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# Modelling EBG Surfaces Using Amended DB Boundary Conditions

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***Abstract*—Recently the so-called DB boundary conditions were introduced that exhibit characteristics similar to that of an isotropic soft surface, except for an anomaly at normal incidence. This anomaly has also been amended, and the amended DB boundary condition has been shown to represent reflection from an EBG surface within the bandgap. The present paper summarizes the calculated characteristics of a practical planar EBG surface when modelled both with CST and the homogenous PMC-amended DB boundary condition. The paper also shows that the surface of an anistrotropic material with zero $\varepsilon_z$ and $\mu_z$ will behave similar to the original DB boundary condition, and this is used to understand the anomaly for normal incidence by studying the case of $\varepsilon_z$ and $\mu_z$ approaching zero.**

## I. Introduction

Ideal canonical surfaces such as PEC (Perfect Electric Conductor) and PMC (Perfect Magnetic Conductor) have been very successfully used in computational electromagnetics in quite a long time. The use of such surfaces significantly simplifies the analysis since the boundary conditions in these cases are well know and very easy to implement. With the development of more complex surfaces, such as corrugations, the soft and hard boundary conditions were defined [1]. They describe in a simple way the properties of these new surfaces for different directions of the wave incidence. Homogenised boundary conditions have been used successfully for corrugated surfaces, strip surfaces and the PEC/PMC boundary conditions for soft and hard conditions [2]-[3].

Recently, with the increased interest for metamaterials, a large number of new surfaces has been developed and presented. Many among these new surfaces are the so-called artificial magnetic conductor (AMC) surfaces which are being used as PMC ground planes for various applications. It is important to mention that the PMC behaviour of these surfaces is in most cases valid only for waves with the incidence angle close to normal incidence. For the grazing incidence the situation is usually quite different - the surface acts as an isotropic soft surface and stops all surface waves in a certain frequency band. That is why when referring to this property; these surfaces are called electromagnetic bandgap (EBG) surfaces. A very well known example of a surface that has these properties is the mushroom surface [4].

As mentioned the properties and the behaviour of the EBG/AMC surfaces are rather complex and it would be impossible to characterize them completely in terms of PEC and PMC or soft and hard boundary conditions. That is why specific boundary conditions, which can take into account all the peculiarities of these surfaces, are needed. However, in order for such boundary conditions to be applicable, they still have to be rather simple and easy to implement.

The first steps towards the generalization of this problem were the recently introduced DB boundary conditions [5]

$$E_n = 0 \;\; \& \;\; H_n = 0 \tag{1}$$

where $E_n$ and $H_n$ are the normal field components. However, as we have demonstrated [6,7], the DB boundary conditions are undefined for normal incidence and therefore incomplete. As a matter of fact, with the DB conditions the structure becomes transparent for normal incidence, which results in unphysical field results.

The EM waves incident on a plane surface can be represented as a sum of TM and TE plane waves. The problem with the TE waves is that they “feel” that the EBG surface acts as a PEC structure, which is not correct for angles close to normal incidence. The anomaly for normal incidence must be removed, and the surface should for incident TE waves transform from working like a PEC for grazing incidence to working like a PMC for normal incidence. It has been shown in [6] and [7] that the following PMC-amended boundary conditions are suitable for TE/TM decomposed waves, respectively, and actually solve the normal incidence problem:

$$H_z - jH_{\tan} = 0, \quad E_z = 0, \tag{2}$$

where $H_z$ and $E_z$ are the normal field components and $H_{tan}$ is the tangential field component with respect to the boundary surface. The factor $-j$ ensures that for $\theta = 0^\circ$ or normal incidence the reflection coefficient of the DB surface is $\Gamma = +1$ (PMC) and $\Gamma = -1$ (PEC) for $\theta = 90^\circ$ or grazing incidence.

