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Positioning Algorithms for Wireless Sensor Networks

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Gothenburg, Sweden 2011

Thesis for the degree of Licentiate of Engineering

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Department of Signals and Systems
Technical Report No. R001/2011
ISSN 1403-266X

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This thesis has been prepared using LaTeX.

Printed by Chalmers Reproservice,
Gothenburg, Sweden, March 2011.

To my parents and Somayeh

To that lofty desire, we cannot attain
Unless your favor advances some paces
Hafez's Divan, Ghazal 182

Abstract

Position information is often one of the key requirements for a wireless sensor network (WSN) to function as intended. Due to drawbacks in using GPS or having to manually setting sensor positions, extracting the position information by means of the network itself has been extensively studied in the literature. In this approach, it is commonly assumed that there are a few sensor nodes with known positions, called reference nodes, and some type of measurements are taken between different nodes.

During the last decade, a number of positioning algorithms have been proposed for the positioning problem. Evaluating the performance of various algorithms based on practical data is an important step in classifying different positioning methods. Paper A studies the performance of different approaches through real range measurements made by two ultra-wideband devices. The main result of Paper A is that no positioning algorithm uniformly shows the best performance in different situations.

Paper B considers the positioning problem using received signal strength measurements when the channel parameters (transmit power and path-loss exponent) are unknown in sensor nodes. Assuming fixed values for the channel parameters, the positioning problem is formulated as finding a point in the intersection of a number of halfplanes (in a 2D network). The well-known approach projection onto convex sets (POCS) is employed to solve the problem. The proposed method gives a good coarse estimate for the positioning problem.

In the literature, it has been shown that cooperation between reference nodes can improve the position estimate. To cope with difficulty in solving the optimal estimator in this type of network, which are mainly due to nonlinearity and nonconvexity issues, a two-step linear estimator is proposed in Paper C to solve the positioning problem. In the first step, a coarse estimate is obtained, and it is refined in the second step. For sufficiently high signal-to-noise-ratios, the proposed estimator attains the optimal performance; i.e., it attains the Cramér-Rao lower bound.

Finally, Paper D formulates the positioning problem as a convex feasibility problem (CFP) for both non-cooperative and cooperative networks. To solve a CFP, two methods based on POCS and outer-approximation are employed (two geometric solutions). The properties of POCS for non-cooperative networks are reviewed and an upper-bound on the position error of POCS is proposed. Simulation results show that the positioning algorithms based on geometric solutions are more robust against non-line-of-sight compared to proposed statistical approaches such as nonlinear least squares.

Keywords: Wireless sensor network, performance metric, measurement model, cooperative and non-cooperative networks, maximum likelihood estimator, nonlinear least squares, linear least squares, projection onto convex sets, outer-approximations

List of Publications

Included papers

This thesis is based on the following papers:

[A] M. R. Gholami, E. G. Ström, and M. Rydström, "Indoor sensor node positioning using UWB range measurments," *in Proc. 17th European Signal Processing Conference (EUSIPCO)*, pp. 1943–1947, 2009.

[B] M. R. Gholami, M. Rydström, and E. G. Ström, "Positioning of node using plane projection onto convex sets," *in Proc. IEEE Wireless Communication and Networking Conference (WCNC)*, Apr. 2010.

[C] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, "Hybrid TW-TOA/TDOA positioning algorithms for cooperative wireless networks," to appear in Proc. *IEEE International Conference on Communications (ICC)*, 2011.

[D] M. R. Gholami, H. Wymeersch, E. G. Ström, and M. Rydström, "Wireless network positioning as a convex feasibility problem ," submitted to *EURASIP Journal on Wireless Communications and Networking* (special issue on Localization in Mobile Wireless and Sensor Networks), Dec. 2010.

Acknowledgments

First and foremost, I would like to express my sincere gratitude to my advisor Prof. Erik G. Ström for the support of my Ph.D study and research, for his patience, and for his motivation. His guidance helped me a lot throughout the research process and the writing of this thesis and a number of papers. Besides my advisor, I warmly thank Dr. Mats Rydström and Dr. Henk Wymeersch for their kind support, guidance, advice, and friendly help. I would like to thank Dr. Sinan Gezici for the invaluable comments on our joint papers. I would also like to thank Prof. Mats Viberg for many fruitful discussions. Finally, I would like to thank our colleagues at Newcom++, an EU FP7 Network of Excellence, for collaborating on measuring the practical data.

I have been lucky to have great friends throughout my life, and I would like to use this opportunity to thank all of my former and current friends. I would like to thank all members of the communication group, in particular Kasra, Lotfollah, Behrooz, Ali, Nima, Dr. Tahmasebi, Dr. Guillermo, Dr. Stylianos, Prof. E. Agrell, and Prof. T. Eriksson, for always being open to discussions about different problems. I would also like to thank my good friends, Majid Akbari, Saeid Haghbin, and Nima Jamali for their support. Finally, I would like to thank the adminstration staff at Department of Signal and Systems who helped me a lot throughout this process with particular thanks to Agneta and Lars.

I owe my deepest gratitude to my family. Their constant inspiration and guidance has kept me focused and motivated. I am grateful to my father for teaching me to be independent and hardworking. My gratitude to my mother, whose unconditional love has been my greatest strength, cannot be expressed in words. The constant love and support of my lovely sister, Shahrbanoo, and my lovely brother, Hamid Reza, are sincerely acknowledged.

Last but not the least, my sincere thanks goes to my wife, Somayeh, for her understanding and love during the past few years. Her support and encouragement was in the end what made this dissertation possible. She has been, always, my pillar, my joy, and my guiding light.

This work has been supported by the Swedish Research Council (VR) under Grant 2007-6363.

Mohammad Reza Gholami.
Gothenburg, March 2011.

Acronyms

WSN	Wireless sensor network
RF	Radio frequency
UWB	Ultra-wideband
LOS	Line-of-sight
NLOS	Non-line-of-sight
CRLB	Cramér-Rao lower bound
PDF	Probability density function
CDF	Cumulative distribution function
RMSE	Root mean square error
NB	Norm of bias
FROB	Frobenius metric
GDOP	Geometric dilution of precision
RSS	Received signal strength
AOA	Angle-of-arrival
TOA	Time-of-arrival
TDOA	Time-difference-of-arrival
TW-TOA	Two-way TOA
ML	Maximum Likelihood
NLS	Nonlinear least squares
WNLS	Weighted nonlinear least squares
SNR	Signal-to-noise-ratio
SDP	Semidefinite programming
SOCP	Second-order cone programming
CFP	Convex feasibility problem
POCS	Projection onto convex sets
OA	Outer-approximation

Contents

Chapter 1

Introduction

Recent advances in technology have instigated the use of tiny devices such as sensors in a large distributed wireless sensor network (WSN). A sensor device is able to sense its environment for monitoring, controlling, or tracking purposes for both civil and military applications [1]. Every sensor can communicate with nearby sensors or with a central processing unit. Wireless sensors commonly use radio frequency bands to communicate with each other. Although no unique physical layer technology has been finalized for WSNs, there are a number of candidates for employment with WSNs, e.g., ultra-wide band (UWB) and code division multiple access [2, 3].

In practical applications, WSNs may have a random or a regular deployment. For data gathered by sensor nodes to be useful in processing procedures, the positions of sensor nodes should be known beforehand, otherwise the data collected by the sensors is meaningless. As an example, consider Fig. 1.1 where a few sensors inside the dashed circle detect a fire event and then report this alarm to a central unit for further processing. Suppose that the short range sensors can find a route to the central unit using a routing protocol. In the central unit, before any decision about the alarm has been made, the positions of the sensors detecting the fire event need to be known.

Position information is often a key requirement for a WSN to function as intended. This information can be obtained by equipping sensors with global positioning system (GPS) devices or manually setting the correct position. Since sensor devices are often very small and cheap, adding GPS receivers increases the cost of the network. In addition, GPS receivers work well in line-of-sight (LOS) conditions, meaning they cannot be used in an indoor scenario or situations in which there is limited visibility to the GPS satellites. Manually setting the sensor positions may be possible for a small and fixed network, but it is quite cumbersome for large networks. In some scenarios, sensors may have a short lifetime and new ones need to be deployed in a possibly random manner in the network. This implies that the positioning process may need to be repeated at certain times. For instance, to monitor the quality of crops on a farm, thousands of tiny sensors may be randomly distributed over a large area. Every sensor may only be operational for a few days, thus creating the need to distribute new sensors.

1 Positioning of nodes

Due to drawbacks in using GPS or manually setting the sensor position, extracting position information by means of the network itself, also called localization, has been ex-

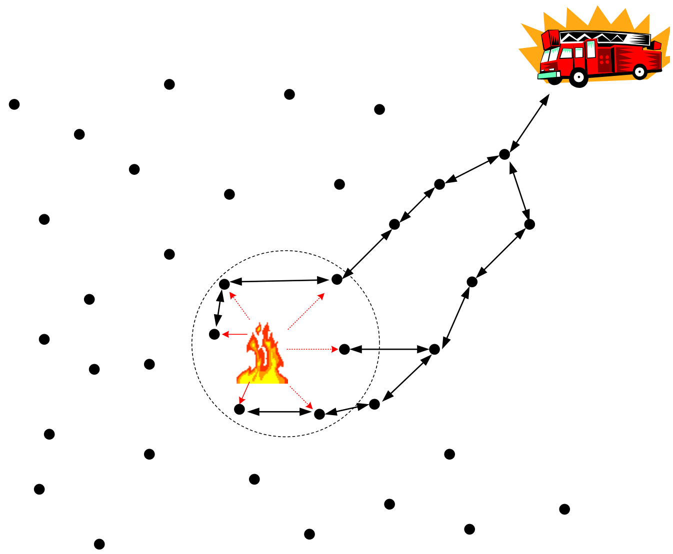

Figure 1.1: An example of the application of a WSN. Solid dots denote wireless sensors. Sensors inside the dashed circle detect a fire event and then report an alarm to a central unit for further processing.

tensively studied in the literature [1, 4–7]. In the literature, it is commonly assumed that there are a number of fixed sensors called reference or anchor nodes whose positions are initially known using GPS receivers or manual settings [8]. Besides reference nodes, there are a number of other sensor nodes at unknown positions, called target, unknown sensor, or agent nodes. In this thesis, we will use the terms reference nodes and target nodes, unless stated otherwise. We assume sensor nodes are able to make some type of measurements. Sensor nodes in WSNs can be either stationary or moving. They can be either transmitters or receivers, or both. To study the positioning problem in a WSN, we need suitable models for the measurements taken between sensor nodes. It is commonly assumed that measurements are made between target and reference nodes, but in some situations measurements between target nodes are also available.

