Radiative Damping of Low Shear Toroidal Alfvén Eigenmodes

R. Nyqvist[1], S. E. Sharapov[2] and M. Lisak[1]

[1]*EURATOM/VR, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden*
[2]*EURATOM/CCFE, Culham Science Centre, OX14 3DB, Abingdon, UK*

I. Introduction In burning ITER plasmas, the number and type of excited Alfvén instabilities, and their properties at high plasma pressure, will be an important factor for alpha particle transport. In particular, core localized modes (CLMs) are of major concern, since extended regions of low shear may be present in sawtoothing plasmas or hybrid regimes [1, 2]. In contrast to the conventional toroidal Alfvén eigenmode (TAE), with its single eigenfrequency per Alfvén gap, low shear toroidal Alfvén eigenmodes (LSTAEs) exhibit a multiple spectrum, and they can exist at considerably higher plasma pressure than ordinary TAEs [3, 4, 5].

In fact, the very development of the CLM concept was prompted by DT experiments on TFTR, where Alfvénic instabilities were present at surprisingly high plasma pressures. More recently, CLMs were recognized as important features of sawtoothing tokamaks. On the Joint European Torus (JET), core localized "tornado" modes were found to precede monster sawtooth crashes [6], and on Alcator C-Mod, frequency sweeping modes associated with very low shear were observed during the sawtooth cycle [7].

In this contribution, we investigate radiative damping of CLMs due to wave tunneling, by incoorporating non-ideal effects such as finite electron inertia and ion Larmor gyroradius into the ideal MHD framework.

II. Ideal MHD Modes In finite-β, ideal MHD theory, the ordering $\epsilon \ll s^2 \ll 1$, with $\epsilon = r/R$ the inverse aspect ratio and $s = r/q\; dq/dr$ the magnetic shear, admits a downshifted TAE with frequency

$$\omega^2 = \omega_0^2 \left[1 - \hat{\epsilon} \left(1 - \frac{\pi^2 s^2}{8} \left[1 - \alpha \frac{1+s}{s^2} \right]^2 \right) \right]$$

Here, $\omega_0 = v_A/2qR$ is the frequency at the center of the toroidicity induced Alfvén gap, $\hat{\epsilon} = 2\,(\epsilon + \Delta')$ (with Δ' the radial derivative of the Shafranov shift Δ), and $\alpha = -(2Rq^2/B^2)\; dp/dr$ is the normalized pressure gradient. This solution is valid only for $\alpha < s^2/(1+s)$, so the mode only exists at moderate plasma pressures.

In the plasma core however, observations suggest that $s^2 \ll \epsilon \ll 1$ might hold. With the ordering $s \sim \epsilon \ll 1$, ideal MHD theory predicts two core-localized modes (CLMs) in the Alfvén gap – one downshifted and one upshifted (see Figure 1). Their respective

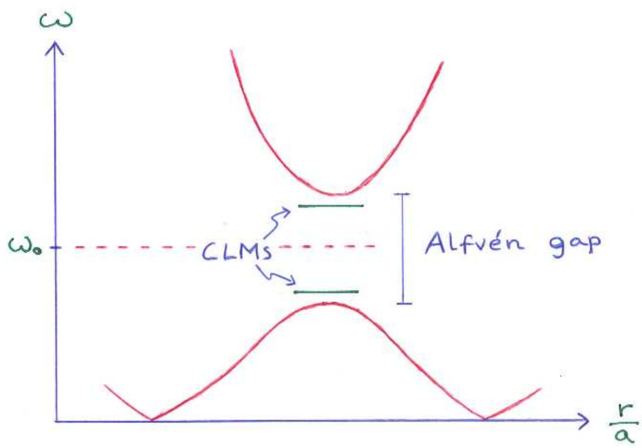

Figure 1: Toroidicity induced Alfvén gap with down- and upshifted CLMs residing close to the upper and lower Alfvén continua.

frequencies are given by

$$\omega_d^2 = \omega_0^2 \left[1 - \hat{\epsilon}\left(1 - \frac{\pi^2 s^2}{8}\left[1 + \frac{\delta - \alpha}{s^2}\right]^2\right)\right]$$

and

$$\omega_u^2 = \omega_0^2 \left[1 + \hat{\epsilon}\left(1 - \frac{\pi^2 s^2}{8}\left[1 - \frac{\delta - \alpha}{s^2}\right]^2\right)\right] ,$$

with $\delta = \epsilon + 2\Delta'$. The downshifted mode exists for $\alpha < \delta + s^2$, and the upshifted mode exists for $\alpha < \delta - s^2$.

III. Radiative Damping of CLMs Adding first order finite ion Larmor gyroradius effects to the ideal MHD theory introduces new terms in the mode equations. The problem is solved by asymptotic matching, with the following Fourier space Shrödinger-type eigenvalue problem in the non-ideal region:

$$\frac{d^2\psi}{dk^2} + [E - V(k)]\,\psi = 0$$

with $E = -(1 - g^2)$, and

$$V(k) = 2g\lambda^2 k^2 - \lambda^4 k^4 \ .$$

Here, $k = \frac{\hat{\epsilon}}{4s}\frac{k_r}{k_\theta}$, with $k_\theta = \frac{m}{r}$ the poloidal wave number, and

$$\lambda^2 = \frac{16 s^2}{\hat{\epsilon}^3}\left[\frac{3}{4} + \tau(1 - i\delta)\right](k_\theta \rho_i)^2 \ll \epsilon \ ,$$

with $\tau = T_e/T_i$, $0 < \delta \ll 1$, and ρ_i the ion Larmor gyroradius. Finally, g is a measure of the mode frequency deviation from ω_0 given by

$$g = \frac{\omega^2/\omega_0^2 - 1}{\hat{\epsilon}} .$$

Note that the Alfvén gap in Figure 1 extends from $\omega^2 = \omega_0^2 (1 - \hat{\epsilon})$ to $\omega^2 = \omega_0^2 (1 + \hat{\epsilon})$, corresponding to $g = \pm 1$. The potential $V(k)$ is plotted for $g \approx \pm 1$ in Figure 2.

