CHALMERS TEKNISKA HÖGSKOLA

CHALMERS UNIVERSITY OF TECHNOLOGY
GÖTEBORG
SWEDEN

Machine and Vehicle Design

Gear Shifting with Retained Power Transfer

Bengt Jacobson

1993

Gear Shifting with Retained Power Transfer

av

Bengt Jacobson

Akademisk avhandling, som för avläggande av teknisk doktorsexamen vid Chalmers tekniska högskola försvaras vid offentlig disputation tisdag 1993–05–11 kl. 13^{15} i sal HC2, Hörsalsvägen 16, Chalmers tekniska högskola, Göteborg.

Fakultetsopponent: Dr.-Ing. Andreas Laschet

Institutionen för maskin- och fordonskonstruktion

Gear Shifting with Retained Power Transfer

by Bengt Jacobson, Machine and Vehicle Design, Chalmers University of Technology, S - 412 96 GÖTEBORG, Sweden

ABSTRACT

The most essential property of a *powershifting* transmission is that power can be transferred even during a gear shift. Automatic transmissions in vehicles are a common application. The gearbox is shifted by simultaneous engaging and disengaging of clutches, connected to a gear train. Analytic simulation models are useful tools when optimizing the control strategy for these clutches.

This dissertation presents the following models for analysis of powershift operations in vehicle drivelines:

- ☐ **Model for dimensionless equations:** A driveline model without too many details generates equations, which are made dimensionless in a lucid way. Through these, a good overview of the principles of powershifting is found. Also, some phenomena of interest are strictly defined (*tie-up, flare, torque* and *inertia phase*, etc.).
- ☐ **Two-clutch model:** This driveline model has a gearbox, which shifts between two gears by controlling two clutches, one for the lower gear and one for the higher gear. The simulation results are verified by tests. Thus the level of detail in the driveline model is found to be appropriate.
- ☐ **Multi-clutch model:** In this model, the gearbox is equipped with as many clutches as the real gearbox has. (In the presented example, this means five clutches.) Therefore, all gear shifts for a certain gearbox can be analyzed with the same model. Also, it is possible to analyze other shift operations than those where exactly one clutch is engaging and one is disengaging.
- ☐ **Model including surrounding systems:** The driveline model is connected to models of the engine block and vehicle. Also, a simple model of a passenger is present, making it possible to study the passenger comfort more properly.

The clutches are often oil-immersed, and so they cannot be properly described with dry friction characteristics. A model of an engaging multi-disc clutch, including squeeze and viscous friction in the oil film, is therefore developed. The influence of the oil film on the gear shift quality is studied.

The modelling is carried out systematically. Therefore, the work also gives some general advice as to how dynamic transmission systems should be modelled. An algorithm is developed to find the appropriate state variables for a constrained dynamic transmission system.

Keywords: transmission, gearbox, powershift, gear shift, ratio change, automatic transmission, analysis, model, dynamic, simulation, clutch, oil-immersed clutch, wet clutch, dry friction, viscous friction

Chalmers University of Technology

Gear Shifting with Retained Power Transfer

by
Bengt Jacobson

Machine and Vehicle Design
Chalmers University of Technology
S – 412 96 GÖTEBORG
Sweden

1993

Published and distributed by:
Machine and Vehicle Design
Chalmers University of Technology
S – 412 96 GÖTEBORG
Sweden

ISBN 91 – 7032 – 808 – 0

Printed in Sweden
Vasastadens Bokbinderi AB, Göteborg
1993

Gear Shifting with Retained Power Transfer

by Bengt Jacobson, Machine and Vehicle Design, Chalmers University of Technology,
S – 412 96 GÖTEBORG, Sweden

ABSTRACT

The most essential property of a *powershifting* transmission is that power can be transferred even during a gear shift. Automatic transmissions in vehicles are a common application. The gearbox is shifted by simultaneous engaging and disengaging of clutches, connected to a gear train. Analytic simulation models are useful tools when optimizing the control strategy for these clutches.

This dissertation presents the following models for analysis of powershift operations in vehicle drivelines:

- ☐ **Model for dimensionless equations:** A driveline model without too many details generates equations, which are made dimensionless in a lucid way. Through these, a good overview of the principles of powershifting is found. Also, some phenomena of interest are strictly defined (*tie-up, flare, torque* and *inertia phase*, etc.).
- ☐ **Two-clutch model:** This driveline model has a gearbox, which shifts between two gears by controlling two clutches, one for the lower gear and one for the higher gear. The simulation results are verified by tests. Thus the level of detail in the driveline model is found to be appropriate.
- ☐ **Multi-clutch model:** In this model, the gearbox is equipped with as many clutches as the real gearbox has. (In the presented example, this means five clutches.) Therefore, all gear shifts for a certain gearbox can be analyzed with the same model. Also, it is possible to analyze other shift operations than those where exactly one clutch is engaging and one is disengaging.
- ☐ **Model including surrounding systems:** The driveline model is connected to models of the engine block and vehicle. Also, a simple model of a passenger is present, making it possible to study the passenger comfort more properly.

The clutches are often oil-immersed, and so they cannot be properly described with dry friction characteristics. A model of an engaging multi-disc clutch, including squeeze and viscous friction in the oil film, is therefore developed. The influence of the oil film on the gear shift quality is studied.

The modelling is carried out systematically. Therefore, the work also gives some general advice as to how dynamic transmission systems should be modelled. An algorithm is developed to find the appropriate state variables for a constrained dynamic transmission system.

Keywords: transmission, gearbox, powershift, gear shift, ratio change, automatic transmission, analysis, model, dynamic, simulation, clutch, oil-immersed clutch, wet clutch, dry friction, viscous friction

Dissertation

This dissertation comprises the following papers. Only printing formats are changed and minor errata corrected in comparison with the previously published versions.

Paper A: JACOBSON, BENGT: *Vehicle Driveline Mechanics during Powershifting*, Report No 1990–05–30 (Licentiate of Engineering thesis), Machine and Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1990

Paper B: JACOBSON, BENGT: Analysis of Shift Operations in Automatic Transmissions, *3rd International EAEC Conference on Vehicle Dynamics and Powertrain Engineering*, Strasbourg, France, June 11–13, 1991, pp 195–202

Paper C: JACOBSON, BENGT: Dynamic Simulation of Powershifting Transmissions, *Eighth World Congress on the Theory of Machines and Mechanisms*, Praha, Czechoslovakia, August 26–31, 1991, pp 573–576

Paper D: JACOBSON, BENGT: Dynamic Vehicle Transmission System — Derivation of Equations on Explicit Form, submitted for publication in *Vehicle System Dynamics* (submitted in December 1992)

Paper E: JACOBSON, BENGT: Engagement of Oil Immersed Multi-disc Clutches, *6th International Power Transmission and Gearing Conference, ASME*, Scottsdale, USA, September 13–16, 1992, Volume 2, pp 567–574

Paper F: JACOBSON, BENGT: Influence of Oil Film in the Clutches on Gear Shifts in Automatic Vehicle Transmissions, accepted for presentation at the *4th International EAEC Conference on Vehicle and Traffic Systems Technology*, Strasbourg, France, June 16–18, 1993

Contents

Acknowledgments

This work has been carried out under the excellent guidance of Professor Göran Gerbert and Doctor Anders Hedman at Machine and Vehicle Design, Chalmers University of Technology in Göteborg, Sweden. Their competence, both in the specific research area and in scientific considerations in general, have been invaluable. Göran Gerbert is also the head of Machine and Vehicle Design, a task that he accomplishes with great success.

Among my colleagues at Machine and Vehicle Design, I want to thank Professor Mart Mägi for many views, and for the idea behind figure 14. Other colleagues[1)] have contributed through really good friendship and profitable discussions. My work has received a great compliance from people at SAAB Automobile AB and Volvo Car Corporation and some invaluable help has been given to me by the mathematicians Bernt Wennberg and Magnus Lundin. Furthermore, Mrs Linda Schenk has done an excellent job in correcting my attempts in the English language.

Titti, my wife, and Nils, our son, help me to constitute a family, for which I am truly grateful. Such things are actually more important than estimating how much the output power from a mechanical transmission drops during a powershift operation.

1) Maud, Bengt$_1$, Bo, Göran$_2$, Hans$_1$, Ronny, Jan, Dag$_1$, Hakan, Lars-Erik, Yatao, Göran$_3$, H}kan, Claes, Peter, Jonas, Hans$_2$, Michael, Johan$_1$, Kjell, Lars, Magnus, Rikard, Per, Fredrik, Filip, Sixten and Dag$_2$

1. Introduction

A powershifting transmission is shifted by engaging and disengaging clutches, connected to a gear train. In practice, this is often an automatic transmission in a vehicle, where each clutch is designed as a multi-disc clutch, a band brake or a one-way clutch. In order to reach good shift quality, an analytic simulation model is useful.

During the last 10–20 years, many models for gear shifts in specific automatic transmissions in vehicles have been presented. Such work is often focused on the design of the control system. Two older references, [1] and [2], present the basic mechanics in an excellent way. The mechanical phenomena during a powershift are discussed. For instance, they discuss the **timing** problem. It is a problem in shift operations where both disengaging and engaging clutches are controlled, i.e. no clutch is designed as a one-way clutch. If the clutches are too loosely engaged, the engine will overspeed (*flare*) and if they are engaged too hard the engine will choke (*tie-up*). Both cases result in unnecessarily large losses of power supply to the vehicle. Furthermore, there is a contradiction between the **wear** of the clutches and passenger **comfort**, which are two other essential aspects of the shift quality.

Papers A–D in this dissertation present work carried out in order to systemize the modelling technique itself. To a great extent, this work is also valid for dynamic transmission systems in general. However, the aim was never to develop a complete modelling technique for dynamic transmission systems, as was, for example, the case in reference [3]. In papers A and B, simulation and test results are compared. The behavior of a specific important component, the engaging oil-immersed multi-disc clutch, is studied in papers E and F.

2. Overview of the System Description

The three words in the concept *dynamic transmission system* are interpreted here as follows:

- ☐ **"dynamic"** means that the system, at each instant in time, changes in a way determined by its present conditions. Therefore, the solution is implicitly defined by sufficient information about the system conditions at some suitable instants in time. Mathematically, this is treated as an *initial value problem.*
- ☐ **"transmission"** means that the conditions in each point (i.e. section of shaft) of the system can be described by one rotational velocity and one torque. Also, these quantities are coaxial, and so the product of them is the mechanical power, which flows through the point in a certain direction.
- ☐ **"system"** means a constellation of components, most suitably described in two steps. Firstly, properties for components are defined. Secondly, the connections between the components are given.

As a further restriction in this dissertation, all component properties are lumped, e.g. there are no shafts with continuously distributed mass and elasticity properties. Therefore, the variables might only be differentiated with respect to time, i.e. the system is an **ordinary** differential equation system.

2.1 Velocity and Torque Variables

It is important to have a lucid convention for where velocities and torques are found and which senses are considered as positive. First, it is assumed that the transmissions are observed in a sketch, where all velocities and torques are horizontal on the paper (i.e. directed from left to right or vice versa). Figure 1 shows a suitable convention for analysis of a single transmission component. Here, all shafts are shaft **ends**, and thus both shaft velocity and shaft torque are vector variables. In figure 1, the positive senses for all shaft velocities and shaft torques are defined through one single positive direction. The product of shaft velocity and shaft torque is then the **input** power on the shaft. When expanding the analyze to a system, the shaft velocities are still vectors, and their positive senses can be defined by one single positive direction. However, a shaft torque cannot be observed unless a shaft is cut off. Then there are two counter-directed torque vectors, one on each shaft end. One possible solution to this problem is to introduce *nodes*, as shown in figure 2. Here, one single positive direction tells the positive sense of all node velocities and all node powers. If node power is defined as the product of node velocity and torque, a positive node torque is implicitly defined. Hereby, *power input nodes* and *power output nodes* can be defined for a component in the system. This is when node power is defined as positive as input and output to the component, respectively. In figure 1, node 2 is an output power node for the single gear transmission, but it is an input power node for the elasticity. In reference [4], another convention for transmission systems is used. It is based on the convention in figure 1 and interaction between components is handled by connection points, referred to as nodes.

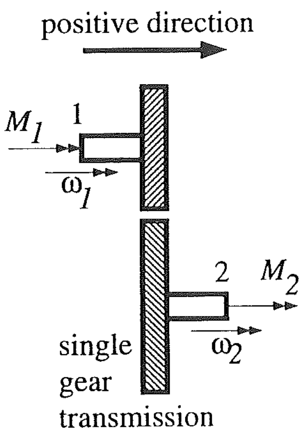


Figure 1: Suitable convention for positive senses of velocities and torques when analyzing a component

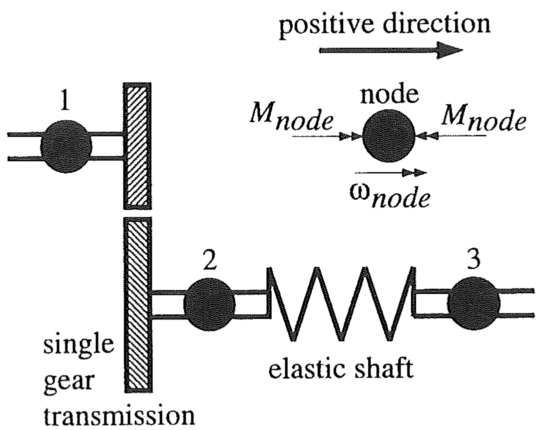


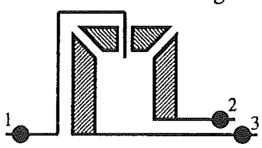
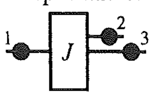
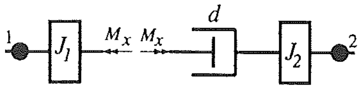
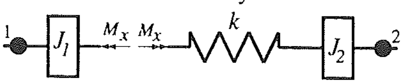
Figure 2: Suitable convention for positive senses of velocities and torques when analyzing a system

The convention in figure 2 is first established in paper A and used in papers A–F. It is sometimes expanded to include nodes with force and linear velocity. (The node concept is also expanded with *relative velocity nodes*, introduced in section 3.3.) The positive direction is assumed from left to right, unless otherwise defined.

2.2 Component and System Equations

The systems studied are composed of components, connected in nodes. The natural way to describe the components is to give as many (independent) equations as there are nodes connected to the component. This hypothesis (or definition of *"natural way"*) is declared in paper D. In paper A, some of the most essential components are presented. Also, in table 1, some examples are given. Note that both examples 4 and 5 are considered as **one** component. This shows that a component can be defined in a rather arbitrary way, as far as it can be connected to nodes in a system. However, for such a component, the number of component equations is only equal to the number of nodes, if we just count *effective* equations. In examples 4 and 5, M_x can be regarded as an intermediate variable, which can be eliminated. This would leave the component with just two equations, i.e. the effective ones. However, in example 5, M_x cannot be eliminated without introducing second order differentials (or integrals). In section 2.3 the equations are formulated as first order differential equations, and so components of this kind are not recommended. It is better to model example 5 as two components, i.e. to introduce a node where M_x is positioned.

Table 1: Examples of components and their component equations

EXAMPLE	COMPONENT	COMPONENT EQUATIONS
1	lossfree differential gear 	$\omega_1 = \frac{\omega_2 + \omega_3}{2}$ $M_1 = M_2 + M_3$ $M_2 = M_3$ (1)
2	inertia (one input and two outputs) 	$\omega_1 = \omega_2$ $\omega_1 = \omega_3$ $J \cdot \dot{\omega}_1 = M_1 - M_2 - M_3$ (2)
3	elasticity (linear) 	$M_1 = M_2$ $\dot{M}_1 = k \cdot (\omega_1 - \omega_2)$ (3)
4	inertia-damper-inerta 	$J_1 \cdot \dot{\omega}_1 = M_1 - M_x$ $M_x = d \cdot (\omega_1 - \omega_2)$ $J_2 \cdot \dot{\omega}_2 = M_x - M_2$ (4)
5	inertia-elasticity-inerta 	$J_1 \cdot \dot{\omega}_1 = M_1 - M_x$ $\dot{M}_x = k \cdot (\omega_1 - \omega_2)$ $J_2 \cdot \dot{\omega}_2 = M_x - M_2$ (5)

2.3 Physical Variables versus State Variables

In general, the component equations are composed of both algebraic[2)] and differential equations, and therefore they cannot be algebraically solved. Instead, the system generates an initial value problem, which is based on the assumption that we know the initial conditions. The solution is obtained by numerical integration. Computer software for such methods is well established for initial value problems that can be formulated in *normal form*, i.e. as shown in equation 6. Here, x and y denote column vectors of *physical variables* (or *algebraic variables*) and *state variables*, respectively. The physical variables are the node quantities, i.e. node velocities and node torques. Superscript iv means *initial value.* Time is denoted by t. The time derivatives of the state variables (dotted form) are referred to as *state derivatives.*

$$\begin{aligned} \mathrm{y}^{\mathrm{iv}} &= \text{ known} \\ [\dot{\mathrm{y}}, \mathrm{x}] &= \text{ system function}(\mathrm{y}, t) \end{aligned} \tag{6}$$

It is supposed that the system function can be derived from the component equations **if** a transformation between state variables and physical variables is given. In paper A, it was found that the most straightforward way to express the state variables is in terms of velocities of inertias and torques of elasticities. These quantities directly describe the level of stored energy in the components, and the storage of energy is the very essence of the dynamic nature of a dynamic transmission system. In paper A, there is also a discussion of the conventional way to describe the conditions in a mechanical system, i.e. with position and velocity of each inertia as state variables. This is found to be an unnecessarily complex way for a dynamic transmission system, where velocities and torques are the quantities of interest. It can also tempt us to introduce small inertias without physical significance, e.g. between the elasticity and the damper in figure 3. Let us study the example in figure 3, for which the component equations can be written as shown in table 2.

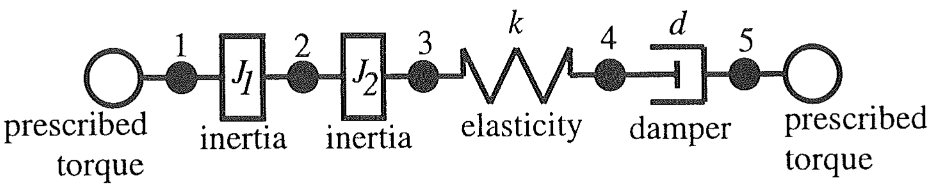


Figure 3: Example with 6 components connected in 5 nodes

It is tempting to choose the state variables as the physical variables that occur in dotted form in the component equations, i.e. $\mathrm{y} = [\omega_1 \quad \omega_2 \quad M_3]^T$. Note that these are velocities of inertias and torques of elasticities. However, it is not physically correct to use all three of them, because there is a constraint between ω_1 and ω_2, namely equation 8, embedded in table 2. Also M_3 is constrained, namely to t through equations 16, 14 and 12. A primitive method for finding the appropriate state variables is, intuitively, to become aware of these constraints and to use $\mathrm{y} = \omega_1$ or $\mathrm{y} = \omega_2$ instead. This method, which often requires some trial and error, is used in all papers except paper D. There, an algorithm is developed for finding

[2)] Here, and in the papers A-F, "algebraic" means non-differentiated. Therefore, also transcendent equations, linear interpolations in tables etc. are included in the expression "algebraic equations".

Table 2: Component equations for the system in figure 3

COMPONENT	COMPONENT EQUATIONS	
(left) prescribed torque	$M_1 = f_1(t)$; where f_1 is a known function of t (7)	
(left) inertia	$\omega_1 = \omega_2$ (8)	$J_1 \cdot \dot{\omega}_1 = M_1 - M_2$ (9)
(right) inertia	$\omega_2 = \omega_3$ (10)	$J_2 \cdot \dot{\omega}_2 = M_2 - M_3$ (11)
elasticity	$M_3 = M_4$ (12)	$\dot{M}_3 = k \cdot (\omega_3 - \omega_4)$ (13)
damper	$M_4 = M_5$ (14)	$M_4 = d \cdot (\omega_4 - \omega_5)$ (15)
(right) prescribed torque	$M_5 = f_5(t)$; where f_5 is a known function of t (16)	

a physically correct set of state variables. The state variables become linear combinations of the physical variables. In this example, the algorithm would suggest $\mathbf{y}$ to be a linear combination of the node velocities in the system. A conclusion is that one should not regard state variables as something basically physical. They are just mathematical variables with ambiguous physical interpretations.

The algorithm presented in paper D is useful mainly for linear equation systems with constant coefficients. Each (scalar) differential equation must be a first order linear equation with constant coefficients. Non-linear algebraic equations cannot be included if they are active as constraints. These restrictions are seldom an obstacle in transmission systems. A future development of the algorithm could be to facilitate interaction to *subsystems*. This might also be a way to allow non-linear differential equations and non-linear constraints, through "hiding" them in the subsystems.

3. Equations for Gear Trains

The gear trains in powershifting gearboxes are almost exclusively designed with gearwheels in planetary arrangements. This can act as a deterrent to the analyst, which would be a pity because planetary gear trains are actually very easy to model. At least, the modelling is easy when losses are neglected, which is the case in this dissertation. The matrix formulation of the component equations quite simply becomes, in inhomogeneous form:

$$\begin{aligned} \omega_{\mathrm{x}} &= \mathbf{A} \cdot \omega_{\mathrm{y}} \\ \mathbf{M}_{\mathrm{y}} &= \mathbf{B} \cdot \mathbf{M}_{\mathrm{x}} \end{aligned} \tag{17}$$

or, in homogeneous form:

$$\begin{aligned} \mathbf{C} \cdot \omega &= \mathbf{0} \\ \mathbf{D} \cdot \mathbf{M} &= \mathbf{0} \end{aligned} \tag{18}$$

Here, **A**, **B**, **C** and **D** are matrices with coefficients determined by the number of teeth in the gearwheels. The column vectors ω and **M** contains all node velocities and torques, respectively. In ω_x and $\mathbf{M}_x$, only some of the nodes are represented. The remaining nodes are represented in ω_y and $\mathbf{M}_y$. Now there are simple ways to obtain the torque matrix from the velocity matrix (or the other way around) for both the inhomogeneous and homogenous form. The methods are valid also for more general mechanisms, where some nodes may have translational velocities and forces.

Equations 19 and 20 show that the matrix formulations in equations 17 and 18 can be obtained for the example in figure 4. These (conventional) derivations are based on kinematics and torque equilibrium for the gearwheels.

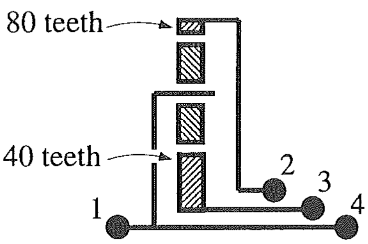


Figure 4: Example of gear train with gearwheels in planetary arrangement

$$\left\{ \begin{array}{c} \dfrac{\omega_2-\omega_1}{\omega_3-\omega_1} = \dfrac{-40}{80} \text{ (Willis' equation)} \\ \omega_1 = \omega_4 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \Rightarrow \underbrace{\begin{bmatrix} \omega_1 \\ \omega_4 \end{bmatrix}}_{\omega_x} = \underbrace{\begin{bmatrix} 8/12 & 4/12 \\ 8/12 & 4/12 \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} \omega_2 \\ \omega_3 \end{bmatrix}}_{\omega_y} \\ \text{or} \\ \Rightarrow \underbrace{\begin{bmatrix} -12 & +8 & +4 & 0 \\ +1 & 0 & 0 & -1 \end{bmatrix}}_{\mathbf{C}} \cdot \underbrace{\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}}_{\omega} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array} \right. \tag{19}$$

$$\left\{ \begin{array}{rcl} M_1 & = & M_2+M_3+M_4 \\ & & \text{(global equilibrium)} \\ 0 & = & M_2/80 - M_3/40 \\ & & \text{(equilibrium for} \\ & & \text{the planet wheel)} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \Rightarrow \underbrace{\begin{bmatrix} M_2 \\ M_3 \end{bmatrix}}_{\mathbf{M}_y} = \underbrace{\begin{bmatrix} +8/12 & -8/12 \\ +4/12 & -4/12 \end{bmatrix}}_{\mathbf{B}} \cdot \underbrace{\begin{bmatrix} M_1 \\ M_4 \end{bmatrix}}_{\mathbf{M}_x} \\ \text{or} \\ \Rightarrow \underbrace{\begin{bmatrix} +1 & -1 & -1 & -1 \\ 0 & -1/8 & +1/4 & 0 \end{bmatrix}}_{\mathbf{D}} \cdot \underbrace{\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix}}_{M} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array} \right. \tag{20}$$

3.1 Inhomogeneous Form

The relationship between matrices **A** and **B** in equation 17 can be formulated in basic matrix manipulations. This was shown in references [5] and [6] and in paper A. If all nodes connected to the gear train are power input nodes, the relationship between the matrices becomes as simple as:

$$\mathbf{B} = -\mathbf{A}^T \tag{21}$$

In equation 21, superscript T denotes transpose of the matrix. If there are power output nodes among the x-nodes, the corresponding columns in $\mathbf{B}$ should change sign. Furthermore, power output nodes among the y-nodes change the signs of the corresponding rows in $\mathbf{B}$. The drawback of the inhomogeneous form is that nodes on the left and right hand sides in equation 17 cannot be chosen arbitrarily. For instance, in the example in figure 4, it is impossible to write equation 17 with $\omega_y = [\omega_1 \quad \omega_4]^T$.

For the example in figure 4, matrix $\mathbf{B}$ in equation 17 can be found using equation 21 and the sign rules. Both rows of $-\mathbf{A}^T$ should change signs because nodes 2 and 3 are output power nodes. The second column of $-\mathbf{A}^T$ should change sign because node 4 is an output power node. The result is shown in equation 22. Note that the resulting $\mathbf{B}$ is identical to the one in equation 20. For a general gear train, derivations using equation 21 give different matrices $\mathbf{B}$ as compared with derivations through torque equilibrium. However, they describe the same torque relationships.

$$\mathbf{B} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{\substack{\text{changes signs} \\ \text{of both rows}}} \cdot \underbrace{\left(-\begin{bmatrix} 8/12 & 4/12 \\ 8/12 & 4/12 \end{bmatrix}^T \right)}_{-\mathbf{A}^T} \cdot \underbrace{\begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}}_{\substack{\text{changes sign of} \\ \text{the 2nd column}}} = \begin{bmatrix} 8/12 & -8/12 \\ 4/12 & -4/12 \end{bmatrix} \qquad (22)$$

3.2 Homogeneous Form

The equations for any gear train can be written in the homogeneous form in equation 18. A method for deriving the torque matrix from the velocity matrix is used, but neither properly presented nor proofed, in papers B and C. Therefore it is presented and proofed in appendix A. In the following, only the result is exemplified.

For the example in figure 4, matrix $\mathbf{D}$ is derived from the matrix C, or actually a preliminary variant, C_0. C_0 is allowed to contain dependent rows. If C_0 is taken as C in equation 19, the matrices in equation 18 are determined as shown in equation 23. Note that the C and D found are not identical with the C and D derived in equations 19 and 20. However, they describe the same velocity and torque relationships.

$$\begin{aligned} &\underbrace{\begin{bmatrix} +0.8031 & -0.5330 & -0.2665 & -0.0036 \\ -0.3032 & -0.3701 & -0.1851 & +0.8584 \end{bmatrix}}_{\mathrm{C}} \cdot \underbrace{\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}}_{\omega} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ &\underbrace{\begin{bmatrix} 0.5130 & -0.6156 & -0.3078 & -0.5130 \\ 0 & +0.4472 & -0.8944 & 0 \end{bmatrix}}_{\mathrm{D}} \cdot \underbrace{\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix}}_{\mathrm{M}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \qquad (23)$$

One drawback of this homogenous formulation is that the derivation of the torque matrix is carried out numerically, and so an analytic expression cannot be obtained.

3.3 Relative Velocity Nodes

So far, the gear train has been assumed to be connected to its surroundings through (*common*) nodes, defined as in figure 2. Figure 5 shows how this works in the case of connection to a clutch. In order to reduce the number of variables, two common nodes can be exchanged to one *relative velocity node.* The velocity and torque of a relative velocity node are defined in figure 6. A clutch connected by means of such a node is shown in figure 7. This connection is mostly useful for clutches connected to gear trains, but can be used between any connected components, provided that the component equations can be expressed in relative velocity and one torque. For example, in papers B and C, an elasticity is connected to a gear train by means of a relative velocity node.

The relationships between velocity and torque matrices, presented earlier in this chapter, can be applied to gear trains with relative velocity nodes. The relative velocity nodes should then be regarded as power **output** nodes.

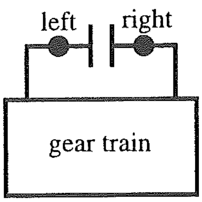


Figure 5: Clutch connected to a gear train by means of two nodes. Four variables are involved: ω_{left}, ω_{right}, M_{left} and M_{right}.

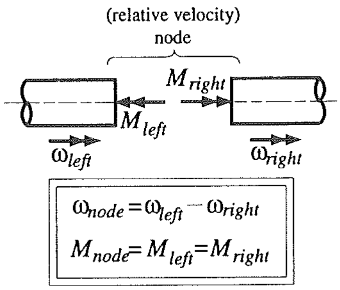


Figure 6: Suitable convention for a relative velocity node. The node power, defined as $M_{\text{node}} \cdot \omega_{\text{node}}$, is a power **loss**.

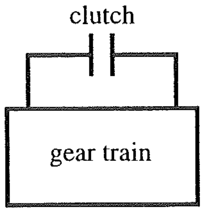


Figure 7: Clutch connected to a gear train by means of a relative velocity node. Two variables are involved: ω_{clutch} and M_{clutch}.

4. Equations for Clutches

In this chapter, the clutches (including brakes and one-way clutches) are treated as components acting with pure dry friction.

4.1 Component Equations

Three operating phases need to be handled: **stick, positive slip** and **negative slip** phases. In the first case, the component restricts the relative velocity to zero. In the two latter cases, the torque is restricted to a function of time, dependent on the actuating force, coefficient of friction etc.. In equation 24 and figure 8 the component equations and characteristics are shown. The variables denoted by ph and fl are referred to as *phase* and *flags.*

$$\begin{cases} M = -c(t) & ; \text{if } ph = -1 \\ \omega = 0 & ; \text{if } ph = 0 \\ M = +c(t) & ; \text{if } ph = +1 \end{cases}$$

$$\begin{cases} ph \text{ switches from } -1 \text{ to } 0 & ; \text{when } fl^{-}_{slip} \text{ tends to become } > 0 \\ ph \text{ switches from } 0 \text{ to } -1 & ; \text{when } fl^{-}_{stick} \text{ tends to become } > 0 \\ ph \text{ switches from } 0 \text{ to } +1 & ; \text{when } fl^{+}_{stick} \text{ tends to become } > 0 \\ ph \text{ switches from } +1 \text{ to } 0 & ; \text{when } fl^{+}_{slip} \text{ tends to become } > 0 \end{cases} \tag{24}$$

$$\begin{cases} fl^{-}_{slip} = +\omega \\ fl^{-}_{stick} = -M - c(t) \\ fl^{+}_{stick} = M - c(t) \\ fl^{+}_{slip} = -\omega \end{cases}$$

The dry friction torque capacity, c, is thought to be given as a direct function of time. When the clutch is a single disc clutch, $c(t) = \mu \cdot F_{act}(t) \cdot$ radius, where μ is the coefficient of friction and F_{act} is the actuating force. The actuating force is given by the hydraulic pressure, which is treated here as a known function of time.

To be more precise, there can be four variables for describing the torque capacity: c^{-}_{slip}, c^{-}_{stick}, c^{+}_{stick} and c^{+}_{slip}. The superscripts refer to slipping direction and the subscripts distinguish between slip and stick friction. The slip friction is also allowed to vary slightly with sliding velocity. The meanings of the four capacities are shown in figure 9. It is now quite possible to obtain characteristics for a one-way clutch, simply by assigning:

$$\begin{Bmatrix} c^{-}_{slip} \equiv -\infty \\ c^{-}_{stick} \equiv -\infty \\ c^{+}_{stick} \equiv 0 \\ c^{+}_{slip} \equiv 0 \end{Bmatrix} \text{ or } \begin{Bmatrix} c^{-}_{slip} \equiv 0 \\ c^{-}_{stick} \equiv 0 \\ c^{+}_{stick} \equiv +\infty \\ c^{+}_{slip} \equiv +\infty \end{Bmatrix} \tag{25}$$

This approach is used, in one way or another, in all the papers for describing the dry friction characteristics of the clutches. Slightly different notations are used, e.g. in paper A-C, a factor, α, is introduced instead of distinguishing between slip and stick capacities. There, $\alpha(\omega) \cdot c^{-}_{stick}$ corresponds to c^{-}_{slip} and $\alpha(\omega) \cdot c^{+}_{stick}$ corresponds to c^{+}_{slip}.

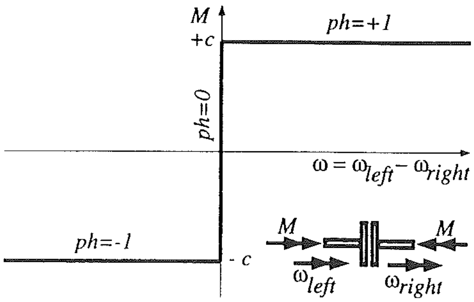


Figure 8: Clutch characteristics

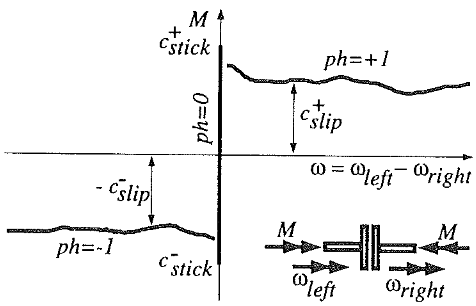


Figure 9: Clutch characteristics, with the four torque capacities: c^{-}_{slip}, c^{-}_{stick}, c^{+}_{stick} and c^{+}_{slip}

4.2 Influence on the Nature of the System

When the clutch is described using the equations in section 4.1, there is another type of dynamics in the system. This is referred to as *discrete dynamics* and the new (discrete) state variables are expressed in the phases of the clutches. There are continuous variables, referred to as *flags*, which tell **when** a phase switch should be carried out. Flags are related to state derivatives, since they give information about changes of the phases. For a system with n clutches, there are 3^n possible phase combinations, since each clutch can operate in 3 phases: negative slip, stick and positive slip. This kind of dynamic system is referred to as *multi-phase system*. If the numerical integration method is described in a flow scheme, it would be like that in figure 10 for an ordinary (*single-phase*) system and as in figure 11 for a multi-phase system. This approach can be regarded as just a systematic way of handling several initial value problems, spliced together.

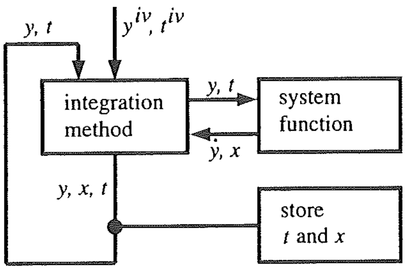


Figure 10: Flow scheme for numerical integration of an ordinary dynamic system (single phase dynamic system)

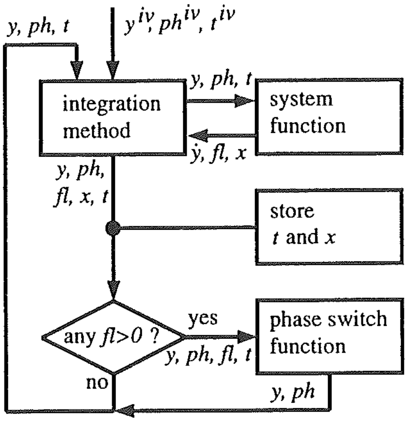


Figure 11: Flow scheme for numerical integration of a multi-phase dynamic system. It is assumed that the last time step before a phase switch is adapted so that the flag just passes zero by a very little, or actually infinite, amount.

As seen in figure 11, the system function has to be extended, as compared with figure 10 and equation 6. Another difference compared with the single-phase system, is that a *phase switch function* has to be given. The normal form, needed for simulation of a multi-phase system, becomes:

$$
\begin{aligned}
\left[\mathrm{y}^{\mathrm{iv}}, \mathrm{ph}^{\mathrm{iv}}, t^{iv}\right] &= \text{ known} \\
[\dot{\mathrm{y}}, \mathrm{fl}, \mathrm{x}] &= \text{ system function}(\mathrm{y}, \mathrm{ph}, t) \\
[\mathrm{y}, \mathrm{ph}] &= \text{ phase switch function}(\mathrm{y}, \mathrm{ph}, \mathrm{fl}, t)
\end{aligned}
\tag{26}
$$

This description can include cases where the transformation between state variables and physical variables has to change between different phase combinations. Furthermore, it is not obvious how the phase switch function should assign new values to the state variables. The answers to these questions are implicitly hidden in the equations of the whole system, not just in the component equations of the clutches. Let us study the example described in figure 12 and table 3.

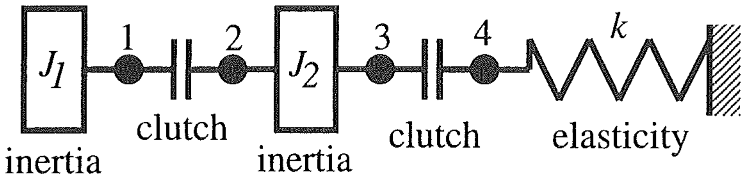


Figure 12: Example of multi-phase system. 5 components connected in 4 nodes

Table 3: Component equations for the system in figure 12

COMPONENT	COMPONENT EQUATIONS	
(left) inertia	$J_1 \cdot \dot{\omega}_1 = 0 - M_1$ (27)	
(left) clutch	$M_1 = M_2$ (28)	$\begin{cases} M_1 = -c_1(t) & ; \text{if } ph_1 = -1 \\ \omega_1 = \omega_2 & ; \text{if } ph_1 = 0 \\ M_1 = +c_1(t) & ; \text{if } ph_1 = +1 \end{cases}$ (29) phase switch conditions as in equation 24
(right) inertia	$\omega_2 = \omega_3$ (30)	$J_2 \cdot \dot{\omega}_2 = M_2 - M_3$ (31)
(right) clutch	$M_3 = M_4$ (32)	$\begin{cases} M_3 = -c_3(t) & ; \text{if } ph_3 = -1 \\ \omega_3 = \omega_4 & ; \text{if } ph_3 = 0 \\ M_3 = +c_3(t) & ; \text{if } ph_3 = +1 \end{cases}$ (33) phase switch conditions as in equation 24
elasticity, connected to ground	$\dot{M}_4 = k \cdot (\omega_4 - 0)$ (34)	

A possible transformation between state variables and physical variables is shown in table 4. As seen here, the transformation has to be changed between different phase combinations. In all phase switches, the state variables should be given new values. These should be chosen with respect to how the physical quantities behave in a phase switch. Velocities ω_1 and ω_2 are velocities of inertias, which cannot be changed discontinuously without the presence of impacts. There can be no impacts, because the clutches always switch phase when both clutch halves have the same velocities. Therefore, the first phase switch requirement is that ω_1 and ω_2 should be continuous in all phase switches. The torque M_4 is the torque of an elasticity, which cannot be changed discontinuously unless the end of the elasticity changes position discontinuously. In a phase switch where the right clutch switches from slip to stick phase, the end of the elasticity starts to follow an inertia, and so its position has to be continuous. Therefore, the second phase switch requirement is that M_4 should be continuous in a phase switch from slip to stick phase in the right clutch. In a phase switch where the right clutch switches from stick to slip phase, the end of the elasticity starts to move without kinematic constraint to the inertia, and so its position can be discontinuous. However, in such a phase switch, we do not need to determine the new value of M_4, since it is not a state variable after the switch. For this system, the phase switch requirements can be concluded as: all physical quantities involved in the new state variables should be continuous.

Table 4: Suitable transformation between state variables and physical variables for different phase combinations in the system in figure 12

PHASES		TRANSFORMATION BETWEEN STATE VARIABLES AND PHYSICAL VARIABLES	CONSTRAINTS	
left clutch	right clutch		ω_1 constrained to ω_2	M_4 constrained to t
stick	stick	$y = [\omega_1 \quad M_4]^T$	•	-
stick	slip	$y = [\omega_1]^T$	•	•
slip	stick	$y = [\omega_1 \quad \omega_2 \quad M_4]^T$	-	-
slip	slip	$y = [\omega_1 \quad \omega_2]^T$	-	•

In the beginning of this section, it was stated that there are 3^n phase combinations. However, it is not necessary to use that many different sets of state variables. For instance, in the example, $3^n = 3^2 = 9$ but there are only $4 = 2^2 = 2^n$ different sets of state variables in table 4, because there is no significant difference between the negative and positive slip phases. Therefore, the number of different sets of state variables can be reduced to 2^n. These questions become less important if the algorithm given in paper D is used, because the algorithm can automatically find an appropriate transformation between state variables and physical variables. No such model has been implemented as computer code, but in paper D it is shown that the algorithm **can** find appropriate transformations for a system with a gearbox with five clutches. (Five clutches make $2^5 = 32$ different phase combinations to consider, which requires a tedious work without the algorithm.)

In a gear train modelled with load dependent losses, there are also phases. These are characterized by the sign of the relative power flow in each gear mesh. Therefore, such models of gear trains make the system act as a multi-phase system. A study of such gear train models might be a subject of future work, although in normal powershift analysis the losses in the gear train are not significant.

5. Models for Simulation of Powershifts

5.1 Model for Dimensionless Equations

With a very simple model, like the one in figure 13, it is possible to make the equations dimensionless in a nice way. This is described in paper A.

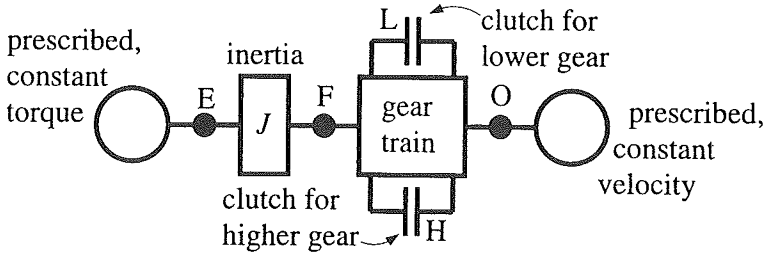


Figure 13: Model, whose equations can be made dimensionless.
(node notations: E=Engine power source, F=engine Flywheel, L=clutch for Lower gear, H=clutch for Higher gear and O=gearbox Output)

The advantages of the dimensionless form are:

- ☐ All speed (and torque) ratios between shafts are eliminated. This information is concluded in one single quantity, the span, s, which is the ratio between the ratios of the lower and higher gears.
- ☐ Engine torque and vehicle velocity are dimensionless. They are scaled to the value 1.
- ☐ The clutch torque capacities are described in dimensionless quantities, c_{*L} and c_{*H}. They are scaled in a way that $c_{*L} = 1$ and $c_{*H} = 1$ correspond to the minimum level necessary for transmitting the engine power on each gear.
- ☐ The engine flywheel inertia is made dimensionless. This requires division by a reference time t_{ref}. It is suitable to chose t_{ref} so that the entire shift operation takes place within the time interval $0 < t_* < 1$, where $t_* = t/t_{ref}$ = dimensionless time.
- ☐ Phenomena such as *tie-up*, *flare*, *torque phase* and *inertia phase* can be strictly defined. It is also shown that one of the clutches always can be designed as a *one-way clutch*, resulting in a shift with perfect timing. When power is transmitted from the engine to the load, this is the clutch of the lower gear. In the case of engine braking, it is the other clutch.
- ☐ The dimensionless equations can be interpreted back to a physical system. Such a system is shown in figure 14. It works in translation instead of rotation. The hands corresponds to the clutches and we can understand the task of the control system by imagining that the hands are our own hands.

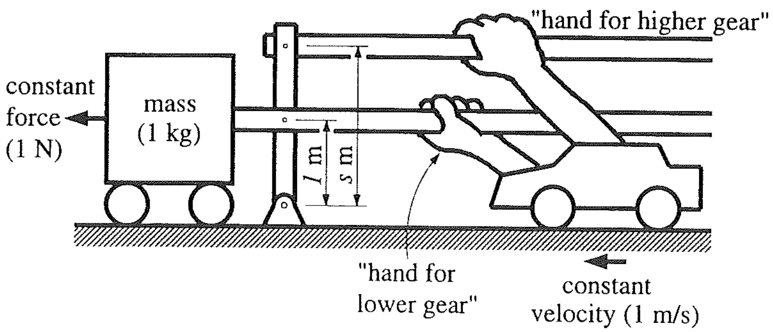


Figure 14: Physical translational system, found by interpretation of the dimensionless equations of the system in figure 13. The span, s, is the ratio between the ratios for the lower and higher gears.

In paper A, there are a large number of diagrams which show how the dimensionless quantities vary with the two dimensionless torque capacities, one for each clutch. In figure 15, about the same is shown for the case when the clutch of the lower gear is a one-way clutch. This gives only one independent variable, namely the dimensionless torque capacity of the higher clutch. Therefore, these diagrams ought to be easier to follow. Equations are not derived here, because they are a special case of the dimensionless equations in paper A.

In figure 15, the following dimensionless variables are used: s=ratio span ($1 < s < \infty$), c_{*H}=dimensionless torque capacity of the clutch for higher gear, M_{*O}=dimensionless output torque, ω_{*E}=dimensionless engine velocity, ω'_{*E}=dimensionless engine acceleration and J_*=dimensionless engine flywheel inertia ($J_* > 0$).

When the engine velocity is constant, the condition is referred to as *torque phase*. In figure 15, it can be seen that the torque phases last until the torque capacity of the higher clutch has passed the value of 1 in either direction. In the torque phase on the lower gear, more and more of the output torque is lost the more the capacity is increased. In the torque phase on the higher gear, the output torque is constant, thanks to the one-way clutch.

When the engine velocity changes, the system state is referred to as *inertia phase*. In figure 15, it can be seen that the engine accelerates or decelerates depending on whether the torque capacity of the higher clutch is smaller or larger than 1. The magnitude of acceleration or deceleration is larger, the further away from the value 1 the capacity is. The output torque behaves similarly. This is not desirable because, for an upshift, the torque has already achieved the final value ($M_{*O} = 1/s$) and for a downshift, the torque change is in the wrong direction.

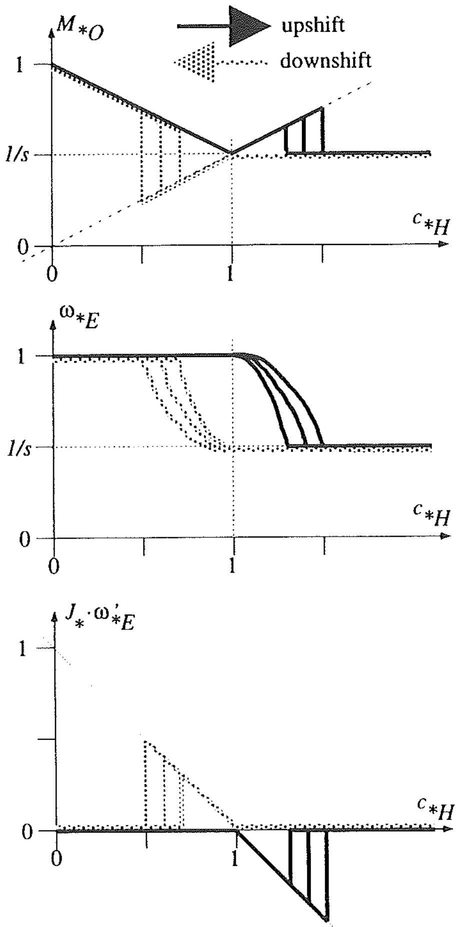


Figure 15: Upshift and downshift operations in dimensionless quantities. There is a one-way clutch on the lower gear

5.2 Two-clutch Model

The model in figure 16 is used in paper A. There are two clutches, one for each gear involved in the gear shift. The essential surrounding components are included. For instance, the engine power source is modelled by the steady state characteristics of the engine, i.e. M_E is an algebraic function of ω_E and the throttle position (or corresponding quantity for other than gasoline engines). The throttle position is assumed to be a known function of time. The exact layout of the gear train component depends on the gearbox studied, and between which two gears the shift is to be carried out. In figure 17, the layout is exemplified for the gearbox ZF 4HP18, shifting between third and fourth gear. Note that the gearbox housing is inside the gear train component, which makes it possible to treat brakes as a special case of clutches.

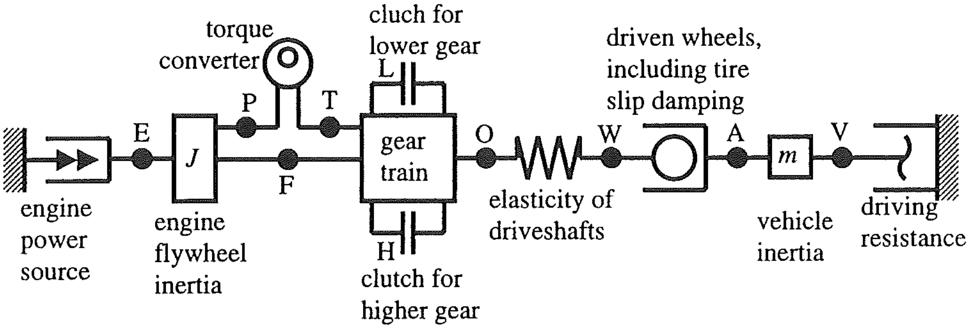


Figure 16: Two-clutch model.
(Node notations: as in figure 13 and P=Pump of torque converter, T=Turbine of torque converter, W=Wheel center, A=driven Axle and V=Vehicle)

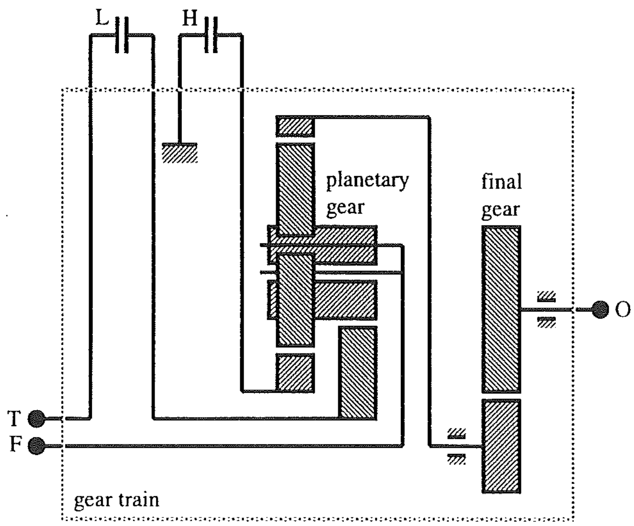


Figure 17: Gear train and clutches in the two-clutch model in figure 16 for the case of gearbox ZF 4HP18, shifted between third and fourth gear. The third gear is a torque-split gear and the fourth is a lock-up gear (or, actually, the torque converter is bypassed).

5.3 Multi-clutch Model

In order to have one gear train component for all gearshifts, it should be equipped with all the clutches present in the real gearbox. Driveline models with such a gear train component are presented in papers B and C. The gear train component can be described as shown in figure 18, in the case of the gearbox ZF 4HP18, which has five clutches. The advantages of including all the clutches are, first, the lucidity of having one component for all gear shifts and, second, that *generalized ratio change operations* can be analyzed. Generalized ratio change operations mean that not exactly one clutch should be disengaged and one engaged. This can, for instance, be a lock-up maneuver, where only one clutch is engaged, or a shift that demands two disengaging clutches and two engaging ones. Such a shift is hardly ever used in practice, unless some intermediate gear is skipped, e.g. when shifting directly from fourth to second gear.

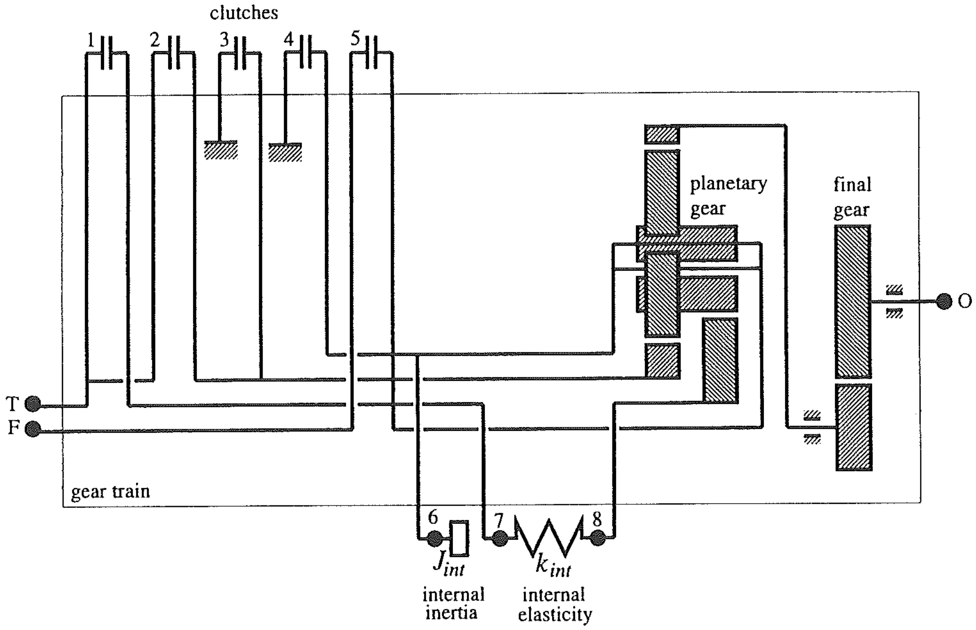


Figure 18: Example of gear train for multi-clutch model (gearbox ZF 4HP18). (Node notations: as in figure 16 and **1–5**=clutches number **1–5** and **6–8**=nodes for connecting internal inertia and elasticity)

As seen in figure 18, the gear train component is connected to an *internal inertia* and an *internal elasticity*. For most phase combinations, these have the values $J_{int} = 0$ and $1/k_{int} = 0$. However, when too many clutches slip, there are too many kinematic degrees of freedom. Then, J_{int} is given a small positive value. The internal inertia is given the physical interpretation as the planet carrier. Actually, such an interpretation is of minor interest. What is important is only that the nodes of the gear train accelerate "very rapidly" in the proper direction, satisfying the velocity equations of the gear train. When too many clutches stick, the gear train becomes statically indeterminate, which requires extra elasticity. Then, $1/k_{int}$ is given a small positive value. The internal elasticity is given the physical interpretation of one of the slenderest shafts in the gearbox. Actually, the physical interpretation is not of any major interest here. The important thing is that the nodes of the gear train change their torques "very rapidly", satisfying the torque equations of the gear train. A question for further work is to study whether these internal dynamic properties can be introduced without any concrete assumptions of exactly which shaft, etc. has inertia or elasticity. The algorithm in paper D can, even in its present state, detect these indeterminate situations. Another way to handle statically indeterminate conditions is, consequently, to introduce a very stiff elasticity in each clutch when it sticks. This is used in reference [3].

5.4 Model Including Surrounding Systems

The inertia of a transversely mounted power unit[3] is likely to participate quite a lot in dynamics of the system. This, and a model of the vehicle body, are included in paper C. In paper F, almost the same model is used, but it is equipped with a simple model of the passenger, see also figures 19-21, in order to estimate passenger comfort. Normally, comfort is defined as some sort of change in acceleration of the human being. Obviously, this is a possible output from this model. However, exactly how to quantify the comfort is beyond the scope of this work. A more general result from this model is that it connects common dynamic models to the driveline model. Note especially how the gear train component has to be equipped with a node, corresponding to the gearbox housing.

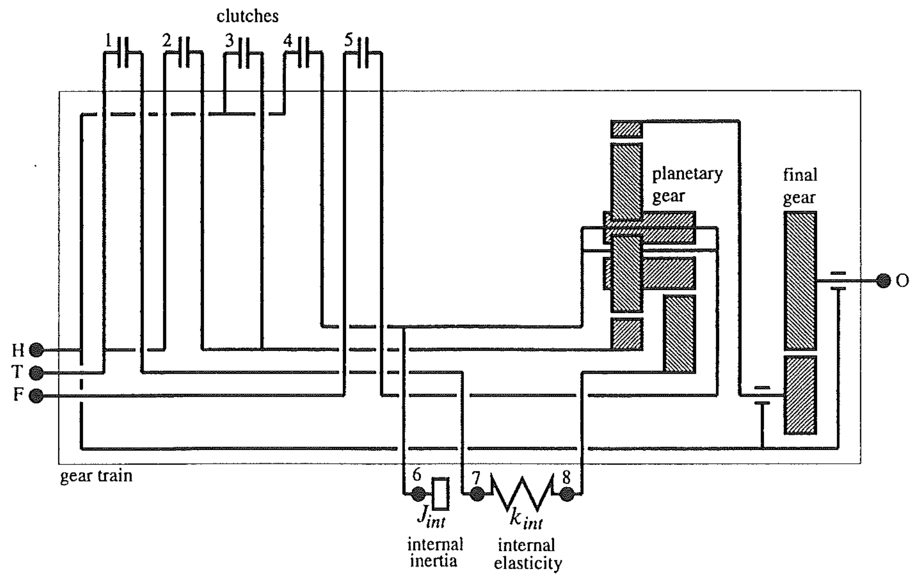


Figure 19: Gear train for the driveline model in figure 20.
(Node notations: as in figure 18 and H=gearbox Housing)

[3] "Power unit" means engine block and gearbox housing, rigidly connected to one another.

