Single node localization in wireless sensor networks

Master of Science Thesis

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Abstract

Nowadays wireless sensor network (WSN) has been emerged in many applications such as environmental monitoring, traffic controlling, and animal tracking. The task of sensors in a WSN is to collect data and information, and send them for further processing. However, this information would not be useful without any knowledge about the locations of sensors. Consequently, localization is one of the most important issues in WSNs. Since localization is a broad subject and each of its aspects requires studies and researches, in the thesis we concentrate on two particular situations in sensor localization.

In the first part, we study the bearing-only target localization problem when the observer positions are subject to error. In this problem, the angle-of-arrival of the transmitted signal between a target and an observer are used to estimate the target position. In this work, we assume that not only the bearing measurements are corrupted by noises but also the exact position of observer is not available to the estimator. The maximum likelihood (ML), the least squares (LS), total least squares algorithms and a new method of localization based on weighted total least squares (WTLS) approach are developed for this problem. The corresponding Cramér-Rao lower bound (CRLB) is also derived. Simulation results show that the new method, i.e., WTLS, outperforms other algorithms and can attain the CRLB for sufficiently large signal-to-noise-ratios (SNRs).

In the second part, received signal strength (RSS) based single source localization when there is not a prior knowledge about the transmit power of the source is investigated. Because the RSS model is a function of transmit power, anchors should have the transmit power of the source to find the location of the source. Because of nonconvex behavior of the ML cost function, it requires intensive computations to achieve its global minimum. Therefore, we propose a novel semidefinite programming (SDP) approach by approximating ML problem to a convex optimization problem which can be solved efficiently. Simulation results show that although the ML estimator outperforms other algorithms, our proposed SDP has a remarkable performance very close to ML estimator. By linearizing the RSS model, we also derive a partly novel LS and WTLS algorithms for this problem. Simulations illustrate that WTLS improves the performance of LS considerably.

Keywords: Wireless sensor network (WSN), localization, received signal strength (RSS), bearing-only, semidefinite programming (SDP), weighted total least squares (WTLS), Cramér-Rao lower bound (CRLB), total least squares (TLS)

List of Publications

This thesis is based on the following papers:

[A] Reza M. Vaghefi, Mohammad Reza Gholami, and Erik G. Ström, "Bearing-only target localization with uncertainties in observer position," *2010 IEEE 21st International Symposium on Personal, Indoor and Mobile Radio Communications Workshops (PIMRC Workshops)*, pp. 238-242, Sep. 2010.

[B] Reza M. Vaghefi, Mohammad Reza Gholami, and Erik G. Ström, "RSS-based sensor localization with unknown transmit power," *2011 IEEE International Conference on on Acoustics, Speech and Signal Processing (ICASSP 2011)*, to appear, May 2011.

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Abbreviations

AOA	Angle-of-arrival
CDF	Cumulative density function
CRLB	Cramér-Rao lower bound
LS	Least squares
ML	Maximum liklihood
MSE	Mean square error
PDF	Probability density function
RMSE	Root mean square error
RSS	Received-signal-strenght
SDP	Semidefinite programming
SNR	Signal-to-noise-ratio
TLS	Total least squares
TDOA	Time-difference-of-arrival
TOA	Time-of-arrival
WLS	Weighted least squares
WSN	Wireless sensor network
WTLS	Weighted total least squares

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Chapter 1

Introduction

Wireless sensor network (WSN) has a wide varieties of applications such as controlling, tracking, and monitoring. A WSN consists of a group of nodes (or sensors) distributed in a area to work cooperatively and collect information such as temperature, pressure, distance, and volume. Different characteristics such as size, cost, sensitivity, memory, speed, and energy consumption are involved in designing a WSN. Each node usually communicate wirelessly with neighbor nodes and a central processor to transmit data. Data and information from sensors would be useless if their locations are not known. Therefore, the central processor needs to know the location of each sensor to make its information valuable. Localization in WSNs means determining the location of sensors in a WSN by using noisy measurements. Equipping each sensor with a Global Positioning System (GPS) device would be a possible solution. However, using GPS devices has many drawbacks which leads us to use localization in sensor networks. For instance, it would be highly expensive to equip all sensors in a network with GPS devices or sensors are too small to have a GPS. In addition, some sensors cannot be fixed at a place and their positions might change, for instance, in underwater networks. Another major problem with a GPS device is that it does not work properly in indoor networks. Consequently localization of some sensors in a WSN is inevitable.

Here some examples of WSN application are mentioned. Assume that the there are some sensors located in a jungle to find the location of accidental fire. Locating the sources that make alarm help us to find the position of the fire. Another example would be the tracking of animals in a herd to study their behaviors and mutual actions [1]. Sensors are usually used to detect the movement of lands and predict possible earthquakes. Sensor may be used underwater [1], e.g., to measure the pressure of different areas of the ocean.

In general, there are some sensors in a network that their locations are available (anchor sensors). They may be equipped with a GPS device or have been placed and fixed at a known position. The anchors try to find the location of sensors whose locations are not known (source sensors) with inaccurate and noisy measurements. Different types of measurements are employed in localization such as received-strength-signal (RSS), time-of-arrival (TOA), and angle-of-arrival (AOA) [2].

In this work, we concentrate only on single source localization, meaning that there is only one sensor with an unknown position should be localized. In some cases, it is required to find the location of many sources. As a result the localization algorithms are divided in two groups; noncooperative localization and cooperative localization. In noncooperative localization, as mentioned earlier, anchors tries to find the location of sources. On the other hand, in cooperative localization, not only anchors but also sources themselves are involved in localization. This means that sources also are connected to each other and exchange information. By using cooperative localization, we can improve localization performance considerably in comparison with noncooperative localization [2].

1.1 Measurement Models

In this section, we describe different types of measurements generally used for the sensor localization. Let $\mathbf{x}_s = [x_s, y_s]^T \in \mathbb{R}^2$ be the coordinates of the source to be determined and $\mathcal{C} = \{1, \ldots, M\}$ be the set of indices of the anchors connected to the source, and $\mathbf{x}_i = [x_i, y_i]^T \in \mathbb{R}^2$, $i \in \mathcal{C}$ be the known location of anchor nodes. Here, we describe the measurement models for single source localization and 2D coordinates, however, the extension for multi-source and cooperative localization, and also 3D coordinate is often straightforward.

1.1.1 Received-Strength-Signal

The average measured power transmitted by a source at the anchors is defined as RSS measurement. The RSS measurements include information from the distances between the anchors and the source [2]. The major sources of error in RSS are shadowing and multipath signal caused by objects and obstructions between the source and anchors.

The average received power (in dB) at ith anchor, P_i, under the log-distance pathloss and log-normal shadowing, is modeled as [3]

$$P_i = P_0 - 10\beta \log_{10} \frac{d_i}{d_0} + n_i, \quad i \in \mathcal{C}, \tag{1.1}$$

where P_0 is the reference power at reference distance d_0 (which depends on transmit power), β is the path loss exponent, $d_i = \|\mathbf{x}_s - \mathbf{x}_i\|_2$ is the true distance between source and the ith anchor, and n_i are the log-normal shadowing term modeled as zero-mean Gaussian random variables with variance σ_{dB}^2. The standard deviation of shadowing is reported in dB unit and is topically between 4 and 12 dB [2]. Moreover, σ_{dB} is related to the environment where a network placed and is constant with range [2]. During RSS measurement, the calibration between the source and anchors is inevitable [2].

1.1.2 Time-of-Arrival

The TOA measurement is considered as the time of signal transmission between a source and anchors T_i. The measured TOA also includes a propagation-induced time delay. The TOA measured at the ith anchor is modeled as [2],

$$T_i = \frac{d_i}{c} + w_i + \tau_i, \quad i \in \mathcal{C}, \tag{1.2}$$

where c is the propagation speed of the signal depending on the environment (e.g., in vacuum electro-magnetic propagation speed is $c \approx 3x10^8$ m/s), w_i is the measurement noise modeled as zero-mean Gaussian random variable with variance $\sigma_{T,i}^2$ which is related to range, and τ_i is the time delay error. Time delays mainly result from the hardware and software in the receiver and transmitter which add to the measured time. The major sources of TOA measurements errors are multipath and addictive noise [2]. Depicting a numerical example, the UWB measurements reported by a Motorola factory showed that the standard deviation of TOA systems is around 0.3 ns and the time delay error is around 1.9 ns [2]. To diminish the effect of time delay error, a source and anchors should work synchronously [2].

1.1.3 Angle-of-Arrival

The angle between a source and an anchor is measured as the AOA measurement. The hardware for measuring AOA is relatively more complex than other methods. In the presence of the measurement noise, the AOA measurements modeled as [2]

$$\alpha_i = \tan^{-1}\frac{y_s - y_i}{x_s - x_i} + v_i, \quad i \in \mathcal{C}, \tag{1.3}$$

where $\tan^{-1}\{\cdot\}$ is four-quadrant inverse tangent, and v_i is the measurement noise modeled as zero-mean Gaussian random variable with variance σ_α^2. In an acoustic-based AOA measurements, the results shows that the standard deviation is between 2 and 6 degrees, depending on range [2].

1.1.4 Other Measurement Types

There are many other types of measurement used for localization considered in the literature. For instance, to eliminate the effect of time delay error in TOA measurement, one can use time-difference-of-arrival (TDOA) [4] in which the differences of arrival times of pairs of sources are measured at anchors[1, 5]. The combinations of two or three mentioned methods such as RSS/TOA are also studied in the literature [6].

1.2 Problem Statement

As mentioned earlier, we will focus on two specific situations in the sensor localization. In this section, two problems are introduced and defined. In the next section, the previous research and studies in the literature are mentioned.

In the first part, we investigate the bearing-only localization with observer position errors. In bearing-only localization, the observer tries to find the location of a observer by measuring the angle between the target and the observer. The measurement model for this problem is similar to the AOA measurement given in the previous section. In classic bearing-only localization, it is assumed that the location of an observer is known (e.g., by equipping the observer with a GPS device), however, in this work, we assume that the exact location of observer is not available and only approximate or noisy values are known to the estimator (e.g., the GPS device in the observer does not work properly).

In the second part of this thesis, we turn to another method of measurement in localization. We study single sensor RSS localization when the transmit power of the source is not available. Since the RSS model is a function of the transmit power of the source, anchors are not able to find the location of the source without having any information about transmit power. As a result the source has to report its transmit power to the anchors [2]. However, this requires an additional cost and space. Here, two methods to deal with unknown transmit power are introduced.

1.3 Literature Survey

Most of the studies done on bearing-only localization in the literature assume that the exact position of the observer is available [7, 8, 9, 10]. The maximum likelihood (ML), the least squares (LS), and the weighted least squares (WLS) were derived and compared in [8]. The Cramer-Rao lower bound (CRLB) of bearing-only localization when the exact position of the observer is known was also investigated in [8]. The WLS first introduced by

Stansfield [7], therefore, WLS solution of the bearing-only localization is called Stansfield estimator. There are some related studies on the bearing-only localization when the position of the observer is subject to error [11, 12]. The novel total least squares (TLS) was first introduced for this problem [11]. Another extension to TLS called constrained total least squares (CTLS) was also mentioned in [11]. In [12], the combination of doppler and bearing localization in the presence of observer position error was studied and ML estimator and CRLB were computed.

We can find many studies in the literature about RSS localization and most of them assume that the transmit power of the source is available to the estimator [3, 13, 14]. Patwari et al. [3] derived the CRLB and ML estimator for RSS localization and they showed that the ML estimator derived for RSS localization is biased. The LS, WLS, and constrained least squares estimators of RSS localization were derived in [5]. The ML estimator is optimal, meaning it attains the CRLB accuracy, when the data record is sufficiently large [1, 3], however, since the ML cost function is severely non-linear and nonconvex, it has not any closed-form solution and finding its global minimum requires difficult computations. One way to make the computation easier is to linearize the corresponding model and apply linear estimators, such as LS, which has a closed-form solution. However, the performance of LS is not as good as the ML estimator [5]. But, it can be improved by using either a correction method [15] or a constrained method [5]. Another way is to approximate the cost function of ML to a convex problem and solve it by convex optimization algorithms such as semidefinite programming (SDP) [16]. The advantage of SDP approaches over the nonconvex ML estimator is that SDP has no local minima and can be solved efficiently. The SDP relaxation for general case of localization in WSN was investigated [17]. The SDP approach for the RSS localization in for both cooperative and noncooperative was studied in [18]. There are only few studies about RSS localization with unknown transmit power [19, 20].

1.4 Thesis Outline

The introduction of the thesis has been given in Chapter 1. The bearing-only localization is investigated in Chapter 2. In this part, first the model and assumptions are defined and then the corresponding CRLB of the proposed model is derived. The ML estimator is derived in two cases; when the observer position error is neglected and when it is considered. It is shown that the two mentioned ML estimators lead to the same estimation for target localization under certain circumstances. Different linear estimators, i.e., LS, TLS, and WLS, are also studied. We introduce a novel method called weighed total least squares (WTLS). By using this method, we can enhance the performance of TLS and LS considerably [21, 22]. It is shown that the cost function of WTLS is nonlinear and there is no closed-form solution for it, consequently, it should be solved using iterative optimization methods [21].