WSNs can be categorized into two groups based on the type of interaction between target nodes: cooperative and non-cooperative networks. In cooperative networks, measurements between target nodes are used in the positioning process, as well as measurements between target nodes and reference nodes. In non-cooperative networks, only the latter type of measurements are used. When there are limited number of reference nods, cooperation effectively improves the performance of positioning algorithms [9]. In summary, the positioning problem studied here can be defined as follows:

The position of target nodes should be estimated based on known positions of a number of reference nodes and using some type of measurements taken between sensor nodes.

Positioning algorithms commonly aim at positioning target nodes as accurately as possible. The performance of a positioning algorithm depends on the type of measurement taken, reference node selection, measurement errors, geometry of network and other factors. Moreover, the practical impairments, e.g., modeling or round-off, that might be unknown also considerably affect the performance of positioning algorithms. During the last decade, a large number of positioning methods have been proposed in the literature.

It is not always straightforward to compare all the different approaches based on a unique criterion. Hence, to evaluate different positioning methods, we use a number of different metrics, e.g., complexity or accuracy.

In this thesis, we study a number of important factors, e.g., accuracy or coverage, in designing an algorithm and propose some methods to solve the positioning problem for both cooperative and non-cooperative networks.

2 Thesis outline

The positioning problem is a wide research area that involves many different topics. Based on one statistic, about 13.3% of recent papers in the WSN area have been focused on tracking and positioning topics [1]. In this thesis, we focus on designing positioning algorithms to be accurate, to have low complexity, and to be robust for both non-cooperative and cooperative networks. To present our contributions, we first give a brief and general description of the positioning problem and then introduce our work. This thesis is organized as follows: in Chapter 2, we formulate the positioning problem and explain some performance measures. In Chapter 3, we review the measurement models commonly used in the positioning literature. Using the measurement models, we study a number of positioning algorithms in Chapter 4. The contributions of this thesis are discussed in Chapter 5. This thesis is based on four papers, i.e., Papers A, B, C, and D, that are attached to the thesis.

Chapter 2

Problem statement

In this chapter, we briefly study the positioning problem. The positioning problem can be formulated as an estimation problem, thus different estimation techniques can be employed to solve it. To evaluate the performance of a positioning algorithm, we study a number of performance metrics. Classification and evaluation may be considered together to assess an algorithm. For instance, one can classify algorithms based on cooperation or non-cooperation, ranged-based or connectivity-based, and then evaluate the performance of different positioning methods. Moreover, if the prior knowledge about the position of target nodes is available, performance metrics may be defined such that to consider that information. In this thesis we assume that there is no prior knowledge about the position of the target nodes.

1 The positioning problem

As mentioned in the previous chapter, there are usually a number of fixed sensors at known positions and some type of measurements taken by sensor nodes. The goal is to estimate the positions of targets as accurately as possible. To position the target nodes, we need a system model for the problem. Throughout this thesis we use a unified system model for both cooperative and non-cooperative networks.

Let us consider a two-dimensional[1] network with $N + M$ sensor nodes. Suppose that M target nodes are at the unknown positions[2] $\mathbf{x}_i = [x_{i1}\ x_{i2}]^T \in \mathbb{R}^2$, $i = 1, ..., M$, and the remaining N reference nodes are located at known positions $\mathbf{a}_j = [a_{j1}\ a_{j2}]^T \in \mathbb{R}^2$, $j = M + 1, ..., N + M$. Every target can communicate with nearby sensor nodes and also with other targets. Let us define

$$\mathcal{A}_i = \{j \mid \text{reference node } j \text{ can communicate with target } i\} \tag{2.1}$$

and

$$\mathcal{B}_i = \{j \mid i \neq j,\ \text{target } j \text{ can communicate with target } i\} \tag{2.2}$$

as the sets of indices of all reference and target nodes that can communicate with target i, respectively. For a non-cooperative network, $\mathcal{B}_i = \emptyset,\ \forall i$.

[1]Generalization to the three-dimensional case is often straightforward, but is not explored in this thesis.

[2]In some appended papers, different notations for reference and target nodes have been used.

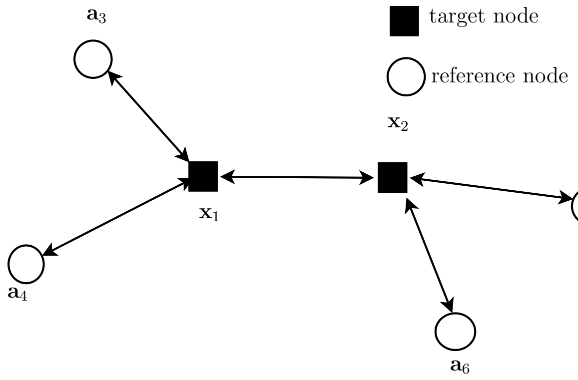

Figure 2.1: A cooperative network with two targets and four reference nodes.

Let us consider the measurements taken between different nodes as a function of their positions. Formally, the measurement between nodes i and j collected at node i is given by

$$m_{ij} = \begin{cases} f(\mathbf{x}_i, \mathbf{a}_j) + \epsilon_{ij}, & j \in \mathcal{A}_i, \\ f(\mathbf{x}_i, \mathbf{x}_j) + \epsilon_{ij}, & j \in \mathcal{B}_i, \end{cases} \tag{2.3}$$

where $f(\boldsymbol{\alpha}, \boldsymbol{\gamma})$ is a deterministic function that defines a type of noiseless measurement between two sensors at positions $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, and ϵ_{ij} is the measurement error. The function $f(\boldsymbol{\alpha}, \boldsymbol{\gamma})$ may have different shapes based on the positions $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$. For instance, for distance measurements, it is the ℓ_2-norm of difference between $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, i.e., $\|\boldsymbol{\alpha}-\boldsymbol{\gamma}\|$. The measurement errors ϵ_{ij} may have any probability density function (PDF). Measurements can be collected at reference nodes, target nodes, or both reference and target nodes. As an example, Fig. 2.1 shows a cooperative network consisting of four reference nodes and two target nodes.

Generally, the positioning problem can be defined as follows: *The positioning problem is to find the position of the M target nodes based on N known sensors' positions and measurements in* (2.3).

Note that sensor nodes can, in general, be moving or stationary. However, in this thesis we only study the positioning problem for fixed reference and target nodes. In the next chapters, we study different types of measurements and various positioning algorithms that are commonly used for the positioning problem in wireless sensor networks.

2 Algorithm evaluation

The means for estimating the position of an unknown target considering measurements (2.3) actually depends on the application. Based on different assumptions, various techniques can be derived to estimate the position of an unknown target. For instance, if the full statistics of measurement errors are assumed to be known, one can apply traditional estimators to solve the problem. On the other hand, one may relax the problem to some degree and design suboptimal[3] algorithms that might be of low complexity. Since po-

[3]Optimality here refers to the accuracy of estimation, normally in root mean square error sense.

sitioning algorithms are expected to be implemented in practical systems, they should satisfy a number of practical requirements as well. To the best of our knowledge, there are no unique criteria to compare and evaluate various proposed approaches in the literature. Moreover, there are different factors in practice, e.g., modeling and round-off, affecting the performance of an algorithm and they might be unknown. Therefore, a good algorithm that shows excellent performance considering a theoretical model through simulation, may have poor performance in practice.

In addition, robustness is another important issue when designing a positioning algorithm or in general for any estimator. Since the measurements are noisy, a positioning algorithm might not be able to localize an unknown target accurately. Due to outliers (measurements with large errors), the performance of a positioning algorithm may change dramatically. Moreover, in practical scenarios for some types of measurements, non-line-of-sight (NLOS) conditions might appear in which a measurement has very large positive errors due to, for instance, an unknown obstacle. There are a number of traditional methods for dealing with outliers and NLOS conditions that may be useful for the positioning problem [10, 11].

The sensor network may consist of a few or many sensor nodes [1]. Thus, one way to assess algorithms is to consider whether an algorithm designed for a small network can be extended to a large network, often referred to as scalability. For example, in centralized processing, a proposed algorithm for a small network can be extended to a large network with more complexity, while for the distributed version, it is not straightforward to answer this question.

There are a number of criteria for the evaluation and comparison between different positioning algorithms [1], among them we consider a few important metrics to compare different methods.

2.1 Accuracy metrics

It is of great importance to have an algorithm that estimates the position of an unknown target as accurately as possible. It is clear that the accuracy requirement changes from one application to another. To evaluate the accuracy of an estimator, we need a reasonable benchmark. One way to evaluate the performance of different algorithms is to compare to a lower bound on the estimation error. However, it might be difficult to compute such a lower bound. There are a number of lower bounds in the literature, e.g., the Cramér-Rao lower bound (CRLB), that can be employed as benchmarks. The CLRB can be computed if the PDF of the measurement error is known and satisfies some regularity conditions [12]. The CRLB is a lower bound on the variance of any unbiased estimator. If the estimator is biased, it is not necessarily bounded by the CRLB and other bounds should be considered [13, 14]. If there is a *priori* information about the unknown target, the CRLB may not be a tight bound. One possibility for deriving a lower bound, when there is a *priori* information, might be Bayesian CRLB [15] that considers the prior distribution of the targets as well as the distribution of the measurement errors.

Different positioning algorithms, regardless of whether a lower-bound is computable or not, can be compared with each other based on various accuracy metrics. Here, we consider a number of such metrics.

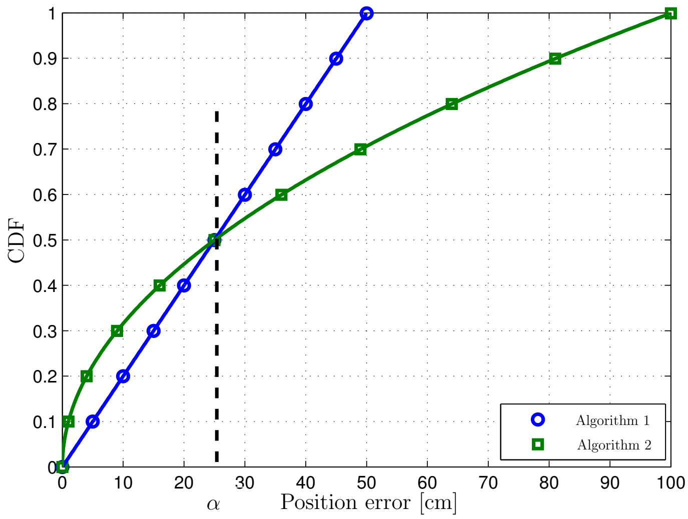

Figure 2.2: Comparison between two CDFs.