The present paper expands the explanation of the DB unphysical results, given in [7] for planar surfaces, also to cylindrical ones using a spectral domain representation of the needed Green’s function. Furthermore, the anomaly is also studied using anisotropic material representation of the DB conditions clearly showing the normal incidence problem. Finally, a small summary of the results obtained using amended DB boundary conditions is given, showing that in cases where TE/TM decomposition is possible, the corrected boundary conditions give excellent results,

## II. STUDY OF THE UNPHYSICAL RESULTS FOR NORMAL INCIDENCE ON DB BOUNDARY

The formulation of the DB boundary conditions states that both vertical D and vertical B field components are zero [5], however, since we limit our practical interpretations to surfaces in vacuum or air the same boundary condition can be applied to vertical E and vertical H field. Essentially, by enforcing this boundary condition all waves propagating along the surface are stopped for both horizontal and vertical polarizations and for all angles of incidence. This means that it describes an isotropic soft surface, realization of which is an EBG surface. However, this DB boundary condition is not defined for planar waves at normal incidence that have no vertical field components. This means that the incident waves at normal incidence will simply pass through the boundary, what makes this boundary condition incomplete. The phase of reflection coefficient of the realized EBG surface always varies with elevation angle for the TE case, from PEC for grazing angle to PMC for normal incidence. Therefore, if the DB conditions are to be used to describe EBG surfaces, the normal incidence correction has to be made in such a way that it corresponds to the practical EBG surface behaviour.

The problem of the vertical incidence when using the original DB boundary conditions can be investigated by observing the radiation of a small horizontal dipole above the planar DB boundary as shown in Fig. 1. For this case it can be shown by applying spectral domain approach similarly as in [8] that the Green's function ($G_{xx}$ component) becomes

$$\widetilde{G}_{xx}(z>h) = -\frac{\eta_0 k_x^2 k_z^2 \cos(k_z h) + j\eta_0 k_0^2 k_y^2 \sin(k_z h)}{\beta^2 k_0 k_z} e^{-jk_z z}\,\widetilde{J}_x \tag{3}$$

where $k_{x,y,z}$ are the wave numbers for the respective directions, $\eta_0$ is the free space impedance and $\beta^2 = k_x^2 + k_y^2$.

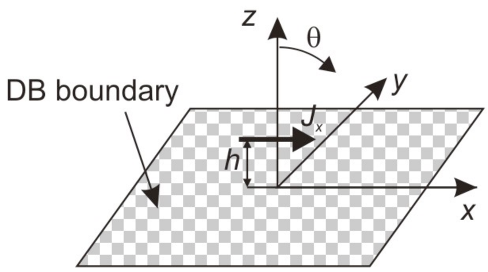



Fig. 1. Geometry of a horizontal dipole over original DB boundary.

In this test case we shall set the frequency at 12 GHz and place the dipole $h$ = 0.5 mm above the boundary. The E-field radiation pattern obtained for this case is shown in Fig. 2. and it clearly shows that E-plane and H-plane do not coincide for the case of $\theta$ = 0° or normal incidence, which is unphysical (E- and H-planes should coincide for $\theta$ = 0°). This behaviour can be readily seen from (2) if we separately consider E-plane ($k_y$ = 0) and H-plane ($k_x$ = 0) and compare this separate terms for the case of normal incidence (when $k_z = k$).

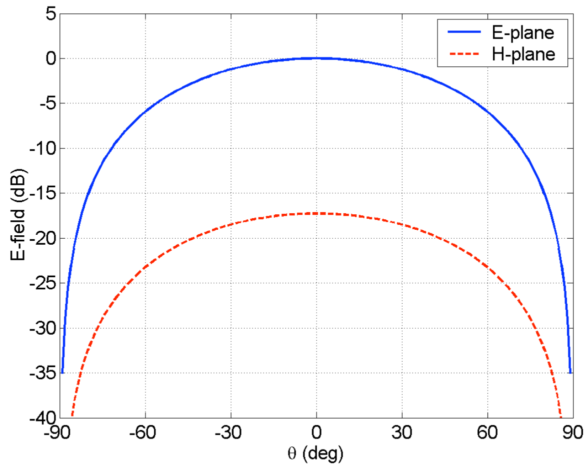



Fig. 2. E-field radiation pattern of a short horizontal dipole above the boundary described with DB boundary conditions.

