Stochastic modeling of an industrial electric load

Ingemar Mathiasson

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Department for Energy and Environment

Division of Electric Power Engineering

Chalmers University of Technology

Content

1 INTRODUCTION

This paper deals with the electric power consumption, where the load consists of the sum of different kind of companies. The aim of the study is to find out a suitable mathematic model that could be used for statistic evaluations of the power consumption when a region consisting of a mix of different companies is on hand. The model is based on results from a measurement campagne with a specific region as measurement object.

Company region: Almås, Lindome
Companies and power consumption according to Table 1.
Measurement interval: 05-10-25 to 05-11-29

Company	Power consumption (kWh/year)	Fuse (A)	Voltage (kV)
HB LINDOME	220929	315	10
HB LINDOME	266298	200	10
MÖLNDAL KOMMUN	22900	35	0.4
MÖLNDAL KOMMUN	111900	100	0.4
SVENSK VÅTRUMSTEKNIK I GÖTEBORG	19400	63	0.4
BILHUSET I LINDOME AB	121600	63	0.4
BILHUSET I LINDOME AB	13100	25	0.4
BILHUSET I LINDOME AB	3000	16	0.4
BILHUSET I LINDOME AB	15300	16	0.4
GISSLÉNS ENTREPRENAD AB	41400	35	0.4
TRIAGON SNICKERI AB	45700	63	0.4
R SEGERS FASTIGHETSKONTOR	67500	80	0.4
STÅLMARIN AB	45600	35	0.4
HULTHÉNS FASTIGHET O FÖRSÄLJNING	102000	35	0.4
STIGS RÖRLÄGGERI AB	9300	25	0.4
STURE JONSSONS PLÅTSLAGERI	11300	25	0.4
MKS INPLASTNING	6600	25	0.4
VAGOTT KB	150500	125	0.4
Total Consumption (kWh/year):	1274327		

Table 1 Name and power consumption for the companies that where involved during the measurement campage in question

Figure 1 illustrates the total power consumption for the company area during the measurement interval in question.

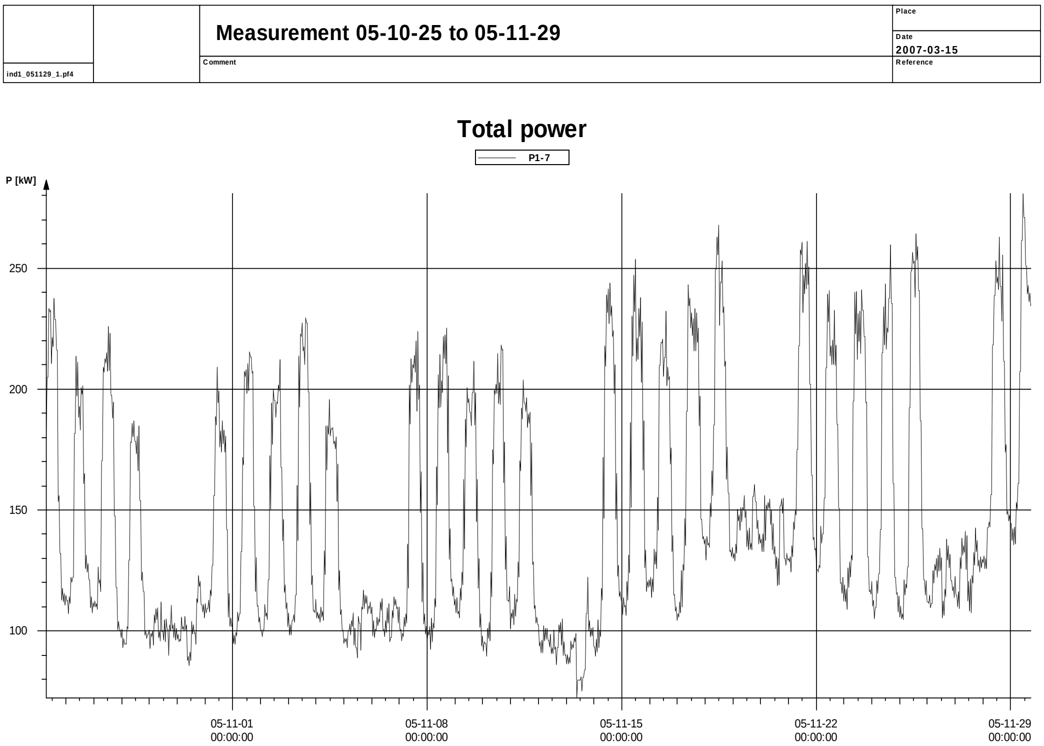

Figure 1 The power consumption for the studied company area during the measurement interval in question

Figure 2 illustrates the total power consumption during seven days. The interval 00.00 o´clock, 26/10 to 00.00 o´clock, 2/11 has been taken as an example. As can be observed there are two days with extremely small consumption. These days correspond to the weekend days, Suterday respectively Sunday.

Back to Figure 1. This figure illustrates that the consumption regarding the weekend differs from week to week. This indicates that there probably are some varying industrial activities in operation during the weekends.