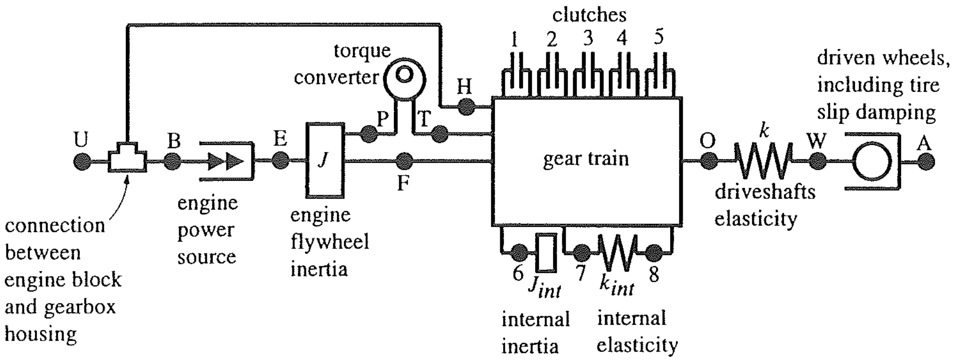


Figure 20: Driveline model, for connection to surrounding systems.
(Node notations: as in figure 19 and U=power Unit block and B=engine Block)

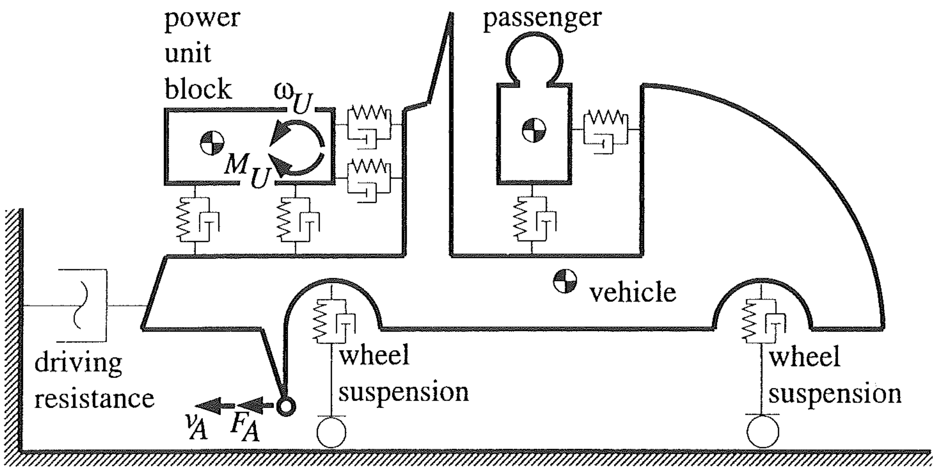


Figure 21: Surrounding systems for the driveline model in figure 20. (Node notations: as in figure 20)

6. Simulation and Test Results

It is important to make the dynamic model detailed enough to obtain the interesting phenomena. It is also important not to include too many details, since then it becomes very time consuming to create the model and to interpret the results. There is also a risk of numerically unstable solutions if there are too many details. To judge whether or not a model is detailed enough, the objective has to be defined, and simulation results must be compared with the reality.
The objective of the models is to estimate shift quality, or at least to simulate the physical quantities that affect the shift quality. A definition of shift quality is given in paper F. In short, shift quality is determined with regard to three aspects:

1. **Power supply** to the vehicle should not drop unnecessarily much during the shift.
2. **Wear of the clutches** should be minimized.
3. **Passenger comfort** should be maximized.

Roughly, the corresponding physical quantities are:

1. **Tractive force** on the vehicle
   (The vehicle normally has approximately constant velocity during the gear shift, and thus the power is proportional to the force.)
2. **Energy loss** in the clutches
   (The wear is mainly due to high temperature caused by this energy loss.)
3. **Acceleration** of the passenger
   (Comfort is assumed to be characterized by changes in acceleration.)

Test results from paper A are used as a comparison to simulations results. Tractive force and engine velocity are measured. In conclusion, they coincide rather well with the results from the simulations, and the computer time used was not annoyingly large. This probably means that the level of detail in the driveline model is appropriate to simulate all velocities and torques in the driveline. Thus, support for estimation of power supply and wear of the clutches can be extracted from the simulations. If passenger acceleration can be expressed in terms of tractive force, support for comfort estimation can also be found. Otherwise, vehicle and passenger models should be included, as is done in paper F.

However, in order to simulate the measured tractive force in paper A, the prescribed torque capacity functions of the clutches had to be modified[4)], as compared with what the hydraulic actuating pressure gave. The reason was guessed to be the oil film in the clutches. In papers E and F, this was verified analytically.

7. Consideration of Oil Film Phenomena

7.1 Engagement of a Single Clutch

Oil Film/Surface Model

Paper E describes a model for the conditions in a pair of friction surfaces. The oil film is described using Reynolds' equation, and the surface asperities are described as elastic springs, with a certain coefficient of dry friction. An engagement can be described by the following items:

- ☐ The actuating force is carried by a **contact pressure** on the asperities and an **oil film pressure**. In paper E, both these pressures are mean pressures over the whole surface. For thick oil films, the contact pressure is zero.
- ☐ The contact pressure, multiplied by the coefficient of friction, yields a **dry friction shear stress**.
- ☐ The oil film pressure **squeezes** out the oil, allowing the surfaces to approach one another. This squeeze makes the engagement into a dynamic problem, whereby the oil film thickness is a suitable state variable. The oil film is also sheared, which causes a **viscous shear stress**, dependent on the oil viscosity, the oil film thickness and the sliding velocity.

[4)] From another point of view, the dry friction based description of the clutches can be regarded as not satisfying. In addition to the dry friction torque capacity, a viscous torque coefficient is needed.

- ☐ Since the surfaces approach one another, the asperities are more and more compressed, and the dry friction shear stress increases. Also the viscous shear stress increases, because the oil film thickness decreases.
- ☐ The dry friction shear stress increases until the whole actuating force is carried by the asperities. Regarding the oil film thickness, the viscous shear stress should also reach a maximum value and stay there. However, the oil viscosity decreases due to the temperature rise, and the viscous shear stress actually fades out to almost zero. (Also, when the oil film/surface model is used in a clutch in a transmission system, the relative velocity becomes zero when the clutch sticks. Then the viscous shear stress becomes exactly zero.)

Model of the Clutch

In paper E, the oil film/surface model is applied to a multi-disc clutch. Knowing the clutch geometry, Reynolds' equation can be solved and the shear stresses can be interpreted as torques, a dry friction torque and a viscous torque. The clutch torque is the sum of these.

A similar study of a band brake would be a subject for future work. Such a study would result in partial differential equations, since the oil film thickness varies both in time and along the drum periphery. In paper A, tests indicate that band brakes have larger time delay[5)] than multi-disc clutches. Disengagement of different clutches and brakes is also of interest to study.

Model of the Actuating System

In order to find the proper solutions, it is important to include the hydraulic actuating system. In the system in paper E, the clutch is actuated by a hydraulic pressure, via an orifice. The engagement is controlled by how quickly the surfaces approach one another. Roughly, this is determined by the orifice at the beginning of the engagement and by the oil film squeeze at the end.

7.2 Influence on Gear Shift Quality

The influence of the oil film phenomena on the gear shift quality is studied in paper F. The model from paper C (extended with a simple model of a passenger) is used for simulations. The dry friction torque capacities for the clutches are input data for this model. Numerical values for these are modified, relative to what the hydraulic pressure gives, in order to simulate the conditions with both dry and viscous friction in the clutches.

It is found that the oil film effects may disturb the timing and also affect the wear and comfort. The following is concluded for the engaging clutch in an upshift. (Note that tie-up and flare are no problem when the clutch of the lower gear is designed as a one-way clutch.)

- ☐ **Multi-disc clutch** with **few discs** give almost **no oil film effects**.
- ☐ **Multi-disc clutch** with **many discs** give **tie-up**, **worse comfort** but **less wear**.
- ☐ **Band brake** tends to give **flare**, **more wear** but **better comfort**.

A subject for future work might be to extend the model used in paper F with the model of the clutch and actuating system from paper E. Such a system is only sketched in paper F.

---

5) Here, "delay" means delay of clutch torque relative to hydraulic pressure.

8. Conclusions

When powershifts are to be simulated, it is essential to include both the gearbox and some of the surrounding components. As a minimum, for automotive applications, it is suggested that the following components should be included: engine power source (with steady state characteristics), engine flywheel inertia, torque converter, planetary gear train with its clutches, elasticity of driveshafts and tire slip damper. Then the boundary conditions are: engine throttle position, dry friction torque capacities for the clutches and vehicle speed.

With the aim of systemizing the modelling, also for dynamic transmission systems in general, the following conclusions are drawn:

- ☐ The ***state variables*** of the system should be interpreted as velocities of inertias and torques of elasticities.
- ☐ Equations for planetary gear trains should be written in **matrix form**, for lucidity. If the gear train is assumed to be free from losses, the torque equations can easily be found from the velocity equations, by means of power equilibrium.
- ☐ Each clutch should be modelled with strict Coulombian friction. This means that a *discrete state variable*, or *phase*, defining whether the clutch sticks or slips, is introduced. The whole system then becomes a ***multi-phase system***, which changes character every time a clutch switches between stick and slip phase.
- ☐ The equations of the system should be written in an **explicit form**, which is a form where the *state derivatives* can be explicitly calculated as functions of the state variables and time. This form is difficult to find when *constraints* are present. An algorithm which handles this problem is developed.
- ☐ The clutches are **oil-immersed**. The oil has to be squeezed out before the dry friction torque can be developed. Also, a viscous torque is present. In order to simulate these phenomena, a model of the oil film and the friction surface is needed. The actuating system of the clutch may also be important to model.

Appendix A: Relationship between Velocity and Torque Equations on Homogeneous Form

In this appendix the relationship between matrices C and D in equation 18 is studied. It is shown how D can be derived from C, or actually from a preliminary variant, C_0. The derivation is based on the matrix manipulation *singular value decomposition*[6), *svd*.

Start from a matrix C_0, satisfying the following velocity equation:

$$C_0 \cdot \omega = 0 \tag{35}$$

---

6) A singular value decomposition makes it possible to write any matrix A as $U \cdot S \cdot V^T$, where S is a diagonal matrix of the same size as A and with **non-negative** diagonal elements in decreasing order. The matrices U and V are both square and orthogonal. Reference [7] describes the singular value decomposition more thoroughly.

Information about the sign conventions is given in the form of a vector s, with elements, $s_i = +1$ or $s_i = -1$, such that $\omega_i \cdot s_i \cdot M_i =$ **input**[7] power in node number i. For the example in figure 4, $\mathrm{s} = [+1 \quad -1 \quad -1]$. Equation 35 is then subject to the following operations:

$$\mathrm{C_0} \cdot \omega = 0$$

$$\text{svd of } \mathrm{C_0} \text{ gives: } \underbrace{\mathrm{U} \cdot \mathrm{S} \cdot \mathrm{V}^T}_{\mathrm{C_0}} \cdot \omega = \mathrm{U} \cdot \mathrm{S} \cdot \mathrm{W} \cdot \omega = 0$$

$$\mathrm{U}^T \cdot \mathrm{U} \cdot \mathrm{S} \cdot \mathrm{W} \cdot \omega = \mathrm{U}^T \cdot 0$$

$$\mathrm{S} \cdot \mathrm{W} \cdot \omega = 0$$

$$\underbrace{\begin{bmatrix} \mathrm{S_1} & 0 \\ 0 & 0 \end{bmatrix}}_{\mathrm{S}} \cdot \begin{bmatrix} \mathrm{W}_{\omega 1} \\ \mathrm{W}_{\omega 2} \end{bmatrix} \cdot \omega = 0 \text{ ; where } \mathrm{S_1} \text{ has positive diagonal elements}$$

$$\mathrm{S_1} \cdot \mathrm{W}_{\omega 1} \cdot \omega = 0$$

$$\mathrm{S}_1^{-1} \cdot \mathrm{S_1} \cdot \mathrm{W}_{\omega 1} \cdot \omega = \mathrm{W}_{\omega 1} \cdot \omega = 0$$

Since $\mathrm{W}_{\omega 1} \cdot \omega = 0$, C can be identified as $\mathrm{C} = \mathrm{W}_{\omega 1}$. Then equation 37 is obtained. The rows of $\mathrm{W}_{\omega 1}$ are orthogonal. Therefore, equation 37 contains no linearly dependent equations, which could have been the case in equation 35.

$$\mathrm{C} \cdot \omega = 0 \tag{37}$$

Introduce a transformed velocity vector ω' such that $\omega = \mathrm{W}_{\omega 2}^T \cdot \omega'$. The columns of $\mathrm{W}_{\omega 2}^T$ are then a set of orthogonal base vectors, describing directions of vector ω, satisfying the kinematic constraints: $\mathrm{W}_{\omega 1} \cdot \omega = 0$. In other words, varying the values of the elements in ω' can give all combinations of velocities, allowed with respect to the kinematic constraints, and no other combinations.

If the torque equations had been given, they could have been manipulated in the same way, yielding: $\mathrm{W_{M1}} \cdot \mathrm{M} = 0$ and $\mathrm{M} = \mathrm{W}_{\mathrm{M2}}^T \cdot \mathrm{M}'$. Here, varying the values of the elements in M' can give all combinations of torques, allowed with respect to the torque equilibrium, and no other combinations.

The power equilibrium is written:

$$\begin{aligned} &\sum \text{input power} = 0 \Rightarrow \sum_{\forall i} \omega_i \cdot s_i \cdot M_i = 0 \Rightarrow \omega^T \cdot \mathrm{diag(s)} \cdot \mathrm{M} = 0 \Rightarrow \\ &\Rightarrow \omega'^T \cdot \mathrm{W}_{\omega 2} \cdot \mathrm{diag(s)} \cdot \mathrm{W}_{\mathrm{M2}}^T \cdot \mathrm{M}' = 0 \end{aligned} \tag{38}$$

The notation diag(s) means a diagonal matrix with the elements of s on the diagonal. Power equilibrium should be satisfied for all allowed combinations of velocities and torques in ω and M. Then, equation 38 should be satisfied for arbitrary values of the elements in ω' and M'. One possibility is $\mathrm{W_{M2}} = \mathrm{W}_{\omega 1} \cdot \mathrm{diag(s)}$. Then, $\omega'^T \cdot \mathrm{W}_{\omega 2} \cdot \mathrm{diag(s)} \cdot \mathrm{W}_{\mathrm{M2}}^T \cdot \mathrm{M}' = \omega'^T \cdot \mathrm{W}_{\omega 2} \cdot \mathrm{diag(s)} \cdot (\mathrm{diag(s)})^T \cdot \mathrm{W}_{\omega 1}^T \cdot \mathrm{M}' = \omega'^T \cdot \mathrm{W}_{\omega 2} \cdot \mathrm{W}_{\omega 1}^T \cdot \mathrm{M}'$, because $\mathrm{diag(s)} \cdot (\mathrm{diag(s)})^T =$ the identity matrix according to the definition of s. Furthermore, the rows of $\mathrm{W}_{\omega 2}$ and $\mathrm{W}_{\omega 1}$ are orthogonal, which yields $\omega'^T \cdot \mathrm{W}_{\omega 2} \cdot \mathrm{W}_{\omega 1}^T \cdot \mathrm{M}' = \omega'^T \cdot 0 \cdot \mathrm{M}' = 0$.

[7] The opposite sign conventions in vector s can also be used.

The matrices $\mathbf{W}_{M1}$ and $\mathbf{W}_{M2}$ should be orthogonal, which most easily is satisfied if $\mathbf{W}_{M1} = \mathbf{W}_{\omega 2} \cdot \mathrm{diag}(\mathbf{s})$. The torque matrix, $\mathbf{D} = \mathbf{W}_{M1}$, can therefore be obtained as:

$$\mathbf{D} = \mathbf{W}_{\omega 2} \cdot \mathrm{diag}(\mathbf{s}) \tag{39}$$

In MATLAB-code, see reference [8], this derivation can be written as the function, listed in figure 22. The function has $\{\mathbf{C}_0, \mathbf{s}\}$ as input parameters and $\{\mathbf{C}, \mathbf{D}\}$ as output parameters.

```
function [C,D]=powequil(C0,s)

[U,S,V]=svd(C0);
W=V';
rankS=rank(S);
Ww1=W([1:rankS],:);
Ww2=W([rankS+1:size(W)],:);
C=Ww1;
D=Ww2*diag(s);
```

Figure 22: MATLAB-code for derivation of torque matrix

References

[1] FÖRSTER, Hans Joachim. Getriebeschaltung ohne Zugkraftunterbrechung. *Automobil Industrie*, pages 60–76, October, 1962.

[2] WINCHELL, F.J. and ROUTE, W.D. Ratio Changing the Passenger Car Automatic Transmission. *Design Practices - Passenger Car Automatic Transmissions*, pages 79–105, SAE, 2nd ed., New York, 1973.

[3] LASCHET, Andreas. Simulation von Antriebssystemen: Modellbildung der Schwingungssysteme und Beispiele aus der Antriebstechnik. Springer-Verlag, Berlin Heidelberg, 1988.

[4] HEDMAN, Anders. *Mechanical Transmission Systems – a General Computer-based Method of Analysis.* PhD thesis, Division of Machine Elements, Chalmers University of Technology, Göteborg, Sweden, 1988.

[5] SANGER, D. J. Matrix methods in the analysis and synthesis of coupled differentials and differential mechanisms. *4th World Congress on the Theory of Machines and Mechanisms, Newcastle, UK*, pages 27–31, 1975.

[6] PICHARD, J. and BESSON, B. Proposition d'un modèle général représentatif des transmissions de puissance. Exemples d'exploation. *Ingenieurs de l'automobile*, pages 183–191, Number 4, 1977.

[7] GOLUB, Gene H. and VAN LOAN, Charles F. Matrix Computations. North Oxford Academic Publishers Ltd, London, 1986.

[8] The MathWorks, Inc. PRO-MATLAB for Sun Workstations. January 31, 1990.

Paper A

JACOBSON, BENGT: *Vehicle Driveline Mechanics during Powershifting*, Report No 1990–05–30 (Licentiate of Engineering thesis), Machine and Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1990

Abstract

The main objective of this study is to simulate the mechanics of a driveline during powershift. A powershifting transmission normally works with two clutches in operation simultaneously, one clutch engaging and the other disengaging. By proper control of the clutches one can make a ratio change very smooth and without too much wear of the clutches.

A model suitable for a passenger car driveline has been constructed. The simulation results of this model are verified in tests. The model simulates the global mechanics, e. g. engine speed and vehicle traction force. Local mechanics, such as squeal in clutches, have not been included.

There is also a simplified, dimensionless model. Using this model it is easy to define and describe the essential phenomena during a powershift.

The work is valid for ordinary gears as well as planetary gears. The clutches may be ordinary clutches, one-way clutches or brakes. Input to the models are the torque capacities of the clutches as functions of time.

The driveline modelling technique is somewhat unconventional. The differential equations are written with velocities of rotating bodies and torques of torsional springs as state variables. This is more convenient since the absolute angular positions are of no interest, but the torques are. In addition, connecting points without inertia are used. This makes the results easier to comprehend and the numerical solutions more stable. Furthermore, the clutches are modelled as components that either stick or slip, i. e. as components with Coulombian friction.

Keywords:
automatic transmission
powershift
gear shift
clutch
Coulombian friction
driveline
model
dynamic
simulation

Acknowledgements

This work has been carried out under the guidance of Professor Göran Gerbert and Dr Anders Hedman at the Division of Machine and Vehicle Design (former Division of Machine Elements), Chalmers University of Technology.

I wish to express a special thank to both my very qualified supervisors Professor Gerbert and Dr Hedman. My friends at SAAB Automobile (former SAAB Scania car division) have also contributed considerably to the realization of this project.

Furthermore, all my colleagues at the Division deserve many thanks. Especially I would like to thank Professor Mart Mägi and M.Sc. Kjell Melkersson for inspiring discussions and Mrs Maud Abrahamsson for the Christmas tree.

Finally I would like to thank my girlfriend Titti, for kindly listening to my endless talk of clutches and planetary gears.

Bengt Jacobson
Göteborg, May 1990

Contents

Notation

φ	Angular position [rad]
ω	Angular velocity [rad/s]
ω_{rel}	Relative angular velocity [rad/s]
M	Torque [Nm]
$i = \begin{cases} \omega_{input}/\omega_{output} \\ M_{output}/M_{input} \end{cases}$	Transmission ratio
$s = i_1/i_2$	Ratio span
c^+	Static torque capacity (of a clutch), positive direction [Nm]
c^-	Static torque capacity (of a clutch), negative direction [Nm]
$\alpha = M_{clutch,dynamic}/c$	Fraction of c, α is weakly dependent of ω_{rel}.
μ	Coefficient of friction in clutch
$\nu = \omega_{turbine}/\omega_{pump}$	Velocity ratio in converter
$\mu = M_{turbine}/M_{pump}$	Torque amplification in converter
$\lambda^* = M_{pump}/\omega_{pump}^2$	Torque capacity in converter $[Nm/(rad/s)^2]$
J	Moment of inertia $[kgm^2]$
k	Torsional linear stiffness [Nm/rad]
d	Torsional linear damping [Nm/(rad/s)]
t	Time [s]

Subscripts

IN	Input shaft
OUT	Output shaft
C	Clutch
L	Clutch for lower gear (with high ratio, i)
H	Clutch for higher gear (with low ratio, i)
ref	Reference quantity, used for dimensionless quantities
$*$	Dimensionless quantity (see pp A32 and A38)

Others

Vectors (≡column matrices) are marked by an arrow above: $\vec{\omega}$
Vectors (≡row matrices) are written within parentheses: (x, y, z)
Matrices are written with bold upper-case: $\mathbf{I}$
Transposed matrices and vectors have the superscript T: $\mathbf{I}^T$, $\vec{\omega}^T$, $(x, y, z)^T$
Differentiation with respect to real time t is marked by a dot: $\dot{\omega}$
Differentiation with respect to dimensionless time t_* is marked by a prime: ω'
Initial values have the superscript IV: ω^{IV}

Chapter 1

Introduction

The subject of this work is to study the change of speed ratio in a vehicle with powershifting transmissions, which in practice means automatic transmissions.

The ratio change in powershifting transmissions is very easy to describe and perform in principal. The gearbox can be regarded as several branches for transmitting the power. There is one branch for each ratio. Each branch includes a clutch, see figure 1.1.

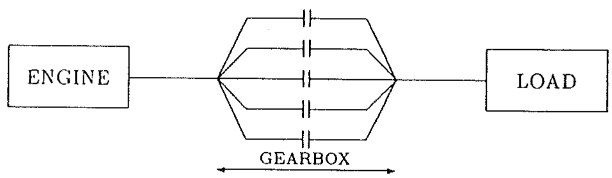


Figure 1.1: *Schematic transmission with several ratios*

In ordinary driving, just one of these clutches is activated. When changing ratio that clutch is disengaged. Simultaneously, the clutch for the next ratio is engaged. This sequence is shown in figure 1.2.

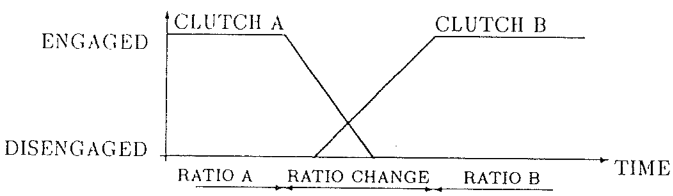


Figure 1.2: *Schematic description of ratio change (A=previous gear, B=new gear)*

A2

It is difficult to fail completely when changing ratio. However, it is complicated to accomplish a ratio change of good quality, i.e. to choose the right disengaging and engaging functions.

A ratio change of good quality requires:

- Long transmission life (low energy losses in the clutches)
- Passenger comfort (smooth output torque)
- Absence of squeal (proper conditions near the clutches)

A complete analysis of the ratio change quality should be based on:

- Dynamics of the control system (control technique, hydraulics)
- Characteristics of the clutches (tribology, chemistry)
- Global dynamics of the driveline (mechanics)
- Local dynamics near the clutches (mechanics)

The present work deals with the global dynamics.

Chapter 2

Literature review

The analysis of the mechanics of ratio changes presumes mathematical models for the gearbox and the surrounding driveline. The most important attributes of the gearbox are the speed ratios and the clutches involved, which are described as components that either stick or slip. The most important attributes of the driveline are normally the torque and inertia of the engine, the elastic components (mainly the drive shafts or half shafts) and the damping components (mainly the torque converter and the tyre slip).

In practice, there are two types of gearboxes: those with ordinary gears and those with gears in planetary arrangements. The latter are undoubtably the most common types in automatic transmissions.

[Förster 1957] and [Förster 1962] treat only gearboxes with ordinary gears. A catalogue of different types of ratio changes is presented. The works are very fundamental and give a good understanding of the phenomena *flare* (German: *negative Überschneidung*) and *tie up* (*positive Überschneidung*). These phenomena occur when the clutches involved are either too weakly engaged or engaged too hard. The engine then overspeeds or chokes. Elasticities of the driveline are neglected and so is the damping.

Similar work has been done by [Winchell 1962], with the difference that planetary gears are treated. A useful discussion on the practical aspects of automatic transmissions is included and describes many aspects of the layout of a planetary gears and clutches.

[Ott 1972] shows equations for two planetary gears in series, which were both shifted. No elastic components were considered. The damping action of the torque converter was included, using the steady state characteristics.

[Kraft 1972] and [Kraft 1974] are follow-up works of [Förster 1957] and [Förster 1962]. They are concerned with planetary gears and how the equations correspond to those for ordinary gears. The difference obtained is that an extra inertia occurs inside the planetary gears. The works also treat thermal and wear aspects of the clutches.

[Ishihara 1968] and [Ishihara 1970] treat planetary gears as well as the equations of the hydraulic control system. The latter work presents verifying tests. [Ito 1972] is a kind of follow-up work with most of its attention directed to the hydraulic control system.

The work of [Shindo 1980] and [Koch 1972] considers the elastic components of the driveline, which have been neglected by the previous authors. They deal with numerical examples of specific transmissions.

As seen from this review, very little has been done in this field during the last ten years. Interest during this period seems to have been attracted to the control technique for automatic transmissions, while basic mechanics seems to have disappeared from the scene.

Chapter 3

Component equations

The driveline is composed of *physical components*, such as engine and drive shafts. There are also *engineering phenomena*, such as tyre slip and driving resistance. Finally there are *mechanical properties*, such as inertia, elasticity and damping. Building blocks from these three groups cooperate to form the mathematical model of the system. The most essential properties of every building block should be described by mathematics, e. g. by equations (algebraic or differential), diagrams or tables. This chapter presents the mathematics for the building blocks used. The building blocks are referred to as "components", although not all building blocks are physical components. They may be regarded as *model components*.

The most important components presented in this chapter are the *gearbox transmission* and the *clutch*. Furthermore, the *torque converter*, the *tyre slip*, the *engine*, the *driving resistance* and the *linear damper* are presented. These components do not store any energy. They just produce, transmit or consume energy. Energy storing components described are *inertia* and *elasticity*. They store kinetic and potential energy, respectively. See table 3.1.

The reason for separation into components that store and do not store energy is perhaps more easily understood in light of the next chapter, which deals with the system equations. The components that do not store energy only generate algebraic equations. The energy storing components generate differential equations.

The subject of this work requires, as described in chapter 1, proper models for the gearbox transmission and the clutches. These components are treated more carefully, as is described in the beginning of this chapter. Moreover, the gearbox with clutches must cooperate with the rest of the driveline components. These components are modelled in a more conventional way, as is described at the end of this chapter.

Table 3.1: *Classification of components*

Components					
Components not storing energy				Components storing energy	
Energy producing components	Energy transmitting components		Energy consuming components	Inertia	Elasticity
	without losses	with losses			
Engine (driving)	Gearbox transmission, Clutch (sticking)	Torque converter, Tyre slip, Clutch (slipping)	Driving resistance, Engine (braking)	Flywheel, Vehicle	Drive shafts

3.0.1 Positive senses of velocity and torque

There are many ways of defining positive senses of vector quantities, such as velocity and torque. The basic rule used in this report is that velocity and torque are defined at the same point, a *node*. Furthermore, the product of velocity and torque is power. Therefore, power is also a quantity defined at a node. The positive senses of velocity and torque are chosen in such a way that the power is positive when energy is transmitted in an assumed direction. This direction is defined as a positive sense, or direction, of the power.

In this report, the power direction is from left to right in the figures. All nodes are therefore drawn on horizontal lines. This basic rules leaves two ways of defining positive senses, as shown in figure 3.1.

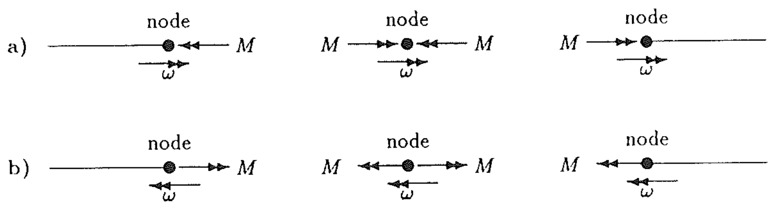


Figure 3.1: *Two ways of defining positive senses of velocities and torques at a node*

There is no reason to exclude either of the two ways given in figure 3.1. However, it is often convenient not to mix them, since the equations for an intermediate component would change then.

This work deals with clutches. When both clutch halves are studied, there will be two nodes, i. e. two velocities and two torques. However, it is suitable also to define relative velocity and clutch torque. The product of these quantities is power. Therefore, clutch power is also defined. The positive senses of relative velocity and clutch torque are chosen as:

$\omega_{rel} = \omega_{left} - \omega_{right}$
$M_{clutch} = M_{left} = M_{right}$

The clutch power is then the power loss in the clutch, since:

$P_{clutch} = M_{clutch} \cdot \omega_{rel} = P_{left} - P_{right} = P_{input} - P_{output} = P_{loss}$

Let us assume an energy consuming friction model, e. g. Coulombian friction. Then the power loss must always be positive (or zero). The clutch torque must then have the same sign as the relative velocity.

3.1 Gearbox transmission

When shifting between two speed ratios in a gearbox, there are two branches in which the power may be transmitted; one for each ratio. In order to control which way the power is transmitted, each branch includes a clutch.

To avoid interruption in the transmission of power, the torque in a clutch is controlled when the clutch is slipping. Therefore, the clutches are *force conditioned* (German: *kraftschlüssig*) rather then *form conditioned* (*formschlüssig*).

3.1.1 Two-clutch gearbox with two power shafts

In the simplest and absolutely most common case there is just one clutch included in each branch. This makes two active clutches in a gear shift. Furthermore, there are usually just one input and one output shaft. Such a gearbox can be drawn schematically as in figure 3.2.

Assume that the transmission is linear with respect to velocity. This implies that the velocity equations can be written as linear equations. Examples of such transmissions are all gear and chain transmissions and, if the slip is neglected also, for example belt transmissions.

The most common design of a transmission results in two degrees of freedom with respect to velocity (ω-dof). The IN- and OUT-shafts have to be linearly independent. This means that the velocity equations can be written as follows:

$$\begin{bmatrix} \omega_{relL} \\ \omega_{relH} \end{bmatrix} = \begin{bmatrix} 1/i_{L,IN} & -i_{L,OUT} \\ 1/i_{H,IN} & -i_{H,OUT} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN} \\ \omega_{OUT} \end{bmatrix} \quad \text{or} \quad \vec{\omega}_{rel} = \mathbf{I}_{\omega} \cdot \vec{\omega} \tag{3.1}$$

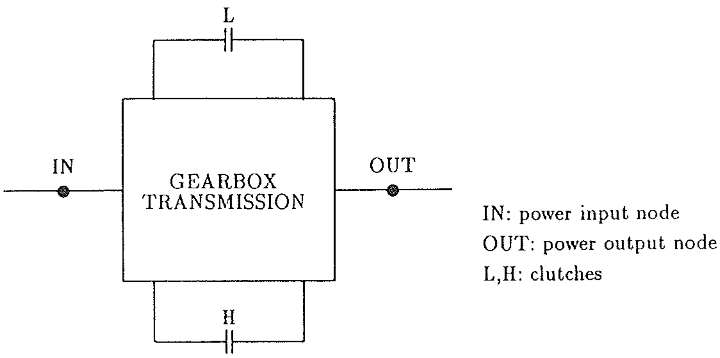


Figure 3.2: *Two-clutch gearbox with one input and one output shaft*

Here, the i-constants are speed ratios. They are determined by ratios of appropriate radii. For gear transmissions this is equivalent to ratios of the corresponding number of teeth.

The gearbox transmission is assumed to contain neither energy storing components (inertias or elasticities) nor energy consuming components (velocity or torque losses). The torque equations can then be written:

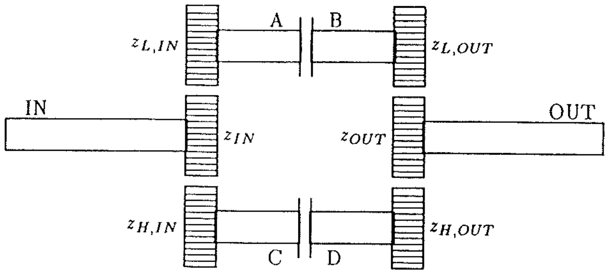
$$\begin{bmatrix} M_{IN} \\ M_{OUT} \end{bmatrix} = \begin{bmatrix} 1/i_{L,IN} & 1/i_{H,IN} \\ i_{L,OUT} & i_{H,OUT} \end{bmatrix} \cdot \begin{bmatrix} M_L \\ M_H \end{bmatrix} \quad \text{or} \quad \vec{M} = \mathbf{I}_M \cdot \vec{M}_C \tag{3.2}$$

The torque matrix $\mathbf{I}_M$ can be derived directly from the velocity matrix $\mathbf{I}_\omega$ as follows. More details are shown in appendix A.

$$\mathbf{I}_M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \mathbf{I}_\omega^T \tag{3.3}$$

3.1.2 Physical interpretation of i-constants

Figure 3.3 shows one possible layout for the schematic gearbox in figure 3.2. In this layout the i-constants can be interpreted as the <u>direct</u> speed ratios between the power shafts and the clutch halves.



$i_{L,IN} = -z_{L,IN}/z_{IN}$

$i_{L,OUT} = -z_{OUT}/z_{L,OUT}$

$i_{H,IN} = -z_{H,IN}/z_{IN}$

$i_{H,OUT} = -z_{OUT}/z_{H,OUT}$

Figure 3.3: *One possible layout of the gearbox. z denotes number of teeth.*

The velocity equations for the gearbox in figure 3.3 become:

$$\omega_A = \omega_{IN}/i_{L,IN}$$
$$\omega_B = \omega_{OUT} \cdot i_{L,OUT}$$
$$\omega_C = \omega_{IN}/i_{H,IN}$$
$$\omega_D = \omega_{OUT} \cdot i_{H,OUT}$$

$$\omega_{relL} = \omega_A - \omega_B = \omega_{IN}/i_{L,IN} - \omega_{OUT} \cdot i_{L,OUT}$$
$$\omega_{relH} = \omega_C - \omega_D = \omega_{IN}/i_{H,IN} - \omega_{OUT} \cdot i_{H,OUT}$$

The relative velocities can be written in matrix form. Then the same matrix equation is obtained as for the gearbox in figure 3.2.

Note that $\omega_A, \omega_B, \omega_C$ and ω_D in figure 3.3 are not necessarily the same as the corresponding velocities of the gearbox in figure 3.2. Only the relative velocities agree fully.

In the general case the gearbox may contain more complex transmissions, for instance planetary gears. Then the i-constants cannot be interpreted as direct speed ratios. They are more probably just constants describing the influence of the velocities of the power shafts on the relative velocities of the clutches. They also describe the influence of the clutch torques on the torques of the power shafts.

In appendix B there is a numerical example of how the i-constants are found for a gearbox with planetary gears.

3.1.3 Multi-clutch gearbox with several power shafts

By writing the equations in matrix form as is the case in equations 3.1 and 3.2, a general approach is achieved to nearly all vehicle transmissions during ratio change. However, the restriction to two power shafts is limiting when any of the speed ratios includes a torque split gear, for example through a torque converter. In that case there are normally two input shafts. Extra power shafts could also be used for considering inertia, elasticity or damping in the gearbox. The restriction to two clutches is limiting when the lock-up operation of a torque converter is analysed. In this case, just one clutch is active. The formulas could also be used for a controlled limited slip differential. In that case there is one input shaft, two output shafts and one clutch.

Let us study gearboxes with several input and output shafts and with no restriction in the number of clutches. The gearbox can be drawn schematically as in figure 3.4.

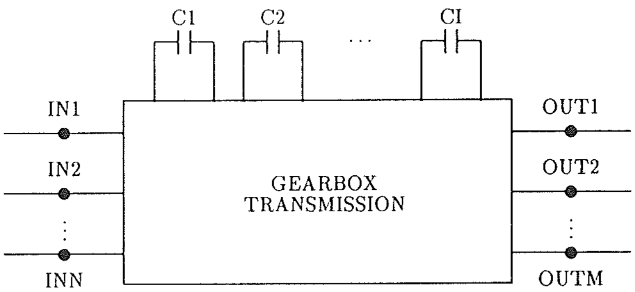


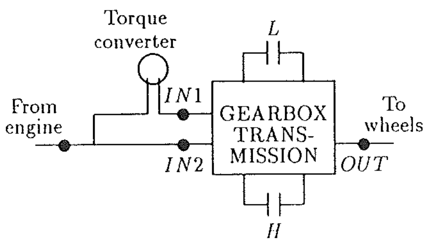
Figure 3.4: *Multi-clutch gearbox with several input and output shafts*

Let us assume N input and M output shafts, according to figure 3.4. Furthermore, there are I clutches. Their relative velocities are assumed to be a linear combination of the velocities of L shafts among the input and output shafts.

The equations can be written in a similar way as for the simpler gearbox in figure 3.2. The general model in figure 3.4 is probably more extensive than is needed in most practical cases. The equations may describe the gearbox transmission during shifts between just two gears, but also a gearbox during all shifts. The complete equations are shown in Appendix A.

For practical use it is seldom necessary to deal with more complex gearboxes than in the following three examples.

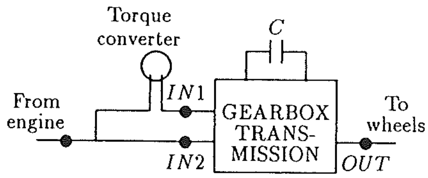
Torque-split example: The gearbox transmission has two input shafts, one output shaft and two clutches. There are three ω-dof with the input and output shafts linearly independent. The equations can be written as follows:



$$\begin{bmatrix} \omega_{relL} \\ \omega_{relH} \end{bmatrix} = \begin{bmatrix} 1/i_{L,IN1} & 1/i_{L,IN2} & -i_{L,OUT} \\ 1/i_{H,IN1} & 1/i_{H,IN2} & -i_{H,OUT} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{3.4}$$

$$\begin{bmatrix} M_{IN1} \\ M_{IN2} \\ M_{OUT} \end{bmatrix} = \begin{bmatrix} 1/i_{L,IN1} & 1/i_{H,IN1} \\ 1/i_{L,IN2} & 1/i_{H,IN2} \\ i_{L,OUT} & i_{H,OUT} \end{bmatrix} \cdot \begin{bmatrix} M_L \\ M_H \end{bmatrix} \tag{3.5}$$

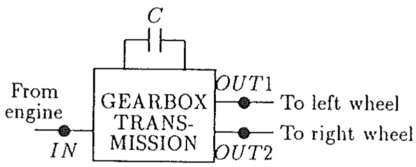
Converter lock-up example: The gearbox transmission has two input shafts, one output shaft and one clutch for locking up. There are two ω-dof with, for instance, one of the input shafts and the output shaft linearly independent. The equations can be written as follows (subscript C for clutch):



$$\begin{bmatrix} \omega_{rel} \\ \omega_{IN1} \end{bmatrix} = \begin{bmatrix} 1/i_{C,IN2} & -i_{C,OUT} \\ 1/i_{IN1,IN2} & -i_{IN1,OUT} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{3.6}$$

$$\begin{bmatrix} M_{IN2} \\ M_{OUT} \end{bmatrix} = \begin{bmatrix} 1/i_{C,IN2} & -1/i_{IN1,IN2} \\ i_{C,OUT} & -i_{IN1,OUT} \end{bmatrix} \cdot \begin{bmatrix} M_C \\ M_{IN1} \end{bmatrix} \tag{3.7}$$

Controlled limited slip differential example: The gearbox transmission (i. e. the differential) has one input shaft, two output shafts and one clutch. There are two ω-dof with, for instance, the two output shafts linearly independent. Then, the equations can be written as follows:



$$\begin{bmatrix} \omega_{rel} \\ \omega_{IN} \end{bmatrix} = \begin{bmatrix} 1/i_{C,OUT1} & -1/i_{C,OUT2} \\ 1/i_{IN,OUT2} & 1/i_{IN,OUT2} \end{bmatrix} \cdot \begin{bmatrix} \omega_{OUT1} \\ \omega_{OUT2} \end{bmatrix} \tag{3.8}$$

$$\begin{bmatrix} M_{OUT1} \\ M_{OUT2} \end{bmatrix} = \begin{bmatrix} -1/i_{C,OUT1} & 1/i_{IN,OUT2} \\ 1/i_{C,OUT2} & 1/i_{IN,OUT2} \end{bmatrix} \cdot \begin{bmatrix} M_C \\ M_{IN} \end{bmatrix} \tag{3.9}$$

3.2 Clutches

The most characteristic feature of a clutch is that it operates in two different phases: the clutch halves either stick or slip. This makes the clutch very difficult to analyse.

There are also other phenomena that make the clutch a complex transmission component, such as:

- VARYING COEFFICIENT OF FRICTION WHILE SLIPPING. The most essential parameter at that phenomenon is the sliding velocity, but the contact pressure and the temperature also influence.
- THE LANDING PHASE. When the clutch engages, the friction surfaces have to reach each other to make contact. This takes some time, since the clutches are oil immersed and the oil has to be squeezed out. The torque, due to Coulombian friction, therefore is delayed with respect to the control pressure.
- THE VISCOUS PART OF THE TORQUE. During the landing phase there is an oil film between the clutch halves. During this time interval, there can be a viscous torque, caused by shear stresses in the oil film. The total torque can be either less or greater than what one should expect if only Coulombian friction were considered. This may be observed as either a positive or a negative time delay, respectively.

A gearbox transmission, as shown above, may have several clutches. The analysis of such compound components leads to similar phases as for the single clutch.

3.2.1 Single clutch

In the model presented below it is assumed that the torque in a slipping clutch is only weakly dependent on the relative velocity. Therefore, the model is <u>not</u> suitable for including the viscous part of the torque.

The static torque capacities in both directions, c^+ and c^- ($c^+ \geq 0$, $c^- \leq 0$) in figure 3.5, are regarded as known functions of time. The capacity may be calculated as proportional to the hydraulic control pressure and the static coefficient of friction. If desired, the capacity may be adjusted, in order to consider phenomena during the landing phase. However, such adjustment is difficult, and is not treated in this work. The difference in torque capacity in the two directions makes it possible to analyse even band brakes and one-way clutches.

The torque during the slipping phase is a given fraction, α ($0 \leq \alpha \leq 1$) in figure 3.5, of the static torque capacity, c. The fraction α is a function of ω_{rel}, but it is only weakly dependent on ω_{rel}. This makes the torque and velocity approximately independent of each other, except for the phase switches. The most proper way to determine the α function should be by means of dynamic testing.

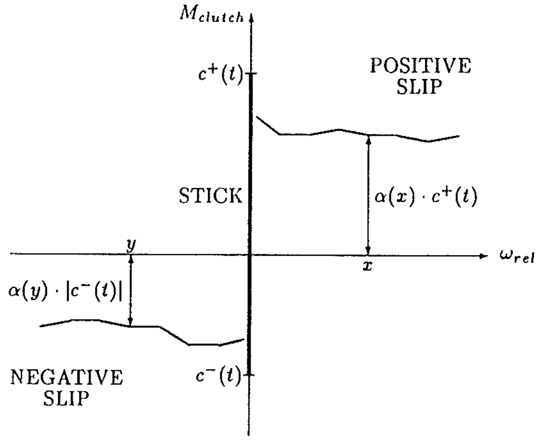


Figure 3.5: *Three phases of a single clutch*

A single clutch is a component with four quantities: two velocities (ω_{left} and ω_{right}) and two torques (M_{left} and M_{right}). In the slip phase there are two ω-dof and no M-dof. This means that the surroundings may determine both velocities. The torques are then prescribed by the clutch. In the stick phase there is one ω-dof and one M-dof. This means that the surroundings may determine one velocity and one torque. The clutch then prescribes the other velocity and the other torque.

Phase switch conditions are shown in figure 3.6.

Figure 3.6: *Phase switch conditions of a single clutch. "$>$" should be read "becomes greater than".*

The following types of friction can be defined:

- *Pure Coulombian friction*, where $\alpha \equiv 1$
- *Velocity independent Coulombian friction*, where $\alpha \neq 1$ and is constant
- *Velocity dependent Coulombian friction*, where $\alpha = \alpha(\omega_{rel})$

The following clutch designs can be identified:

- *Disc-clutch*, where $c^+ = -c^-$
- *One-way clutch*, where either $c^+ = +\infty$ and $c^- = 0$ or $c^- = -\infty$ and $c^+ = 0$
- *Band brake*, where $c^+/c^- = -e^{\mu \cdot angle}$ or $c^+/c^- = -e^{-\mu \cdot angle}$

3.2.2 Two-clutch gearbox

The models for gearbox and clutches described above may be linked. Let us consider a gearbox with two clutches. It could be described with the $3^2 = 9$ gearbox phases. The phases are shown in figure 3.7 and the phase switch conditions in figure 3.8. Notation is given in table 3.2. Figures 3.7 and 3.8 should be compared with figures 3.5 and 3.6, which are drawn for a single clutch.

Table 3.2: *Notation for gearbox phases of a two-clutch gearbox*

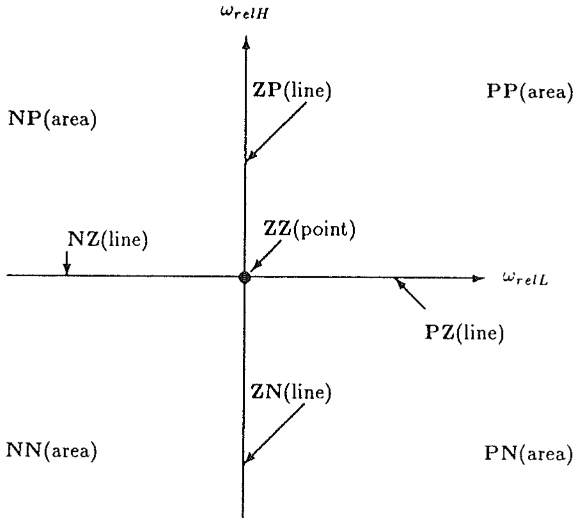
type of phase	phase	clutch L	clutch H
Slip phases	PP	positive slip	positive slip
	NP	negative slip	positive slip
	NN	negative slip	negative slip
	PN	positive slip	negative slip
Stick phases	ZP	stick	positive slip
	NZ	negative slip	stick
	ZN	stick	negative slip
	PZ	positive slip	stick
Lock-up phase	ZZ	stick	stick

The first four phases in table 3.2 are called slip phases. They agree very well with the slip phases of a single clutch. If there is one input and one output shaft, there will be two ω-dof but no M-dof, as in the single clutch case.

The next four phases in table 3.2 are called stick phases. They agree very well with the stick phases of a single clutch. If the gearbox has one input and one output shaft, there will be one ω-dof and one M-dof, as in the single clutch case.

The last phase in table 3.2 is called the lock-up phase. It has no correspondence with the single clutch case. A gearbox with one input and one output shaft, will have no ω-dof (i. e. no shaft is able to rotate) but two M-dof.

A gearbox during a powershift normally just passes through some of the nine phases. For instance, an upshift may start in stick phase ZP, pass through slip phase NP and end up in stick phase NZ.



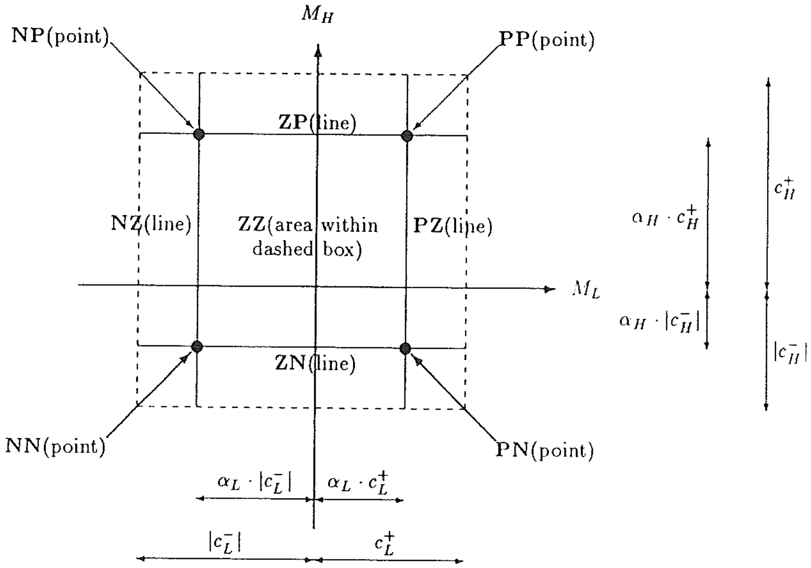


Figure 3.7: *Nine phases of a two-clutch gearbox. The lower diagram is only valid when α is constant for both clutches.*

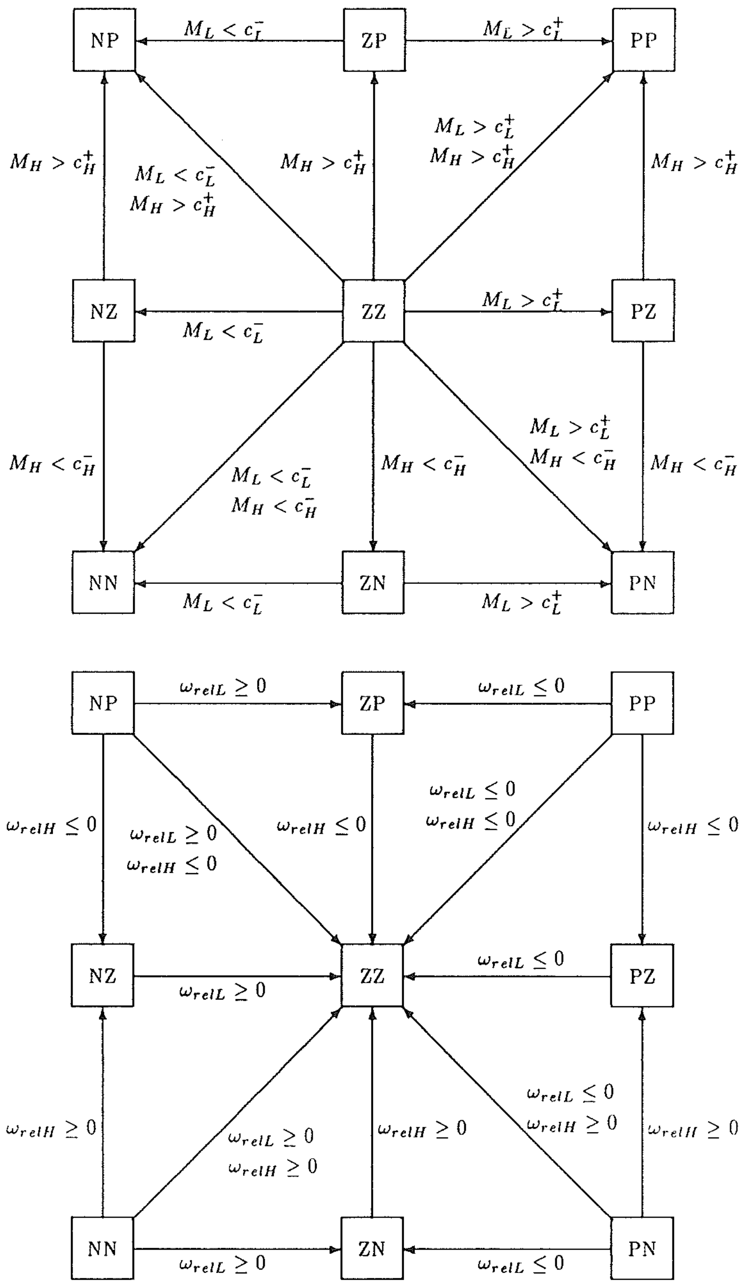


Figure 3.8: *Phase switches of a two-clutch gearbox. "$>$" should be read "becomes greater than".*

3.3 Torque converter

The hydrodynamic torque converter adopted here has two active shafts. The input shaft P is connected to the *pump*. The output shaft T is connected to the *turbine*. The *Trilok* type of converter is very common. In a Trilok converter there is a third impeller, the *reactor*, which is connected to ground via a one-way clutch.

Definition of quantities:
$\nu = \omega_T/\omega_P$
$\mu = M_T/M_P$
$\lambda^* = M_P/\omega_P^2$

Available equations:
$\mu = \mu(\nu)$
$\lambda^* = \lambda^*(\nu)$

There are two velocities and two torques. In almost every case, we may prescribe any two of them and calculate the other two using algebraic equations.

The torque converter as a physical component also has essential inertias, especially the pump. The torque converter as a model component has no inertia. However, this can be considered by placing inertias outside the model component.

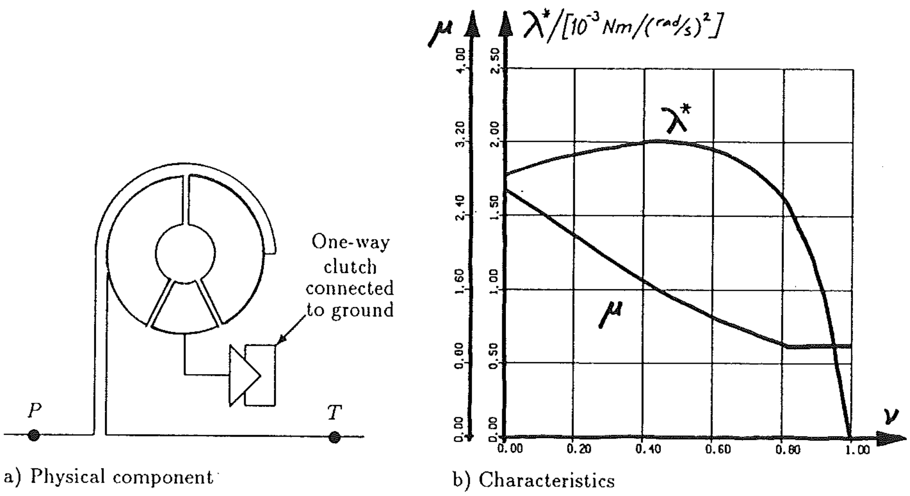


Figure 3.9: *Torque converter of Trilok type*

3.4 Tyre slip

When dealing with the tyre, there are two velocities to consider: the angular velocity of the wheel and the linear velocity of the road (or the wheel translation). There are two loads: the torque about the wheel centre and the traction force from the road. All velocities and loads are transformed to angular quantities.

If we want to consider the mass properties of the wheel, we should place inertias outside the tyre slip component. The rotational inertia of the wheel should be placed on the drive shaft side (to the left in figure 3.11 a). The translational inertia of the wheel should be placed on the road side (to the right in figure 3.11 a).

Definition of quantities:
M_{wheel} =torque from drive shafts
F_{road} =wheel traction force
$M_{road} = F_{road} \cdot R_{wheel}$
ω_{wheel} =angular velocity of wheel
v_{road} =linear velocity of wheel centre
$\omega_{road} = v_{road}/R_{wheel}$
$s = (\omega_{wheel} - \omega_{road})/\omega_{wheel} = slip$

Available equations:
$M_{road} = M_{road}(s)$ (figure 3.11 b)
$M_{wheel} = M_{road}$

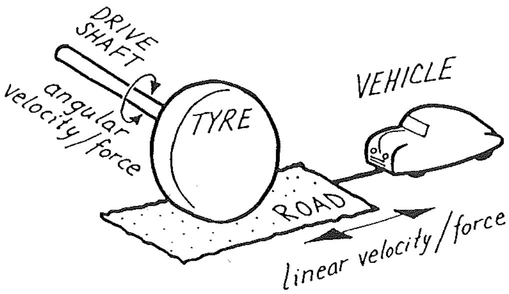
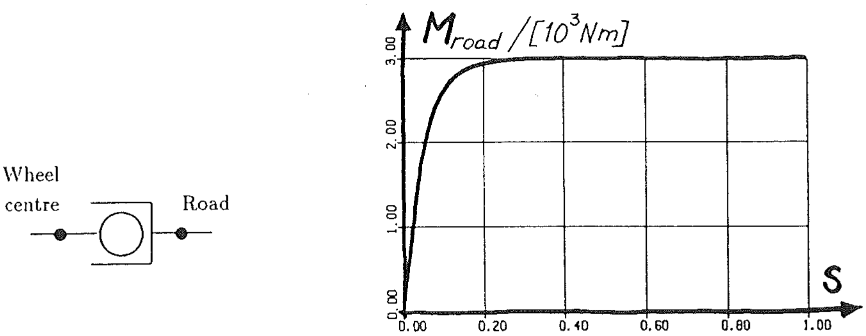


Figure 3.10: *Tyre slip as a physical component*



a) Model

b) Characteristic

Figure 3.11: *Tyre slip as a model component*

3.5 Engine

The steady state characteristics of the engine are used. This means that the engine torque can be calculated if the velocity and the throttle opening are known.

Definition of quantities:
ω_E = velocity of engine
M_E =torque of engine
x =throttle opening $(0 \leq x \leq 1)$

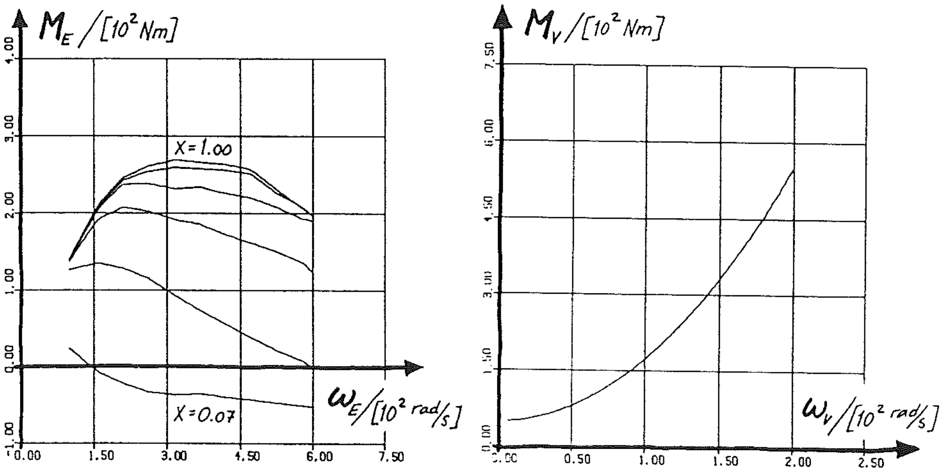
Available equations:
$M_E = M_E(\omega_E, x)$ (figure 3.12 a)
$x = x(t)$

3.6 Driving resistance

The driving resistance of the vehicle is a function of velocity. All quantities are transformed into angular quantities.

Definition of quantities:
v_V =linear velocity of the vehicle
$\omega_V = v_V / R_{wheel}$
F_V =driving resistance (force)
$M_V = F_V \cdot R_{wheel}$

Available equation:
$M_V = M_V(\omega_V)$ (figure 3.12 b)



a) Engine b) Driving resistance

Figure 3.12: *Engine and driving resistance characteristics*

3.7 Inertia

Moving bodies are able to store kinetic energy. In this work, every moving body is transformed to appear as a rotating one and is always characterized by its moment of inertia J. For translating inertias, i. e. the wheel or the vehicle, the equivalent moment of inertia is calculated as $J = m \cdot R^2_{wheel}$, where m is the mass.