In Chapter 3, the RSS localization is studied. In this work, we deal with the unknown transmit power with two methods. In the first method, the unknown transmit power is eliminated from the RSS model. This can be done by using computing RSS difference between two anchors [2]. The second method is that we consider the transmit power of the source as a nuisance parameters and estimate it along with source location [2, 23]. There is also another method to deal with this problem in which we estimate the location of the source only by comparing RSS measurements without using any model [24]. However, we have not included this method in our work. First, we define the system model and assumption. Although the RSS localization is generally biased [3], the CRLB of the model is derived and used as a benchmark for evaluating the proposed algorithms. We introduce a new SDP approach to solve the RSS localization with unknown transmit power. Since the transmit power of the source is not available, previously studied SDP approaches [18, 25] are not applicable here. The proposed SDP approach is applied to two mentioned methods. Linear estimators having closed-form solutions are also obtained. LS is also derived for two methods; eliminating and estimating source transmit power. To improve the performance of LS, we derived the corresponding WTLS algorithm [21, 22, 26]. Finally, the conclusion and possible future works are given in Chapter 4.

Chapter 2

Bearing-only Target Localization

2.1 Introduction

In bearing-only localization problem, a moving observer is used to find the location of a fixed target or to track a moving target. In this work, we concentrate on fixed target localization. Bearing measurements are obtained from different points along the trajectory line of the moving observer. The location of target is estimated from the intersection point of bearing lines among different positions of an observer and target [10].The bearing-only is frequently applied in military application such as locating a ground target by an aircraft [7].

The bearing measurements can be acquired by several observers at distinct locations. In this case, therefore, the problem can be interpreted as the locating a sensor in wireless sensor network using AOA measurements [2]. Hence, the target and the observer in previous problem turn into source sensor and several anchors (sensor nodes with known positions) in distinct positions respectively and consequently the angle of transmitted signal from the source in anchor nodes are measured. Since the formulation for both problems are the same, we mention all procedures for the bearing-only localization, then it can be extended to AOA sensor localization easily.

Various works have been done on the bearing-only localization [5, 7, 10]. In [10], the performance of ML estimator, LS estimator, and WLS estimator, also called the Stansfield algorithm, were examined. The Stansfield and ML estimators for different observer trajectories were also indicated in [8] where the performances of the estimators are enhanced by finding the optimal observer trajectories. Moreover, total least squares estimator (TLS) and an iterative two-stage approach involving TLS and Kalman filtering were surveyed in [27]. The bearing-only localization was converted geometrically to another problem and solved by optimization techniques [28].

Most of the works done in the literature are based on the assumption that the exact position of the observer is available. However, this assumption is not realistic in practice. For instance, wind can change the direction of aircraft from what it is supposed to be or unpredictable errors appears in aircraft's GPS (Global Positioning System). Recently, some works have been carried out into bearing-only localization problem with uncertainties about observer position [11, 29]. In [11], TLS and ML estimators were developed for this kind of problem. The doppler-bearing tracking problem in the presence of observer position error in the case of one and two observers was also investigated in [12].

In the this work, we assume that the exact position of the observer is not available. ML estimator is investigated in two cases; when the estimator does not know about existence of the observer position errors and also when it does. To find the ML solution, it is required to solve a nonlinear problem which is computationally intensive. In addition, having a

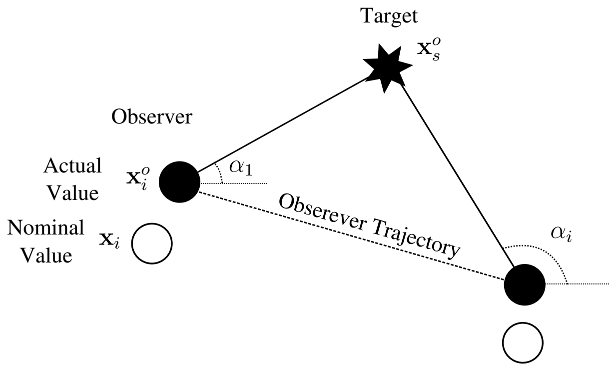

Figure 2.1: The bearing-only localization model. The star denotes the target, the solid circle shows the actual position of observer which are unknown to estimator, and the hollow circle shows the nominal position of observer available to estimator. The observer moves along the observer trajectory and collects bearing measurements.

good initial guess is required to guarantee that the algorithm converges to the global minimum of the ML cost function. Therefore, we also focus on some linear algorithms. First, LS, WLS, and TLS, an extension to LS, are applied for the problem. Then, we apply the novel technique based on weighted TLS (WTLS) estimator for our problem in order to improve the performance of TLS. The CRLB of bearing-only localization with uncertainties in observer position is obtained and comparison among the proposed algorithms and CRLB is made.

The rest of the chapter is organized as follows. In Section 2.2, the model of bearing-only localization and the corresponding CRLB are described. In Section 2.4, we derive different algorithms formulation for bearing-only localization. In Section 2.5, computer simulations are presented to evaluate the performance of the proposed algorithms.

2.2 Localization Model

In this work, we consider 2D network. Fig. 2.1 demonstrates the bearing-only localization model. Let $\mathbf{x}_s^o = [x_s^o, y_s^o]^T \in \mathbb{R}^2$ be the coordinate of the target to be estimated. The observer collects bearing measurements at M distinct points $\mathbf{x}_i^o = [x_i^o, y_i^o]^T \in \mathbb{R}^2$, $i = 1, 2, \ldots, M$. In the absence of the measurement noise, the relation between the true bearing angle (in radians) and the true location of the target is

$$\alpha_i^o = \tan^{-1} \frac{y_s^o - y_i^o}{x_s^o - x_i^o}. \tag{2.1}$$

where $\tan^{-1}\{\cdot\}$ is four-quadrant inverse tangent. Let $\boldsymbol{\alpha}$ be the bearing measurement vector consisting of the true bearing corrupted by additive noise,

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_M]^T = \boldsymbol{\alpha}^o + \mathbf{n}, \tag{2.2}$$

where $\mathbf{n}$ is the bearing measurement error vector modeled as zero mean Gaussian random vector with covariance matrix $\boldsymbol{\Psi}_{\boldsymbol{\alpha}}$. In the current model, we assume that the exact position of the observer is not available. Let $\mathbf{x}_i = [x_i, y_i]^T$ be the nominal value of the observer position at the ith point and $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_M^T]^T$ be the vector of nominal observer positions available to the estimator, then,

$$\mathbf{x} = \mathbf{x}^o + \mathbf{v}, \tag{2.3}$$

where $\mathbf{v}$ is observer position error vector assumed to be zero mean Gaussian random vector with covariance matrix of $\boldsymbol{\Psi}_{\mathbf{x}}$. Note that $\boldsymbol{\Psi}_{\mathbf{x}} = \text{blkdiag}[\boldsymbol{\Psi}_{\mathbf{x}_1}, \boldsymbol{\Psi}_{\mathbf{x}_2}, \ldots, \boldsymbol{\Psi}_{\mathbf{x}_M}]$, where blkdiag $\{\cdot\}$ denotes the block diagonal matrix, and $\boldsymbol{\Psi}_{\mathbf{x}_i}$ is covariance matrix of noise over ith position of observer. We assume that the bearing measurement and observer position errors, i.e., $\mathbf{n}$ and $\mathbf{v}$, are statistically independent. This assumption has been previously considered for similar cases in [11, 12], but might not be valid for all bearing measurement systems.

2.3 Performance Bound and Optimal Estimator

The CRLB expresses a lower bound on the variance of any unbiased estimators which is usually applied for the performances comparison of different unbiased estimators [23]. To compute CRLB, we consider the same approach used in [12] in which they have derived the CRLB for a similar problem. Let $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T]^T = [\mathbf{x}_s^{oT}, \mathbf{x}^{oT}]^T$ be the unknown parameter vector to be estimated. Note that since the true position of the observer is not known for the estimator, it should also be estimated. Let $\boldsymbol{\beta} = [\boldsymbol{\alpha}^T, \mathbf{x}^T]^T$ be the data vector consisting of bearing measurements and nominal positions of the observer. The bearing measurements and nominal positions are statistically independent, therefore, the probability density function (PDF) of the data vector is the product of their individual PDFs,

$$f(\boldsymbol{\beta}; \boldsymbol{\theta}) = f(\boldsymbol{\alpha}; \boldsymbol{\theta}) f(\mathbf{x}; \boldsymbol{\theta}). \tag{2.4}$$

The CRLB of the unknown parameters is computed by the inverse of the Fisher information matrix [23],

$$\text{CRLB}(\theta_k) = [\mathbf{I}^{-1}(\boldsymbol{\theta})]_{kk}, \quad k = 1, 2, \ldots, 2 + 2M, \tag{2.5}$$

where $\mathbf{I}(\boldsymbol{\theta})$ is the Fisher information matrix and its elements are given as,

$$\mathbf{I}(\boldsymbol{\theta}) = -E\left[\frac{\partial^2 \ln f(\boldsymbol{\beta}; \boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T}\right]. \tag{2.6}$$

The Fisher matrix can be partitioned as,

$$\mathbf{I}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix}, \tag{2.7}$$

where,

$$\mathbf{X} = \mathbf{A}^T \boldsymbol{\Psi}_{\boldsymbol{\alpha}}^{-1} \mathbf{A}, \tag{2.8a}$$
$$\mathbf{Y} = \mathbf{A}^T \boldsymbol{\Psi}_{\boldsymbol{\alpha}}^{-1} \mathbf{B}, \tag{2.8b}$$
$$\mathbf{Z} = \mathbf{B}^T \boldsymbol{\Psi}_{\boldsymbol{\alpha}}^{-1} \mathbf{B} + \boldsymbol{\Psi}_{\mathbf{x}}^{-1}, \tag{2.8c}$$

where $\mathbf{A} = \partial \boldsymbol{\alpha}^o / \partial \mathbf{x}_{\boldsymbol{s}}^{\boldsymbol{o}}$ and $\mathbf{B} = \partial \boldsymbol{\alpha}^o / \partial \mathbf{x}^{\boldsymbol{o}}$. Taking the inverse of partitioned matrix (2.7), we have [30],

$$\text{CRLB}(\mathbf{x}_s^o) = \mathbf{X}^{-1} + \mathbf{X}^{-1}\mathbf{Y}\, \text{CRLB}(\mathbf{x}^o)\, \mathbf{Y}^T \mathbf{X}^{-1}, \tag{2.9a}$$
$$\text{CRLB}(\mathbf{x}^o) = (\mathbf{Z} - \mathbf{Y}^T \mathbf{X}^{-1} \mathbf{Y})^{-1} = \boldsymbol{\Psi}_{\mathbf{x}}, \tag{2.9b}$$

Let $\mathbf{a}_i^T$ and $\mathbf{b}_i^T$ be the ith row of matrix $\mathbf{A}$ and matrix $\mathbf{B}$ respectively,

$$\mathbf{a}_i^T = \left[-\tfrac{y_s^o - y_i^o}{d_i} \quad \tfrac{x_s^o - x_i^o}{d_i}\right], \tag{2.10a}$$

$$\mathbf{b}_i^T = -\begin{bmatrix}\mathbf{0}_{(2i-2)\times 1}^T & \mathbf{a}_i^T & \mathbf{0}_{(2M-2i)\times 1}^T\end{bmatrix}, \tag{2.10b}$$

where $d_i = \|\mathbf{x}_s^o - \mathbf{x}_i^o\|_2$ is the Euclidian distance between target and observer at the ith point. It can be seen from (2.9b) that the CRLB of the observer position is equal to the covariance matrix of the observer position error. Moreover, (2.9a) shows that the CRLB of the target location depends on the covariance matrix of the observer position error $\mathbf{\Psi}_{\mathbf{x}}$. By setting $\mathbf{\Psi}_{\mathbf{x}} = \mathbf{0}$ (i.e., the exact position of the observer is known), the CRLB of the target location reduces to $\text{CRLB}(\mathbf{x}_s^o) = \mathbf{X}^{-1}$ which is the same as the CRLB derived in [10] when the exact position of the observer is available to the estimator.

2.3.1 Maximum Likelihood

ML estimator is obtained by maximizing the likelihood function of unknown parameters [23]. For some cases, the ML estimator is asymptotically (for large data records) unbiased and efficient meaning that it can achieve CRLB accuracy [23]. The ML estimator for bearing-only localization problem with observer position error can be defined in two cases. In the first case, uncertainties about observer position are present but the ML estimator ignores the noise on the observer position (or does not know about it) and tries to maximize only the likelihood function of bearing measurement $\boldsymbol{\alpha}$. In the second case, the ML estimator takes also observer position errors into account and tries to maximize the joint PDF of the bearing measurement and the observer position. Here, we will show that the ML derived for two mentioned cases yield the same estimate for target position in our particular problem under certain circumstances.

