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New Analytical and Numerical Solutions for the Short-term Analysis of Vertical Ground Heat Exchangers

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ABSTRACT

This paper presents the background, development and the validation of new analytical and numerical solutions for the modeling of short-term response of borehole heat exchangers. The new analytical solution studies the borehole's heat transfer and the related boundary conditions in the Laplace domain. A set of equations for the Laplace transforms for the boundary temperatures and heat-fluxes is obtained. These equations are represented by a thermal network. The use of the thermal network enables swift and precise evaluation of any thermal or physical setting of the borehole. Finally, very concise formulas of the inversion integrals are developed to obtain the time-dependent solutions. The new analytical solution considers the thermal capacities, the thermal resistances and the thermal properties of all the borehole elements and provides a complete solution to the radial heat transfer problem in vertical boreholes. The numerical solution uses a special coordinate transformation. The new solutions can either be used as autonomous models or easily be incorporated in any building energy simulation software.

INTRODUCTION

Long-term response of a borehole represents the development of ground temperatures over time in response to the overall ground heat injections and extractions. On the other hand, short-term response of the borehole shows the variations in circulating fluid temperatures not associated to the long-term response of the ground. The short-term response of the borehole corresponds to time periods ranging from a few minutes to a number of days. Today many commercial buildings, like super markets and shopping centers, have simultaneous heating and cooling demands. Many other commercial and office buildings have a cooling demand during the day, even in cold climates, and a heating demand during the night. For such buildings, a significant amount of thermal energy is just pumped up and down the borehole system with heat transfer mainly occurring in the borehole. Similarly, the circulating fluid temperature of a borehole system operating under peak load conditions depends largely on the internal heat transfer of the borehole. For these cases, the borehole exit fluid temperature depends on the short-term thermal response of the borehole. As operation and performance of a heat pump both depend on the fluid temperature from the borehole system, the thermal energy use and electrical demands of the heat pump and the overall ground source heat pump (GSHP) system are considerably affected by the short-term response of the borehole. Therefore, when optimizing the operation, control and performance of a GSHP system, the short term response of the borehole is quite important. The evaluation of thermal response tests (TRTs) and heat-flux build-up analysis of the borehole are also conducted using models based on the short-term response of the borehole.

Various numerical, analytical and semi-analytical solutions have been developed to model the short-term response of the borehole. Analytical solutions, because of their flexibility and superior computational time efficiencies, are of particular

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interest; however, numerical solutions are also required to obtain precise solutions and for parametric analysis. The first major contribution came from Yavuzturk (1999), who extended Eskilson's concept of non-dimensional temperature response functions (1987) to include the short-term analysis. As with Eskilson's approach, the work of Yavuzturk also requires the response to be pre-computed for individual cases. Shonder and Beck (1999) and Austin (1998) also developed solutions which numerically solve the heat transfer in the borehole. However, both these solutions are aimed at evaluation of TRTs. Young (2004) modified the classical Buried Electric Cable (BEC) solution (Carslaw and Jaeger, 1959) to develop his analytical Borehole Fluid Thermal Mass (BFTM) solution. The solution is based on an analogy between a buried electric cable and a vertical ground heat exchanger and needs to be optimized for individual cases. Lamarche and Beauchamp (2007) presented an exact solution assuming two legs of the U-tube as a hollow equivalent-diameter cylinder. The solution solves the heat transfer problem assuming a steady heat-flux condition across the hollow cylinder boundary. However, it ignores the thermal capacity of the fluid present in the U-tube. Bandyopadhyay et al. (2008) also presented an exact solution for the case of boreholes backfilled with the borehole cuttings. This solution has limited practical application as most of the boreholes are backfilled with a material quite different from the borehole cuttings. Gu and O'Neal (1995) developed an analytical short-term response solution assuming a cylindrical source in an infinite composite region. The solution solves the borehole transient heat transfer problem using the generalized orthogonal expansion technique which requires calculation of multiple eigenvalues. Beier and Smith (2003) also developed a semi-analytical solution, which first solves the borehole heat transfer problem in the Laplace domain and then uses a numerical inversion to obtain the time domain solution. Bandyopadhyay et al. (2008) have also reported a similar solution. Javed et al. (2010) studied the existing short-term solutions in detail and noted the need of an analytical solution which should consider the thermal capacities, the thermal resistances and the thermal properties of all the borehole elements and which could be easily incorporated in building energy simulation software to optimize the operation and control of GSHP systems.

