the electron density is zero. There are three simple ways of choosing the electric field strength. Either one uses the maximum electric field located on the surfaces, the minimum field located in the middle of the system, or one takes the voltage between the cylinders and divides by the gap distance. The third option is the most complicated due to the fact that the voltage is connected to the field strength through a complicated formula. We will use the two first approaches, and then compare with the full solution for this system. It will be seen that these two solutions show the wrong functional dependence on pressure as compared to the full solution. This also eliminates the third option as a good approximation for the breakdown threshold, since the relation between field strength and voltage is independent of pressure.

In a homogeneous field corresponding to the maximum field strength in the full geometry, $E_{\max}$, between two parallel plates, Eq. (8) becomes

$$\frac{\partial^2 n}{\partial x^2} + n(\lambda - q) = 0 \tag{8}$$

The boundary conditions are $n(-d/2) = n(d/2) = 0$, where $d = D - 2R$ is the gap width between the cylinder surfaces. Using the normalization, $\tilde{x} = xd/2$, we get

$$n(\tilde{x}) = n_0 \cos(\tilde{x}\frac{d}{2}\sqrt{\lambda - q}) \tag{9}$$

$$\frac{d}{2}\sqrt{\lambda - q} = \frac{\pi}{2} \quad \Leftrightarrow \quad \lambda = q + \frac{\pi^2}{d^2} \tag{10}$$

If we instead wish to find the solution where the electric field has the same strength as the minimum electric field in the full problem, we need to use Eq. (5), and we get

$$\lambda_{\min} = \lambda \left(\frac{D + 2R}{4R} \right)^{5.33} \tag{11}$$

These solutions are portrayed in Fig. (2).

SOLUTION FOR PARALLEL PLATES WITH INHOMOGENEOUS FIELD PROFILE

A more refined, but still simple model, can be constructed by taking into account the inhomogeneity of the electric field between the cylinders, but approximating their surfaces as parallel infinite plates. Using Eq. (1), the electric field strength between the cylinders will be

$$E(\tilde{x}) = \frac{E_0}{1 - \dfrac{\tilde{x}^2}{1 + 2\tilde{R}}} \tag{12}$$

where $R = \tilde{R}d/2$ and $x = \tilde{x}d/2$. The continuity equation becomes

$$\frac{\partial^2 n}{\partial \tilde{x}^2} + n(\Lambda s(\tilde{x}) - Q) = 0 \tag{13}$$

where $\Lambda = \nu_{i,\max} d^2/(4D_e)$, $Q = \nu_a d^2/(4D_e)$, and $s(\tilde{x}) = (2\tilde{R}/(1 + 2\tilde{R} - \tilde{x}^2))^{5.33}$. The easiest way to deal with this set of equation is just to perform a numerical solution. This can be done in several ways. One of the most convenient ways is to use a relaxation method, as described in the next section. However, if a relaxation method is chosen, it is necessary to use a low value for the relaxation parameter α, at least below unity, otherwise the system is quite unstable.

FULL TWO DIMENSIONAL SOLUTION

The full solution of the complete geometry and field is only achievable through direct numerical techniques. There are several different ways of doing this, but the simplest to implement are probably the basic relaxation schemes. They start with the discretization of the normalized continuity equation on a finite grid with spacing h, and using a time step τ. If we use the normalization $\tilde{x} = xD$, $\tilde{y} = yD$ we get

$$\frac{n'[i] - n[i]}{\tau} \approx \frac{n[i+1, j] + n[i-1, j] + n[i, j+1] + n[i, j-1] - 4n[i, j]}{h^2} + n[i](\tilde{\lambda} s(\tilde{x}[i], \tilde{y}[i]) - \tilde{q}) \tag{14}$$

where $\tilde{\lambda} = \nu_i(E_{\max})D^2/D_e$, $\tilde{q} = \nu_a D^2/D_e$, and $n'[i]$ is the value of $n[i]$ after the small time τ has elapsed. We can rearrange to get the new value of the density on the left hand side, and all the old quantities on the right.

$$n'[i] \approx n[i](1-\alpha) + \frac{\alpha}{4}(n[i+1,j] + n[i-1,j] + n[i,j+1] + n[i,j-1] + h^2 n[i](\tilde{\lambda} s(\tilde{x}[i],\tilde{y}[i]) - \tilde{q})) \quad (15)$$

where $\alpha = 4\tau / h^2$. By stepping through all the interior grid points and using this formula one can update the value of the density in the domain. One then monitors the total integrated density, or the maximum value, and look for growth or decay. Since we are looking for the solution to the continuity equation where the time derivative is zero, we should adjust the value of $\tilde{\lambda}$ until the solution converges to a stable density function. In this way, the eigenvalue $\tilde{\lambda}$ as a function of $\tilde{q}$ will be found simultaneously as the eigenfunction for this system is found. The value of α that should be used is in some sense undefined. By setting $\alpha = 1$ one gets the classical Jacobi scheme, which will provide the correct answer eventually, but the convergence rate is quite low [14]. By using a number larger than unity the scheme becomes overrelaxed and the convergence rate increases, but a to high value will result in numerical instability. There are formulas to estimate the "best" value for α, but as a rule of thumb one can say that the finer the grid, the closer α should be to 2.