First, it is assumed that the exact positions of the observer are available to the ML estimator . Since the bearing measurement has a Gaussian PDF, ML problem turns into the following nonlinear minimization problem [23],

$$\hat{\boldsymbol{\theta}}_{1,\text{ML}} = \arg\min_{\boldsymbol{\theta}_1} (\boldsymbol{\alpha} - \mathbf{g}_1(\boldsymbol{\theta}_1))^T \mathbf{C}_1 (\boldsymbol{\alpha} - \mathbf{g}_1(\boldsymbol{\theta}_1)), \tag{2.11}$$

where $\mathbf{C}_1 = \mathbf{\Psi}_{\boldsymbol{\alpha}}^{-1}$, $\mathbf{g}_1(\boldsymbol{\theta}_1) = [g_{1,1}(\boldsymbol{\theta}_1), g_{1,2}(\boldsymbol{\theta}_1), \ldots, g_{1,M}(\boldsymbol{\theta}_1)]^T$, and

$$g_{1,i}(\boldsymbol{\theta}_1) = \tan^{-1}(y_s - y_i)/(x_s - x_i). \tag{2.12}$$

Above minimization can be approximated by the Gauss-Newton (GN) method [23],

$$\boldsymbol{\theta}_1^{k+1} = \boldsymbol{\theta}_1^k + \left(\mathbf{H}_{1,k}^T \mathbf{C}_1 \mathbf{H}_{1,k}\right)^{-1} \mathbf{H}_{1,k}^T \mathbf{C}_1 (\boldsymbol{\alpha} - \mathbf{g}_1(\boldsymbol{\theta}_1^k)), \tag{2.13}$$

where $\mathbf{H}_{1,k} = \partial \mathbf{g}_1(\boldsymbol{\theta}_1)/\partial \boldsymbol{\theta}_1|_{\boldsymbol{\theta}_1 = \boldsymbol{\theta}_1^k}$. Note that $\mathbf{H}_{1,k}$ is equal to $\mathbf{A}$ when $\mathbf{x}_s^o = \boldsymbol{\theta}_1^k$ and $\mathbf{x}^o = \mathbf{x}$. Now assume that the ML estimator tries to estimate the observer position as well as the target location using the joint PDF of bearing measurement and observer position. Consequently, the ML estimate is

$$\hat{\boldsymbol{\theta}}_{\text{ML}} = \arg\min_{\boldsymbol{\theta}} (\boldsymbol{\beta} - \mathbf{g}(\boldsymbol{\theta}))^T \mathbf{C} (\boldsymbol{\beta} - \mathbf{g}(\boldsymbol{\theta})), \tag{2.14}$$

where $\mathbf{C} = \text{blkdiag}[\mathbf{C}_1, \mathbf{C}_2] = \text{blkdiag}[\mathbf{\Psi}_{\boldsymbol{\alpha}}^{-1}, \mathbf{\Psi}_{\mathbf{x}}^{-1}]$, and $\mathbf{g}(\boldsymbol{\theta}) = [\mathbf{g}_1(\boldsymbol{\theta})^T, \boldsymbol{\theta}_2]^T$. Similar to (2.11), the minimization of (2.14) can be approximated using GN method [23], therefore,

$$\boldsymbol{\theta}^{k+1} = \boldsymbol{\theta}^k + \left(\mathbf{H}_k^T \mathbf{C} \mathbf{H}_k\right)^{-1} \mathbf{H}_k^T \mathbf{C} (\boldsymbol{\beta} - \mathbf{g}(\boldsymbol{\theta}^k)), \tag{2.15}$$

where $\mathbf{H}_k = \partial \mathbf{g}(\boldsymbol{\theta})/\partial \boldsymbol{\theta}|_{\boldsymbol{\theta} = \boldsymbol{\theta}^k}$. Partitioning the second term of right hand side of (2.15) for $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ yields

$$\begin{bmatrix} \hat{\mathbf{X}}_k^{-1} \mathbf{H}_{1,k}^T \mathbf{C}_1 (\boldsymbol{\alpha} - \mathbf{g}_1(\boldsymbol{\theta}^k)) + \hat{\mathbf{X}}_k^{-1} \mathbf{C}_2 (\mathbf{x} - \boldsymbol{\theta}_2^k) \\ \mathbf{U}_k^{-1} \mathbf{C}_2 (\mathbf{x} - \boldsymbol{\theta}_2^k) \end{bmatrix}, \tag{2.16}$$

where $\mathbf{U}_k = \hat{\mathbf{Z}}_k - \hat{\mathbf{Y}}_k^T \hat{\mathbf{X}}_k^{-1} \hat{\mathbf{Y}}_k$, and $\hat{\mathbf{X}}_k$, $\hat{\mathbf{Y}}_k$, and $\hat{\mathbf{Z}}_k$ are equal to $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ respectively by setting $\mathbf{x}_s^o = \boldsymbol{\theta}_1^k$ and $\mathbf{x}^o = \boldsymbol{\theta}_2^k$. Based on our computer simulations, for any initialization of $\boldsymbol{\theta}_2$ sufficiently close to $\mathbf{x}$, $\boldsymbol{\theta}_2^k$ converges to $\mathbf{x}$ after some iterations, therefore, the term $\mathbf{x} - \boldsymbol{\theta}_2^k$ in (2.16) vanishes and the final solution of (2.15) for $\boldsymbol{\theta}_2$ would be identical to the nominal position of observer. In addition, hereafter, the updating terms for $\boldsymbol{\theta}_1$ in (2.16) would be the same as given in (2.13) and eventually after convergence, (2.15) reaches to the same estimate for the target location as (2.13). It should be mentioned that the minimization of (2.11) and (2.14) using MATLAB routine `fminsearch` (a derivative-free method) also yields the same estimate for target location. In conclusion, according to our simulations, we think that both ML estimators ignoring and considering observer position uncertainties achieve the same result for target location. Furthermore, by applying the joint PDF, we are unable to find a better estimate for the observer position than the nominal value.

2.4 Localization Algorithms

In this section, we develop different algorithms for solving the bearing-only localization problem defined in Section II. We start with ML algorithm then we continue with introducing the linear algorithms, i.e., LS, and TLS.

2.4.1 Least Squares

ML estimator cost function is non-linear and finding its global minimum needs convoluted computations. The solution of the ML can be approximately calculated using iterative optimization techniques such as GN method mentioned in the previous section [23]. Iterative algorithms like GN requires a good initialization to make certain that the algorithm converges to the global minimum of the cost function. Otherwise, the algorithms would find either a saddle point or local minimum and stops iteration which leads to a high estimation error. One way to deal with this problem is that the localization model is linearize by introducing some assumptions. The linearizing of bearing-only localization model of (2.1) is based on assumption that the bearing measurement errors are sufficiently small [10]. Consider (2.1), it can be written as

$$\tan(\alpha_i^o) = \frac{\sin(\alpha_i^o)}{\cos(\alpha_i^o)} = \frac{y_s^o - y_i^o}{x_s^o - x_i^o}. \tag{2.17}$$

By cross multiplying

$$x_s^o \sin(\alpha_i^o) - y_s^o \cos(\alpha_i^o) = x_i^o \sin(\alpha_i^o) - y_i^o \cos(\alpha_i^o). \tag{2.18}$$

In the presence of noise, (2.18) can be expressed in matrix form

$$\mathbf{G}\boldsymbol{\theta}_1 = \mathbf{h}, \tag{2.19}$$

where $\boldsymbol{\theta}_1$ defined earlier is location of target and

$$\mathbf{G} = \begin{bmatrix} \sin\alpha_1 & -\cos\alpha_1 \\ \vdots & \vdots \\ \sin\alpha_M & -\cos\alpha_M \end{bmatrix}, \mathbf{h} = \begin{bmatrix} x_1 \sin\alpha_1 - y_1 \cos\alpha_1 \\ \vdots \\ x_M \sin\alpha_M - y_M \cos\alpha_M \end{bmatrix}. \tag{2.20}$$

The least squares solution of (2.19) is (if $\mathbf{G}$ is full rank) [23],

$$\hat{\boldsymbol{\theta}}_{1,\mathrm{LS}} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{h}. \tag{2.21}$$

Unlike the ML estimator, the LS estimator has a closed-form solution and does not need iterative computations.

2.4.2 Weighted Least Squares

The disturbance due to bearing measurement and observer position noises are unequally sized. Therefore, the performance of LS algorithm can be enhanced by defining a weighting matrix to minimization problem. Like ML estimator, we also derive weighted least squares expressions in two case; ignoring and considering observer position error.

In the former case, the estimator neglects the noise in observer position. In the presence of bearing measurement noise, substituting (2.3) in (2.18), we have,

$$x_s^o \sin(\alpha_i - n_i) - y_s^o \cos(\alpha_i - n_i) = x_i \sin(\alpha_i - n_i) - y_i \cos(\alpha_i - n_i). \quad (2.22)$$

Expanding trigonometry elements, and using the approximations $\sin(n_i) \approx n_i$ and $\cos(n_i) \approx 1$ which are valid if the bearing measurement noises are sufficiently small, equation (2.22) turns into,

$$x_s^o \sin(\alpha_i) - y_s^o \cos(\alpha_i) = x_i \sin(\alpha_i) - y_i \cos(\alpha_i) + \epsilon_{1,i}, \quad (2.23)$$

where $\epsilon_{1,i}$ is residual error in (2.22),

$$\epsilon_{1,i} = n_i\big[(x_s^o - x_i)\sin(\alpha_i) - (y_s^o - y_i)\cos(\alpha_i)\big] = n_i d_i, \quad (2.24)$$

Expressing (2.23) into matrix form,

$$\mathbf{G}\boldsymbol{\theta}_1 = \mathbf{h} + \boldsymbol{\epsilon}_1. \quad (2.25)$$

The weighted least squares solution of (2.25) is [23],

$$\hat{\boldsymbol{\theta}}_{1,\mathrm{WLS1}} = (\mathbf{G}^T\mathbf{W}_1\mathbf{G})^{-1}\mathbf{G}^T\mathbf{W}_1\mathbf{h}, \quad (2.26)$$

where $\mathbf{W}_1$ is the weighting matrix which is equal to the inverse of the covariance matrix of residual error (2.24),

$$\mathbf{W}_1 = \mathrm{E}[\boldsymbol{\epsilon}_1\boldsymbol{\epsilon}_1^T]^{-1} = (\mathbf{B}_1\boldsymbol{\Psi}_{\boldsymbol{\alpha}}\mathbf{B}_1^T)^{-1}, \quad (2.27)$$

where $\mathbf{B}_1 = \mathrm{diag}(d_1, d_2, \ldots, d_M)$. This algorithm was previously introduced as Stansfield estimator in [10]. The above algorithm was also indicated in [11] where the authors concluded that if the distance information d_i are not available, WLS algorithm cannot be applied. It should be noted that the weighting matrix $\mathbf{W}$ depends on the true position of the target $\mathbf{x}_s^o$ which is not available for estimator. Therefore, the WLS estimator can be approximated in two steps. In the first step, we use identity matrix for WLS algorithm, i.e., $\mathbf{W} = \mathbf{I}_{M\times M}$. Indeed, WLS estimator changes to LS estimator defined in (2.21). For next step, we use estimated target location for computing the weighing matrix (2.34) for WLS algorithm.

In the second case, we also take the observer position error into account. Replacing true values with noisy ones in (2.18), we obtain,

$$x_s^o \sin(\alpha_i - n_i) - y_s^o \cos(\alpha_i - n_i) = (x_i - v_{x,i})\sin(\alpha_i - n_i) - (y_i - v_{y,i})\cos(\alpha_i - n_i). \quad (2.28)$$

By expanding trigonometry elements and rearranging, the equation (2.28) becomes,

$$x_s^o \sin(\alpha_i) - y_s^o \cos(\alpha_i) = x_i \sin(\alpha_i) - y_i \cos(\alpha_i) + \epsilon_{2,i}, \quad (2.29)$$

where $\epsilon_{2,i}$ is residual error in (2.28),

$$\epsilon_{2,i} = n_i d_i + v_{x,i}(n_i\cos(\alpha_i) + \sin(\alpha_i)) + v_{y,i}(n_i\sin(\alpha_i) - \cos(\alpha_i)). \quad (2.30)$$

Simplifying,

$$\epsilon_{2,i} = n_i d_i + n_i \dot{\mathbf{g}}_i^T \mathbf{v}_i + \mathbf{g}_i^T \mathbf{v}_i, \tag{2.31}$$

where $\dot{\mathbf{g}}_i = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right] \mathbf{g}_i$, and $\mathbf{v}_i = [v_{x,i}, v_{x,i}]^T$ the noise vector of the ith position of the observer. Expressing (2.29) into matrix form,

$$\mathbf{G}\boldsymbol{\theta}_1 = \mathbf{h} + \boldsymbol{\epsilon}_2. \tag{2.32}$$

The weighted least squares solution of (2.32) is,

$$\hat{\boldsymbol{\theta}}_{1,\text{WLS2}} = (\mathbf{G}^T \mathbf{W}_2 \mathbf{G})^{-1} \mathbf{G}^T \mathbf{W}_2 \mathbf{h}, \tag{2.33}$$

where $\mathbf{W}_2$ is the weighting matrix which is equal to the inverse of the covariance matrix of residual error (2.30),

$$\mathbf{W}_2 = \mathrm{E}[\boldsymbol{\epsilon}_2 \boldsymbol{\epsilon}_2^T]^{-1} = (\mathbf{B}_1 \boldsymbol{\Psi}_{\boldsymbol{\alpha}} \mathbf{B}_1^T + \boldsymbol{\Psi}_{\boldsymbol{\alpha}} \mathbf{D}_2 \boldsymbol{\Psi}_{\mathbf{x}} \mathbf{D}_2^T + \mathbf{D}_1 \boldsymbol{\Psi}_{\mathbf{x}} \mathbf{D}_1^T)^{-1}, \tag{2.34}$$

where $\mathbf{D}_1 = \text{blkdiag}[\mathbf{g}_1^T, \mathbf{g}_2^T, \ldots, \mathbf{g}_M^T]$, $\mathbf{D}_2 = \text{blkdiag}[\dot{\mathbf{g}}_1^T, \dot{\mathbf{g}}_2^T, \ldots, \dot{\mathbf{g}}_M^T]$. Like previous case, the weighting matrix $\mathbf{W}_2$ requires the position of the target. Therefore, it can be solved with two-step method as it is mentioned before. Note that if we assume that the true position of the observer is known (i.e., $\boldsymbol{\Psi}_{\mathbf{x}} = \mathbf{0}$), (2.34) reduces to the expression given in [10] for the weighting matrix of WLS when the exact position of the observer is available to the estimator (the so-called Stansfield estimator).

There are some interesting points should to be noted. In both (2.32) and (2.25), since the weighting matrix is a function of the target location, the exact value of the weighting matrix is not known to the estimator and it should be approximated by available data. As mentioned before, the elements of weighting matrix can be approximately calculated by the estimated target location obtained in (2.19). Similar to this technique was previously considered in the literatures and they have been shown that the performance does not diminish remarkably [29]. We have also compare the WLS algorithm with the exact weighting matrix and the WLS estimator which uses the approximate weighting matrix obtained by the mentioned approach. Our computer simulations show that the there is no significant deviation between them.