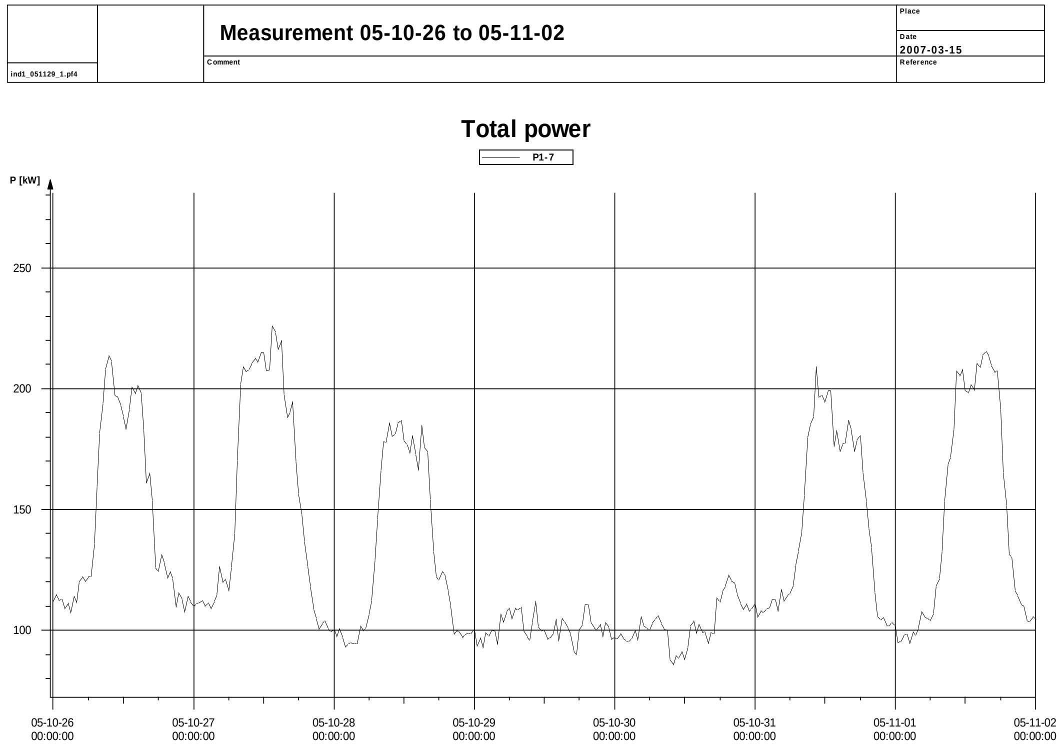

Figure 2 The power consumption for the studied company area during a sample period of seven days

Figure 3 illustrates the total power consumtion for the company area during a sample period of 24 hours. The example in question is typical regarding the power profile as a function of time for a working day. There are two main time points; “The start of the working day” and “The end of the working day”. In the figure the start point is about 06.30 and the end about 18.00. Between these two time points there are two other time points. Namely “The time point when the start up is finished” respectively “The time point when the end of the working day begins”. From Figure 3 it could be noticed that the corresponding time points are about 08.30 respectively 15.00. Figure 4 shows the principle of the modeling the type of load in question. See [1].

Comparering Figure 3 with Figure 4 results in the following corresponding points:

“The start of the working day” → TP_2

“The end of the working day” → TP_5

The time points between these ponts, namely:

“The time point when the start up is finished” → TP_3

“The time point when the end of the working day begins” → TP_4

According to above this gives:

TP_2 ≈ 06.30

TP_5 ≈ 18.00

TP_3 ≈ 08.30

TP_4 ≈ 15.00

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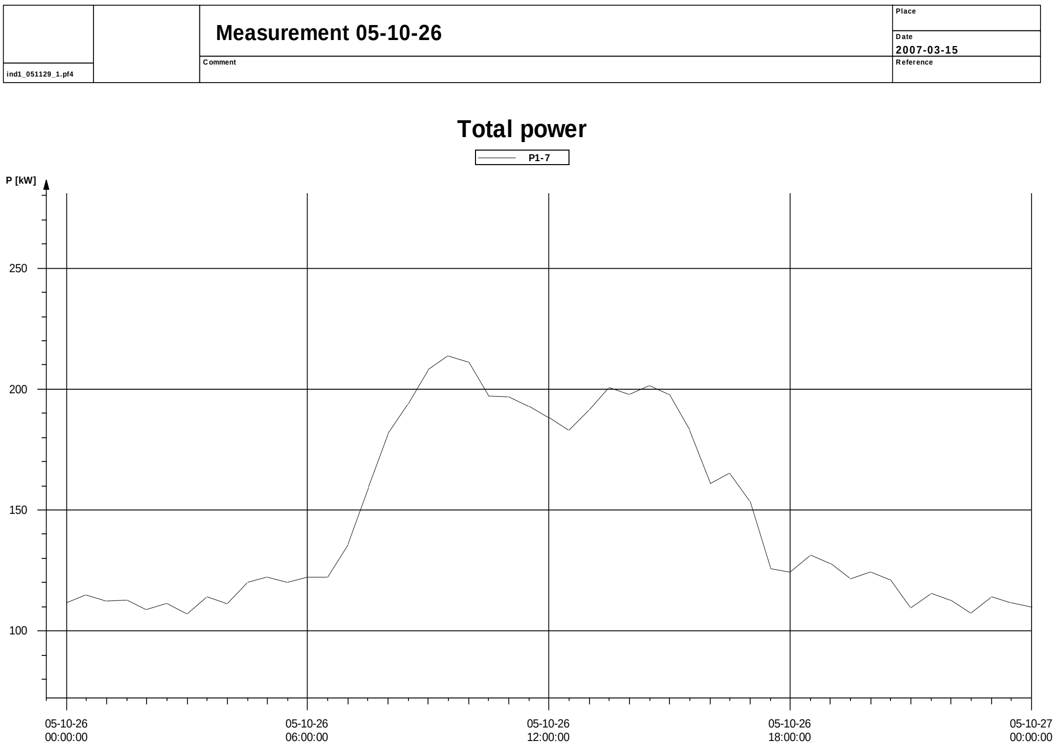

Figure 3 The power consumption for the studied company area during a sample period of 24 hours (26/10, 00.00 – 27/10, 00.00)

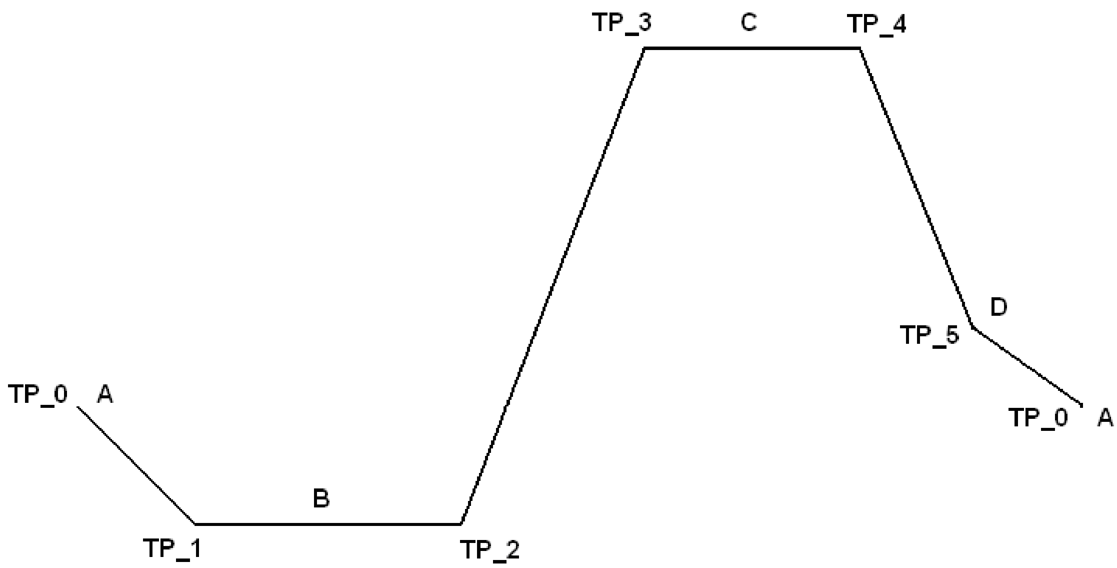

Figure 4 An illustration of the base power shifts between the different time points during a 24 hours cycle

2 MEASUREMENT RESULTS

The measurement results follow according to Figure 5 to Figure 38. These figures illustrate the total power consumption for every single day during the measurement period in question.