Definition of quantities:	Available equations:
See figures 3.13 a) and 3.1.	$\dot{\omega}_1 = (M_1 - M_2)/J$ (Newton's second law)
	$\omega_1 = \omega_2$

3.8 Elasticity

Elastic components are able to store potential energy. As for inertias, all elasticities can be transformed to torsional elasticities.

Definition of quantities:	Available equations:
See figures 3.13 b) and 3.1.	$\dot{M}_1 = \dot{M}_1(M_1, \omega_1 - \omega_2)$ (constitutive equation)
	$M_1 = M_2$

In order to explain the constitutive equation, let us study some different kinds of elastic components. A linearly elastic component is characterized by the constitutive equation:

$M = M_k + k \cdot (\varphi_1 - \varphi_2)$ where M_k and k are constants

or its derivative:

$\dot{M} = k \cdot (\omega_1 - \omega_2)$

A quadratic constitutive equation is:

$M = M_k + k \cdot (\varphi_1 - \varphi_2) \cdot |\varphi_1 - \varphi_2|$

Differentiating with respect to time and using the inverse constitutive equation yields:

$\dot{M} = 2 \cdot \sqrt{k \cdot |M - M_k|} \cdot (\omega_1 - \omega_2)$

A more general constitutive equation reads:

$M = M(\varphi_1 - \varphi_2)$

$\dot{M} = \dot{M}(M, \omega_1 - \omega_2)$

a) Inertia b) Elasticity

Figure 3.13: *Inertia and elasticity*

3.9 Linear damper

Torque converter, tyre slip and driving resistance are examples of non-linear dampers. In fact the engine also acts like a damper, but with negative damping (production of energy). In the examples in the next chapter linear dampers are introduced. Very few real dampers are linear ones.

Definition of quantities:
See figures 3.14 and 3.1.

Available equation:
$M_1 = d \cdot (\omega_1 - \omega_2)$ (constitutive equation)
$M_1 = M_2$

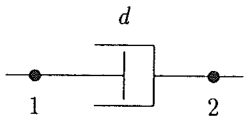


Figure 3.14: *Linear damper*

Chapter 4

System equations

4.1 A set of initial value problems

The equations of the complete driveline system constitute an initial value problem (*IV-problem*), or in fact a set of IV-problems. The deviation from an ordinary (single) IV-problem is caused by the presence of clutches. Clutches are components working in different phases, see chapter 3.

A single IV-problem can be formulated as follows:
Determine $\vec{y}(t)$ for $t^{IV} \leq t \leq t_{stop}$ when $\dot{\vec{y}} = \vec{f}(\vec{y}, t)$ and $\vec{y}(t^{IV}) = \vec{y}^{IV}$

When the differential equation and initial values are defined, the solution is obtained using suitable numerical methods. There is a great deal of computer software available for this purpose.

The function $\vec{f}$ is formally written as an explicit function. Theoretically, it may be implicitly determined, as long as there is an unambiguous derivative ($\dot{\vec{y}}$) for each state ($\vec{y}, t$). In the case of linear equations, it is usually possible to write $\vec{f}$ in explicit form. In the case of nonlinear equations, this is not always possible. Then one has to accept an implicit form, supplied with a strategy for an iterative solution. A common situation for drivelines including clutches is that the iteration may be carried out recursively, starting from an estimated torque of a slipping clutch. This is mostly a satisfactory recursive formula, since the torque is well known in advance, thanks to Coulombian friction and known direction of the slip.

Because of the phase switches, we have to write a <u>set</u> of IV-problems. This means that different (single) IV-problems have to be solved in different time intervals. The difficulty is that we do not know either t_{stop} or $\vec{y}^{IV}$ for each single IV-problem in advance.

For each phase we have to determine the following:

- Differential equation of the system ($\dot{\vec{y}} = \vec{f}(\vec{y}, t)$)
- Phase switch conditions
  (($\vec{y}, t$) determines if the phase should be switched and to what phase)
- New initial values
  ($\vec{y}^{IV}$ should be determined by previous ($\vec{y}, t$) and phase)

It should be noted that the components of the vector $\vec{y}$ do not necessarily have to be the same in the different phases.

4.2 Reference example

The theory of modelling dynamic systems may be formulated in a very general way. The present report deals with models for driveline mechanics. Therefore, an illustrative example is introduced, see figure 4.1. The reader should be aware that certain terms will be used which have not yet been defined. They are defined later in this chapter.

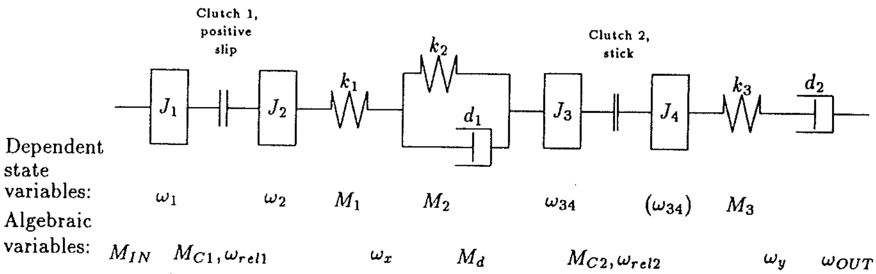


Figure 4.1: *Reference example of driveline system*

The equations of the system in figure 4.1 can be written:

Differential equations (level 0):

$\dot{\omega}_1 = (M_{IN} - M_{C1})/J_1$
$\dot{\omega}_2 = (M_{C1} - M_1)/J_2$
$\dot{\omega}_{34} = (M_1 - M_3)/(J_3 + J_4)$
$\dot{M}_1 = k_1 \cdot (\omega_2 - \omega_x)$
$\dot{M}_2 = k_2 \cdot (\omega_x - \omega_{34})$
$\dot{M}_3 = k_3 \cdot (\omega_{34} - \omega_y)$

Algebraic equations (level 1):

$M_{IN} = M_{IN}(t)$
$M_{C1} = \alpha(\omega_{rel1}) \cdot c_1^+(t)$
$\omega_x = \omega_{34} + M_d/d_1$
$\omega_y = \omega_{OUT} + M_3/d_2$

Algebraic equations (level 2):

$\omega_{rel1} = \omega_1 - \omega_2$
$M_d = M_1 - M_2$
$\omega_{OUT} = \omega_{OUT}(t)$

The equations above define the six state derivatives from the six dependent state variables and time. The nine algebraic variables are calculated as intermediate results. The calculations should be carried out in the following order: level 2, level 1 and finally level 0. This order makes the calculation explicit, as is needed in practice. The order within a level is not stipulated.

The equations are valid as long as no clutch switches phase. The phase switches can be handled in the following way:

- For clutch No 1, a phase switch is detected when ω_{rel1} becomes ≤ 0. The clutch sticks and the two state variables ω_1 and ω_2 are no longer accurate. Instead the state variable ω_{12} should be used. ω_{12} is the common velocity of inertias 1 and 2.
- For clutch No 2, a phase switch is detected when M_3 becomes $> c_2^+$ or $< c_2^-$. The clutch slips and the state variable ω_{34} is no longer accurate. Instead the state variables ω_3 and ω_4 should be used. ω_3 and ω_4 are the velocities of inertias 3 and 4.

4.3 Algebraic variables and functions

To handle the problem in practice, there are sometimes reasons to introduce *algebraic variables*, $\vec{x}$. In principal we get $\dot{\vec{y}} = f(\vec{x})$, where $\vec{x} = g(\vec{y}, t)$. The equation $\vec{x} = \vec{g}$ is an algebraic equation. Also the phase switch conditions and the initial values may be expressed in $\vec{x}$.

The following types of variables may be defined:

- Independent (state) variable: The time t
- (Dependent) state variables: The elements in $\vec{y}$
- (Dependent) state derivatives: The elements in $\dot{\vec{y}}$
- Algebraic variables: The elements in $\vec{x}$

The state variables $(\vec{y}, t)$ should contain just enough information needed to examine everything about the system. The algebraic variables are a kind of auxiliary variables and they may be as many as desired.

The algebraic variables and equations may be introduced at several levels and the functions may use information that is evaluated at higher levels:

level 0: $\dot{\vec{y}} = \vec{f}(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \ldots, \vec{x}_N, \vec{y}, t)$

level 1: $\vec{x}_1 = \vec{g}_1(\vec{x}_2, \vec{x}_3, \vec{x}_4, \ldots, \vec{x}_N, \vec{y}, t)$

level 2: $\vec{x}_2 = \vec{g}_2(\vec{x}_3, \vec{x}_4, \ldots, \vec{x}_N, \vec{y}, t)$

$\vdots$

level N: $\vec{x}_N = \vec{g}_N(\vec{y}, t)$

Some reasons for introducing algebraic variables could be:

- To reach variables that are not state variables
- To structure the calculation of the derivatives
- To solve a problem with varying sets of state variables, without the computer software knowing about it

Levels are introduced to show in which order the equations should be evaluated. Evaluating the levels in backward order makes the calculation explicit, or at least defines a recursive iteration strategy. The forward order of the levels often shows a logical order of thinking when deriving the equations.

4.4 Constraints

Constraints are algebraic equations that define relationships between state variables, both the dependent ($\vec{y}$) and the independent one (t). If there are constraints, we cannot integrate the state variables involved separately.

In fact, constraints tell us that we have chosen too many state variables. For each constraint, the number of state variables should be reduced by one. The physical quantity, corresponding to the former state variables, may then become an algebraic variable. This new algebraic variable can be calculated using the former constraint which now is an algebraic equation.

Let us study how constraints can appear in a system of torsional mechanics. Assume that velocities of inertias and torques of elasticities are chosen as state variables. State constraints occur for instance when two inertias (or two elasticities) are connected to each other by a gear transmission. It could also be exemplified by an inertia with prescribed velocity or an elasticity with prescribed torque.

In such simple systems that are treated in this work, it is usually no trouble with eliminating the constraints.

4.5 Physical quantities versus state variables

The dependent state variables are determined by the differential equations. The variables that appear as first derivatives of time are the dependent state variables. A higher order differential equation is thought to be written as a system of first order differential equations.

When making a mathematical description of the model, we use equations such as Newton's second law of motion, constitutive equations for elasticities and dampers and prescribed time functions. We should make use of all equations for all components in some way. However, it is not clear which equations should be used as differential equations. It will depend on which variables we want as state variables.

As seen in the reference example, we use the state variables **velocities of inertias** and **torques in elasticities**. The common feature of these state variables is that they describe the level of energy stored in the components. With this approach, only energy storing components become dynamic (i. e. generate differential equations).

It should be noted that the conventional way of modelling mechanical systems is to use **positions and their corresponding velocities of inertias** as state variables. In the case of driveline mechanics, torques are of more interest than positions. Therefore, the conventional modelling technique is not so attractive.

There is also another reason for not using the conventional technique. Using that approach it is difficult (not impossible) to model connecting points without inertia (for instance, nodes x and y in the reference example). Positions must be used as state variables, without using the corresponding velocities as state variables. (φ_x and φ_y in the reference example.)

With the conventional approach the following state variables should have been used in the reference example:

φ_1, φ_2, φ_x, φ_{34}, φ_y, ω_1, ω_2 and ω_{34} (ω_x and ω_y should not be state variables!)

Of course it would be possible to use relative positions:

φ_1, $(\varphi_2 - \varphi_x)$, $(\varphi_x - \varphi_{34})$, $(\varphi_{34} - \varphi_y)$, ω_1, ω_2 and ω_{34}

Multiplying by the stiffnesses yields:

φ_1, M_1, M_2, M_3, ω_1, ω_2 and ω_{34}

In this case conventional modelling is just a complex way of using the velocities and torques as state variables. There is no doubt that it is easier to start from velocities and torques as state variables.

4.6 Number of state variables

By increasing the number of state variables, we should get a more relevant description of the real world. However, there are many reasons for not using too many state variables. For example, the numerical calculations become more uncertain. It should always be considered whether the numerical method used is qualified to solve the mathematical model. The results may also be more difficult to interpret.

A common reason for increasing the number of state variables is the possibility of systematizing the building of the model and the derivation of equations. This is sometimes a good reason, for example when dealing with more complex systems. However, in the case of drivelines this is not so.

An example, which calls for care, is elasticity directly connected to a damper (for instance k_3 and d_2 in the reference example). The systematized approach forces us to introduce a small inertia ($\varepsilon \approx 0$) between these two components. Then the velocity (ω) of this small inertia is made into a state variable, with a corresponding differential equation: $\dot{\omega} = (M_k - M_d)/\varepsilon$. This leads to a *stiff* differential equation, which is often difficult to solve by means of numerical methods. To avoid such problems, simply set the inertia as zero. Then the differential equation becomes an algebraic equation: $0 = M_k - M_d$ and the velocity becomes an algebraic variable. The problem is no longer stiff.

Another example is a clutch connecting two inertias (e. g. both the clutches in the reference example). In order to systematize, the natural way is to use the velocities of the two inertias as state variables all the time. The torque of the clutch then has to be modelled as a continuous function of the relative velocity. When the relative velocity is close to zero the torque is almost discontinuous. It is calculated as $M = d \cdot (\omega_1 - \omega_2)$, where d is large and this makes the problem stiff for small relative velocities. Separating the phases leads to a set of non-stiff differential equations, which are more easily solved. In the stick phase it is only necessary to use one state variable: the common velocity of the inertias. In the slip phase two state variables have to be used: the velocities of both inertias.

4.7 Examples of systems including clutches

The equations for systems including clutches are shown below. Two examples are shown here: one with a single clutch and the other with a gearbox. More examples can be seen in appendix C.

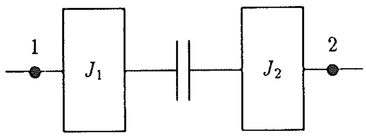
There are three main problems for each phase:

- Determine the differential equation
- Determine the phase switch condition
- Calculate initial values

The phase switch conditions are discussed in chapter 3. When the clutch torque exceeds the torque capacity, the phase should be switched from stick to slip. When the relative velocity changes sign, the phase should be switched from slip to stick.

In the examples we just study one of the slip phases and one of the stick phases. The structure of the equations are the same for the other slip and stick phases.

4.7.1 Inertia – clutch – inertia



This system demands two state variables (ω_1 and ω_2) when the clutch is slipping. In the stick phase just one state variable is required (ω_{12}). M_1 and M_2 are considered as known variables from an outer system.

Slip phase:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M)/J_1$
$\dot{\omega}_2 = (M - M_2)/J_2$

Algebraic equations (level 1):
$M = \alpha(\omega_{rel}) \cdot c(t)$

Algebraic equations (level 2):
$\omega_{rel} = \omega_1 - \omega_2$

Initial values:
$\omega_1^{IV} = \omega_{12}$
$\omega_2^{IV} = \omega_{12}$

Stick phase:

Differential equation (level 0):
$\dot{\omega}_{12} = (M_1 - M_2)/(J_1 + J_2)$

Algebraic equation (level 1):
$M = (J_2 \cdot M_1 + J_1 \cdot M_2)/(J_1 + J_2)$
$\omega_{rel} = 0$

Initial value:
$\omega_{12}^{IV} = (J_1 \cdot \omega_1 + J_2 \cdot \omega_2)/(J_1 + J_2)$

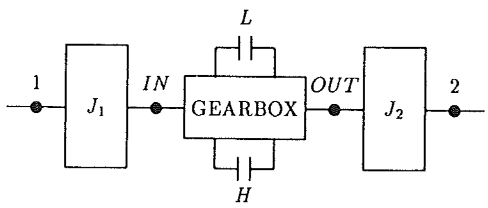
As seen above, evaluating the levels in backward order, yields an explicit calculation of the state derivatives.

In the stick phase, the clutch torque M should be calculated at the algebraic level 1. The torque is not needed in the differential equation, but it is needed to detect a phase switch to slip phases. The formula for calculating M in the stick phase, is derived from the constraint of equal acceleration of the two inertias:

$\frac{M_1-M}{J_1} = \frac{M-M_2}{J_2}$

It should be noted that ω_{12}^{IV} is calculated based on conservation of momentum. Theoretically we could have used either $\omega_{12}^{IV} = \omega_1$ or $\omega_{12}^{IV} = \omega_2$, because $\omega_1 \equiv \omega_2$ when switching to stick phase. However, numerical errors can cause $\omega_1 \neq \omega_2$ and, therefore, conservation of momentum seems to be the best approach.

4.7.2 Inertia – gearbox – inertia



The gearbox is assumed here to have two clutches, one input shaft and one output shaft. The lock-up phase (both clutches sticking) is not considered.

In a slip phase there are two state variables (ω_1 and ω_2). In the stick phase there is just one and it is a velocity. For instance we may choose the velocity of inertia 2. This state variable will be named ω_{20}. The same notation will be used as in chapter 3. The notation i_L means $i_{L,IN} \cdot i_{L,OUT}$, which is the overall speed ratio of the lower gear. M_1 and M_2 are known variables from an outer system.

Slip phase (both clutches slip):

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M_{IN})/J_1$
$\dot{\omega}_2 = (M_{OUT} - M_2)/J_2$

Algebraic equations (level 1):
$$\begin{bmatrix} M_{IN} \\ M_{OUT} \end{bmatrix} = \mathbf{I}_M \cdot \begin{bmatrix} M_L \\ M_H \end{bmatrix}$$

Algebraic equations (level 2):
$M_L = \alpha_L(\omega_{relL}) \cdot c_L(t)$
$M_H = \alpha_H(\omega_{relH}) \cdot c_H(t)$

Algebraic equations (level 3):
$$\begin{bmatrix} \omega_{relL} \\ \omega_{relH} \end{bmatrix} = \mathbf{I}_\omega \cdot \begin{bmatrix} \omega_{IN} \\ \omega_{OUT} \end{bmatrix}$$

Algebraic equations (level 4):
$\omega_{IN} = \omega_1$
$\omega_{OUT} = \omega_2$

Initial values (if L was sticking in the previous phase):
$\omega_1^{IV} = i_L \cdot \omega_{20}$
$\omega_2^{IV} = \omega_{20}$

Stick phase (if L sticks):

Differential equations (level 0):
$\dot{\omega}_{20} = (M_{OUT} - M_2)/J_2$

Algebraic equations (level 1):
$(M_{IN}, M_{OUT}, M_L)^T = \mathbf{A} \cdot (M_H, M_1, M_2)^T$

Algebraic equations (level 2):
$M_H = \alpha(\omega_{relH}) \cdot c_H(t)$

Algebraic equations (level 3):
$(\omega_{IN}, \omega_{relH})^T = \mathbf{B} \cdot (\omega_{relL}, \omega_{OUT})^T$

Algebraic equations (level 4):
$\omega_{relL} = 0$
$\omega_{OUT} = \omega_{20}$

Initial values:
$\omega_{20}^{IV} = (i_L^2 \cdot J_1 \cdot \omega_1 + J_2 \cdot \omega_2)/(i_L^2 \cdot J_1 + J_2)$

In the stick phase, we use the matrices **A** and **B**. The matrix **A** can be derived from the torque matrix equation and the constraint between ω_1 and ω_2. This constraint demands that the acceleration of J_1 is i_L times the acceleration of J_2, which yields:

$$\frac{M_1 - M_{IN}}{J_1} = i_L \cdot \frac{M_{OUT} - M_2}{J_2} \Rightarrow J_2 \cdot M_1 + i_L \cdot J_1 \cdot M_2 = J_2 \cdot M_{IN} + i_L \cdot J_1 \cdot M_{OUT}$$

Insertion of the torque matrix equation gives:

$$\begin{bmatrix} 1/i_{L,IN} & 1/i_{H,IN} & 0 & 0 \\ i_{L,OUT} & i_{H,OUT} & 0 & 0 \\ 0 & 0 & J_2 & i_L \cdot J_1 \end{bmatrix} \cdot \begin{bmatrix} M_L \\ M_H \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ J_2 & i_L \cdot J_1 \end{bmatrix} \cdot \begin{bmatrix} M_{IN} \\ M_{OUT} \end{bmatrix}$$

From this we can derive **A** by using elementary matrix manipulations. In the same way, matrix **B** can be derived from the velocity matrix equation. **A** become a 3×3-matrix and contain J_1, J_2 and i-constants. **B** will become a 2×2-matrix and contain just i-constants.

The calculation of ω_{20}^{IV} is based on conservation of momentum, as for the case "inertia – clutch – inertia". The term $i_1^2 \cdot J_1$ is the inertia J_1 being transformed to the output shaft.

Chapter 5

Dimensionless equations

Both the gearbox equations and the system equations can be written in a dimensionless form. The gearbox studied in this chapter has one input shaft, one output shaft and two clutches. The clutches are assumed to act with pure Coulombian friction. The driveline used is highly simplified. The model is shown in figure 5.1.

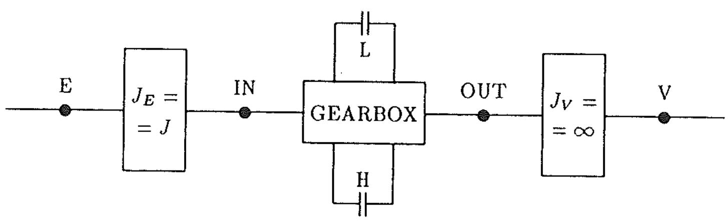


Figure 5.1: *Simple driveline model*

In this chapter the gearbox equations are presented first, and then the system equations are presented. In junction with the gearbox equations, some phenomena are defined and discussed, viz.: *tie-up*, *flare* and *ratio change with one-way clutch*. In junction with the system equations *torque phases* and *inertia phases* are defined and discussed. Finally, two numerical examples are presented.

5.1 Dimensionless gearbox equations

We start from the equations in chapter 3. The following applies to the gearbox in figure 3.2.

$$\begin{bmatrix} \omega_{relL} \\ \omega_{relH} \end{bmatrix} = \begin{bmatrix} 1/i_{L,IN} & -i_{L,OUT} \\ 1/i_{H,IN} & -i_{H,OUT} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN} \\ \omega_{OUT} \end{bmatrix} \tag{5.1}$$

$$\begin{bmatrix} M_{IN} \\ M_{OUT} \end{bmatrix} = \begin{bmatrix} 1/i_{L,IN} & 1/i_{H,IN} \\ i_{L,OUT} & i_{H,OUT} \end{bmatrix} \cdot \begin{bmatrix} M_L \\ M_H \end{bmatrix} \tag{5.2}$$

Equations for each clutch:

$$\begin{array}{rl}
\text{Negative slip} : & \left\{ \begin{array}{l} M = -c(t) \\ \omega_{rel} < 0 \\ \text{stick if } \omega_{rel} \text{ becomes} \geq 0 \end{array} \right. \\
\text{Stick} : & \left\{ \begin{array}{l} \omega_{rel} = 0 \\ -c(t) \leq M \leq +c(t) \\ \text{positive slip if } M \text{ becomes} > +c(t) \\ \text{negative slip if } M \text{ becomes} < -c(t) \end{array} \right. \\
\text{Positive slip} : & \left\{ \begin{array}{l} M = +c(t) \\ \omega_{rel} > 0 \\ \text{stick if } \omega_{rel} \text{ becomes} \leq 0 \end{array} \right.
\end{array} \tag{5.3}$$

The following defines our dimensionless quantities. Two new variables s and P_{ref} are also defined. Reference variables ω_{ref} and M_{ref} are used. They will, later on, be set as ω_{OUT} and M_{IN}, respectively.

$$\begin{aligned}
\omega_{*IN} &= \omega_{IN}/(i_{L,IN} \cdot i_{L,OUT} \cdot \omega_{ref}) \\
\omega_{*OUT} &= \omega_{OUT}/\omega_{ref} \\
\omega_{rel*L} &= \omega_{relL}/(i_{L,OUT} \cdot \omega_{ref}) \\
\omega_{rel*H} &= \omega_{relH}/((i_{LIN} \cdot i_{L,OUT}/i_{H,IN}) \cdot \omega_{ref}) \\
M_{*IN} &= M_{IN}/M_{ref} \\
M_{*OUT} &= M_{OUT}/(i_{L,IN} \cdot i_{L,OUT} \cdot M_{ref}) \\
M_{*L} &= M_L/(i_{L,IN} \cdot M_{ref}) \\
M_{*H} &= M_H/(i_{H,IN} \cdot M_{ref}) \\
c_{*L} &= (c_L/|i_{L,IN} \cdot M_{ref}|) \cdot sign(P_{ref}) \\
c_{*H} &= (c_H/|i_{H,IN} \cdot M_{ref}|) \cdot sign(P_{ref}) \\
P_{ref} &= i_{L,IN} \cdot i_{L,OUT} \cdot M_{ref} \cdot \omega_{ref} \\
s &= \frac{i_{L,IN} \cdot i_{L,OUT}}{i_{H,IN} \cdot i_{H,OUT}}
\end{aligned}$$

Note that $s = i_{lower\ gear}/i_{higher\ gear}$, which means the *ratio span* of the two gears involved. If the gears involved are both forward, or both reverse, the span will be $s > 1$. When one forward and one reverse gear are involved, let us define the one with the highest $|i|$ as the gear H. The span will then be $s < -1$. However, the case of negative span is of very little practical interest. As seen below, dimensionless velocity and torque equations, the span is the only parameter needed to determine the velocity and torque matrices.

The sign of P_{ref} will tell us the direction of power flow at the gear L. This will be made clearer later, when we decide which physical quantities should be reference quantities.

Both torque capacities c_L and c_H are positive. This is not necessarily true for the dimensionless torque capacities c_{*L} and c_{*H}. The dimensionless torque capacities will have the same sign as P_{ref}.

Insertion of the dimensionless quantities yields:

$$\begin{bmatrix} \omega_{rel*L} \\ \omega_{rel*H} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1/s \end{bmatrix} \cdot \begin{bmatrix} \omega_{*IN} \\ \omega_{*OUT} \end{bmatrix} \tag{5.4}$$

$$\begin{bmatrix} M_{*IN} \\ M_{*OUT} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1/s \end{bmatrix} \cdot \begin{bmatrix} M_{*L} \\ M_{*H} \end{bmatrix} \tag{5.5}$$

and for each clutch:

$$\begin{array}{rcl}
\text{Negative slip} & : & \left\{ \begin{array}{l} M_* = -c_*(t) \\ \omega_{rel*} < 0 \\ \text{stick if } \omega_{rel*} \text{ becomes } \geq 0 \end{array} \right. \\
\text{Stick (if } P_{ref} > 0) & : & \left\{ \begin{array}{l} \omega_{rel*} = 0 \\ -c_*(t) \leq M_* \leq +c_*(t) \\ \text{positive slip if } M_* \text{ becomes } > +c_*(t) \geq 0 \\ \text{negative slip if } M_* \text{ becomes } < -c_*(t) \leq 0 \end{array} \right. \\
\text{Stick (if } P_{ref} < 0) & : & \left\{ \begin{array}{l} \omega_{rel*} = 0 \\ +c_*(t) \leq M_* \leq -c_*(t) \\ \text{positive slip if } M_* \text{ becomes } < +c_*(t) \leq 0 \\ \text{negative slip if } M_* \text{ becomes } > -c_*(t) \geq 0 \end{array} \right. \\
\text{Positive slip} & : & \left\{ \begin{array}{l} M_* = +c_*(t) \\ \omega_{rel*} > 0 \\ \text{stick if } \omega_{rel*} \text{ becomes } \leq 0 \end{array} \right.
\end{array} \tag{5.6}$$

It should be noted that the sign of the dimensionless slip does not always coincide with the sign of the real slip.

Equations 5.4 and 5.5 can be interpreted in a specific gearbox layout shown in figure 5.2. The speed ratios are not drawn as gears but as belts.

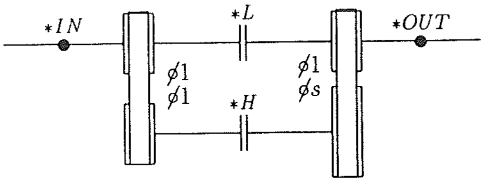


Figure 5.2: *A specific gearbox layout, interpretated from the dimensionless velocity and torque equations. The sign ϕ means diameter of belt pulley.*

Let us establish the following definitions of power:

Real powers:

$$\begin{cases} P_{IN} &= M_{IN} \cdot \omega_{IN} \\ P_{OUT} &= M_{OUT} \cdot \omega_{OUT} \\ P_L &= M_L \cdot \omega_{relL} \\ P_H &= M_H \cdot \omega_{relH} \end{cases}$$

Dimensionless powers:

$$\begin{cases} P_{*IN} &= M_{*IN} \cdot \omega_{*IN} &= P_{IN}/P_{ref} \\ P_{*OUT} &= M_{*OUT} \cdot \omega_{*OUT} &= P_{OUT}/P_{ref} \\ P_{*L} &= M_{*L} \cdot \omega_{rel*L} &= P_L/P_{ref} \\ P_{*H} &= M_{*H} \cdot \omega_{rel*H} &= P_H/P_{ref} \end{cases}$$

In the driveline model we will choose the engine torque as reference torque and the vehicle velocity as reference velocity. These quantities are approximately constant during the ratio change. Before introducing the driveline model, it is therefore suitable to set $M_{ref} = M_{IN}$ and $\omega_{ref} = \omega_{OUT}$. Table 5.1 shows a study of the quantities when the transmission works solely at each gear.

P_{ref} can now be interpreted as the power transmitted from the engine to the vehicle at gear L. Let us then consider the fact that the dimensionless input and output powers are positive for the gear L, as shown in table 5.1. The sign of P_{ref} indicates the direction of the real power flow at that gear. If $P_{ref} > 0$, the gearbox transmits power from the engine to the wheels. If $P_{ref} < 0$, the gearbox transmits power from the wheels to the engine i. e. braking by engine.

5.1.1 Tie-up and flare

The terms *tie-up* (German: *positive Überschneidung*) and *flare* (*negative Überschneidung*) are used frequently in the literature. They can be strictly defined by the dimensionless torque capacities c_{*L} and c_{*H}.

Let us first establish the fact that ratio changes can be described as paths in a c_{*H}, c_{*L}-diagram. Furthermore, in such a diagram, the first quadrant is for ratio changes with $P_{ref} > 0$. The third quadrant is for ratio changes with $P_{ref} < 0$. The second and fourth quadrant are never used, since c_{*H} and c_{*L} never have different signs.

The torque capacity path for an upshift starts from a point where $c_{*H} = 0$ and

Table 5.1: *Dimensionless quantities when the transmission works solely at each gear.* $\omega_{ref} = \omega_{OUT}$ *and* $M_{ref} = M_{IN}$.

GEAR	L	H
$\lvert c_{*L}\rvert$	≥ 1	0
$\lvert c_{*H}\rvert$	0	≥ 1
ω_{*IN}	1	$1/s$
ω_{*OUT}	1	1
ω_{rel*L}	0	$1/s - 1$
ω_{rel*H}	$1 - 1/s$	0
M_{*IN}	1	1
M_{*OUT}	1	$1/s$
M_{*L}	1	0
M_{*H}	0	1
P_{*IN}	1	$1/s$
P_{*OUT}	1	$1/s$
P_{*L}	0	0
P_{*H}	0	0

$|c_{*L}| \geq 1$. The end point is $|c_{*H}| \geq 1$ and $c_{*L} = 0$. A downshift will have the start and end points reversed.

When changing ratio the two clutches can be either too much or too little engaged with respect to $M_{ref} = M_{IN}$. The first condition is known as *tie-up* and the other *flare*. Tie-up results in loss of torque, since one clutch counteracts the other. Flare results in loss of velocity, since both clutches slip.

Strictly, we define:

- TIE-UP implies $|c_{*L} + c_{*H}| > 1$, but $c_{*L} \cdot c_{*H} \neq 0$
- FLARE implies $|c_{*L} + c_{*H}| < 1$

Ratio changes may pass through both the tie-up and flare areas. Examples of ratio changes that are strictly tie-up or flare are shown in figure 5.3.

5.1.2 Shift with one-way clutch

Neither tie-up nor flare is desirable. The way of balancing between them is to follow the line between $(0, 1)$ and $(1, 0)$. Of course this is very hard to do with two hydraulically controlled clutches. However, replacing one clutch with a one-way

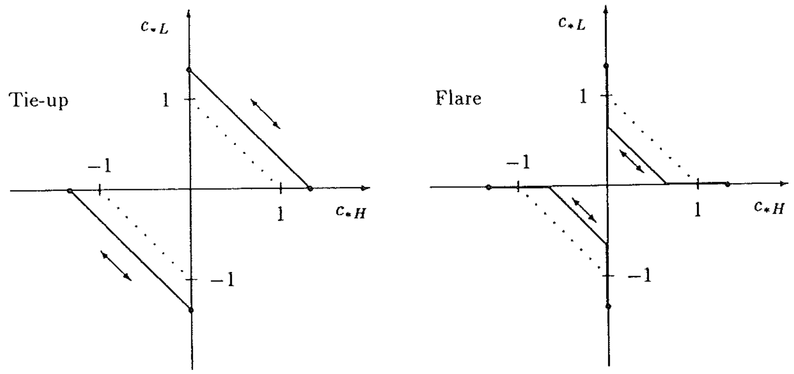


Figure 5.3: *Tie-up and flare*

clutch means that this clutch never counteracts the ordinary clutch. This is an easy way to balance between tie-up and flare. Then only the ordinary clutch has to be controlled.

Studying table 5.1 and assuming $s > 1$, the following demands on the one-way clutch can be established:

- A one-way clutch replacing clutch L demands locking for positive $M_{*L} = M_L/(i_{L,IN} \cdot M_{ref})$ and unlocking for negative $\omega_{rel*L} = \omega_{relL}/(i_{L,IN} \cdot \omega_{ref})$.
- A one-way clutch replacing clutch H demands locking for positive $M_{*H} = M_H/(i_{H,IN} \cdot M_{ref})$ and unlocking for positive values of $\omega_{rel*H} = \omega_{relH}/((i_{L,IN} \cdot i_{L,OUT}) \cdot i_{H,IN} \cdot \omega_{ref})$

Now, if a one-way clutch locks for real torque in one direction, it will unlock for real relative velocity in the other direction. Studying the signs, we find that the one-way clutch should replace clutch L when $P_{ref} > 0$, and clutch H when $P_{ref} < 0$.

Most commercial automatic transmissions use one-way clutches in the lower gear in some of their ratio changes. The advantages are obvious; correct change without tie-up or flare and easier means of control. The drawbacks are the higher price and weight, but also that ratio changes with reversed power flow will have a phase of totally interrupted power transmission. This means that braking by engine is not possible at the lower gear. If engine braking is desired, it is necessary to engage an ordinary clutch connected in parallel to the one-way clutch.

5.2 Dimensionless system equations

The model is shown in figure 5.1. This model cannot be expected to give anything but a rough picture of the ratio change. For instance, no oscillations will be seen because of the lack of elasticity. Furthermore, the torque variation will be a bit nonsmooth because of the absence of elasticity and damping. However, thanks to the simplicity of the model, it is easy to write the system equations in dimensionless form.

The system equations become very similar to the example "inertia – gearbox – inertia" in chapter 4. However, the velocity of the vehicle is not a state variable, since $J_V = \infty$. In the slip phase (both clutches slip) we will have one state variable ω_{IN} or in the dimensionless case ω_{*IN}. In the stick phase (one clutch sticks) we will have no state variables at all. The lock-up phase (both clutches stick) is not treated.

As reference quantities we use: $\omega_{ref} = \omega_V$ and $M_{ref} = M_E$. The interesting quantities are the velocity of the engine (ω_{IN} or ω_{*IN}) and the traction force of the vehicle (M_{OUT} or M_{*OUT}).

5.2.1 Torque phases

In the literature, the stick phases (one of the clutches sticks) are referred to as *torque phases* because, with this simple model, it is only the torques that change. The changes are governed by the following algebraic equations:

If L sticks

Insertion of $\omega_{rel*L} = 0$ and $\omega_{*OUT} = 1$ in equation 5.4 yields:

$$\begin{cases} \omega_{*IN} &= 1 \\ \omega_{*OUT} &= 1 \\ \omega_{rel*L} &= 0 \\ \omega_{rel*H} &= 1 - \frac{1}{s} \end{cases}$$

Insertion of $M_{*H} = c_{*H} \cdot sign(\omega_{rel*H})$ and $M_{*IN} = 1$ in equation 5.5 yields:

$$\begin{cases} M_{*L} &= 1 - c_{*H} \cdot sign(1 - \frac{1}{s}) \\ M_{*H} &= c_{*H} \cdot sign(1 - \frac{1}{s}) \\ M_{*IN} &= 1 \\ M_{*OUT} &= 1 - |1 - \frac{1}{s}| \cdot c_{*H} \end{cases}$$

If H sticks

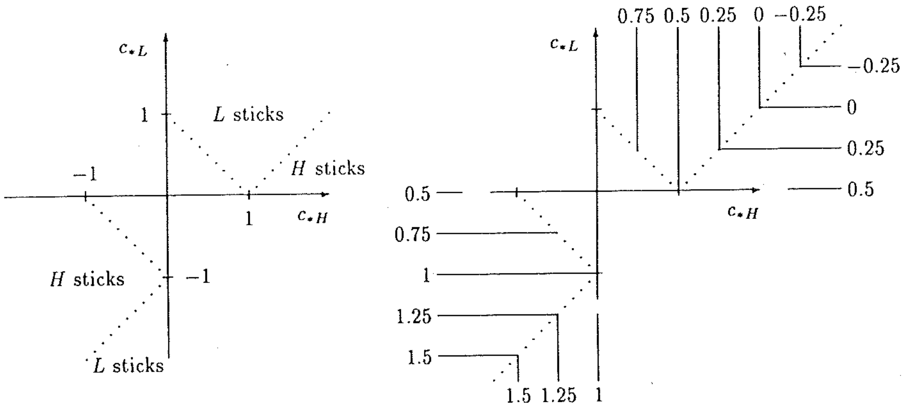
Insertion of $\omega_{rel*H} = 0$ and $\omega_{*OUT} = 1$ in equation 5.4 yields:

$$\begin{cases} \omega_{*IN} &= \frac{1}{s} \\ \omega_{*OUT} &= 1 \\ \omega_{rel*L} &= \frac{1}{s} - 1 \\ \omega_{rel*H} &= 0 \end{cases}$$

Insertion of $M_{*L} = c_{*L} \cdot sign(\omega_{rel*L})$ and $M_{*IN} = 1$ in equation 5.5 yields:

$$\begin{cases} M_{*L} &= c_{*L} \cdot sign(\frac{1}{s} - 1) \\ M_{*H} &= 1 - c_{*L} \cdot sign(\frac{1}{s} - 1) \\ M_{*IN} &= 1 \\ M_{*OUT} &= \frac{1}{s} - |\frac{1}{s} - 1| \cdot c_{*L} \end{cases}$$

Figure 5.4 shows possible areas for both stick phases when $s > 1$. They are obtained by inserting the last equations for M_{*L} and M_{*H} in the phase switch condition from equation 5.6: $-|c_*| < |M_*| < +|c_*|$. Iso-torque curves for M_{*OUT} when $s = 2$ are also shown in figure 5.4.



Possible areas for stick phases when $s > 1$ Iso-torque curves for M_{*OUT} when $s = 2$

Figure 5.4: *Stick phases*

5.2.2 Inertia phases

The slip phases (both clutches slip) are often referred to as *inertia phases* because, with this simple model, it is mainly the velocities that change. The changes are governed by the differential equation of the inertia.

Let us introduce:
$t_* = t/t_{ref}$ ($\frac{d}{dt_*}$ is marked with a prime)
$J_* = J \cdot i_{L,IN} \cdot i_{L,OUT} \cdot \omega_{ref}/(M_{ref} \cdot t_{ref})$
It is convenient to chose t_{ref} as the total time for the ratio change. Then the t_*-axis will extend from 0 to 1. Note that $sign(J_*) = sign(P_{ref})$. If the reference velocity is constant, the dimensionless equations are as follows:

$$\left\{ \begin{array}{rcl} \omega'_{*IN} & = & (1 - M_{*IN})/J_* \\ \omega_{*OUT} & = & 1 \\ \omega_{rel*L} & = & \omega_{*IN} - 1 \\ \omega_{rel*H} & = & \omega_{*IN} - 1/s \\ M_{*L} & = & c_{*L} \cdot sign(\omega_{*IN} - 1) \\ M_{*H} & = & c_{*H} \cdot sign(\omega_{*IN} - 1/s) \\ M_{*IN} & = & c_{*L} \cdot sign(\omega_{*IN} - 1) + c_{*H} \cdot sign(\omega_{*IN} - 1/s) \\ M_{*OUT} & = & c_{*L} \cdot sign(\omega_{*IN} - 1) + (c_{*H}/s) \cdot sign(\omega_{*IN} - 1/s) \end{array} \right.$$

The sign of ω'_{*IN} can be presented in diagrams displaying c_{*H} versus c_{*L} as shown in figure 5.5, where $s > 1$. Iso-torque curves for M_{*OUT} are shown in figure 5.6. They are drawn for $s = 2$.

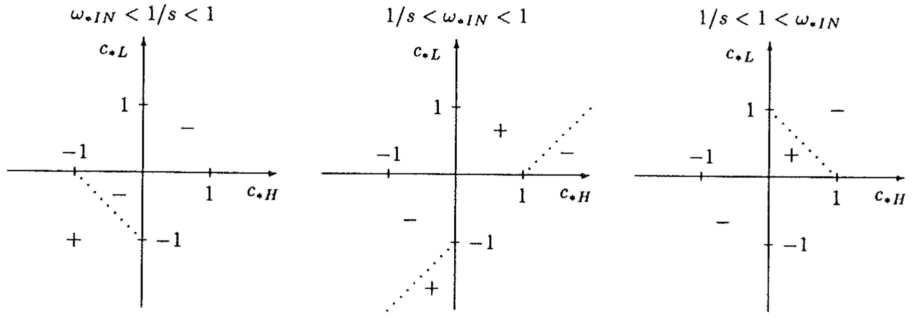
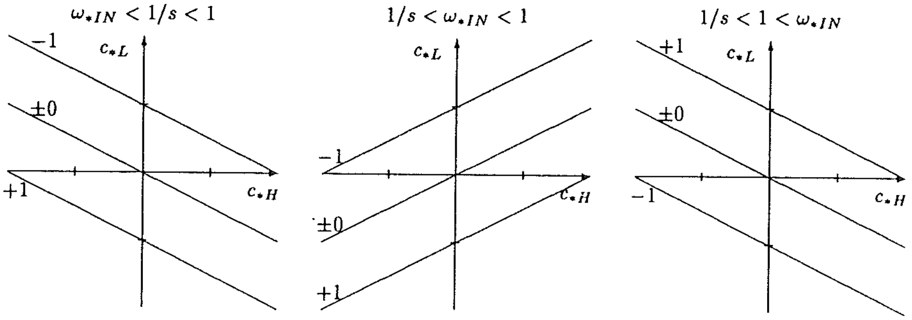


Figure 5.5: *Signs of ω'_{*IN} when $s > 1$*



$M_{*OUT} = -c_{*L} - c_{*H}/s$ $\quad$ $M_{*OUT} = -c_{*L} + c_{*H}/s$ $\quad$ $M_{*OUT} = +c_{*L} + c_{*H}/s$

Figure 5.6: *Iso-torque curves for M_{*OUT} when $s = 2$*

5.2.3 "Aquarium" diagram

One way of illustrating the path of a ratio change is to draw it in a three-dimensional diagram with c_{*L}, c_{*H} and ω_{*IN} on the axes, as shown in figure 5.7. Such a diagram is referred to here as an *aquarium diagram*.

A shift can be described as travelling on platforms (torque phases or stick phases) and jumping between them (inertia phases or slip phases). In the stick phases the velocities are constant but the torques are determined by the clutches. In the slip phases the velocities change according to Newton's second law of motion; acceleration takes place towards a stable position (a stick phase) or towards infinite velocities.

The ways of travelling in the diagram are now to be explained. Firstly, the coordinates c_{*H} and c_{*L} are known, since the hydraulic control pressure to each clutch is regarded as a known function of time. Thus, we just have to determine whether ω_{*IN} is changing, and if so, up or down. Let us imagine the drawn volumes as full of water. Place a table tennis ball somewhere at the known coordinates c_{*H}, c_{*L}. The ball will float upwards if it is within the volumes with water. The ball will drop down if it is outside the volumes. However, the amount of acceleration $|\omega'_{*IN}|$ cannot be determined only through this diagram.

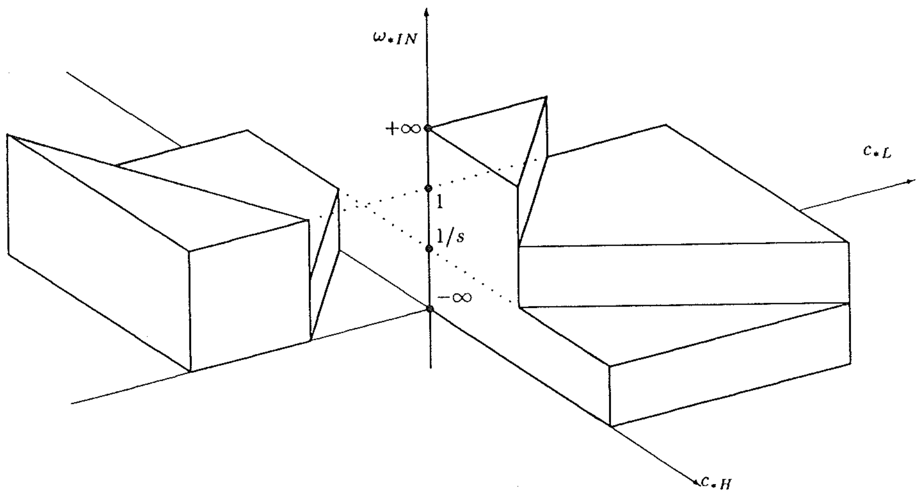


Figure 5.7: *Aquarium diagram when $s > 1$. The scale on the ω_{*IN}-axis is not linear.*

5.2.4 Numerical examples

There are, of course, an infinite number of different ways of performing ratio changes. When making them dimensionless, the number is reduced, but still infinite. Broadly speaking, one should discuss different combinations of:

- $s > 1$ and $s < -1$ (negative ratio span s is of very little interest)
- Up- and downshifts
- Power direction (different signs of P_{ref})
- Tie-up, flare and one-way clutch
- Total time of the ratio change

There are too many such combinations to fit in this report. As numerical examples presented below, just two ratio changes are chosen, viz.: upshift with flare and downshift with tie-up. The power direction is from engine to vehicle, $P_{ref} > 0$. The ratio span $s = 2$ and the dimensionless inertia $J_* = 0.5$ are used. These are representative for a passenger car shifting between gear 1 and 2, with engine at medium to high speed and wide open throttle.

Upshift with flare

The solution in the time domain is shown in figure 5.10. The path in the aquarium diagram is shown in figure 5.8 and described below:

1. Torque phase, where L sticks
2. Inertia phase, where ω'_{*IN} is positive. The engine accelerates.
3. When c_{*H} becomes > 1, another inertia phase is valid. In this phase ω'_{*IN} is negative. The engine decelerates.
4. When ω_{*IN} passes from > 1 to < 1, yet another inertia phase enters, but ω'_{*IN} is still negative.
5. We reach the final torque phase, where H sticks.

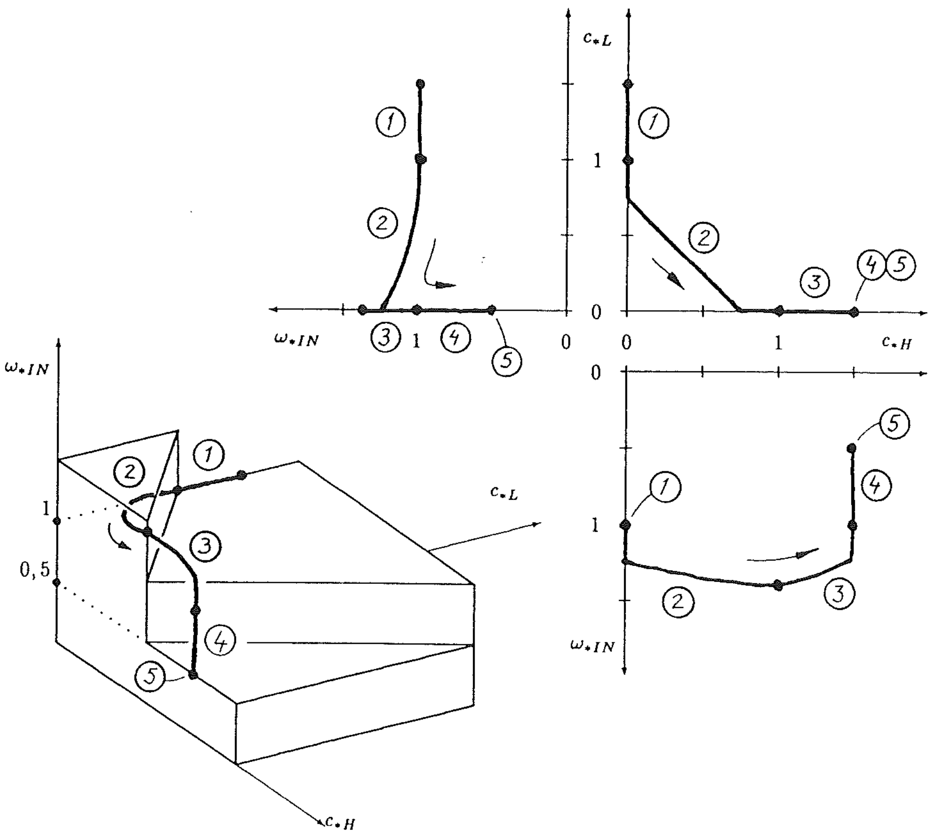


Figure 5.8: *Aquarium diagram for an upshift with flare*

Downshift with tie-up

The solution in the time domain is shown in figure 5.11. The path in the aquarium diagram is shown in figure 5.9 and described below:

1. Torque phase where H sticks
2. When c_{*L} becomes $> -1 + c_{*H}$, an inertia phase takes place. The engine accelerates ($\omega'_{*IN} > 0$) until ω_{*IN} reaches the value of 1.
3. A torque phase is entered, where L sticks.

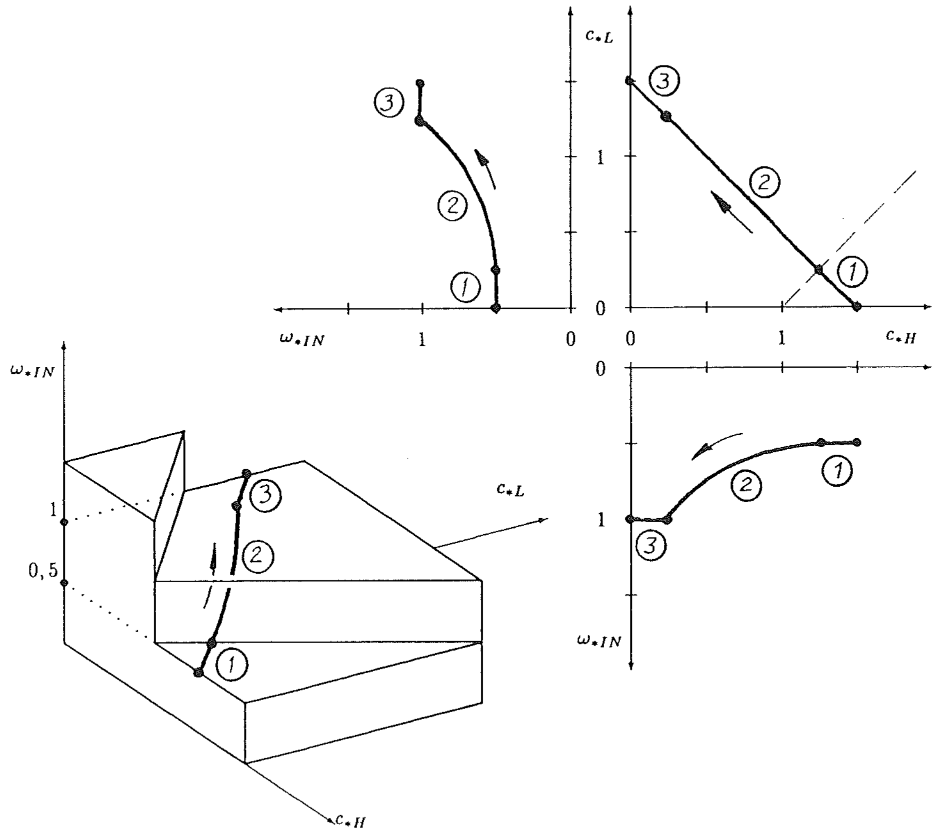


Figure 5.9: *Aquarium diagram for a downshift with tie-up*

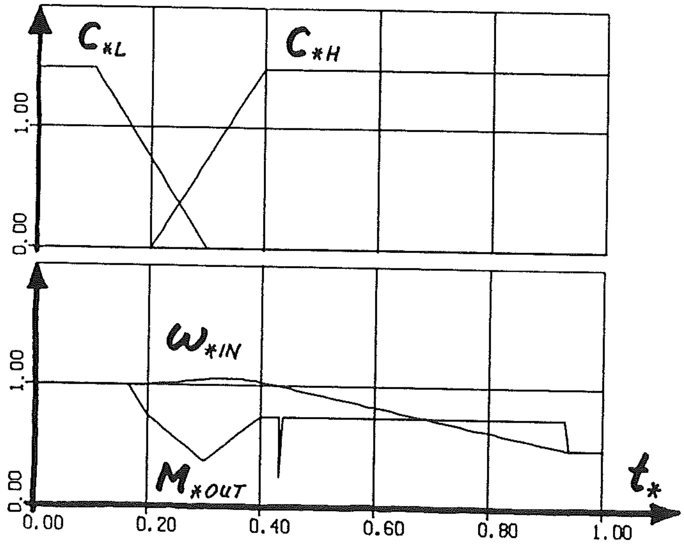


Figure 5.10: *Solution in time domain for upshift with flare*

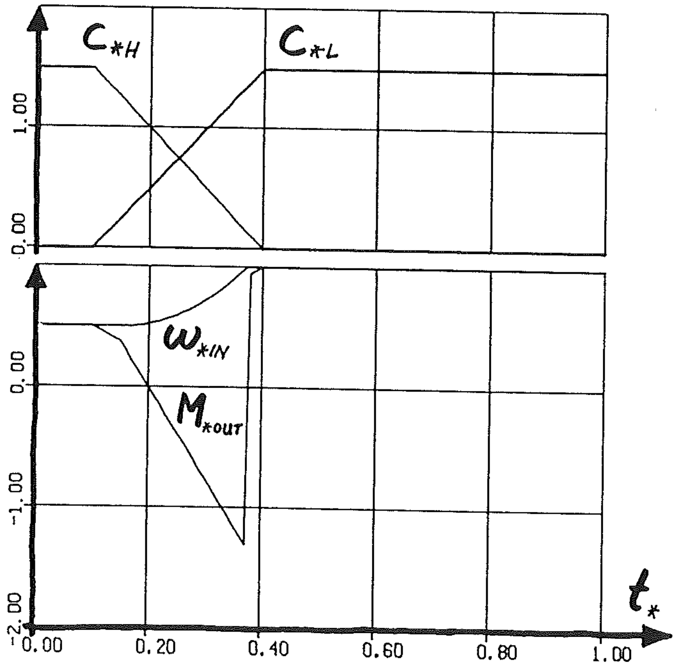
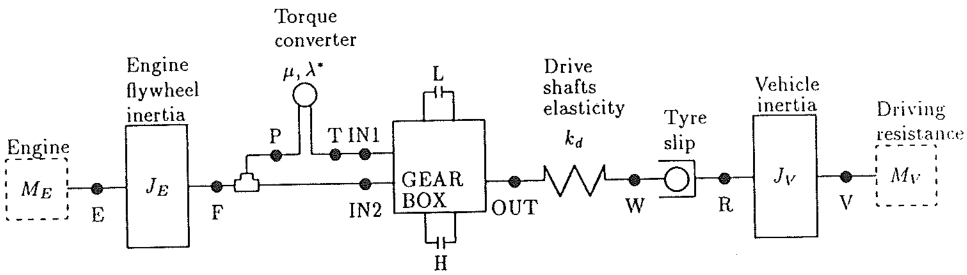


Figure 5.11: *Solution in time domain for downshift with tie-up*

Chapter 6

Final driveline model



E : Engine
F : Flywheel
P : Pump
T : Turbine

IN1 : Input No 1 to gearbox
IN2 : Input No 2 to gearbox
OUT : Output from gearbox

W : Wheel
R : Road
V : Vehicle

Figure 6.1: *Final driveline model*

In this chapter simulations of a complete driveline are shown. The model shown in figure 6.1 has been used. The system equations are very similar to those in the last example (C.5) in appendix C. The differences are:

In this chapter	In example C.5	Model description
Torque converter	Linear damper, d_1	See section 3.3
Tyre slip	Linear damper, d_2	See section 3.4
Engine, M_E	Outer torque, M_1	See section 3.5
Driving resistance, M_V	Outer torque, M_6	See section 3.6

Verifying tests have been carried out. Numerical data for the model are chosen to correspond to the vehicle used in these tests. The following should be particularly noted:

- The inertia J_E is the inertia of the moving parts of the engine with the inertia of the torque converter pump with its oil included.

- The inertia of the torque converter turbine is neglected. It is only approximately 10% of J_E, including its oil content.
- The stiffness k_d is the stiffness of three elasticities in series: the drive shafts, the tyre (in torsion) and the wheel suspension (in the longitudinal direction of the vehicle).
- Neither the rotational nor translational inertia of the wheel are considered, since they hardly affect the most essential oscillation modes. These modes are the rigid body mode and the mode with one single node very near J_V.

6.1 Tests and simulations

The most important data are the torque capacities of the clutches (c_L and c_H), the engine velocity (ω_E) and the vehicle traction torque (M_R). These are the only variables presented in the examples below. In addition, the time instants for phase switches are marked with notation according to table 3.2. Three tests with their corresponding simulations are presented (see table 6.1).

Table 6.1: *Tested and simulated ratio changes*

Case No. and shift direction	Gear before shift: Gear No. and use of converter	Gear before shift: Clutch	Gear after shift: Gear No. and use of converter	Gear after shift: Clutch	Ratio span	Throttle opening
1 Upshift	1 Common	One-way brake	2 Common	Multi-disc and band brake in parallel	≈ 1.8	1.0
2 Upshift	3 Torque split	Multi-disc clutch	4 Lock-up (bypassed)	Band brake	≈ 1.3	1.0
3 Downshift	3 Torque split	Multi-disc clutch	2 Common	One-way brake	≈ 1.4	0.1

6.1.1 Tests

The tests were performed in a passenger car with front wheel drive driving on the road. The automatic transmission was the same as in appendix B. Test results for the cases in table 6.1 are shown in figures 6.2, 6.4, 6.6 and 6.8.