Similar investigation can be performed for the equivalent cylindrical case and for that we have calculated the radiation of an axially ($z$) oriented dipole over the cylindrical DB surface. For example, it can be shown by applying spectral domain approach that the Green's function ($G_{zz}$ component) becomes

$$\widetilde{G}_{zz}(\rho > h) = -\frac{\pi b k_0}{2\eta_0} k_\rho^2 \left(J_m(k_\rho b) + R \cdot H_m^{(2)}(k_\rho b)\right) H_m^{(2)}(k_\rho \rho)$$

$$R = -\frac{\dfrac{k_z^2}{k_0\eta_0}\dfrac{\rho}{m}\dfrac{H'^{(2)}_m(k_\rho a)}{H_m^{(2)}(k_\rho a)} J'_m(k_\rho a) - \dfrac{k_0}{\eta_0}\dfrac{m}{\rho k_\rho^2} J_m(k_\rho a)}{\dfrac{k_z^2}{k_0\eta_0}\dfrac{\rho}{m}\dfrac{H'^{(2)}_m(k_\rho a)}{H_m^{(2)}(k_\rho a)} H'^{(2)}_m(k_\rho a) - \dfrac{k_0}{\eta_0}\dfrac{m}{\rho k_\rho^2} H_m^{(2)}(k_\rho a)} \tag{4}$$

The considered cylindrical surface has a radius of 21.6 mm and the dipole is placed 0.5 mm above its surface. The analysis is again performed using spectral domain approach. Similarly as before the radiation pattern for both planes, azimuthal and elevation is shown in Fig. 3., calculated for the frequency of 12 GHz. However, unlike the previous case, here the radiation patterns match for the normal incidence, but show a rather peculiar behaviour. Namely, the PEC component of the DB boundary prevails over the PMC component and the structure for the normal incidence acts more like a PEC, which is not expected from an EBG surface.

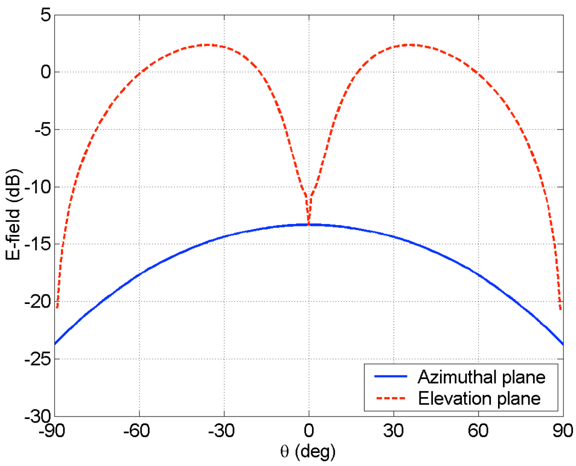



Fig. 3. Far-field radiation pattern of an axially-directed dipole over the cylindrical original DB boundary.

## III. ANISOTROPIC MATERIAL REPRESENTATION OF DB BOUNDARY

Here we will study the reflection coefficient at the surface of an anisotropic material with the following permittivity and permeability matrices:

$$\varepsilon = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \ \mu = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_x & 0 \\ 0 & 0 & \mu_z \end{bmatrix}. \quad (5)$$

It can readily be shown that the boundary conditions of this surface will become the DB boundary conditions by letting $\varepsilon_z$ and $\mu_z$ go to zero. Using this material we can relatively simply study the anomaly for normal incidence by using the plane wave spectral domain method for finite $\varepsilon_z$ and $\mu_z$ and then letting them approach zero. Using similar procedure as in [9] we can obtain the reflection coefficient for a plane wave incidence on the surface of the material described with (5) is

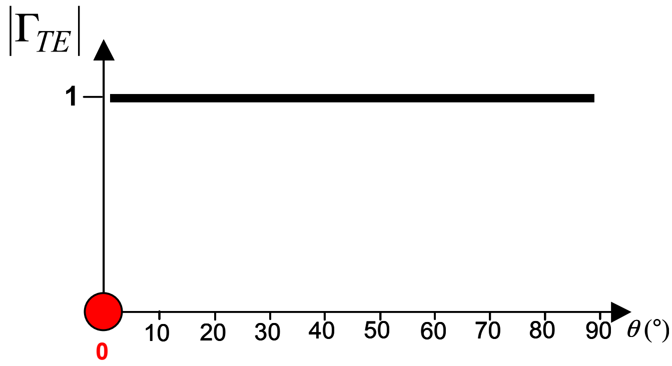
$$\Gamma^{TE} = \frac{k_z - k_z^{TE}}{k_z + k_z^{TE}}, \quad (6)$$