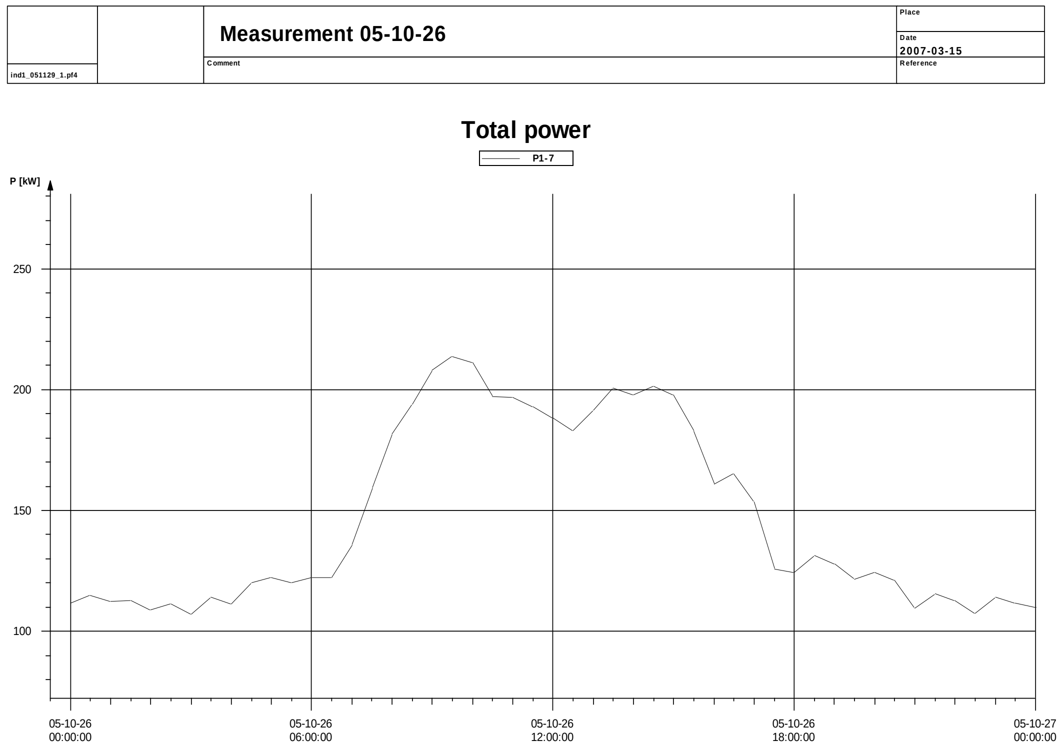

Figure 5 The power consumption 26/10, 00.00 – 27/10, 00.00

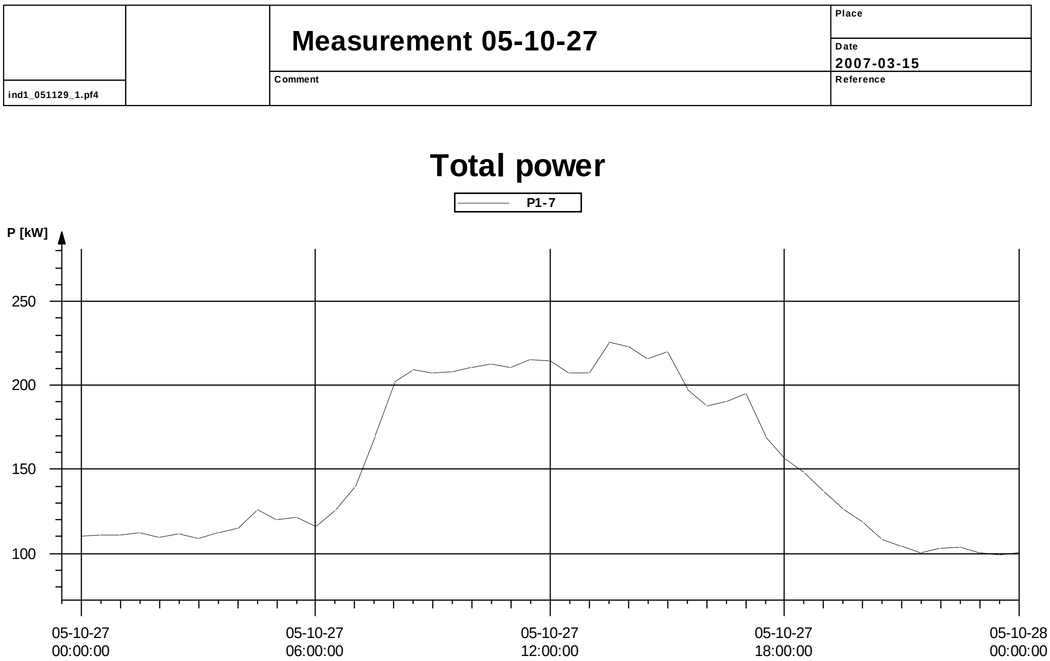

Figure 6 The power consumption 27/10, 00.00 – 28/10, 00.00

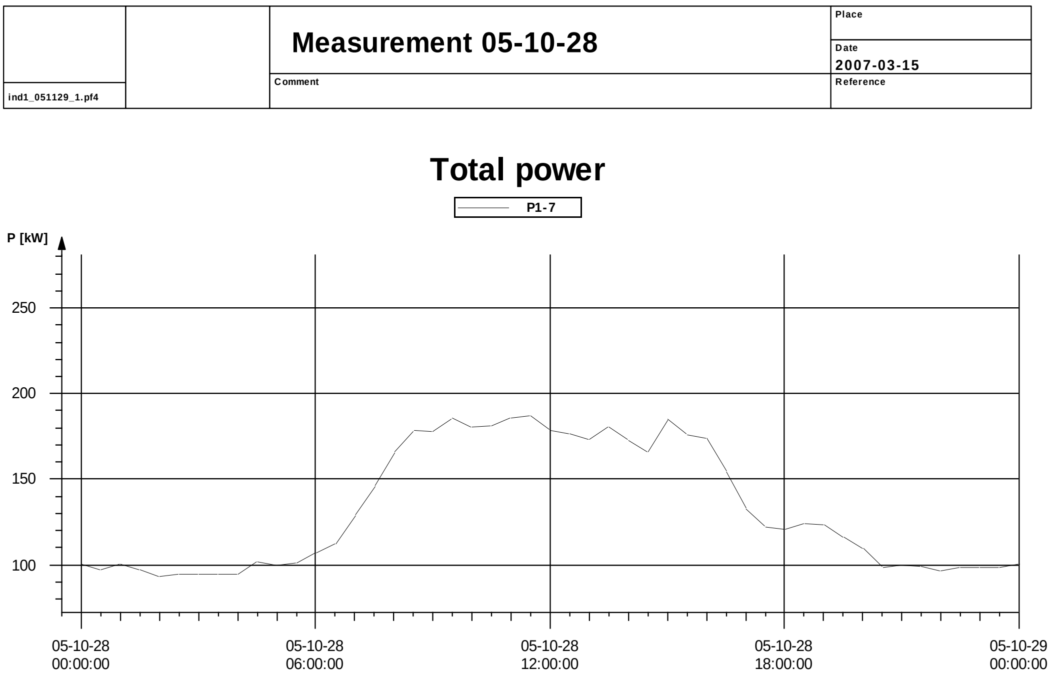