Let us define the error $\mathbf{e}_i$ as

$$\mathbf{e}_i = \hat{\mathbf{x}}_i - \mathbf{x}, \quad i = 1, \ldots, M, \tag{2.4}$$

where $\hat{\mathbf{x}}_i$ is an estimate of the position of target i given by a positioning algorithm. Due to randomness in measurement errors or network deployment, the vector $\mathbf{e}_i$ is random.

There are different ways to study the performance of a positioning algorithm through the position error $\|\mathbf{e}_i\|$. In the following, we study a number of accuracy metrics such as the cumulative distribution function (CDF), root mean square error (RMSE), and norm of bias (NB).

Note that the positioning information derived by a positioning algorithm constructs a geometrical layout of the actual layout. Therefore, one way to assess the performance of a positioning algorithm is to consider how exactly the layout represented by a positioning algorithm matches to the actual layout [1].

Cumulative distribution function (CDF): One way to evaluate the performance of an algorithm is to investigate the PDF or CDF of $\|\mathbf{e}_i\|$. The CDF of the position error for target i is defined as

$$P_{\|\mathbf{e}_i\|}(\alpha) = \Pr(\|\mathbf{e}_i\| \leq \alpha), \quad i = 1, 2, ..., M. \tag{2.5}$$

The CDF gives more insight into the performance of positioning algorithms than, for instance, root mean square error, which gives one value. For example, two different algorithms may have relatively different performances for different error intervals. One algorithm might be superior in small errors while other may perform better for medium errors. Let us consider Fig. 2.2 which shows the CDF of the position error for two Algorithm 1 and 2. It demonstrates that if the position error is less than α, Algorithm 1 outperforms Algorithm 2. If the position error is larger than α, Algorithm 2 shows better performance.

Root mean square error(RMSE): The RMSE for target i is defined as

$$\text{RMSE}_i = \sqrt{\mathbb{E}\big(\|\hat{\mathbf{x}}_i - \mathbf{x}_i\|^2\big)} \approx \sqrt{\frac{1}{K}\sum_{k=1}^{K} \|\hat{\mathbf{x}}_i(k) - \mathbf{x}_i\|^2}, \quad i = 1, \ldots, M, \tag{2.6}$$

where $\hat{\mathbf{x}}_i(k)$, $k = 1, 2, \ldots, K$, are the estimates of target i at position $\mathbf{x}_i$ given by a positioning algorithm for the kth realization of noise or network deployment. The RMSE for a network consisting of M targets can then be computed as

$$\mathrm{RMSE} = \frac{1}{M} \sum_{i=1}^{M} \mathrm{RMSE}_i. \tag{2.7}$$

Instead of the average error, we can investigate maximum or median error, which may be useful in some scenarios. For instance, the maximum error is defined as [16]

$$\text{Max-Error}_i = \max_{k=1,\ldots,K} \|\hat{\mathbf{x}}_i(k) - \mathbf{x}_i\|, \quad i = 1, 2, \ldots, M. \tag{2.8}$$

The maximum position error for the network can be obtained as

$$\text{Max-Error} = \max_{i=1,\ldots,M} \text{Max-Error}_i. \tag{2.9}$$

Norm of bias (NB): Bias is another factor that is sometimes considered to evaluate an algorithm. The bias of an estimator is defined as

$$\mathbf{B}_i = \mathbb{E}\left(\mathbf{e}_i\right), \quad i = 1, 2, \ldots, M. \tag{2.10}$$

Moreover, to evaluate the bias, we define the NB as

$$\mathrm{NB}_i = \|\mathbb{E}\left(\mathbf{e}_i\right)\|, \quad i = 1, 2, \ldots, M. \tag{2.11}$$

Frobenius metric (FROB): Suppose that the distance $\tilde{d}_{ij}$ is the distance between a located target i, i.e., $\hat{\mathbf{x}}_i$, and node j. The FROB, which has been considered as a method for evaluation of positioning algorithms in the literature, is defined as [17]

$$\mathrm{FROB} = \sqrt{\frac{1}{\sum_{i=1}^{M} |\mathcal{A}_i \cup \mathcal{B}_i|} \sum_{i=1}^{M} \sum_{j \in \mathcal{A}_i \cup \mathcal{B}_i} (\tilde{d}_{ij} - d_{ij})^2}, \tag{2.12}$$

where $|\mathcal{X}|$ denotes the cardinality of set $\mathcal{X}$ and d_{ij} is the actual distance between target i and node j.

There are other accuracy metrics including the global energy ratio, which involves the geometry of the network, the global distance error, the average relative deviation, and the boundary alignment ratio [1, 17].

In all accuracy metrics described above, we need to know the position of the target to compute performance metrics. For instance, to compute the CDF of the position error, we should subtract the target's estimated position from the exact position of the target. In simulation scenarios, we know the targets positions and then it is straightforward to compute different metrics. However, in a practical scenario prior knowledge of the targets is not always initially available. In fact, the geometry of network is not initially known. For example, in range-based localization, an algorithm estimates the position of targets based on measurements between different nodes. In such a scenario, the accuracy metric should be defined regardless of the geometry of the network. In [18] a metric based on average distance error (ADE), which can be considered as an accuracy measure, was defined as

$$\mathrm{ADE} = \frac{1}{\sum_{i=1}^{M} |\mathcal{A}_i \cup \mathcal{B}_i|} \sum_{i=1}^{M} \sum_{j \in \mathcal{A}_i \cup \mathcal{B}_i} (\hat{d}_{ij} - \tilde{d}_{ij}) \tag{2.13}$$

where $\tilde{d}_{ij}$ is distance between node i and j after positioning and $\hat{d}_{ij}$ is the observed (measured distance) distance.

2.2 Cost metrics

Another evaluation criterion, which is mainly a practical issue, is the cost of implementing an algorithm. Cost is often a trade-off against accuracy. Different parameters determine the cost of a positioning algorithm, such as power consumption, overhead, time taken to obtain a position estimate, and the required amount of RAM (memory) [1, 19]. In general, cost is commonly studied using the following metrics:

- **Reference to node ratio** is defined as the number of reference nodes divided by the number of sensor nodes in a network. It is commonly used to investigate the trade-off on accuracy of algorithms. For instance, it determines how the accuracy of an algorithm changes if the percentage of references nodes decreases [1];
- **Communication overhead** is defined as the number of packets transmitted or the actual power consumed to reach the positioning goal;
- **Power consumption** determines the lifetime of a sensor node. Power consumption is a combination of the power required to perform local processing, e.g., the task of a sensor node for the distributed processing, and the power used to send and receive packets;
- **Algorithm complexity** determines the computational complexity in time and space. It is common to use standard notation, i.e., big O, to express the computational complexity for both time and space [19];
- **Convergence time** is defined both based on the time taken to gather measurements and the time needed for the positioning algorithm to converge. This metric is mostly studied as a function of the size of a network. For instance, this metric measures how much time takes for collecting the measurement or position changes if the size of a network increases.

2.3 Coverage metrics

The coverage metric is the percentage of target nodes in a WSN that can be positioned, regardless of accuracy. The geometry and the node density have the most effect on coverage results. For a target to be positioned successfully, there should be enough reference nodes around it and sufficient measurements taken by sensor nodes. Density can be determined as the minimum number of neighbors required for target nodes to be positioned considering a certain level of accuracy [1]. If the density of the deployment is low, it is possible that a number of nodes cannot be positioned, due to lack of enough reference nodes around a target node. In this case, cooperation between target nodes can remedy the problem and then improve the coverage metric. Increasing the density also improves the coverage metric, but this might not be an option due to increased message collisions and energy consumption. In addition to node density, the reference node placement has a great impact on positioning error. The effect of the geometry of reference nodes has been studied using the Geometric Dilution of Precision (GDOP) [1, 20]. GDOP analysis shows that if target nodes are located inside the convex hull of the reference nodes, they can be localized with lower error.

2.4 Composite metrics

Instead of considering different metrics individually, one can consider a composite metric where different criteria are combined [1, 21]. For instance, the cost metric defined in [21] is such a composite metric which merges accuracy and complexity in one performance measure.

In conclusion, although there is no unique way to compare different positioning algorithms, various positioning approaches can be evaluated based on a number of metrics, e.g., the performance measures considered in this chapter. A comprehensive assessment of an algorithm may require that a hardware implementation of the algorithm is tested in a real world scenario.

Chapter 3

Measurement models

The type and quality of measurements taken have a considerable effect on the performance of a positioning algorithm in WSNs. Different types of measurements have been considered for the positioning problem, e.g., received signal strength (RSS), angle-of-arrival (AOA), time-of-arrival (TOA), and time-difference-of-arrival (TDOA). Since designing an estimator for the positioning problem strongly depends on the model of measurements, it is of great importance to use an accurate model for measurements. We assume a sensor node either transmits or receives a signal, or both. The measurement, in general, between two nodes i and j follows the model introduced in (2.3). The sensor nodes can be either stationary or moving. They might also be able to make more than one type of measurement. In this chapter we briefly study three types of measurement based on RSS, AOA, and TOA.

1 Received signal strength

The following model for received power in wireless channels is widely used to model RSS in WSNs. The average received power from transmitter i at receiver j, in dB, can be expressed as

$$P_{ij} = P_{0i} - 10\beta \log\left(\frac{d(\mathbf{x}_i, \mathbf{x}_j)}{d_0}\right) + n_{ij}, \tag{3.1}$$

where P_{0i} denotes the power at distance d_0, $d(\mathbf{b}, \mathbf{c}) = \|\mathbf{b} - \mathbf{c}\|$ is the Euclidean distance between $\mathbf{a}$ and $\mathbf{b}$, β is a path-loss exponent that is commonly between 2 and 6 [20], and n_{ij} is modeled as a zero mean Gaussian random variable with variance σ_{ij}^2 [22], i.e., $n_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)$.

As can be seen, the model is nonlinearly dependent on the position of the transmitter i. Here, $f(\mathbf{x}_i, \mathbf{x}_j) = P_{0i} - 10\beta \log\left(\frac{\|\mathbf{x}_j - \mathbf{x}_i\|}{d_0}\right)$. If channel parameters, i.e., P_{0i} and β, are known, the maximum likelihood estimate of the distance between node i and j can be obtained as

$$\hat{d}_{ij} = d_0 10^{\frac{P_{0i} - P_{ij}}{10\beta}}. \tag{3.2}$$

It can be shown that the distance estimate obtained in (3.2) is biased. An unbiased estimate for the distance can be derived as [20]

$$\hat{d}_{ij}^u = d_0 10^{\frac{P_{0i} - P_{ij}}{10\beta}} e^{-\frac{10\beta}{\sigma_{ij} \ln 10}}. \tag{3.3}$$

The Cramér-Rao lower-bound for the variance of any unbiased distance estimator based on RSS measurements can be obtained as [5]

$$\mathbb{E}\big(\hat{d}_{ij} - \mathbb{E}(\hat{d}_{ij})\big)^2 \geq \Big(\frac{\sigma_{ij} d(\mathbf{x}_i, \mathbf{x}_j) \ln 10}{10\beta}\Big)^2. \tag{3.4}$$

It is observed that the distance estimate accuracy deteriorates with the distance between two nodes as well as the standard deviation σ_{ij} of measurement noise in (3.1). It also shows that the larger a path-loss exponent β, the more accurate the distance estimate, the reason being that the average power is more sensitive to distance for the larger path-loss [5].