The number of iterations that is needed to find a stable solution is dependent on the grid spacing and the initial guess for the density function. If the grid is coarse the convergence will be fast, but the truncation error will be quite large, rendering a poor approximation for the correct continuous solution. Therefore one needs to relax the solution on a fine grid. But if the initial guess for the function is far from the actual solution, the number of iterations will have to be large. This dilemma can be overcome by using grids of different sizes. There are several interesting methods for using a complicated routine to go between different grids to achieve maximum convergence rate. These methods are generally called multigrid techniques. For this problem we chose the very simplest form, which consists of relaxing the solution on a coarse grid, interpolate the obtained solution to a finer grid, and repeat the routine until the truncation error becomes sufficiently small.

One question which should be dealt with before running lengthy codes is; what parameter range is interesting? It is quite clear that the approximate formulas used to calculate things will fail to be accurate at certain pressures and dimensions. For example, we cannot expect to get a correct answer when the typical dimensions of the system are smaller than the typical dimensions of the discharge region, since the approximation for the ionization frequency is only valid over a certain range of field strength, and because the boundary conditions imposed in the numerical routine becomes more important than the antenna geometry. To get a rough estimate of when this happens we state that the characteristic dimension of the breakdown region is equal to the attachment length, $L_a \equiv \sqrt{D_e / \nu_a}$. The characteristic dimension of the antenna system is roughly D. We already defined $\tilde{q} = D^2 \nu_a / D_e$, so $\tilde{q} = D^2 / L_a^{\,2}$. We should expect the model to start to fail around the point where $L_a \sim D$, equivalent to $\tilde{q} \sim 1$. Therefore, going lower than for example $\tilde{q} = 0.1$ will be a waste of computer time. The maximum value of $\tilde{q}$ is a different matter, typically when one increases $\tilde{q}$ one will eventually reach a region where $\tilde{\lambda} \sim \tilde{q}$ and there will be no point in looking at higher values for $\tilde{q}$.

COMPARISON OF THE RESULTS

When comparing the results from the different approaches it is necessary to use the same normalization. The solution for the parallel plates case was $\lambda = q + \pi^2 / d^2$. We multiply with D^2, and define $r \equiv R/D$, which leads to $\tilde{\lambda} = \tilde{q} + \pi^2 / (1-2r)^2$. In the analysis of the 1-D model with an inhomogeneous field profile we used $\Lambda = \nu_{i,\max} d^2 / (4D_e)$ and $Q = \nu_a d^2 / (4D_e)$, which means that we can get the correct normalization to compare results by using $\tilde{\lambda} = 4\Lambda / (1-2r)^2$, and $\tilde{q} = 4Q / (1-2r)^2$.

The result of the comparisons between the predictions made by the parallel plates model, the 1-D inhomogeneous model, and the full numerical solution are shown in Fig. (2).