with

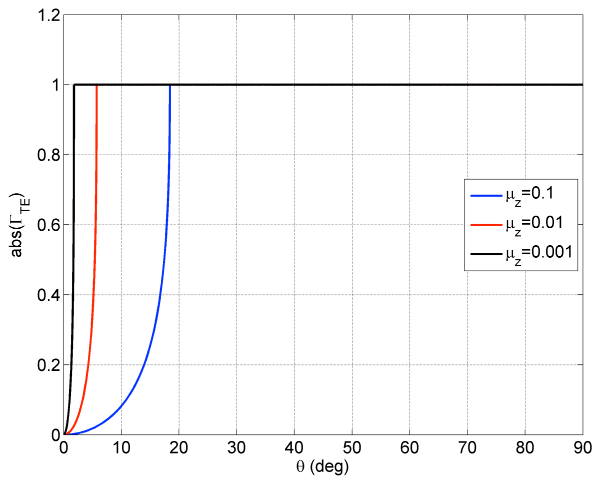
$$\left(k_z^{TE}\right)^2 = \varepsilon_x \mu_x k_0^2 - \frac{\mu_x}{\mu_z}\beta^2. \quad (7)$$

Similar can be also obtained for the TM case. If we now plot this reflection coefficient with respect to the incidence angle we should get a clearer image of what is happening in the limiting cases. This is shown in Fig. 4 and indicates the problem which was already highlighted, i.e., in the case of normal incidence the DB boundary actually becomes transparent.

We will now show that the amended DB boundary condition removes this anomaly and represents a useful homogeneous boundary condition for the EBG surface.




(a)




(b)

Fig. 4. Reflection coefficient of the DB boundary; (a) ideal DB boundary, (b) DB boundary realized by anisotropic uniaxial medium.

## IV. PMC-AMENDED DB BOUNDARY CONDITIONS FOR EBG SURFACE

Using the amended set of boundary conditions in (2) the analysis of the case shown in Fig. 1 results in radiation patterns shown in Fig. 5. The working frequency is again 12 GHz and the dipole is placed 0.5 mm above the surface. Unlike the original DB case, here the two planes coincide for normal incidence and the entire radiation pattern corresponds to that what is expected from an EBG surface. To verify that this theoretical surface really can be used to model practical EBG surfaces, we will now compare its results with the results obtained for a practical mushroom type EBG surface.

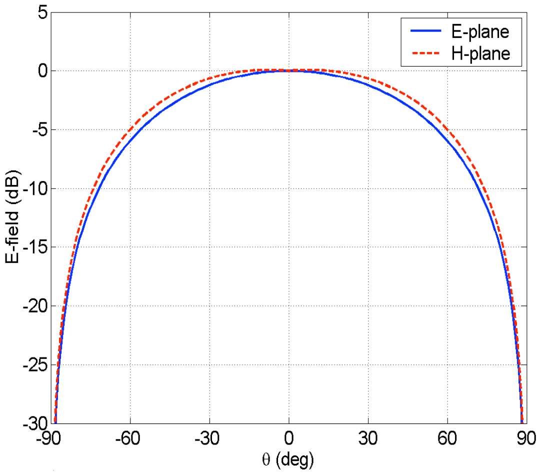



Fig. 5. Far-field radiation pattern of a horizontal dipole over a surface described with amended DB boundary conditions.

The geometry of the considered mushroom surface is shown in Fig. 6. The periodicity is 2.4 mm, the width of the patches is 2.25 mm and the metallized via hole diameter is 0.36 mm. The substrate has permittivity $\varepsilon_r$ = 2.2, and its thickness is $t$ = 1.6 mm. The frequency is 12 GHz and the radius of the PEC cylinder is 20 mm.