Note that if the path-loss exponent β or reference power P_{0i} are unknown, they must be treated as nuisance parameters when estimating distance based on RSS [23, 24].

2 Angle-of-arrival

Using an array of antennas in a sensor node, we are able to measure the angle-of-arrival for a signal received from a sensor i. The measured angle can be written, in radians, as:

$$\hat{\theta}_{ij} = \tan^{-1}\left(\frac{x_{j2} - x_{i2}}{x_{j1} - x_{i1}}\right) + n_{ij}, \tag{3.5}$$

where $\tan^{-1}$ denotes four-quadrant inverse tangent and n_{ij} is often modeled by a zero mean Gaussian random variable. Then, $f(\mathbf{x}_i, \mathbf{x}_j) = \tan^{-1}\left(\frac{x_{j2}-x_{i2}}{x_{j1}-x_{i1}}\right)$. Let us consider a uniform linear array with N_a elements with distance r between elements as shown in Fig. 3.1. Assuming the same fading coefficient α for all signal arriving at the array elements, the Cramér-Rao lower bound on variance of any unbiased estimator of AOA, also called direction-of-arrival or bearing, is given by [5]

$$\mathbb{E}\big(\hat{\theta}_{ij} - \mathbb{E}(\hat{\theta}_{ij})\big)^2 \geq \frac{\sqrt{3}c}{\sqrt{2}\pi\sqrt{\mathrm{SNR}}B_e r\sqrt{N_a(N_a^2-1)}\sin(\theta_{ij})}, \tag{3.6}$$

where θ_{ij} is the actual angle, c is the speed of propagation, $\mathrm{SNR} = \frac{\alpha^2 E}{N_0}$ is the received signal-to-noise-ratio, N_0 denotes the spectral density of additive white Gaussian noise, and B_e is the effective bandwidth defined by

$$B_e = \left(\frac{1}{E}\int_{-\infty}^{\infty} f^2 |S(f)|^2 df\right)^{\frac{1}{2}}, \tag{3.7}$$

with $S(f)$ and E representing the Fourier transform and the energy of the transmitted signal $s(t)$ [5].

Eq. (3.6) shows that the performance of AOA estimation can be improved by increasing the SNR, the number of array elements, or the distance between elements. It also shows that the performance depends on the angle of θ_{ij}. In a 2D network and in the absence of noise, AOA measurements from at least two receivers can be used to estimate the location of a transmitter, while in the presence of measurement noise, more than two AOA measurements are needed to find the position of a transmitter [1].

Note that AOA measurements are obtained versus local coordinates, therefore, we need to know the orientation of reference nodes with respect to global coordinates.

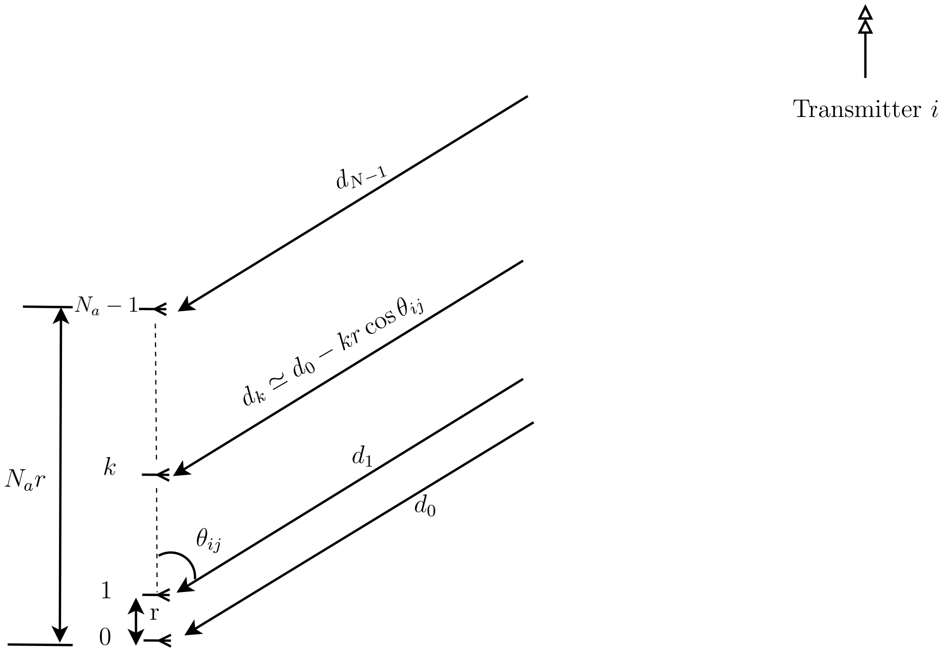

Figure 3.1: A uniform linear array that measures the angle between two nodes i and j.

3 Time-of-arrival

Time-of-arrival (TOA) measurements are one of the most popular techniques used to solve the positioning problem. To calculate the distance between two sensor nodes based on the time the signal spends traveling from one node to another node, we need a time synchronized network that can be achieved by using a number of techniques [25–29]. The TOA estimate is commonly obtained by employing correlator or matched filter receivers [30–32]. In this thesis, we review three strategies to compute the TOA measurements: one-way TOA, two-way TOA (TW-TOA), and time-difference-of-arrival (TDOA) measurements.

3.1 One-way time-of-arrival

Suppose sensor nodes are synchronized with a common clock and assume a line-of-sight transmission. The TOA estimate for the signal transmitted from sensor i at the $j$the sensor can be modeled by

$$\hat{t}_{ij} = \frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{c} + n_{ij}, \tag{3.8}$$

where n_{ij} is often assumed to be a zero mean Gaussian random variable, i.e., $n_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)$ [1, 31, 33]. The distance estimate is then obtained by

$$\hat{d}_{ij} = c\hat{t}_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\| + c\,n_{ij}. \tag{3.9}$$

Considering the effective bandwidth B_e defined in (3.7), the CRLB for the TOA estimate is computed as [5]

$$\mathbb{E}\big(\hat{t}_{ij} - \mathbb{E}(\hat{t}_{ij})\big)^2 \geq \frac{1}{2\sqrt{2}\pi\sqrt{\mathrm{SNR}}B_e}. \tag{3.10}$$

It is seen that increasing the SNR or effective bandwidth improves the performance of TOA estimation.

3.2 Two-way time-of-arrival

In two-way TOA (TW-TOA), the distance between two nodes is computed using the round-trip delay estimation without the need for a common time reference. In this method, a sensor node i sends a signal to a node j and waits for a response from it. Node j, then, replies with an acknowledgment after a turn-around time T_j^{ar}. Therefore, the estimated distance using TW-TOA can be obtained as

$$\hat{d}_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\| + cT_j^{\mathrm{ar}} + c\left(\frac{n_{ij}}{2} + \frac{n_{ji}}{2}\right), \tag{3.11}$$

where n_{ij} and n_{ji} are TOA estimation errors at node j and node i for the signals transmitted from node i and j, respectively. It is seen that the T_j^{ar} should be known or at least accurately estimated. Although TW-TOA removes the error due to imperfect synchronization between a reference node and a target, it still suffers from clock drift in the reference node when measuring TW-TOA [31]. The drawback of this method is that we need to send two signals for every range measurement compared to the TOA approach, which of course increases the complexity.

For error-free measurements, distances derived from RSS, one-way TOA, or TW-TOA define a number of circles around the reference nodes, and the target node is found in the intersection of them.

A practical measurement: To study the effect of practical impairments, let us consider real distance measurements collected by UWB devices. A measurement campaign was performed on the second floor of the Department of Electronic, Information and Systems at the Cesena campus of the University of Bologna, Italy [34]. The network deployment, shown in Fig. 3.2, consisted of 20 sensor nodes. Two ultra-wideband devices were used to measure the distance between every pair of nodes using a TW-TOA technique.

Measurements for different distances are plotted in Fig. 3.3. We also depict the CDF of all measurement errors in Fig. 3.4. As can be seen from both Fig. 3.3 and Fig. 3.4, there are positive and negative outliers. Fig. 3.4 shows that the Gaussian assumption for the measurement errors is not accurate in this case.

3.3 Time-difference-of-arrival

Instead of measuring the absolute distance between two nodes, time-difference-of-arrival (TDOA) alternatively measures the distance difference between an unknown node and two synchronized reference nodes. This method is used by the GPS system where a receiver at an unknown position measures the TDOA of received signals from two synchronized satellites. For example, the TDOA between target node $\mathbf{x}_i$ and synchronized sensor nodes at positions $\mathbf{x}_j$ and $\mathbf{x}_k$ can be written as

$$\hat{t}_{jk}^{i} = \hat{t}_{ij} - \hat{t}_{ik} = \frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{c} - \frac{\|\mathbf{x}_i - \mathbf{x}_k\|}{c} + n_{ij} - n_{ik}. \tag{3.12}$$

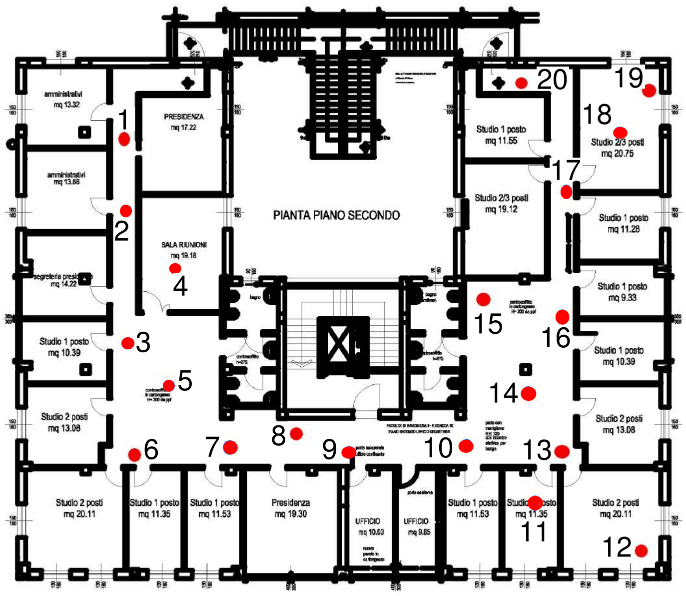

Figure 3.2: The measurement environment where the red points indicate the sensor node positions.