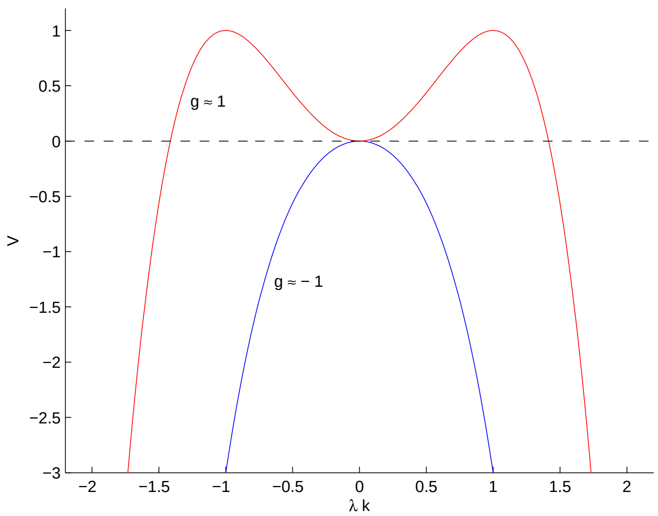

Figure 2: Schrödinger potentials for $g \approx \pm 1$.

For modes with frequencies in the Alfvén gap, $|g|$ is smaller than 1, so that $1 - g^2 > 0$. Hence, $E < 0$, and the problem at hand is of the tunneling type (see Figure 3). The transmitted wave propagates into the $k \gg 1$ region, where it develops rapid radial oscillations. In real space this corresponds to parts of the wave propagating away from the gap location towards regions of small $k_{\|}$. We assume that the tunneled wave amplitude is lost, and that this mechanism accounts for the radiative damping of the CLMs.

To get all numerical factors right, we formulate the problem in balloning space, and match the solution in the kinetic region asymptotically with ideal MHD solutions [8]. The resulting damping rates are

$$\left|\frac{\gamma}{\omega}\right| = \hat{\epsilon}\frac{\pi^2 s^2}{8}\left[1 + \frac{\delta - \alpha}{s^2}\right]^2 \exp\left(-\frac{\pi^3 s^2}{2^{7/2}\lambda}\right)$$

for the downshifted mode, and

$$\left|\frac{\gamma}{\omega}\right| = \hat{\epsilon}\frac{\pi^2 s^2}{2}\left[1 - \frac{\delta - \alpha}{s^2}\right]^2 \exp\left(-\frac{2^{5/2}}{3\lambda}\right)$$

for the upshifted mode. Hence, the upper mode is seen to be much less radiatively damped than the lower mode, especially in regions of very low shear.

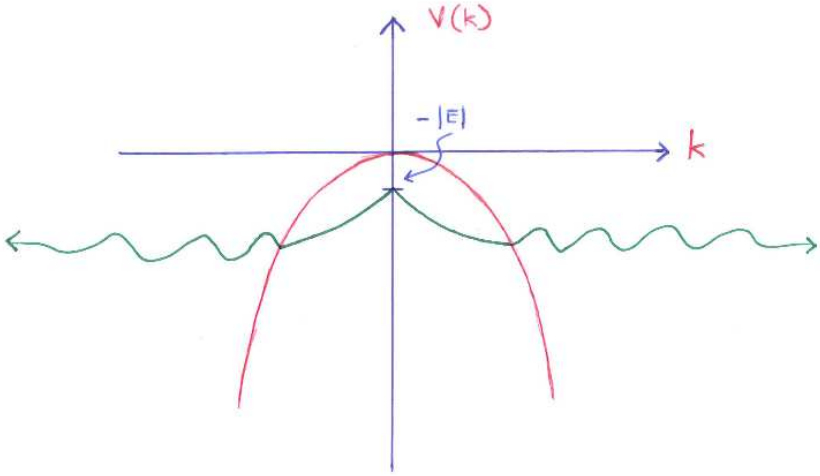

Figure 3: Illustration of the tunneling. The wave amplitude is attenuated until it reaches the turning points, where $V(k) = E$. From there it propagates outwards, only slightly damped due to $\delta > 0$.

Finally, we note that the non-ideal effects included here also give rise to kinetic modes with frequencies above the upper continuum. These modes have $g \gtrsim 1$, so that $E > 0$. Hence, $V(k)$ acts as a local potential well with a finite barrier width, providing boundary conditions for a discrete mode spectrum and once again permitting wave tunneling into the $k \gg 1$ region. Also, in the ideal MHD limit $\lambda^2 = 0$, where the effects of finite electron inertia and ion Larmor radius are completely neglected, the potential is given by $V(k) = 0$, and it extends out to infinity. Hence, there is no wave tunneling present in ideal MHD theory, and therefore no radiative damping.

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