Figure 7 The power consumption 28/10, 00.00 – 29/10, 00.00

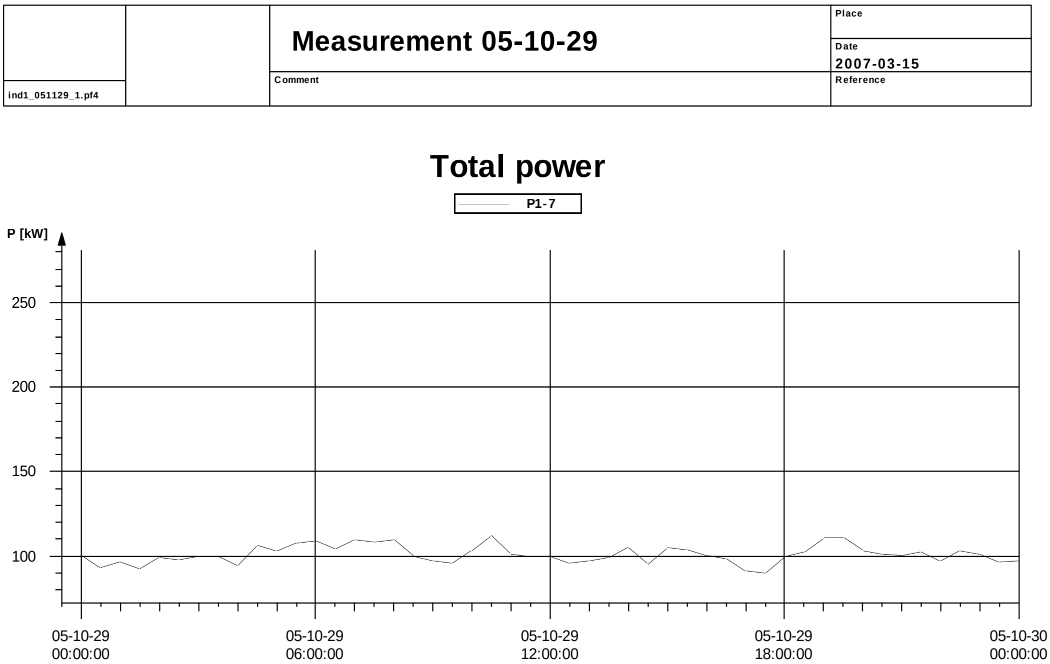

Figure 8 The power consumption 29/10, 00.00 – 30/10, 00.00

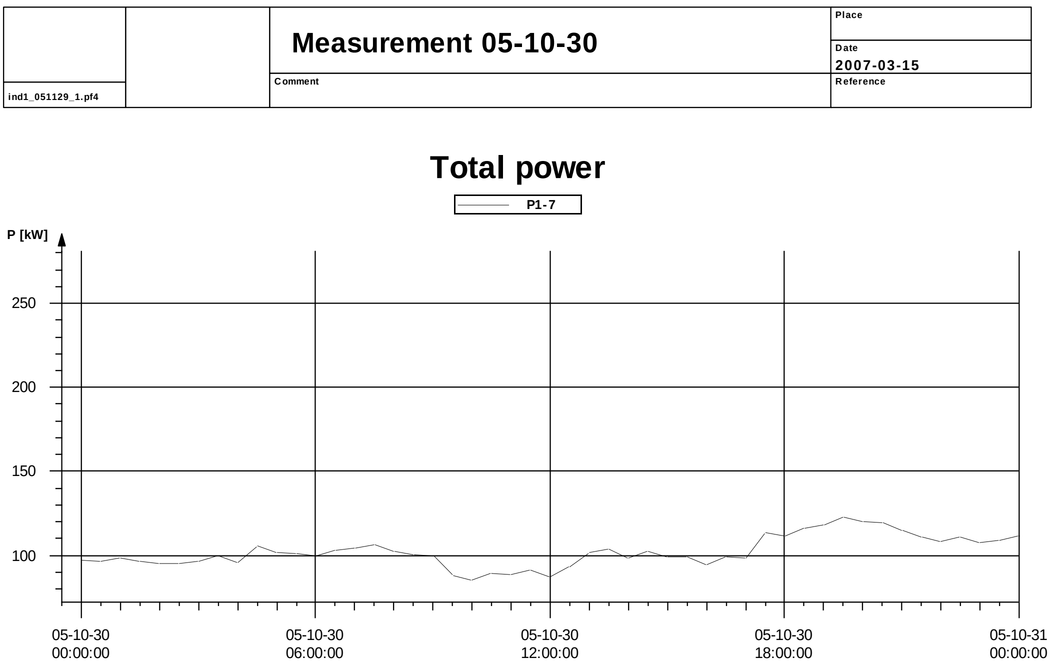

Figure 9 The power consumption 30/10, 00.00 – 31/10, 00.00

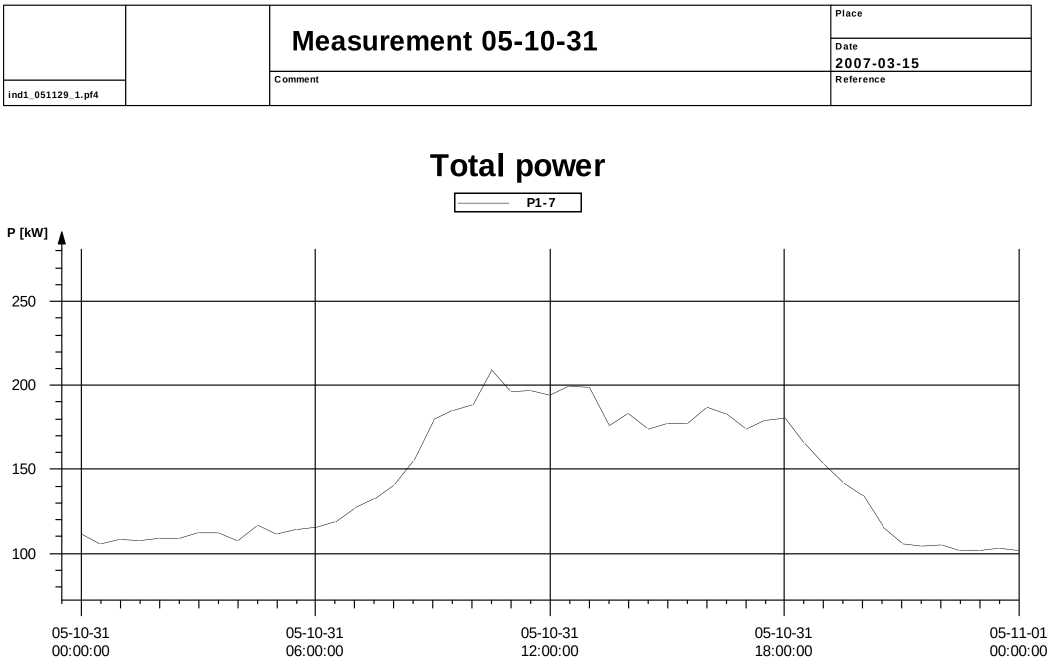