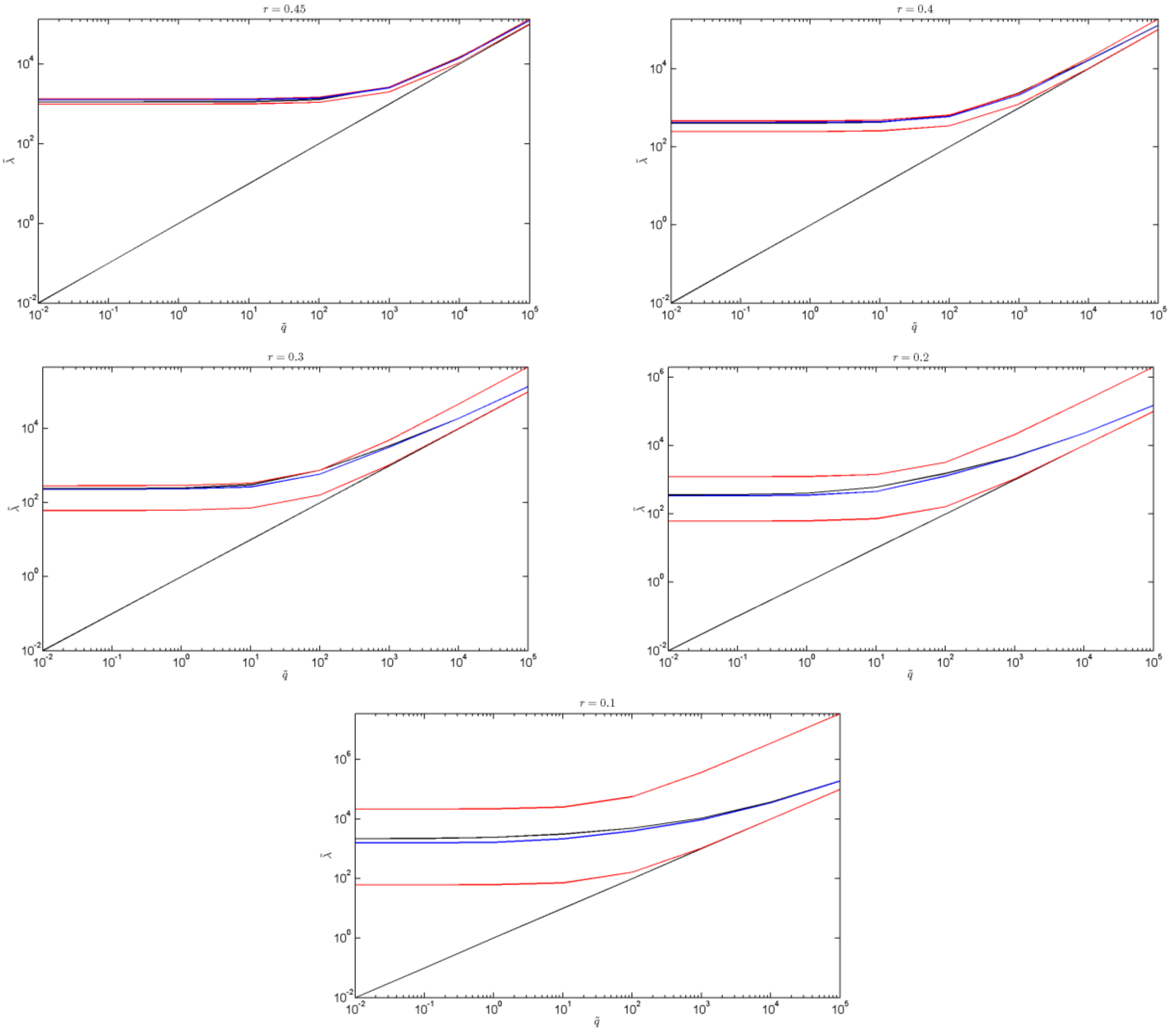

Fig. 2: Comparisons between the three different models. The red curves are the results from the parallel plates approximation, where the upper one corresponds to a homogeneous field equal in strength to the field minima in the full geometry, and the lower line corresponds to the field maxima. The blue curve is the result from the 1-D model with inhomogeneous field strength. The black straight line represents the solution in the high pressure limit, $\tilde{\lambda} = \tilde{q}$. The curved black lines are the results from the full 2-D solution. The five panels from the upper left to the lower right shows the solutions for $r = R/D = 0.45, 0.4, 0.3, 0.2$ and 0.1 respectively.

It is quite clear that for large radii, the two simple models show quite good agreement with the full solution, but as the radii decreases, the disagreement between the parallel plates model and the two other solutions become larger. The two red curves separate quite rapidly as the radii decreases, illustrating the growing inhomogeneity of the field. In the high $\tilde{q}$ limit of the graphs, all curves except the upper red one approaches the equality $\tilde{\lambda} = \tilde{q}$, which is to be expected, since high $\tilde{q}$ is equivalent to high pressure, and at high pressures the breakdown condition is simply the equality between the attachment and ionization frequencies. Another feature which is typical for these types of solutions is that in the low pressure region (left end) of the curves, they level out and reach a stationary value. This value is determined by the gap width in some manner, for the parallel plates case it is simply $\tilde{\lambda} \sim \pi^2/(1-2r)^2$, but for the other cases it has to be determined numerically, or using approximate analytical techniques. The most interesting and useful conclusion we can draw from these comparisons is that the 1-D inhomogeneous model gives a good estimate of the full solution even for low values of the radii. This implies that we need not run lengthy simulations to get a good order of magnitude estimate for the breakdown threshold, but that it is sufficient to solve the 1-dimensional problem, which is far simpler.