NEW ANALYTICAL SOLUTION

A new analytical solution has been developed to model the short-term response of the borehole (Claesson, 2010). The new solution studies the heat transfer and the related boundary conditions in the Laplace domain. The solution assumes radial heat transfer in the borehole. To meet this requirement, the U-tube in the borehole is approximated as a single equivalent-diameter pipe. The fluid temperatures entering and leaving the U-tube are represented using a single average value. The resulting problem is shown in Figure 1. The heat flux q_{inj} is injected to the circulating fluid with temperature $T_f(\tau)$. The fluid has a thermal capacity of C_p. The pipe thermal resistance is R_p, and the pipe outer boundary temperature is $T_p(\tau)$. The heat flux $q_p(\tau)$ flows through the pipe wall to the grout. The thermal conductivity and the thermal diffusivity of the grout are λ_g and a_g, respectively. The heat flux $q_b(\tau)$ flows across the borehole boundary to the surrounding ground (soil). The borehole boundary temperature is $T_b(\tau)$. The thermal conductivity and the thermal diffusivity of the ground (soil) are λ_s and a_s, respectively.

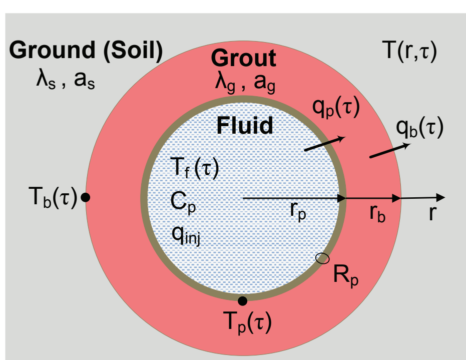


Figure 1 *Geometry, temperatures, heat fluxes and thermal properties of the borehole.*

The temperature $T(r, \tau)$ satisfies the radial heat conduction equation in the grout and the ground (soil) regions.

$$\frac{1}{a(r)} \cdot \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r}, \qquad a(r) = \begin{cases} a_g, & r_p < r < r_b \\ a_s, & r > r_b \end{cases}. \tag{1}$$

The radial heat flux in the grout and the soil regions is:

$$q(r,\tau) = 2\pi r[-\lambda(r)] \cdot \frac{\partial T}{\partial r}, \qquad \lambda(r) = \begin{cases} \lambda_g, & r_p < r < r_b \\ \lambda_s, & r > r_b \end{cases}. \tag{2}$$

The heat flux at the grout-soil interface is continuous and hence the boundary condition from Equation 2 at $r=r_b$ is:

$$\lambda_g \cdot \left.\frac{\partial T}{\partial r}\right|_{r=r_b-0} = \lambda_s \cdot \left.\frac{\partial T}{\partial r}\right|_{r=r_b+0}. \tag{3}$$

The boundary condition at the pipe-grout interface is:

$$T_f(\tau) - T(r_p,\tau) = R_p \cdot q(r_p,\tau). \tag{4}$$

The heat balance of the fluid in the pipe with the injected heat q_{inj} is:

$$q_{inj} = C_p \cdot \frac{dT_f}{d\tau} + q(r_p,\tau), \qquad \tau \geq 0. \tag{5}$$

The initial temperatures in the pipe, the grout and the ground (soil) are all taken as zero.

$$T_f(0) = 0, \qquad T(r,0) = 0, \qquad r > r_p. \tag{6}$$

The mathematical problem defined above (i.e. Equations 1 to 6) is solved using Laplace transforms of temperatures and heat fluxes at $r = r_p$, $r_p \leq r \leq r_b$, $r = r_b$ and $r > r_b$. For $r = r_p$, two relation between the Laplace transforms of temperature and heat flux are readily obtained by taking Laplace transforms of Equations 4 and 5.