Figure 10 The power consumption 31/10, 00.00 – 1/11, 00.00

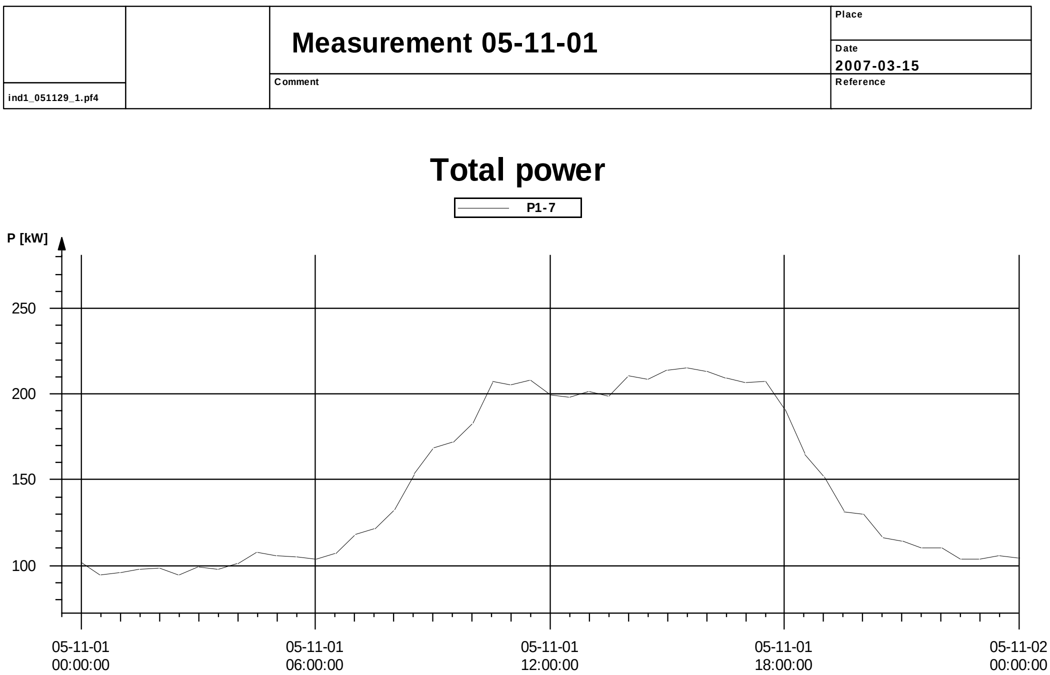

Figure 11 The power consumption 1/11, 00.00 – 2/11, 00.00

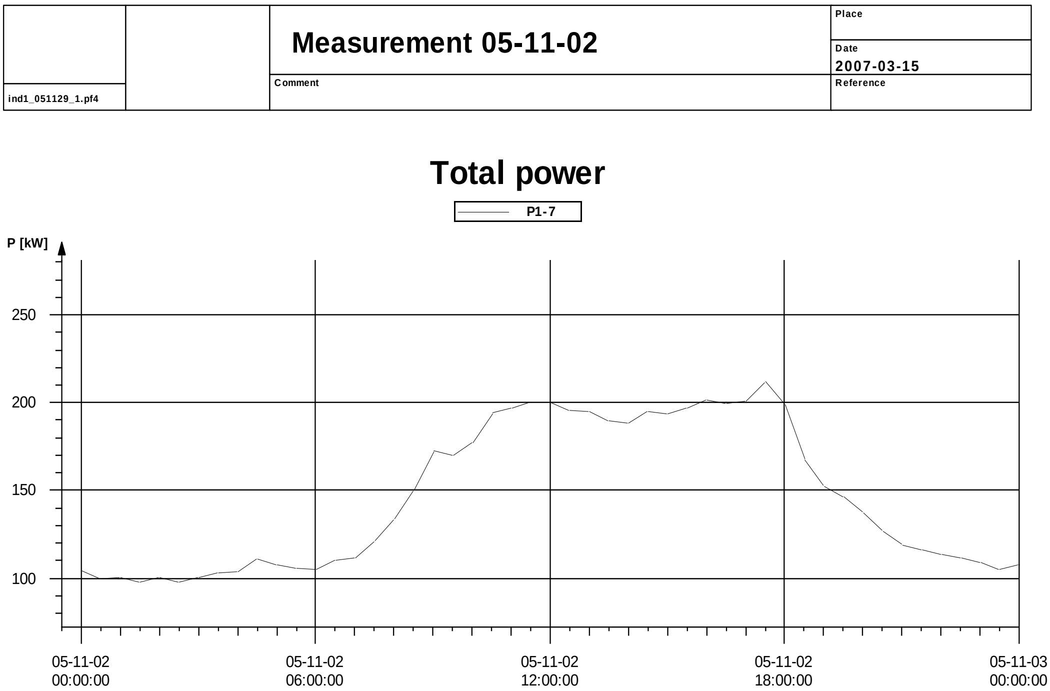

Figure 12 The power consumption 2/11, 00.00 – 3/11, 00.00