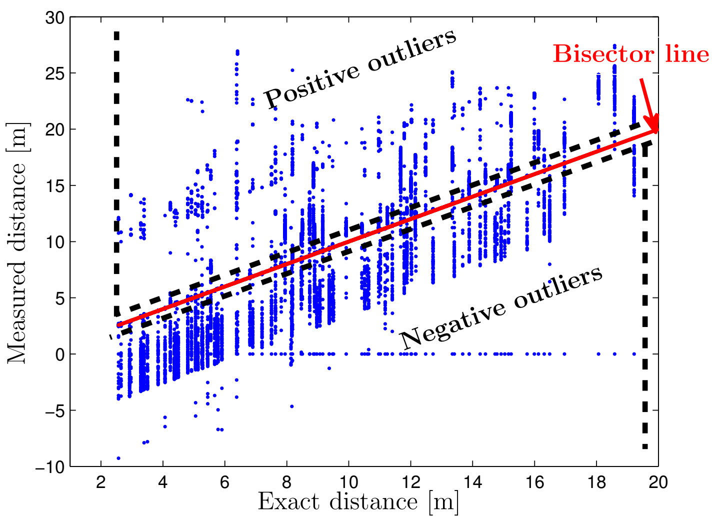

Figure 3.3: UWB measurements for indoor scenario. There are both positive and negative outliers.

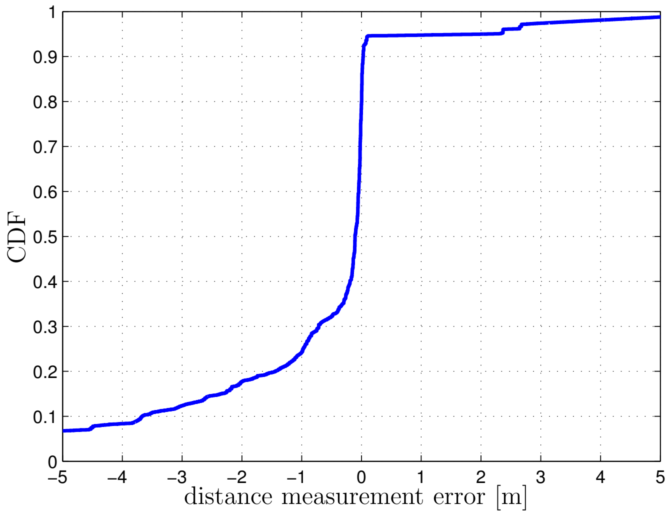

Figure 3.4: CDF of measurement errors for all pair of nodes.

Therefore, an estimation of the distance differences between node j and k to node i can be written as

$$\hat{d}^i_{jk} = c\,\hat{t}^i_{jk} = \|\mathbf{x}_i - \mathbf{x}_j\| - \|\mathbf{x}_i - \mathbf{x}_k\| + c(n_{ij} - n_{ik}). \tag{3.13}$$

As can be seen from (3.13), this technique creates correlation between measurements: e.g., $\hat{t}^i_{jk}$ and $\hat{t}^i_{lk}$ are correlated through n_{ik}. Each TDOA measurement defines a hyperbola where every point on the hyperbola has a constant distance difference to nodes j and k.

Generally, the main sources of error in time-based ranging are [31]; *propagation effects*, *clock imperfections*, and *interference*. The propagation effects include multipath fading, direct-path delay, and direct-path blockage. Imperfect synchronization between nodes causes range estimates to have large errors [27–29]. Finally, the interference from other signals using the same frequency band (or neighboring band) will deteriorate the range estimate.

In general, the different techniques can be summarized as [1, 31]:

- **RSS** is simple to implement and not sensitive to timing. It requires an accurate model of the RSS-distance dependency. However, the estimation using RSS is not accurate enough compared to, e.g., TOA-based approach;

- **AOA** is strongly affected by NLOS conditions. The accuracy depends on RF bandwidth and SNR.

- **TOA/TDOA** is an accurate technique that suffers from NLOS conditions. For perfectly synchronized networks, the accuracy depends on RF bandwidth and SNR.

4 Hybrid measurements

It is also possible to use hybrid measurements for positioning. A number of hybrid schemes have been studied in the literature, e.g., TOA/AOA [35], TDOA/AOA [36], TDOA/TW-TOA [37, 38], and TOA(TDOA)/RSS [39, 40].

Chapter 4

Positioning algorithms

The most obvious and easy solution to the positioning problem of a sensor node is probably to equip the sensor with a GPS receiver. Although GPS is a reliable solution, it may not be suitable for some scenarios. In an advanced GPS technology, called differential GPS, it is possible to reach accuracies of 1 to 3 meters [1]. Thus, the GPS accuracy is not a drawback to be used for positioning in some networks. The main drawbacks of the GPS receiver when used in sensor nodes are that it is an expensive solution for tiny low cost sensor nodes and it is not applicable in scenarios where there is a limited visibility of GPS satellites, e.g., in indoor scenarios. Alternatively, self-positioning by the network itself has found a growing interest during the last few years [1, 4–6, 8, 9, 41].

A positioning algorithm commonly takes the position of a number of reference nodes and some types of measurements between reference nodes and an unknown target, and locates the unknown target. Fig. 4.1 shows a high level implementation of a positioning algorithm. Reference nodes (or even unknown target nodes) obtain some type of measurements, e.g., TOA, TDOA, RSS, or hybrid measurements, based on a received RF signal[1] from an unknown target. A positioning algorithm is consequently applied to estimate the unknown target position. Positioning algorithms can be either centralized or distributed; therefore, the measurements need to be sent to a center (centralized) or can be locally processed (distributed). During the last decades, various positioning algorithms have been proposed for WSNs. For instance, as long as the model and the statistics of measurement errors are known, classical estimators can be employed to solve the problem. When the distribution of measurement errors are unknown or the complexity of classical estimation algorithms is extensive, a number of simplified techniques can be applied to the problem. One important factor in evaluating a positioning algorithm is its robustness against outliers, especially for non-line-of-sight conditions. In this thesis, we study a number of positioning algorithms with a focus on range-based methods. We categorize the positioning algorithms in three families: classic algorithms, convex relaxation techniques, and set theoretic approaches.

1 Classic estimators

In this section, we review the maximum likelihood (ML) estimator and the least squares approximation (both nonlinear and linear) for the positioning problem.

[1]Other type of signals such as acoustic or laser signal can also be used for positioning.

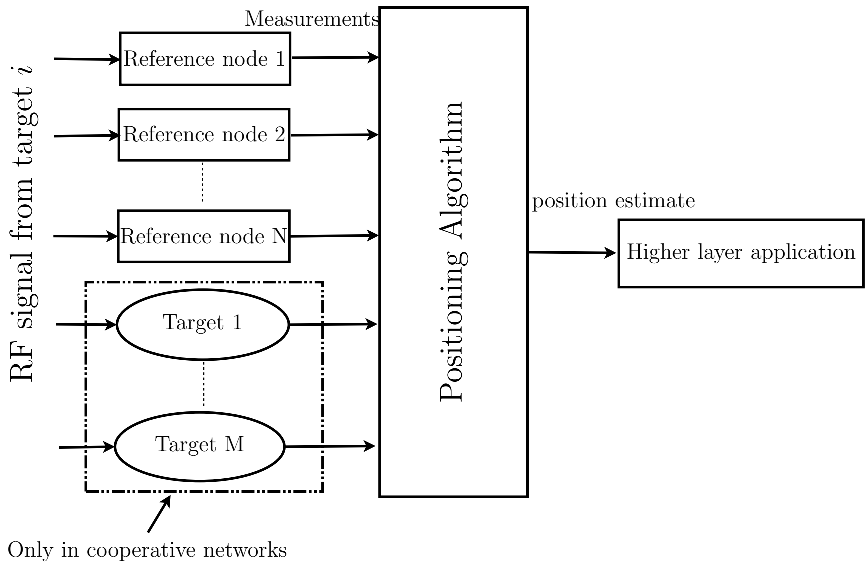

Figure 4.1: A high level description of positioning.

1.1 Maximum likelihood

Suppose measurement errors are independent and identically distributed (i.i.d.). Let the PDF of the measurements in (2.3) be $p_{M_{ij}}(m_{ij}; \mathbf{X})$, where $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_M]^T \in \mathbb{R}^{2\times M}$. The maximum likelihood estimator can be derived as [12, 42]

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X}\in\mathbb{R}^{2\times M}} \sum_{i=1}^{M} \sum_{j\in\mathcal{A}_i\cup\mathcal{B}_i} \log p_{M_{ij}}(m_{ij}; \mathbf{X}). \tag{4.1}$$

In general, the optimization problem in (4.1) is a nonconvex problem and thus is difficult to solve. For i.i.d. Gaussian measurement errors, (4.1) is

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}\in\mathbb{R}^{2\times M}} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} \frac{1}{\sigma_{ij}^2} \big(m_{ij} - f(\mathbf{x}_i, \mathbf{a}_j)\big)^2 + \sum_{j\in\mathcal{B}_i} \frac{1}{\sigma_{ij}^2} \left(m_{ij} - f(\mathbf{x}_i, \mathbf{x}_j)\right)^2 \right). \tag{4.2}$$

where σ_{ij}^2 is the variance of measurement errors.

For distance measurements, using the TOA approach for synchronized networks, (4.2) can be written as

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}\in\mathbb{R}^{2\times M}} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} \frac{1}{\sigma_{ij}^2} \left(\hat{d}_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|\right)^2 + \sum_{j\in\mathcal{B}_i} \frac{1}{\sigma_{ij}^2} \left(\hat{d}_{ij} - \|\mathbf{x}_i - \mathbf{a}_j\|\right)^2 \right). \tag{4.3}$$

Note that for TDOA measurements, the correlation between different measurements also needs to be included in the ML objective function.