The most important point is behind the philosophy of the LS [23, 31]. In classic LS problem, an overdetermined set of linear equations $\mathbf{Gx} = \mathbf{h}$ are available. It is also assumed that the data matrix $\mathbf{G}$ to be free of noise and exactly known and all errors are confined in the vector $\mathbf{h}$. Moreover, the weighting matrix in WLS problem is defined as the inverse of covariance matrix of error in vector $\mathbf{h}$. But, in two proposed algorithms mentioned above and also algorithms given in [10, 11, 29], the data matrix $\mathbf{G}$ is also subject to errors. Consequently, both LS and WLS do not respect disturbances in data matrix $\mathbf{G}$. One way regularly used in the studies [10, 29] is that the errors in data matrix $\mathbf{G}$ are moved from left-hand side of equations to the right-hand side and combined with errors in vector $\mathbf{h}$. Although by using this method the performance might be improved, it might be not sufficient. Hence, introducing an algorithm considering errors in both data matrix $\mathbf{G}$ and observation vector $\mathbf{h}$ separately is necessary. In the next section, we introduce TLS algorithm which tries to find the target position considering errors in both sides of linear equations.

2.4.3 Total Least Squares

The TLS is an extension to the classic least squares [21]. Consider (2.19), the disturbance of bearing measurement as well as observer position error affect both matrix $\mathbf{G}$ and vector

$\mathbf{h}$. The LS algorithm only respects disturbance in vector $\mathbf{h}$, while the TLS takes errors in both vector $\mathbf{h}$ and matrix $\mathbf{G}$ into account [21, 31]. The TLS solution of (2.19) is [21],

$$\hat{\boldsymbol{\theta}}_{1,\mathrm{TLS}} = (\mathbf{G}^T\mathbf{G} - \sigma_s^2\,\mathbf{I})^{-1}\mathbf{G}^T\mathbf{h}, \tag{2.35}$$

where σ_s is the smallest singular value of matrix $[\mathbf{G}\ \ \mathbf{h}]$. It has been stated that the TLS algorithm has better performance than LS algorithm if we have errors in both data matrix and observation vector [21]. The TLS was developed for bearing localization in [11] where simulation results were used to compare the TLS and LS algorithms and it has been showed that TLS has better performance than LS. In simulation section, we will see this conclusion is not true for every situation.

2.4.4 Weighted Total Least Squares

In TLS algorithm, we presume that the errors in both matrix $\mathbf{G}$ and vector $\mathbf{h}$ are independent and identically distributed (IID). This assumption is not valid in the bearing-only localization problem. Hence, we introduce WTLS estimator which considers correlated noises with different statistical properties for the matrix $\mathbf{G}$ and vector $\mathbf{h}$. The classification of the WTLS was mentioned in [21] based on the structure of the weighting matrix. In contrast to the classic TLS, the WTLS has no closed-form solution. Currently WTLS is formulated as an optimization problem and solved by iterative algorithms. In this section, for the first time we apply WTLS algorithm for bearing-only localization model. In the WTLS algorithm, we not only have to compute the covariance matrix of the residual error in the vector $\mathbf{h}$, but also we require the covariance matrix of residual error in the matrix $\mathbf{G}$, and the covariance between residual errors in $\mathbf{h}$ and $\mathbf{G}$. Consider the ith row of (2.19), substituting true parameters with noisy ones and extracting residual errors, we have

$$\boldsymbol{\epsilon}_{\mathbf{g},i} = [n_i\cos(\alpha_i^o),\, n_i\sin(\alpha_i^o)]^T = n_i\dot{\mathbf{g}}_i, \tag{2.36}$$

which is residual error of the ith row of matrix $\mathbf{G}$. The covariance matrix of (2.36) would be

$$\boldsymbol{\Psi}_{\mathbf{g},i} = \mathrm{E}[\boldsymbol{\epsilon}_{\mathbf{g},i}\boldsymbol{\epsilon}_{\mathbf{g},i}^T] = \dot{\mathbf{g}}_i\boldsymbol{\Psi}_{\boldsymbol{\alpha},i}\dot{\mathbf{g}}_i^T, \tag{2.37}$$

where $\boldsymbol{\Psi}_{\boldsymbol{\alpha},i} = [\boldsymbol{\Psi}_{\boldsymbol{\alpha}}]_{ii}$. The residual error of the ith element of vector $\mathbf{h}$ is

$$\epsilon_{h,i} = n_i\dot{\mathbf{g}}_i^T\mathbf{x}_i + n_i\dot{\mathbf{g}}_i^T\mathbf{v}_i + \mathbf{g}_i^T\mathbf{v}_i. \tag{2.38}$$

The covariance matrix of (2.38) becomes

$$\boldsymbol{\Psi}_{h,i} = \mathrm{E}[\epsilon_{h,i}^2] = \mathbf{x}_i^T\dot{\mathbf{g}}_i\boldsymbol{\Psi}_{\boldsymbol{\alpha},i}\dot{\mathbf{g}}_i^T\mathbf{x}_i + \boldsymbol{\Psi}_{\boldsymbol{\alpha},i}\dot{\mathbf{g}}_i^T\boldsymbol{\Psi}_{\mathbf{x},i}\dot{\mathbf{g}}_i + \mathbf{g}_i^T\boldsymbol{\Psi}_{\mathbf{x},i}\mathbf{g}_i, \tag{2.39}$$

where $\boldsymbol{\Psi}_{\mathbf{x},i} = \boldsymbol{\Psi}_{\mathbf{x}_i}$. Moreover, it is required to derive the covariance matrix between residual errors in $\mathbf{h}$ and $\mathbf{G}$,

$$\boldsymbol{\Psi}_{\mathbf{g}h,i} = \mathrm{E}[\boldsymbol{\epsilon}_{\mathbf{g},i}\epsilon_{h,i}] = \dot{\mathbf{g}}_i\boldsymbol{\Psi}_{\boldsymbol{\alpha},i}\dot{\mathbf{g}}_i^T\mathbf{x}_i. \tag{2.40}$$

It should be noted that in the above derivations we have used the approximations applied in (2.31). Now, we will define the WTLS solution based on the algorithm developed in [26]. First, we rewrite (2.19) as

$$\mathbf{F}\boldsymbol{\Theta} = \mathbf{0}, \tag{2.41}$$

where $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1^T, -1]^T$, and $\mathbf{F} = [\mathbf{G}, \mathbf{h}]$. Let $\boldsymbol{\Psi}_{\mathbf{f},i}$ be the covariance matrix of the ith row of $\mathbf{F}$, then

$$\boldsymbol{\Psi}_{\mathbf{f},i} = \begin{bmatrix} \boldsymbol{\Psi}_{\mathbf{g},i} & \boldsymbol{\Psi}_{\mathbf{g}h,i} \\ \boldsymbol{\Psi}_{\mathbf{g}h,i}^T & \boldsymbol{\Psi}_{h,i} \end{bmatrix}. \tag{2.42}$$

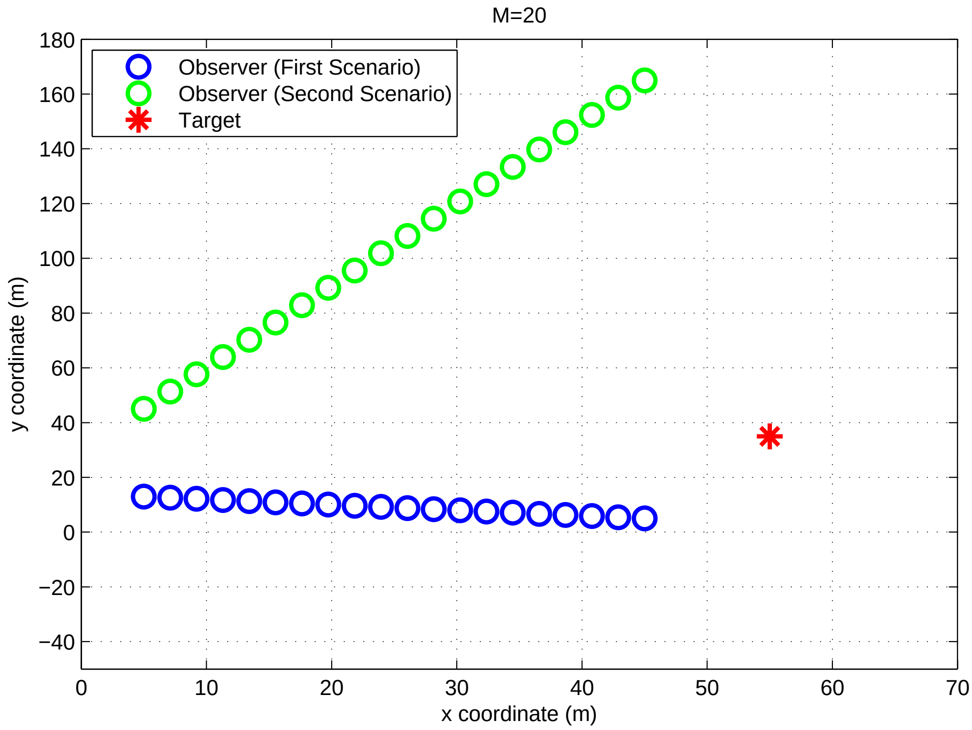

Figure 2.2: The configuration of the target and observer. The distance between the observer and target in the second scenario is much longer than the first scenario.

Therefore, the WTLS problem is defined as [26],

$$\hat{\boldsymbol{\theta}}_{1,\mathrm{WTLS}} = \arg\min_{\boldsymbol{\theta}_1, \Delta\mathbf{f}_i} \sum_{i=1}^{M} \|\boldsymbol{\Psi}_{\mathbf{f},i}^{-1/2} \Delta\mathbf{f}_i\|_2^2 \tag{2.43a}$$

$$\text{subject to } (\mathbf{F} + \Delta\mathbf{F})\boldsymbol{\Theta} = \mathbf{0}, \tag{2.43b}$$

where $\Delta\mathbf{F}$ is a correction matrix trying to compensate errors in matrix $\mathbf{F}$, $\Delta\mathbf{f}_i$ is ith row of matrix $\Delta\mathbf{F}$, and $\|\cdot\|_2$ denotes 2-norm. We have assumed that residual errors in each row of $\mathbf{F}$ are statistically independent. This type of WTLS problem is classified as row-wise WTLS [21]. The problem in (2.43) is an optimization problem. The full details of minimization procedure is given in [26]. After some computations, (2.43) turns into the following minimization problem

$$\hat{\boldsymbol{\theta}}_{1,\mathrm{WTLS}} = \arg\min_{\boldsymbol{\theta}_1} f(\boldsymbol{\theta}_1) = \arg\min_{\boldsymbol{\theta}_1} \sum_{i=1}^{M} \frac{r_i^2(\boldsymbol{\theta}_1)}{q_i(\boldsymbol{\theta}_1)}, \tag{2.44}$$

where,

$$[r_1(\boldsymbol{\theta}_1), r_2(\boldsymbol{\theta}_1), \ldots, r_M(\boldsymbol{\theta}_1)]^T = \mathbf{G}\boldsymbol{\theta}_1 - \mathbf{h}, \tag{2.45}$$

$$q_i(\boldsymbol{\theta}_1) = \boldsymbol{\Theta}^T \boldsymbol{\Psi}_{\mathbf{f},i} \boldsymbol{\Theta}. \tag{2.46}$$

Indeed, $f(\boldsymbol{\theta}_1)$ is the cost function of WTLS should be minimized. To find the minimum of the cost function, the derivative of $f(\boldsymbol{\theta}_1)$ is equated to zero, i.e., $f'(\boldsymbol{\theta}_1) = \partial f(\boldsymbol{\theta}_1)/\partial \boldsymbol{\theta}_1 = 0$, where

$$f'(\boldsymbol{\theta}_1) = 2\sum_{i=1}^{M} \left[\mathbf{g}_i \frac{r_i(\boldsymbol{\theta}_1)}{q_i(\boldsymbol{\theta}_1)} - (\boldsymbol{\Psi}_{\mathbf{g},i}\boldsymbol{\theta}_1 - \boldsymbol{\Psi}_{\mathbf{g}h,i}) \frac{r_i^2(\boldsymbol{\theta}_1)}{q_i^2(\boldsymbol{\theta}_1)} \right]. \tag{2.47}$$

(2.47) has probably several roots but the root corresponding to the global minimum of (2.44) is the WTLS estimation of target location. In [26] an iterative linear approximation

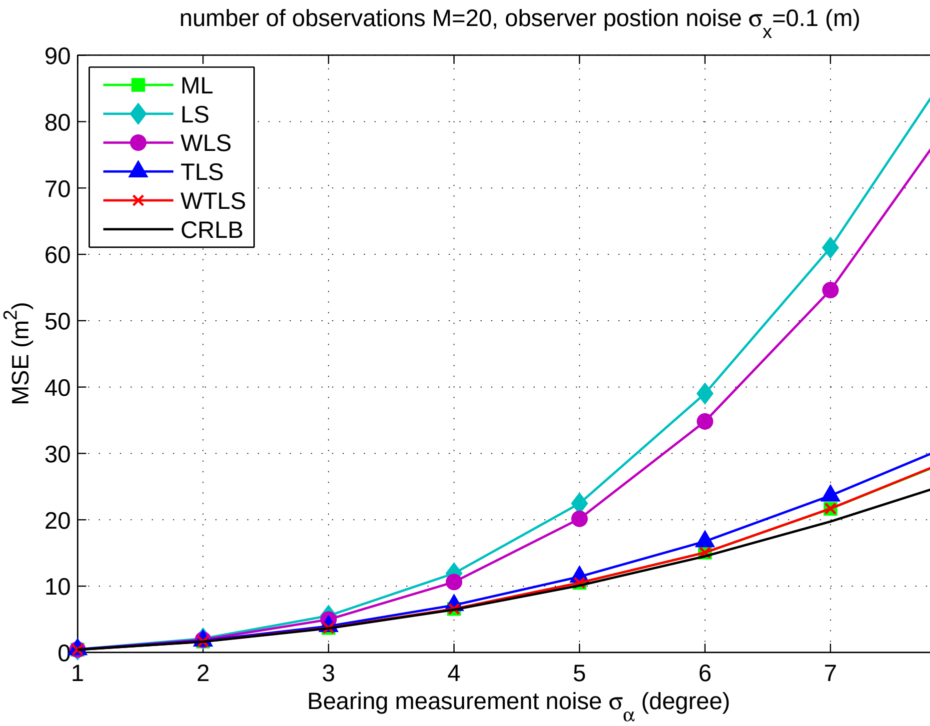

Figure 2.3: The MSE performance of the proposed algorithms versus standard deviation of the bearing measurement noise (the first scenario).

algorithm has been suggested for solving (2.47) which seems to be inappropriate in some conditions. Effective numerical methods for finding the roots of (2.47) can be found in [32]. In our computer simulations, we have employed MATLAB routine `fsolve` with default settings, which uses *Dogleg* algorithm. Like the ML estimator, WTLS also has convergence problem due to the nonlinearity behavior of the cost function [26]. Although it has been shown that for large sample size and sufficiently close initialization, the algorithm converges certainly to the global minimum of the cost function [26], it is still possible that the algorithm either converges to a local minimum or diverges.