The torque capacities are calculated from the measured hydraulic control pressures to the clutches:
$c = \mu \cdot (p \cdot A - F_{return}) \cdot R \cdot N$; multi-disc clutches and multi-disc brakes

$c = (e^{\mu \cdot \beta} - 1) \cdot (p \cdot A - F_{return}) \cdot R$; band brakes in self-wrapping direction
$c = (1 - e^{-\mu \cdot \beta}) \cdot (p \cdot A - F_{return}) \cdot R$; band brakes in the other direction
where p = measured hydraulic pressure, A = piston area, F_{return} = force in the return spring and centrifugal forces in the oil, R = friction radius, N = number of friction surfaces, β = wrapping angle of band brake

The traction torque is measured by special wheels, equipped with force sensors between hub and rim. The torque presented is an arithmetic mean value of the left and right wheel torque.

The phase switches are estimated from measured velocities of the planetary gear in the automatic transmission. From these velocities, the relative velocities can be calculated.

6.1.2 Simulations

The torque capacities from the tests were used as input data. The engine velocity and the traction torque are the output data. A constant coefficient of dynamic friction was used. Simulation results are shown in figures 6.3, 6.5, 6.7 and 6.9.

6.1.3 Tests versus simulations

Case 1: Upshift with one-way clutch (Figures 6.2 and 6.3)
The torque in the clutch of the higher gear is split between the multi-disc and the band brake. The multi-disc brake takes approximately 70% of the clutch torque.

There are two main differences between the test and the simulation results:

- The oscillations of approximately 11 Hz in the traction torque is due to a first order imbalance in the wheels, which rotate at 11 rps.
- There is a peak in the measured traction torque at $t = 1.25$ s. This peak is not seen in the simulations. It probably comes from a viscous part of the clutch torque, which has not been included in the model. Such a viscous part may be seen in [Henriksson 1989], which is an experimental work on the very same multi-disc clutch as is used here. The viscous part of the clutch torque is briefly discussed in section 3.2.

Case 2: Upshift to lock-up gear (Figures 6.4, 6.5, 6.6 and 6.7)
The shift is done with a weak tie-up, which may be observed as a slight decrease of the engine velocity for $1.15 < t < 1.25$ s.

The system is much less damped at the higher gear because of the lock-up of the torque converter. This results in some oscillations in the engine velocity and the traction force, which is seen for $1.8 < t < 2.1$ s (test) and $1.75 < t < 1.9$ s (simulation).

We will discuss three differences between the tests and the simulations:

- Analogously with the previous case, the imbalance in the wheels introduces an oscillation in the traction force. In this case it is of approximately 24 Hz, which coincides very well with the velocity of the wheels.
- The oscillations directly after the inertia phase differ in frequency. The test shows 8 Hz and the simulation 16 Hz. A possible explanation is that the engine block with its suspension participates in the oscillations. This is not considered in the model. Simply introducing the wheel rotational and translational inertia in the model would not decrease that frequency sufficiently.
- The third difference is the most serious one. The simulation shows a very quick response from hydraulic pressure to clutch torque in the clutch of the higher gear. The test shows that the clutch torque responds as if there were a delay in the torque capacity. There is probably such a delay because the oil has to be squeezed out before the clutch can land and develop Coulombian friction. A very rough correction of the torque capacity of the clutch of the higher gear is shown in figure 6.7. This shows that the model is able to produce results more like the tests.

Case 3: Downshift from torque-split gear (Figures 6.8 and 6.9)
The test and simulations mainly differ in the following four points:

- The imbalance in the wheel is noticed as an oscillation of 4 to 5 Hz. This coincides well with the velocity of the wheels.
- For $t < 0.9$ s, we also notice a higher frequency, 42 Hz. The engine velocity is 21 rps. It is then quite obvious that this is the combustion frequency of the four cylinder four-stroke engine. This disturbance is not seen at higher engine velocities because the signals have been filtered.
- There is also an oscillation of 20 Hz, for $t > 1$ s. One possible explanation of this is that some higher mode oscillation of the driveline is excited by the wheel velocity of 4 rps=4 Hz, which is a fifth of the oscillation frequency, i. e. 4 Hz/20 Hz=1/5.
- The traction torque for $t > 1.1$ s also differs. The test results show a slower rise of the torque ($1.1 < t < 1.4$ s) but to a higher torque level ($t > 1.4$ s). A reasonable explanation of this is found in incorrect characteristics of the engine. The engine characteristics at low velocities and low throttle openings are difficult to estimate.

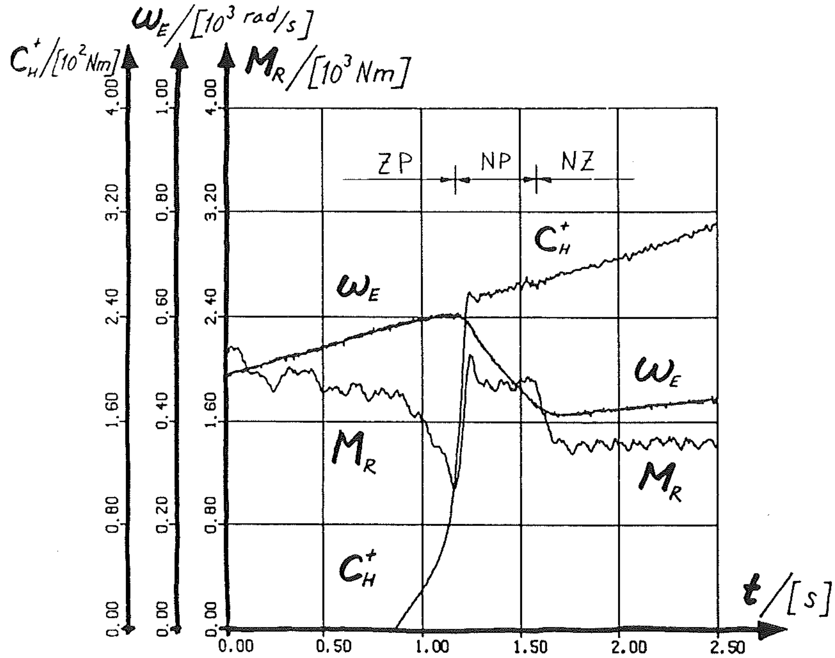


Figure 6.2: *Case 1: Upshift with one-way clutch. Test results*

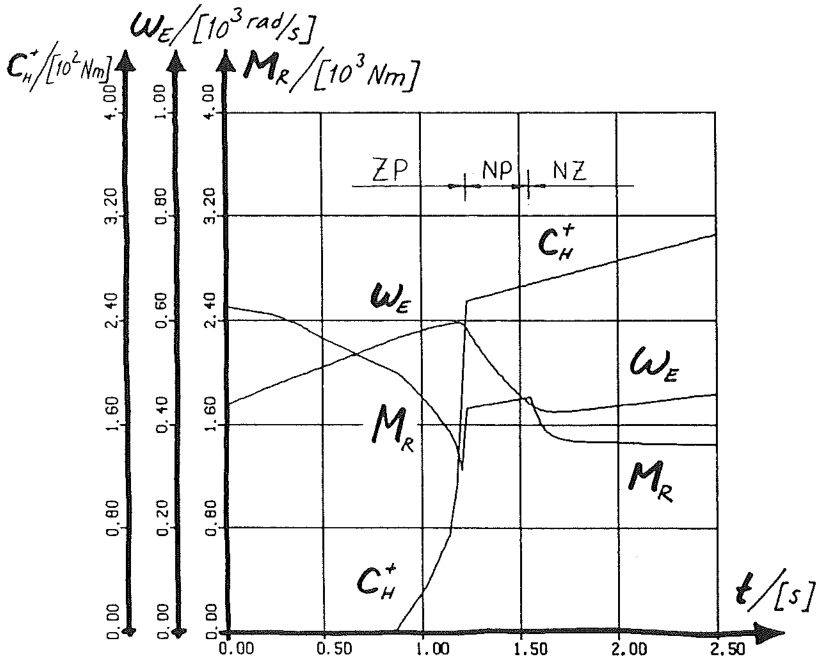


Figure 6.3: *Case 1: Upshift with one-way clutch. Simulation results*

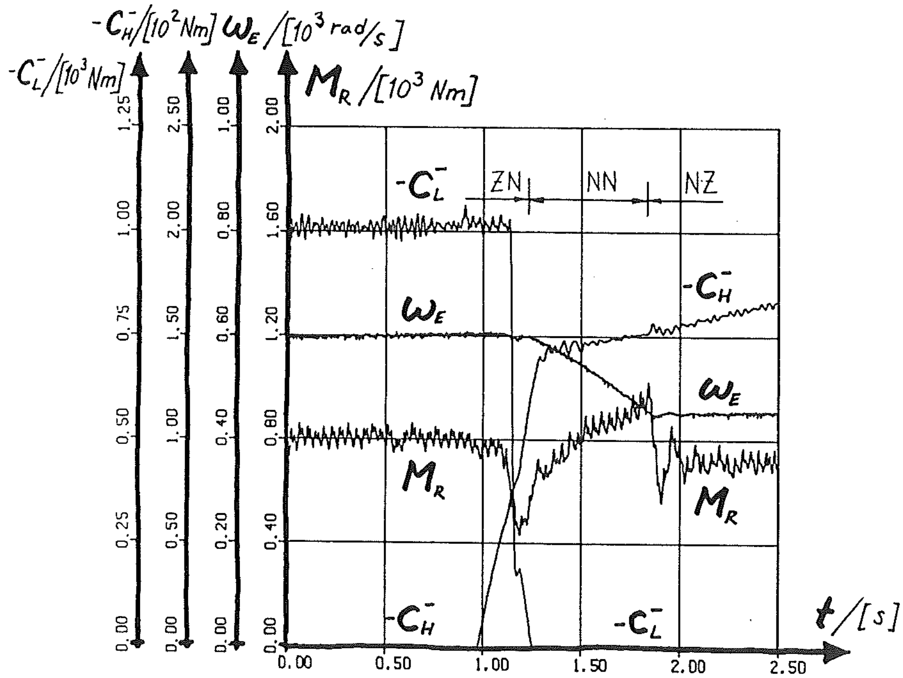


Figure 6.4: *Case 2: Upshift to lock-up gear. Test results*

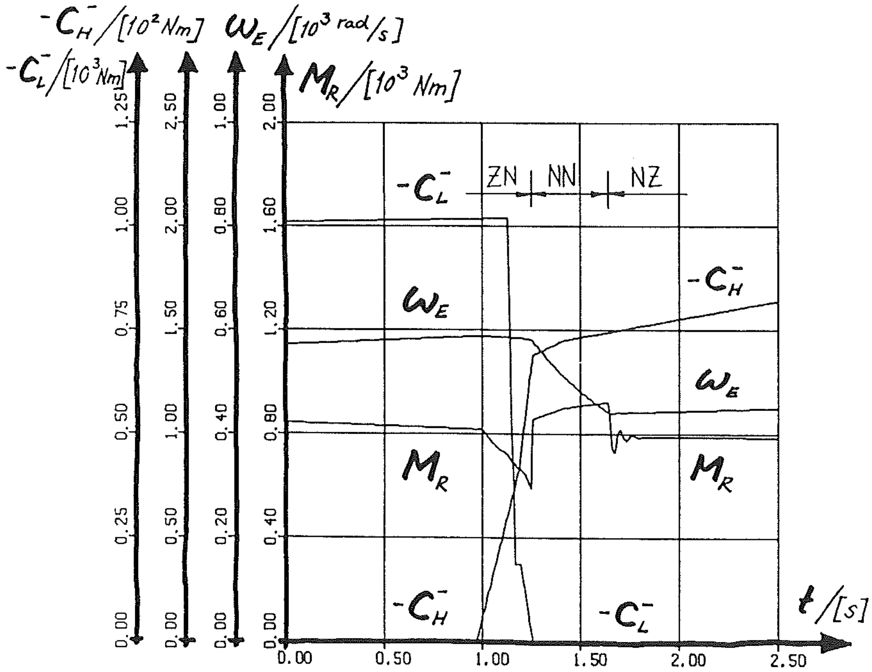


Figure 6.5: *Case 2: Upshift to lock-up gear. Simulation results*

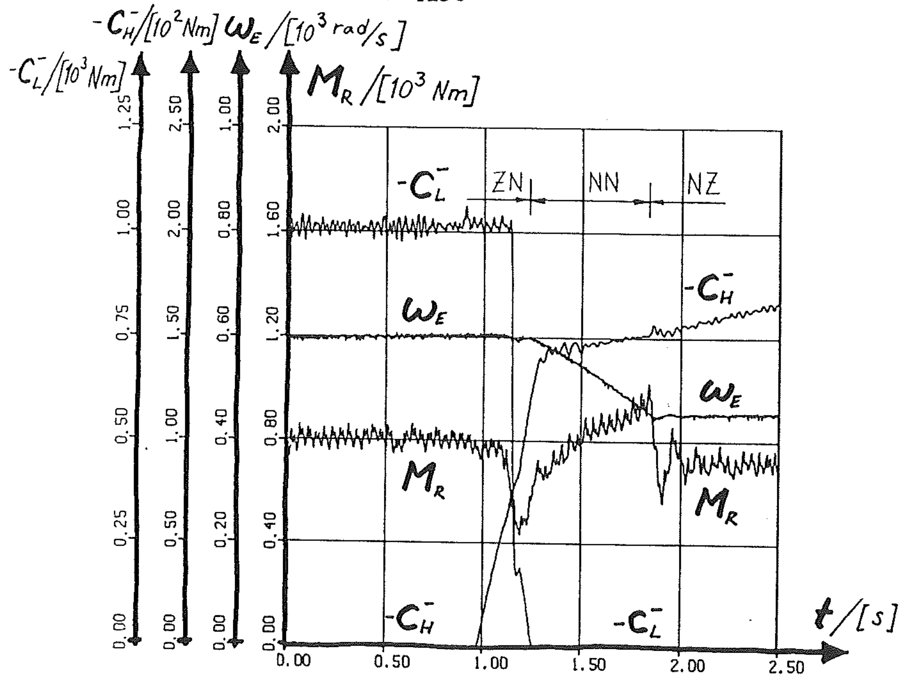


Figure 6.6: *Case 2: Upshift to lock-up gear. Test results*

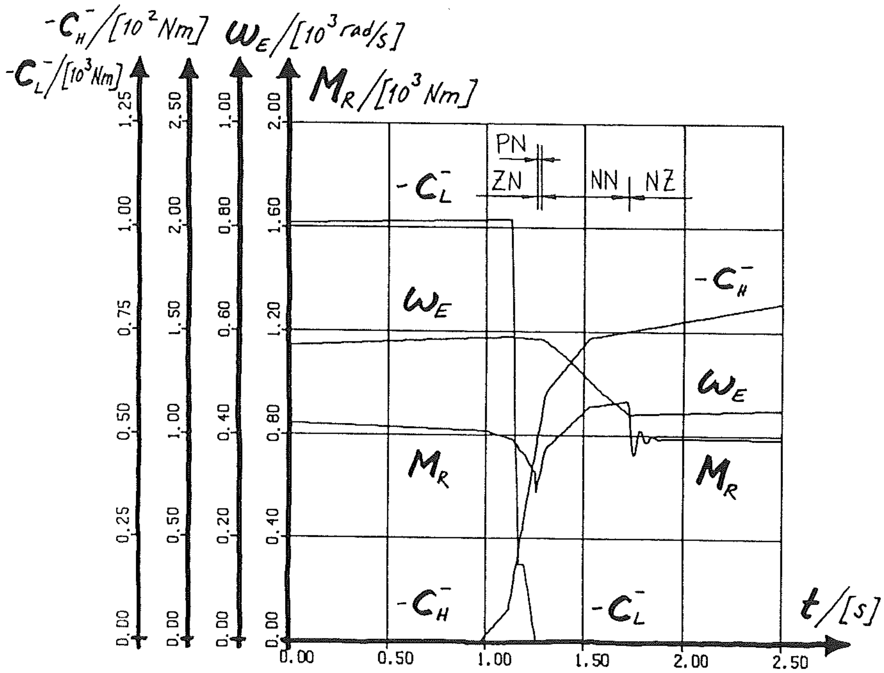


Figure 6.7: *Case 2: Upshift to lock-up gear. Simulation results with corrected torque capacity,* c_H

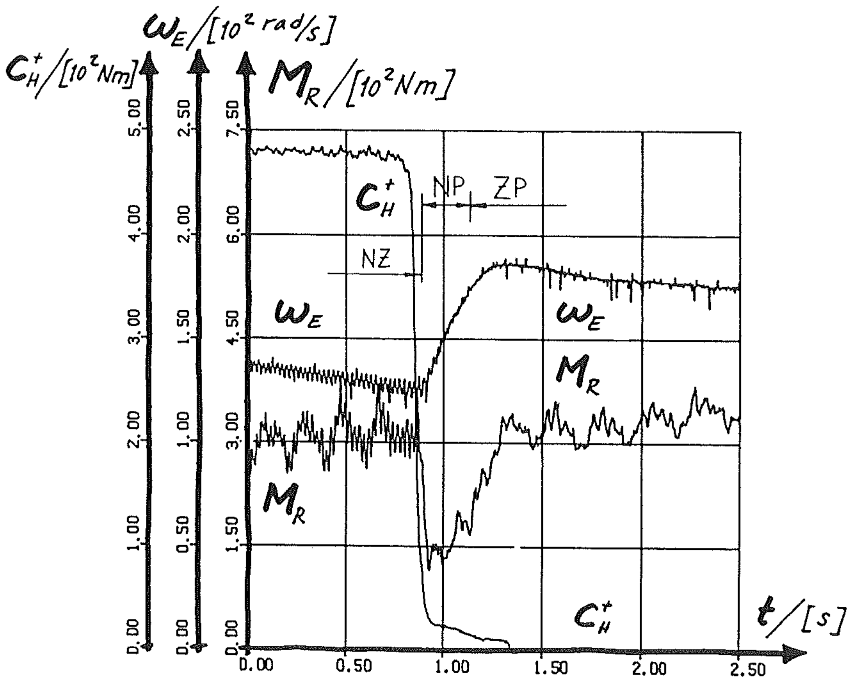


Figure 6.8: *Case 3: Downshift from torque-split gear. Test results*

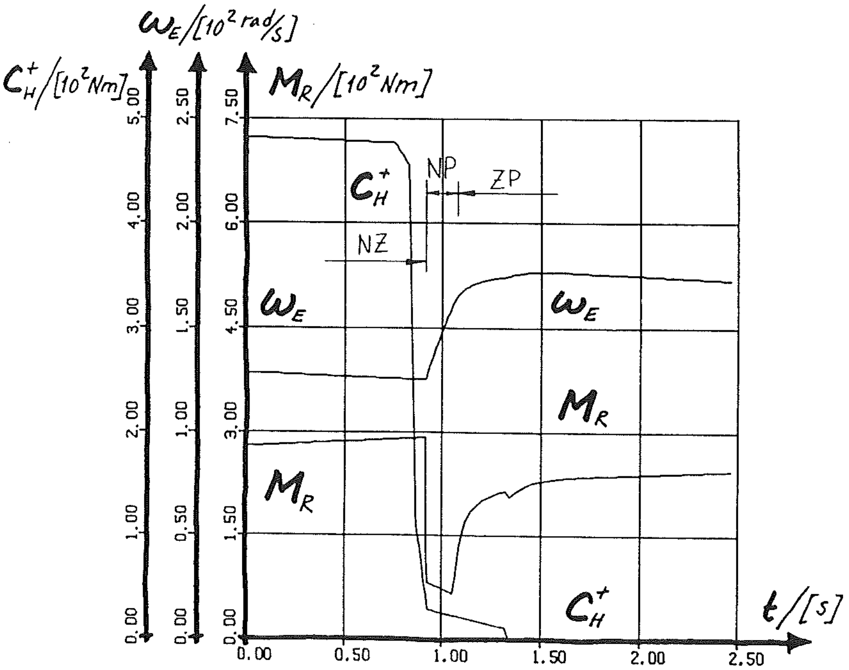


Figure 6.9: *Case 3: Downshift from torque-split gear. Simulation results*

Conclusions

The model presented in this study is suitable for simulating the global mechanics of a ratio change. The differences between test and simulation results can mainly be explained on the basis of incorrect input data. There is a particular need for further work that makes it possible to translate the hydraulic pressure to a torque capacity during the landing phase of the clutch.

A simplified model can be analysed in a dimensionless way. The essential phenomena can thus easily be explained. A ratio change can be studied in an "aquarium diagram".

The following three points are useful when simulating ratio changes in powershifting transmissions. They are presented in a general form, since they ought to be of interest for other dynamic transmission problems as well.

- **Matrix equations**
Matrix equations are very powerful when dealing with complex transmission systems. The requirement is transmissions that are linear with respect to velocity and torque. This is the case with most transmissions, when losses are neglected.

- **Velocity and torque as state variables**
Velocity of inertias and torques of elasticities are suitable to use as state variables in transmission dynamics. The differential equations are easily derived from this point of view. This is also an easy way to handle systems without inertias between all the components. It leads to more reliable numerical solutions than if a small inertia had been used between the components.

- **Separating phases in dynamic systems**
Certain components operate in different phases. This constitutes different differential equations, which are valid in different time intervals. One example is systems including clutches with Coulombian friction. It is convenient to solve such a problem as a <u>set</u> of initial value problems. This also leads to more reliable numerical solutions than if the system had been approximated by one with just a single phase.

Bibliography

[Förster 1957] FÖRSTER, Hans Joachim. 'Das kraftschlüssige Schalten von Übersetzungsstufen in Fahrzeug-Getrieben'. *VDI-Zeitung*, 1957, No 27, pp 1319-1331.

[Förster 1962] FÖRSTER, Hans Joachim. 'Getriebeschaltung ohne Zugkraftunterbrechung'. *Automobil Industrie*, 1962, October, pp 60-76.

[Winchell 1962] WINCHELL, F. J. and ROUTE, W. D. 'Ratio Changing the Passenger Car Automatic Transmission'. *Design Practices - Passenger Car Automatic Transmissions* (SAE), 1962, pp 79-105.

[Ott 1972] OTT, Anton. 'Simulation von Gruppengetrieben mit Hilfe elektronischer Rechenanlage'. *Automobiltechnische Zeitschrift*, 1972, September, No 9, pp 343-348 and pp 452-455.

[Kraft 1972] KRAFT, Karl-Friedrich. *Zugkraftschaltungen in automatischen Fahrzeuggetrieben*. Universität Karlsruhe, 1972 (Dissertation).

[Kraft 1974] KRAFT, Karl-Friedrich. 'Zugkraftschaltungen in automatischen Fahrzeuggetrieben'. *Automobil Industrie*, 1974, No 1, pp 41-74.

[Ishihara 1968] ISHIHARA, T., OYA, M., NISHIKAWA, H. and SUZUKI, K. 'Transient Characteristics of Automatic Transmission during Gear Change'. *12th FISITA congress* (Barcelona), 1968, pp 1-13.

[Ishihara 1970] ISHIHARA, T. and INUI, S. 'Testing Method for Automatic Transmission during Gear Change'. *13th FISITA congress* (Bruxelles), 1970, paper 16 C.

[Ito 1972] ITO, Shin and SUZUKI, Katsuya. 'Transient Characteristics of Automatic Transmission during Gear Ratio Change'. *Bulletin of JSAE*, 1972, No 4, pp 75-82.

[Shindo 1980] SHINDO, Y., ITO, H. and ISHIHARA, T. 'A Fundamental Consideration on Shift Mechanism of Automatic Transmission'. *SAE Transactions*, section 1, Volume 88, pp 219-229 (SAE790043).

[Koch 1972] KOCH, L. G. 'Power Train-Vehicle Modeling to Simulate Shifting Transients of Off-Highway Vehicles'. *SAE Transactions*, section 1, Volume 72, pp 234-248 (SAE720044).

[Winkelmann 1985] WINKELMANN, S. and HARMUTH, H. *Schaltbare Reibkupplungen*. Springer-Verlag, 1985.

[Sanger 1975] SANGER, D. J. 'Matrix methods in the analysis of coupled differentials and differential mechanisms'. *4th World Congress on the Theory of Machines and Mechanisms*, 1975, pp 27-31.

[Pichard 1977] PICHARD, J. and BESSON, B. 'Proposition d'un modélé général représentatif des transmissions de puissance exemples d'exploitation'. *Ingenieurs de l'automobile*, No. 4, 1977, pp 183-191.

[Henriksson 1989] HENRIKSSON, Magnus. *Measurements of the friction characteristics of a multi-disc brake in an automatic transmission*, Master degree thesis, Machine & Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1989. In swedish.

Appendix A

Transmission matrix equations

Start with a transmission as shown in figure A.1

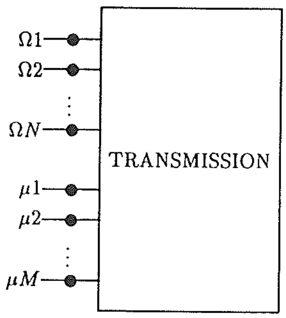


Figure A.1: *Transmission with input shafts only*

Assume that:

- The transmission is linear with respect to velocity.
- There are N ω-dof.
- The Ω-nodes are linearily independent with respect to velocity.

Then the following velocity equation can be written:

$$\vec{\omega}_\mu = \begin{bmatrix} \omega_{\mu 1} \\ \omega_{\mu 2} \\ \vdots \\ \omega_{\mu M} \end{bmatrix} = \begin{bmatrix} (M \times N)\text{-} \\ \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \omega_{\Omega 1} \\ \omega_{\Omega 2} \\ \vdots \\ \omega_{\Omega N} \end{bmatrix} = \mathbf{I}_\omega \cdot \vec{\omega}_\Omega \tag{A.1}$$

Assume that the torque equations can be written as:

$$\vec{M}_\Omega = \begin{bmatrix} M_{\Omega 1} \\ M_{\Omega 2} \\ \vdots \\ M_{\Omega N} \end{bmatrix} = \begin{bmatrix} (N \times M)- \\ \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} M_{\mu 1} \\ M_{\mu 2} \\ \vdots \\ M_{\mu M} \end{bmatrix} = \mathbf{I}_M \cdot \vec{M}_\mu \tag{A.2}$$

This last assumption can be verified by checking the power equilibrium. Power equilibrium of all $N + M$ nodes should be valid. The power input in every node is $M \cdot \omega$.

$\sum P = 0 \;\Rightarrow\; \vec{M}_\mu^T \cdot \vec{\omega}_\mu + \vec{M}_\Omega^T \cdot \vec{\omega}_\Omega = 0$

Substituting the velocity and torque equations into the power equilibrium gives:

$$\vec{M}_\mu^T \cdot \mathbf{I}_\omega \cdot \vec{\omega}_\Omega + \vec{M}_\mu^T \cdot \mathbf{I}_M^T \cdot \vec{\omega}_\Omega = 0 \Rightarrow \mathbf{I}_M = -\mathbf{I}_\omega^T \tag{A.3}$$

The same kind of fundamental correlation between velocity and torque may also be seen in [Sanger 1975] and [Pichard 1977].

Below the same procedure is carried out for a transmission with both input nodes, output nodes and clutches.

Output nodes are treated in a very similar way to input nodes. The difference is just that $P = M \cdot \omega$ means <u>output</u> power.

Clutches are treated somewhat differently from input nodes and output nodes. In a first approach, we note that a clutch has two shafts and therefore two velocities and two torques. However, it is appropriate to use only one torque, since the end torques are equal. Furthermore, it is appropriate to use the relative velocity. The positive senses of clutch torque (M_C) and relative velocity (ω_{rel}) are chosen in such a way that $P_C = M_C \cdot \omega_{rel}$ means <u>output</u> power, i. e. power loss. The clutches are placed among the μ-nodes, which normally is the case in a gearbox.

The complete transmission could be drawn as in figure A.2.

Velocity equations:

$$\vec{\omega}_\mu = \begin{bmatrix} \vec{\omega}_{rel} \\ \vec{\omega}_{\mu IN} \\ \vec{\omega}_{\mu OUT} \end{bmatrix} = \begin{bmatrix} \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \vec{\omega}_{\Omega IN} \\ \vec{\omega}_{\Omega OUT} \end{bmatrix} = \mathbf{I}_\omega \cdot \vec{\omega}_\Omega \tag{A.4}$$

Torque equations:

$$\vec{M}_\Omega = \begin{bmatrix} \vec{M}_{\Omega IN} \\ \vec{M}_{\Omega OUT} \end{bmatrix} = \begin{bmatrix} \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \vec{M}_C \\ \vec{M}_{\mu IN} \\ \vec{M}_{\mu OUT} \end{bmatrix} = \mathbf{I}_M \cdot \vec{M}_\mu \tag{A.5}$$

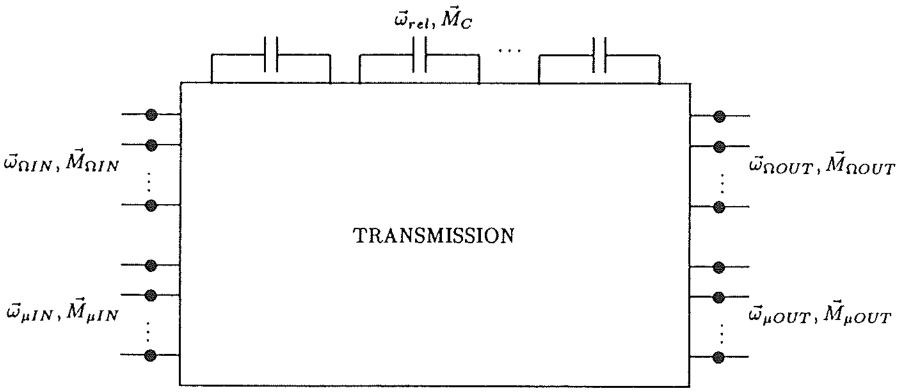


Figure A.2: *Transmission with input shafts, output shafts and clutches*

Power equilibrium requires:

$$P_{IN} = P_{OUT}$$

$$\vec{M}^T_{\Omega IN} \cdot \vec{\omega}_{\Omega IN} + \vec{M}^T_{\mu IN} \cdot \vec{\omega}_{\mu IN} = \vec{M}^T_{\Omega OUT} \cdot \vec{\omega}_{\Omega OUT} + \vec{M}^T_{\mu OUT} \cdot \vec{\omega}_{\mu OUT} + \vec{M}^T_C \cdot \vec{\omega}_{rel}$$

$$\vec{M}^T_{\Omega IN} \cdot \vec{\omega}_{\Omega IN} - \vec{M}^T_{\Omega OUT} \cdot \vec{\omega}_{\Omega OUT} = \vec{M}^T_C \cdot \vec{\omega}_{rel} - \vec{M}^T_{\mu IN} \cdot \vec{\omega}_{\mu IN} + \vec{M}^T_{\mu OUT} \cdot \vec{\omega}_{\mu OUT}$$

$$\left(\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} \end{bmatrix} \cdot \begin{bmatrix} \vec{M}_{\Omega IN} \\ \vec{M}_{\Omega OUT} \end{bmatrix} \right)^T \cdot \begin{bmatrix} \vec{\omega}_{\Omega IN} \\ \vec{\omega}_{\Omega OUT} \end{bmatrix} =$$
$$= \left(\begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix} \cdot \begin{bmatrix} \vec{M}_C \\ \vec{M}_{\mu IN} \\ \vec{M}_{\mu OUT} \end{bmatrix} \right)^T \cdot \begin{bmatrix} \vec{\omega}_{rel} \\ \vec{\omega}_{\mu IN} \\ \vec{\omega}_{\mu OUT} \end{bmatrix}$$

where $\mathbf{E}$ and $\mathbf{0}$ are identity and null matrices, respectively. This can be written:

$$\left(\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} \end{bmatrix} \cdot \vec{M}_\Omega \right)^T \cdot \vec{\omega}_\Omega = \left(\begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix} \cdot \vec{M}_\mu \right)^T \cdot \vec{\omega}_\mu$$

Substituting the velocity and torque equations (A.4 and A.5) gives:

$$\left(\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} \end{bmatrix} \cdot \mathbf{I}_M \cdot \vec{M}_\mu \right)^T \cdot \vec{\omega}_\Omega = \left(\begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix} \cdot \vec{M}_\mu \right)^T \cdot \mathbf{I}_\omega \cdot \vec{\omega}_\Omega$$

$$\mathbf{I}_M^T \cdot \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix} \cdot \mathbf{I}_\omega$$

Finally: $$\mathbf{I}_M = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} \end{bmatrix} \cdot \mathbf{I}_\omega^T \cdot \begin{bmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix} \tag{A.6}$$

Appendix B

Examples of gearbox matrices

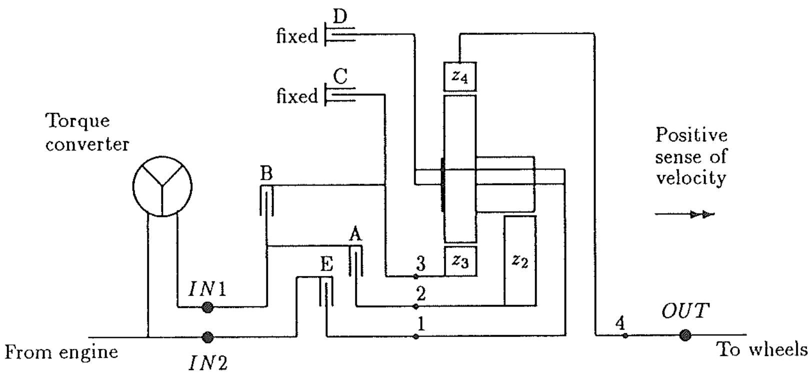


Figure B.1: *Automatic transmission*

Table B.1: *Gear/clutch table*

Clutch→ Gear↓	E	A	C	D	B	Speed ratio if no slip in converter	Use of converter
1		•		•		2.579	Common
2		•	•			1.407	Common
3'		•			•	1	Common
3	•	•				1	Torque split
3"	•	•			•	1	Lock-up
4	•		•			0.742	Lock-up (bypassed)
R				•	•	−2.882	Common

This appendix shows how the gearbox matrices in chapter 3 are derived. An automatic transmission with planetary gears is used as the example. The layout is shown in figure B.1. Table B.1 shows which clutches are engaged at each gear.

B.1 Equations

Conventional analysis of planetary gears (Willis' equation) yields:

$(\omega_2 - \omega_1)/(\omega_4 - \omega_1) = z_4/z_2 = R_{24}$
$(\omega_3 - \omega_1)/(\omega_4 - \omega_1) = -z_4/z_3 = R_{34}$

Node 4 is directly connected to node OUT, which yields:

$\omega_4 = \omega_{OUT}$

Define the following relative velocities:

$\omega_{relA} = \omega_{IN1} - \omega_2$
$\omega_{relB} = \omega_{IN1} - \omega_3$
$\omega_{relC} = 0 - \omega_3$
$\omega_{relD} = 0 - \omega_1$
$\omega_{relE} = \omega_{IN2} - \omega_1$

These equations may be written on matrix form:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_{relA} \\ \omega_{relB} \\ \omega_{relC} \\ \omega_{relD} \\ \omega_{relE} \end{bmatrix} = \begin{bmatrix} R_{24}-1 & 1 & 0 & -R_{24} & 0 & 0 & 0 \\ R_{34}-1 & 0 & 1 & -R_{34} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.1}$$

For numerical calculations the following numbers are used:

$R_{24} = z_4/z_2 = 98/38 \approx 2.57895$
$R_{34} = -z_4/z_3 = -98/34 \approx -2.88235$

B.2 Matrix manipulations

The velocities ω_1, ω_2, ω_3 and ω_4 can be eliminated from the equation B.1:

$$\begin{bmatrix} \frac{R_{34}-1}{R_{24}-1} & -1 & 0 & 0 & 0 \\ \frac{R_{34}-1}{R_{24}-1} & 0 & -1 & 0 & 0 \\ \frac{-1}{R_{24}-1} & 0 & 0 & -1 & 0 \\ \frac{-1}{R_{24}-1} & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \omega_{relA} \\ \omega_{relB} \\ \omega_{relC} \\ \omega_{relD} \\ \omega_{relE} \end{bmatrix} = \begin{bmatrix} \frac{R_{34}-1}{R_{24}-1} - 1 & 0 & 1 - \frac{R_{34}-1}{R_{24}-1} \\ \frac{R_{34}-1}{R_{24}-1} & 0 & 1 - \frac{R_{34}-1}{R_{24}-1} \\ \frac{-1}{R_{24}-1} & 0 & \frac{R_{24}}{R_{24}-1} \\ \frac{-1}{R_{24}-1} & -1 & \frac{R_{24}}{R_{24}-1} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.2}$$

Inserting numerical values gives:

$$\begin{bmatrix} -2.4588 & -1 & 0 & 0 & 0 \\ -2.4588 & 0 & -1 & 0 & 0 \\ -0.6333 & 0 & 0 & -1 & 0 \\ -0.6333 & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \omega_{relA} \\ \omega_{relB} \\ \omega_{relC} \\ \omega_{relD} \\ \omega_{relE} \end{bmatrix} = \begin{bmatrix} -3.4588 & 0 & 3.4588 \\ -2.4588 & 0 & 3.4588 \\ -0.6333 & 0 & 1.6333 \\ -0.6333 & -1 & 1.6333 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix}$$

B.2.1 One clutch steadily engaged

Ratio changes where exactly one clutch is steadily engaged during the shift can be analysed by putting the corresponding relative velocity $\omega_{rel} = 0$. For instance, ratio changes between gears 1, 2, 3' and 3 should use $\omega_{relA} = 0$. Equation B.2 may then be rewritten in the following way for each of the five clutches steadily engaged:

$$\omega_{relA} = 0 \quad \Rightarrow (\omega_{relB}, \omega_{relC}, \omega_{relD}, \omega_{relE})^T =$$

$$= \begin{bmatrix} 1 - \frac{R_{34}-1}{R_{24}-1} & 0 & \frac{R_{34}-1}{R_{24}-1} - 1 \\ -\frac{R_{34}-1}{R_{24}-1} & 0 & \frac{R_{34}-1}{R_{24}-1} - 1 \\ \frac{1}{R_{24}-1} & 0 & \frac{-R_{24}}{R_{24}-1} \\ \frac{1}{R_{24}-1} & 1 & \frac{-R_{24}}{R_{24}-1} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} = \begin{bmatrix} 3.4588 & 0 & -3.4588 \\ 2.4588 & 0 & -3.4588 \\ 0.6333 & 0 & -1.6333 \\ 0.6333 & 1 & -1.6333 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.3}$$

$$\omega_{relB} = 0 \quad \Rightarrow (\omega_{relA}, \omega_{relC}, \omega_{relD}, \omega_{relE})^T =$$

$$= \begin{bmatrix} 1 - \frac{R_{24}-1}{R_{34}-1} & 0 & \frac{R_{24}-1}{R_{34}-1} - 1 \\ -1 & 0 & 0 \\ \frac{1}{R_{34}-1} & 0 & \frac{-1}{R_{34}-1} - 1 \\ \frac{1}{R_{34}-1} & 1 & \frac{-1}{R_{34}-1} - 1 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} = \begin{bmatrix} 1.4067 & 0 & -1.4067 \\ -1 & 0 & 0 \\ -0.2576 & 0 & -0.7424 \\ -0.2576 & 1 & -0.7424 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.4}$$

$$\omega_{relC} = 0 \quad \Rightarrow (\omega_{relA}, \omega_{relB}, \omega_{relD}, \omega_{relE})^T =$$

$$= \begin{bmatrix} 1 & 0 & \frac{R_{24}-1}{R_{34}-1} - 1 \\ 1 & 0 & 0 \\ 0 & 0 & \frac{-1}{R_{34}-1} - 1 \\ 0 & 1 & \frac{-1}{R_{34}-1} - 1 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1.4067 \\ 1 & 0 & 0 \\ 0 & 0 & -0.7424 \\ 0 & 1 & -0.7424 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.5}$$

$$\omega_{relD} = 0 \quad \Rightarrow (\omega_{relA}, \omega_{relB}, \omega_{relC}, \omega_{relE})^T =$$

$$= \begin{bmatrix} 1 & 0 & -R_{24} \\ 1 & 0 & -R_{34} \\ 0 & 0 & -R_{34} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2.5789 \\ 1 & 0 & 2.8824 \\ 0 & 0 & 2.8824 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.6}$$

$$\omega_{relE} = 0 \quad \Rightarrow (\omega_{relA}, \omega_{relB}, \omega_{relC}, \omega_{relD})^T =$$

$$= \begin{bmatrix} 1 & R_{24}-1 & -R_{24} \\ 1 & R_{34}-1 & -R_{34} \\ 0 & R_{34}-1 & -R_{34} \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} = \begin{bmatrix} 1 & 1.5789 & -2.5789 \\ 1 & -3.8824 & 2.8824 \\ 0 & -3.8824 & 2.8824 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \tag{B.7}$$

Identification of the $\mathbf{I}_\omega$-matrix

When shifting between gears 1 and 2 clutch A is steadily engaged. This means $\omega_{relA} = 0$ and equation B.3 is valid. Clutch D is renamed clutch L, since it is engaged at the lower gear. Clutch C is renamed clutch H, since it is engaged at the higher gear.

The essential parts of equation B.3 are:

$$\begin{bmatrix} \omega_{relL} \\ \omega_{relH} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_{24}-1} & 0 & \frac{-R_{24}}{R_{24}-1} \\ -\frac{R_{34}-1}{R_{24}-1} & 0 & \frac{R_{34}-1}{R_{24}-1} - 1 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix}$$

This 2×3-matrix is the $\mathbf{I}_\omega$-matrix and the i-constants may be identified as:

$i_{L,IN1} = R_{24} - 1 = 1.5789$
$i_{L,IN2} = 1/0 = \pm\infty$
$i_{L,OUT} = R_{24}/(R_{24} - 1) = 1.6333$
$i_{H,IN1} = -(R_{24} - 1)/(R_{34} - 1) = 0.4067$
$i_{H,IN2} = 1/0 = \pm\infty$
$i_{H,OUT} = 1 - (R_{34} - 1)/(R_{24} - 1) = 3.4588$

The gearbox may now be treated as simply a box containing the $\mathbf{I}_\omega$-matrix. This is shown in figure B.2. Figure B.3 shows what is inside the box in figure B.2.

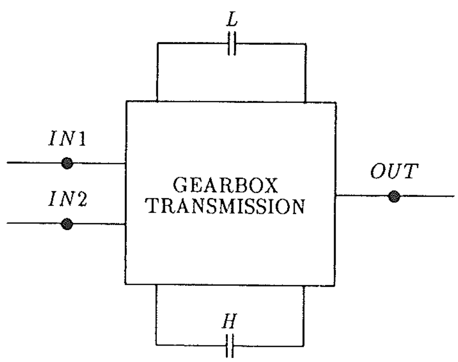


Figure B.2: *Two-clutch gearbox with two input shafts and one output shaft*

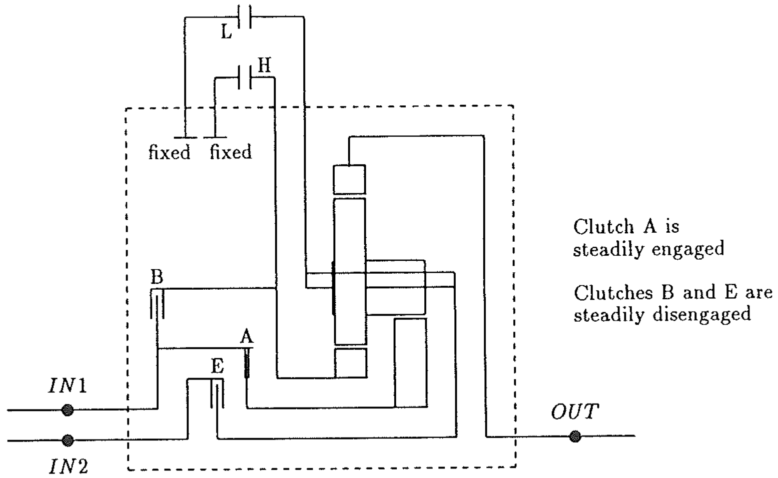


Figure B.3: *Shifting between gears* 1 *and* 2

B.2.2 Two clutches steadily engaged

Ratio changes where two clutches are steadily engaged during the shift, can be analysed by setting their relative velocities equal to zero. However, this reduces the number of ω-dof from three to two. Therefore, we should place one of the velocities of the power shafts among the relative velocities. For instance, ratio changes between the gears 3 and 3" should use $\omega_{relA} = \omega_{relE} = 0$. Here it is convenient to place ω_{IN1} among the relative velocities. Equation B.8 below corresponds to the previous equation B.2.

$$\omega_{relA} = \omega_{relE} = 0 \quad \Rightarrow (\omega_{relB}, \omega_{relC}, \omega_{relD}, \omega_{IN1})^T =$$

$$= \begin{bmatrix} R_{34} - R_{24} & R_{24} - R_{34} \\ R_{34} - 1 & -R_{34} \\ -1 & 0 \\ 1 - R_{24} & R_{24} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} = \begin{bmatrix} -5.4613 & 5.4613 \\ -3.8824 & 2.8824 \\ -1 & 0 \\ -1.5789 & 2.5789 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \qquad \text{(B.8)}$$

Identification of the $\mathbf{I}_\omega$-matrix

When shifting between gears 3 and 3", clutch B is the only active clutch. This ratio change is a lock-up operation of the torque converter.

The essential parts of equation B.8 are:

$$\begin{bmatrix} \omega_{relB} \\ \omega_{IN1} \end{bmatrix} = \begin{bmatrix} R_{34} - R_{24} & R_{24} - R_{34} \\ 1 - R_{24} & R_{24} \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN2} \\ \omega_{OUT} \end{bmatrix}$$

This 2×2-matrix is the $\mathbf{I}_\omega$-matrix.

B.2.3 No clutches steadily engaged

In attempting to treat all ratio changes by the same matrix equation a problem arises, since there are to many ω-dof. It is impossible to determine all relative velocities only by the velocities of the three power shafts. It is then convenient to introduce an additional power shaft. It is convenient to chose the shaft with the largest amount of inertia. In this case it is likely to be the planet carrier (shaft 1 in figure B.1). Let us call the corresponding node $IN3$. Equation B.1 may then be extended by the equation:

$\omega_1 = \omega_{IN3}$

The equation corresponding to equation B.2 is then:

$$\begin{bmatrix} \omega_{relA} \\ \omega_{relB} \\ \omega_{relC} \\ \omega_{relD} \\ \omega_{relE} \end{bmatrix} = \begin{bmatrix} 1 & 0 & R_{24}-1 & -R_{24} \\ 1 & 0 & R_{34}-1 & -R_{34} \\ 0 & 0 & R_{34}-1 & -R_{34} \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{IN3} \\ \omega_{OUT} \end{bmatrix} \tag{B.9}$$

Insertion of numerical values gives:

$$\begin{bmatrix} \omega_{relA} \\ \omega_{relB} \\ \omega_{relC} \\ \omega_{relD} \\ \omega_{relE} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.5789 & -2.5789 \\ 1 & 0 & -3.8824 & 2.8824 \\ 0 & 0 & -3.8824 & 2.8824 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{IN3} \\ \omega_{OUT} \end{bmatrix}$$

Identification of the $\mathbf{I}_\omega$-matrix

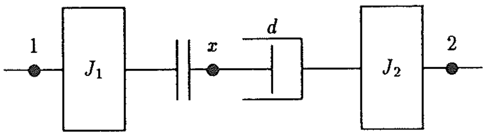
When shifting between all gears the 5×4-matrix in equation B.9 is the $\mathbf{I}_\omega$-matrix. The gearbox may now be treated as simply a box containing this matrix. Using this approach it would be possible to handle all ratio changes, not only those between the gears presented in table B.1. However, it is very difficult to handle all phases. The gearbox would be able to work in $3^5 = 243$ gearbox phases! (Each clutch has 3 phases as discussed in chapter 3.)

Appendix C

Examples of systems including clutches

Some examples of systems including clutches where shown in chapter 4. This appendix gives further examples.

C.1 Inertia – clutch – damper – inertia



This case generates very little trouble because the same state variables (ω_1 and ω_2) may be used in both the slip and the stick phases. No new initial values have to be calculated. M_1 and M_2 may be thought of as known variables, calculated from an outer system.

Both phases:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M)/J_1$
$\dot{\omega}_2 = (M - M_2)/J_2$

Slip phase:

Algebraic equations (level 1):
$\alpha = \alpha(\omega_{rel})$

Algebraic equations (level 2):
$\omega_{rel} = \omega_1 - \omega_x$

Algebraic equations (level 3):
$\omega_x = \omega_2 + M/d$

Algebraic equations (level 4):
$M = \alpha \cdot c(t)$

Stick phase:

Algebraic equations (level 1):
$M = d \cdot (\omega_x - \omega_2)$

Algebraic equations (level 2):
$\omega_x = \omega_1 - \omega_{rel}$

Algebraic equations (level 3):
$\omega_{rel} = 0$

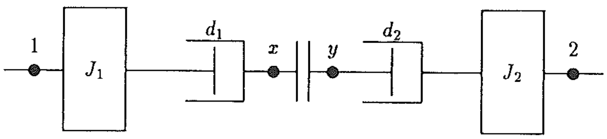
In the slip phase the algebraic equations define α implicitly. Evaluating levels 1-4 in backward order makes a recursive iteration. The following recursive formula can be obtained:

$\alpha = \alpha(\omega_1 - (\omega_2 + \alpha \cdot c(t)/d))$ (α is a function of itself)

Fast convergence will be achieved, since α is only weakly dependent on ω_{rel}. A starting value of α may, for instance, be α from the previous time instant. When α is determined, then level 1 should be evaluated.

Using velocity independent Coulombian friction (constant α) there is no need to iterate. The torque M could be determined directly by the level 4, but the other levels must be evaluated anyway in order to get a value of ω_{rel}, to check the phase switch condition.

C.2 Inertia – damper – clutch – damper – inertia



This example is very similar to the previous one. The difference obtained is an iteration in the stick phase as well. This iteration is somewhat different from the slip phase iteration. State variables in both phases are ω_1 and ω_2. M_1 and M_2 may be thought of as known variables, calculated from an outer system.

Both phases:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M)/J_1$
$\dot{\omega}_2 = (M - M_2)/J_2$

Slip phase:

Algebraic equations (level 1):
$\alpha = \alpha(\omega_{rel})$

Algebraic equations (level 2):
$\omega_{rel} = \omega_x - \omega_y$

Algebraic equations (level 3):
$\omega_x = \omega_2 - M/d_1$
$\omega_y = \omega_2 + M/d_2$

Algebraic equations (level 4):
$M = \alpha \cdot c(t)$

Stick phase:

Algebraic equations (level 1):
$M = d_1 \cdot (\omega_1 - \omega_x)$

Algebraic equations (level 2):
$\omega_x = \omega_y + \omega_{rel}$

Algebraic equations (level 3):
$\omega_y = \omega_2 + M/d_2$
$\omega_{rel} = 0$

The slip phase is almost identical with the previous example. The same kind of recursive iteration may be carried out. The stick phase also needs an iteration, which is not the case in the previous example. This iteration is in principal different from the one in the slip phase. The recursive formula defines M as a function of M. It is more difficult to guess a good starting value for the torque M, when the clutch is not slipping.

It is usually possible to determine the equation for both dampers together. This is especially easy when the dampers are linear. The total damping constant d can be calculated as:

$d = \frac{1}{1/d_1 + 1/d_2}$ (two linear dampers in series)

Then the stick phase equations become explicit:

(Alternative) Stick phase:

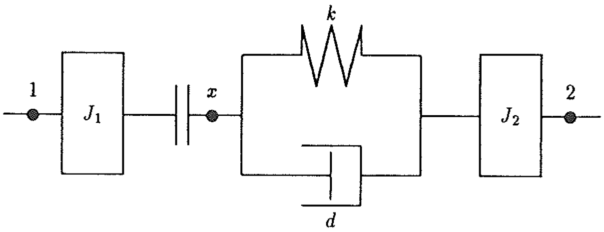
Algebraic equations (level 1):
$\omega_x = \omega_y + \omega_{rel}$

Algebraic equations (level 2):
$\omega_y = \omega_2 + M/d_2$
$\omega_{rel} = 0$

Algebraic equations (level 3):
$M = d \cdot (\omega_1 - \omega_2)$

Levels 1-2 are only added to calculate ω_x, ω_y and ω_{rel}. They are not really needed to calculate the state derivatives at level 0.

C.3 Inertia – clutch – elasticity/damper – inertia



This system demands three state variables when sticking (ω_1, ω_2 and M). The same three state variables can be used when slipping. Therefore, this system demands no new initial values. There are three torques involved: the clutch torque (M_C), the damper torque (M_d) and the elastic torque (M). Torques M_1 and M_2 may be thought of as known variables, calculated from an outer system.

Both phases:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M_C)/J_1$
$\dot{\omega}_2 = (M_C - M_2)/J_2$
$\dot{M} = k \cdot (\omega_1 - \omega_x)$

Slip phase:

Algebraic equations (level 1):
$\alpha = \alpha(\omega_{rel})$

Algebraic equations (level 2):
$\omega_{rel} = \omega_1 - \omega_x$

Algebraic equations (level 3):
$\omega_x = \omega_2 + M_d/d$

Algebraic equations (level 4):
$M_d = M_C - M$

Algebraic equations (level 5):
$M_C = \alpha \cdot c(t)$

Stick phase:

Algebraic equations (level 1):
$M_C = M + M_d$

Algebraic equations (level 2):
$M_d = d \cdot (\omega_x - \omega_2)$

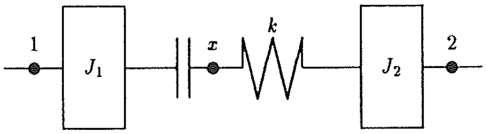
Algebraic equations (level 3):
$\omega_x = \omega_1 - \omega_{rel}$

Algebraic equations (level 4):
$\omega_{rel} = 0$

In the slip phase, α should be determined by recursive iteration of the algebraic equations at levels 1-5.

As long as d is large enough there are no problems. If $d \to 0$, ω_x may no longer be determined in the slip phase, since d appears in the denominator. This problem is solved in the following example.

C.4 Inertia – clutch – elasticity – inertia



This system demands three state variables when sticking (ω_1, ω_2 and M). When slipping the clutch torque is time governed, since $c = c(t)$. Therefore, the slip phase just requires two state variables (ω_1 and ω_2). The torques M_1 and M_2 may be thought of as known variables, calculated from an outer system.

Slip phase:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M)/J_1$
$\dot{\omega}_2 = (M - M_2)/J_2$

Algebraic equations (level 1):
$M = \alpha \cdot c(t)$

Algebraic equations (level 2):
$\alpha = \alpha(\omega_{rel})$

Algebraic equations (level 3):
$\omega_{rel} = \omega_1 - \omega_x$

Algebraic equations (level 4):
$\omega_x = \omega_2 + \dot{M}/k$

Algebraic equations (level 5):
$\dot{M} \approx \alpha \cdot \dot{c}(t)$

Initial values:
$\omega_1^{IV} = \omega_1$
$\omega_2^{IV} = \omega_2$

Stick phase:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M)/J_1$
$\dot{\omega}_2 = (M - M_2)/J_2$
$\dot{M} = k \cdot (\omega_x - \omega_2)$

Algebraic equations (level 1):
$\omega_x = \omega_1 - \omega_{rel}$

Algebraic equations (level 2):
$\omega_{rel} = 0$

Initial values:
$\omega_1^{IV} = \omega_1$
$\omega_2^{IV} = \omega_2$
$M^{IV} = M$

In the slip phase we again have a recursive formula for iteration. The start value needed is an estimated α to put into the equation at level 5. Levels 2-5 then constitute the recursive formula. Note that there is an approximation at level 5: $\frac{d}{dt}(\alpha \cdot c(t)) \approx \alpha \cdot \dot{c}(t)$. This approximation can be made since α is weakly dependent on ω_{rel}. Furthermore, we assume ω_{rel} does not vary too quickly.

The problem could (in principal) be solved without any approximations. This is achieved if we also let M be a state variable in the slip phase:

(Alternative) Slip phase:

Differential equations (level 0):
$\dot{\omega}_1 = (M_1 - M)/J_1$
$\dot{\omega}_2 = (M - M_2)/J_2$
$\dot{M} = k \cdot (\omega_x - \omega_2)$

Algebraic equations (level 1):
$\omega_x = \omega_1 - \omega_{rel}$

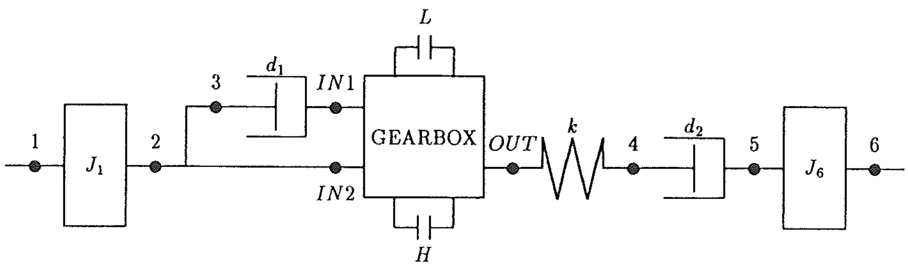
Algebraic equations (level 2):
$\omega_{rel} = \alpha^{-1}(M/c(t))$

Initial values:
$\omega_1^{IV} = \omega_1$
$\omega_2^{IV} = \omega_2$
$M^{IV} = ?$

The alternative slip phase uses the inverse α-function. This means we try to determine the relative velocity in the clutch from a known clutch torque. This is not physically correct for a component with Coulombian friction. Another problem is to find a proper initial value for M. Therefore the torque should not be used as a state variable.

Another observation is that M changes discontinuously when switching from the stick phase to the slip phase. The velocity ω_x must therefore be treated as infinite during the phase switch. When analysing the same system, but with an inertia at the node x, *stick-slip* occurs. Node x oscillates back so that ω_{rel} changes sign, which leads us back to the stick phase. If there is enough damping in the system, the oscillation ceases and the stick-slip vanishes. When $J_x \to 0$ the amount of damping needed to prevent stick-slip is also $\to 0$. Assuming inertia and damping $\to 0$ in the right way, this stick-slip phenomenon may be neglected. Without this assumption, there would be no possibility of remaining in the slip phase, when entering from the stick phase.

C.5 Inertia – damper – gearbox – elasticity – damper – inertia



The gearbox is assumed here to have two clutches, two input shafts and one output shaft. The damper d_1 is comparable with a torque converter.

In a slip phase there are two state variables (ω_1 and ω_6). In the stick phase there must be three state variables (ω_1, ω_6 and M_4). In the lock-up phase (both clutches stick) we use the same three state variables as in the stick phase. If there had been just two power shafts, the lock-up phase would imply that no shaft was able to rotate. This is not the case here.