$$\bar{T}_f(s) - \bar{T}_p(s) = R_p \cdot \bar{q}_p(s), \qquad r = r_p. \tag{7}$$

$$\frac{q_{inj}}{s} = C_p \cdot s \cdot \left[\bar{T}_f(s) - 0\right] + \bar{q}_p(s), \qquad r = r_p. \tag{8}$$

Here, s is the complex-valued variable in the Laplace domain. For the annular grout region defined by $r_p \leq r \leq r_b$, the Laplace transform of the radial heat equation is considered. General solutions of the resulting equation involving Bessel functions are then obtained together with the Laplace transform of the radial heat flux. From this, two equations between the Laplace transforms of the boundary temperatures and boundary fluxes are obtained. The full derivation of the formulas below is presented in Claesson (2010).

$$\bar{q}_p(s) = \bar{K}_p(s) \cdot \left(\bar{T}_p(s) - 0\right) + \bar{K}_t(s) \cdot \left(\bar{T}_p(s) - \bar{T}_b(s)\right), \tag{9}$$

$$-\bar{q}_b(s) = \bar{K}_b(s) \cdot (\bar{T}_b(s) - 0) + \bar{K}_t(s) \cdot \left(\bar{T}_b(s) - \bar{T}_p(s)\right). \tag{10}$$

Here $\bar{K}_t(s)$ may be interpreted as a transmittive conductance for the problem in Laplace domain, whereas $\bar{K}_p(s)$ and $\bar{K}_b(s)$ are absorptive conductances. The values of these conductances (and their inverse, the resistances) become:

$$\bar{K}_t(s) = \frac{1}{\bar{R}_t(s)} = \frac{2\pi\lambda_g}{K_0(\sigma_p)\cdot I_0(\sigma_b) - I_0(\sigma_p)\cdot K_0(\sigma_b)}, \tag{11}$$

$$\bar{K}_p(s) = \frac{1}{\bar{R}_p(s)} = \frac{\sigma_p[I_1(\sigma_p)\cdot K_0(\sigma_b) + K_1(\sigma_p)\cdot I_0(\sigma_b)] - 1}{\bar{R}_t(s)}, \tag{12}$$

$$\bar{K}_b(s) = \frac{1}{\bar{R}_b(s)} = \frac{\sigma_b[I_1(\sigma_b)\cdot K_0(\sigma_p) + K_1(\sigma_b)\cdot I_0(\sigma_p)] - 1}{\bar{R}_t(s)}, \tag{13}$$

$$\sigma_p = r_p\sqrt{s/a_g}, \qquad \sigma_b = r_b\sqrt{s/a_g}. \tag{14}$$

There is a corresponding relation between the Laplace transforms of the temperature and flux at $r = r_b$ to account for the region outside the borehole, which is derived from the Laplace transforms for the soil region.

$$\bar{q}_b(s) = \bar{K}_s(s)\cdot(\bar{T}_b(s) - 0), \qquad r = r_b. \tag{15}$$

Here $\bar{K}_s(s)$ is the ground thermal conductance and its value is:

$$\bar{K}_s(s) = \frac{1}{\bar{R}_s(s)} = \frac{2\pi\lambda_s\cdot\sigma_s\cdot K_1(\sigma_s)}{K_0(\sigma_s)}, \qquad \sigma_s = r_b\sqrt{s/a_s}. \tag{16}$$

The heat transfer problem, shown in Figure 1, can now be represented by means of the thermal network shown in Figure 2. This network for the equivalent-diameter pipe, the circulating fluid, the borehole annulus region and the infinite ground outside the borehole is drawn using Equations 7 and 8 for the pipe region, Equations 9 and 10 for the annular region and Equation 15 for the soil region. The network involves a sequence of composite resistances.