2.5 Simulation Results

In this section, computer simulations are conducted to evaluate the performance of the proposed algorithms. Two scenarios for the simulations were considered. Fig. 2.2 depicts the configuration of the target and observer in both scenarios. In the first scenario which is the same as the configuration in [11], the target is located at $[55, 35]^T$, the observer trajectory is $y = -0.2x+14$ for $5 < x < 45$, and observer obtains M bearing measurements in equal distant points. In the second scenario, the target location remains as the first scenario and the observer trajectory is $y = 3x + 30$ for $5 < x < 45$. The bearing measurements and the nominal observer position are generated by adding zero mean Gaussian random variables with covariance matrix $\mathbf{\Psi}_{\boldsymbol{\alpha}} = \sigma_\alpha^2 \mathbf{I}_{M\times M}$ and $\mathbf{\Psi}_{\mathbf{x}} = \sigma_x^2 \mathbf{I}_{2M\times 2M}$, respectively to true values. The values of σ_α^2 and σ_x^2 are indicated in each figure. The mean square error (MSE) of each algorithm is computed by averaging of 10,000 independent realizations as follows

$$\text{MSE} = \frac{1}{10000} \sum_{i=1}^{10000} \|\hat{\mathbf{x}}_s^i - \mathbf{x}_s^o\|_2^2, \tag{2.48}$$

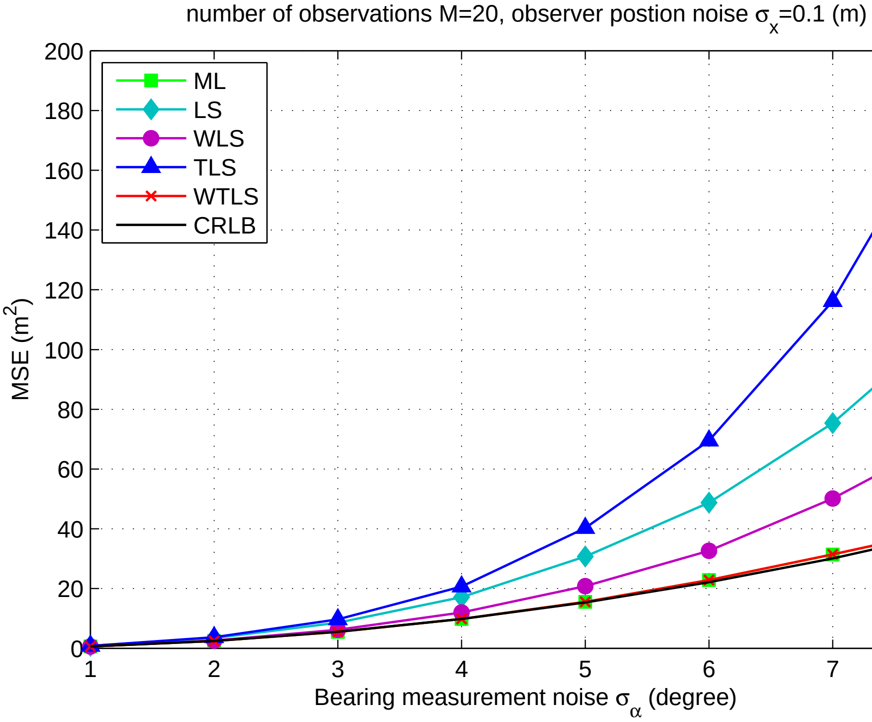

Figure 2.4: The MSE performance of the proposed algorithms versus standard deviation of the bearing measurement noise (the second scenario).

where $\hat{\mathbf{x}}_s^i$ is the estimated source location at ith noise realization. The plotted CRLB is computed as trace[CRLB($\mathbf{x}_s^o$)] in (2.9a).

Fig. 2.3 shows the MSE of the proposed algorithms versus the standard deviation of bearing measurement noise when the number of observations is $M = 20$ and the standard deviation of the observer position noise is $\sigma_x = 0.1$ m. The ML estimator is calculated using GN method [9]. We have used the true position of target as the initialization of ML and WTLS to increase the probability that the algorithms converge to the global minimum. It can be seen that the WLS algorithm performs better than LS. The TLS has remarkably better performance than LS and WLS since it respects disturbances in the data matrix. Furthermore, WTLS, and ML have very close performance and can attain the CRLB accuracy for bearing noise standard deviation under 5°.

The MSE of proposed algorithms as a function of the standard deviation of bearing measurement noise for the second scenario is shown in Fig. 2.4. The number of observations and observer position noise remain as Fig. 2.3. The WTLS and ML show a similar performance and achieve the CRLB for bearing noise standard deviation under 5°. The LS has higher MSE than WLS. In this case, the TLS has not better MSE than the LS. The reason is that, for TLS it is assumed that the errors in matrix $\mathbf{G}$ and vector $\mathbf{h}$ are independent and equally sized, however, (2.36) and (2.38) show that the errors in $\mathbf{G}$ and $\mathbf{h}$ depend on the observer position and since the distance between the first and the last observation in the second scenario is almost three times more than the first scenario, the errors in $\mathbf{G}$ and $\mathbf{h}$ of the latter will be unequally sized more severely than the former. Consequently, the assumption in TLS is not valid anymore and its performance will degrade.

In Fig. 2.5, we compare the MSE of the proposed algorithms in the first scenario versus standard deviation of observer position noise. The number of observations is $M = 20$ and the standard deviation of the bearing noise is $\sigma_\alpha = 2°$. The MSE of all algorithms get worse as the noise on observer position increases. The order of LS, WLS, and TLS is the

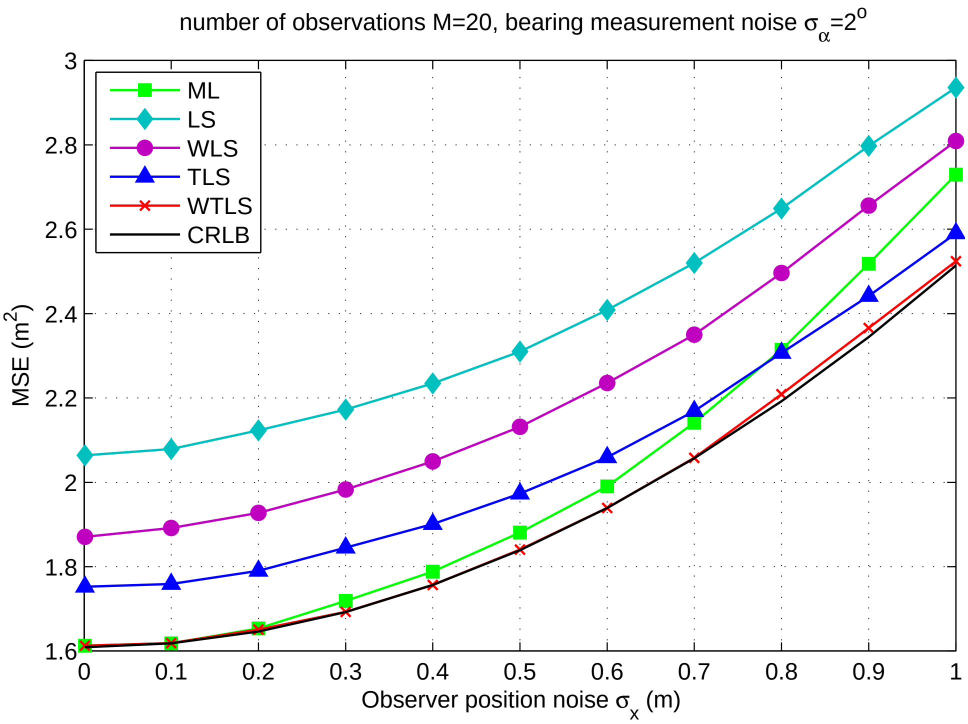

Figure 2.5: The MSE performance of the proposed algorithms versus standard deviation of the observer position noise (the first scenario).

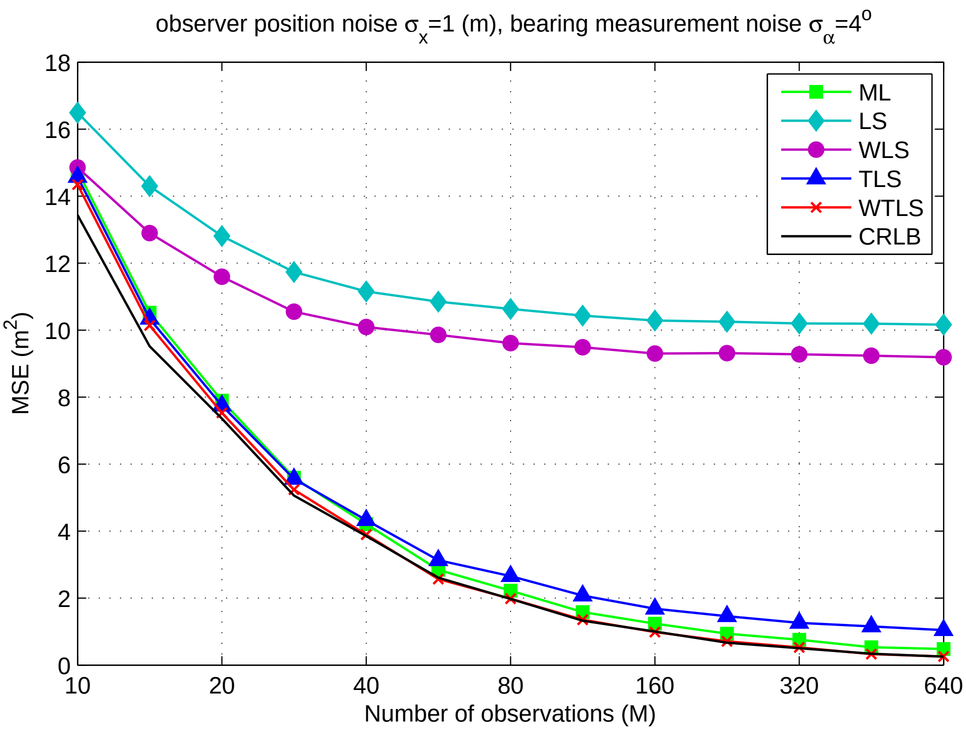

Figure 2.6: The MSE performance of the proposed algorithms versus the number of observations (the first scenario).

same as Fig. 2.3. The ML and WTLS have the optimum performance for lower noise (less than 0.2 m). However, the MSE of ML intensifies as the noise on the observer position increases, which is consistent with the results in [11], while the WTLS performance stays close to the CRLB. Although the ML estimator is expected to be asymptotically efficient, efficiency is not guaranteed for a finite number of observations [23]. Therefore, we expect the ML gets back to an efficient estimator for sufficiently large data records as indicated in Fig. 2.6.

Fig. 2.6 depicts the MSE of proposed algorithms for different number of observations in the first scenario when the standard deviation of bearing measurement and observer position noises are 4° and 1 m respectively. It can be seen that when the number of observations increases, the MSE of all algorithms diminishes. However, the MSE decline for LS and WLS is very slow and almost flat for the large number of observations (i.e., greater than 160) because they do not consider the disturbances in the data matrix $\mathbf{G}$ in (2.19). On the other hand, the WTLS obtains the CRLB performance by increasing the number of observations presenting asymptotically efficient behavior. The MSE of the ML is also interesting. It has inferior performance compared to TLS for the number of observations less than 20, but it surpasses TLS for greater number of observations. We can conclude that it might be optimal for large data records.

Chapter 3

Received Signal Strength-Based Sensor Localization

3.1 Introduction

As mentioned in previous sections, there are many types of measurement used for localization in a WSN. Among them, the RSS measurement is always an interesting method because of its low complexity and costs of devices [2].

Anchor sensor is equipped with a receivers received signal strength indicator (RSSI) circuit to measure the power transmitted by sources. The RSS measurements contain information from distance between anchors and the source. There are many localization techniques based on RSS measurements in the literature. The ML estimator and the CRLB were derived in [3]. In addition, RSS linear estimators such as LS and constrained least squares (CLS) were studied in [5]. The RSS localization with unknown path loss was investigated in [33] where the corresponding CRLB and ML estimator were introduced.