The same notation is used here as in chapter 3. Torques M_1 and M_6 may be regarded as known variables, calculated from an outer system.

Slip phase:

Differential equations (level 0):

$\dot{\omega}_1 = (M_1 - M_2)/J_1$
$\dot{\omega}_6 = (M_5 - M_6)/J_6$

Initial values:

$\omega_1^{IV} = \omega_1$
$\omega_6^{IV} = \omega_6$

Stick phase or **Lock-up phase:**

Differential equations (level 0):

$\dot{\omega}_1 = (M_1 - M_2)/J_1$
$\dot{\omega}_6 = (M_5 - M_6)/J_6$
$\dot{M}_4 = k \cdot (\omega_{OUT} - \omega_4)$

Initial values:

$\omega_1^{IV} = \omega_1$
$\omega_6^{IV} = \omega_6$
$M_4^{IV} = M_4$

Slip phase (both clutches slip):

Algebraic equations (level 1):
$M_2 = M_3 + M_{IN2}$

Algebraic equations (level 2):
$M_3 = M_{IN1}$

Algebraic equations (level 3):
$\alpha_L = \alpha_L(\omega_{relL})$
$\alpha_H = \alpha_H(\omega_{relH})$

Algebraic equations (level 4):
$(\omega_{relL}, \omega_{relH})^T = \mathbf{I}_\omega \cdot (\omega_{IN1}, \omega_{IN2}, \omega_{OUT})^T$

Algebraic equations (level 5):
$\omega_{IN1} = \omega_3 - \dot{M}_{IN1}/d_1$
$\omega_{OUT} = \omega_4 + \dot{M}_{OUT}/k$

Algebraic equations (level 6):
$\omega_4 = \omega_5 + M_5/d_2$

Algebraic equations (level 7):
$M_5 = M_4$

Algebraic equations (level 8):
$M_4 = M_{OUT}$

Algebraic equations (level 9):
$(M_{IN1}, M_{IN2}, M_{OUT})^T = \mathbf{I}_M \cdot (M_L, M_H)^T$
$(\dot{M}_{IN1}, \dot{M}_{IN2}, \dot{M}_{OUT})^T = \mathbf{I}_M \cdot (\dot{M}_L, \dot{M}_H)^T$

Algebraic equations (level 10):
$M_L = \alpha_L \cdot c_L(t)$
$M_H = \alpha_H \cdot c_H(t)$
$\dot{M}_L \approx \alpha_L \cdot \dot{c}_L(t)$
$\dot{M}_H \approx \alpha_H \cdot \dot{c}_H(t)$

Algebraic equations (level 11):
$\omega_3 = \omega_2$
$\omega_{IN2} = \omega_2$

Algebraic equations (level 12):
$\omega_2 = \omega_1$
$\omega_5 = \omega_6$

Stick phase (if L sticks):

Algebraic equations (level 1):
$M_2 = M_3 + M_{IN2}$

Algebraic equations (level 2):
$M_3 = M_{IN1}$

Algebraic equations (level 3):
$\alpha_H = \alpha_H(\omega_{relH})$

Algebraic equations (level 4):
$(\omega_{relH}, \omega_{OUT})^T = \mathbf{B} \cdot (\omega_{relL}, \omega_{IN1}, \omega_{IN2})^T$

Algebraic equations (level 5):
$\omega_{IN1} = \omega_3 - M_{IN1}/d_1$

Algebraic equations (level 6):
$(M_{IN1}, M_{IN2}, M_L)^T = \mathbf{A} \cdot (M_H, M_{OUT})^T$

Algebraic equations (level 7):
$M_H = \alpha_H \cdot c_H(t)$

Algebraic equations (level 8):
$\omega_3 = \omega_2$
$\omega_{IN2} = \omega_2$
$\omega_4 = \omega_5 + M_4/d_2$

Algebraic equations (level 9):
$\omega_2 = \omega_1$
$\omega_5 = \omega_6$
$M_{OUT} = M_4$
$M_5 = M_4$
$\omega_{relL} = 0$

Lock-up phase (No clutch slips):

Algebraic equations (level 1):
$M_2 = M_3 + M_{IN2}$

Algebraic equations (level 2):
$M_3 = M_{IN1}$

Algebraic equations (level 3):
$(M_{IN2}, M_L, M_H)^T = \mathbf{C} \cdot (M_{IN1}, M_{OUT})^T$

Algebraic equations (level 4):
$M_{IN1} = d_1 \cdot (\omega_3 - \omega_{IN1})$

Algebraic equations (level 5):
$(\omega_{IN1}, \omega_{OUT})^T = \mathbf{D} \cdot (\omega_{relL}, \omega_{relH}, \omega_{IN2})^T$

Algebraic equations (level 6):
$\omega_3 = \omega_2$
$\omega_{IN2} = \omega_2$
$\omega_4 = \omega_5 + M_4/d_2$

Algebraic equations (level 7):
$\omega_2 = \omega_1$
$\omega_5 = \omega_6$
$M_{OUT} = M_4$
$M_5 = M_4$
$\omega_{relL} = 0$
$\omega_{relH} = 0$

Matrices **A**, **B**, **C** and **D** are used in the algebraic equations. They should be derived from the respective torque and velocity matrix equations. This is in analogy with the example "inertia – gearbox – inertia" given in section 4.7.2.

In the slip phase an iteration is needed for α_L and α_H. It should be carried out recursively using levels 3-10. In the stick phase an iteration is needed for α_H. It should be carried out recursively using levels 3-7. In the lock-up phase no iteration is needed.

Paper **B**

JACOBSON, BENGT: Analysis of Shift Operations in Automatic Transmissions, *3rd International EAEC Conference on Vehicle Dynamics and Powertrain Engineering*, Strasbourg, France, June 11–13, 1991, pp 195–202

B1

Analysis of Shift Operations in Automatic Transmissions

Bengt Jacobson
Machine & Vehicle Design,
Chalmers University of Technology
S-412 96 Göteborg, Sweden

Abstract

The different gears in an automatic transmission are obtained by engaging different clutches. It is difficult to analyse the shift operation, since there are discontinuous properties involved in the clutch principle. These properties are Coulombian friction (common clutches) and locking in one direction (one-way clutches).

A model of a passenger car driveline is presented along with some simulation results. Those results are verified in practical tests. The model is based on a theory that deals with the discontinuities in a systematic and simple way. It is therefore easy to rearrange the model to be valid for other drivelines.

The different phases of the clutch operation, stick phase and slip phase, are strictly separated. This makes the numerical solution stable and reliable. Another advantage is that all clutches in the gearbox can be separately controlled. This means that simulations can be carried out for shifts between any combinations of clutches. For instance, a shift directly from first to fourth gear is easily analysed.

KEYWORDS: automatic transmission, powershift, gear shift, clutch, Coulombian friction, driveline, model, dynamic, simulation

1 Introduction

An automatic transmission can be drawn schematically, as in figure 1. Different gears are obtained by engaging different combinations of clutches, e.g. according to table 1.

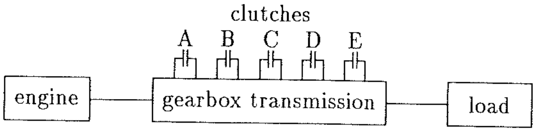


Figure 1: *Schematic driveline including an automatic transmission with five clutches*

Table 1: *Gear/clutch table*

Clutch→ Gear↓	A	B	C	D	E
1	•			•	
2	•		•		
3	•				•
4			•		•
R		•		•	

Ratio changes in automatic transmissions are most often performed as *powershifts*, which means that the traction force of the vehicle does not need to be interrupted during the shift operation. This demands that the disengaging and engaging of the clutches can be carried out simultaneously. In practice this is realized by hydraulically controlled clutches.

Most often a ratio change is performed by disengaging just **one** clutch and engaging another **one**. In that case we could label the clutches after the corresponding gear: clutch L (lower gear) and clutch H (higher gear). See figure 2. The ratio change operation can then be drawn schematically, as in figure 3.

The fundamentals of powershifting are well described in reference [1] and [2]. These papers also include dynamic analyses, but a quite simplified driveline was used. Neither elasticity nor damping were included. The present work deals with modelling and analysis of ratio change operations. Two models will be used:

- **The two-clutch model** uses just two clutches, as in figure 2. Simulations and corresponding test results will be presented. The model is also described in reference [3].
- **The five-clutch model** treats a drivline where the gearbox has five clutches. All five clutches can be controlled separately. Simulations are shown, but no corresponding tests are available. A more detailed description is given in reference [4].

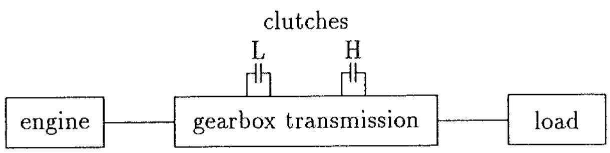


Figure 2: *Schematic driveline including an automatic transmission with ratio changes between two gears and just two active clutches. L and H are the clutches of lower and higher gear, respectively.*

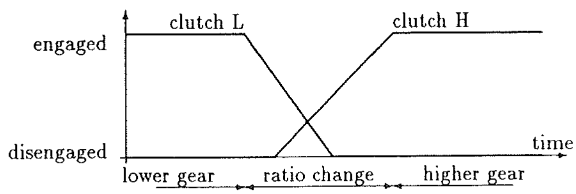


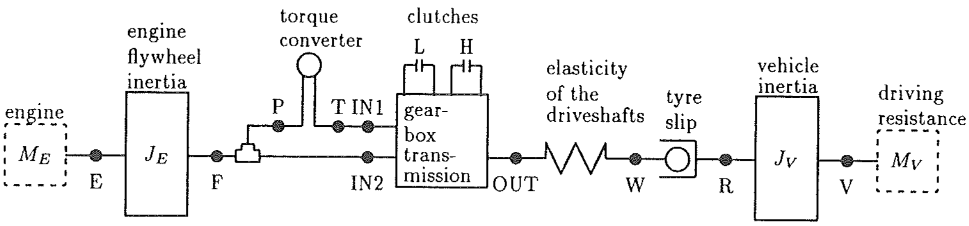
Figure 3: *Schematic ratio change from lower to higher gear, with two active clutches*

2 The two-clutch model

2.1 Presentation of the model

When modelling and analysing the ratio change operation we have to include both the gearbox and the surrounding driveline. Simply speaking, the driveline represents the dynamic properties and the gearbox introduces the excitation.

The model is shown in figure 4. It is drawn as a system with just rotational parts. Even the translating inertia of the vehicle is converted to a rotating inertia, i.e. a flywheel.



E : Engine
F : Flywheel
P : Pump
T : Turbine

IN1 : Input No 1 to gearbox
IN2 : Input No 2 to gearbox
OUT : Output from gearbox

W : Wheel
R : Road
V : Vehicle

Figure 4: *Two-clutch model*

The most important component is the gearbox. **The gearbox transmission** is modelled as a pure transmission, i.e. neither velocity nor torque losses are considered. There are neither internal inertias nor internal elasticities. The final gear is included in the gearbox transmission. The gearbox is also equipped with **clutches**. These can operate in two phases: sticking and slipping. When sticking, the relative velocity is known as zero. When slipping, the torque is known as $\alpha \cdot c$. Here, the static torque capacity c is proportional to the hydraulic control pressure of the clutch. Therefore $c(t)$ is considered as input data. α is a fraction implying the sliding torque divided by the static torque capacity. Since Coulombian friction is used, α is normally slightly less than 1, and varies weakly with the sliding velocity.

There are only three dynamic components, i.e. components able to store energy. They are **the engine flywheel inertia** (including torque converter pump inertia), **the vehicle inertia** and **the elasticity of the driveshafts** (including the elasticity in the tyres and in the longitudinal suspension of the wheels).

Other components are:
The engine, modelled as a pure energy producer, where the torque depends on the velocity and the throttle position. The throttle position, as a function of time, is input data.
The torque converter, modelled as a non-linear damper, where two relationships

describing the steady state characteristics couple the four variables: velocities and torques of pump and turbine shaft.
The tyre slip, modelled as a non-linear damper, similar to the torque converter.
The vehicle resistance, modelled as a pure energy consumer, where the torque depends on the velocity.

There are three main points of interest:

$\boxed{\text{a}}$ The velocity and torque equations of the gearbox transmission are **matrix-formulated**. That makes the model more general, since the gearbox may be considered as just a "black box" as in figure 4. The equations are:

$$\begin{bmatrix} \omega_{relL} \\ \omega_{relH} \end{bmatrix} = \mathbf{I}_\omega \cdot \begin{bmatrix} \omega_{IN1} \\ \omega_{IN2} \\ \omega_{OUT} \end{bmatrix} \text{ and } \begin{bmatrix} M_{IN1} \\ M_{IN2} \\ M_{OUT} \end{bmatrix} = \mathbf{I}_M \cdot \begin{bmatrix} M_L \\ M_H \end{bmatrix}$$

The matrices $\mathbf{I}_\omega$ and $\mathbf{I}_M$ can be derived from one another by using power equilibrium. The matrices will just differ in numerical value when analysing ratio changes between different gears. Drawing a gearbox just like a box with clutches is may be better clarified in figure 5, which shows a possible design of the gearbox transmission.

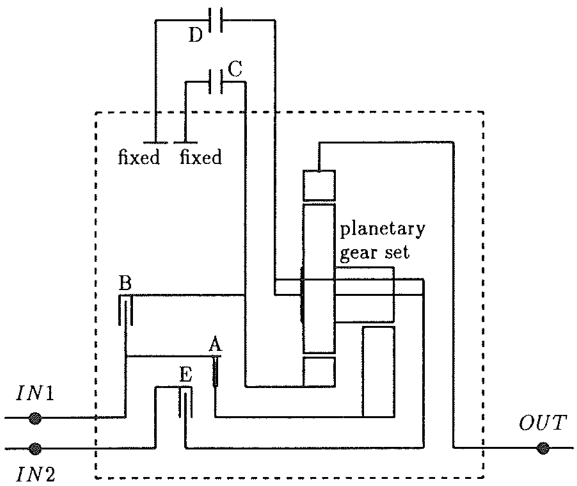


Figure 5: *A possible layout of the gearbox shifting between two gears. Clutch A is steadily engaged. Clutches B and E are steadily disengaged. Clutch D = clutch of lower gear = clutch L. Clutch C = clutch of higher gear = clutch H.*

$\boxed{\text{b}}$ The differential equations use velocities of inertias and torques of elasticities as **state variables**. (State variables are variables that are integrated in an initial value problem.) Roughly speaking, the differential equations become:
$\dot{\omega} = (M_1 - M_2)/J$ for each inertia and
$\dot{M} = k \cdot (\omega_1 - \omega_2)$ for each elasticity
where subscripts 1 and 2 mean before and after the component.
These equations are based on Newton's second law of motion and the constitutive equation of the elasticity, respectively. In mechanical dynamic systems the velocity and the position of inertias are often used as state variables. The proposed approach

is more convenient when dealing with transmission dynamics, since the torques are of more interest than the positions (angles) in the system.

$\boxed{\text{c}}$ The clutches in the model can operate in different **phases**: sticking and slipping. In order to strictly separate these phases, different sets of equations have to be used during the simulation. Thus the physical model is described by several mathematical systems. The advantage of this approach is that the result is more reliable and the calculation time becomes shorter.
A positively slipping clutch will start sticking if ω_{rel} tends to become < 0 and vice versa for a clutch in the negative slip phase. A sticking clutch will start to slip positively when M tends to become $> c^+$ and to slip negatively when M tends to become $< c^-$.

2.2 Simulations versus tests

Two simulations with corresponding tests are presented. The vehicle is a front wheel driven passenger car. The tests are performed in the car driving on the road. The measured hydraulic pressure to the clutches defines the function $c(t)$, which is taken as input data to the simulations.

The results from the simulations and the tests are presented in figures 6 and 7, where c_L, c_H, ω_E and M_R is the static torque capacity of clutch L and H, the engine velocity and the traction torque of the vehicle.

The first case (figure 6) is a ratio change from the first to the second gear. The clutch of the lower gear is a one-way clutch (easily simulated by putting $|c| \equiv \infty$ in one direction and $c \equiv 0$ in the other direction). The clutch of the higher gear is mainly a multi-disc brake (brakes come easily out as a special case of clutches when treating the gearbox transmission as a "black box"). The throttle is wide open.

The most significant difference between simulation and test is the little peak in traction torque that appears when time is approximately 1.25 s in figure 6. Probably, this is caused by an additional viscous part of the clutch torque (the model only treats Coulombian friction). The viscous part arises since the clutches are oil immersed. In the beginning of the engagement, the oil film is a carrier of shear stresses. This has also been observed in reference [5].

The second case (figure 7) is a ratio change from the third to the fourth gear. The clutch of the lower gear is a multi-disc clutch. The clutch of the higher gear is a band brake. The lower gear is a gear with torque split through the converter and the higher gear is a lock-up gear. The throttle is wide open.

The simulation and test results are shown in figure 7. The simulations show a very quick torque response during the time interval of 1.25 to 1.5 s, but the tests do not. One reason could be that the oil has to be squeezed out of the band brake before Coulombian friction can be developed.

B6

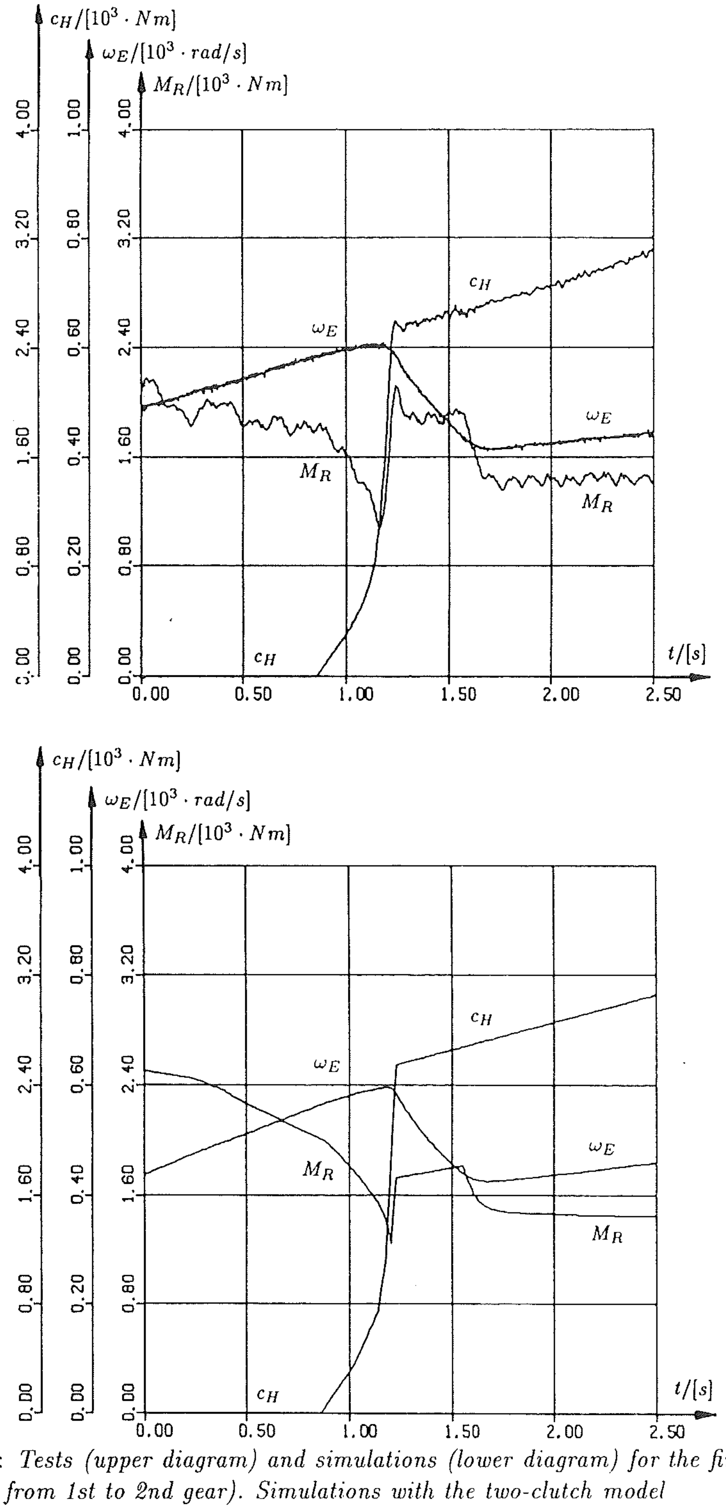
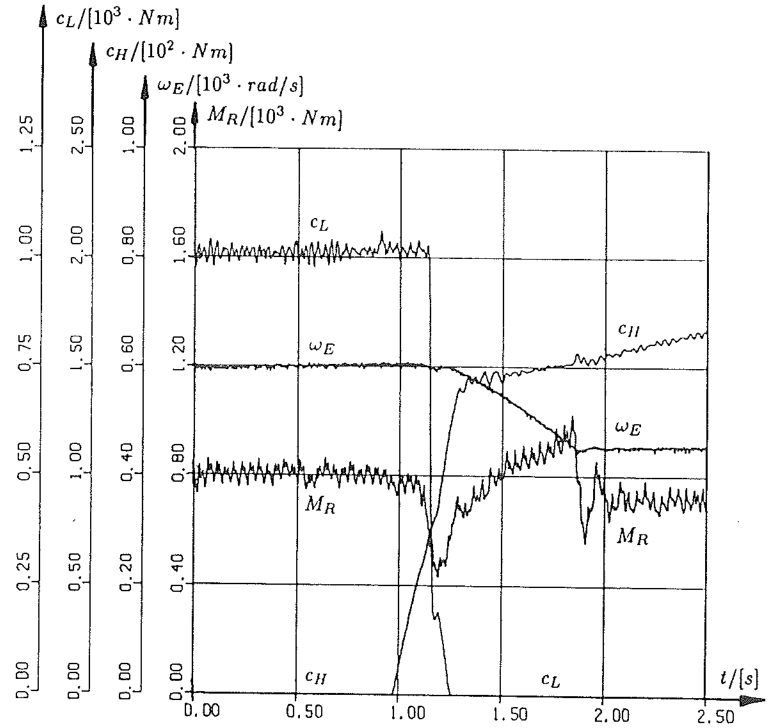


Figure 6: *Tests (upper diagram) and simulations (lower diagram) for the first case (shifting from 1st to 2nd gear). Simulations with the two-clutch model*



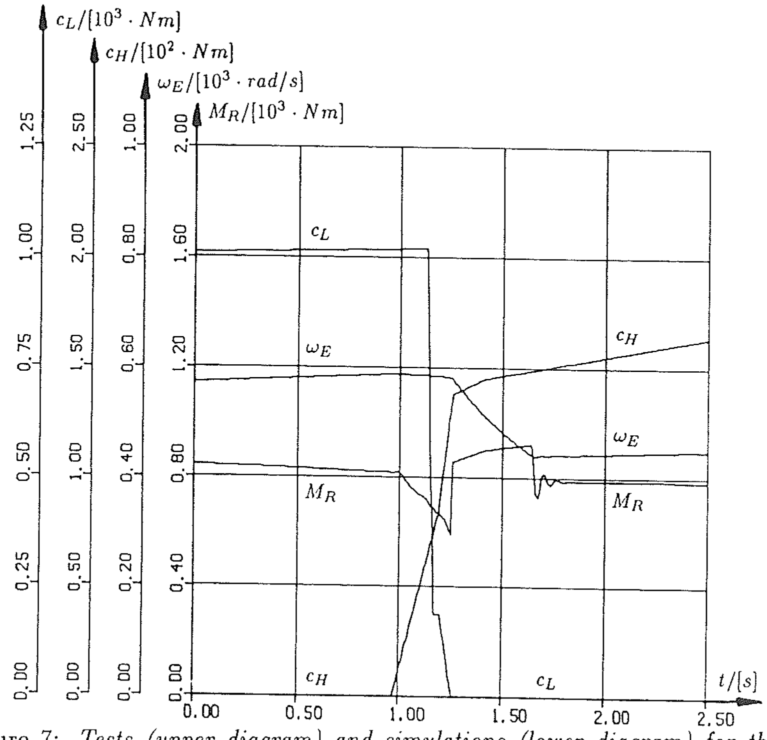


Figure 7: *Tests (upper diagram) and simulations (lower diagram) for the second case (shifting from 3rd to 4th gear). Simulations with the two-clutch model*

3 The five-clutch model

3.1 Presentation of the model

The five-clutch model is is based on the same surrounding driveline as the two-clutch model. The difference is that each of the five clutches can be separately controlled. This means that the input is no longer $c_L(t)$ and $c_H(t)$, but $c_A(t)$, $c_B(t)$, $c_C(t)$, $c_D(t)$ and $c_E(t)$.

When studying the original gearbox it was found that it had four ω-dof (degrees of freedom with respect to velocity) and four M-dof (degrees of freedom with respect to torque). This is enough for most phase combinations. However, it is not enough for the combinations **all clutches slip** and **all clutches stick**. If these cases are to be handled, an extra M-dof and an extra ω-dof must be introduced. This is made through the inertia at node 11 and the elasticity at node 12 in figure 8. The inertia J_{11} and the stiffness k_{12} correspond to internal shafts inside the gearbox transmission. Normally $J_{11} = 1/k_{12} = 0$, but when an extra M-dof or ω-dof is needed, J_{11} or $1/k_{12}$ slightly larger than zero is used. The velocity and torque equations are used on the form:

$$\mathbf{I}_\omega \cdot \begin{bmatrix} \omega_4 \\ \omega_5 \\ \vdots \\ \omega_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{I}_M \cdot \begin{bmatrix} M_4 \\ M_5 \\ \vdots \\ M_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

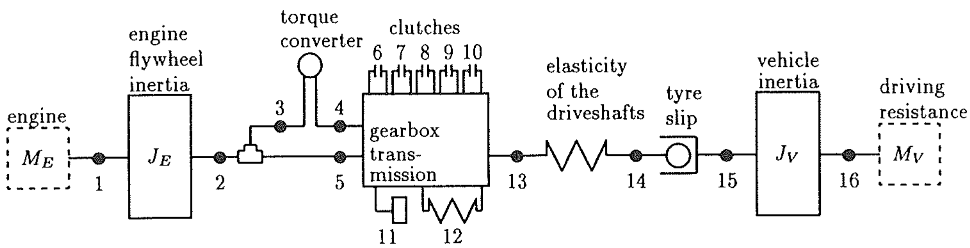


Figure 8: *Five-clutch model*

3.2 Simulations

The simulation in figure 9 is carried out mainly to show that the model can handle some difficult controlling of the clutches. It is not compared with any tests, since there are no control systems available that would actually control the clutches in such a pointless way. In the diagrams, the notations ω_1, ω_{15} and M_{15} are used for engine velocity, vehicle velocity and traction torque of the vehicle, respectively. The simulations are carried out with wide open throttle.

Some of the operations are not conventional ratio change operations, e.g. the lock-up operation and the engagement of all clutches. Such operations could be called *generalized ratio change operations*. The following presents what happens in figure 9:

$\boxed{0 < t < 1s}$ The vehicle is started in reverse gear (clutches B and D). The engine speed is then, $\omega_1 = 100\ rad/s$. The vehicle accelerates backwards.

$\boxed{1 < t < 5s}$ During this interval, the clutches are changed to the position of the first gear (clutches A and D). The vehicle starts to accelerate forward, and its velocity becomes positive at $t = 3s$. The engine velocity firstly increases because clutch B starts to slip. Then it decreases when clutch A gets more torque capacity and finally sticks at $t = 3.5s$.

$\boxed{5 < t < 8s}$ The next combination of clutches headed for is clutches A and B. In the real gearbox, this combination is not used as a gear. The speed ratio is 1 (converter slip neglected), which is the same as the real third gear. Therefore, this gear may be called 3'. The shift is made relatively quick. From $t = 6s$ on, the whole system accelerates rather calmly.

$\boxed{8 < t < 9s}$ Now, another clutch (E) is engaged, without any other being disengaged. This leads to a lock-up of the converter. This gear could be called 3". The engine velocity is forced down.

$\boxed{9 < t < 10s}$ Clutch E is now disengaged. The transmission comes back into gear 3'. The velocity of the engine increases again.

$\boxed{10 < t < 12s}$ During this interval the gear is changed from 3' to 4. This ratio change is unusual since it requires two clutches to be disengaged (A and B) and two other to be engaged (D and E).

$\boxed{12 < t < 14s}$ Here, all clutches are disengaged which, of course, is not very useful in practice, but the model works, since the engine flares up in velocity. The vehicle starts to slowly decelerate because no engine power will reach the driving wheels.

$\boxed{14 < t < 20s}$ All clutches are now engaged. Like the previous disengagement of all clutches, this operation is not very useful in practice. It is, however, a challenge for the model. First a slightly positive traction force is obtained. Then the clutches are engaged too much. This results in a deceleration of the vehicle. The slip in all clutches decreases and equals zero at $t = 17.5s$. The engine and the vehicle also decelerate towards a zero velocity. The engine reaches exactly a zero velocity. The vehicle is connected to the output shaft of the gearbox via an elasticity, which makes the vehicle oscillate.

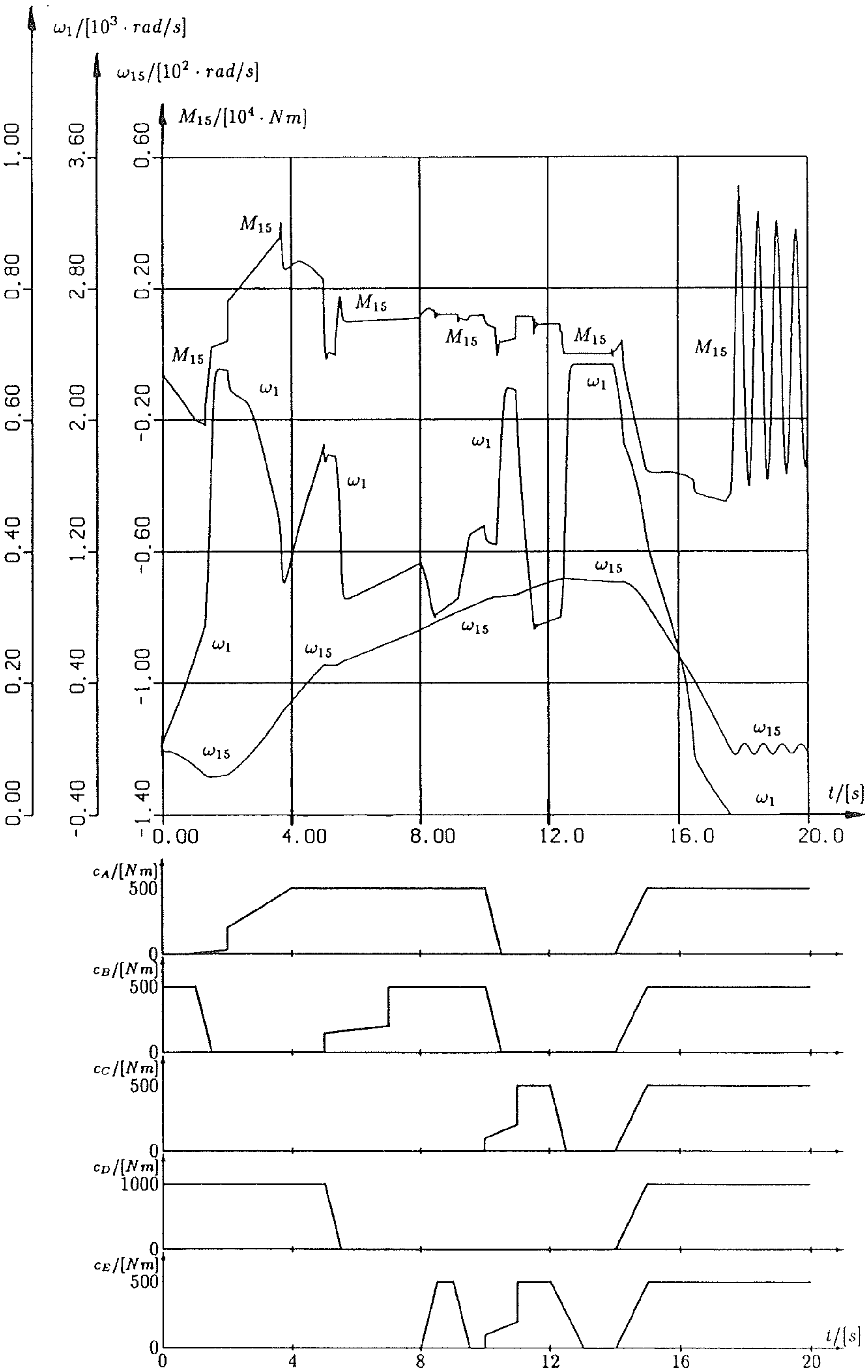


Figure 9: *Simulations with five-clutch model*

4 Conclusions

The oil in the clutches of an automatic transmission can develop shear stresses, making the clutch torque higher than predicted by the Coulombian friction model. When this happens in the beginning of an engagement of a clutch, it can be observed as a negative time delay between hydraulic pressure and clutch torque. This phenomenon was observed when engaging a clutch of multi-disc type.

On the other hand, the oil has to be squeezed out before Coulombian friction can develop. This can be observed as a positive time delay. This phenomenon was observed when engaging a clutch of band brake type.

Some more general advice for modelling may also be formulated. They are not only valid for models of ratio change operations but also for other problems in the field of transmission dynamics.
$\boxed{\text{a}}$ It is convenient to use equations on matrix form for the gearbox transmission.
$\boxed{\text{b}}$ Velocities of inertias and torques of elasticities are suitable state variables.
$\boxed{\text{c}}$ The clutches can operate in different phases: sticking and slipping, which constitute different equations. The problem can be solved by using a set of initial value problems and splicing them together.

Based upon this advice, a model is presented, in which the gearbox has five clutches that can be separately controlled. This model can simulate even *generalized ratio change operations*, e.g. lock-up operations or when all the clutches are engaged at the same time.

5 Notations

ω	Rotational velocity
M	Torque
c	Static torque capacity (c^+ and c^- for positive and negative direction)
α	Fraction = slipping torque divided by static torque capacity
t	Time
J	Moment of inertia
k	Stiffness

Subscripts

rel	Relative, used as ω_{rel} in a clutch
A, B, C, D, E	Clutches A,B,C,D,E
L, H	Clutches for lower and higher gear

Differentiation with respect to time is marked with a dot, e.g. $\dot{\omega}$.

References

[1] FÖRSTER, Hans Joachim. 'Getriebeschaltung ohne Zugkraftunterbrechung'. *Automobil Industrie*, 1962, October, pp 60-76.

[2] WINCHELL, F. J. and ROUTE, W. D. 'Ratio Changing the Passenger Car Automatic Transmission'. *Design Practices - Passenger Car Automatic Transmissions* (SAE), 1962, pp 79-105.

[3] JACOBSON, Bengt. *Vehicle Driveline Mechanics during Powershifting*, Machine & Vehicle Design, Report No 1990-05-30, Chalmers University of Technology, Göteborg, Sweden, 1990

[4] JACOBSON, Bengt. 'Dynamic Simulation of Powershifting Transmissions', to be presented at *Eighth World Congress on the Theory of Machines and Mechanisms*, Prague, Czechoslovakia, August 26-31, 1991.

[5] HENRIKSSON, Magnus. *Measurements of the friction characteristics of a multi-disc brake in an automatic transmission*, Master degree thesis, Machine & Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1989. In swedish.

Paper C

JACOBSON, BENGT: Dynamic Simulation of Powershifting Transmissions, *Eighth World Congress on the Theory of Machines and Mechanisms*, Praha, Czechoslovakia, August 26–31, 1991, pp 573–576

C1

DYNAMIC SIMULATION OF POWERSHIFTING TRANSMISSIONS

Bengt Jacobson, Machine & Vehicle Design,
Chalmers University of Technology, S-412 96 Göteborg, SWEDEN

ABSTRACT: Powershifting transmissions, e.g. automatic vehicle transmissions, accomplish changes in ratio without interrupting the power supply. A dynamic driveline model is connected to a model of the chassis and the power unit housing. The powershifting transmission has five clutches, which can all be controlled separately during a ratio change operation.

The clutches can operate in strict stick or slip phases. This is handled as a *multi-phase system*. Mathematically, this means a set of initial value problems, spliced together. The equations of the planetary gearbox mechanism are matrix formulated. Velocities of inertias and torques of elasticities are used as state variables.

KEYWORDS: automatic transmission, powershift, gear shift, clutch, Coulombian friction, driveline, model, dynamic, simulation

1 INTRODUCTION

A driveline with a powershifting transmission is schematically drawn in figure 1. Different gears are obtained by engaging different combinations of clutches, e.g. according to table 1. The ratio change operation can be performed as a *powershift*, which means that the transmitted power is not interrupted. The disengaging and engaging of the clutches must then be carried out simultaneously. In practice powershifting transmissions are used as automatic transmissions in vehicles. The clutches are usually hydraulically controlled or one-way clutches.

Table 1: *Gear/clutch table*

Clutch→ Gear↓	A	B	C	D	E
1	•			•	
2	•		•		
3'	•	•			
3	•				•
3"	•	•			•
4			•		•
Reverse		•		•	

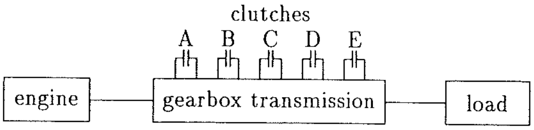


Figure 1: *Schematic driveline including a powershifting transmission with five clutches*

The fundamentals of powershifting are well described in [Förster 1962] and [Winchell 1962]. Those papers assume only two active clutches in a ratio change; one clutch disengaging and one engaging. Furthermore, in the dynamic analyses quite simplified driveline models are used.

[Jacobson 1990] treats general modelling techniques for transmission dynamics and presents a rather detailed driveline model, but there are only two clutches involved in each ratio change operation. [Jacobson 1991] presents an extended driveline model briefly, where five clutches are involved. The present work describes the model in [Jacobson 1991] in greater detail. An extension is that the chassis and the power unit housing are included.

2 PRESENTATION OF THE MODEL

The present model consists of two main parts; the **driveline model** and the **vehicle model**. This paper focuses on the driveline model. The important aspect of the vehicle model is how it cooperates with the driveline model, rather than all the details.

2.1 Basic Mathematics and Physics

A dynamic system is a system governed by ordinary differential equations. These are written in the form $\dot{\vec{y}} = \vec{f}(\vec{y}, t)$. When solving by means of a computer, the state variables $\vec{y}$ are updated in every time step by numerical integration: $\vec{y} = \int \dot{\vec{y}} \cdot dt$.

As state variables, velocities (translational or rotational) of inertias and forces (or torques) of elasticities are used. This is often very suitable in systems where the absolute position is of no interest. The differential equation used for each inertia is Newton's second law: $\dot{y} = force/mass$, where y is a velocity. For each elasticity, the differentiated constitutive equation is used, i.e. for a linear elasticity: $\dot{y} = stiffness \cdot velocity$, where y is a force.

So inertias and elasticities can generate state variables and differential equations, in which case they are *dynamic components*. However, they are not always dynamic. The following example treats an inertia. Normally it is dynamic, which means that $velocity \leftarrow y$, where $y \leftarrow \int (force/mass) \cdot dt$. (The notation $a \leftarrow b$ should be read "a is assigned the value of b".) This is the case when the force can be calculated algebraically from $\vec{y}$ and/or t. However, if the velocity can be calculated algebraically from t, there is no need to integrate. The force can then be calculated as $force \leftarrow mass \cdot acceleration$, where the acceleration is known since the velocity is known as a function of time. Another case, when the inertia is not a dynamic component, is when its velocity can be calculated algebraically from $\vec{y}$. Then there is a *constraint* in the system. For example two inertias, directly connected to each other, must be treated as only one dynamic component.

A multi-phase dynamic system: Inertias and elasticities can be dynamic components. The clutches are also some kind of dynamic components, but *discontinuously dynamic*. This is because they can operate in different phases, sticking or slipping. The function $\vec{f}$ above becomes different depending on in which phase $\vec{ph}$ the clutches operate. This can be written as $\dot{\vec{y}} = \vec{f}(\vec{y}, \vec{ph}, t)$. Therefore, the phases are some kind of *discrete state variables*.

The common state variables are updated by integration. The phases should also be updated, but they cannot be integrated since they are discrete. Instead, flags $\vec{fl}$ are used to indicate **when** the phases should be (discontinuously) changed. When a flag passes from negative to positive, a certain phase switch is indicated. The flags should be defined by $\vec{fl} = f(\vec{y}, \vec{ph}, t)$. Table 2 shows phases and flags for a single clutch.

In figure 2a the calculation flow is sketched for a multi-phase system. The algebraic variables $\vec{x}$ are included as results of the function $\vec{f}$, so that $(\dot{\vec{y}}, \vec{fl}, \vec{x}) = \vec{f}(\vec{y}, \vec{ph}, t)$. They are calculated in order to plot the results and for communication between subsystems,

Table 2: *Phases, flags and phase switches for a single clutch*

phase ph	flags $\vec{fl}$	phase switch indicated
$ph = -1$ (negative slip)	$fl = +\omega_{rel}$	from $ph = -1$ to 0
$ph = +1$ (positive slip)	$fl = -\omega_{rel}$	from $ph = +1$ to 0
$ph = 0$ (stick)	$fl_1 = M - c^+$	from $ph = 0$ to $+1$
	$fl_2 = c^- - M$	from $ph = 0$ to -1

as discussed below. In the present model all velocities and forces (including torques) are algebraic variables.

<u>**Separation into subsystems:**</u> In practice, the function $\vec{f}$ is written as a computer program. It is convenient to split the program into smaller modules, each of which corresponds to a physical part of the system, a *subsystem.*

Figure 2b shows the function $\vec{f}$, separated in two subsystems. Note that subsystem 1 has to be evaluated before 2, because the algebraic variables $\vec{x}_1$ are input to subsystem 2. If $\vec{f}_2$ also is a function of $\vec{x}_2$ the subsystem 2 is a *closed loop subsystem.* Provided $\vec{f}_2$ is sufficiently weakly dependent on $\vec{x}_2$, the solution can be obtained by iteration.

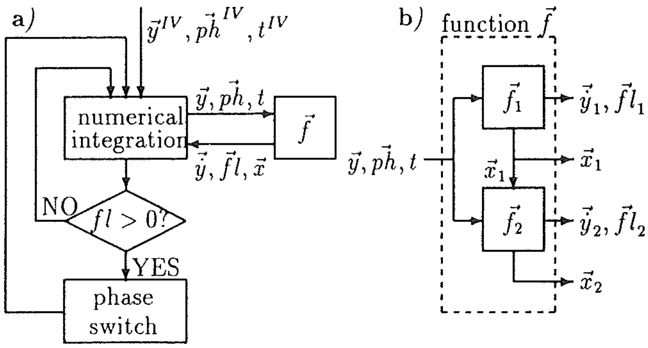


Figure 2: *Calculation flow scheme.* a) *Multi-phase dynamic system. A single-phase system would neither need the check $fl > 0$ nor the phase switch box.* b) *Function $\vec{f}$ separated in subsystems*

In the models presented below, the driveline model is subsystem 1 and the vehicle model subsystem 2. The state variables from the vehicle model used in the driveline model are velocities of the power unit housing and the chassis. These are converted to driveline variables ω_{12} and ω_{19}, see figure 4. The algebraic variables delivered from the driveline model to the vehicle model are reaction torque on power unit housing (M_{12}) and traction torque on the wheels (M_{19}). These are converted to vehicle model variables F_{81} and F_{51}, respectively, see figure 3. The vehicle model is a single-phase system, why neither $\vec{ph}_2$ nor $\vec{fl}_2$ in figure 2b actually exist in the present model.

When the function $\vec{f}$ is formulated, the solution is obtained using some suitable computer software for numerical integration. The program must be able to integrate numerically as well as to adapt the time step to match the phase switches correctly. In this work the *ACSL* program has been used.

2.2 Vehicle Model

The model in figure 3 is suitable for a frontwheel driven passenger car with transversally mounted power unit. Each inertia contributes with three state variables, two translational velocities and one rotational velocity.

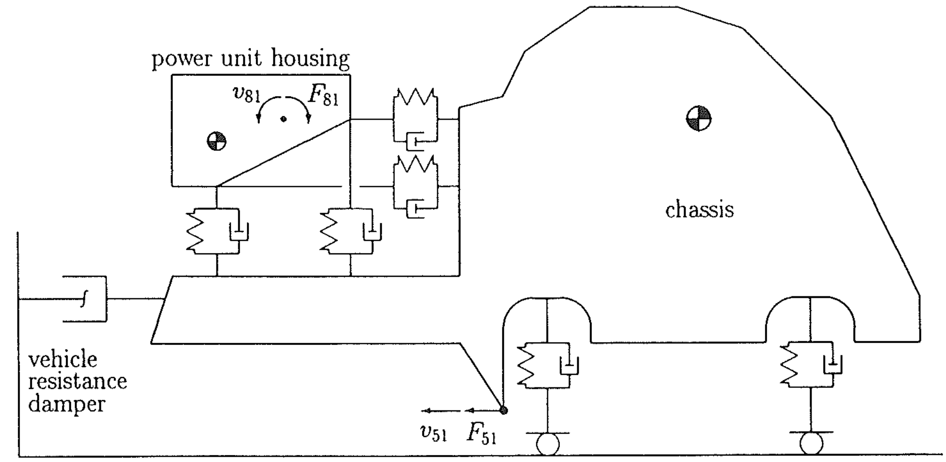


Figure 3: *Vehicle model. The* ⊕ *marks center of gravity for an inertia movable in the two-dimensional plane.*

2.3 Driveline Model

The driveline model includes the gearbox and its surrounding driveline, as shown in figure 4. There are 19 numbered *nodes*. Each node has two variables, velocity and torque. Nodes 1,2,3,4,5 and 11 are special in having **relative** velocity. Furthermore, there are *components* between the nodes. Every component must produce as many equations as nodes connected to it, see table 3.

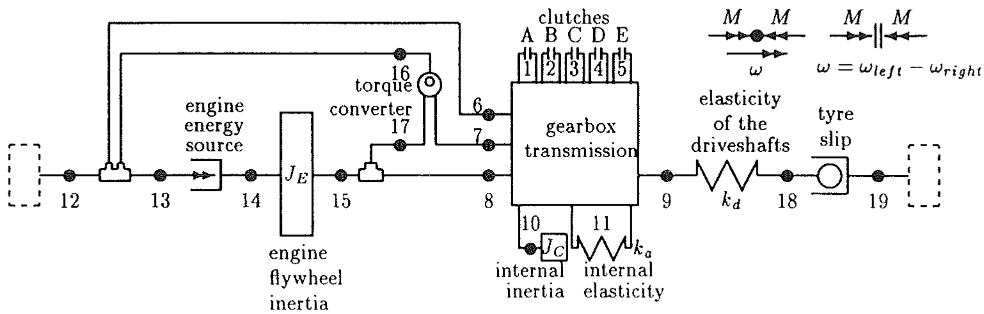


Figure 4: *Driveline model. The dashed boxes are* interface components, *which handle the communication with the driveline model.*

Table 3: *Equations of components in driveline model. The variables* y_{51}, y_{53} *and* y_{62} *are state variables corresponding to velocities in vehicle model. The* Λ *functions should actually read* $\Lambda(\omega_6, \omega_7, \omega_{17})$ *to strictly use the steady state characteristics of the torque converter. However,* $\Lambda(\omega_7/\omega_{17})$ *are the characteristics usually given and they are used although they are valid only when* $\omega_6 = 0$.

Component	Equations
Interface at node 12	$\omega_{12} = y_{62}$
Connection between nodes 12,6,16 and 13	$\omega_{12} = \omega_{13} = \omega_{16} = \omega_6$ $M_{12} = M_{13} + M_{16} + M_6$
Engine energy source	$M_{13} = K_{14} = M_E((\omega_{14} - \omega_{13}), t)$
Engine flywheel inertia when dynamic	$\omega_{14} = y_1$ where $y_1 = \int((M_{14} - M_{15})/J_E) \cdot dt$ $\omega_{15} = \omega_{14}$
Engine flywheel inertia when not dynamic	$J_E \cdot \dot{\omega}_{14} = M_{14} - M_{15}$ where $J_E = 0$ is used $\omega_{15} = \omega_{14}$
Connection between nodes 15,17 and 8	$\omega_{15} = \omega_{17} = \omega_8$ $M_{15} = M_{17} + M_8$
Torque converter	$M_{16} + M_{17} = M_7$ $M_{17}/\omega_{17}^2 = \Lambda_P(\omega_7/\omega_{17})$ $M_7/\omega_{17}^2 = \Lambda_T(\omega_7/\omega_{17})$
Gearbox transmission	$\underbrace{\mathbf{I}_\omega}_{5\times 11} \cdot \vec{\omega}_{1,2,3,4,5,6,7,8,9,10,11} = \vec{0}$ $\underbrace{\mathbf{I}_M}_{6\times 11} \cdot \vec{M}_{1,2,3,4,5,6,7,8,9,10,11} = \vec{0}$
Clutch A	$\omega_1 = \omega_{relA}(t) = 0$ or $M_1 = \alpha_A \cdot c_A(t)$
Clutch B	$\omega_2 = \omega_{relB}(t) = 0$ or $M_2 = \alpha_B \cdot c_B(t)$
Clutch C	$\omega_3 = \omega_{relC}(t) = 0$ or $M_3 = \alpha_C \cdot c_C(t)$
Clutch D	$\omega_4 = \omega_{relD}(t) = 0$ or $M_4 = \alpha_D \cdot c_D(t)$
Clutch E	$\omega_5 = \omega_{relE}(t) = 0$ or $M_5 = \alpha_E \cdot c_E(t)$
Internal inertia when dynamic	$\omega_{10} = y_2$ where $y_2 = \int(M_{10}/J_C) \cdot dt$
Internal inertia when not dynamic	$J_C \cdot \dot{\omega}_{10} = M_{10}$ where $J_C = 0$ is used
Internal elasticity when dynamic	$M_{11} = y_3$ where $y_3 = \int(k_a \cdot \omega_{11}) \cdot dt$
Internal elasticity when not dynamic	$\dot{M}_{11} = k_a \cdot \omega_{11}$ where $1/k_a = 0$ is used
Elasticity of driveshafts when dynamic	$M_9 = y_4$ where $y_4 = \int(k_d \cdot (\omega_9 - \omega_{18})) \cdot dt$ $M_9 = M_{18}$
Elasticity of driveshafts when not dynamic	$\dot{M}_9 = k_d \cdot (\omega_9 - \omega_{18})$ $M_9 = M_{18}$
Tyre slip	$M_{18} = M_{19} = s((\omega_{18} - \omega_{19})/\omega_{18})$
Interface at node 19	$\omega_{19} = (y_{51} - h_{trac} \cdot y_{53})/R_{wheel}$

3 SOLUTION METHOD

The gearbox transmission is a planetary gear mechanism, see figure 5. Its velocity and torque equations may be written as shown in table 3: $\mathbf{I}_\omega \cdot \vec{\omega} = \vec{0}$ and $\mathbf{I}_M \cdot \vec{M} = \vec{0}$. Here $\mathbf{I}_\omega$ and $\mathbf{I}_M$ are 5×11 and 6×11 matrices, respectively. The matrices $\mathbf{I}_\omega$ and $\mathbf{I}_M$ can easily be derived from each other, using power equilibrium. The homogeneous form of the equations is not useful in a direct evaluation. Instead we use:

$\vec{\omega}_{five\ nodes} \leftarrow \underbrace{\mathbf{I}_\omega}_{5\times6} \cdot \vec{\omega}_{six\ nodes}$ and $\vec{M}_{six\ nodes} \leftarrow \underbrace{\mathbf{I}_M}_{6\times5} \cdot \vec{M}_{five\ nodes}$

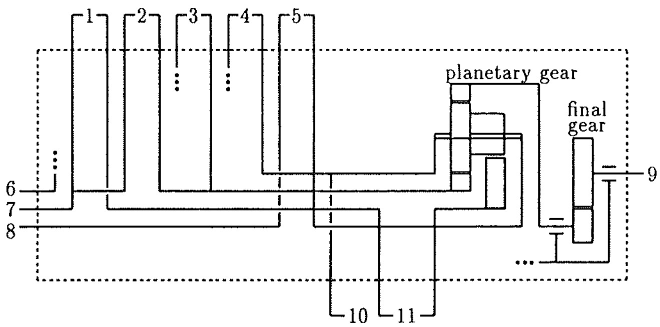


Figure 5: *Planetary gear mechanism. Three dots ($\cdots$ or $\vdots$) mark shafts connected to node 6, which is the transmission housing.*

In this work the derivation of these new matrices is done numerically using the computer program *MATLAB*. The matrix operations used include inversion, which is where singularities are discovered. These force us to chose different functions $\vec{f}$ in different phase combinations; different components become dynamic. See table 4.

The internal inertia and elasticity are only used as dynamic components when they are needed as such. That is, when there are too many degrees of freedom with respect to velocity or torque, respectively. Otherwise, J_C and/or $1/k_a$ is set $= 0$.

Also, J_E is set $= 0$ when the engine flywheel is not dynamic. Practically speaking, the flywheel inertia is neglected when it is kinematically coupled directly to the power unit housing. This is probably a minor error. The problem with taking it into account was that a constraint to the vehicle model would have had been developed. This would have made it impossible to split the model into subsystems as chosen.

In the following, just two examples of different phase combinations are studied.

Evaluation order when (A,B) stick: The procedure to the left in table 5 calculates all algebraic variables if (A,B) stick. The calculation will be similar if clutch combinations (A,C), (A,D), (A,E), (B,D), (B,E), (C,D), or (C,E) stick. The difference will only be a permutation of clutch nodes and a numerical difference in the matrices $\mathbf{I}_\omega$ and $\mathbf{I}_M$.

Evaluation order when (B,C) stick: The evaluation order when (B,C) stick is shown to the right in table 5. If we tried to use a calculation similar to *(A,B) stick*, the matrix

$\mathbf{L}_\omega$ would be needed so that: $\vec{\omega}_{1,4,5,9,10} = \mathbf{L}_\omega \cdot \vec{\omega}_{2,3,6,7,8,11}$ However, this matrix cannot be derived, since it should include inversion of a singular matrix.

This can be understood practically by studying figure 5. The velocities $\vec{\omega}_{2,3,6,7,8,11}$ cannot determine the velocities $\vec{\omega}_{1,4,5,9,10}$ because nodes 7 and 9 are kinematically uncoupled. Neither can the corresponding torque matrix be derived.

The fact that nodes 7 and 9 are kinematically uncoupled must then be used. The matrix $\mathbf{L}_\omega$ is derived so that: $\vec{\omega}_{1,4,5,7,10} = \mathbf{L}_\omega \cdot \vec{\omega}_{2,3,6,8,9,11}$. With this equation ω_7 can be calculated without knowing ω_9.

4 SIMULATIONS

[Jacobson 1990] and [Jacobson 1991] show simulations and verifying tests with a vehicle. The simulations in figure 6 are carried out mainly to show that the model can handle some difficult controlling of the clutches. It is a ratio change sequence with the following order of "gears": **Reverse, 1, 3', 3", 3', 4, all clutches disengaged**, and finally **all clutches engaged**. Some of these ratio change operations are not conventional, e.g. the lock-up operation (**3'** to **3"**) and the engagement and disengagement of all clutches. Such operations could be called *generalized ratio change operations* and are special in not having exactly two active clutches.

5 CONCLUSIONS

A theory has been developed for making a dynamic driveline model of powershifting transmissions. The model can handle *generalized ratio changes*, i.e. when several clutches are active. How such a model can be extended with *subsystems*, e.g. a model of the chassis and the power unit housing, is also shown.

The driveline is mathematically described as a *multi-phase system*. This is actually a way of splicing several initial value problems to one another. The gearbox transmission is described using matrix equations. Different kinds of singularities in these matrices show different ways of arranging the equations, depending on which clutches stick.