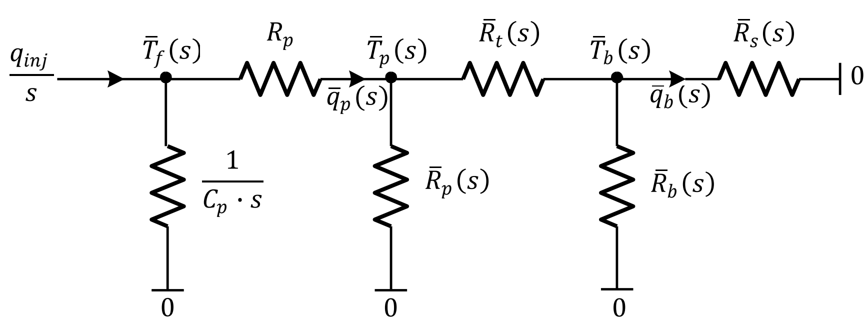


Figure 2 *The thermal network for a borehole in the Laplace domain.*

The Laplace transform for the fluid temperature is readily obtained from the thermal network. Starting from the right in Figure 2, the conductances $\bar{K}_b(s)$ and $\bar{K}_s(s)$ lie in parallel and are added. The inverse of this composite conductance is added to the resistance $\bar{R}_t(s)$. This composite resistance lies in parallel with $\bar{R}_P(s) = 1/\bar{K}_P(s)$ and their inverses are added. This composite resistance lies in series with the resistance of the pipe wall R_P. The total composite resistance from R_P and rightwards lies in parallel with the thermal conductance $C_P\cdot s$. The Laplace transform of the fluid temperature becomes:

$$\bar{T}_f(s) = \frac{q_{inj}}{s} \cdot \cfrac{1}{C_p \cdot s + \cfrac{1}{R_p + \cfrac{1}{\bar{K}_p(s) + \cfrac{1}{\bar{R}_t(s) + \cfrac{1}{\bar{K}_b(s) + \bar{K}_s(s)}}}}}. \tag{17}$$

In the type of problems considered here, the inversion formula to get $T_f(\tau)$ from $\bar{T}_f(s)$ is:

$$T_f(\tau) = \frac{2}{\pi} \cdot \int_0^\infty \frac{1 - e^{-u^2 \cdot \frac{\tau}{\tau_0}}}{u} \cdot L(u) du. \tag{18}$$

The function *L(u)* in the above equation is given by:

$$L(u) = \mathrm{Im}\left[-s \cdot \bar{T}_f(s)\right]_\Gamma, \qquad \Gamma: \ \tau_0 \cdot s = -u^2 + i \cdot 0,$$
$$\sqrt{\tau_0 \cdot s} = i \cdot u, \qquad 0 < u < \infty. \tag{19}$$

Here, τ_0 is an arbitrary time constant, and Im[...] denotes the imaginary part. The first factor in the integral depends on the dimensionless time τ/τ_0, and it is independent of the particular Laplace transform $\bar{T}_f(s)$. The second factor, the function $L(u)$, is independent of τ and represents the particular Laplace transform for the considered case. The inversion integral in Equation 18 is obtained by considering a closed loop in the complex s-plane. The original integral along a vertical line is replaced by an integral along the negative real axis. The following conditions have to be fulfilled:

$$T_f(0) = 0, \qquad \frac{dT_f}{d\tau} \to 0, \qquad \tau \to \infty. \tag{20}$$

The Laplace transform for the fluid temperature is given by Equation 17. When taken for *s* on the negative real axis, we get:

$$L(u) = \mathrm{Im} \cfrac{-q_{inj}}{C_p \cdot \cfrac{-u^2}{\tau_0} + \cfrac{1}{R_p + \cfrac{1}{\bar{K}_p(u) + \cfrac{1}{\bar{R}_t(u) + \cfrac{1}{\bar{K}_b(u) + \bar{K}_s(u)}}}}}. \tag{21}$$