Note that if there are additional unknown parameters besides the targets' positions, we can involve them as nuisance parameters in the ML objective function. For instance, for positioning based on RSS measurement, if the path-loss exponent and transmission powers are unknown, one way to obtain an estimator is to minimize the following cost function over all unknown transmission powers and path-loss exponents

$$\hat{\mathbf{X}} = \underset{\substack{\mathbf{X}\in\mathbb{R}^{2\times M},\beta>0\\ P_{0i}\in\mathbb{R}}}{\arg\min} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} \frac{1}{\sigma_{ij}^2} \left(P_{ij} - P_{0i} + 10\beta \log\left(\frac{d_{ij}(\mathbf{x}_i, \mathbf{a}_j)}{d_0}\right)\right)^2 \right.$$
$$\left. + \sum_{j\in\mathcal{B}_i} \frac{1}{\sigma_{ij}^2} \left(P_{ij} - P_{0i} + 10\beta \log\left(\frac{d_{ij}(\mathbf{x}_i, \mathbf{x}_j)}{d_0}\right)\right)^2 \right). \quad (4.4)$$

Here, we assumed that the path-loss factor is the same for all links.

The ML estimator is asymptotically efficient; i.e., it attains the CRLB when the statistics of the measurement errors are available. In practice, however, it is difficult to obtain a *priori* knowledge of the full statistics of measurement errors.

1.2 Nonlinear least squares

The least squares approximation is commonly used in the positioning literature as a benchmark to compare different algorithms. A nonlinear least squares algorithm for positioning tries to minimize the following cost function [43]:

$$\hat{\mathbf{X}} = \underset{\mathbf{X}\in\mathbb{R}^{2\times M}}{\arg\min} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} (m_{ij} - f(\mathbf{x}_i, \mathbf{a}_j))^2 + \sum_{j\in\mathcal{B}_i} (m_{ij} - f(\mathbf{x}_i, \mathbf{x}_j))^2 \right). \quad (4.5)$$

When the variances of measurement errors are available, the NLS can be formulated as a weighted nonlinear least squares (WNLS):

$$\hat{\mathbf{X}} = \underset{\mathbf{X}\in\mathbb{R}^{2\times M}}{\arg\min} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} \frac{1}{\sigma_{ij}^2} (m_{ij} - f(\mathbf{x}_i, \mathbf{a}_j))^2 + \sum_{j\in\mathcal{B}_i} \frac{1}{\sigma_{ij}^2} (m_{ij} - f(\mathbf{x}_i, \mathbf{x}_j))^2 \right). \quad (4.6)$$

In general, the WNLS solution coincides with the ML estimate if measurement errors are i.i.d. Gaussian; e.g., for the range-based positioning, WNLS is similar to (4.3). There are a number of techniques that can be employed to solve the (W)NLS problem, e.g., the Gauss-Newton approach [12].

An alternative approach in the positioning literature is applying the squared-range NLS squares (SR-NLS) [44–47] for the range-based positioning. Then

$$\hat{\mathbf{X}} = \underset{\mathbf{X}\in\mathbb{R}^{2\times M}}{\arg\min} \sum_{i=1}^{M} \left(\sum_{j\in\mathcal{A}_i} \left(\hat{d}_{ij}^2 - \|\mathbf{x}_i - \mathbf{a}_j\|^2\right)^2 + \sum_{j\in\mathcal{B}_i} \left(\hat{d}_{ij}^2 - \|\mathbf{x}_i - \mathbf{x}_j\|^2\right)^2 \right). \quad (4.7)$$

Note that the SR-NLS approach is suboptimal in the maximum likelihood sense, but it can be shown that a global solution to (4.7) can be obtained efficiently [46].

1.3 Linear least squares

Due to the nonlinear and often nonconvex objective function in NLS, we need to resort to numerical methods to find the position, e.g., an iterative search with a good initial point. A method that often gives good estimates at high SNR is to linearize the measurements versus the position of the target nodes and then to employ linear least squares (LLS) [48–52]. To form a linear least squares problem, we need to find a signal model that is linear in the unknown parameters [49]. Here, we derive the LLS for the non-cooperative network, i.e., $\mathcal{B}_i = \emptyset$, and for distance measurements. Suppose that there are at least three non co-linear nodes with known positions around a target. Let the measurement error be small compared to the true distances, and assume that the distance measurement between target i and reference node j is

$$\hat{d}_{i,j} = d(\mathbf{x}_i, \mathbf{a}_j) + \epsilon_{ij}, \qquad j \in \mathcal{A}_i \tag{4.8}$$

where ϵ_{ij} is measurement noise with variance σ_{ij}^2. Here, there are no particular assumptions about noise statistics other than variance.

Squaring both sides of (4.8), after dropping small terms and some manipulations, yields:

$$\tilde{d}_{ij} = \hat{d}_{ij}^2 - \|\mathbf{a}_j\|^2 \approx [-2\mathbf{a}_j{}^T \ 1]\boldsymbol{\psi}_i + 2d(\mathbf{x}_i, \mathbf{a}_j)\epsilon_{ij} \ , \qquad j \in \mathcal{A}_i, \tag{4.9}$$

where $\boldsymbol{\psi}_i = \left[\mathbf{x}_i^T, \ \|\mathbf{x}_i\|^2\right]^T$. Now a set of linear equations can be written as

$$\mathbf{d}_i = \mathbf{A}_i\boldsymbol{\psi}_i + \boldsymbol{\nu}_i \ , \tag{4.10}$$

where

$$\mathbf{d}_i = \left[\tilde{d}_{ij_1} \ \tilde{d}_{ij_2} \ \ldots \ \tilde{d}_{ij_k}\right]^T, \tag{4.11a}$$

$$\mathbf{A}_i = \begin{bmatrix} -2\mathbf{a}_{j_1}^T & 1 \\ \vdots & \vdots \\ -2\mathbf{a}_{j_k}^T & 1 \end{bmatrix} \tag{4.11b}$$

$$\boldsymbol{\nu}_i = \left[2d(\mathbf{x}_i, \mathbf{a}_{j_1})\,\epsilon_{ij_1} \ \ldots \ 2d(\mathbf{x}_i, \mathbf{a}_{j_k})\,\epsilon_{ij_k}\right]^T \tag{4.11c}$$

where $\mathcal{A}_i = \{j_1, \ldots, j_k\}$, and $k = |\mathcal{A}_i|$ is the cardinality of set $\mathcal{A}_i$.

If the matrix $\mathbf{A}_i$ is full rank, then the unknown parameter $\boldsymbol{\psi}_i$ can be estimated by [12]

$$\hat{\boldsymbol{\psi}}_i = (\mathbf{A}_i^T\mathbf{C}_{\boldsymbol{\nu}_i}^{-1}\mathbf{A}_i)^{-1}\mathbf{A}_i^T\mathbf{C}_{\boldsymbol{\nu}_i}^{-1}\mathbf{d}_i \ , \tag{4.12}$$

where the weighting matrix $\mathbf{C}_{\boldsymbol{\nu}_i}$, for i. i. d. measurement noise, is given by [48]

$$\mathbf{C}_{\boldsymbol{\nu}_i} = \mathrm{diag}\left(4d^2(\mathbf{x}_i, \mathbf{a}_{j_1})\sigma_{ij_1}^2, \ldots, 4d^2(\mathbf{x}_i, \mathbf{a}_{j_k})\sigma_{ij_k}^2\right). \tag{4.13}$$

The covariance matrix of $\hat{\boldsymbol{\psi}}_i$ can be computed as [12]

$$\mathrm{cov}(\hat{\boldsymbol{\psi}}_i) = \left(\mathbf{A}_i^T\mathbf{C}_{\boldsymbol{\nu}_i}^{-1}\mathbf{A}_i\right)^{-1}. \tag{4.14}$$

To compute the weighting matrix $\mathbf{C}_{\boldsymbol{\nu}_i}$, the real distances between known nodes to the target i are required. Since in practice the real distances are not available, we instead use the measured distances in (4.13). Since the linear estimator derived in the positioning literature is suboptimal [52], a number of techniques can be used to improve the estimate, e.g., correction techniques [50, 53] or constrained least squares approaches [54].

It is also possible to derive the LLS by removing the quadratic term $\|\mathbf{x}_i\|^2$ [34, 49, 52, 55].

2 Convex relaxation techniques

Since ML and NLS methods yield nonconvex optimization problems that are in general NP-hard [56], convex relaxation techniques can be employed to solve them in an efficient way. In this section, we briefly study two techniques based on semidefinite programming (SDP) and second-order cone programming (SOC) for TOA range-based positioning [44, 56–58]. For TDOA, RSS, and AOA see [59–61]. SDP relaxation techniques are suitable for small and medium sized networks [56]. For large networks, a distributed version of SDP is required [62]. Alternatively, SOCP techniques have been considered to solve positioning problems in large networks [56]. It has been shown that the SOCP relaxation approach is always weaker than the SDP relaxation [56], thus SDP gives more accurate estimates compared to SOCP.

2.1 Semidefinite programming

Let us consider the ML problem in (4.3) and reformulate it as

$$\begin{aligned}\underset{\mathbf{X}\in\mathbb{R}^{2\times M},\alpha_{ij},\gamma_{ij}\in\mathbb{R}}{\text{minimize}}\ & \sum_{i=1}^{M}\left(\sum_{j\in\mathcal{A}_i}\frac{1}{\sigma_{ij}^2}\alpha_{ij}+\sum_{j\in\mathcal{B}_i}\frac{1}{\sigma_{ij}^2}\gamma_{ij}\right)\\ \text{subject to}\ & \left(\hat{d}_{ij}-\|\mathbf{x}_i-\mathbf{a}_j\|\right)^2=\alpha_{ij},\quad j\in\mathcal{A}_i,\\ & \left(\hat{d}_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|\right)^2=\gamma_{ij},\quad j\in\mathcal{B}_i, i=1,\dots,M.\end{aligned}\tag{4.15}$$

To solve the nonconvex optimization problem in (4.15), we can use relaxation techniques and reformulate it as an SDP problem. The basic idea behind formulating an SDP relaxation is to remove the quadratic term in distance constraints by adopting a relaxation [57]. The constraints in (4.15) can be written as

$$\begin{aligned}\left(\hat{d}_{ij}-\|\mathbf{x}_i-\mathbf{a}_j\|\right)^2&=\left[\hat{d}_{ij},-1\right]\mathbf{D}_{ij}\left[\hat{d}_{ij},-1\right]^T=\alpha_{ij},\quad j\in\mathcal{A}_i,\\ \left(\hat{d}_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|\right)^2&=\left[\hat{d}_{ij},-1\right]\mathbf{V}_{ij}\left[\hat{d}_{ij},-1\right]^T=\gamma_{ij},\quad j\in\mathcal{B}_i,\end{aligned}\tag{4.16}$$

where

$$\mathbf{D}_{ij}=\begin{bmatrix}1 & \|\mathbf{x}_i-\mathbf{a}_j\|\\ \|\mathbf{x}_i-\mathbf{a}_j\| & \|\mathbf{x}_i-\mathbf{a}_j\|^2\end{bmatrix},\quad j\in\mathcal{A}_i,$$

$$\mathbf{V}_{ij}=\begin{bmatrix}1 & \|\mathbf{x}_i-\mathbf{x}_j\|\\ \|\mathbf{x}_i-\mathbf{x}_j\| & \|\mathbf{x}_i-\mathbf{x}_j\|^2\end{bmatrix},\quad j\in\mathcal{B}_i.$$