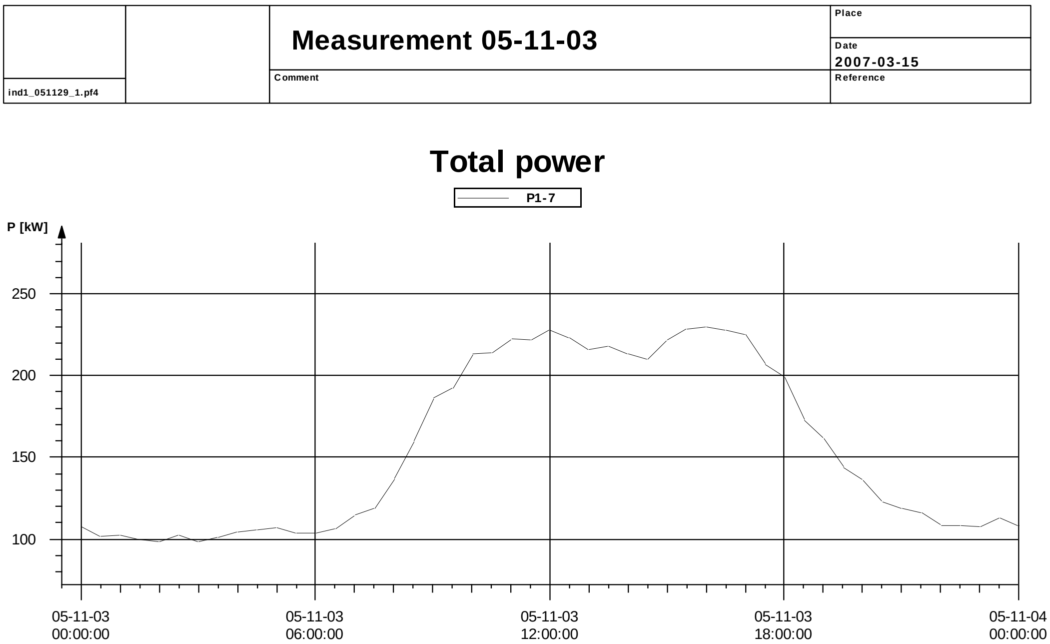

Figure 13 The power consumption 3/11, 00.00 – 4/11, 00.00

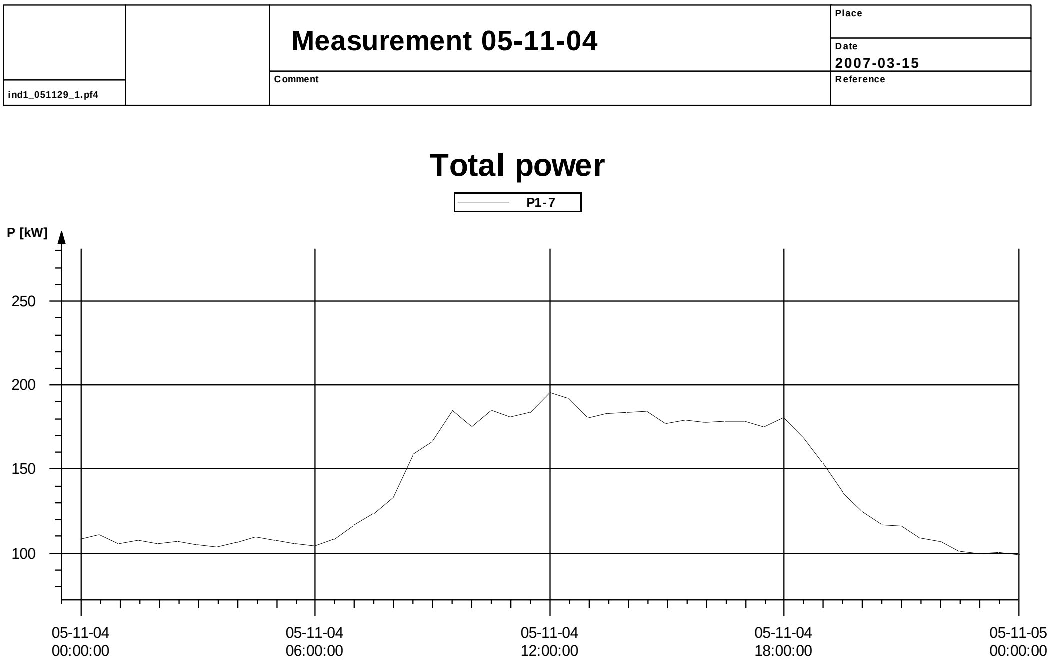

Figure 14 The power consumption 4/11, 00.00 – 5/11, 00.00

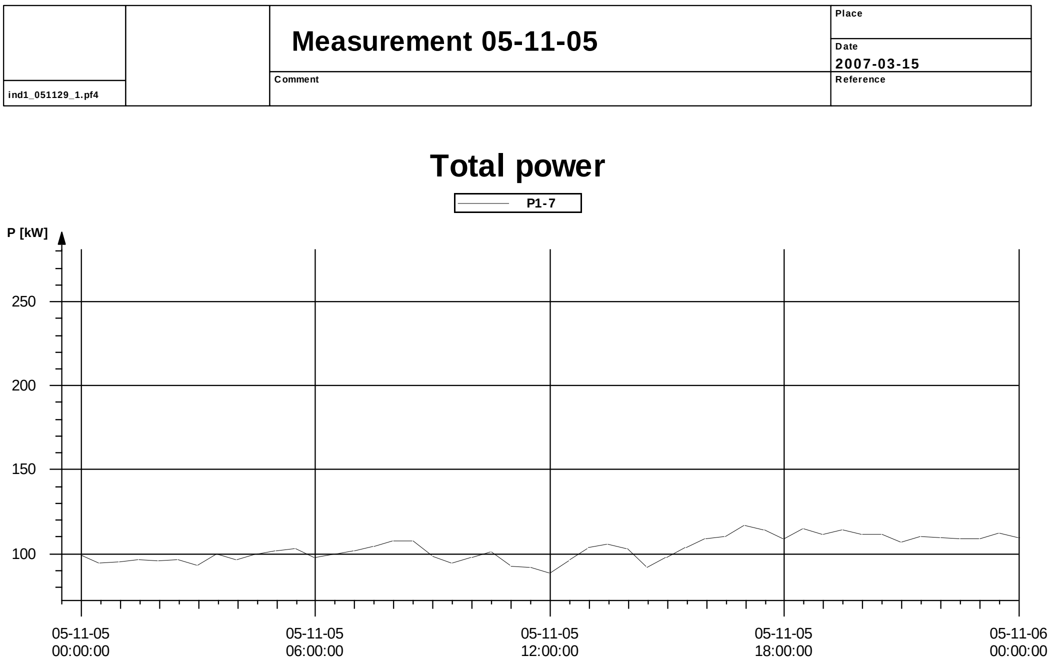