The final values of the conductances (and their inverse, the resistances), when taken on the negative real axis are expressed using ordinary Bessel functions:

$$\bar{K}_s(u) = \frac{1}{\bar{R}_s(u)} = \frac{2\pi\lambda_s \cdot p_s u \cdot [J_1(p_s u) - i \cdot Y_1(p_s u)]}{J_0(p_s u) - i \cdot Y_0(p_s u)}, \tag{22}$$

$$\bar{K}_t(u) = \frac{1}{\bar{R}_t(u)} = \frac{4\lambda_g}{J_0(p_p u) \cdot Y_0(p_b u) - Y_0(p_p u) \cdot J_0(p_b u)}, \tag{23}$$

$$\bar{K}_p(u) = \frac{1}{\bar{R}_p(u)} = \frac{0.5\,\pi\, p_p u \cdot \left[J_1(p_p u)\, Y_0(p_b u) - Y_1(p_p u)\, J_0(p_b u)\right] - 1}{\bar{R}_t(u)}, \tag{24}$$

$$\bar{K}_b(u) = \frac{1}{\bar{R}_b(u)} = \frac{0.5\,\pi\, p_b u \cdot \left[J_1(p_b u)\, Y_0(p_p u) - Y_1(p_b u)\, J_0(p_p u)\right] - 1}{\bar{R}_t(u)}, \tag{25}$$

$$p_p = r_p/\sqrt{a_g \cdot \tau_0}, \qquad p_b = r_b/\sqrt{a_g \cdot \tau_0}, \qquad p_s = r_b/\sqrt{a_s \cdot \tau_0}. \tag{26}$$

NEW NUMERICAL SOLUTION

A new numerical solution to determine the short-term response of the borehole heat exchanger has also been developed. The objective of the new numerical solution is to validate the new analytical solution and to have a simple and rapid numerical solution. The new numerical solution uses a special coordinate transformation for which the heat flux has the simplest possible form. The radial heat equation in its general form is:

$$\rho(r) \cdot c(r) \cdot \frac{\partial T}{\partial \tau} = \frac{1}{r} \cdot \frac{\partial}{\partial r}\left[r \cdot \lambda(r) \cdot \frac{\partial T}{\partial r}\right], \qquad r \geq r_p. \tag{27}$$

The heat equation can also be rewritten as:

$$\rho \cdot c \cdot 2\pi \cdot r \cdot \frac{\partial T}{\partial \tau} = -\frac{\partial q}{\partial r}, \qquad r \geq r_p.$$
$$q(r,\tau) = -2\pi \cdot r \cdot \lambda \cdot \frac{\partial T}{\partial r}. \tag{28}$$

The transient heat conduction for variable thermal conductivity *λ(r)* may be simplified by using the steady-state thermal resistance of an annular region as a new coordinate *u*:

$$u(r) = \int_{r_p}^{r} \frac{\lambda_g}{\lambda(r') \cdot r'} dr', \qquad u(r_p) = 0, \qquad u_b = u(r_b) = \ln\left(\frac{r_b}{r_p}\right). \tag{29}$$

With the new coordinate, the heat equation behaves as a case with constant thermal conductivity.

$$q(r,\tau) = 2\pi \cdot r[-\lambda(r)] \cdot \frac{\partial T}{\partial u} \cdot \frac{du}{dr} = -2\pi \cdot \lambda_g \cdot \frac{\partial T}{\partial u}. \tag{30}$$

The heat equation for *T(u,τ)* becomes:

$$\rho(r) \cdot c(r) \cdot \frac{d}{du}[\pi \cdot (r(u))^2] \cdot \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial u}\left(2\pi \cdot \lambda_g \cdot \frac{\partial T}{\partial u}\right), \qquad r = r(u). \tag{31}$$

The radius as function of *u* becomes:

$$r(u) = \begin{cases} r_p \cdot \exp(u), & 0 \leq u \leq u_b, \\ r_b \cdot \exp[(u - u_b) \cdot \lambda_s/\lambda_g], & u \geq u_b. \end{cases} \tag{32}$$

The borehole region and the soil outside the borehole are divided into N_b and N_s cells, respectively. The total number of cells is $N = N_b + N_s$. A constant cell width Δu and a time step of $\Delta\tau$ are used.

$$\Delta u = \frac{u_b}{N_b}, \qquad N_s = 1 + \text{int}\left[\frac{\lambda_s}{\Delta u \cdot \lambda_g} \cdot \ln\left(\frac{\sqrt{p_{max} \cdot 4a_s\tau_{max}}}{r_b}\right)\right] \tag{33}$$