USING THE SOLUTIONS

So far we have concluded that it should be sufficient to use the 1-D model with an inhomogeneous field profile to get an estimate of the breakdown threshold, but it might be hard to see how the solutions above can be used to estimate the actual breakdown field strength. However, by using empirical expressions for the ionization and attachment frequencies, as well as the diffusion coefficient it is possible to form more useful expressions. It is known that good physical fits can be achieved with the formulas below

$$\nu_i \approx 4\times 10^7 p\left[\frac{E_{eff}}{100p}\right]^{5.33} \quad (s^{-1}) \tag{16}$$

$$\nu_a \approx 6.4\times 10^4 p \quad (s^{-1}) \tag{17}$$

$$D_e \approx \frac{10^6}{p} \quad (cm^2 s^{-1}) \tag{18}$$

where $E_{eff} = E/\sqrt{1+\omega^2/\nu_{coll}{}^2}$ is the effective electric field for energy transfer in Volts/cm, and p is the effective pressure in Torr. Keeping in mind the definitions for Λ and Q it is possible to form the expressions

$$\frac{E_{eff}}{p} \approx 30\left(\frac{\Lambda}{Q}\right)^{1/5.33}, \quad pd \approx 7.9\sqrt{Q} \tag{19}$$

Or equivalently

$$E_{eff} d \approx 237\sqrt{Q}\left(\frac{\Lambda}{Q}\right)^{1/5.33}, \quad pd \approx 7.9\sqrt{Q} \tag{20}$$

Once again it is necessary to know to how low values of Q it is necessary to investigate. Using the same line of reasoning as above one can show that the approximations should start to fail when

$$Q \sim (1+\tilde{R})^{-2} \tag{21}$$

To illustrate how one can analyze a system we will give a specific example. An actual helix antenna with a working frequency of 8 GHz has $R = 0.06$ cm, and $d = 0.38$ cm. This means that $\tilde{R} \approx 0.316$, and we should not investigate values much lower than $Q \sim 0.6$. The solution is displayed in Fig. (3), where we have used $\nu_{coll} \approx 5.3\times 10^9$ (s^{-1}) for the electron-neutral collision frequency in air.

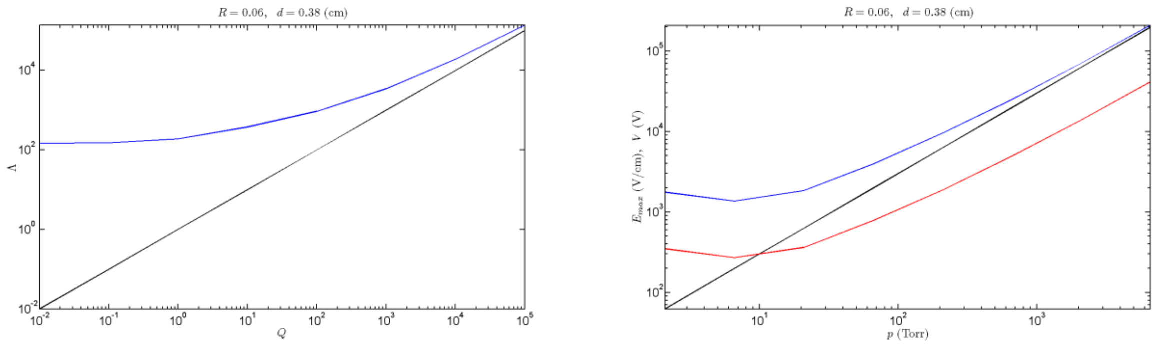

Fig. 3: The 1-D solution with inhomogeneous field profile for $R = 0.06$ cm, and $d = 0.38$ cm. The straight line is the high pressure solution, which in the left panel is the equality $\Lambda = Q$, and in the right panel $E_{\max} = 30p$ (V/cm). The blue curve in the left panel is $E_{\max}$, and the red curve is V, the voltage between the cylinders.

Fig. (3) shows how it is possible to go from the normalized system of parameters to more physical quantities. In this case we plot the maximum electric field strength, blue curve, and the voltage (using Eq. (4)), red curve, as a function of pressure. The right panel clearly shows the well known Paschen minimum, and the high pressure approximation for the breakdown field $E_{\max} / p \approx 30$ (V/cm/Torr). It should be mentioned that the curves cannot be trusted in the far left region (in this particular case, left of the Paschen minimum), since we are below the value of Q which was estimated as the limit for this model.

CONCLUSIONS

By solving the continuity equation numerically in 2-D and comparing this with more approximate methods of calculating the breakdown threshold, we have shown that it is sufficient to consider a simple 1-D model when calculating the threshold for the simplified two-wire system, chosen to qualitatively represent a helix antenna. The simple model consists of two parallel plates, where the field profile between the plates is the same as that on a line drawn between cylinder centres in the full geometry of the two-wire system. Using this model one can easily calculate the breakdown threshold using standard numerical techniques. However, when pressures are very weak, and the attachment length is larger than the distance between conductors, this model should not give accurate results, simply because it does not include the surrounding space, which will be the most important factor in determining the breakdown criterion.

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