We can further write the Euclidian distance $\|\mathbf{x}_i-\mathbf{x}_j\|^2$ and $\|\mathbf{x}_i-\mathbf{a}_j\|^2$ as

$$\begin{aligned}[\mathbf{0},\mathbf{u}_i-\mathbf{u}_j]\ \mathbf{Z}\ [\mathbf{0},\mathbf{u}_i-\mathbf{u}_j]^T&=\|\mathbf{x}_i-\mathbf{x}_j\|^2,\quad j\in\mathcal{B}_i,\\ [-\mathbf{a}_j,\mathbf{u}_i]\ \mathbf{Z}\ [-\mathbf{a}_j,\mathbf{u}_i]^T&=\|\mathbf{x}_i-\mathbf{a}_j\|^2,\quad j\in\mathcal{A}_i,\end{aligned}\tag{4.17}$$

where the matrix $\mathbf{Z}$ is

$$\mathbf{Z}=\begin{bmatrix}\mathbf{I}_2 & \mathbf{X}\\ \mathbf{X}^T & \mathbf{X}^T\mathbf{X}\end{bmatrix},\tag{4.18}$$

and $\mathbf{I}_2$ denotes the 2×2 identity matrix. The vector $\mathbf{u}_i$ is a vector with all zeros except its ith entry, which is one.

Now, we obtain the SDP relaxation as

$$\begin{aligned}
\underset{\mathbf{X},\alpha_{ij},\gamma_{ij},\mathbf{D}_{ij},\mathbf{V}_{ij},\mathbf{Z}}{\text{minimize}} \quad & \sum_{i=1}^{M} \left(\sum_{j \in \mathcal{A}_i} \frac{1}{\sigma_{ij}^2} \alpha_{ij} + \sum_{j \in \mathcal{B}_i} \frac{1}{\sigma_{ij}^2} \gamma_{ij} \right) \\
\text{subject to} \quad & \left[\hat{d}_{ij}, -1\right] \mathbf{D}_{ij} \left[\hat{d}_{ij}, -1\right]^T = \alpha_{ij}, \quad j \in \mathcal{A}_i, \\
& \left[\hat{d}_{ij}, -1\right] \mathbf{V}_{ij} \left[\hat{d}_{ij}, -1\right]^T = \gamma_{ij}, \quad j \in \mathcal{B}_i, \\
& \left[\mathbf{0}, e_i - e_j\right] \mathbf{Z} \left[\mathbf{0}, e_i - e_j\right]^T = \|\mathbf{x}_i - \mathbf{x}_j\|^2, \quad j \in \mathcal{B}_i, \\
& \left[-\mathbf{x}_j, e_i\right] \mathbf{Z} \left[-\mathbf{x}_j, e_i\right]^T = \|\mathbf{x}_i - \mathbf{a}_j\|^2, \quad j \in \mathcal{A}_i, \\
& \mathbf{D}_{ij} \succeq 0, \quad j \in \mathcal{A}_i, \\
& \mathbf{V}_{ij} \succeq 0, \quad j \in \mathcal{B}_i, \\
& \mathbf{Z} \succeq 0.
\end{aligned} \tag{4.19}$$

The SDP relaxation given in (4.19) originates from the MLE cost function. It is also possible to derive an SDP relaxation for other cost functions in positioning problem, e.g., the sum of squared method [44, 56].

2.2 Second-order cone programming

Let the positioning estimate be the solution to the following nonconvex optimization problem [56] (to formulate the ML estimator as a SOCP see [59]):

$$[\hat{\mathbf{x}}_1, ..., \hat{\mathbf{x}}_M]^T = \arg \min_{\mathbf{X} \in \mathbb{R}^{2\times M}} \sum_{i=1}^{M} \left(\sum_{j \in \mathcal{A}_i} \left|\hat{d}_{ij}^2 - \|\mathbf{x}_i - \mathbf{a}_j\|^2\right| + \sum_{j \in \mathcal{B}_i} \left|\hat{d}_{ij}^2 - \|\mathbf{x}_i - \mathbf{x}_j\|^2\right| \right). \tag{4.20}$$

By introducing new variables t_{ij} and l_{ij}, (4.20) can be written as

$$\begin{aligned}
\underset{\mathbf{X} \in \mathbb{R}^{2\times M}, t_{ij}, l_{ij} \in \mathbb{R}}{\text{minimize}} \quad & \sum_{i=1}^{M} \left(\sum_{j \in \mathcal{A}_i} \left|\hat{d}_{ij}^2 - t_{ij}\right| + \sum_{j \in \mathcal{B}_i} \left|\hat{d}_{ij}^2 - l_{ij}\right| \right) \\
\text{subject to} \quad & \|\mathbf{x}_i - \mathbf{a}_j\|^2 = t_{ij}, \quad j \in \mathcal{A}_i, \\
& \|\mathbf{x}_i - \mathbf{x}_j\|^2 = l_{ij}, \quad j \in \mathcal{B}_i.
\end{aligned} \tag{4.21}$$

Now, relaxing the equality constraints to $\|\mathbf{x}_i - \mathbf{x}_j\|^2 \le l_{ij}$ and $\|\mathbf{x}_i - \mathbf{a}_j\|^2 \le t_{ij}$, we get the SOCP relaxation

$$\begin{aligned}
\underset{\mathbf{X} \in \mathbb{R}^{2\times M}, t_{ij}, l_{ij} \in \mathbb{R}}{\text{minimize}} \quad & \sum_{i=1}^{M} \left(\sum_{j \in \mathcal{A}_i} |\hat{d}_{ij}^2 - t_{ij}| + \sum_{j \in \mathcal{B}_i} |\hat{d}_{ij}^2 - l_{ij}| \right) \\
\text{subject to} \quad & \|\mathbf{x}_i - \mathbf{a}_j\|^2 \le t_{ij}, \quad j \in \mathcal{A}_i \\
& \|\mathbf{x}_i - \mathbf{x}_j\|^2 \le l_{ij}, \quad j \in \mathcal{B}_i.
\end{aligned} \tag{4.22}$$

Alternatively (4.22) can also be written as

$$\begin{aligned}
&\underset{\mathbf{X}\in\mathbb{R}^{2\times M}, t_{ij}, l_{ij}, y_{ij}, p_{ij}\in\mathbb{R}}{\text{minimize}} \sum_{i=1}^{M}\left(\sum_{j\in\mathcal{A}_i} y_{ij} + \sum_{j\in\mathcal{B}_i} p_{ij}\right)\\
&\text{subject to} \quad \|\mathbf{x}_i - \mathbf{a}_j\|^2 \le t_{ij}, \quad j \in \mathcal{A}_i\\
&\qquad\qquad \|\mathbf{x}_i - \mathbf{x}_j\|^2 \le l_{ij}, \quad j \in \mathcal{B}_i,\\
&\qquad\qquad |\hat{d}_{ij}^2 - t_{ij}| \le y_{ij},\\
&\qquad\qquad |\hat{d}_{ij}^2 - l_{ij}| \le p_{ij}.
\end{aligned} \tag{4.23}$$

Note that it is also possible to obtain a distributed version of SOCP that is suitable for large networks.

3 Set theoretic approach

Let us consider the ranged-based positioning problem as an optimization problem

$$\begin{aligned}
&\underset{\mathbf{X}\in\mathbb{R}^{2\times M}}{\text{minimize}}\, 0\\
&\text{subject to } \|\mathbf{x}_i - \mathbf{a}_j\| = \hat{d}_{ij}, \quad j \in \mathcal{A}_i\\
&\qquad\qquad \|\mathbf{x}_i - \mathbf{x}_j\| = \hat{d}_{ij}, \quad j \in \mathcal{B}_i.
\end{aligned} \tag{4.24}$$

In fact, in the absence of measurement errors, the target i, at position $\mathbf{x}_i$ can be found at the intersection of a number of circles with distances d_{ij} and centers $\mathbf{a}_j$ and $\mathbf{x}_j$, i.e., $d_{ij} = \hat{d}_{ij}$.

The nonconvex problem in (4.24) renders to a *convex feasibility problem* (CFP)[2], more precisely a SOCP feasibility problem, by relaxing the nonconvex constraints to the convex constraints as follows:

$$\begin{aligned}
&\underset{\mathbf{X}\in\mathbb{R}^{2\times M}}{\text{minimize}}\, 0\\
&\text{subject to } \|\mathbf{x}_i - \mathbf{a}_j\| \le \hat{d}_{ij}, \quad j \in \mathcal{A}_i\\
&\qquad\qquad \|\mathbf{x}_i - \mathbf{x}_j\| \le \hat{d}_{ij}, \quad j \in \mathcal{B}_i.
\end{aligned} \tag{4.25}$$

When the measurement errors are all positive [64], the relaxation in (4.25) makes sense; i.e., the actual distance cannot be larger than the measured distance.

To get some insight into problem (4.25), let us consider Fig. 4.2 for a cooperative network consisting of two targets and four reference nodes. For positive measurement errors, we obtain two intersection areas where target one and target two can be found.

To find a point in the intersection area, we can apply the well-known approach projection onto convex sets (POCS) [65] or a technique based on outer-approximation (OA). In the this section, we study POCS and OA for non-cooperative networks. For cooperative versions, we refer the reader to Paper D in this thesis.

[2]The CFP is to find a point in the nonempty intersection of convex sets $\mathcal{Q}_i = \{\mathbf{z} \in \mathbb{R}^n \mid f_i(\mathbf{z}) \le 0\}$, i.e., $\mathcal{Q} = \cap_{i=1}^m \mathcal{Q}_i$, where $f_i(\cdot)$ is a convex function [63].