To compute ML solution, it is required to minimize a nonconvex cost function which is computationally intensive. The ML cost function should be solved by iterative optimization algorithms that requires a very good initialization to make sure that the algorithm converges to the global minimum [23]. Convergence problems of the ML estimator can be addressed by using semidefinite programming (SDP) techniques, in which the ML cost function is approximated with a convex function [18, 19, 25, 34, 35]. In [34, 35], the general localization problem in WSN was solved. Their SDP algorithm is applicable if we have noisy distances between anchors and sensor which can be easily obtained from time-of-arrival measurements. However, in the case of RSS localization, we first have to estimate the distances from RSS measurements [2] and then apply their algorithm. Another SDP approach was introduced in [18] where the authors introduced an SDP technique which can be directly applied on RSS measurements and it is not required to find the distances.

RSS-based localization requires a calibration between the source and anchors [2]. Since, in the RSS model, the measurement is a function of transmit power, finding the location of the source is not feasible as far as its transmit power is not available at anchors. Consequently, the source must transfer its transmit power to the anchors during RSS measurements which needs additional hardware in both source and anchors [2].

In this work, we assume that the anchors are not aware of the source transmit power. Dealing to this problem, in general, we introduce two methods. In the first method, we estimate the unknown transmit power along with source location (we call this method URSS). In the second method, the dependency of the unknown transmit power in RSS model is eliminated from all measurements by using RSS difference between two anchors and a suitable estimator is applied in consequence, hereafter we call this method DRSS.

For both methods, we propose a novel SDP approach to transform the ML or the nonlinear least-squares (NLS) cost function to a convex one by using approximations and relaxation techniques. Our proposed SDP approach is different from those studied in [18, 25] since there is no information about transmit power. Certainly, for the proposed problem, it is also not possible to use the algorithms derived in [34, 35] because we cannot extract any knowledge about distances from RSS measurement when there is a total lack of information about source transmit power [2]. We further linearize the proposed measurement model and apply the least squares (LS) solution to our linear model. We also derive the novel weighted total least squares estimator to enhance the performance of LS [22, 26]. Although RSS localization is generally biased [3], we employ the corresponding CRLB as a benchmark to compare the performance of proposed algorithms.

The rest of chapter is organized as follows. In Section II, we describe the proposed RSS localization model. In Section III, the CRLB as a performance bound and ML estiamtor as an optimal estimator are derived. The proposed localization algorithms are derived in Section IV. The performances of proposed algorithm are evaluated through computer simulations in Section V.

3.2 System Model

Through this chapter, we have used the following notations. $\|\cdot\|_2$ denotes ℓ_2 norm, $\operatorname{diag}\{\cdot\}$ denotes the diagonal matrix. $\mathbf{I}_M$ and $\mathbf{0}_M$ denote M by M identity and zero matrices respectively. For arbitrary symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, we write $\mathbf{A} \succeq \mathbf{B}$ if $\mathbf{A} - \mathbf{B} \succeq 0$ that means $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

Let $\mathbf{x}_s = [x_s, y_s]^T \in \mathbb{R}^2$ be the coordinates of the source to be determined. Denote by $\mathcal{C} = \{1, \ldots, M\}$ the set of indices of the anchors connected to the source and by $\mathbf{x}_i = [x_i, y_i]^T \in \mathbb{R}^2$, $i \in \mathcal{C}$ the known location of anchor nodes. Under the log-distance path loss and log-normal shadowing model, the average received power (in dB) at ith anchor, P_i, is modeled as [3]

$$P_i = P_0 - 10\beta \log_{10} \frac{d_i}{d_0} + n_i, \qquad i \in \mathcal{C}, \tag{3.1}$$

where P_0 is the reference power at reference distance d_0 (which depends on transmit power), β is the path loss exponent, $d_i = \|\mathbf{x}_s - \mathbf{x}_i\|_2$ is the true distance between source and ith anchor, and n_i for $i \in \mathcal{C}$ are the log-normal shadowing term modeled as iid zero-mean Gaussian random variables with variance σ_{dB}^2. Without loss of generality, we can assume that $d_0 = 1$m. We also assume that other calibration parameters such as antenna gain are included in P_0.

3.3 Performance Bound and Optimal Estimator

As mentioned before, the CRLB defines a lower bound for the variance (or MSE) of any unbiased estimators which is usually applied for the performances comparison of different unbiased estimators [23]. To compute the CRLB, let $\boldsymbol{\theta} = [\mathbf{x}_s^T, P_0]^T$ be the unknown parameter vector to be estimated. We have to compute the Fisher information matrix of measurements model to compute the corresponding CRLB. The Fisher information matrix of model (3.1) is [23]

$$\boldsymbol{\mathcal{I}}_{m,n} = \left[\frac{\partial \mathbf{g}(\boldsymbol{\theta})}{\partial \theta_m}\right]^T \mathbf{C}^{-1} \left[\frac{\partial \mathbf{g}(\boldsymbol{\theta})}{\partial \theta_n}\right], \quad m, n = 1, 2, 3, \tag{3.2}$$

where $\mathbf{C} = \sigma_{dB}^2 \mathbf{I}_M$, $\mathbf{g}(\boldsymbol{\theta}) = [g_1(\boldsymbol{\theta}), g_2(\boldsymbol{\theta}), \ldots, g_M(\boldsymbol{\theta})]^T$, $g_i(\boldsymbol{\theta}) = P_0 - 10\beta \log_{10} d_i$, and

$$\frac{\partial \mathbf{g}(\boldsymbol{\theta})}{\partial \theta_i} = \begin{bmatrix} \frac{\partial g_1}{\partial \theta_i} & \frac{\partial g_2}{\partial \theta_i} & \cdots & \frac{\partial g_M}{\partial \theta_i} \end{bmatrix}^T, \tag{3.3}$$

where

$$\frac{\partial g_i}{\partial \theta_1} = \frac{\partial g_i}{\partial x_s} = \frac{10\beta}{\ln 10} \frac{x_i - x_s}{d_i^2}, \tag{3.4a}$$

$$\frac{\partial g_i}{\partial \theta_2} = \frac{\partial g_i}{\partial y_s} = \frac{10\beta}{\ln 10} \frac{y_i - y_s}{d_i^2}, \tag{3.4b}$$

$$\frac{\partial g_i}{\partial \theta_3} = \frac{\partial g_i}{\partial P_0} = 1. \tag{3.4c}$$

The CRLB of unknown parameters are the diagonal elements of the inverse of Fisher information matrix (3.2) as [23]

$$\mathrm{CRLB}(\theta_k) = [\boldsymbol{\mathcal{I}}^{-1}]_{kk}, \quad k = 1, 2, 3 \tag{3.5}$$

3.3.1 Maximum Likelihood

Let $\boldsymbol{\theta} = [\mathbf{x}_s^T, P_0]^T$ be the unknown parameter vector to be estimated, the ML estimator based on the measurements in (3.1) is computed by following nonconvex optimization problem [23]

$$\hat{\boldsymbol{\theta}}_{\mathrm{ML}} = \arg\min_{\boldsymbol{\theta}} \sum_{i \in \mathcal{C}} \left(P_i - P_0 + 10\beta \log_{10} d_i\right)^2. \tag{3.6}$$

We can express (3.6) alternatively as

$$\hat{\boldsymbol{\theta}}_{\mathrm{ML}} = \arg\min_{\boldsymbol{\theta}} \sum_{i \in \mathcal{C}} \log_{10}^2 \frac{h_i \lambda_i}{\alpha}, \tag{3.7}$$

where $h_i \triangleq d_i^2$, $\lambda_i \triangleq 10^{P_i/5\beta}$, and $\alpha \triangleq 10^{P_0/5\beta}$. The solution of (3.7) is not closed-form, but can be approximated, for instance, by the Gauss-Newton (GN) method [23]as [23]

$$\boldsymbol{\theta}^{k+1} = \boldsymbol{\theta}^k + \left(\mathbf{H}_k^T \mathbf{H}_k\right)^{-1} \mathbf{H}_k^T (\mathbf{p} - \mathbf{g}(\boldsymbol{\theta}^k)), \tag{3.8}$$

where $\mathbf{H}_k = \partial \mathbf{g}(\boldsymbol{\theta}) / \partial \boldsymbol{\theta}|_{\boldsymbol{\theta} = \boldsymbol{\theta}^k}$, and $\mathbf{p} = [P_1, P_2, \ldots, P_M]^T$ is observation vector. The drawback of GN method is that it requires a good initialization to make sure that the algorithm converges to the global minimum.

3.4 Localization Algorithms

3.4.1 Semidefinite Programming

The cost function of ML is severely nonlinear and nonconvex and finding its global minimum is computationally intensive. By using SDP relaxation, we convert the ML problem to a convex optimization problem. The advantage of SDP problem over ML is that it can be solved with efficient computational methods that certainly converge to its global minimum [36]. As we mentioned earlier, we have two methods to deal with our problem.

Let us start with the first method. Consider (3.1), by rearranging and diving both side by 5β, it can be reformulated as

$$\log_{10} d_i^2 \lambda_i = \frac{P_0}{5\beta} + \frac{n_i}{5\beta}, \qquad i \in \mathcal{C}. \tag{3.9}$$

Taking power of 10 from both side yields

$$d_i^2\lambda_i = \alpha 10^{n_i/5\beta}, \qquad i \in \mathcal{C}. \tag{3.10}$$

For sufficiently small noise, the the right hand side of (3.10) can be approximated using the first-order Taylor series expansion as

$$d_i^2\lambda_i = \alpha\left(1 + \frac{\ln 10}{5\beta}n_i\right), \qquad i \in \mathcal{C}. \tag{3.11}$$

This can be rewritten as

$$h_i\lambda_i = \alpha + n_i', \qquad i \in \mathcal{C}, \tag{3.12}$$

where n_i' is a zero-mean Gaussian random variable with variance $(\ln 10)^2\alpha^2\sigma_{dB}^2/25\beta^2$. Now, corresponding ML estimator of (3.12) is

$$\hat{\mathbf{x}}_s = \arg\min_{\mathbf{x}_s,\alpha} \sum_{i\in\mathcal{C}} \left(h_i\lambda_i - \alpha\right)^2. \tag{3.13}$$

To progress, we have to use another approximation. The ML estimator of (3.13) tries to minimize the ℓ_2 norm of the residual error. For sufficiently small residual error we can approximate (3.13) by using ℓ_1 norm rather than ℓ_2 norm [36]

$$\hat{\mathbf{x}}_s = \arg\min_{\mathbf{x}_s,\alpha} \sum_{i\in\mathcal{C}} \left|h_i\lambda_i - \alpha\right|. \tag{3.14}$$

Indeed, we approximately turn the original ML cost function of (3.7) to another cost function (3.14). The cost function (3.14) is still nonlinear and noncovex. In the next step, we define an auxiliary variable y as

$$h_i = d_i^2 = \|\mathbf{x}_s - \mathbf{x}_i\|_2^2 = y - 2\mathbf{x}_i^T\mathbf{x}_s + \mathbf{x}_i^T\mathbf{x}_i, \quad i \in \mathcal{C} \tag{3.15}$$

where $y = \mathbf{x}_s^T\mathbf{x}_s$. The minimization problem (3.14) can be relaxed to an SDP optimization problem as [36]

$$\min_{\mathbf{x}_s,\alpha,t_i,h_i,y} \sum_{i\in\mathcal{C}} t_i \tag{3.16a}$$

$$\text{s.t.} \; -t_i < h_i\lambda_i - \alpha < t_i, \tag{3.16b}$$

$$h_i = y - 2\mathbf{x}_i^T\mathbf{x}_s + \mathbf{x}_i^T\mathbf{x}_i, \tag{3.16c}$$

$$y \geq \mathbf{x}_s^T\mathbf{x}_s. \tag{3.16d}$$

Solution of (3.16) can be found effectively with optimal algorithms such as interior point method [36]. Moreover, convergence to the global minimum is guaranteed in SDP optimization problems [36].

Note, in (3.16), we have used the inequality constraint (3.16d) instead of the equality to relax our problem to a convex problem [36]. The inequality (3.16d) can be written as a linear matrix inequality (LMI) using the Schur complement [36]

$$\begin{bmatrix} y & \mathbf{x}_s^T \\ \mathbf{x}_s & \mathbf{I}_2 \end{bmatrix} \succeq \mathbf{0}_3. \tag{3.17}$$

Here, we continue with describing the SDP optimization for the second method. We select an anchor as a reference (with index $r \in \mathcal{C}$) and calculate DRSS measurements. Hence (3.1) is expressed as

$$P_{r,i} = P_r - P_i = 10\beta\log_{10}\frac{d_i}{d_r} + m_i\,, \qquad i \in \mathcal{C}, i \neq r, \tag{3.18}$$

where P_r is the received power at the reference anchor, d_r is distance between the reference anchor and the source, and $m_i = n_r - n_i$ is a zero-mean Gaussian random variable with variance $2\sigma_{dB}^2$. Since the noise of reference anchor appears in all DRSS measurements, they are correlated, which makes it difficult to relax the ML problem into an SDP problem. For this reason, we proceed with the LS estimator instead. The LS solution of (3.18) is [23]

$$\hat{\mathbf{x}}_s = \arg\min_{\mathbf{x}_s} \sum_{i \in \mathcal{C}, i \neq r} \left(P_{r,i} - 10\beta \log_{10} \frac{d_i}{d_r} \right)^2. \tag{3.19}$$

Using the procedure mentioned for previous case, we can approximate solution of (3.19) with the following optimization problem,

$$\hat{\mathbf{x}}_s = \arg\min_{\mathbf{x}_s} \sum_{i \in \mathcal{C}, i \neq r} \left| d_i^2 \vartheta_i - d_r^2 \right|. \tag{3.20}$$

where $\vartheta_i = 10^{P_{r,i}/5\beta}$. The minimization problem (3.20) can be relaxed to an SDP optimization problem as [36]

$$\min_{\mathbf{x}_s, t_i, h_i, h_r, y} \sum_{i \in \mathcal{C}, i \neq r} t_i \tag{3.21a}$$

$$\text{s.t.} \; -t_i < h_i \vartheta_i - h_r < t_i, \tag{3.21b}$$

$$h_i = y - 2\mathbf{x}_i^T \mathbf{x}_s + \mathbf{x}_i^T \mathbf{x}_i, \tag{3.21c}$$

$$h_r = y - 2\mathbf{x}_r^T \mathbf{x}_s + \mathbf{x}_r^T \mathbf{x}_r, \tag{3.21d}$$

$$y \geq \mathbf{x}_s^T \mathbf{x}_s. \tag{3.21e}$$

Now, we have to pick up one of anchors as a reference. Note that the effect of log-normal shadowing is multiplicative to the distance in (3.1) [3], hence, long measured distances have higher error than short ones [3]. Consequently we select the nearest anchor to the source (the anchor with the highest RSS) as a reference anchor to prevent raising more errors in equations.