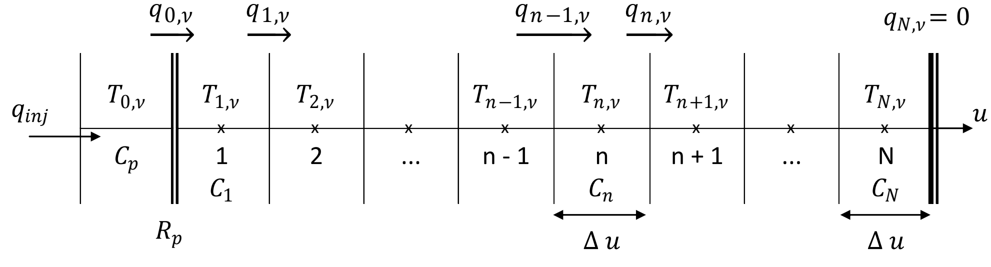


Figure 3 *Notations for the numerical solution.*

Here, τ_{max} is the end time for the computations, and int[…] denotes the integer part. The number N_s is chosen so that the heat flux at the outer boundary is sufficiently small for $\tau \leq \tau_{max}$. The particular expression is obtained from the Line Source solution (Ingersoll et al., 1954) in soil. The criterion is that the heat flux at the outer boundary is smaller than $(e^{-p_{max}} \cdot q_{inj})$ up to the maximum time τ_{max}. The choice $p_{max}=4$ should be sufficient as $e^{-4}=0.02$. Figure 3 shows the notations that are used. The temperature at the midpoint of cell n at time step ν is $T_{n,\nu}$ and the heat flux from cell n to $n+1$ is $q_{n,\nu}$. The initial temperatures for $\nu = 0$ are zero: $T_{n,0} = 0$, $n = 0, 1, 2, \ldots N$. The heat fluxes are given by:

$$q_{0,\nu} = K_0 \cdot (T_{0,\nu} - T_{1,\nu}), \qquad q_{N,\nu} = 0,$$
$$q_{n,\nu} = K_u \cdot (T_{n,\nu} - T_{n+1,\nu}), \qquad n = 1, 2, \ldots, N-1. \tag{34}$$

The thermal conductances in the numerical solution become:

$$K_0 = \frac{1}{R_p + 0.5\,\Delta u\,/(2\pi \cdot \lambda_g)}, \qquad K_u = \frac{2\pi \cdot \lambda_g}{\Delta u}. \tag{35}$$

The temperatures at the new time step, $\tau = (\nu + 1) \cdot \Delta\tau$, are given by:

$$T_{0,\nu+1} = T_{0,\nu} + \frac{q_{inj} - q_{0,\nu}}{C_p} \cdot \Delta\tau,$$
$$T_{n,\nu+1} = T_{n,\nu} + \frac{q_{n-1,\nu} - q_{n,\nu}}{C_n} \cdot \Delta\tau, \qquad n = 1, 2, \ldots, N. \tag{36}$$

The C_n i.e. the heat capacity of cell n, is equal to the area of the annular cell times the volumetric heat capacity:

$$C_n = \pi\{[r(n \cdot \Delta u)]^2 - [r(n \cdot \Delta u - \Delta u]^2\} \cdot \begin{cases} \rho_g c_g & n = 1, \ldots, N_b \\ \rho_s c_s & n = N_b + 1, \ldots N \end{cases} \tag{37}$$

To ensure numerical stability, the time step must satisfy the inequalities:

$$\Delta t \leq min\left(\frac{C_{min}}{2K_u}, \frac{C_p}{K_0}\right), \qquad C_{min} = \underbrace{min}_{1 \leq n \leq N}(C_n) \tag{38}$$

Equations 34 and 36 give the iterative numerical calculation procedure. The full numerical solution is defined by Equations 32 to 37.

The new analytical and numerical solutions can be used to accommodate any prescribed heat injection/extraction function using the superposition principle. The solutions can then be implemented in building energy simulation software or used as a standalone program.