Table 4: *Different components become dynamic during different phases of the clutches.*

Sticking clutches					velocity of engine	velocity of internal inertia	torque of internal elasticity	torque of driveshafts
A	B	C	D	E				
					●	●		
○					●			
	○				●			
		○			●			
			○		●			
				○	●			
○	○				●			●
○		○			●			●
○			○		●			●
○				○	●			●
	○	○			●			
	○		○		●			●
	○			○	●			●
		○	○		●			●
		○		○	●			●
			○	○				
○	○	○			●			●
○	○		○		●			●
○	○			○	●			●
○		○	○		●			●
○		○		○	●			●
○			○	○				●
	○	○	○		●			●
	○	○		○	●			●
	○		○	○				●
		○	○	○				●
○	○	○	○		●		●	●
○	○	○		○				●
○	○		○	○				●
○		○	○	○				●
	○	○	○	○				●
○	○	○	○	○			●	●

Table 5: *Evaluation order when (A,B) and (B,C) stick. The evaluation order between the groups of equations is essential, but within a group it is not. The parenthesis notation* $\vec{\omega}_{(1,4,5),7,(10)} \leftarrow \mathbf{I}_\omega \cdot \vec{\omega}_{2,3,6,8,(9),11}$ *means that* ω_7 *(not* $\vec{\omega}_{1,4,5,10}$*) is calculated from* $\vec{\omega}_{2,3,6,8,11}$ *(*ω_9 *is not needed).*

(A,B) stick
$\omega_{12} \leftarrow y_{62}$ $\omega_{19} \leftarrow (y_{51} - h_{trac} \cdot y_{53})/R_{wheel}$
$\omega_{14} \leftarrow y_1$ $M_9 \leftarrow y_4$
$\omega_{13} \leftarrow \omega_{12}$ $\omega_{16} \leftarrow \omega_{12}$ $\omega_6 \leftarrow \omega_{12}$
$\omega_1 \leftarrow \omega_{relA}(t)$ $\omega_2 \leftarrow \omega_{relB}(t)$ $M_3 \leftarrow \alpha_C \cdot c_C(t)$ $M_4 \leftarrow \alpha_D \cdot c_D(t)$ $M_5 \leftarrow \alpha_E \cdot c_E(t)$
$\omega_{15} \leftarrow \omega_{14}$ $M_{18} \leftarrow M_9$ $M_{13} \leftarrow M_E((\omega_{14} - \omega_{13}), t)$ $M_{10} \leftarrow 0$ $\omega_{11} \leftarrow 0$
$\omega_{17} \leftarrow \omega_{15}$ $\omega_8 \leftarrow \omega_{15}$ $M_{14} \leftarrow M_{13}$ $M_{19} \leftarrow M_{18}$ $\vec{M}_{1,2,6,7,8,11} \leftarrow \mathbf{I}_M \cdot \vec{M}_{3,4,5,9,10}$ $\omega_{18} \leftarrow \omega_{19}/(1 - s^{-1}(M_{18}))$
$\omega_7 \leftarrow \omega_{17} \cdot \Lambda_T^{-1}(M_7/\omega_{17}^2)$
$\vec{\omega}_{3,4,5,9,10} \leftarrow \mathbf{I}_\omega \cdot \vec{\omega}_{1,2,6,7,8,11}$ $M_{17} \leftarrow \omega_{17}^2 \cdot \Lambda_P(\omega_7/\omega_{17})$
$M_{15} \leftarrow M_{17} + M_8$ $M_{16} \leftarrow M_7 - M_{17}$
$M_{12} \leftarrow M_{13} + M_{16} + M_6$

(B,C) stick
$\omega_{12} \leftarrow y_{62}$ $\omega_{19} \leftarrow (y_{51} - h_{trac} \cdot y_{53})/R_{wheel}$
$\omega_{14} \leftarrow y_1$
$\omega_{13} \leftarrow \omega_{12}$ $\omega_{16} \leftarrow \omega_{12}$ $\omega_6 \leftarrow \omega_{12}$
$M_1 \leftarrow \alpha_A \cdot c_A(t),\ \dot{M}_1 \leftarrow \alpha_A \cdot \dot{c}_A(t)$ $\omega_2 \leftarrow \omega_{relB}(t)$ $\omega_3 \leftarrow \omega_{relC}(t)$ $M_4 \leftarrow \alpha_D \cdot c_D(t),\ \dot{M}_4 \leftarrow \alpha_D \cdot \dot{c}_D(t)$ $M_5 \leftarrow \alpha_E \cdot c_E(t),\ \dot{M}_5 \leftarrow \alpha_E \cdot \dot{c}_E(t)$
$\omega_{15} \leftarrow \omega_{14}$ $M_{13} \leftarrow M_E((\omega_{14} - \omega_{13}), t)$ $M_{10} \leftarrow 0,\ \dot{M}_{10} \leftarrow 0$ $\omega_{11} \leftarrow 0$
$\omega_{17} \leftarrow \omega_{15}$ $\omega_8 \leftarrow \omega_{15}$ $M_{14} \leftarrow M_{13}$
$\vec{\omega}_{(1,4,5),7,(10)} \leftarrow \mathbf{I}_\omega \cdot \vec{\omega}_{2,3,6,8,(9),11}$
$M_{17} \leftarrow \omega_{17}^2 \cdot \Lambda_P(\omega_7/\omega_{17})$ $M_7 \leftarrow \omega_{17}^2 \cdot \Lambda_T(\omega_7/\omega_{17})$
$M_{16} \leftarrow M_7 - M_{17}$ $\vec{M}_{2,3,6,8,9,11} \leftarrow \mathbf{I}_M \cdot \vec{M}_{1,4,5,7,10}$ $\vec{\dot{M}}_{(2,3,6,8),9,(11)} \leftarrow \mathbf{I}_M \cdot \vec{\dot{M}}_{1,4,5,(7),10}$
$M_{12} \leftarrow M_{13} + M_{16} + M_6$ $M_{15} \leftarrow M_{17} + M_8$ $M_{18} \leftarrow M_9$
$M_{19} \leftarrow M_{18}$ $\omega_{18} \leftarrow \omega_{19}/(1 - s^{-1}(M_{18}))$
$\omega_9 \leftarrow \omega_{18} + \dot{M}_9/k_d$
$\vec{\omega}_{1,4,5,(7),10} \leftarrow \mathbf{I}_\omega \cdot \vec{\omega}_{2,3,6,8,9,11}$

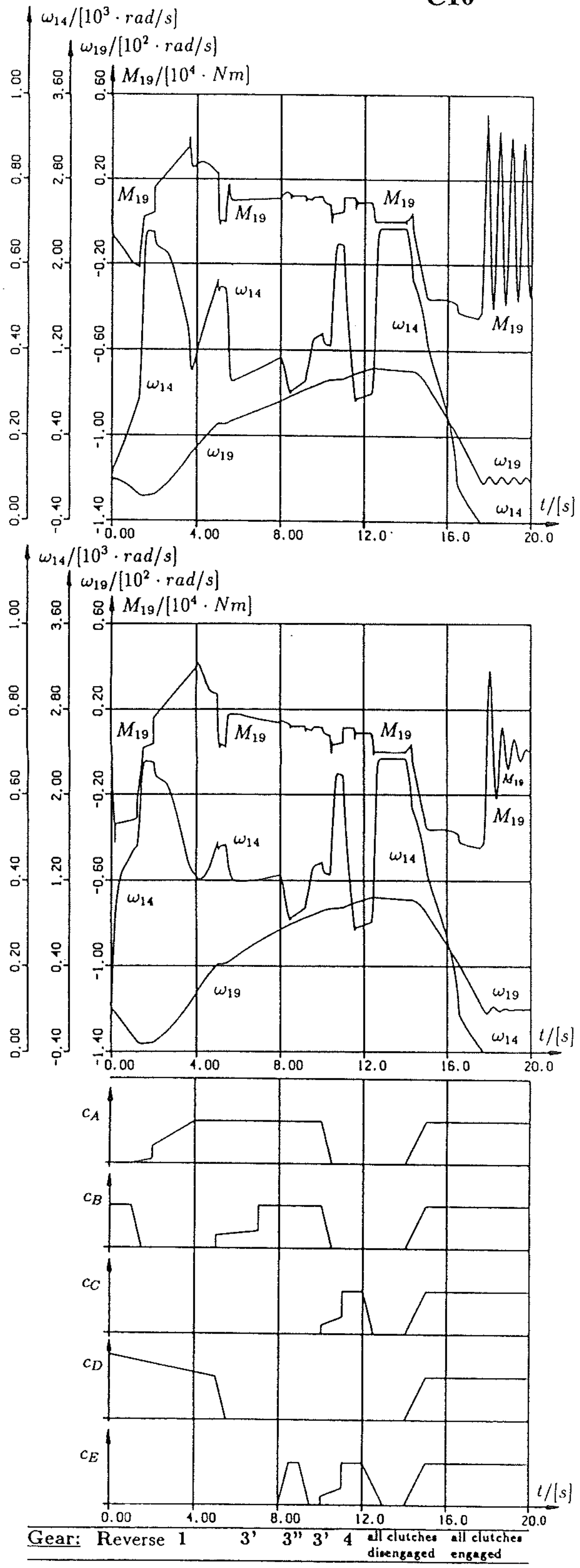


Figure 6:
Simulations of some generalized ratio change operations.
Upper: *Simulation without vehicle model.*
Middle: *Simulation with vehicle model.*
Lower: *Clutch torque capacities (input data)*

6 NOTATIONS

t	Time
y	State variable
x	Algebraic variable
ph	Phase, indicates if a clutch is slipping or sticking
fl	Flag, indicates when a phase should be switched
ω	Velocity (rotational), driveline model
v	Velocity (translational or rotational), vehicle model
M	Force (torque) in driveline model
F	Force (or torque) in vehicle model
c	Static torque capacity for a clutch (c^+ and c^- for positive and negative direction)
α	Fraction = slipping torque divided by static torque capacity. α is weakly dependent on ω_{rel}
J	Moment of inertia, driveline model
m	Mass (or moment of inertia), vehicle model
k	Stiffness
R_{wheel}	Radius of wheel
h_{trac}	Height from wheel center to center of gravity of the chassis

Sub- and superscripts

rel	Relative, used as ω_{rel} in a clutch
IV	Initial value
T	Transpose of matrices or vectors

Differentiation with respect to time is marked with a dot, e.g. $\dot{\omega}$.
Matrices are written in bold, e.g. $\mathbf{I}$.
Column vectors are marked with an arrow above, e.g. $\vec{\omega}$.
Subscripts for elements of a vector may be placed as vector subscripts, e.g. $\vec{\omega}_{1,2,3} = (\omega_1, \omega_2, \omega_3)^T$.

References

[Förster 1962] FÖRSTER, Hans Joachim. 'Getriebeschaltung ohne Zugkraftunterbrechung'. *Automobil Industrie*, 1962, October, pp 60-76.

[Winchell 1962] WINCHELL, F. J. and ROUTE, W. D. 'Ratio Changing the Passenger Car Automatic Transmission'. *Design Practices - Passenger Car Automatic Transmissions* (SAE), 1962, pp 79-105.

[Jacobson 1990] JACOBSON, Bengt. *Vehicle Driveline Mechanics during Powershifting*, Machine & Vehicle Design, Report No 1990-05-30, Chalmers University of Technology, Göteborg, Sweden, 1990.

[Jacobson 1991] JACOBSON, Bengt. 'Analysis of Shift Operations in Automatic Transmissions', *3rd International EAEC Conference on Vehicle Dynamics and Powertrain Engineering*, Strasbourg, France, June 11-13, 1991.

Paper D

JACOBSON, BENGT: Dynamic Vehicle Transmission System — Derivation of Equations on Explicit Form, submitted for publication in *Vehicle System Dynamics* (submitted in December 1992)

Dynamic Vehicle Transmission Systems —
— Derivation of Equations on Explicit Form

Bengt Jacobson

Machine and Vehicle Design
Chalmers University of Technology
S - 412 96 Göteborg, Sweden

SUMMARY

Dynamic models of vehicle transmission systems include inertias (flywheels), elasticities (elastic shafts), gear transmissions, etc.. Such models can be mathematically described as first order initial value problems. Most numerical solution methods require that the *state derivatives* can be expressed as a function of the *state variables* and time. In a first approach, such an explicit form of the equations is easily obtained using the velocity of each inertia and torque of each elasticity as state variables. However, there are often algebraic relationships (*constraints*) between these quantities, making the explicit form more complicated to find. An algorithm for this is presented.

KEYWORDS: transmission, dynamic, analysis, simulation, initial value problem, ordinary differential equation, ODE, constraint, differential-algebraic equation system, DAE

Chapter 1: INTRODUCTION AND OVERVIEW

Many transmission systems can be dynamically analyzed as initial value problems. Finding the numerical solution can be described in the following four steps:

1. **Modelling:** It should be decided how the real system should be described with model components, e.g. if a wheel should be modelled as an inertia or an elasticity. Computer aid for this step is non-existent or at least very rare.
2. **Generating equations on implicit form:** Equations on implicit form should be found, e.g. $\omega_1 + i \cdot \omega_2 = 0$ for a simple gear transmission. Computer programs of *MultiBody System* type (e.g. *ADAMS*^{TM} and *DADS*^{TM}) have built-in knowledge of these equations for certain predetermined types of components.
3. **Finding explicit form of equations:** How the state derivatives can be calculated from the state variables and time should be found, i.e. $\dot{\mathbf{y}}_{\mathbf{state}} = \text{function}(\mathbf{y}_{\mathbf{state}}, t)$. Computer aid for this step is available in programs of MultiBody System type. There are also "sorting programs" (e.g. *ACSL*^{TM} and *CSSL*^{TM}), which can handle this to some extent. Such programs just rearrange the order of (scalar) equations, each given on the proper explicit form. A third way is to use numerical or symbolic solvers for equation systems (examples of symbolic solvers are *MAPLE*^{TM} and *Mathematica*^{TM}). However, using sorting programs or solvers requires knowledge of which variables are suitable as state variables. The *Bond Graph* method (see e.g. references [1] and [2]) can find the explicit form of equations, but when constraints are present it becomes quite complicated.
4. **Numerical integration:** The solution is found by integration: $\mathbf{y}_{\mathbf{state}} = \mathbf{y}^{iv}_{\mathbf{state}} + \int \dot{\mathbf{y}}_{\mathbf{state}} \cdot dt$, where superscript iv denotes initial values. A lot of computer software is available for such numerical methods.

This paper describes an algorithm to find the explicit form of equations. It is based on a matrix formulation of linear differential and/or algebraic equations. Suitable state variables are found, as well as the proper explicit form of possible non-linear equations. The algorithm is developed with vehicle transmission systems in mind, but can also be used for other systems generating the same kind of equations. The systems studied are characterized as follows:

1. There are *components* connected in *nodes*.
2. The components have known characteristics, described by *component equations*. There can also be additional equations, not originating from any particular component. In each node, one (rotational) *velocity* and one *torque* is defined.
3. The equations can either be *algebraic* or *differential*. The differential equations are linear, first order differential equations in velocity and torque with constant coefficients[1]. The derivatives are with respect to time.

All node velocities and torques form a column matrix x. The equations can be written on matrix form as in equation 1. The desired solution is $\mathrm{x}(t)$, i.e. how all velocities and torques vary with time. In order to solve it as an initial value problem, an explicit form is desired, e.g. as in equation 2. Here, $\mathrm{y_{state}}$ is a column vector with *state variables*, $\dot{\mathrm{y}}_{\mathrm{state}}$ is a column vector with *state derivatives*, and x contains the *algebraic variables*.

$$\left\{\begin{array}{ll} \mathbf{A}\cdot\dot{\mathrm{x}}+\mathbf{B}\cdot\mathrm{x}=\mathbf{F}\cdot\mathrm{f}(t) & \text{...component equations, linear in x} \\ \mathrm{g}_0(\mathrm{x},t)=\mathbf{0} & \text{...component equations, non-linear in x} \end{array}\right\} \tag{1}$$

$$\left\{\begin{array}{l} \dot{\mathrm{y}}_{\mathrm{state}} = \text{function of } (\mathrm{y}_{\mathrm{state}}, t) \\ \mathrm{x} = \text{function of } (\mathrm{y}_{\mathrm{state}}, t) \end{array}\right\} \tag{2}$$

Chapter 2: CONVENTIONAL DERIVATION — AN EXAMPLE

An example of a dynamic transmission system is given in figure 1. The equations are given as component equations in table 1. The total number of (scalar) equations should always equal twice the total number of nodes in the system. Then there will be as many unknowns as equations. The "natural way" to obtain this, is that each component generates as many (scalar) component equations as there are nodes connected to it.

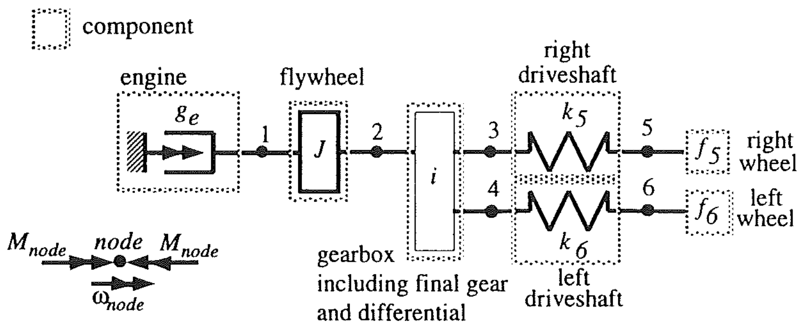


Figure 1: The driveline in a two-wheel driven vehicle, as an example of dynamic transmission system. Here, M and ω denote torque and rotational velocity, respectively.

[1] Time varying coefficients could have been taken into account by adding some terms in the algorithm. However, systems with time varying coefficients are not suitable for this method, since the algorithm would have to be carried out every time the state derivatives were calculated.

Table 1: Component equations for the example system.

COMPONENT	COMPONENT EQUATIONS
engine (g_e describes the steady state characteristics, if throttle position is prescribed as a function of time)	$M_1 = g_e(\omega_1, t)$ (3)
flywheel	$\omega_1 = \omega_2$ (4) $J \cdot \dot{\omega}_1 = M_1 - M_2$ (5)
gearbox including final gear and differential (losses are neglected)	$\omega_2 = i \cdot \frac{\omega_3 + \omega_4}{2}$ (6) $M_3 = i \cdot M_2/2$ (7) $M_4 = i \cdot M_2/2$ (8)
right driveshaft (the second equation is the differentiated form of the constitutive equation)	$M_3 = M_5$ (9) $\dot{M}_3 = k_5 \cdot (\omega_3 - \omega_5)$ (10)
left driveshaft (the second equation is the differentiated form of the constitutive equation)	$M_4 = M_6$ (11) $\dot{M}_4 = k_6 \cdot (\omega_4 - \omega_6)$ (12)
right wheel (prescribed velocity)	$\omega_5 = f_5(t)$ (13)
left wheel (prescribed velocity)	$\omega_6 = f_6(t)$ (14)

In a first approach to finding the explicit form of this equation system, it is tempting to use $\mathbf{y}_{\mathbf{state}} = [\omega_1 \; M_3 \; M_4]^T$ because their time derivatives occur in the component equations in table 1. There is a constraint between M_3 and M_4. Therefore, no more than two state variables can be used, e.g. $\mathbf{y}_{\mathbf{state}} = [\omega_1 \; M_3]^T$. An explicit order of calculation, obtained **without** the algorithm developed in this work is presented in the equations 15–19, which should be evaluated in the same order as they are written. These equations are derived from the equations in table 1 and the choice of state variables. For instance, the matrix equation in equation 18 is obtained by combining equations 6, 7, 8, 10 and 12.

$$\left\{ \begin{array}{c} \begin{bmatrix} \omega_1 \\ M_3 \end{bmatrix} = \mathbf{y}_{\mathbf{state}} \\ \omega_5 = f_5(t) \\ \omega_6 = f_6(t) \end{array} \right\} \tag{15}$$

$$\left\{ \begin{array}{c} M_1 = g_e(\omega_1, t) \\ \omega_2 = \omega_1 \\ M_2 = 2 \cdot M_3/i \\ M_5 = M_3 \end{array} \right\} \tag{16}$$

$$\left\{ \begin{array}{c} \dot{\omega}_1 = \frac{M_1 - M_2}{J} \\ M_4 = i \cdot M_2/2 \end{array} \right\} \tag{17}$$

$$\left\{ \begin{array}{c} M_6 = M_4 \\ \begin{bmatrix} \omega_3 \\ \omega_4 \\ \dot{M}_2 \\ \dot{M}_3 \\ \dot{M}_4 \end{bmatrix} = \begin{bmatrix} i/2 & i/2 & 0 & 0 & 0 \\ 0 & 0 & -i/2 & 1 & 0 \\ 0 & 0 & -i/2 & 0 & 1 \\ -k_5 & 0 & 0 & 1 & 0 \\ 0 & -k_6 & 0 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -k_5 & 0 \\ 0 & 0 & -k_6 \end{bmatrix} \cdot \begin{bmatrix} \omega_2 \\ \omega_5 \\ \omega_6 \end{bmatrix} \end{array} \right\} \tag{18}$$

$$\left\{ \begin{array}{c} \dot{\mathrm{y}}_{\mathrm{state}} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{M}_3 \end{bmatrix} \\ \mathrm{x} = [\omega_1 \ \cdots \ \omega_6 \ M_1 \ \cdots \ M_6]^T \end{array} \right\} \tag{19}$$

Note that the component equations cannot be used exactly as they are originally written, i.e. as written in table 1. The following changes are required:

- ☐ The component equations cannot be evaluated in the same **order** as they are written in table 1.
- ☐ Sometimes, the equations should be **solved for another variable**. For instance, the equation 7, $M_3 = i \cdot M_2/2$, is originally solved for M_3, but in equation 16 it should be solved for M_2, i.e. $M_2 = 2 \cdot M_3/i$.
- ☐ Some equations must be **combined**, e.g. as in the matrix formulation in equation 18, which includes five component equations that have to be combined to find the solution.
- ☐ The component equations may have to be **differentiated**. Examples of that are equations 7 and 8, which are differentiated in the matrix formulation in equation 18.

The method described in this work shows a way to carry out such changes of the component equations automatically. It also identifies constraints and describes how they reduce the number of state variables.

Chapter 3: EXPLICIT FORM OF EQUATIONS

The explicit form of the equations is based on an orthogonal transformation between the *physical quantities* x and the *calculation variables* y. This is shown in equation 20. The state variables are placed in a part of y called $\mathrm{y}_{\mathrm{n-1}}$. The other parts of y, i.e. $\mathrm{y}_1, \cdots, \mathrm{y}_{\mathrm{n-2}}$ and y_{n} are algebraic variables in the calculation variable domain. Starting from the matrices **A**, **B** and **F**, an algorithm presented in appendix A generates the matrices in equations 20 and 21.

$$\mathrm{x} = \underbrace{[\mathbf{T}_1 \ \cdots \ \mathbf{T}_{\mathrm{n}}]}_{\mathrm{T}} \cdot \underbrace{\begin{bmatrix} \mathrm{y}_1 \\ \vdots \\ \mathrm{y}_{\mathrm{n}} \end{bmatrix}}_{\mathrm{y}} \quad \text{and} \quad \begin{bmatrix} \mathrm{y}_1 \\ \vdots \\ \mathrm{y}_{\mathrm{n}} \end{bmatrix} = \mathbf{T}^{-1} \cdot \mathrm{x} = \underbrace{\begin{bmatrix} \mathbf{T}_1^T \\ \vdots \\ \mathbf{T}_{\mathrm{n}}^T \end{bmatrix}}_{\mathrm{T}^T} \cdot \mathrm{x} \tag{20}$$

$$\left\{\begin{array}{c} \mathbf{y}_1 = \mathbf{F}_{1,0} \cdot \mathbf{f}(t) \\ \mathbf{y}_2 = \mathbf{F}_{2,0} \cdot \mathbf{f}(t) + \mathbf{F}_{2,1} \cdot \dot{\mathbf{f}}(t) \\ \vdots \\ \mathbf{y}_{n-2} = \mathbf{F}_{n-2,0} \cdot \mathbf{f}(t) + \cdots + \mathbf{F}_{n-2,n-3} \cdot \mathbf{f}^{(n-3)}(t) \\ \mathbf{y}_n = \mathbf{g}\left(\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{n-1} \end{bmatrix}, t\right) \\ \dot{\mathbf{y}}_{n-1} = \mathbf{C} \cdot \mathbf{y}_{n-1} + \mathbf{D} \cdot \mathbf{y}_n + \mathbf{F}_{n-1,0} \cdot \mathbf{f}(t) + \cdots + \mathbf{F}_{n-1,n-2} \cdot \mathbf{f}^{(n-2)}(t) \end{array}\right\} \tag{21}$$

The state derivatives, $\dot{\mathbf{y}}_{n-1}$, can now be calculated from $\mathbf{y}_{n-1}$ and t, by evaluation of equation 21 in the order in which it is written. This also gives $\mathbf{y}$, through which $\mathbf{x}$ can be calculated by means of equation 20. Only function $\mathbf{g}$ is missing. It is implicitly defined in equation 22, where $\mathbf{g}_0$ is the non-linear function in equation 1.

$$\mathbf{g}_0(\mathbf{x}, t) = \mathbf{g}_0\left(\begin{bmatrix} \mathbf{T}_1 & \cdots & \mathbf{T}_{n-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{n-1} \end{bmatrix} + \mathbf{T}_n \cdot \mathbf{y}_n, t\right) = \mathbf{0} \tag{22}$$

The system was originally described by as many equations as variables. Therefore, equation 22 has equally many (scalar) equations as there are unknowns, i.e. elements in $\mathbf{y}_n$. However, it may be impossible to solve equation 22 for $\mathbf{y}_n$. This can be interpreted as the system is including non-linear constraints, which means that it cannot be solved with this method. Sometimes the explicit formulation of $\mathbf{g}$ can be required in terms of variables in $\mathbf{x}$ instead of $\mathbf{y}$. This is shown in appendix B.

Chapter 4: INITIAL VALUES

The initial conditions should be expressed as initial values of the state variables, i.e. $\mathbf{y}_{n-1}^{iv}$. However, the initial conditions are normally known in the physical variables, i.e. in velocities and torques. To find appropriate $\mathbf{y}_{n-1}^{iv}$ from required initial conditions, expressed as $[\mathbf{x}^{iv}]_{req}$, equations 21 and 23 should be satisfied. Equation 23 is certain rows of equation 20. ($[\mathbf{A}]_b$ denotes certain rows from $\mathbf{A}$, with ordinals described by b.)

$$[\mathbf{x}^{iv}]_{req} = \underbrace{\begin{bmatrix} \mathbf{T}_1 & \cdots & \mathbf{T}_{n-2} & \mathbf{T}_n \end{bmatrix}_{req} \cdot \begin{bmatrix} \mathbf{y}_1^{iv} \\ \vdots \\ \mathbf{y}_{n-2}^{iv} \\ \mathbf{y}_n^{iv} \end{bmatrix} + [\mathbf{T}_{n-1}]_{req} \cdot \mathbf{y}_{n-1}^{iv}}_{[\mathbf{T}]_{req} \cdot \mathbf{y}^{iv}} \tag{23}$$

A recursive iteration that will work if the non-linear equations are not too dependent on the state variables, is:

1. Give required initial values, i.e. $[\mathbf{x}^{iv}]_{req}$, and t^{iv}.
2. Calculate $\mathbf{y}_1^{iv}, \cdots, \mathbf{y}_{n-2}^{iv}$ by means of equation 21.
3. Guess values of $\mathbf{y}_{n-1}^{iv}$. Let $\tilde{\mathbf{y}}_{n-1}^{iv}$ denote the guess.
4. Calculate $\mathbf{y}_n^{iv}$ from equation 21, using $\tilde{\mathbf{y}}_{n-1}^{iv}$ instead of $\mathbf{y}_{n-1}$.
5. If $[\mathbf{T}_{n-1}]_{req}$ can be inverted, calculate $\mathbf{y}_{n-1}^{iv}$ from equation 23. Otherwise, the required initial values in step 1 are not possible to enforce. If so, return to step 1.
6. If $\mathbf{y}_{n-1}^{iv}$ and $\tilde{\mathbf{y}}_{n-1}^{iv}$ differ too much, let $\tilde{\mathbf{y}}_{n-1}^{iv} = \mathbf{y}_{n-1}^{iv}$ and go to step 3.

Chapter 5: A NUMERICAL EXAMPLE

The same example as in chapter 2 is studied. The start for the derivation algorithm is the matrices in equation 1. For instance, $\mathrm{x} = [\omega_1 \ \cdots \ \omega_6 \ M_1 \ \cdots \ M_6]^T$. Then equations 4–14 form the linear part. The matrix $\mathbf{A}$ becomes an 11×12–matrix and $\mathrm{f}(t) = [f_5(t) \ \ f_6(t)]^T$. The non-linear part of equation 1 is constructed from equation 3 and shown in equation 24.

$$\mathrm{g_0}(\mathrm{x},t) = \mathrm{g_0}\Big([\omega_1 \ \cdots \ \omega_6 \ M_1 \ \cdots \ M_6]^T, t\Big) = M_1 - g_e(\omega_1, t) = 0 \qquad (24)$$

The component data are: $J = 0.2$ kgm^2, $i = 8$, $k_5 = 5000$ Nm/rad and $k_6 = 6000$ Nm/rad.

Equation 25 shows $f_5(t)$ and $f_6(t)$, which model a vehicle with constant velocity for $t < 0.5$ s. At $t = 0.5$ s, one of the wheels starts to increase its velocity exponentially which, in practice, could be a case where the tire loses its inflation pressure due to a puncture (flat tire).

$$\begin{bmatrix} f_5(t) \\ f_6(t) \end{bmatrix} = \left\{ \begin{array}{ll} = \begin{bmatrix} 50 \text{ rad/s} \\ 50 \text{ rad/s} \end{bmatrix} & \text{if } t < 0.5 \text{ s} \\ = \begin{bmatrix} 50 \text{ rad/s} \\ 100 - 50 \cdot e^{-5\cdot(t-0.5)} \text{ rad/s} \end{bmatrix} & \text{if } t \geq 0.5 \text{ s} \end{array} \right\} \qquad (25)$$

In equation 26, the non-linear equation is defined. It models a very simplified steady state characteristics for a combustion engine. The dependence on time models a decreasing throttle position.

$$\left\{ \begin{array}{c} M_1 = g_e(\omega_1, t) = \left(1 - 0.1 \cdot t - \left(\frac{\omega_1}{600}\right)^3\right) \cdot 200 \\ \text{units: } M_1 \text{ in Nm, } \omega_1 \text{ in rad/s and } t \text{ in s} \end{array} \right\} \qquad (26)$$

Section 5.1: Explicit Form of Equations

The algorithm is implemented in a computer (in *MATLAB™* code, see reference [3]). Numerical values of matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{F}$ in equation 1 have to be given, as well as information about which variables affect the non-linear function $\mathrm{g_0}$ in equation 1. The result is shown in figure 2. As an example of an algebraic variable, the sixth variable in $\mathrm{y_1}$ is studied. The sixth row of $\mathbf{T}_1^T$ inserted in equation 20 gives: $[\mathrm{y_1}]_6 = +0.6882\cdot M_3 - 0.6882\cdot M_4 - 0.1625\cdot M_5 + 0.1625\cdot M_6$. This linear combination of torques should always be zero, considering component equations 7, 8, 9 and 11, giving $M_3 = M_4 = M_5 = M_6$. This is also satisfied, since the sixth row in $\mathbf{F}_{1,0}$ includes only zeros, cf. equation 21. A similar discussion can be carried out for the other variables in y.

```
n=4

[F1,0]=                     0       0
                            1       0
                            0       1
                            0       0
                            0       0
                          . 0       0
                            0       0
                            0       0

[F2,0 F2,1]=        0.6428 -0.7713  0  0

[F3,0 F3,1 F3,2]=        0        0  0  0  0  0
                    5504.0   5488.6  0  0  0  0

[C]=          0 -0.8839
       964.1752       0

[D]=   -7.1265
             0

[T1]=           0  0  0  -0.7125        0        0        0  -0.0013
                0  0  0   0.6906        0        0        0   0.1754
                0  0  0   0.0877        0        0        0  -0.6961
                0  0  0   0.0877        0        0        0  -0.6961
                0  1  0        0        0        0        0        0
                0  0  1        0        0        0        0        0
                0  0  0        0        0        0        0        0
                0  0  0        0   0.1240        0   0.9845        0
          -0.1625  0  0        0   0.4883   0.6882  -0.1240        0
           0.1625  0  0        0   0.4883  -0.6882  -0.1240        0
          -0.6882  0  0        0  -0.5038  -0.1625   0.0009        0
           0.6882  0  0        0  -0.5038   0.1625   0.0009        0

[T2]= -0.0079            [T3]= -0.7016        0           [T4]= 0
      -0.0079                  -0.7016        0                 0
       0.7061                  -0.0957        0                 0
      -0.7081                  -0.0797        0                 0
            0                        0        0                 0
            0                        0        0                 0
            0                        0        0                 1
            0                        0  -0.1240                 0
            0                        0  -0.4961                 0
            0                        0  -0.4961                 0
            0                        0  -0.4961                 0
            0                        0  -0.4961                 0

[P]= 0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    1 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
     0 0 0 0 0 0    0 0 0 0 0 0
```

Figure 2: The resulting matrices from the algorithm applied on the example system

The variables in $\mathbf{y}_{n-1} = \mathbf{y}_3$ are of special interest. From $\mathbf{T}_3^T$ and equation 20 it can be seen that: $[\mathbf{y}_3]_1$ is a linear combination of velocities and $[\mathbf{y}_3]_2$ is a linear combination of torques. They correspond to the two state variables in $\mathbf{y}_{\text{state}}$, i.e. ω_1 and M_3, found in the example in chapter 2.

The non-linear equation should be solved for $\mathrm{y_n}$, i.e. $\mathrm{y_4}$:

$$\left\{\begin{aligned}
&\mathrm{g_0}(\mathrm{x},t) = M_1 - g_e(\omega_1,t) = [\mathrm{x}]_7 - g_e([\mathrm{x}]_1,t) = 0 \Rightarrow \\
&\Rightarrow [\mathbf{T}]_7 \cdot \mathrm{y} - g_e([\mathbf{T}]_1 \cdot \mathrm{y},t) = 0 \Rightarrow \\
&\Rightarrow [\,\mathbf{T}_1 \quad \mathbf{T}_2 \quad \mathbf{T}_3\,]_7 \cdot \begin{bmatrix} \mathrm{y_1} \\ \mathrm{y_2} \\ \mathrm{y_3} \end{bmatrix} + [\mathbf{T}_4]_7 \cdot \mathrm{y_4} + \\
&+(-1)\cdot g_e\left([\,\mathbf{T}_1 \quad \mathbf{T}_2 \quad \mathbf{T}_3\,]_1 \cdot \begin{bmatrix} \mathrm{y_1} \\ \mathrm{y_2} \\ \mathrm{y_3} \end{bmatrix} + [\mathbf{T}_4]_1 \cdot \mathrm{y_4}, t\right) = 0 \Rightarrow \\
&\Rightarrow 0 + 1\cdot \mathrm{y_4} - g_e\left([\,\mathbf{T}_1 \quad \mathbf{T}_2 \quad \mathbf{T}_3\,]_1 \cdot \begin{bmatrix} \mathrm{y_1} \\ \mathrm{y_2} \\ \mathrm{y_3} \end{bmatrix} + 0\cdot \mathrm{y_4}, t\right) = 0 \Rightarrow \\
&\Rightarrow \underbrace{\mathrm{y_4}}_{\mathrm{y_n}} = \underbrace{g_e\left([\,\mathbf{T}_1 \quad \mathbf{T}_2 \quad \mathbf{T}_3\,]_1 \cdot \begin{bmatrix} \mathrm{y_1} \\ \mathrm{y_2} \\ \mathrm{y_3} \end{bmatrix}, t\right)}_{\mathrm{g}\left(\begin{bmatrix} \mathrm{y_1} \\ \vdots \\ \mathrm{y_{n-1}} \end{bmatrix}, t\right)}
\end{aligned}\right\} \tag{27}$$

Section **5.2: Initial Values**

The following initial conditions are expected at $t^{iv} = 0$ s:

$$\omega_1^{iv} = 400 \text{ rad/s},\ \omega_2^{iv} = 400 \text{ rad/s},\ M_3^{iv} = 560 \text{ Nm and } M_4^{iv} = 560 \text{ Nm} \tag{28}$$

If we try to require all these initial conditions then $[\mathrm{x}^{iv}]_{req} = [\mathrm{x}^{iv}]_{1,2,9,10} = [400 \quad 400 \quad 560 \quad 560]^T$. The iteration procedure in chapter 4 will result in a $[\mathbf{T}_{\mathrm{n}-1}]_{req}$, that cannot be inverted (It is not even square!) and so we understand that we cannot require all these initial values. Instead trying $[\mathrm{x}^{iv}]_{req} = [\mathrm{x}^{iv}]_{1,2} = [400 \quad 400]^T$, we will observe that $[\mathbf{T}_{\mathrm{n}-1}]_{req}$ is square, but still cannot be inverted. Then we try $[\mathrm{x}^{iv}]_{req} = [\mathrm{x}^{iv}]_{1,9} = [400 \quad 560]^T$. This works and, in this example, just one iteration is needed, since $[\mathbf{T}_\mathrm{n}]_{req} = \mathbf{0}$. The following initial values are found:

$$\mathrm{y}_{\mathrm{n}-1}^{iv} = \begin{bmatrix} 1128.7 \\ 570.1 \end{bmatrix} \tag{29}$$

Section 5.3: Simulation

Figure 3 shows the simulation results. A simulation with the manually derived equations (equations 15–19) gives identical results.

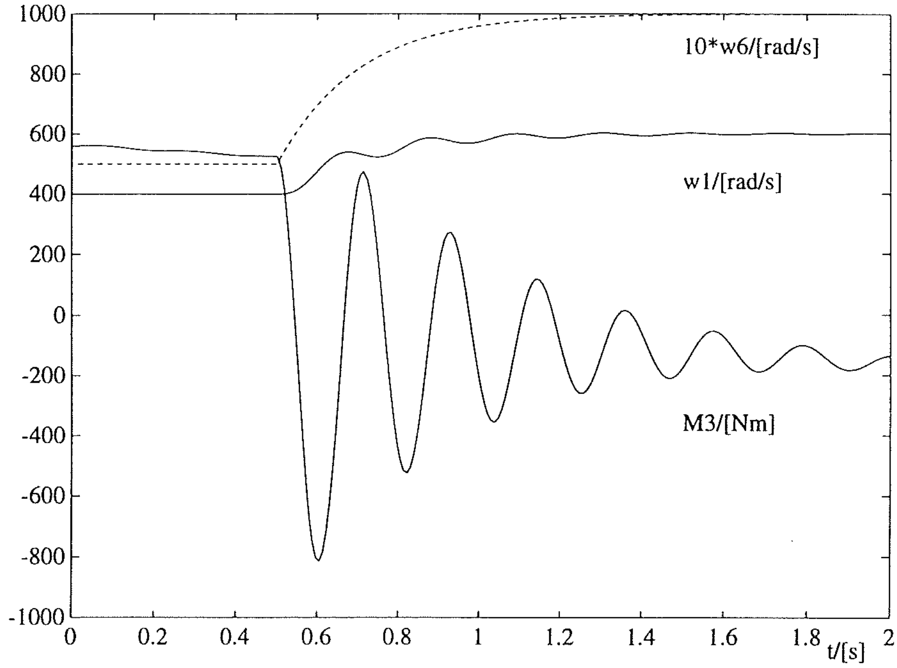


Figure 3: Simulation results. Engine velocity (ω_1), wheel velocity of the flatted tire (ω_6) and driveshaft torque (M_3) are visualized.

Chapter 6: CONCLUSIONS

The explicit form of equations is derived in an automated way. The advantages are mainly that the proper state variables are chosen with respect to constraints and that the proper explicit form of non-linear equations is found. With conventional derivation, the explicit form of equations is found after a lot of tedious work, and many errors can easily occur. The drawbacks with the method are that only linear differential equations with constant coefficients can be treated, and that non-linear constraints cannot be present.

This paper also presents a systematic modelling technique, based on components with component equations, connected in nodes where one velocity and one torque are defined. This, and the described algorithm of equation derivation, could be implemented in computer code to form a very user friendly simulation tool for dynamic transmission systems in vehicles.

Appendix A: ALGORITHM FOR EXPLICIT FORM

The overall purpose of this algorithm is to arrange the linear part of equation 1 to the form in equations 20 and 21, which also can be written as in equation 31. This appendix first treats how the form in equation 30 is derived and then how it is converted to equation 31. The (square) identity matrix is referred to as $\mathbf{I}$.

$$\left\{\begin{array}{l}
\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ \mathbf{A}_{2,1} & 0 & \cdots & 0 & 0 & 0 \\ \mathbf{A}_{3,1} & \mathbf{A}_{3,2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{A}_{n-2,1} & \mathbf{A}_{n-2,2} & \cdots & 0 & 0 & 0 \\ \mathbf{A}_{n-1,1} & \mathbf{A}_{n-1,2} & \cdots & \mathbf{A}_{n-1,n-2} & \mathbf{I} & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{y}}_1 \\ \dot{\mathbf{y}}_2 \\ \vdots \\ \dot{\mathbf{y}}_n \end{bmatrix} + \\
+ \begin{bmatrix} \mathbf{I} & 0 & \cdots & 0 & 0 & 0 \\ \mathbf{B}_{2,1} & \mathbf{I} & \cdots & 0 & 0 & 0 \\ \mathbf{B}_{3,1} & \mathbf{B}_{3,2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{B}_{n-2,1} & \mathbf{B}_{n-2,2} & \cdots & \mathbf{I} & 0 & 0 \\ \mathbf{B}_{n-1,1} & \mathbf{B}_{n-1,2} & \cdots & \mathbf{B}_{n-1,n-2} & \mathbf{B}_{n-1,n-1} & \mathbf{B}_{n-1,n} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_n \end{bmatrix} = \\
= \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_{n-2} \\ \mathbf{F}_{n-1} \end{bmatrix} \cdot \mathbf{f}(t) \text{ ; where } \mathbf{x} = \underbrace{\begin{bmatrix} \mathbf{T}_1 & \cdots & \mathbf{T}_n \end{bmatrix}}_{\mathbf{T}} \cdot \underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_n \end{bmatrix}}_{\mathbf{y}}
\end{array}\right\} \tag{30}$$

$$\left\{\begin{array}{l}
\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \dot{\mathbf{y}}_{n-1} \end{bmatrix} + \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{n-2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ -\mathbf{C} & -\mathbf{D} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}_{n-1} \\ \mathbf{y}_n \end{bmatrix} = \\
= \begin{bmatrix} \mathbf{F}_{1,0} & 0 & 0 & \cdots & 0 \\ \mathbf{F}_{2,0} & \mathbf{F}_{2,1} & 0 & \cdots & 0 \\ \mathbf{F}_{3,0} & \mathbf{F}_{3,1} & \mathbf{F}_{3,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{n-1,0} & \mathbf{F}_{n-1,1} & \mathbf{F}_{n-1,2} & \cdots & \mathbf{F}_{n-1,n-2} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{f}(t) \\ \dot{\mathbf{f}}(t) \\ \vdots \\ \mathbf{f}^{(n-2)}(t) \end{bmatrix} \\
\text{where } \mathbf{x} = \underbrace{\begin{bmatrix} \mathbf{T}_1 & \cdots & \mathbf{T}_n \end{bmatrix}}_{\mathbf{T}} \cdot \underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_n \end{bmatrix}}_{\mathbf{y}}
\end{array}\right\} \tag{31}$$

Section A.1: Algorithm for Equation 30

The matrices in equation 30 can be determined using the following algorithm.

1. First, the linear part of equation 1 is written in the form shown in equation 32. This is made through the assignments: $\mathbf{A}^*_{22} = \mathbf{A}$, $\mathbf{B}^*_{22} = \mathbf{B}$, $\mathbf{F}^*_2 = \mathbf{F}$ and $\mathbf{T}^*_x = \mathbf{I}$. Further, $\mathbf{A}^*_{11}, \mathbf{B}^*_{11}, \mathbf{F}^*_1$, 0 and $\mathbf{y}^*$ have no rows and $\mathbf{A}^*_{11}, \mathbf{A}^*_{21}, \mathbf{B}^*_{11}, \mathbf{B}^*_{21}$ and $\mathbf{T}^*_y$ have no columns.

$$\left\{\begin{array}{c}
\begin{bmatrix} \mathbf{A}^*_{11} & 0 \\ \mathbf{A}^*_{21} & \mathbf{A}^*_{22} \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{y}}^* \\ \dot{\mathbf{x}}^* \end{bmatrix} + \begin{bmatrix} \mathbf{B}^*_{11} & 0 \\ \mathbf{B}^*_{21} & \mathbf{B}^*_{22} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^* \end{bmatrix} = \begin{bmatrix} \mathbf{F}^*_1 \\ \mathbf{F}^*_2 \end{bmatrix} \cdot \mathbf{f}(t) \\
\mathbf{x} = \begin{bmatrix} \mathbf{T}^*_y & \mathbf{T}^*_x \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^* \end{bmatrix}
\end{array}\right\} \tag{32}$$

The principal idea for the remaining algorithm is to process $\mathbf{A}^*_{22}$ and $\mathbf{B}^*_{22}$ and move suitable parts to $\mathbf{A}^*_{11}$, $\mathbf{A}^*_{21}$, $\mathbf{B}^*_{11}$ and $\mathbf{B}^*_{21}$. After a finite number of loops through steps 2–4, $\mathbf{A}^*_{11}$ and $\mathbf{B}^*_{11}$ will look like submatrix rows and columns 1 to $n-2$ in equation 30. Finally, steps 5 and 6 transform $[\mathbf{A}^*_{21} \quad \mathbf{A}^*_{22}]$ and $[\mathbf{B}^*_{21} \quad \mathbf{B}^*_{22}]$ to submatrix row $n-1$ in equation 30.

2. Equation 32 can be written as equation 34. First, a *singular value decomposition*[2] is carried out for $\mathbf{A}^*_{22} \Rightarrow \mathbf{A}^*_{22} = \mathbf{U}^{**} \cdot \begin{bmatrix} 0 \\ \mathbf{S}^{**} \end{bmatrix} \cdot \mathbf{V}^{**}$, where $\mathbf{S}^{**}$ has no zero rows. Then, the first matrix equation in equation 32 is multiplied by $\begin{bmatrix} \mathbf{I} & 0 \\ 0 & (\mathbf{U}^{**})^T \end{bmatrix}$ from the left, which yields equation 33. Then $\mathbf{x}^*$ is substituted by $(\mathbf{V}^{**})^T \cdot \mathbf{x}^{**}$, which yields equation 34.

$$\left\{\begin{array}{l} \begin{bmatrix} \mathbf{A}^*_{11} & 0 \\ (\mathbf{U}^{**})^T \cdot \mathbf{A}^*_{21} & \underbrace{(\mathbf{U}^{**})^T \cdot \mathbf{U}^{**}}_{\mathbf{I}} \cdot \begin{bmatrix} 0 \\ \mathbf{S}^{**} \end{bmatrix} \cdot \mathbf{V}^{**} \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{y}}^* \\ \dot{\mathbf{x}}^* \end{bmatrix} + \\ + \begin{bmatrix} \mathbf{B}^*_{11} & 0 \\ (\mathbf{U}^{**})^T \cdot \mathbf{B}^*_{21} & (\mathbf{U}^{**})^T \cdot \mathbf{B}^*_{22} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^* \end{bmatrix} = \begin{bmatrix} \mathbf{F}^*_1 \\ (\mathbf{U}^{**})^T \cdot \mathbf{F}^*_2 \end{bmatrix} \cdot \mathbf{f}(t) \\ \mathbf{x} = [\mathbf{T}^*_y \quad \mathbf{T}^*_x] \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^* \end{bmatrix} \end{array}\right\} \tag{33}$$

$$\left\{\begin{array}{l} \begin{bmatrix} \mathbf{A}^*_{11} & 0 \\ \mathbf{A}^{**}_{21} & 0 \\ \mathbf{A}^{**}_{31} & \mathbf{S}^{**} \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{y}}^* \\ \dot{\mathbf{x}}^{**} \end{bmatrix} + \begin{bmatrix} \mathbf{B}^*_{11} & 0 \\ \mathbf{B}^{**}_{21} & \mathbf{B}^{**}_{22} \\ \mathbf{B}^{**}_{31} & \mathbf{B}^{**}_{32} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^{**} \end{bmatrix} = \begin{bmatrix} \mathbf{F}^*_1 \\ \mathbf{F}^{**}_2 \\ \mathbf{F}^{**}_3 \end{bmatrix} \cdot \mathbf{f}(t) \\ \mathbf{x} = [\mathbf{T}^*_y \quad \mathbf{T}^{**}_x] \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^{**} \end{bmatrix} \end{array}\right\} \tag{34}$$

Now if the middle row of submatrices is empty ($\mathbf{A}^{**}_{21}$, $\mathbf{B}^{**}_{21}$, $\mathbf{B}^{**}_{22}$ and $\mathbf{F}^{**}_2$ have no rows): go to step 5.

3. Equation 34 can be written as equation 35. Singular value decomposition of $\mathbf{B}^{**}_{22}$ gives $\mathbf{B}^{**}_{22} = \mathbf{U}^{***} \cdot [\mathbf{S}^{***} \quad 0] \cdot \mathbf{V}^{***}$, where $\mathbf{S}^{***}$ has no zero columns[3]. The first matrix equation in equation 34 is multiplied by $\begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & (\mathbf{S}^{***})^{-1} \cdot (\mathbf{U}^{***})^T & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix}$ from left and $\mathbf{x}^{**}$ is substituted by $(\mathbf{V}^{***})^T \cdot \begin{bmatrix} \mathbf{x}^{***}_1 \\ \mathbf{x}^{***}_2 \end{bmatrix}$.

$$\left\{\begin{array}{l} \begin{bmatrix} \mathbf{A}^*_{11} & 0 & 0 \\ \mathbf{A}^{***}_{21} & 0 & 0 \\ \mathbf{A}^{**}_{31} & \mathbf{A}^{***}_{32} & \mathbf{A}^{***}_{33} \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{y}}^* \\ \dot{\mathbf{x}}^{***}_1 \\ \dot{\mathbf{x}}^{***}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}^*_{11} & 0 & 0 \\ \mathbf{B}^{***}_{21} & \mathbf{I} & 0 \\ \mathbf{B}^{**}_{31} & \mathbf{B}^{***}_{32} & \mathbf{B}^{***}_{33} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^{***}_1 \\ \mathbf{x}^{***}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}^*_1 \\ \mathbf{F}^{***}_2 \\ \mathbf{F}^{**}_3 \end{bmatrix} \cdot \mathbf{f}(t) \\ \mathbf{x} = [\mathbf{T}^*_y \quad \mathbf{T}^{***}_{x1} \quad \mathbf{T}^{***}_{x2}] \cdot \begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^{***}_1 \\ \mathbf{x}^{***}_2 \end{bmatrix} \end{array}\right\} \tag{35}$$

---

[2] A singular value decomposition makes it possible to write any matrix $\mathbf{A}$ as $\mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}$, where $\mathbf{S}$ is a diagonal matrix of the same size as $\mathbf{A}$, with non-negative diagonal elements in increasing order and $\mathbf{U}$ and $\mathbf{V}$ are both square and orthogonal. Reference [4] describes the singular value decomposition more thoroughly.

[3] Such $\mathbf{S}^{***}$ can only be found if rows in $\mathbf{B}^{**}_{22}$ are independent. If not, the original equations are dependent, i.e. $[\mathbf{A} \quad \mathbf{B}]$ has not full row rank, which should cause the algorithm to give us a warning and then stop.

4. Let i denote the ordinal for which time this step is executed. Then, the number of components in y_i can be found as the number of elements in x_1^{***} in equation 35. New variants of the submatrices in equation 32 are also identified as indicated in equation 36. Go to step 2.

$$\left\{\begin{array}{c}\begin{bmatrix} A_{11}^{*} & 0 \\ A_{21}^{*} & A_{22}^{*} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11}^{*} & 0 \\ A_{21}^{***} & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ [A_{31}^{**} \quad A_{32}^{***}] & A_{33}^{***} \end{bmatrix} \\ \begin{bmatrix} B_{11}^{*} & 0 \\ B_{21}^{*} & B_{22}^{*} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} B_{11}^{*} & 0 \\ B_{21}^{***} & I \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ [B_{31}^{**} \quad B_{32}^{***}] & B_{33}^{***} \end{bmatrix} \\ \begin{bmatrix} F_1^{*} \\ F_2^{*} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} F_1^{*} \\ F_2^{***} \end{bmatrix} \\ F_3^{**} \end{bmatrix} \text{ and } [T_y^{*} \quad T_x^{*}] = \begin{bmatrix} [T_y^{*} \quad T_{x1}^{***}] & T_{x2}^{***} \end{bmatrix} \end{array}\right\} \tag{36}$$

5. Equation 34 can be written as equation 37. First, note that the middle row of submatrices in equation 34 has no rows. Therefore, this row of submatrices is neglected. Also, S^{**} is a diagonal matrix with positive elements and can be separated into $S^{**} = [S^{***} \quad 0]$, where S^{***} can be inverted. The first matrix equation in equation 34 is multiplied by $\begin{bmatrix} I & 0 \\ 0 & (S^{***})^{-1} \end{bmatrix}$ from left and x^{**} is substituted by $\begin{bmatrix} x_1^{***} \\ x_2^{***} \end{bmatrix}$.

$$\left\{\begin{array}{c}\begin{bmatrix} A_{11}^{*} & 0 & 0 \\ A_{21}^{***} & I & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{y}^{*} \\ \dot{x}_1^{***} \\ \dot{x}_2^{***} \end{bmatrix} + \begin{bmatrix} B_{11}^{*} & 0 & 0 \\ B_{21}^{***} & B_{22}^{***} & B_{23}^{***} \end{bmatrix} \cdot \begin{bmatrix} y^{*} \\ x_1^{***} \\ x_2^{***} \end{bmatrix} = \begin{bmatrix} F_1^{*} \\ F_2^{***} \end{bmatrix} \cdot f(t) \\ x = [T_y^{*} \quad T_{x1}^{***} \quad T_{x2}^{***}] \cdot \begin{bmatrix} y^{*} \\ x_1^{***} \\ x_2^{***} \end{bmatrix} \end{array}\right\} \tag{37}$$

6. Equation 37 has the same form as equation 30. The numbers of components in $y_1, \cdots, y_{n-2}$ were found in step 4. The numbers of components in y_{n-1} and y_n are found as the numbers of elements in x_1^{***} and x_2^{***}, respectively. Then, the numbers of components in all y_i are known and all submatrices in equation 30 can be identified from equation 37.

Section **A.2: Algorithm for Equation 31**

All submatrices under the diagonal of submatrices on the left hand side in equation 30 can be moved to the right hand side. In the remaining left hand side, **C** and **D** from equation 31 can be identified as $\mathbf{C} = -\mathbf{B}_{n-1,n-1}$ and $\mathbf{D} = -\mathbf{B}_{n-1,n}$. Then, the proper left hand side of equation 31 is obtained. So far, equation 30 is changed to equation 38.

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \dot{y}_{n-1} \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_{n-2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ -C & -D \end{bmatrix} \cdot \begin{bmatrix} y_{n-1} \\ y_n \end{bmatrix} = \begin{bmatrix} F_1 \\ \vdots \\ F_{n-1} \end{bmatrix} \cdot f(t) +$$

$$+(-1) \cdot \begin{bmatrix} 0 & 0 & \cdots & 0 \\ A_{2,1} & 0 & \cdots & 0 \\ A_{3,1} & A_{3,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{n-1,1} & A_{n-1,2} & \cdots & A_{n-1,n-2} \end{bmatrix} \cdot \begin{bmatrix} \dot{y}_1 \\ \vdots \\ \dot{y}_{n-2} \end{bmatrix} - \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B_{2,1} & 0 & \cdots & 0 \\ B_{3,1} & B_{3,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_{n-1,1} & B_{n-1,2} & \cdots & B_{n-1,n-2} \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ \vdots \\ y_{n-2} \end{bmatrix} \quad (38)$$

Subsequently, all y_i and $\dot{y}_i$ will then be eliminated on the right hand side. The first row of submatrices gives: $y_1 = [F_1 \quad 0] \cdot [f(t) \quad \dot{f}(t)]^T$ and, differentiated, $\dot{y}_1 = [0 \quad F_1] \cdot [f(t) \quad \dot{f}(t)]^T$. This can be used to eliminate y_1 and $\dot{y}_1$ on the right hand side. The result is shown in equation 39.

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \dot{y}_{n-1} \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_{n-2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ -C & -D \end{bmatrix} \cdot \begin{bmatrix} y_{n-1} \\ y_n \end{bmatrix} = \left(\begin{bmatrix} F_1 & 0 \\ \vdots & \vdots \\ F_{n-1} & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ B_{2,1} & A_{2,1} \\ B_{3,1} & A_{3,1} \\ \vdots & \vdots \\ B_{n-1,1} & A_{n-1,1} \end{bmatrix} \cdot \begin{bmatrix} F_1 & 0 \\ 0 & F_1 \end{bmatrix} \right) \cdot \begin{bmatrix} f(t) \\ \dot{f}(t) \end{bmatrix} +$$

$$+(-1) \cdot \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ A_{3,2} & 0 & \cdots & 0 \\ A_{4,2} & A_{4,3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{n-1,2} & A_{n-1,3} & \cdots & A_{n-1,n-2} \end{bmatrix} \cdot \begin{bmatrix} \dot{y}_2 \\ \vdots \\ \dot{y}_{n-2} \end{bmatrix} - \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ B_{3,2} & 0 & \cdots & 0 \\ B_{4,2} & B_{4,3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_{n-1,2} & B_{n-1,3} & \cdots & B_{n-1,n-2} \end{bmatrix} \cdot \begin{bmatrix} y_2 \\ \vdots \\ y_{n-2} \end{bmatrix} \quad (39)$$

The second row of submatrices in equation 39 can be used to eliminate y_2 and $\dot{y}_2$ on the right hand side. Similar eliminations can be carried out until equation 31 is obtained. In the form of an algorithm, the eliminations of all y_i and $\dot{y}_i$ can be written:

1. Introduce a counter as $i = 1$. Define matrices $F_1^*, \cdots, F_{n-1}^*$ from $F_1, \cdots, F_{n-1}$ in equation 30 as $[F_1^* \quad \cdots \quad F_{n-1}^*] = [F_1 \quad \cdots \quad F_{n-1}]$.
2. Calculate matrices $F_1^{**}, \cdots, F_{n-1}^{**}$ as:

$$\begin{bmatrix} F_1^{**} \\ \vdots \\ F_{n-1}^{**} \end{bmatrix} = \begin{bmatrix} F_1^* & 0 \\ \vdots & \vdots \\ F_{n-1}^* & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ B_{i+1,i} & A_{i+1,i} \\ \vdots & \vdots \\ B_{n-1,i} & A_{n-1,1} \end{bmatrix} \cdot \begin{bmatrix} F_i^* & 0 \\ 0 & F_i^* \end{bmatrix} \quad (40)$$

3. If $i < n-1$, let $i = i + 1$ and $[F_1^* \quad \cdots \quad F_{n-1}^*] = [F_1^{**} \quad \cdots \quad F_{n-1}^{**}]$. Then, still if $i < n-1$, go to step 2.
   If $i = n-1$, equation 31 is obtained by identifying:

$$\begin{bmatrix} F_{1,0} & 0 & \cdots & 0 \\ F_{2,0} & F_{2,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ F_{n-1,0} & F_{n-1,1} & \cdots & F_{n-1,n-2} \end{bmatrix} = \begin{bmatrix} F_1^{**} \\ F_2^{**} \\ \vdots \\ F_{n-1}^{**} \end{bmatrix} \quad (41)$$

Appendix B: EXPLICIT FORM OF NON-LINEAR EQUATIONS

In general, the function g can be implicitly defined in terms of variables in y, i.e. g is a function $y_n = g\left([y_1 \;\cdots\; y_{n-1}]^T, t\right)$, derived from equation 22. Since the non-linear equations in equation 22 are originally written in x, it would be better if g could be defined in terms of these variables. This appendix shows how this is sometimes possible.