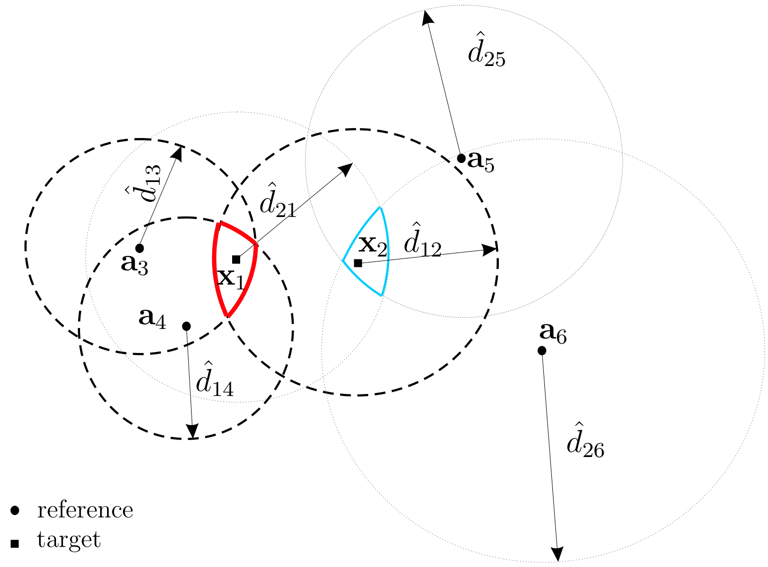

Figure 4.2: Deriving the intersections for target one and target two. Discs with dashed boundaries (circles) are used to derive the intersection for target one and discs with dot boundaries (circle) are used to determine the intersection for target two. The boundary of intersections for target one and target two are red and blue colored, respectively.

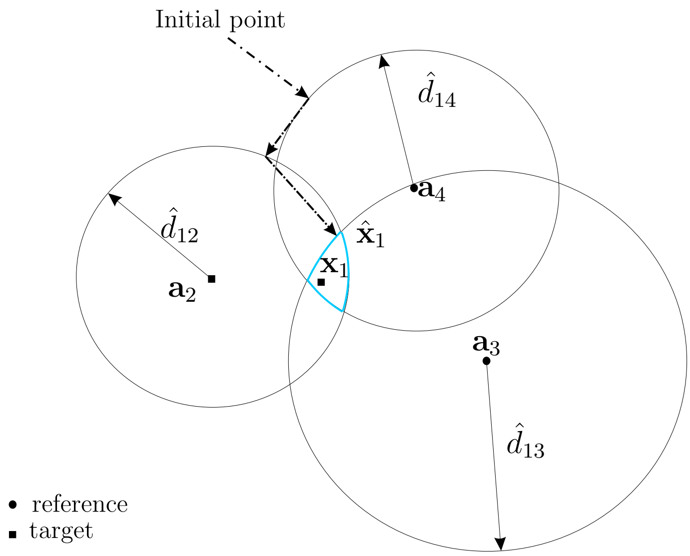

Figure 4.3: POCS converges to a point in the intersection. For positive measurement errors, target $\mathbf{x}_1$ is always found in the intersection of three discs.

3.1 Projections onto convex sets

POCS, also called successive orthogonal projections onto convex sets [63] or alternating projection [66], was first introduced to solve the convex feasibility problem in [67]. POCS has then been applied for different problems in various fields, e.g., in image restoration problems [68–70]. There are generally two versions of POCS: sequential and simultaneous. In this thesis, we only consider sequential POCS and refer the reader to [63] for a study of both sequential and simultaneous projection algorithms. POCS is implemented by sequential projection onto discs. First an arbitrary initial point is chosen. The projection of that point to one disc is computed and then this new point is projected onto another disc. This procedure is continued until a stopping rule is fulfilled. To see how POCS works, let us consider Fig. 4.3, where three reference nodes measure distances to target one. If the measurement errors are all positive, then target one can be found in the intersection shown in this figure. Choosing an arbitrary initial point and after three sequential projections, we end up in one point at boundary of the intersection.

The convergence of POCS has been extensively studied in the literature [63, 71, 72]. It has been shown that for the consistent case, i.e., with a non-empty intersection, the POCS estimate converges to a point in the intersection. For the inconsistent case, using suitable relaxation parameters, POCS converges to a point that minimizes the *sum of squared distances to the convex sets* (here a number of discs). The performance of POCS evaluated through practical data confirms theoretical claims [34, 55]. In positioning problems, POCS has been proposed for a number of scenarios [38, 72–74].

3.2 Bounding the feasible set

As mentioned in the previous section, for positive measurement errors, the intersection of discs is not empty and the target definitely is found there[3]. This assumption can be fulfilled for range estimation based on, e.g., TW-TOA for a reasonable signal-to-noise ratio [64]. The intersection in general may have any convex shape and every point in the intersection can potentially be an estimate of the target position. POCS gives one point as an estimate. In contrast to POCS, OA tries to approximate the feasible set by a suitable shape and then one point inside of it is taken as an estimate, e.g, the middle of the approximated set. The main problem is how the intersection can be accurately approximated. Generally speaking two kinds of approximations, i.e., inner-approximation and outer-approximation, have been extensively studied in the literature. In inner-approximation family, the maximum volume ellipsoid contained in an intersection of ellipsoids and the maximum volume ellipsoid contained in a polyhedron given as a set of linear equalities are tractable [75]. A number of outer-approximation problems are known to be tractable, such as the minimum volume ellipsoid containing a polyhedron and the minimum volume ellipsoid containing a union of ellipsoids [75]. Note that the minimum volume ellipsoid enclosing the intersection of a number of ellipsoids seems to be an intractable problem. There is work in the literature to approximate the intersection by convex regions such as polytopes, ellipsoids, or discs [55, 76–78].

In this thesis, we consider a disc approximation of the feasible set. In a kD network, $k = 2$ or 3, when the number of discs is less than $k - 1$, it is possible to efficiently find the

[3]It is also possible to have a non-empty intersection for mixed positive and negative measurement errors.

smallest disc covering the intersection [78] by solving an SDP problem. In other cases, finding the smallest disc enclosing the intersection is difficult. Using simple geometry, we are able to find all intersection points between different discs and finally find a disc that passes through them and covers the intersection. However, there is no guarantee that this disc is the minimum disc enclosing the intersection. We can solve this problem by, for instance, a heuristic method as explained in Paper D. The generalization of bounding feasible sets to the cooperative scenario is also investigated in Paper D.

Chapter 5

Conclusions and future work

This thesis aims at proposing robust and reliable algorithms with acceptable complexity for the WSN positioning problem. To reach this goal, a number of contributions were introduced for different situations. However, it is still needed to do further explorations for future work.

1 Contributions

The main contributions of this thesis are found in four appended papers.

1.1 Paper A- Indoor sensor node positioning using UWB range measurements

In this paper the performance of a number of positioning algorithms was evaluated based on RMSE and position error CDF for practical distance measurement. Two UWB devices were used to estimate the distance between every pair of nodes in an indoor scenario. We applied different positioning approaches directly on the raw data without any modifications and compared different methods. The main conclusion of this paper is that no method uniformly performs best.

1.2 Paper B- Positioning of node using plane projection onto convex sets

Since obtaining an accurate model for RSS is difficult in practice, in this paper we assume that no prior information about channel models, transmission power and path-loss factor, is available. We only consider a fixed value for these channel parameters for all links and assume that power decreases with distance on average. We modeled the positioning problem as a convex feasibility problem (CFP) and obtained a new method based on projection onto halfplanes (halfspaces). Although this method is not very accurate, it can be considered as a coarse estimate for positioning based on RSS when channel parameters are unknown.

1.3 Paper C- Hybrid TW-TOA/TDOA positioning algorithms for cooperative wireless networks

In this paper we consider the positioning problem using hybrid TW-TOA/TDOA measurements in a cooperative network. Two different reference nodes, called primary and secondary nodes, measure TW-TOA and TDOA, respectively. The MLE derived for this problem is nonconvex and difficult to solve. We propose a two-step linear estimator that has closed-form solution in each step. In the first step, a nonlinear processing of data yields a linear model and a traditional least squares approach can be employed to solve the problem. In the second step, another linear estimator is derived using a first-order Taylor-series expansion and a regularized linear estimator is used to refine the first step estimation. Simulation results show that the proposed estimator attain the CRLB for sufficiently large SNRs.

1.4 Paper D- Wireless network positioning as a convex feasibility problem

In this paper, we study how the positioning problem can be modeled as a CFP. To solve the CFP, we considered two different approaches: POCS and OA. The properties of POCS for non-cooperative networks were reviewed and a method for finding an upper-bound on the position error of POCS has been shown. We also proposed two techniques based on POCS and OA for cooperative networks as well as a constrained nonlinear least squares methods. Simulation results show that the positioning algorithms based on geometric solutions are more robust against NLOS compared to proposed statistical approaches such as NLS.

2 Future work

In Paper D, we proposed a variant of POCS for cooperative networks. The convergence properties of this approach are not known and need more exploration in future work. We also proposed a method for finding the upper-bound on the POCS estimate by solving a nonconvex problem. It seems that convex relaxation techniques can be applied to solve this nonconvex problem. In the literature, it is commonly assumed that the sensor nodes are synchronized and the positioning algorithms are obtained considering a perfect synchronized network. For an unsynchronized network, synchronization can be done before the positioning or it can be jointly done with the positioning. Moreover, other impairments such as uncertainty of the position of reference nodes exist in most practical scenarios. For future work, we will focus on joint positioning and synchronization approaches and we will also consider practical impairments in designing positioning algorithms.

3 Related contributions

Other related publications by the author, which are not included in this thesis, are listed below.

[C1] M. R. Gholami, S. Gezici, M. Rydström, and E. G. Ström, “A distributed positioning algorithm for cooperative active and passive sensors,” in Proc. *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 1711-1716, Sep. 2010.

[C2] M. R. Gholami, E. G. Ström, F. Sottile, D. Dardari, A. Conti, S. Gezici, M. Rydström, and M. Spirito, “Static positioning using UWB range measurements,” in Proc. *Future Network and Mobile Summit*, Florence, Italy, Jun. 16-18, 2010.

[C3] R. M. Vaghefi, M. R. Gholami, and E. G. Ström, “Bearing-only target localization with uncertainties in observer position,” in Proc. *IEEE international Workshop on Advances in positioning and location-enabled communications (in conjunction with PIMRC'10)*, pp. 237-241, Sep. 2010.

[C4] R. M. Vaghefi, M. R. Gholami, and E. G. Ström, “RSS-based sensor localization with unknonw transmit power,” to appear in Proc. *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2011.

[C5] M. R. Gholami, S. Gezici, E. G. Ström, and M. Rydström, “Positioning Algorithms for Cooperative networks in the presence of an unknown Turn-around time,” submitted to *the 12th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2011.

[C6] M. R. Gholami, H. Wymeersch, E. G. Ström, and M. Rydström, “Robust Distributed Positioning Algorithms for Cooperative Networks,” submitted to *the 12th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2011.

[J1] M. R. Gholami, S. Gezici, and E. G. Ström, “Improved Position Estimation Using Hybrid TW-TOA and TDOA in Cooperative Networks,” submitted to *IEEE Transaction on signal processing.*

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