In summary, to apply the SDP solution for our localization problem, we have approximated the original cost function of ML (or NLS) to another cost function and then relaxed it to a convex problem. In the first step, we have substituted the function $\sum |\lambda_i h_i - \alpha|$ for the function $\sum \log_{10}^2(\lambda_i h_i/\alpha)$. Fig. 3.1a depicts two mentioned functions versus unknown parameters h and α (λ is a known parameter). To compare the cost functions of (3.7) and (3.14), we have used one realization. Five anchors are randomly placed in a square of 20×20 meters and a source located at $[10, 10]^T$. The standard deviation of the log-normal shadowing is 3 dB. Fig. 3.1b shows the cost function of the ML estimator given in (3.7) versus x and y coordinates when we have fixed the value of P_0 at the true value. It can be seen that the ML cost function has a global minimum at $[10.5, 11.5]^T$ (the step of mesh grid is 0.5) and some local minima and saddle points (e.g., a local minimum at $[2.5, 17.5]^T$). The cost function of (3.14) is shown in Fig. 3.1c which is much smoother than (3.7) and has a global minimum at $[10, 11.5]^T$. Fig. 3.1c still requires to be relaxed to a convex shape. In the next step, by using SDP relaxation of (3.16d), we transform function (3.14) to a convex function (3.16). Solution of (3.14) and (3.16) for source location will coincide, if the minimum of (3.16) occurs for $y = \mathbf{x}_s^T \mathbf{x}_s$ or if rank 1 condition for y is satisfied.

3.4.2 Least Squares

In this section, we describe linear estimators for our localization model (3.1). Similar to the previous cases, we have two methods to deal with the unknown transmit power.

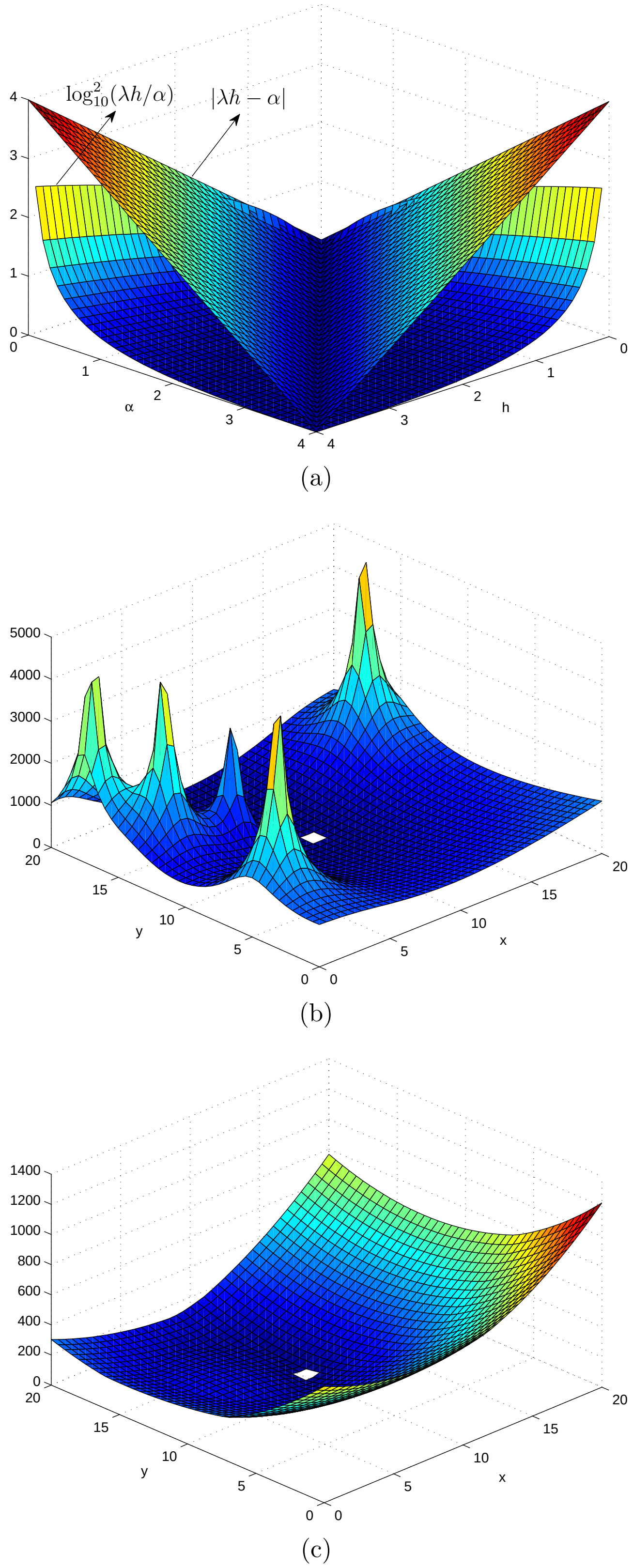

Figure 3.1: (a) depiction of functions $|\lambda h - \alpha|$ and $\log_{10}^2(\lambda h/\alpha)$ versus unknown variables h and α (for simplicity, $\lambda = 1$). (b) cost function of (3.7), (c) cost function of (3.14) versus x and y coordinates, the minimum of the cost functions is indicated with white color.

Consider (3.1), in the absence of noise, we can reformulate it as

$$d_i^2 = \zeta_i \alpha, \qquad i \in \mathcal{C}, \tag{3.22}$$

where $\zeta_i \triangleq 10^{-P_i/5\beta}$. Let $\boldsymbol{\theta}_1 = [\mathbf{x}_s^T, \mathbf{x}_s^T\mathbf{x}_s, \alpha]^T$ be the unknown vector to be estimated, and $k_i = \mathbf{x}_i^T\mathbf{x}_i$. Expanding and rearranging (3.22), we can express (3.22) in matrix form as

$$\mathbf{A}\boldsymbol{\theta}_1 = \mathbf{b}, \tag{3.23}$$

where

$$\mathbf{A} = \begin{bmatrix} 2\mathbf{x}_1^T & -1 & \zeta_1 \\ \vdots & \vdots & \vdots \\ 2\mathbf{x}_M^T & -1 & \zeta_M \end{bmatrix}, \mathbf{b} = \begin{bmatrix} k_1 \\ \vdots \\ k_M \end{bmatrix}. \tag{3.24}$$

The LS solution of (3.23) is [23]

$$\hat{\boldsymbol{\theta}}_{1,\text{LS1}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}. \tag{3.25}$$

Now, we derive the LS estimator for the second method. Consider (3.18), we pick up an anchor as a reference and calculate the DRSS from the other anchors. In the absence of noise, (3.18) would be expressed as

$$d_i^2\vartheta_i = d_r^2, \qquad i \in \mathcal{C}, i \neq r, \tag{3.26}$$

Let $\boldsymbol{\theta}_2 = [\mathbf{x}_s^T, \mathbf{x}_s^T\mathbf{x}_s]^T$ be the unknown vector to be estimated, then (3.26) can be expressed in matrix form as

$$\mathbf{P}\boldsymbol{\theta}_2 = \mathbf{q}, \tag{3.27}$$

where

$$\mathbf{P} = \begin{bmatrix} \vdots & \vdots \\ (2\vartheta_i\mathbf{x}_i - 2\mathbf{x}_r)^T & (1-\vartheta_i) \\ \vdots & \vdots \end{bmatrix}, \mathbf{q} = \begin{bmatrix} \vdots \\ \vartheta_i k_i - k_r \\ \vdots \end{bmatrix}, i \in \mathcal{C}, i \neq r. \tag{3.28}$$

The LS solution of (3.27) is [23]

$$\hat{\boldsymbol{\theta}}_{2,\text{LS2}} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{q}. \tag{3.29}$$

The reference anchor is selected as mentioned before for SDP-DRSS algorithm.

3.4.3 Weighed Total Least Squares

When we have measurement noise in the formulation of LS estimators, the disturbances appear in both data matrix and observation vector. For instance, in (3.27), the value of ϑ_i is subject to measurement error which emerges in both data matrix $\mathbf{P}$ and observation vector $\mathbf{q}$, therefore, both of them corrupted by noises. LS only respects disturbances in the observation vector [23]. The more general case of LS is total least squares (TLS) which can tolerate disturbances in both data matrix and observation vector [21]. TLS assumes that noises in the data matrix and observation vector are equally size and independent and identically distributed. However, this assumption is not valid in our expressions (3.23) and (3.27). The new approach, called weighted total least squares (WTLS) allows us to have unequally sized noises in both data matrix and observation vector [21]. The full details about finding the solution of a WTLS problem is given in [22, 26]. Briefly, the WTLS solution of (3.23) is obtained by the following optimization problem [26]

$$\hat{\boldsymbol{\theta}}_{1,\text{WTLS1}} = \arg\min_{\boldsymbol{\theta}_1} \sum_{i\in\mathcal{C}} \frac{r_i^2}{u_i}, \tag{3.30a}$$

$$\text{s.t.} \quad r_i = \mathbf{a}_i^T \boldsymbol{\theta}_1 - b_i, \tag{3.30b}$$

$$u_i = \boldsymbol{\theta}_1^T \mathbf{W}_{11,i} \boldsymbol{\theta}_1 - 2\boldsymbol{\theta}_1^T \mathbf{W}_{12,i} + \mathbf{W}_{22,i}, \tag{3.30c}$$

where $\mathbf{a}_i$ and b_i are ith row of $\mathbf{A}$ and ith element of $\mathbf{b}$ respectively, and covariance matrices are

$$\mathbf{W}_{11,i} = \mathrm{E}[\mathbf{a}_i^T \mathbf{a}_i] = \mathrm{Var}(\hat{\zeta}_i)\,\mathrm{diag}[0,0,0,1], \tag{3.31a}$$

$$\mathbf{W}_{12,i} = \mathrm{E}[b_i \mathbf{a}_i] = [0,0,0]^T, \tag{3.31b}$$

$$\mathbf{W}_{22,i} = \mathrm{E}[b_i^2] = 0, \qquad i \in \mathcal{C}. \tag{3.31c}$$

Consider (3.23), the noise only appears in ζ_i. Let $\hat{\zeta}_i$ be the value of ζ_i corrupted by the noise given (3.1), then we have $\hat{\zeta}_i = \zeta_i 10^{n_i/5\beta}$. Since n_i is Gaussian random variable, $\hat{\zeta}_i$ has log-normal distribution with variance,

$$\mathrm{Var}(\hat{\zeta}_i) = \hat{\zeta}_i^2 \left(e^{2\sigma_\zeta^2} + e^{\sigma_\zeta^2}\right), \quad \sigma_\zeta = \frac{\sigma_{dB} \ln 10}{5\beta}, \quad i \in \mathcal{C}. \tag{3.32}$$

The cost function of WTLS (3.30) is nonlinear and has not any closed-form solution [26]. Solution of (3.30) can be obtained approximately by iterative algorithms [26].

The corresponding WTLS solution of (3.27) can be derived in a similar manner. The noisy parameter in (3.27) would be ϑ_i. Denote $\hat{\vartheta}_i$ by the value of ϑ_i corrupted by noise in (3.18) which has log-normal distribution with parameters

$$\mu_{\vartheta,i} = \ln \vartheta_i, \quad \sigma_{\vartheta,i} = \frac{2\ln 10}{5\beta}\sigma_{dB}, \tag{3.33}$$

and therefore the variance of $\hat{\vartheta}_i$

$$\mathrm{Var}(\hat{\vartheta}_i) = e^{2\mu_{\vartheta,i}} \left(e^{2\sigma_{\vartheta,i}^2} + e^{\sigma_{\vartheta,i}^2}\right), \qquad i \in \mathcal{S}. \tag{3.34}$$

The WTLS solution of (3.27) is given by

$$\hat{\boldsymbol{\theta}}_{2,\text{WTLS2}} = \arg\min_{\boldsymbol{\theta}_2} \sum_{i\in\mathcal{S}} \frac{r_i^2}{u_i}, \tag{3.35a}$$

where

$$r_i = \mathbf{p}_i^T \boldsymbol{\theta}_2 - q_i, \qquad i \in \mathcal{S}, \tag{3.35b}$$

$$u_i = \boldsymbol{\theta}_2^T \mathbf{V}_{11,i} \boldsymbol{\theta}_2 - 2\boldsymbol{\theta}_2^T \mathbf{V}_{12,i} + \mathbf{V}_{22,i}, \qquad i \in \mathcal{S}, \tag{3.35c}$$

and covariance matrices are

$$\mathbf{V}_{11,i} = \mathrm{E}[\mathbf{p}_i^T \mathbf{p}_i] = \mathrm{Var}(\hat{\vartheta}_i)\,\mathbf{z}_i \mathbf{z}_i^T, \qquad i \in \mathcal{S}, \tag{3.36a}$$

$$\mathbf{V}_{21,i} = \mathrm{E}[q_i \mathbf{p}_i] = \mathrm{Var}(\hat{\vartheta}_i)\,k_i \mathbf{z}_i, \qquad i \in \mathcal{S}, \tag{3.36b}$$

$$\mathbf{V}_{22,i} = \mathrm{E}[q_i^2] = \mathrm{Var}(\hat{\vartheta}_i)\,k_i^2, \qquad i \in \mathcal{S}, \tag{3.36c}$$

where $\mathbf{z}_i = \begin{bmatrix} 2\mathbf{x}_i^T & -1 \end{bmatrix}^T$.