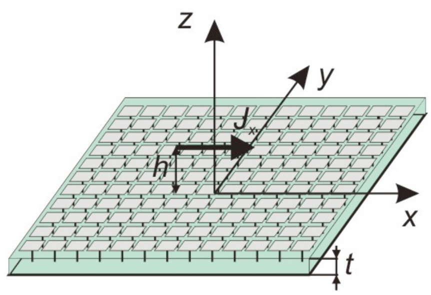



Fig. 6. Geometry of the practical planar mushroom surface.

Fig. 7 shows comparison between the radiation patterns calculated for the planar EBG surface shown in Fig. 6. using amended DB boundary conditions and using CST Microwave Studio [10] to model the actual physical surface. The CST results are shown for frequencies at the beginning, middle and the end of the surface bandgap (the bandgap is approximately between 11 and 16 GHz). In both planes there is an excellent agreement between the patterns at 12 GHz which is in the beginning of the bandgap. At higher frequencies the properties of the bandgap start to change and therefore cannot be captured by the ideal amended DB boundary conditions. On the other hand, the radiation pattern calculated with amended DB boundary conditions varies only slightly when changing the frequency [7]. The deviation between the patterns for ϕ=+/−90 deg in E plane is due to the fact that in the CST calculation the structure of finite length was used.

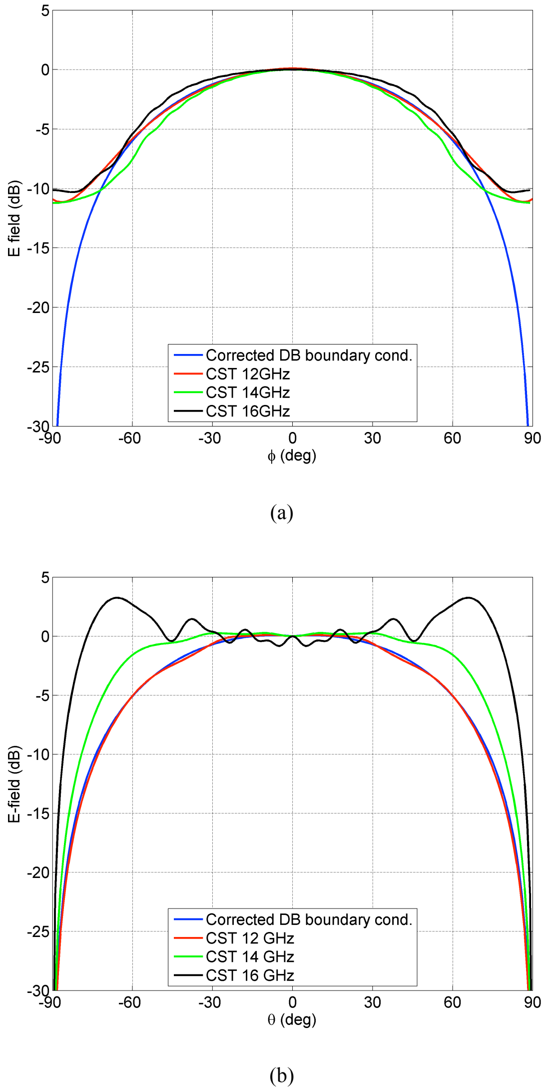



Fig. 7. E-field radiation pattern of a horizontal dipole over the amended DB and EBG surfaces; (a) E-plane, (b) H-plane.

## V. Conclusions

Relatively recently introduced DB boundary conditions were suggested as a possible solution for modeling practical EBG surfaces. Unfortunately this is not possible due to the fact that these boundary conditions are not defined for normal incidence, which results in anomalous field solutions in some cases. This was demonstrated here using spectral domain representation of Green's functions and also using anisotropic material representation of DB boundary. Thereafter we used the PMC-amended DB boundary conditions in [7], which are based on TE/TM decomposition of the field, to model a dipole above such planar surface. The results were similar to a CST model of a dipole above a practical mushroom-type EBG surface in the beginning of the bandgap. The general conclusion is that the new PMC-amended DB boundary conditions provide a very good prediction of the field patterns at the beginning of the bandgap, i.e. in that frequency region where practical EBG surface has the best bandgap characteristics. Still, it has to be pointed out that it is not possible to use these new PMC-amended boundary conditions in the present form in general numerical codes due to the fact that it requires the field decomposition into TE/TM waves.


## ACKNOWLEDGMENT

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