Figure 15 The power consumption 5/11, 00.00 – 6/11, 00.00

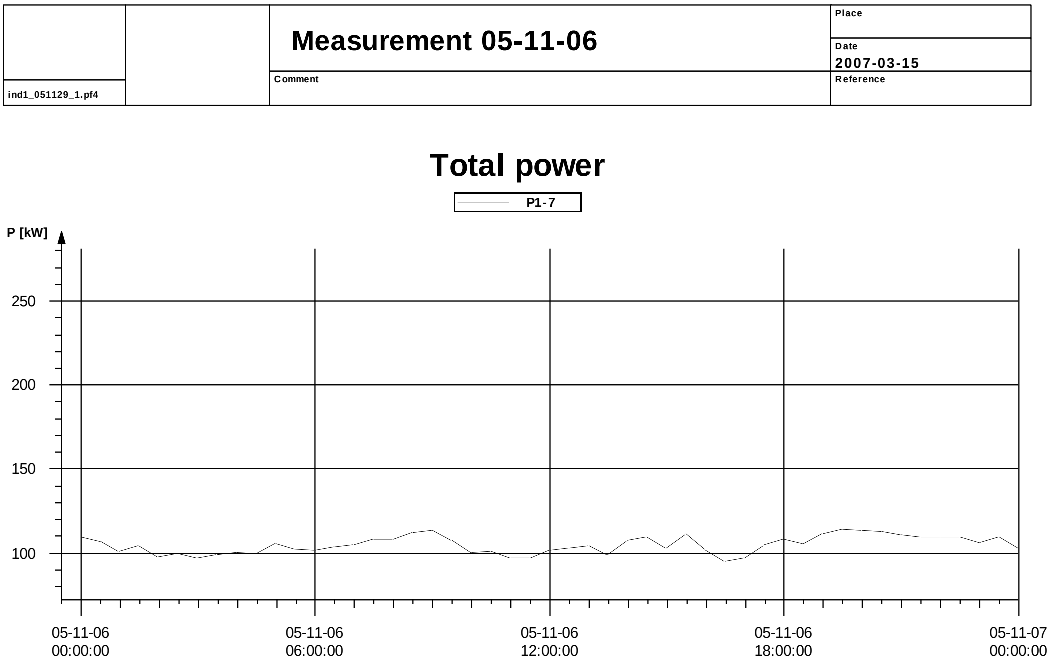

Figure 16 The power consumption 6/11, 00.00 – 7/11, 00.00

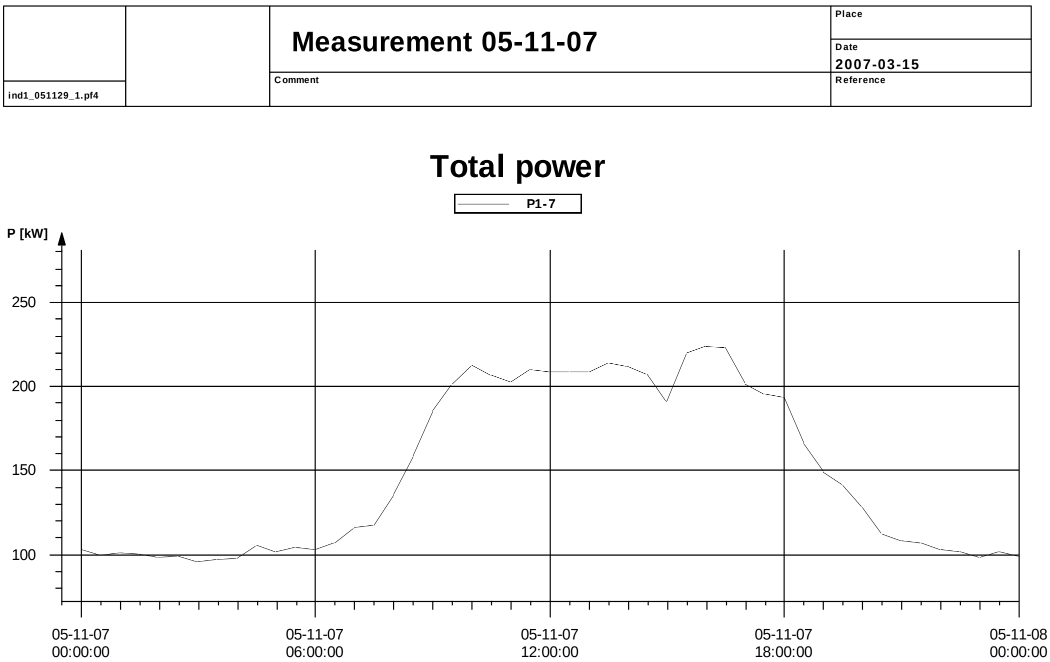

Figure 17 The power consumption 7/11, 00.00 – 8/11, 00.00

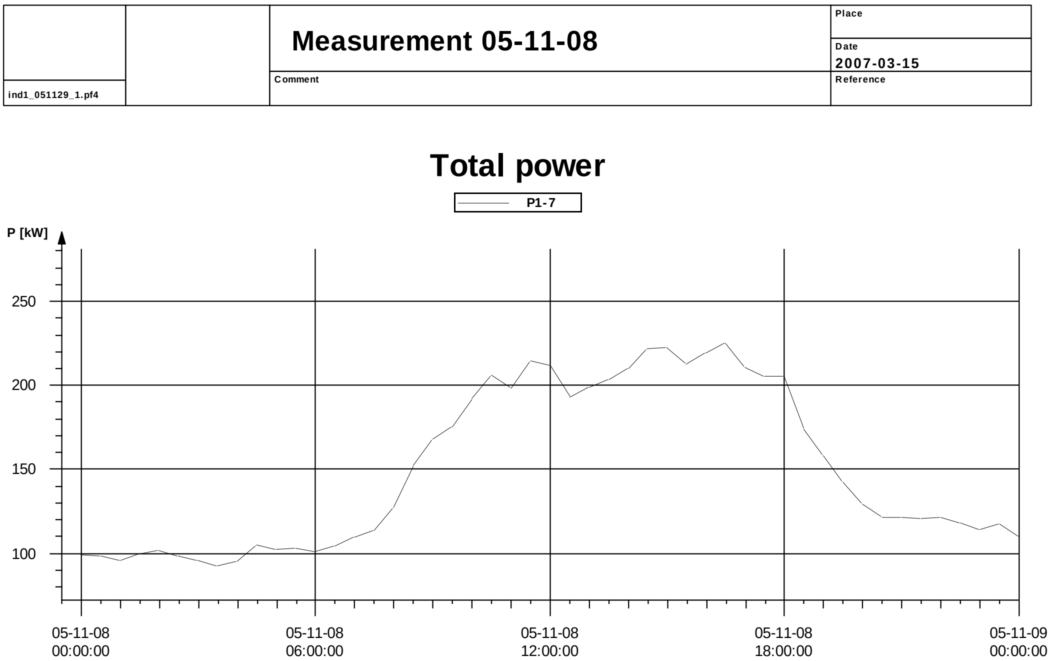