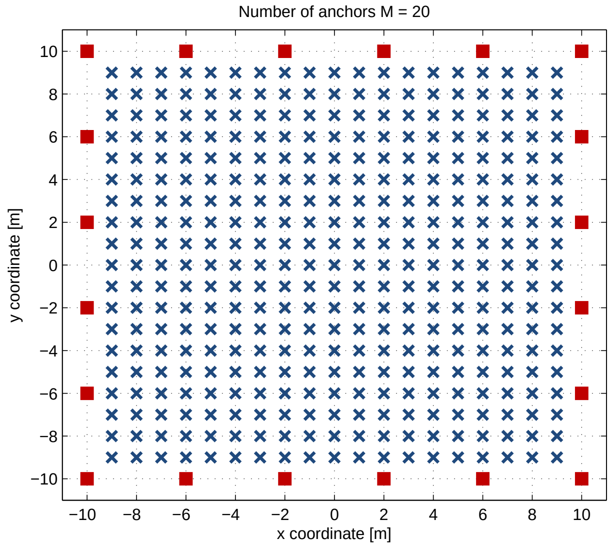

Figure 3.2: The proposed network configuration. The red squares indicate the location of anchors and blue crosses show the location of sensor in each setup.

3.5 Simulation Results

In this section, we compare the performance of proposed algorithms through computer simulations. Fig. 3.2 shows the configuration of the proposed network. Twenty anchors are placed on the sides of a square of $20\,\text{m} \times 20\,\text{m}$ in equal distances and a source is distributed in a square area of $19\,\text{m} \times 19\,\text{m}$. 361 networks is generated and in each network 100 noise realizations are done. The root mean square error (RMSE) of the proposed algorithms and CRLB are computed by averaging over all experiments. The cost function of ML and WTLSs are minimized by MATLAB routine `fminsearch`, with default setting, which uses Nelder-Mead Simplex method. The proposed SDP problems is solved by `CVX` toolbox [37] as the interference and SeDuMi as the solver [38].

The RMSE of the proposed algorithm versus the standard deviation of the log-normal shadowing is depicted in Fig. 3.3. The ML and WTLS algorithms are initialized with the true values to increase the probability of convergence to the global minimum. Fig. 3.3 shows that the performance of the LS algorithms are very poor since they do not respect errors appearing in the data matrix. As we expected, the WTLS estimators perform substantially better than LSs because they respect unequally sized disturbances in both the data matrix and observation vector. However, RMSE of WTLS-URSS is slightly lower than WTLS-DRSS. The reason is that in the derivation of WTLS, we assume that disturbances in each row of data matrix and observation vector are independent (row-wise WTLS [21]), but this assumption is not valid for WTLS-DRSS algorithm since the measurement noise of reference anchor appears in all rows and consequently the rows of data matrix and observation vector are correlated. Furthermore, Fig. 3.3 demonstrate that the ML has a superior performance to other algorithms and only slightly worse than CRLB at low SNR. SDP-URSS performs very well, having a negligible gap with ML. SDP-DRSS performance is moderately worse than SDP-URSS because our SDP-DRSS does not consider the noise correlation due to the reference anchor. Fig. 3.4 depicts the cumulative density function (CDF) of the location error $\|\hat{\mathbf{x}}_s - \mathbf{x}_s\|_2$ of the proposed algorithms when the log-normal shadowing standard deviation is fixed at 3 dB. The order of the proposed algorithms is the same as in Fig. 3.3.

Showing the advantage of SDP approach over other methods, we have considered the previous configuration expect that the ML and WTLSs algorithms were randomly

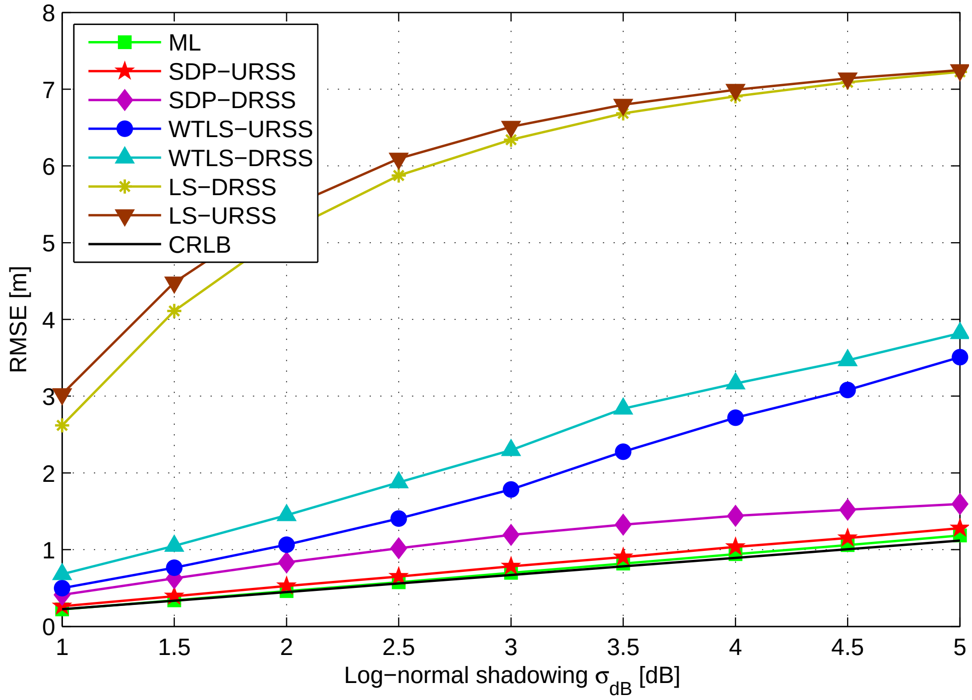

Figure 3.3: The RMSE of the proposed algorithms.

initialized. Fig. 3.5 shows the CDF of proposed algorithms when the configuration is the same as Fig. 3.4 except ML and WTLSs algorithms are randomly initialized. It can be easily seen the performances of ML and WTLSs decrease by using random initialization because they either diverge or converge to a local minimum about 10 percent of times.

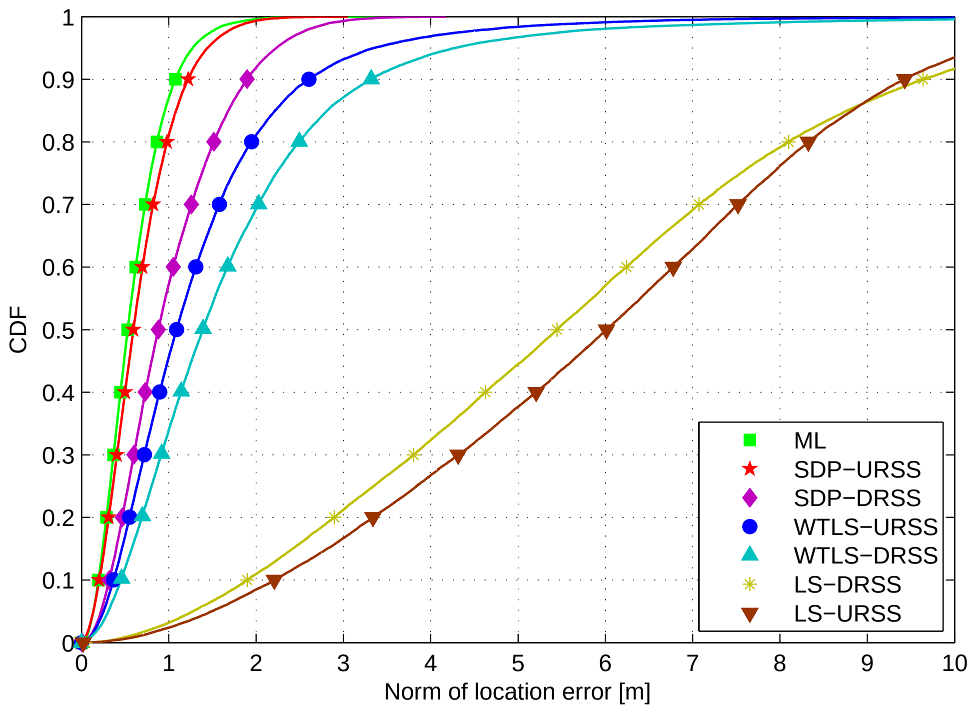

Figure 3.4: The CDF of the proposed algorithms, $\sigma_{dB} = 3$ dB. Iterative algorithms are initialized with true values.

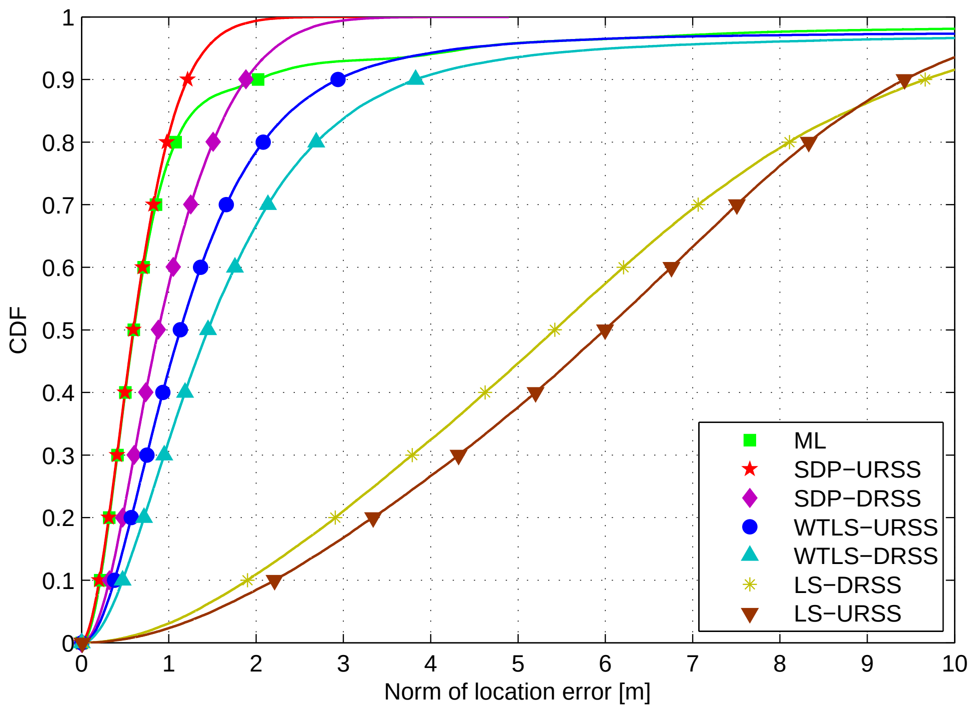

Figure 3.5: The CDF of the proposed algorithms, $\sigma_{dB} = 3$ dB. Iterative algorithms are initialized randomly.

Chapter 4

Conclusions and Future work

4.1 Conclusions

The aspect of localization in WSN was investigated in this thesis. Since the merit of sensors information depends on their location, localization is one of the most important subject in WSNs. Through this thesis, two specific situations in localization were studied. The bearing-only target localization when the observer position is subject to error was surveyed in the first part. Classic bearing-only localization assumes that the exact location of the observer is available, however, in this work it is assumed that the estimator does not know the exact location of observer and only the noise values are accessible. The CRLB of the proposed localization model was derived. The ML, linear LS, WLS, and TLS estimators were developed for this problem and additionally a novel method of positioning based on the weighted total least squares (WTLS) was introduced. It was shown that under some circumstances if the ML estimator does not know about observer position errors, it achieves the same estimate for the target location as the ML estimator knowing about observer position error. Computer simulations were conducted to assess the performance of the proposed algorithms. Simulation results demonstrated that the proposed WTLS method outperforms other methods and obtains the CRLB accuracy asymptotically.

The single source RSS-based localization with unknown transmit power was studied in the second part of the thesis. The transmit power of the source is required in RSS localization to find the location of the source. Dealing with this problem, we introduced two methods. Eliminating the transmit power from RSS measurement by considering differences between pairs of RRS measurements is the first method. The second method is that the transmit power of source is considered as a nuisance parameter and estimated along with source location. The corresponding CRLB was derived for this problem. A novel SDP approach was derived and a partly novel WTLS algorithm was introduced. The simulation results illustrated that the proposed SDP has a significant accuracy very close to the CRLB. Moreover, the proposed WTLS outperforms classic LS algorithms.

4.2 Future Work

In both two studied subjects, the single sensor (or target) localization was investigated. For the first part, we work to extend the algorithms for multi-target localization and also tracking of a target. Using recursive TLS and WTLS would also be another possible future work. Extension to multi-sensor localization and also cooperative localization in RSS with unknown transmit power is the second task we are currently working on.

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