VALIDATION

The new analytical solution has been tested under various conditions and validated using different approaches. The first approach was the comparison of the results from the analytical solution to the results of the numerical solution. Second, the new analytical solution was compared to the semi-analytical solution of Beier and Smith (2003). The thermal properties of the fluid, pipe, grout and soil regions considered for these two approaches are given in Table 1. Finally, the analytical solution was validated using experimental results from a medium-scale laboratory setup which simulates the physical and the thermal characteristics of a borehole in controlled laboratory conditions.

Validation by inter-model comparison of analytical and numerical solutions

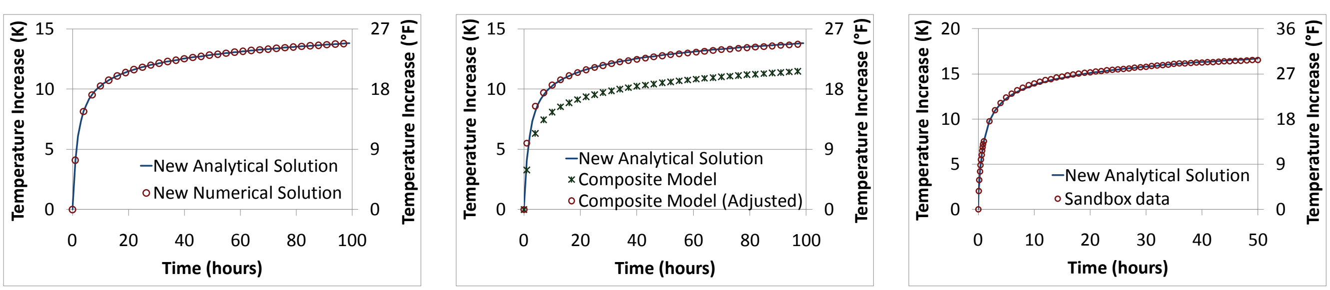
A series of simulations were done to compare the new analytical and the numerical solutions. Such comparisons showed that the new solutions agree to an accuracy of higher than 0.01 K (0.018 °F) for all the simulated cases. As an illustration, the results from one of the comparisons using typical conditions are presented here. For the comparison, a heat injection rate of 50 W/m (15 W/ft) to an equivalent diameter pipe of radius 17.7 mm (0.7 in.) was considered. For these conditions, the increase in the fluid temperatures for the first 100 hours as predicted by the analytical and the numerical methods is shown in Figure 4(a). The difference in the predicted temperatures is remarkably small. The maximum absolute difference in fluid temperatures predicted by two very different approaches is 0.004 K (0.007 °F) while the average absolute difference is smaller than 0.002 K (0.004 °F). For the numerical solution, the temperature increase was simulated using 5 and 38 cells for grout and soil, respectively.

Table 1. Thermal Properties of Fluid, Pipe, Grout and Soil Considered for the Comparison.

Element	Fluid	Pipe	Grout	Soil
Thermal conductivity, W/m·K (Btu/h·ft·°F)	-	0.47 (0.27)	1.5 (0.87)	3.0 (1.73)
Volumetric heat capacity, MJ/m^3·K (Btu·ft^3·°F)	4.18 (62.4)	-	3.1 (46.2)	1.88 (28.0)

Validation against a semi-analytical model

Beier and Smith (2003) also used the Laplace transformation approach to develop their semi-analytical 'Composite model'. However, there are two fundamental differences between their model and the new analytical solution. First, Beier and Smith uses a numerical inversion algorithm (Stehfest, 1970) to invert the Laplace transforms in the real time. Using numerical methods for the inversion of the Laplace transforms complicates the solution and makes its implementation in building energy simulation software more difficult. The second issue with the Composite model is that it assumes same temperature for the fluid and the pipe boundary. In other words, the solution does not account for the fluid and the pipe resistances (i.e. $R_p=0$). These can, however, be implicitly added by adjusting the radius of the equivalent diameter pipe accordingly or by adding the fluid temperature increase, because of the pipe and fluid resistances, to the predicted fluid temperatures. The comparison of the new analytical solution with the Composite model, for the test case described above, is shown in Figure 4(b). As the fluid and the pipe resistances are ignored by the Composite model, the model under-predicts the increase in the fluid temperatures. However, the results from the Composite model become similar to the results from the analytical solution if the effects of the pipe and the resistances are implicitly added (i.e. by adding $R_p \cdot q_{inj}$) to the predicted fluid temperatures.