Define a diagonal square matrix, $\mathbf{P}$, with the ith diagonal element $= 1$ if the ith row of $\mathbf{T_n}$ has any non-zero elements **and** the ith element of x affects g_0. Otherwise, the diagonal element is $= 0$. Then, the non-zero elements in $\mathbf{P} \cdot x$ are the elements in x, through which y_n affect g_0. If possible, solve $g_0 = 0$ for these elements. This solution defines a function g_1 such that:

$$\mathbf{P} \cdot x = g_1([\mathbf{I} - \mathbf{P}] \cdot x, t) \tag{42}$$

Some of the rows in equation 42 are just $0 = 0$. The function g_1 is not dependent on y_n because of the definition of $\mathbf{P}$. Therefore, equation 42 can be written:

$$\mathbf{P} \cdot x = g_1\left([\mathbf{I} - \mathbf{P}] \cdot [\mathbf{T_1} \;\cdots\; \mathbf{T_{n-1}}] \cdot \begin{bmatrix} y_1 \\ \vdots \\ y_{n-1} \end{bmatrix}, t\right) \tag{43}$$

Then g can be identified in equation 44, which also requires that $[\mathbf{P} \cdot \mathbf{T_n}]_{nzr}$ can be inverted. (The subscript nzr denotes the ordinals of all non-zero rows in matrix $\mathbf{P} \cdot \mathbf{T_n}$ or matrix $\mathbf{P}$.)

$$\left\{ \begin{array}{l} \underbrace{g_1([\mathbf{I} - \mathbf{P}] \cdot \tilde{x}, t)}_{\mathbf{P}\cdot x} = \mathbf{P} \cdot \underbrace{(\tilde{x} + \mathbf{T_n} \cdot y_n)}_{x} \Rightarrow \\ \Rightarrow y_n = \underbrace{[[\mathbf{P} \cdot \mathbf{T_n}]_{nzr}]^{-1} \cdot [g_1([\mathbf{I} - \mathbf{P}] \cdot \tilde{x}, t) - \mathbf{P} \cdot \tilde{x}]_{nzr}}_{g\left(\begin{bmatrix} y_1 \\ \vdots \\ y_{n-1} \end{bmatrix}, t\right)} \\ \text{where } \tilde{x} = [\mathbf{T_1} \;\cdots\; \mathbf{T_{n-1}}] \cdot \begin{bmatrix} y_1 \\ \vdots \\ y_{n-1} \end{bmatrix} \end{array} \right\} \tag{44}$$

In summary, the function g is explicitly defined in terms of another function, g_1, by equation 44. The function g_1 is implicitly defined, in terms of variables in x, by equation 42 and being the solution for $\mathbf{P} \cdot x$ to $g_0(x, t) = 0$.

Section B.1: A Numerical Example

In the example in chapter 5, only the seventh diagonal element of $\mathbf{P}$ is $= 1$. Therefore, the only non-zero row of g_1 is found by solving $[x]_7 = M_1$, from the equation $g_0 = 0$, i.e. equation 24. The function g_1 will be as defined in equation 45.

$$\underbrace{[0 \;\cdots\; 0 \;\; M_1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T}_{\mathbf{P}\cdot x} = \underbrace{[0 \;\cdots\; 0 \;\; g_e(\omega_1, t) \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T}_{g_1([\mathbf{I}-\mathbf{P}]\cdot x, t)} \tag{45}$$

Appendix C: AUTOMATIC TRANSMISSION GEAR SHIFT EXAMPLE

This chapter shows a more complex example than the one in chapter 5. Almost the same system is modelled in references [5] and [6], where the explicit form was derived without the algorithm. That required a great deal of tedious manual handling with equations. Gear shift operations are to be analyzed. Therefore, the system should be studied for all combinations of engaged or disengaged clutches. There are $2^5 = 32$ such combinations because there are 5 clutches. Here, "engaged" means sticking (relative velocity in clutch is known as zero) and "disengaged" means slipping (clutch torque is known as a function of time). The system is presented as two subsystems: gearbox and driveline.

Section C.1: Gearbox Subsystem

In this subsystem nodes with relative velocity are introduced. Such a node is actually two shaft ends, where the torques are equal and only the relative velocity is of interest. Nodes with relative velocity are just a way of reducing the number of variables.

As seen in figure 4, the gearbox has internal inertia and elasticity. As long as possible, no inertia ($J_{internal} = 0$) and no elasticity ($1/k_{internal} = 0$) are used. However, in some combinations of engaged clutches, this would cause the equations to become dependent. Such cases are detected by the warning mentioned in step 3 in section A.1. Then, $J_{internal}$ is given a small positive number or $k_{internal}$ a large positive number.

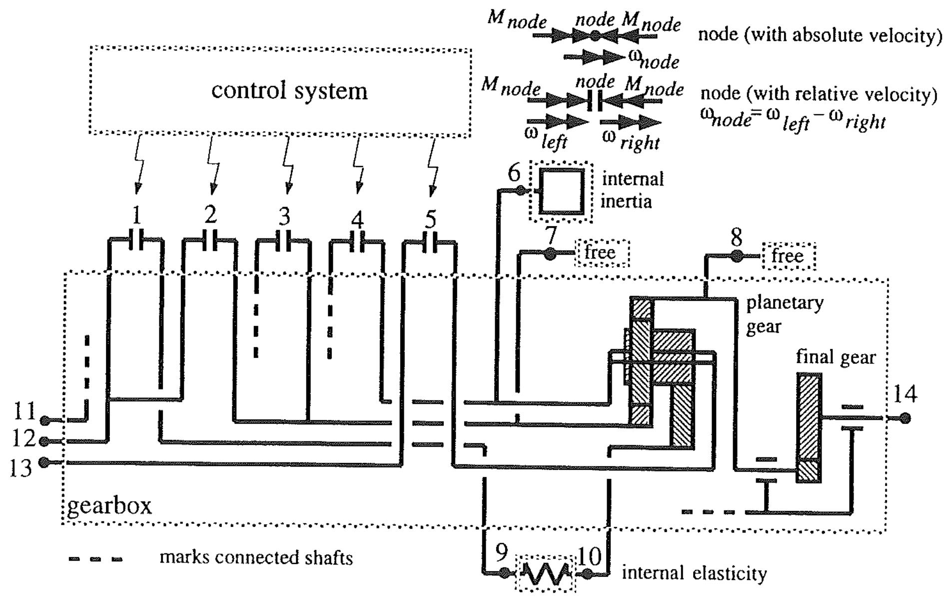


Figure 4: Layout of gearbox subsystem. Nodes 11–14 are connected to the driveline subsystem.

Table 2: Component equations of the gearbox subsystem

COMPONENTS	COMPONENT EQUATIONS
gearbox (includes everything within the rectangle in the figure, i.e. a component that connects nodes 1, 2, ..., 14), 14 linear equations	$$\left\{ \begin{array}{c} \dfrac{\omega_8 - \omega_6}{\omega_7 - \omega_6} = R_{8,7} = \text{known value} \\ \dfrac{\omega_8 - \omega_6}{\omega_{10} - \omega_6} = R_{8,10} = \text{known value} \\ \dfrac{\omega_8 - \omega_{11}}{\omega_{14} - \omega_{11}} = R_{8,14} = \text{known value} \\ \omega_1 = \omega_{12} - \omega_9 \\ \omega_2 = \omega_{12} - \omega_7 \\ \omega_3 = \omega_{11} - \omega_7 \\ \omega_4 = \omega_{11} - \omega_6 \\ \omega_5 = \omega_{13} - \omega_6 \end{array} \right\} \quad (46)$$ From these 8 velocity equations, 14–8=6 corresponding torque equations can be derived from power equilibrium, see reference [7].
control system (an engaged clutch sticks and has time controlled velocity ($\omega_i = f_{\omega i}(t) = 0$), otherwise the clutch is slipping and has time controlled torque ($M_i = f_{Mi}(t)$)), 5 linear equations	$$\left\{ \begin{array}{c} \omega_1 = f_{\omega 1}(t) \text{ or } M_1 = f_{M1}(t) \\ \omega_2 = f_{\omega 2}(t) \text{ or } M_2 = f_{M2}(t) \\ \omega_3 = f_{\omega 3}(t) \text{ or } M_3 = f_{M3}(t) \\ \omega_4 = f_{\omega 4}(t) \text{ or } M_4 = f_{M4}(t) \\ \omega_5 = f_{\omega 5}(t) \text{ or } M_5 = f_{M5}(t) \end{array} \right\} \quad (47)$$
internal inertia, 1 linear equation	$$J_{internal} \cdot \dot{\omega}_6 = M_6 - 0 \quad (48)$$
internal elasticity, 2 linear equations	$$\left\{ \begin{array}{c} \dot{M}_9 / k_{internal} = \omega_9 - \omega_{10} \\ M_9 = M_{10} \end{array} \right\} \quad (49)$$
free shafts at node 7 and 8 2 linear equations	$$\left\{ \begin{array}{c} M_7 = 0 \\ M_8 = 0 \end{array} \right\} \quad (50)$$

Section C.2: Driveline Subsystem

In this subsystem, a translational node is introduced. In such nodes a translational velocity and a force are defined instead of rotational velocity and torque. As seen in figure 5, the velocity of the engine block and the vehicle are treated as prescribed as functions of time. This is a very simple way of modelling the engine block and the vehicle, but it is sufficient for this study. More detailed models are used in references [5] and [6].

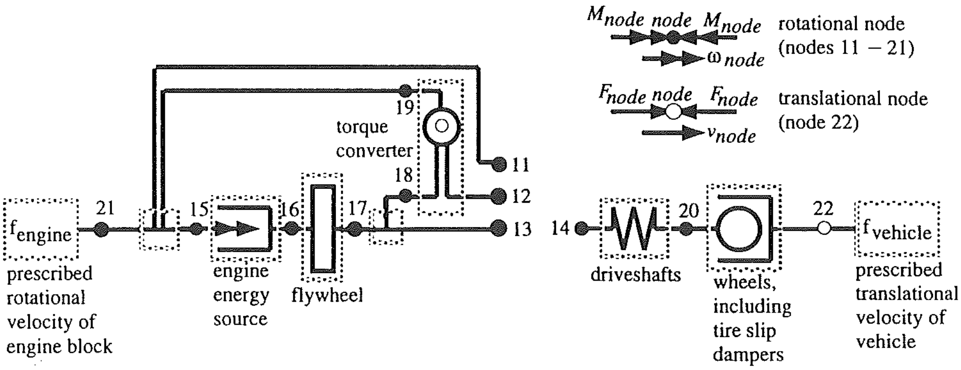


Figure 5: Driveline subsystem. Nodes 11–14 are connected to the gearbox subsystem.

Table 3: Component equations of the driveline subsystem

COMPONENT	COMPONENT EQUATIONS	
prescribed rotational velocity of engine block 1 linear equation	$\omega_{21} = f_{engine}(t)$	(51)
connection between nodes 11, 15, 19 and 21 4 linear equations	$\left\{ \begin{array}{c} \omega_{21} = \omega_{11} = \omega_{19} = \omega_{15} \\ M_{21} = M_{11} + M_{19} + M_{15} \end{array} \right\}$	(52)
engine energy source 1 linear and 1 non-linear equation	$\left\{ \begin{array}{c} M_{15} = g_{engine}(\omega_{16} - \omega_{15}, t) \\ M_{15} = M_{16} \end{array} \right\}$	(53)
flywheel 2 linear equations	$\left\{ \begin{array}{c} J_{engine} \cdot \dot{\omega}_{16} = M_{16} - M_{17} \\ \omega_{16} = \omega_{17} \end{array} \right\}$	(54)
connection between nodes 13, 17 and 18 3 linear equations	$\left\{ \begin{array}{c} \omega_{17} = \omega_{13} = \omega_{18} \\ M_{17} = M_{18} + M_{13} \end{array} \right\}$	(55)
torque converter 1 linear and 2 non-linear equations	$\left\{ \begin{array}{c} M_{12} = g_{conv1}(\omega_{12}, \omega_{18}, \omega_{19}) \\ M_{18} = g_{conv2}(\omega_{12}, \omega_{18}, \omega_{19}) \\ M_{18} + M_{19} = M_{12} \end{array} \right\}$	(56)
driveshafts 2 linear equations	$\left\{ \begin{array}{c} \dot{M}_{14} = k_{driveshafts} \cdot (\omega_{14} - \omega_{20}) \\ M_{14} = M_{20} \end{array} \right\}$	(57)
wheels, including tire slip dampers 1 linear and 1 non-linear equation	$\left\{ \begin{array}{c} F_{22} = g_{tire}(\omega_{20}, v_{22}) \\ M_{20} = R_{wheels} \cdot F_{22} \end{array} \right\}$	(58)
prescribed translational velocity of vehicle 1 linear equation	$v_{22} = f_{vehicle}(t)$	(59)

Section C.3: Complete System

In total, the system has 22 nodes, 40 linear equations and 4 non-linear equations. So in equation 1, the size of matrices **A** and **B** is $40 \times (2 \cdot 22) = 40 \times 44$ and the function g_0 consists of 4 scalar equations. There are 7 functions of time, which means that **F** is a 40×7 matrix. The algorithm in appendix A is applied on the system, in different combinations of engaged clutches. Table 4 shows the results.

Table 4: Results from automatic transmission model, with different combinations of clutches engaged (marked with *). Combinations where internal inertia of gearbox has to be non-zero is marked with * in the J column. The use of non-zero value of $1/k$ for the internal elasticity is, similarly, marked with * in the $1/k$ column.

engaged clutches 12345	J	1/k	number of states	unknown variables in x, when solving the nonlinear equations
-----	*	-	2	w12 w20 M15 M18
*----	-	-	1	w12 w20 M15 M18
-*---	-	-	1	w12 w20 M15 M18
--*--	-	-	1	w12 w20 M15 M18
---*-	-	-	1	w12 w20 M15 M18
----*	-	-	1	w12 w20 M15 M18
**---	-	-	2	w12 w20 M15 M18
---	-	-	2	w12 w20 M15 M18
---	-	-	2	w12 w20 M15 M18
---	-	-	2	w12 w20 M15 M18
-**--	-	-	1	w20 M12 M15 M18
-*-*-	-	-	2	w12 w20 M15 M18
-*--*	-	-	2	w12 w20 M15 M18
--**-	-	-	2	w12 w20 M15 M18
--*-*	-	-	2	w12 w20 M15 M18
---**	-	-	0	w12 w20 M15 M18
***--	-	-	2	w20 M12 M15 M18
**-*-	-	-	2	w20 M12 M15 M18
**--*	-	-	2	w20 M12 M15 M18
*-**-	-	-	2	w20 M12 M15 M18
--*	-	-	2	w20 M12 M15 M18
*--**	-	-	1	w12 w20 M15 M18
-***-	-	-	2	w20 M12 M15 M18
-**-*	-	-	2	w20 M12 M15 M18
-*-**	-	-	1	w12 w20 M15 M18
--***	-	-	1	w12 w20 M15 M18
****-	-	*	3	w20 M12 M15 M18
***-*	-	-	1	w20 M12 M15 M18
-	-	-	1	w20 M12 M15 M18
*-***	-	-	1	w20 M12 M15 M18
-****	-	-	1	w20 M12 M15 M18
*****	-	*	2	w20 M12 M15 M18

In some combinations in table 4, the internal inertia or elasticity is needed. Roughly, few engaged clutches require the extra inertia in order to avoid some accelerations to become infinite. Analogously, many engaged clutches require the extra elasticity in order to avoid a statically undetermined structure.

Table 4 also shows that the system requires different numbers of state variables when different sets of clutches are engaged. For example, when clutches 1 and 2 are engaged, two state variables are needed, while engagement of clutches 2 and 3 only requires one. When clutches 4 and 5 are engaged, no state variable at all is needed. In the case of engagement of clutches 1 and 2, one of the state variables is a linear combination of torques and the other is a linear combination of velocities. These state variables correspond to the state variables obtained in references [5] and [6], i.e. torque of the driveshafts and velocity of the flywheel, respectively. When clutches 2 and 3 are engaged, the torque state disappears, which can be explained as follows: the torque of the driveshafts is an algebraic function of the time-dependent torque in some of the disengaged (or slipping) clutches. When clutches 4 and 5 are engaged the gearbox is kinematically locked to the engine block. Therefore the velocity state disappears, too.

It can also be seen from table 4, that the non-linear equations should be used in different explicit forms. In all cases, the formulation in appendix B can be used. Therefore, the required explicit form can be described as which variables in x that $\mathrm{g}_0 = 0$ should be solved for. The original, implicit, formulation can be:

$$\mathrm{g}_0(\mathrm{x}, t) = \begin{bmatrix} M_{15} - g_{engine}(\omega_{16} - \omega_{15}, t) \\ M_{12} - g_{conv1}(\omega_{12}, \omega_{18}, \omega_{19}) \\ M_{18} - g_{conv2}(\omega_{12}, \omega_{18}, \omega_{19}) \\ F_{22} - g_{tire}(\omega_{20}, v_{22}) \end{bmatrix} = 0 \tag{60}$$

Table 4 shows that when clutches 1 and 2 are engaged, the function g_1 should be found by solving $\mathrm{g}_0 = 0$ for $\omega_{12}, \omega_{20}, M_{15}$ and M_{18}. The non-zero rows of g_1 become:

$$\underbrace{\begin{bmatrix} \omega_{12} \\ \omega_{20} \\ M_{15} \\ M_{18} \end{bmatrix}}_{[\mathrm{P}\cdot\mathrm{x}]_{nzr}} = \underbrace{\begin{bmatrix} g_{conv1}^{-1}(\omega_{18}, \omega_{19}, M_{12}) \\ g_{tire}^{-1}(v_{22}, F_{22}) \\ g_{engine}(\omega_{16} - \omega_{15}, t) \\ g_{conv2}(\omega_{12}, \omega_{18}, \omega_{19}) \end{bmatrix}}_{[\mathrm{g}_1([\mathrm{I}-\mathrm{P}]\cdot\mathrm{x}, t)]_{nzr}} \tag{61}$$

If, instead, clutches 2 and 3 are engaged, $\omega_{20}, M_{12}, M_{15}$ and M_{18} should be solved for. Then, the non-zero rows of g_1 become:

$$\underbrace{\begin{bmatrix} \omega_{20} \\ M_{12} \\ M_{15} \\ M_{18} \end{bmatrix}}_{[\mathrm{P}\cdot\mathrm{x}]_{nzr}} = \underbrace{\begin{bmatrix} g_{tire}^{-1}(v_{22}, F_{22}) \\ g_{conv1}(\omega_{12}, \omega_{18}, \omega_{19}) \\ g_{engine}(\omega_{16} - \omega_{15}, t) \\ g_{conv2}(\omega_{12}, \omega_{18}, \omega_{19}) \end{bmatrix}}_{[\mathrm{g}_1([\mathrm{I}-\mathrm{P}]\cdot\mathrm{x}, t)]_{nzr}} \tag{62}$$

LIST OF SYMBOLS

ω, v	rotational velocity, translational velocity
M, F	torque, force
i	transmission ratio (of velocity or torque)
$\mathbf{x}, \mathbf{y}$	vectors of physical variables and calculation variables
t	time
J, m	moment of inertia, mass
k, d	stiffness and damping coefficient (rotational or translational)
$\mathbf{I}$	square identity matrix
$[\mathbf{X}]_y$	certain rows of matrix $\mathbf{X}$, with ordinals described by y
bold capital letters	matrices
bold lower-case letters	column matrices (column vectors)
one dot above variable	differentiated once with respect to time
two dots above variable	differentiated twice with respect to time
superscript (n)	differentiated n times with respect to time
superscript iv	initial value
superscript T	transpose of matrix
superscript -1	inverse of matrix or inverse function

REFERENCES

[1] ROSENBERG, Ronald C. and Karnopp, Dean C. Introduction to Physical System Dynamics. McGraw-Hill Book Company, 1983.

[2] RINDERLE, James R. and BALASUBRAMANIAM L. Automated Modeling to Support Design. *The 1990 ASME Design Technical Conferences – 2nd International Conference on Design Theory and Methodology, Chicago, Illinois*, pages 281–290, 16-19 September 1990.

[3] The MathWorks, Inc. PRO-MATLAB for Sun Workstations. January 31, 1990.

[4] GOLUB, Gene H. and VAN LOAN, Charles F. Matrix Computations. North Oxford Academic Publishers Ltd, London, 1986.

[5] JACOBSON, Bengt. Analysis of Shift Operations in Automatic Transmissions. *3rd International EAEC Conference on Vehicle Dynamics and Powertrain Engineering, Strasbourg, France*, pages 195–202, June 11-13 1991.

[6] JACOBSON, Bengt. Dynamic Simulation of Powershifting Transmissions. *Eighth World Congress on the Theory of Machines and Mechanisms, Prague, Czechoslovakia*, pages 573–576, August 26-31 1991.

[7] JACOBSON, Bengt. *Vehicle Driveline Mechanics during Powershifting*. Report no 1990-05-30 (Licentiate of Engineering thesis), Machine and Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1990.

Paper E

JACOBSON, BENGT: Engagement of Oil Immersed Multi-disc Clutches, *6th International Power Transmission and Gearing Conference, ASME*, Scottsdale, USA, September 13–16, 1992, Volume 2, pp 567–574

ENGAGEMENT OF OIL IMMERSED MULTI-DISC CLUTCHES

Bengt Jacobson

Machine & Vehicle Design,
Chalmers University of Technology,
S-412 96 Göteborg, SWEDEN

Chapter 1: ABSTRACT

Oil immersed clutches are often treated without consideration of the oil present, i.e. the torque is assumed to be a pure dry friction torque, directly controlled by the actuating force. During engagement, however, this is sometimes not enough. The oil film acts in two ways: first, the oil has to be squeezed out before dry friction can develop. Second, before the friction surfaces have reached contact, the torque is viscous. This work proposes equations that take these phenomena into account. A simple model for the transition from viscous to dry friction is used. Compensation is made for grooves in the friction surface. The hydraulic actuating equipment (piston, return spring, orifice etc.) is also modelled. Simulation results are verified by experiments.

Keywords: *oil immersed, wet clutch, multi-disc clutch, engagement, squeeze, viscous friction, dry friction, simulation*

Chapter 2: INTRODUCTION

In references [1], [2] and [3] the shift operation in automatic automotive transmissions was studied. Tests showed that the torque in an engaging wet multi-disc clutch increased faster than dry friction could explain. Figure 1 shows test results. If the measured torque could be explained by a dry friction model, the torque curve should have the same shape as the pressure curve minus a constant (corresponding to the return spring force). Such curves are drawn as dotted in the figure and we can see that none of the dotted curves has the same shape as the torque curve. The deviation at low sliding velocities is not the problem. It could be explained by a velocity dependent coefficient of friction. However, that explanation is not likely at the higher velocities.

The system of two annular discs, pressed together with an oil film between was analyzed in e.g. references [4], [5] and [6]. The surfaces in these three works are porous, which affects the squeeze motion mostly for thin oil films. Centrifugal forces are also taken into account. Grooves in the friction surfaces are not treated. The two later works include transition between viscous and dry friction. In [5] an abrupt transition is made at a certain critical film thickness. In [6] a mixed asperity contact phase between viscous and dry friction is discussed but not analyzed. More practical analyses of multi-disc clutches are carried out, e.g. by [7]. In particular, spline friction is taken into account, which is why the squeeze motion is separately simulated in each pair of friction surfaces. None of the mentioned references include any actuating equipment in their models.

The present work neglects porosity and centrifugal forces. It also neglects the differences between each pair of contact surfaces in the multi-disc clutch. The asperities are treated as elastic springs, which gives a continuous transition between viscous and dry friction. The asperities also disturb the squeeze flow, which is roughly taken into account. Compensation is made for grooves in the surfaces. The hydraulic actuating device is also included in the model.

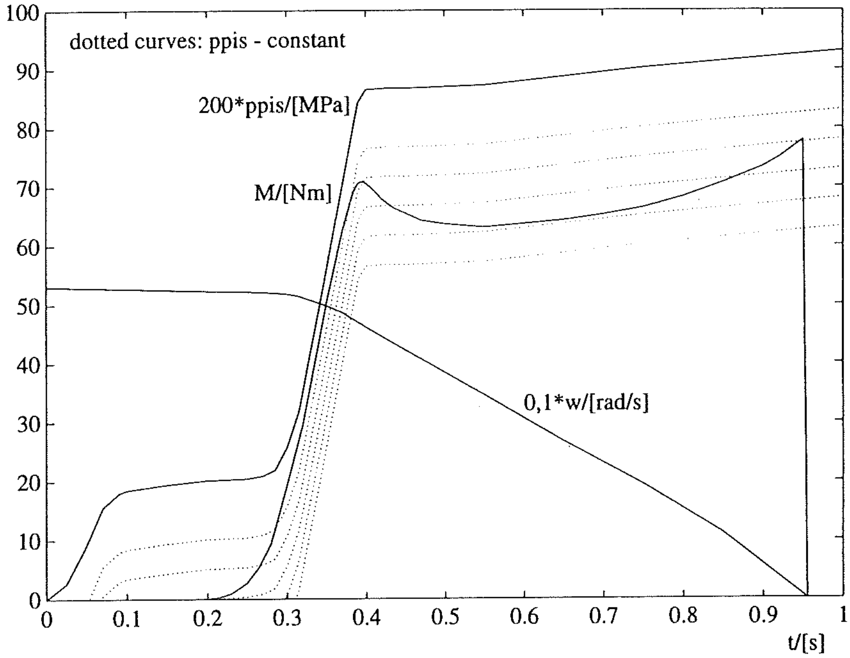


Figure 1: Test results with grooved multi-disc clutch. Pressure in actuating piston (p_{pis}) makes clutch torque (M) increase, which decreases the velocity of a flywheel (ω).

Chapter 3: BASIC ASSUMPTIONS

This chapter treats wet friction surfaces in general. The equations can be used for friction surfaces in multi-disc clutches, cone clutches, band brakes, etc.. There are basically two types of equations: one for the shear stresses and one for the squeeze motion. Together, these equations form a *film/surface model* with a physical interpretation as shown in figure 2.

3.1 Asperity Model

In each pair of friction surfaces there is one steel surface and one paper-lined surface. The steel surface is relatively smooth, while the lined surface is rougher. Figure 3 illustrates the situation.

The asperities are considered as elastic and fully compressible, with the constitutive equation: $p_{cont} = p_{cont}(h)$, where p_{cont} is the **mean** pressure over a surface element, dA in figure 2. It is reasonable that $p_{cont} = 0$ for $h > H_{asp}/2$, because the asperities are then not yet in contact. The contact pressure should start to rise when $h = H_{asp}/2$. Here, the derivative (dp_{cont}/dh) ought to be zero. At $h = -H_{asp}/2$ it is reasonable that the contact pressure increases towards infinity. One constitutive relationship that satisfies these requirements is

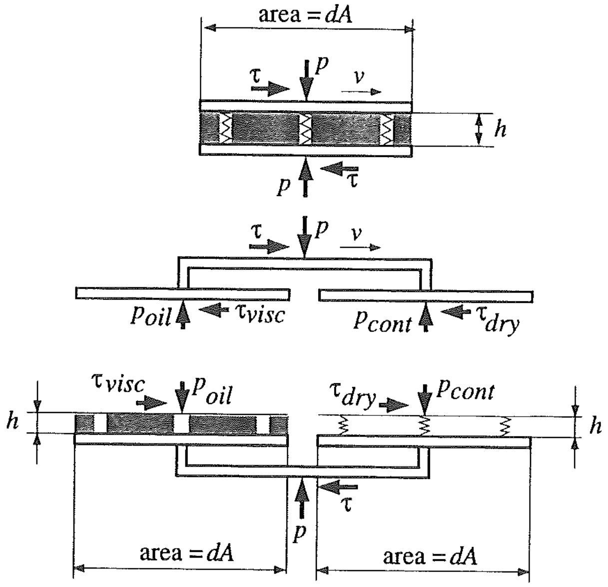


Figure 2: An area element, dA, of the film/surface model. Shaded area mark oil film, which controls squeeze and viscous shear stress. Springs correspond to asperities in the surfaces, which counteract squeeze motion and cause dry friction shear stress.

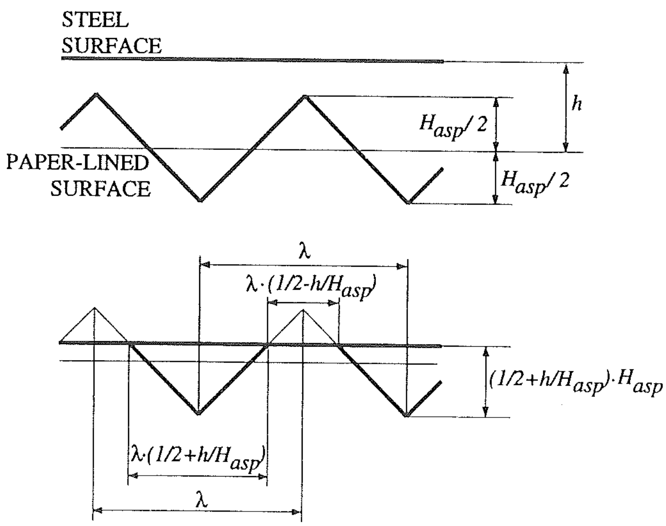


Figure 3: Situation in a pair of friction surfaces. Asperities are considered as pyramides, which can actually not be seen in this two-dimensional sketch.

shown in figure 4 and follows the equation:

$$\frac{dp_{cont}}{dh} = \frac{-E_{asp}}{H_{asp}} \cdot \frac{\left(\frac{1}{2} - \frac{h}{H_{asp}}\right)^2}{\frac{1}{2} + \frac{h}{H_{asp}}} \tag{1}$$

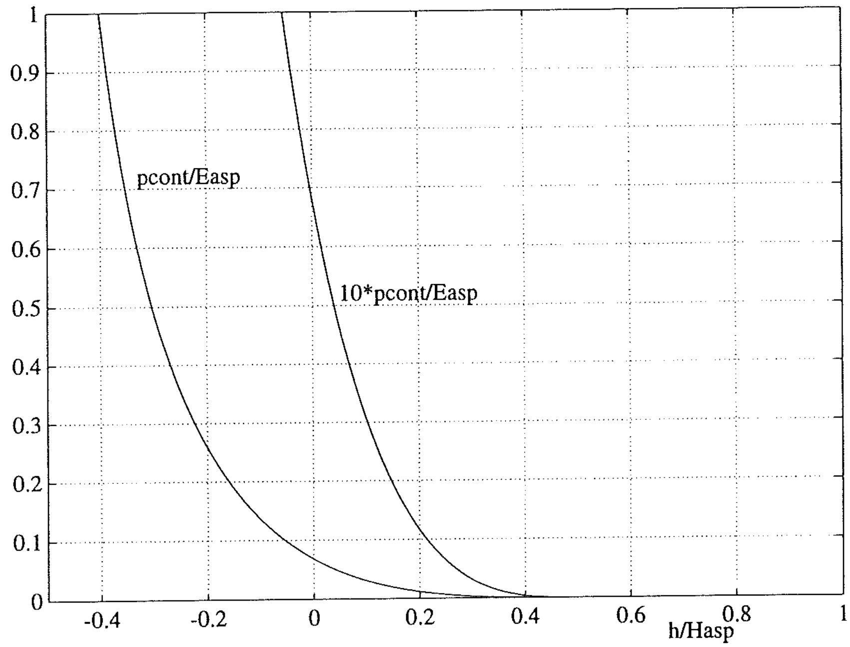


Figure 4: Constitutive relationship for asperities, i.e. function $p_{cont}(h)$. The quantity E_{asp} can be seen as the elasticity modulus of asperities.

3.2 Shear Stress Model

The **mean** shear stress (τ) in the film/surface is the sum of a dry friction **mean** shear stress (τ_{dry}) and a viscous **mean** shear stress (τ_{visc}). Actually, the surfaces under the asperities will probably never be quite dry, but the shear stress will be limited by the strength of the oil. Therefore, the shear stresses under the asperities will **act as dry friction**, governed by the common law of Coulombian friction (with μ =coefficient of friction). The viscous shear stress assumes Newtonian fluid (with η =viscosity) and a thin oil film. Two situations occur:

1. $h > H_{asp}/2$ implies $p_{cont} = 0$, why only viscous shear stress is present:

$$\tau = \underbrace{\eta \cdot v/h}_{\tau_{visc}} \tag{2}$$

2. $h < H_{asp}/2$ implies mixed lubrication, which means that the dry friction shear stress is also present. The area for the viscous shear stress is reduced to the area fraction $\left[1 - (1/2 - h/H_{asp})^2\right]$ of the area element. The oil film thickness in the denominator in equation 2 is replaced with the mean oil film thickness, which is $(1/2 + h/H_{asp}) \cdot H_{asp}/2$. Thus:

$$\tau = \underbrace{\mu \cdot p_{cont}}_{\tau_{dry}} + \underbrace{\eta \cdot v \cdot \frac{2}{H_{asp}} \cdot \frac{1 - \left(\frac{1}{2} - \frac{h}{H_{asp}}\right)^2}{\frac{1}{2} + \frac{h}{H_{asp}}}}_{\tau_{visc}} \tag{3}$$

3.3 Squeeze Model

The pressure is modelled as the sum of the pressure in the asperity contacts and the pressure in the oil film: $p = p_{cont} + p_{oil}$. The contact pressure is determined by the oil film thickness (h) and the constitutive equation for elastic asperities: $p_{cont} = p_{cont}(h)$. The oil film pressure is the pressure that determines the landing velocity ($-\partial h/\partial t$), which makes it possible to follow the landing in the time domain.

First, if $h >> H_{asp}/2$, Reynolds' equation should be solved over the whole surface. The thickness h is inserted as oil film thickness. This will constitute an equation, which, **roughly**, can be written ($\propto$ means "proportional to"):

$$\frac{\partial h}{\partial t} \propto p_{oil} \cdot h^3/\eta \Leftrightarrow \frac{\partial h}{\partial t} = -C_1 \cdot p_{oil} \cdot h^3/\eta \tag{4}$$

Second, when $h << H_{asp}/2$, the squeeze takes place in channels between the asperities. In appendix A there is a discussion with the conclusion:

$$\frac{\partial h}{\partial t} \propto p_{oil} \cdot \left(h + \frac{H_{asp}}{2}\right)^2/\eta \Leftrightarrow \frac{\partial h}{\partial t} = -C_3 \cdot p_{oil} \cdot \left(h + \frac{H_{asp}}{2}\right)^2/\eta \tag{5}$$

As a reasonable approximation we prescribe that the first case is valid for $h > H_{asp}/2$ and the second one for $h < H_{asp}/2$. Furthermore, we demand continuity in $\partial h/\partial t$. Then the constants C_1 and C_3 in equations 4 and 5 have to be coupled to each other:

$$C_3 = \frac{H_{asp}}{8} \cdot C_1 \tag{6}$$

3.4 Grooves in the Paper-lining

In practice, the paper lining is often equipped with grooves, in order to reduce the squeeze resistance. Let θ denote the fraction of non-grooved area. The depth of the grooves is H_{gro}. We introduce the groove film thickness, an effective film thickness and an effective contact pressure:

$$\begin{aligned} h_{gro} &= h + H_{gro} \\ h_{eff} &= \theta \cdot h + (1-\theta) \cdot h_{gro} \\ p_{cont,eff} &= \theta \cdot p_{cont}(h) + (1-\theta) \cdot p_{cont}(h_{gro}) \end{aligned} \tag{7}$$

The shear stresses in the grooved area can be calculated quite analogously as shown in section 3.1 and 3.2. The only difference is that h_{gro} should replace h. Then:

$$\tau = \theta \cdot \tau_{non-grooved} + (1-\theta) \cdot \tau_{grooved} \tag{8}$$

The premises for Reynolds' equation are not very obvious when we introduce grooves. We can differ between two extreme cases:

1. **Thick oil film:** The boundary condition for Reynolds' equation is that the oil pressure is zero at the boundaries of the whole friction surface. The oil film thickness in Reynolds' equation should be set to the effective oil film thickness h_{eff}.
2. **Thin oil film:** The oil pressure in the grooves is zero, which constitutes new boundary conditions. The oil film thickness in Reynolds' equation should be set to the oil film thickness h.

The two solutions will be spliced when they give the same landing velocity. This is assumed to happen when $h = H_{switch}$. Combined with the switch at $h = H_{asp}/2$, the relationship between the oil film thickness (h) and the landing velocity ($-\partial h/\partial t$) is shown in figure 5.

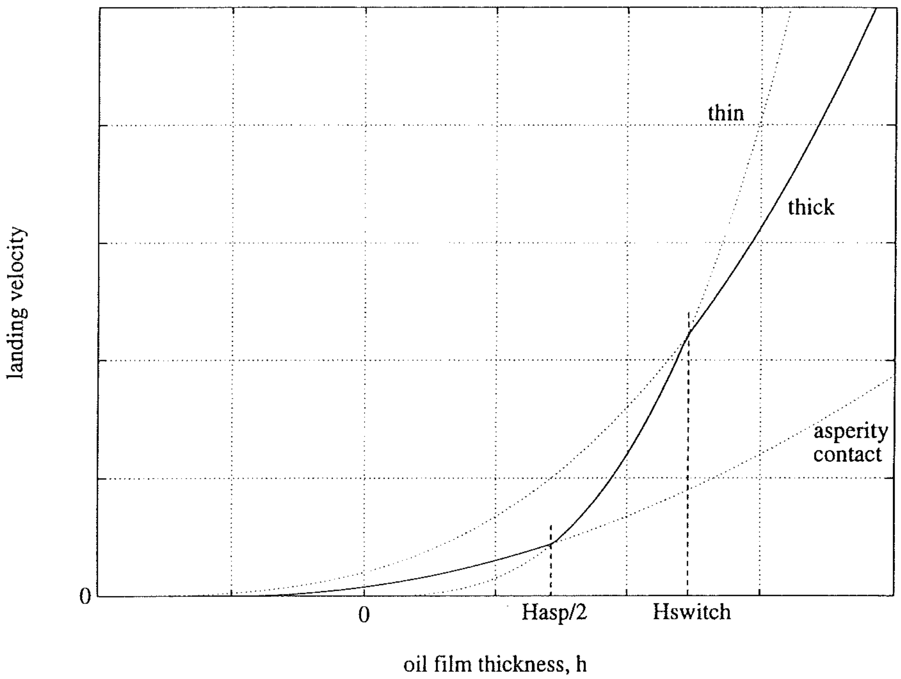


Figure 5: Schematic relationship between oil film thickness and landing velocity, for p_{oil} and η regarded as constants. It is here assumed that $H_{switch} > H_{asp}/2$.

3.5 Viscosity and Temperature Model

The temperature T_{oil} in the oil increases during the engagement. This is essential to consider, since the viscosity is strongly dependent on the temperature, which is the main reason for the decrease of the viscous shear stress. In this work, the following empirical equation is used:

$$\eta = \eta_0 \cdot e^{-\beta \cdot T_{oil}} \tag{9}$$

The temperature in the oil film is governed by differential equations for heat accumulation and conduction, schematically drawn in figure 6. Each pair of friction surfaces consists of one steel surface and one paper surface. The paper lining has low heat conductivity as compared to steel. Therefore, "disc" in figure 6 means only steel discs. The oil film itself is thin, giving low heat accumulation and good heat conduction. Since steel has good conductivity, heat is transferred to the steel discs very rapidly. Therefore, the oil temperature is taken as the disc temperature. Figure 6 and equations 10–12 describe the heat transfer and conduction model.

$$\begin{aligned} P &= \text{thermal power [Nm/s]} \\ C &= \text{heat capacity [Nm/K]} \\ \Lambda &= \text{heat conductivity [(Nm/s)/K]} \end{aligned} \tag{10}$$

$$
\begin{aligned}
P_{surface} &= \int_A \tau \cdot v \cdot dA = M \cdot \omega \\
P_{conduct} &= \Lambda \cdot \left(T_{disc} - T_{housing}\right)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
T_{oil} &= T_{disc} \\
\dot{T}_{disc} &= \left(P_{surface} - P_{conduct}\right) / C_{disc} \\
T_{housing} &= \text{constant}
\end{aligned}
\tag{12}
$$

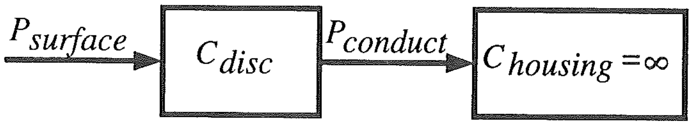


Figure 6: Model for heat accumulation and conduction

3.6 Torques

For all clutches, the torque can be separated into two parts:

$$
M = \int_A r \cdot \tau \cdot dA = \underbrace{\int_A r \cdot \tau_{dry} \cdot dA}_{M_{dry}} + \underbrace{\int_A r \cdot \tau_{visc} \cdot dA}_{M_{visc}} \tag{13}
$$

The velocity can be written as: $v = r \cdot \omega$. We consider the coefficient of friction and the viscosity as constant over the surface. Actually this is an approximation, since the coefficient of friction is supposed to be a function of sliding velocity, see figure 7. In case of high sliding velocities, the asperities will probably be covered by a thin oil film, which can be a reason for the decreasing coefficient of friction.

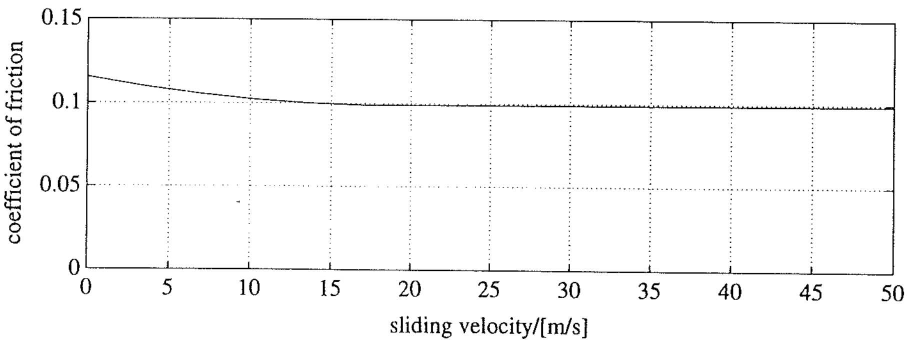


Figure 7: Coefficient of (dry) friction as function of velocity

Chapter 4: MULTI-DISC CLUTCH EQUATIONS

Figure 8 shows the design of a multi-disc clutch schematically. We assume rotational symmetry, plane discs and equal conditions in each pair of friction surfaces. This yields $h = h(t)$, $p_{oil} = p_{oil}(r, t)$ and $p_{cont} = p_{cont}(h) = p_{cont}(t)$.

The relationship between the velocities of the end discs, which is not dependent on the oil film thickness is:

$$\dot{\delta}_{left} - \dot{\delta}_{right} = -N \cdot \dot{h} \tag{14}$$

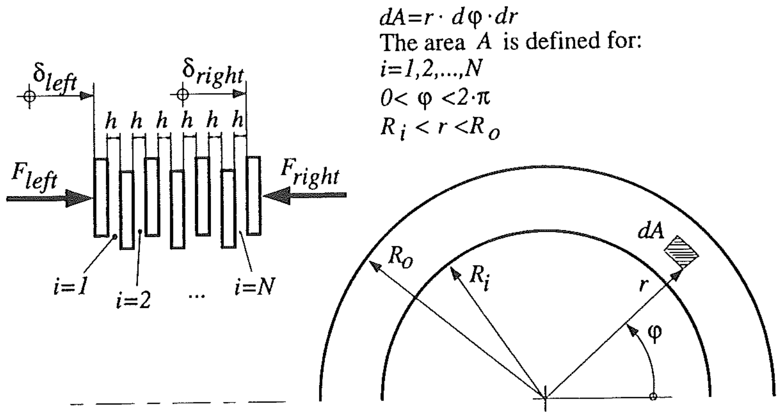


Figure 8: Multi-disc clutch

4.1 Clutch Equations for Smooth Discs

The dry friction torque can be derived from equations 3 and 13:

$$M_{dry} = \mu \cdot \sum_{i=1}^{N} \int_{r=R_i}^{R_o} \int_{\varphi=0}^{2\cdot\pi} p_{cont} \cdot r^2 \cdot d\varphi \cdot dr = \mu \cdot N \cdot \frac{R_o^3 - R_i^3}{3} \cdot 2 \cdot \pi \cdot p_{cont} \tag{15}$$

Force equilibrium for the discs makes:

$$\begin{aligned} F_{left} = F_{right} &= \int_{r=R_i}^{R_o} \int_{\varphi=0}^{2\cdot\pi} (p_{oil} + p_{cont}) \cdot r \cdot d\varphi \cdot dr = \\ &= \int_{r=R_i}^{R_o} \int_{\varphi=0}^{2\cdot\pi} p_{oil} \cdot r \cdot d\varphi \cdot dr + \underbrace{p_{cont} \cdot \pi \cdot \left(R_o^2 - R_i^2\right)}_{F_{cont}} \end{aligned} \tag{16}$$

No Asperity Contact ($h > H_{asp}/2$)

The viscous torque can be derived from equations 2 and 13:

$$M_{visc} = \eta \cdot \omega \cdot \sum_{i=1}^{N} \int_{r=R_i}^{R_o} \int_{\varphi=0}^{2\cdot\pi} \frac{r^3}{h} \cdot d\varphi \cdot dr = \eta \cdot \omega \cdot N \cdot \frac{R_o^4 - R_i^4}{4} \cdot 2 \cdot \pi / h \tag{17}$$

Reynolds' equation in polar coordinates with rotational symmetry reads:

$$\frac{\partial^2 p_{oil}}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial p_{oil}}{\partial r} = \frac{12 \cdot \eta}{h^3} \cdot \dot{h} \quad ; \text{where } \dot{h} = \frac{dh}{dt} = \frac{\partial h}{\partial t} \tag{18}$$

Boundary conditions ($p_{oil}(R_i,t) = p_{oil}(R_o,t) = 0$) gives:

$$p_{oil} = \frac{3\cdot\eta}{h^3}\cdot\left[r^2 + \frac{R_o^2\cdot\ln(r/R_i) - R_i^2\cdot\ln(r/R_o)}{\ln(R_i/R_o)}\right]\cdot\dot{h} \tag{19}$$

Insertion into equation 16 and integration gives:

$$\begin{aligned}\dot{h} &= -C_1\cdot h^3\cdot F_{oil}/\eta\\ \text{where: } F_{oil} &= F_{left} - F_{cont}\\ \text{and: } C_1 &= \frac{2/(3\cdot\pi)}{R_o^4 - R_i^4 - \left(R_o^2 - R_i^2\right)^2/\ln(R_o/R_i)}\end{aligned} \tag{20}$$

Asperity Contact ($h < H_{asp}/2$)

The viscous torque can be derived from equations 3 and 13:

$$\begin{aligned}M_{visc} &= \eta\cdot\omega\cdot\sum_{i=1}^{N}\int_{r=R_i}^{R_o}\int_{\varphi=0}^{2\cdot\pi}\frac{2}{H_{asp}}\cdot\frac{1-\left(\frac{1}{2}-\frac{h}{H_{asp}}\right)^2}{\frac{1}{2}+\frac{h}{H_{asp}}}\cdot r^3\cdot d\varphi\cdot dr =\\ &= \eta\cdot\omega\cdot N\cdot\frac{2}{H_{asp}}\cdot\frac{1-\left(\frac{1}{2}-\frac{h}{H_{asp}}\right)^2}{\frac{1}{2}+\frac{h}{H_{asp}}}\cdot\frac{R_o^4 - R_i^4}{4}\cdot 2\cdot\pi\end{aligned} \tag{21}$$

According to equation 6, equation 20 can be adopted until asperity contact. After that:

$$\dot{h} = -C_3\cdot\left(h + \frac{H_{asp}}{2}\right)^2\cdot F_{oil}/\eta \ \text{; where: } C_3 = \frac{H_{asp}}{8}\cdot C_1 \tag{22}$$

4.2 Clutch Equations for Grooved Discs

The dry friction torque equation will be almost like equation 15:

$$M_{dry} = \mu\cdot N\cdot\frac{R_o^3 - R_i^3}{3}\cdot 2\cdot\pi\cdot p_{cont,eff} \tag{23}$$

The switch between thick and thin oil films (see section 3.4) is carried out when $h = H_{switch}$. At the end of this section, an equation for H_{switch} is derived. Furthermore, it is assumed that $H_{switch} > H_{asp}/2$, in which case we do not have to treat the case with thick oil film and asperity contact. It is also assumed that $H_{gro} > H_{asp}$. This means that asperity contact in the grooved area does not have to be considered. In the following, *asperity contact* means *asperity contact in the non-grooved area, but not in the grooved area.*

No Asperity Contact ($h > H_{asp}/2$)

The viscous torque can be derived from equations 2 and 13 and introducing the area fraction, θ:

$$M_{visc} = \eta\cdot\omega\cdot N\cdot\frac{R_o^4 - R_i^4}{4}\cdot 2\cdot\pi\cdot\left[\frac{\theta}{h} + \frac{1-\theta}{h_{gro}}\right] \tag{24}$$

Thick Oil Film ($h > H_{switch}$)

Equation 20 can be transferred to the grooved case, if the effective oil film thickness, h_{eff}, is introduced:

$$\begin{gathered}\dot{h} = -C_1 \cdot h_{eff}^3 \cdot F_{oil}/\eta \\ \text{where: } F_{oil} = F_{left} - F_{cont,eff} = F_{left} - p_{cont,eff} \cdot \pi \cdot \left(R_o^2 - R_i^2\right)\end{gathered} \tag{25}$$

Thin Oil Film ($h < H_{switch}$)

However, for **thin** oil film, the squeeze equation must be derived again. Reynolds' equation should now be solved under smaller plates, the non-grooved areas. A solution from [8] is used. This reference treats squeeze of the oil film under rectangular plates with the sides L_{short} and L_{long}. We consider the friction surface to be built up by N_{plates} such rectangular plates. In order to maintain the area, we should determine N_{plates} as $N_{plates} = \theta \cdot \pi \cdot \left(R_o^2 - R_i^2\right)/\left(L_{short} \cdot L_{long}\right)$. Furthermore, [8] does not include a sliding velocity of the surfaces, but when the oil film thickness is constant over the whole surface, this will not influence the solution. Then, according to [8]:

$$\begin{gathered}\dot{h} = -C_2 \cdot h^3 \cdot F_{oil}/\eta \\ \text{where: } C_2 = \frac{1}{L_{long}^3 \cdot L_{short} \cdot g \cdot N_{plates}} \\ \text{where: } g = g\left(L_{short}/L_{long}\right)\end{gathered} \tag{26}$$

The function g is given in diagram in [8].

Asperity Contact ($h < H_{asp}/2$)

The viscous torque can be derived from equations 13, 2 and 3 and introducing the area fraction θ:

$$M_{visc} = \eta \cdot \omega \cdot N \cdot \frac{R_o^4 - R_i^4}{4} \cdot 2 \cdot \pi \left[\theta \cdot \frac{2}{H_{asp}} \cdot \frac{1 - \left(\frac{1}{2} - \frac{h}{H_{asp}}\right)^2}{\frac{1}{2} + \frac{h}{H_{asp}}} + \frac{1-\theta}{h_{gro}}\right] \tag{27}$$

Thin Oil Film ($h < H_{switch}$)

According to equation 6, equation 26 can be adapted to asperity contact:

$$\dot{h} = -C_4 \cdot h^3 \cdot F_{oil}/\eta \text{ ; where: } C_4 = \frac{H_{asp}}{8} \cdot C_2 \tag{28}$$

Condition for Switch between Thin and Thick Oil Film

The condition for the switch between **thick** and **thin** film formulas can be expressed as $h = H_{switch}$:

$$\left|\dot{h}_{thick}\right| = \left|\dot{h}_{thin}\right| \Leftrightarrow C_1 \cdot h_{eff}^3 = C_2 \cdot h^3 \Leftrightarrow h = H_{switch} = \frac{(1-\theta) \cdot H_{gro}}{\sqrt[3]{C_2/C_1} - 1} \tag{29}$$

Chapter 5: Multi-disc Clutch System Equations

With the equations derived, we can simulate an engagement if the actuating force is known as a direct function of time. However, in many applications we do not know it in that form. In the actuating system described in figure 9, we rather know the hydraulic pressure, p_{acc}, before an orifice as a direct function of time. The *elasticity in discs* corresponds to both elastic deformations in the friction lining (not the asperities) and in the discs themselves. In addition to what the figure shows, there is a flywheel connected to the clutch shaft.

The component equations in table 1 are available. These equations constitute a system of differential and algebraic equations. It can be solved as an initial value problem with four state variables, e.g. $\{h, \delta_{right}, T_{oil}, \omega\}$. However, by certain conditions during the engagement, this would make the system numerically difficult to solve. It becomes a *stiff* system. The physical interpretation of this mathematical phenomenon is the behavior of the orifice damper and the squeeze damper. With a very tiny flow through the orifice the damping, or pressure difference over the orifice, is very small. With a very thick oil film or when the discs have landed, the damping, or resistance to the squeeze motion, is very low in the squeeze damper. If this is detected during the simulation, the damping is set to zero in these components. For the orifice this means $d_{turb} = d_{lam} = 0$ and for the squeeze damper $C_1 = C_2 = C_3 = C_4 = \infty$. This makes the system lose some state variables, e.g., in the end of the simulations we just use $\{T_{oil}, \omega\}$ as state variables.

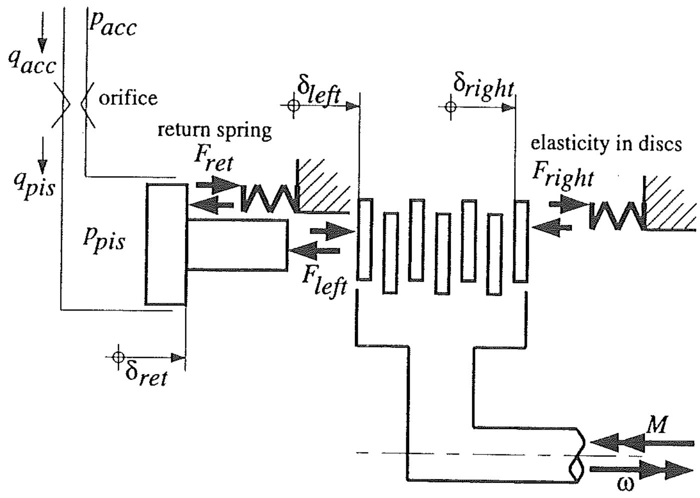


Figure 9: Multi-disc clutch with actuating device

Table 1: Component equations

COMPONENT	EQUATIONS
The **accumulator**, which controls the pressure as an explicit function of time	$p_{acc} = p_{acc}(t) =$ known function
The **orifice**, through which the flow is either laminar (subscript *lam*) or turbulent (subscript *turb*)	$p_{acc} - p_{pis} = \mathrm{sign}(q_{acc}) \cdot$ $\cdot \max\left\{(d_{lam} \cdot \eta \cdot \lvert q_{acc} \rvert); (d_{turb} \cdot q_{acc}^2)\right\}$ $q_{acc} = q_{pis}$
The **piston** (force equilibrium and kinematic relation)	$p_{pis} \cdot A_{pis} = F_{ret} + F_{left}$ $q_{pis} = \dot{\delta}_{ret} \cdot A_{pis}$ $\delta_{ret} = \delta_{left}$
The **return spring**	$F_{ret} = F_{ret}(\delta_{ret}) =$ known function
The **clutch**	Equations from chapter 4, e.g., if smooth discs, $h < H_{switch}$ and no asperity contact: equations 14, 15, 17 and 20
The **viscosity and temperature** model	Equations 9, 11 and 12
The **elasticty in discs**	$F_{right} = F_{right}(\delta_{right}) =$ known function
The **flywheel**	$J \cdot \dot{\omega} = 0 - M$

Chapter 6: Simulations

The simulation results in figure 10 are from a simulation of engagement of a radially grooved multi-disc clutch. Data are presented in table 2.

Table 2: Data for the simulation

$R_i = 60$ mm $R_o = 71.5$ mm $N = 6$ $A_{pis} = 4750$ mm^2	$d_{lam} = 2.00 \cdot 10^9$ m^{-3} $d_{turb} = 1.22 \cdot 10^{14}$ Ns^2/m^8
$\theta = 0.697$ $H_{gro} = 0,15$ mm $L_{long} = R_o - R_i$ $L_{short} = 6$ mm $H_{switch} = 0.046$ mm $N_{plates} = 48$ $g = 0.1830$	0 N $< F_{ret} <$ 500 N (approximate values, return spring is a degressive disc spring)
	$F_{right}/\delta_{right} = 2 \cdot 10^6$ N/m (elasticity in discs is a linear spring)
	$C_{disc} = 90.0$ Nm/K $\Lambda = 2.54$ (Nm/s)/K
$H_{asp} = 0.010$ mm $E_{asp} = 30 \cdot 10^6$ N/m^2	$\eta_0 = 158$ Ns/m^2 $\beta = 0.027$ K^{-1} $J = 0.07$ kgm^2

Figure 1 shows the corresponding test from [3]. A comparison between the test and simulation in figure 10 shows a good agreement. The torque curve disagrees slightly in shape. The simulated torque curve rises too steeply in the interval 10 Nm < M < 50 Nm and does not rise steeply enough in the interval 50 Nm < M < 70 Nm. The shape would probably agree better if the elasticity in discs had been modelled as a progressive spring, which is also very likely in practice.

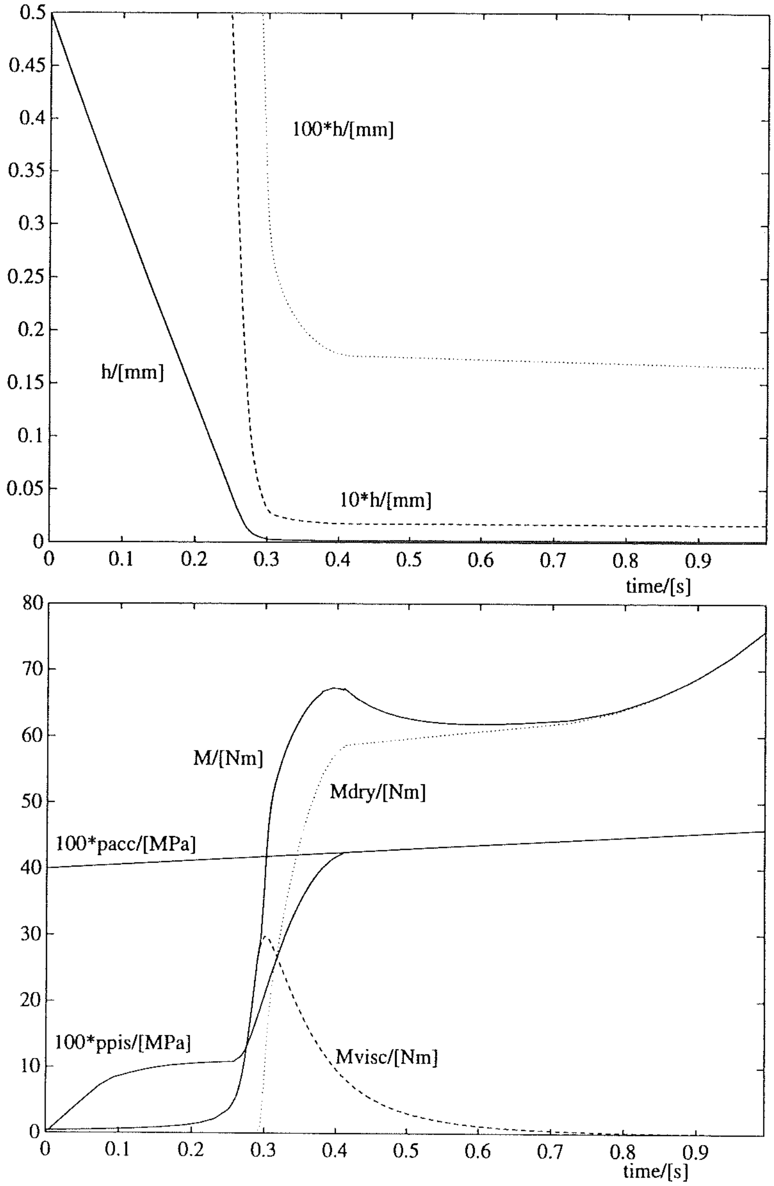


Figure 10: Simulation results of engagement of a multi-disc clutch. This is the reference case, which is a grooved clutch with 6 pairs of friction surfaces. (Continued . . .)