(a) (b) (c)

Figure 4 *Validation of new analytical solution against a) new numerical solution, b) Composite model and c) sandbox data.*

Validation using experimental data

The new analytical solution has also been validated using data from a medium-scale laboratory setup. The setup has been used by various Oklahoma State University researchers (Austin, 1998; Beier and Smith, 2003; Yavuzturk, 1999) to simulate and validate their models under controlled conditions. The setup consists of a wooden box of dimensions 1.8 m x 1.8 m x 18 m (6 ft x 6 ft x 60 ft). An aluminum pipe of diameter 133 mm (5 in.) is centered along the length of the wooden box. The thermal resistance of the aluminum pipe is negligible. The aluminum pipe has a U-tube inserted in it. The U-tube is surrounded by bentonite-based grout and is kept centered in the aluminum pipe by means of spacers. The wooden box is filled with homogeneous sand and the whole setup is hence called a sandbox. Thermal conductivity and the volumetric heat capacity values for the grout and the soil in the sandbox are measured independently. The thermal conductivity values for grout and soil are 0.73 W/m·K (0.42 Btu/h·ft·°F) and 2.82 W/m·K (1.63 Btu/h·ft·°F) respectively and the volumetric heat capacity values for the grout and the soil are 3.84 MJ/m^3·K (57.2 Btu/ft^3·°F) and 1.92 MJ/m^3·K (28.6 Btu/ft^3·°F), respectively. The comparison of the new analytical solution with the experimentally measured mean fluid temperatures from a sandbox test of 50 hours is shown in Figure 4(c). The length of the sandbox test was kept to 50 hours to avoid edge effects. The results from the new analytical solution and the sandbox test overlap each other. The maximum absolute difference between the temperatures predicted by the new solution and the experimentally measured temperature is 0.2 K (0.36 °F) while the average absolute difference between the between the predicted and the measured temperatures is less than 0.1 K (0.18 °F).

CONCLUSION

Short-term response of borehole heat exchanger has significant effects on the energy consumption and the performance of the heat pump and overall ground source heat pump system. The existing analytical solutions don't account for thermal properties of all the borehole elements. A new analytical solution has been developed to complement the existing solutions. The new analytical solution is tested and validated using different approaches. The comparison with a new numerical solution shows that the results from the two solutions agree to an accuracy of higher than 0.01 K (0.018 °F). The results from the new analytical solution are also consistent with the results from a state-of-the-art semi-analytical method adjusted for pipe and fluid resistances. Finally, the fluid temperature predicted by the new analytical solution also agrees with experimental results. A maximum difference of 0.2 K (0.36 °F) is observed between the simulated and the experimental results.

NOMENCLATURE

a	=	thermal diffusivity (m^2/s or ft^2/h)
C	=	thermal capacity per unit length (J/m·K or Btu/ft·°F)
c	=	specific heat capacity (J/kg·K or Btu/lbm·°F)
I_n	=	nth-order modified Bessel function of first kind
J_n	=	nth-order Bessel function of first kind
K	=	thermal conductance (W/m·K or Btu/h·ft·°F)
K_n	=	nth-order modified Bessel function of second kind
$\bar{K}$	=	thermal conductance in Laplace domain (W/m·K or Btu/h·ft·°F)
λ	=	thermal conductivity (W/m·K or Btu/h·ft·°F)
N	=	total number of cells for the numerical model
q	=	rate of heat transfer per unit length (W/m or W/ft)
R	=	thermal resistance (m·K/W or h·ft·°F/Btu)
$\bar{R}$	=	thermal resistance in Laplace domain (m·K/W or h·ft·°F/Btu)

r = radius (m or ft)

ρ = density (kg/m^3 or lbm/ft^3)

s = Laplace transform variable

T = temperature (K or °F)

$\bar{T}$ = Laplace transform of T (K·s or °F·h)

τ = time (s or h)

Y_n = nth-order Bessel function of second kind

Subscripts

b = borehole

f = fluid

g = grout

p = pipe

s = soil (ground)

t = transmittive

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