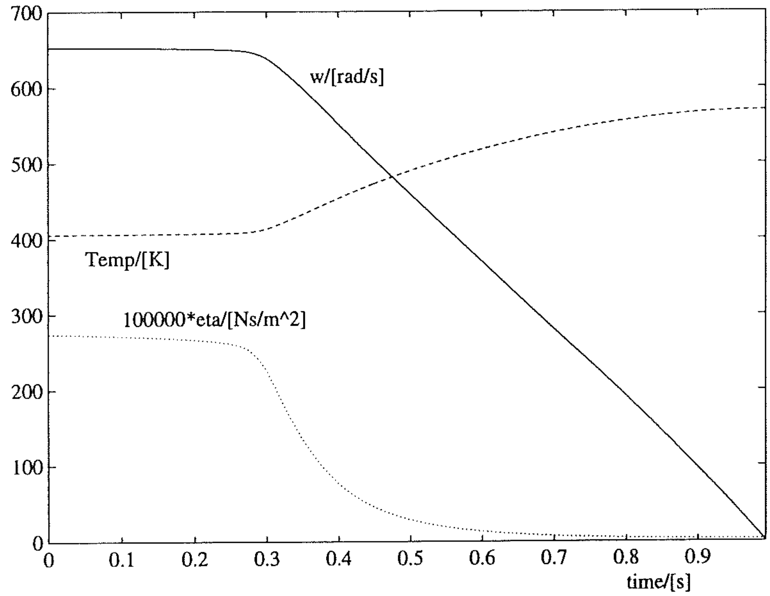


Figure 10: Simulation results of engagement of a multi-disc clutch. This is the reference case, which is a grooved clutch with 6 pairs of friction surfaces.

Figure 11 shows engagements of the same clutch, but with different pressure levels. [Both tests and simulations indicate a slightly smaller viscous torque for the lower pressure level, since the oil is then more heated. Therefore, it is likely that the influence of the pressure level is well described by the model.][1] However, there is certainly a need for some more tests and simulations to completely verify the model. It would be especially interesting to vary the initial velocity of the flywheel and number of discs and to test a clutch with smooth discs.

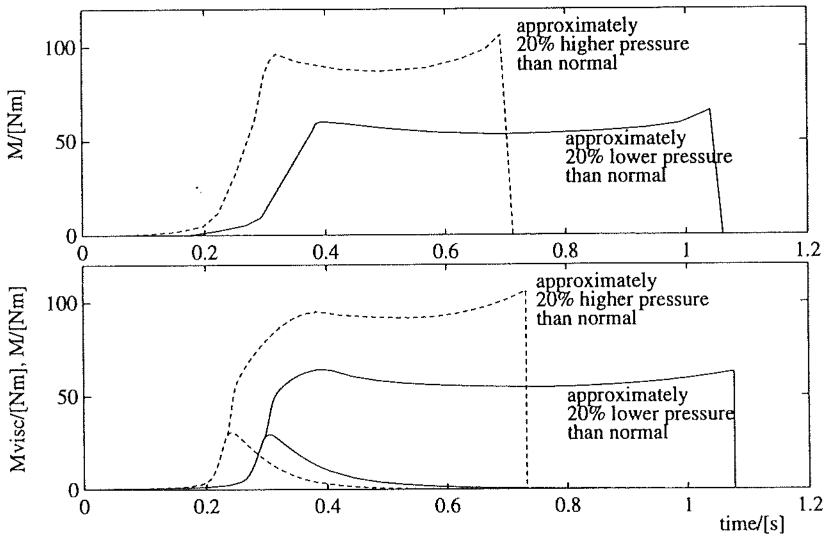


Figure 11: Test results (upper diagram) and simulation results (lower diagram) of engagement of grooved multi-disc clutch with 6 pairs of friction surfaces. Two different levels of the hydraulic pressure p_{acc}.

[1] See "Correction" at the end of this paper.

Figure 12 shows the simulation results for the same clutch but with only half the number of pairs of friction surfaces. In order to reach approximately the same torque, the accumulator pressure is doubled. Comparing this simulation with the simulation in figure 10 shows that the torque peak increases with the number of discs.

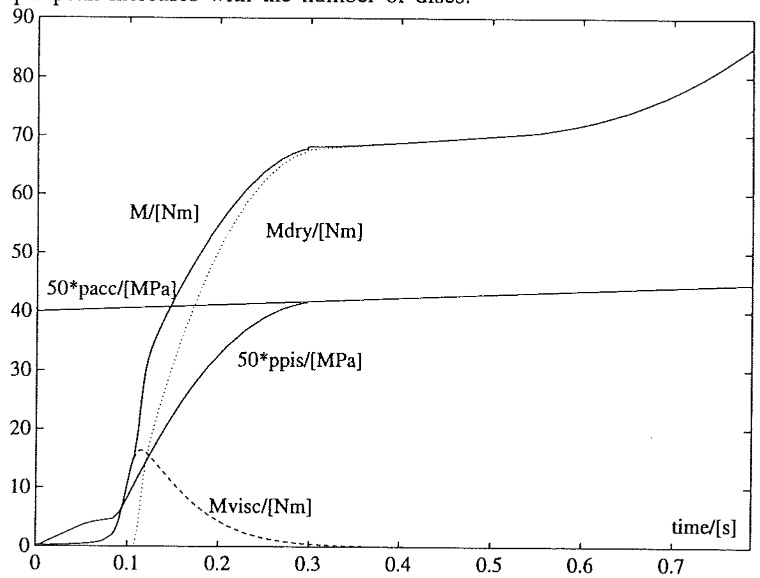


Figure 12: Simulation results of engagement of grooved multi-disc clutch with 3 pairs of friction surfaces

The simulation results in figure 13 show the same clutch but without grooves. Comparing this simulation with the simulation in figure 10 shows that the torque peak decreases if the friction surfaces are grooved.

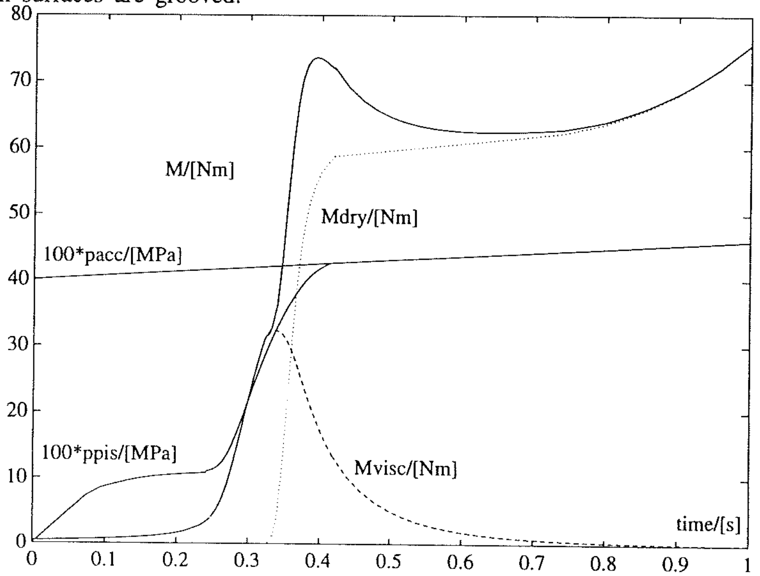


Figure 13: Simulation results of engagement of smooth multi-disc clutch with 6 pairs of friction surfaces

The model for squeeze flow at asperity contact is very simplified. However, the simulations become almost identical if the squeeze model derived without asperity contact is used during the whole simulation. The reason for this is that the asperities almost immediately take all the pressure when they reach contact. This leaves the squeeze damper almost inactive at asperity contact.

It is difficult to estimate a numerical value of the heat conductivity Λ in section 3.5. Anyway, with a reasonable value, the increase of T_{oil} during a simulation is found to be 163 K. Without conduction, the corresponding value is 166 K. The difference is negligible.

Furthermore, simulations have been carried out with the disc separated in more heat accumulating and conducting elements in series. These show a slightly (approximately 2 Nm) smaller peak value for the viscous torque, which is also negligible.

Chapter 7: CONCLUSIONS

Equations are proposed for simulation of the engagement of oil immersed multi-disc clutches. The asperities are treated as elastic springs, which gives a continuous transition between viscous and dry friction. Consideration is taken of the effects of grooved friction surfaces. Tests are compared with the simulations and show good agreement. However, more tests are needed to fully confirm the model.

Simulations show that the grooves in the surface reduce the influence of the viscous torque. They also show that this influence increases with increasing number of discs.

When the oil film is very thin, the oil has to be squeezed out in channels between the asperities. However, simulations shows that a squeeze model, derived without asperity contact, can be used during the whole simulation.

Appendix A: Squeeze in Channels between Asperities

The channels between the asperities are all supposed to have the same cross-section, shown in figure 14. The mean flow in a channel is governed by the following differential equation, based on volume continuity in the fluid:

$$\frac{dq}{dx} = \frac{-dA}{dt} = \frac{-d}{dt}\,\frac{\lambda\cdot\left(\frac{1}{2}+\frac{h}{H_{asp}}\right)\cdot\left(h+\frac{H_{asp}}{2}\right)}{2} \propto \left(\frac{1}{2}+\frac{h}{H_{asp}}\right)\cdot\dot{h} \qquad (36)$$

where q = flow and x = coordinate along the channel

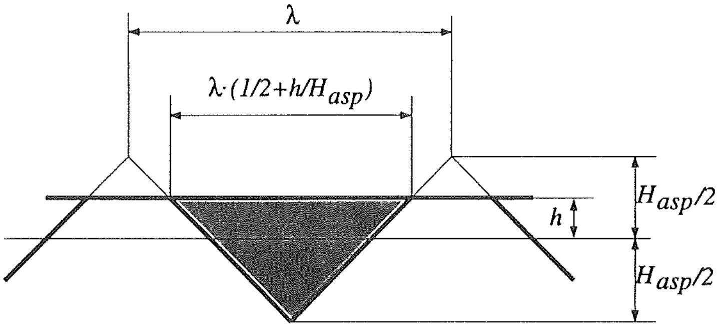


Figure 14: Cross-section of the channels. The material in the asperities is supposed to be fully compressible, which causes the side walls of the asperities not to move. Note that all length dimensions of the cross-section are proportional to $(1/2 + h/H_{asp})$.

Laminar flow through a circular pipe with radius proportional to $1/2 + h/H_{asp}$ gives another differential equation. That equation is based on force equilibrium and on the constitutive equation for a Newtonian liquid. The same equation can be derived from laminar flow in a rectangular pipe with width and height proportional to $1/2 + h/H_{asp}$ and the height much smaller than the width.

$$\frac{dp}{dx} \propto \frac{\eta \cdot q}{(1/2 + h/H_{asp})^4} \tag{37}$$

Equation 36 is solved along a pipe where the coordinate x is measured from a point with zero flow ($q = 0$ at $x = 0$):

$$q \propto \left(\frac{1}{2} + \frac{h}{H_{asp}}\right) \cdot \dot{h} \cdot x \tag{38}$$

Equation 37 is solved with the boundary condition $p = 0$ at $x = L$:

$$p \propto \frac{x^2 - L^2}{(1/2 + h/H_{asp})^3} \cdot \eta \cdot \dot{h} \tag{39}$$

The mean pressure in the oil, p_{oil}, is:

$$\begin{gathered} p_{oil} = \frac{\int_{x=0}^{L} p \cdot \lambda \cdot (1/2 + h/H_{asp}) \cdot dx}{\lambda \cdot L} \propto \frac{\eta \cdot \dot{h}}{(1/2 + h/H_{asp})^2} \Rightarrow \\ \Rightarrow \dot{h} \propto p_{oil} \cdot \left(\frac{1}{2} + \frac{h}{H_{asp}}\right)^2 / \eta \end{gathered} \tag{40}$$

This proportionality can also be derived for a mesh of channels, built up of one long main channel with short side channels, as shown in figure 15. The squeeze in any macro-geometry should be thought of as built up of such meshes. An example of macro-geometry is given in figure 16.

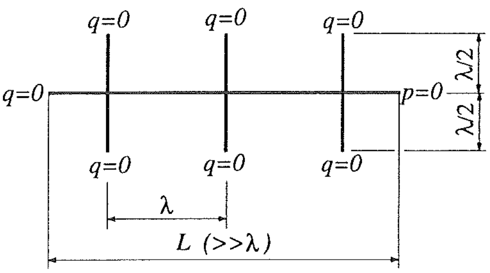


Figure 15: A mesh of channels. Since $\lambda << L$, there are much more side channels than drawn in the sketch.

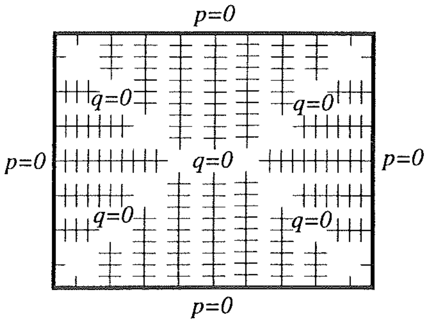


Figure 16: An example of macro-geometry, a rectangle in which a mesh of channels is drawn. At the boundary of the rectangle the pressure is zero and in the inner ends of the meshed channels the flow is zero.

BIBLIOGRAPHY

[1] JACOBSON, Bengt. *Vehicle Driveline Mechanics during Powershifting.* Report No 1990-05-30 (Licentiate of Engineering thesis), Machine and Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1990.

[2] JACOBSON, Bengt. Analysis of Shift Operations in Automatic Transmissions. *3rd International EAEC Conference on Vehicle Dynamics and Powertrain Engineering, Strasbourg, France,* pages 195–202, June 11-13 1991.

[3] HENRIKSSON, Magnus. *Measurements of the friction characteristics of a multi-disc brake in an automatic transmission.* Machine and Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1989. (Master degree thesis, in swedish).

[4] WU, Hai. The Squeeze Film Between Rotating Porous Annular Disks. *Wear,* pages 461–470, Number 18, 1971.

[5] WU, Hai. An Analysis of the Engagement of Wet-clutch Plates. *Wear,* pages 23–33, Number 24, 1973.

[6] TING, L.L. Engagement Behavior of Lubricated Porous Annular Disks. *Wear,* pages 159–182, Number 34, 1975.

[7] BARDZIMASHVILI, N.G. and YASHVILI, S.G. Study of Multidisc Oil Friction Clutch Engagement. *Sov J Frict Wear,* 8(2):57–64, 1987.

[8] HAYS, F. Donald. Squeeze Films for Rectangular Plates. *Journal of Basic Engineering,* pages 243–246, June 1963.

Correction

March 24, 1993

The sentences within brackets in the text on page E14 are inadequately formulated. They should be replaced with the following:

In common for tests and simulations in figure 11 is that the torque peak is more dominating for small pressure levels. The most likely explanation is that the viscous torque is about the same size independent of the pressure level, and so it will dominate when the dry friction torque is small, i.e. for small pressure levels.

Paper F

JACOBSON, BENGT: Influence of Oil Film in the Clutches on Gear Shifts in Automatic Vehicle Transmissions, accepted for presentation at the *4th International EAEC Conference on Vehicle and Traffic Systems Technology*, Strasbourg, France, June 16–18, 1993

Influence of Oil Film in the Clutches on Gear Shifts in Automatic Vehicle Transmissions

Bengt Jacobson

Machine and Vehicle Design
Chalmers University of Technology
S-412 96 Göteborg, Sweden

ABSTRACT

In automatic vehicle transmissions, shift operations are carried out by engaging and disengaging oil-immersed clutches (including brakes). Thereby, the transmission can be made powershifting (no interruption of the power during the gear shift). It is desirable not to drop the **power supply** to the vehicle. The shift operations are studied in order to estimate this power supply, along with **wear** of clutches and passenger **comfort**. Analytic models are useful tools in such estimations. In this paper a model is briefly described. The influence of the oil film between the friction surfaces in the clutches is studied by means of simulations.

The oil film gives a viscous torque, which is added to the dry friction torque. Also, the dry friction torque is delayed because the surfaces cannot reach contact before the oil film has been squeezed away. It is found that these effects can influence the gear shift quality considerably. First, the power supply can be disturbed, because the timing between the disengaging and engaging clutches is disturbed. This can be eliminated by designing the clutch of the lower gear as a one-way clutch. Second, the wear and comfort are affected.

Keywords: automatic transmission, powershift, gear shift, shift quality, comfort, wear, clutch, wet clutch, oil-immersed clutch, model, dynamic, simulation

Chapter 1: INTRODUCTION

Models for simulation of powershifts can be found, for instance in references [1], [2], [3] and [4]. The first two references mainly discuss different mechanical phenomena during powershifts. The latter two mainly discuss the modelling technique itself. There are also many later references that describe models for specific gearboxes or the gearbox control system. However, the basic mechanics in these are not emphasized. In all references mentioned so far is the clutches are modelled with pure dry friction characteristics.

Comparisons of tests and simulations have been made. They show that the dry friction characteristics cannot sufficiently describe the behavior of oil-immersed clutches (including brakes) during engagement. This is noted in reference [3]. In reference [5], a closer study of an engaging oil-immersed multi-disc clutch is carried out. The mismatch between tests and simulations is explained by a viscous torque developed in the oil film. In the present paper, simulations of gear shifts, with and without this viscous torque, are compared.

Chapter 2: SYSTEM DESCRIPTION

The physical system can be described by five subsystems: *control*, *clutch*, *gearbox*, *driveline* and *vehicle*. The models of these are described in appendices A and B. Schematically, the interaction between the subsystems can be described as figure 1 shows.

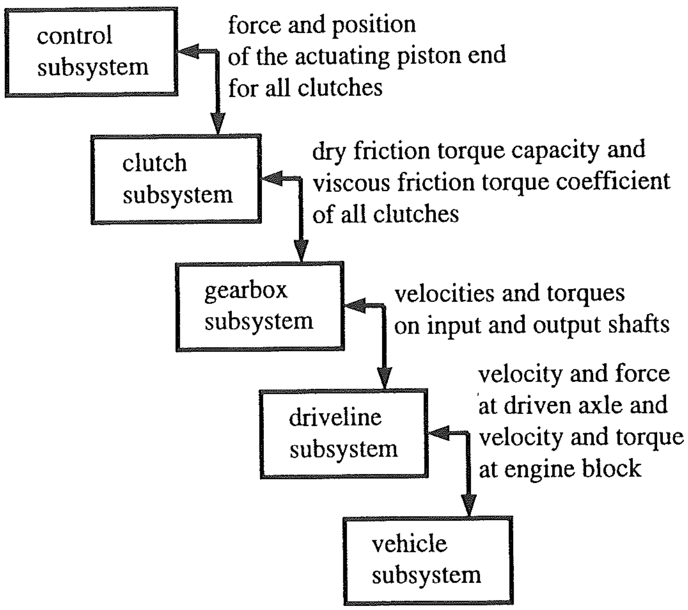


Figure 1: The interaction between the five subsystems

Chapter 3: CLUTCH CHARACTERISTICS

First, an engaging clutch with just dry friction characteristics is studied. The following working principles can be sketched:

1. The interaction between control and clutch subsystems can be expressed in the force of the actuating piston end, F_{act}. (The position of the piston end, δ_{act}, is not needed when the clutch has pure dry friction characteristics.)
2. In the clutch subsystem, F_{act} is interpreted as the contact force, $F_{cont} = F_{act}$. The dry friction torque capacity, c, is calculated from the coefficient of friction, μ, and F_{cont}. In the case of a single-disc clutch this is: $c = \mu \cdot F_{cont} \cdot radius$.
3. The clutch and gearbox subsystems interact through the clutch characteristics, expressed in *dry friction torque capacity*, denoted by c. The gearbox uses c to express the clutch (relative) velocity, ω, or the clutch torque, M, as shown in equation 1 and figure 2. The phase, ph, is an integer variable, used for distinguishing between stick and slip phases.

$$
\begin{cases} M = -c & ; \text{if } ph = -1 \\ \omega = 0 & ; \text{if } ph = 0 \\ M = +c & ; \text{if } ph = +1 \end{cases}
$$

$$
\begin{cases} ph \text{ switches from } -1 \text{ to } 0 & ; \text{if } \omega \text{ would tend to become } > 0 \\ ph \text{ switches from } 0 \text{ to } -1 & ; \text{if } M \text{ would tend to become } < -c \\ ph \text{ switches from } 0 \text{ to } +1 & ; \text{if } M \text{ would tend to become } > c \\ ph \text{ switches from } +1 \text{ to } 0 & ; \text{if } \omega \text{ would tend to become } < 0 \end{cases} \qquad (1)
$$

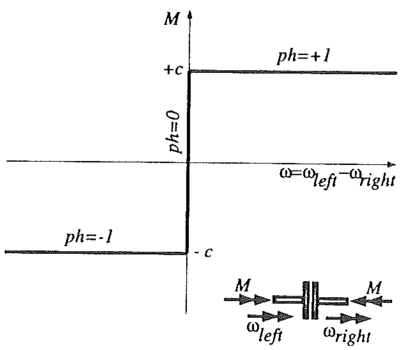


Figure 2: Characteristics of a clutch, modelled with pure dry friction

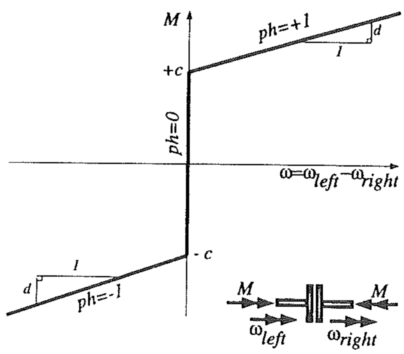


Figure 3: Characteristics of a clutch, modelled with both dry and viscous friction

Normally, c can be calculated algebraically from a measurable physical quantity, often the hydraulic pressure, p, in an actuating cylinder. The conventional way to simulate gear shifts in automatic transmissions is to use these dry friction characteristics, with c calculated algebraically from a measured p.

Now, introduce an oil film. The working principles then become as follows:

1. The control and clutch subsystem interact with the force and position of the actuating piston end, F_{act} and δ_{act}, respectively.
2. The clutch subsystem converts δ_{act} to an oil film thickness, h. Then, h corresponds to a certain contact force, F_{cont}, through a constitutive relation for the friction surface asperities. Furthermore, $F_{cont} = 0$ until the asperities have reach contact. The difference $F_{act} - F_{cont} = F_{oil}$ is the force carried by the oil film. It squeezes out the oil, i.e. controls how h decreases during the engagement. The dry friction torque capacity, c, is calculated as in the case without oil film. The *viscous friction torque coefficient* is defined as $d =$ viscous torque/relative velocity. It can be calculated from oil viscosity, η, and h. For a single-disc clutch, this is: $d = \eta \cdot radius^2 \cdot area/h$.
3. The clutch characteristics, c and d, are used in the gearbox in the following way, which is also shown in figure 3:

$$
\begin{gathered}
M = M_{visc} + M_{dry} \\
M_{visc} = d \cdot \omega \\
\begin{cases} M_{dry} = -c & ; \text{if } ph = -1 \\ \omega = 0 & ; \text{if } ph = 0 \\ M_{dry} = +c & ; \text{if } ph = +1 \end{cases} \\
\begin{cases} ph \text{ switches from } -1 \text{ to } 0 & ; \text{if } \omega \text{ would tend to become } > 0 \\ ph \text{ switches from } 0 \text{ to } -1 & ; \text{if } M_{dry} \text{ would tend to become } < -c \\ ph \text{ switches from } 0 \text{ to } +1 & ; \text{if } M_{dry} \text{ would tend to become } > c \\ ph \text{ switches from } +1 \text{ to } 0 & ; \text{if } \omega \text{ would tend to become } < 0 \end{cases}
\end{gathered}
\tag{2}
$$

In accordance with figures 4 and 5, the effects of the oil film will now be discussed. First, it prevents the friction surfaces from reaching contact, which delays the dry friction torque relative to the pressure. Secondly, a viscous torque is developed in the oil film. This torque can start to rise slightly before the pressure. In conclusion, this might result in either a **torque peak** or a **time delay** in clutch torque relative to the pressure[1)]. Experiments in reference [3] indicates the form of the curves in figure 4 and 5. The analysis in reference [5] also verifies the curves in figure 4.

1) The torque peak can be seen as a "too quick" torque response, i.e. a **negative time delay** relative to the pressure.

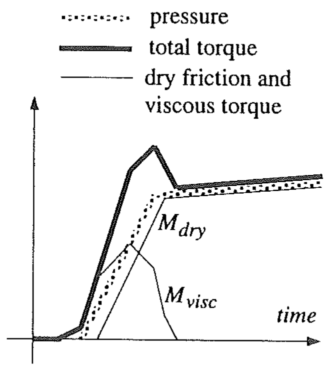


Figure 4: Engagement of a multi-disc clutch (short delay of M_{dry} relative to p and large M_{visc} gives a torque peak in M)

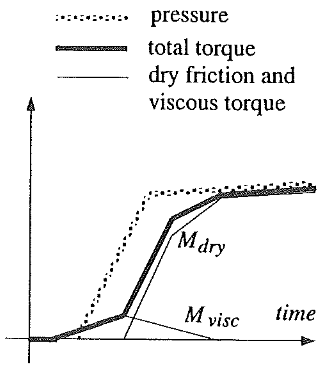


Figure 5: Engagement of a band brake (long delay of M_{dry} relative to p and small M_{visc} gives a time delay of M relative to p)

Chapter 4: SIMULATIONS

The model used for simulations in this paper does not handle the viscous torque strictly as presented in chapter 3. The model actually has neither control nor clutch subsystems. Furthermore, the gearbox subsystem only models the clutches with dry friction, i.e. the input data is $c(t)$ for each clutch. The viscous friction torque coefficient is taken into account simply by using a modified $c(t)$. The modification is relative to a reference case, which has $c(t)$ corresponding to a measured $p(t)$. As long as the clutch slips, this is similar to prescribing the clutch torque itself. Therefore, the solutions in gearbox, driveline and vehicle subsystems are correct, but we do not know exactly to which behavior of the clutch the solutions correspond. So, in spite of simplification, the model used is appropriate to study the oil film influence on the shift quality in principle.

An upshift from a torque split gear to a lock-up gear is studied below. The disengaging clutch is modelled in two ways: as a one-way clutch (figures 6–8) and as a controlled clutch (figures 9–11). For both types of disengaging clutch, one reference case and two modified cases of oil film influence in the engaging clutch are studied. The modifications are made in correspondence with the peak and delay cases presented in chapter 3. The torque capacity as a function of time of the disengaging, controlled clutch is chosen so that almost perfect timing is achieved in the reference case.

Shift quality is discussed in terms of retained **power supply** to the vehicle, **life** of the clutches and passenger **comfort**.:

- ☐ The **power supply** to the vehicle is measured as the traction force, because the velocity is quite constant during the shift. The traction force has to vary between the two levels corresponding to the two gears. Further dips in traction force are regarded as failures.
- ☐ The **life** of the clutches is supposed to be determined by the wear, which is measured as the energy loss in the engaging clutch. (The disengaging clutch has hardly any energy losses at all.) This energy loss is a measure of the temperature rise in the clutch. The temperature is the main problem, not the abrasive wear.
- ☐ Exactly how to estimate passenger **comfort** is questionable, and beyond the scope of this paper. However, a subjective estimation of the variation of passenger acceleration is used here. (Changes in acceleration can be caused by dips in traction force, which is why power supply and comfort are very closely related.)

The simulation results are plotted in figures 6–11. Notations are explained in table 2. A comparison is made in table 1.

Table 1: Comparison between simulations.
Torque peak is likely for a multi-disc clutch with many discs. Time delay is likely for a band brake.
Tie-up is a state, occurring when the clutches are engaged too hard and *flare* occurs when the clutches are too loosely engaged. The engine chokes or overspeeds, respectively.

<table>
<tr><th rowspan="2">clutch of the lower gear</th><th rowspan="2">clutch of the higher gear</th><th colspan="3">comparison to the reference case</th></tr>
<tr><th>power supply to the vehicle</th><th>life of the engaging clutch</th><th>passenger comfort</th></tr>
<tr><td rowspan="2">one-way clutch (always gives perfect timing)</td><td>clutch with torque peak</td><td rowspan="2">as reference case (as good as possible thanks to perfect timing)</td><td>good</td><td>bad</td></tr>
<tr><td>clutch with time delay</td><td>bad</td><td>good</td></tr>
<tr><td rowspan="2">controlled clutch</td><td>clutch with torque peak</td><td>bad (because of tie-up)</td><td>good</td><td>bad</td></tr>
<tr><td>clutch with time delay</td><td>bad (because of flare)</td><td>bad</td><td>good</td></tr>
</table>

Table 2: Notations in figures 6–11

CLO, CHI	dry friction capacity of clutch for lower and higher gear, respectively
PHLO, PHHI	phase in clutch for lower and higher gear, respectively
MLO, MHI	torque in clutch for lower and higher gear, respectively
WENG, FVEH	engine shaft velocity and traction force on vehicle, respectively
LOSS, ACC	loss energy in engaging clutch and acceleration of passenger, respectively

Chapter 5: CONCLUSIONS

The oil film in the clutches can give either a peak in clutch torque or a time delay from actuating force (normally measured as pressure in the hydraulic actuating cylinder) to clutch torque. Multi-disc clutches with many discs are likely to give torque peaks. Band brakes seem to give delays.

Analytic comparisons are made with a reference case without any oil film effects. In both the torque peak case and the time delay case, the timing between the engaging and disengaging clutch is disturbed. This causes the power supply to the vehicle to drop unnecessarily much. However, in a shift where the clutch of the lower gear is designed as a one-way clutch, the timing problem is not present. It is also found that wear of the clutches decreases in the case of a torque peak, but the comfort is also worse. With a delay, it is the other way around.

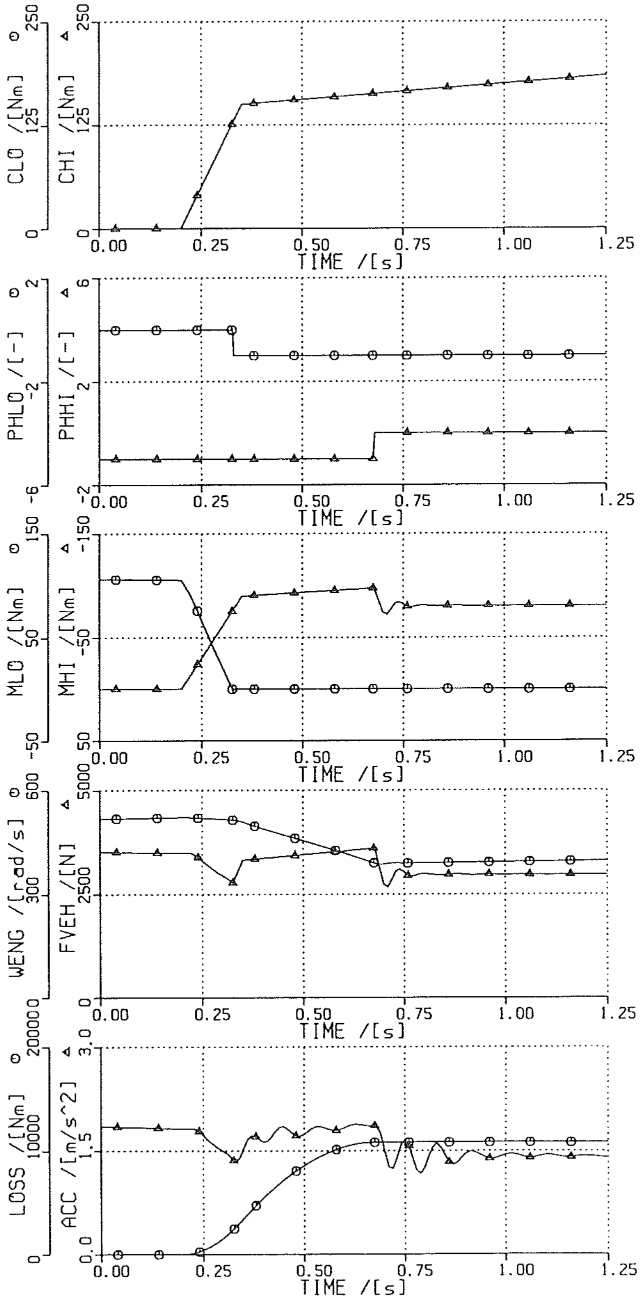


Figure 6: Simulation results for one-way clutch on the disengaging gear — reference case, i.e. without viscous torque in the engaging clutch. Likely to be a good model when the engaging clutch is a multi-disc clutch with few discs

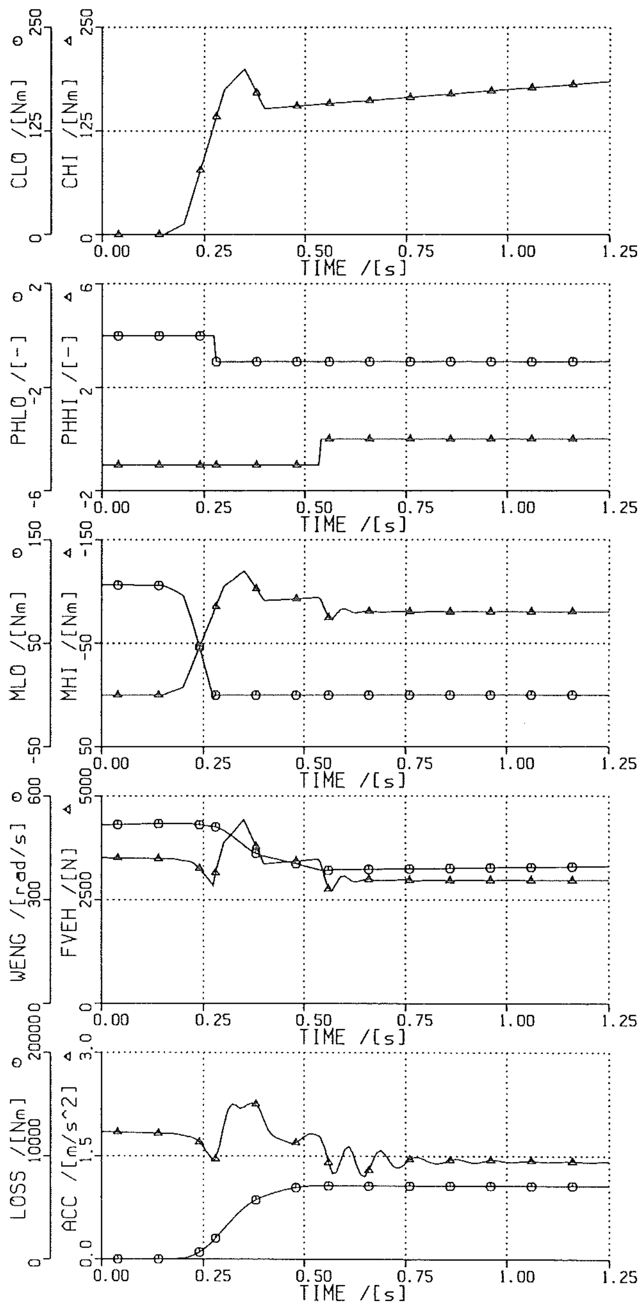


Figure 7: Simulation results for one-way clutch on the disengaging gear — torque peak case. Likely to be a good model when the engaging clutch is a multi-disc clutch with many discs

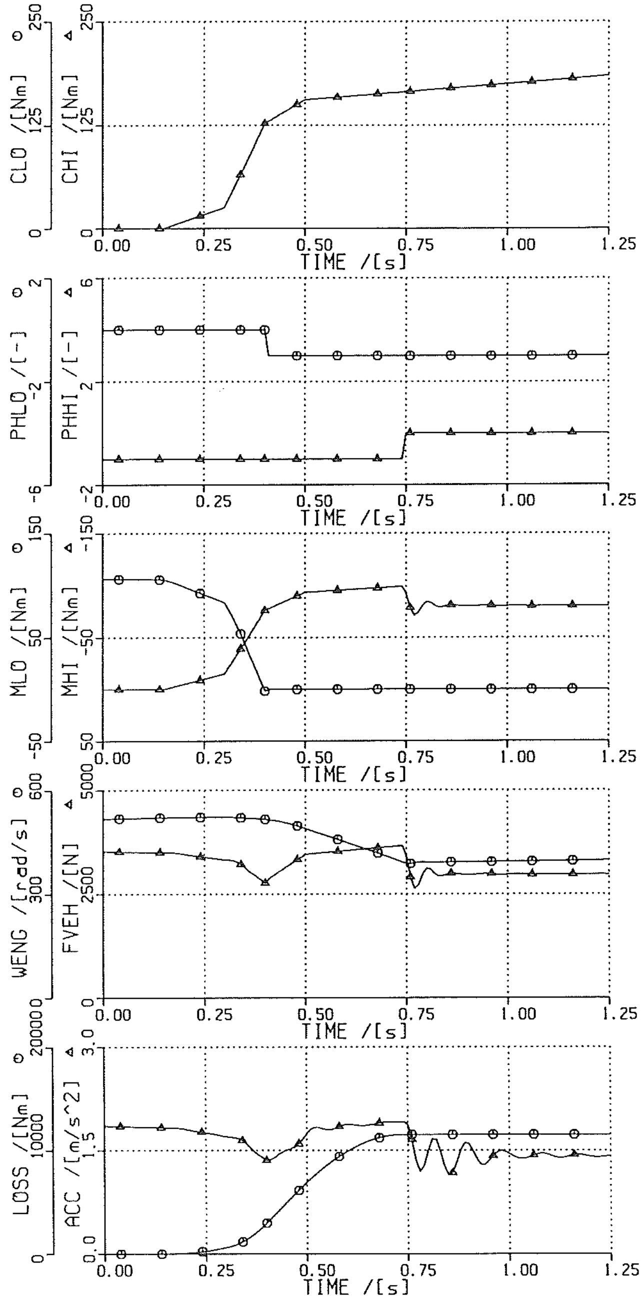


Figure 8: Simulation results for one-way clutch on the disengaging gear — time delay case. Likely to be a good model when the engaging clutch is a band brake

F11

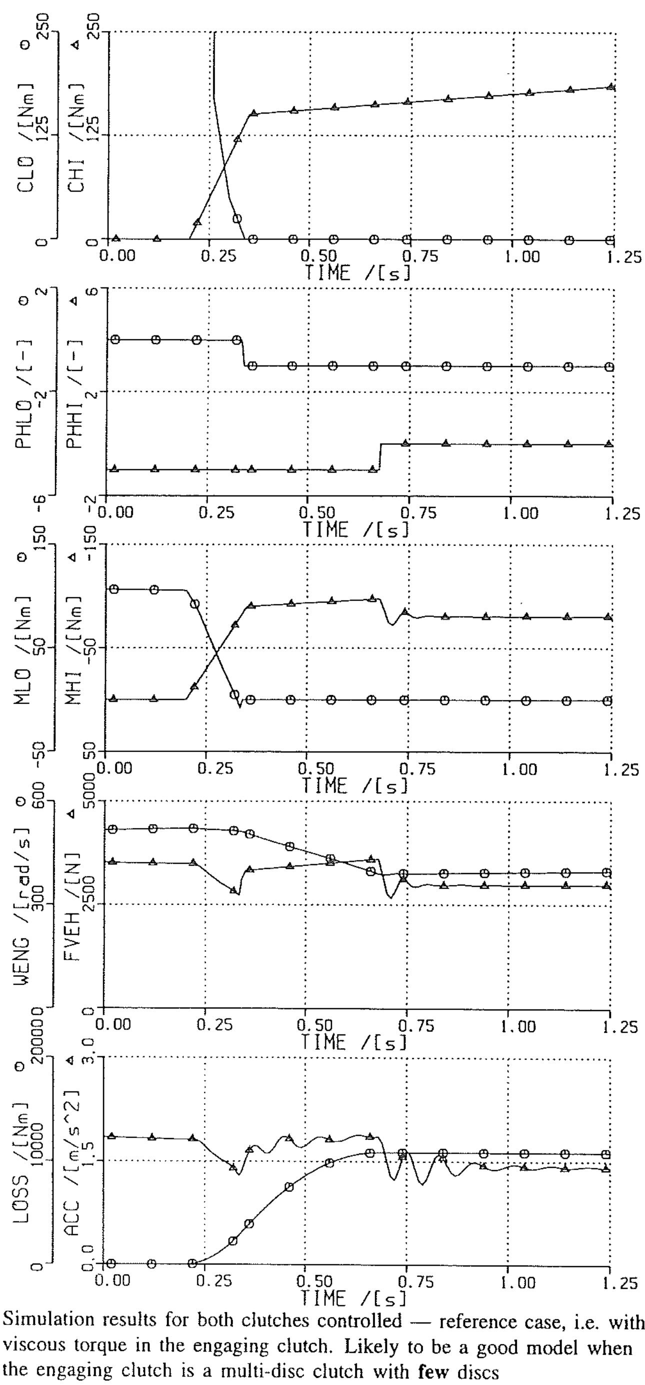


Figure 9: Simulation results for both clutches controlled — reference case, i.e. without viscous torque in the engaging clutch. Likely to be a good model when the engaging clutch is a multi-disc clutch with **few** discs

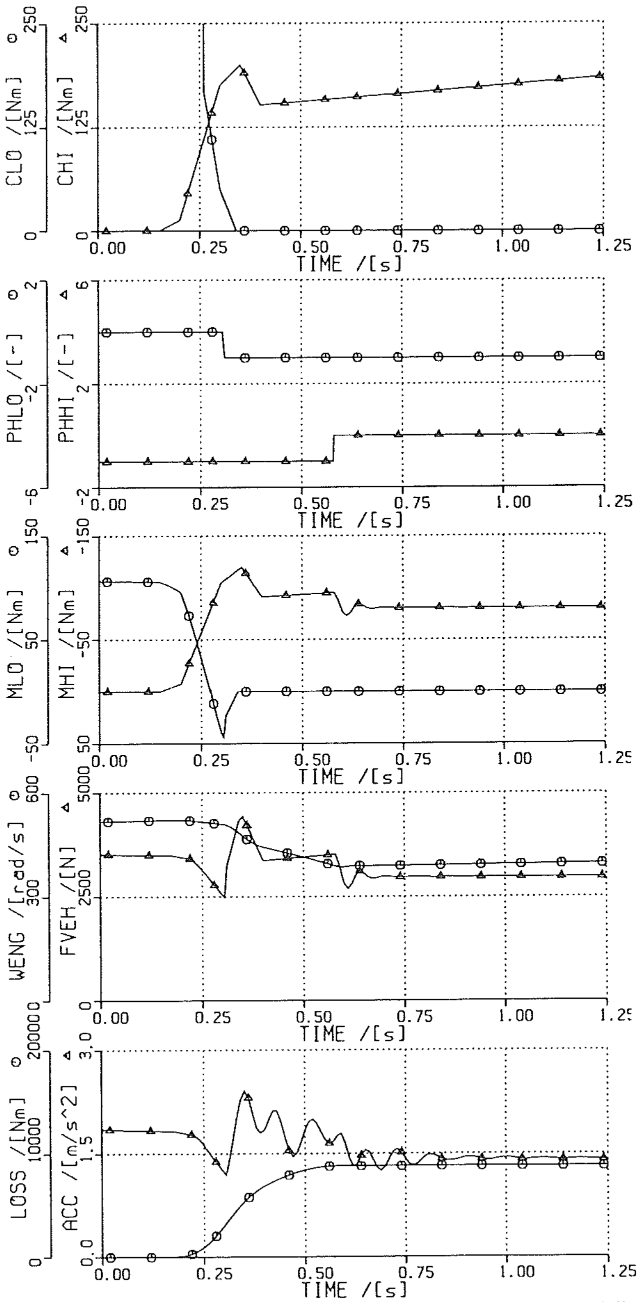


Figure 10: Simulation results for both clutches controlled — torque peak case. Likely to be a good model when the engaging clutch is a multi-disc clutch with many discs

F13

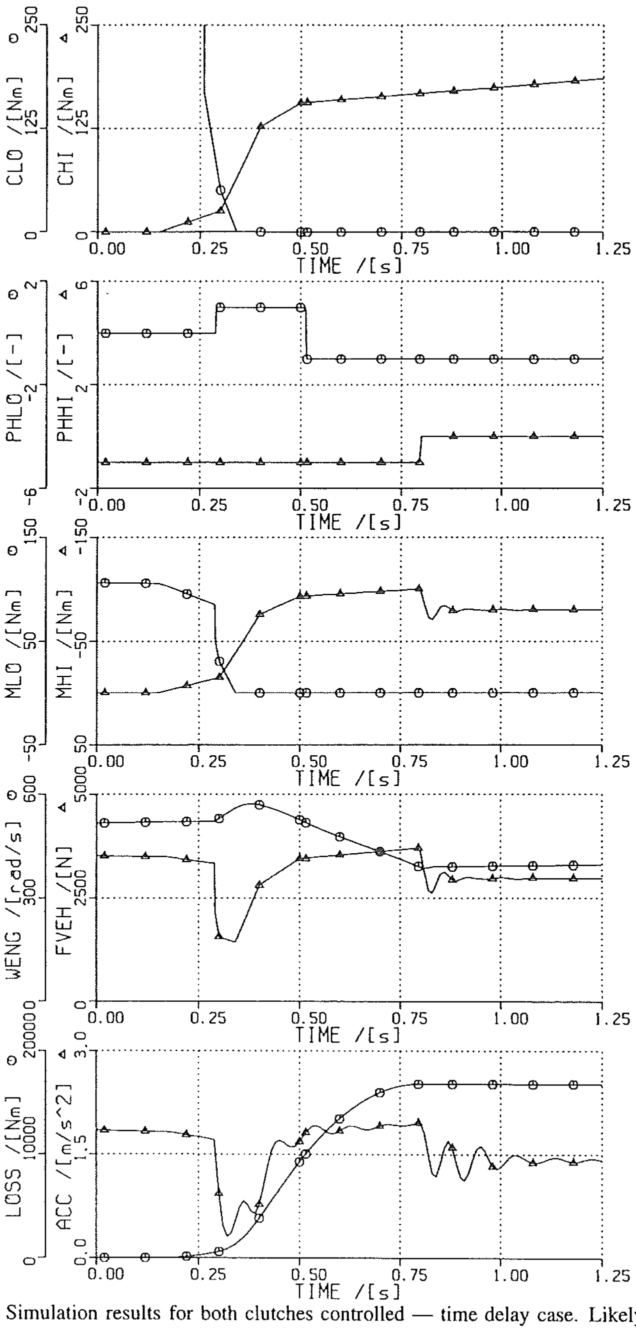


Figure 11: Simulation results for both clutches controlled — time delay case. Likely to be a good model when the engaging clutch is a band brake

Appendix A: CONTROL AND CLUTCH SUBSYSTEM

The control and clutch subsystems can be modelled as shown in reference [5], where simulation results are also presented. The models are briefly described below and the simulation results are given in figure 12.

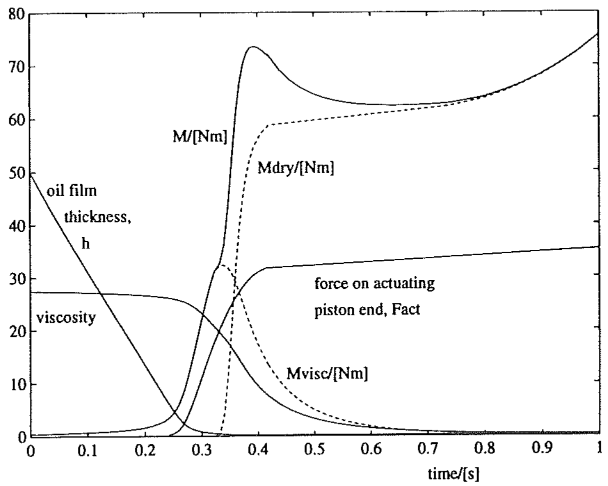


Figure 12: Simulation results for engagement of multi-disc clutch with 6 pairs of smooth friction surfaces (no grooves). From reference [5]

Section A.1: Control Subsystem

The control subsystem is modelled as shown in figure 13. A hydraulic pressure is prescribed before an orifice. In practice, this is the pressure from a hydraulic accumulator, which is relatively well known as a function of time.

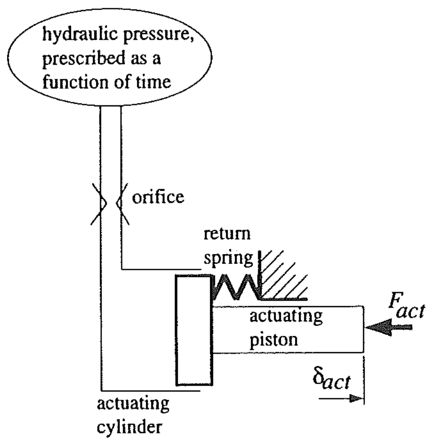


Figure 13: Control subsystem. It interacts with the clutch subsystem through the force and position of the hydraulic piston end, F_{act} and δ_{act}, respectively.

Section A.2: Clutch Subsystem

The clutch subsystem, in the case of a multi-disc clutch, is shown in figure 14. The resulting elasticity in the discs is modelled as one single spring. The oil film thickness, h, is assumed to be the same in every pair of friction surfaces. It is important to analyze the heat accumulation and conduction because the viscosity varies strongly with the temperature. This heat model is not shown in the figure.

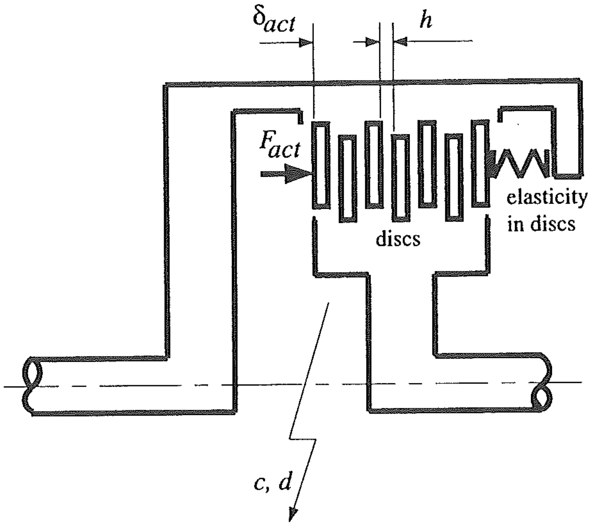


Figure 14: Clutch subsystem, in case of a multi-disc clutch. It interacts with the clutch subsystem through the force and position of the hydraulic piston end, F_{act} and δ_{act}, respectively. It interacts with the gearbox subsystem through the clutch characteristics: dry friction torque capacity, c, and viscous friction torque coefficient, d.

Different dry friction torque capacities are used for stick and slip phases, and for different slipping directions. So instead of just c, there are c^-_{slip}, c^-_{stick}, c^+_{stick} and c^+_{slip}. Also, the slip capacities are slightly dependent on the sliding velocity, due to velocity dependent coefficient of friction. The coefficient of friction increases for small sliding velocities, which is referred to as *rooster tail* characteristics. This is the reason the torque rises so steeply after 0.8 s in figure 12. The sliding velocity is then near zero.

Appendix B: GEARBOX, DRIVELINE AND VEHICLE SUBSYSTEM

Almost the same models are used as in reference [4]. The only difference is that the vehicle subsystem model is equipped with a model of a passenger.

Section B.1: Gearbox Subsystem

The gearbox subsystem is modelled as shown in figure 15. The gear transmission component has linear and homogenous equations in both velocity and torque. It is modelled as free of losses. The internal inertia, $J_{internal}$, and elasticity , $1/k_{internal}$, are normally zero. However, in some combinations of sticking/slipping clutches, one of them has to be slightly larger than zero. For instance, when all clutches are sticking, it is necessary that $1/k_{internal}$ has a small positive value.

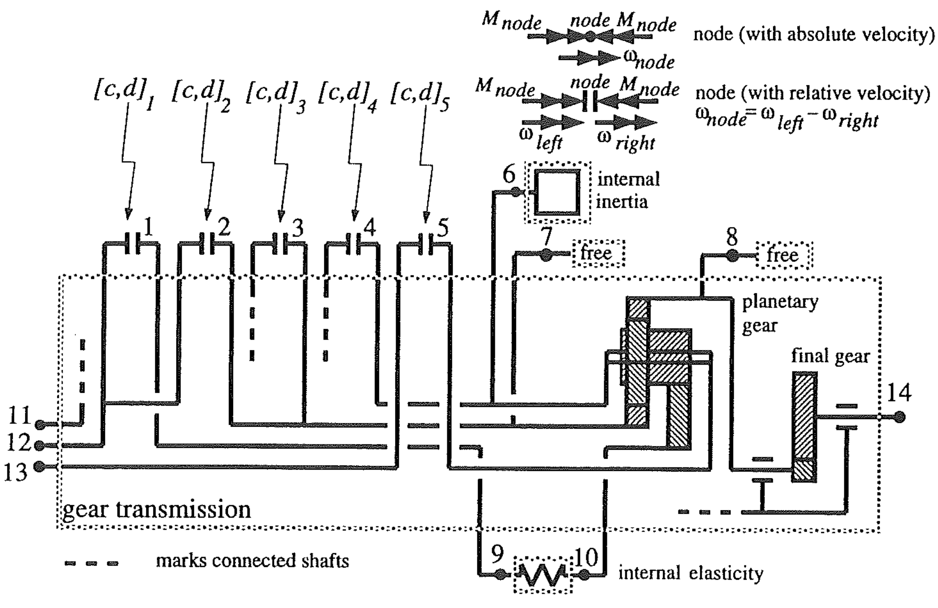


Figure 15: Gearbox subsystem. Nodes 1,2,3,4 and 5 are connected to the clutch subsystems, which is marked with broken arrows. The clutches characteristics, c and d are interaction variables here. Nodes 11,12,13 and 14 interact with the driveline subsystem through velocities and torques.

Section B.2: Driveline Subsystem

The driveline subsystem is modelled as shown in figure 16. There are only two dynamic components, the inertia of the flywheel and the elasticity of the driveshafts.

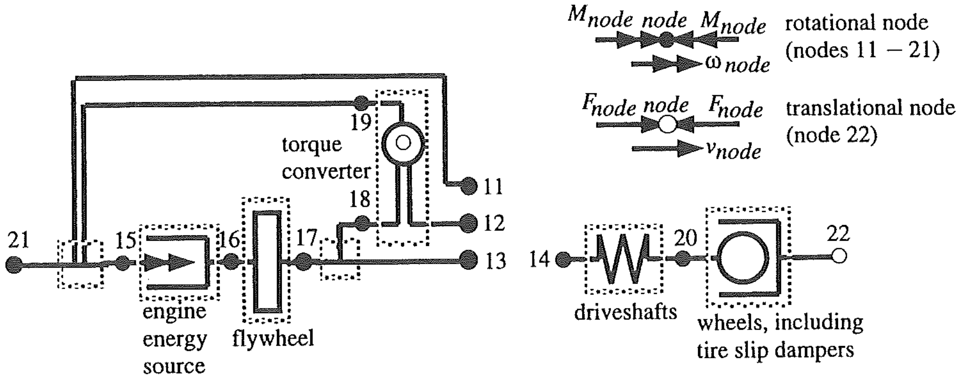


Figure 16: Driveline model. Nodes 11, 12, 13 and 14 are connected to the gearbox subsystem. Nodes 21 and 22 are connected to the vehicle subsystem. Velocities and forces (or torques) are interaction variables.

Section B.3: Vehicle Subsystem

The vehicle subsystem is modelled as shown in figure 17. It is modelled with a front wheel driven passenger car, with transversely mounted power unit. The vehicle and the engine block are modelled as bodies with three dynamic degrees of freedom, i.e. two translations in the plane and one rotation perpendicular to the plane. The human passenger only has two dynamic degrees of freedom, i.e. he is a point mass.

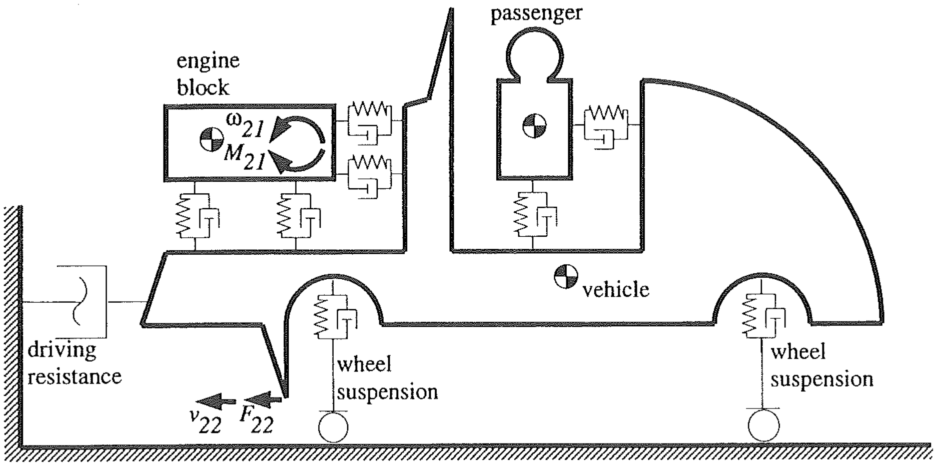


Figure 17: Vehicle model. The interactions variables to the driveline model are marked as ω_{21}, M_{21}, v_{22} and F_{22}.

Appendix C: BIBLIOGRAPHY

[1] FÖRSTER, Hans Joachim. Getriebeschaltung ohne Zugkraftunterbrechung. *Automobil Industrie*, pages 60–76, October, 1962.

[2] WINCHELL, F.J. and ROUTE, W.D. Ratio Changing the Passenger Car Automatic Transmission. *Design Practices - Passenger Car Automatic Transmissions*, pages 79–105, SAE, 2nd ed., New York, 1973.

[3] JACOBSON, Bengt. *Vehicle Driveline Mechanics during Powershifting.* Report no 1990-05-30 (Licentiate of Engineering thesis), Machine and Vehicle Design, Chalmers University of Technology, Göteborg, Sweden, 1990.

[4] JACOBSON, Bengt. Dynamic Simulation of Powershifting Transmissions. *Eighth World Congress on the Theory of Machines and Mechanisms, Prague, Czechoslovakia*, pages 573–576, August 26-31 1991.

[5] JACOBSON, Bengt. Engagement of Oil Immersed Multi-disc Clutches. *6th International Power Transmission and Gearing Conference, ASME, Scottsdale, USA*, vol 2:pp 567–574, September 13-16 1992.

Tryckt & Bunden
Vasastadens Bokbinderi AB
Göteborg 1993

ISBN 91 – 7032 – 808 – 0
Vasastadens Bokbinderi AB
Göteborg 1993