Figure 18 The power consumption 8/11, 00.00 – 9/11, 00.00

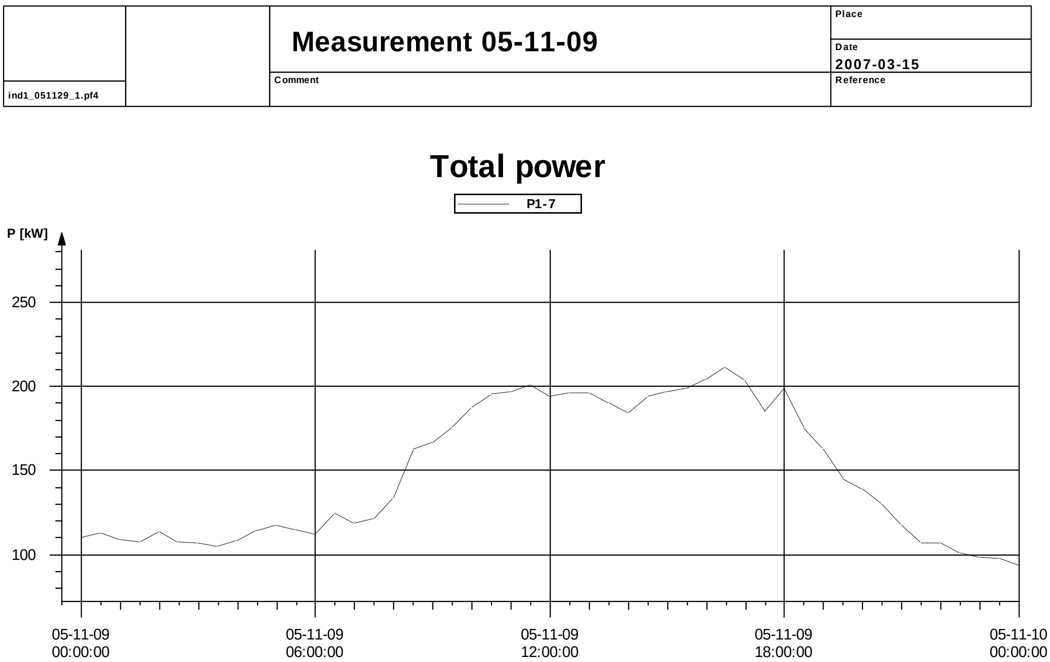

Figure 19 The power consumption 9/11, 00.00 – 10/11, 00.00

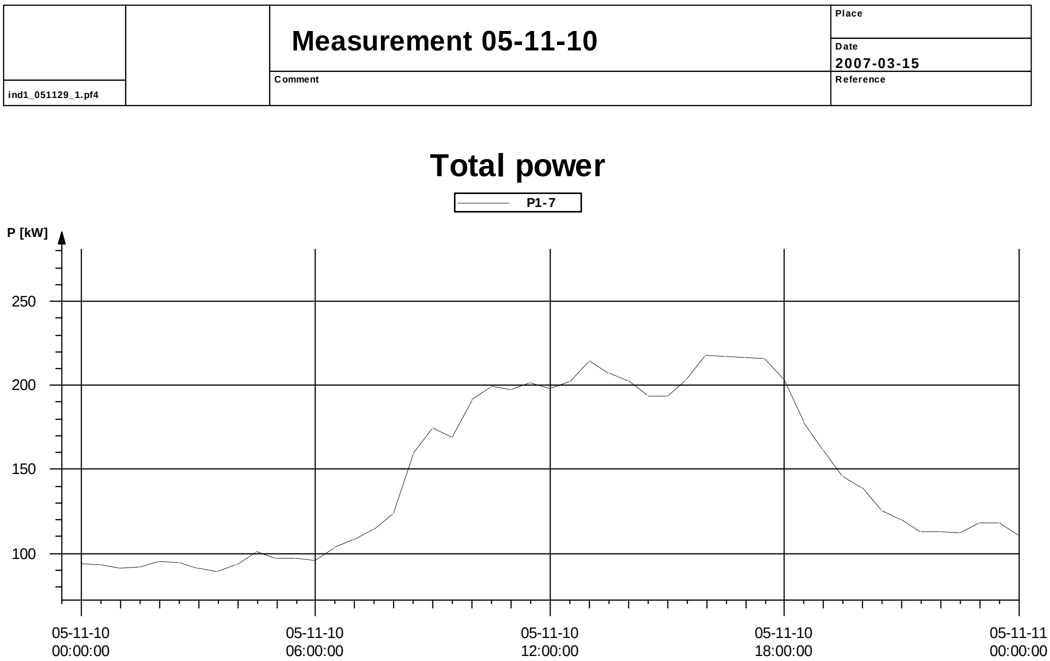

Figure 20 The power consumption 10/11, 00.00 – 11/11, 00.00

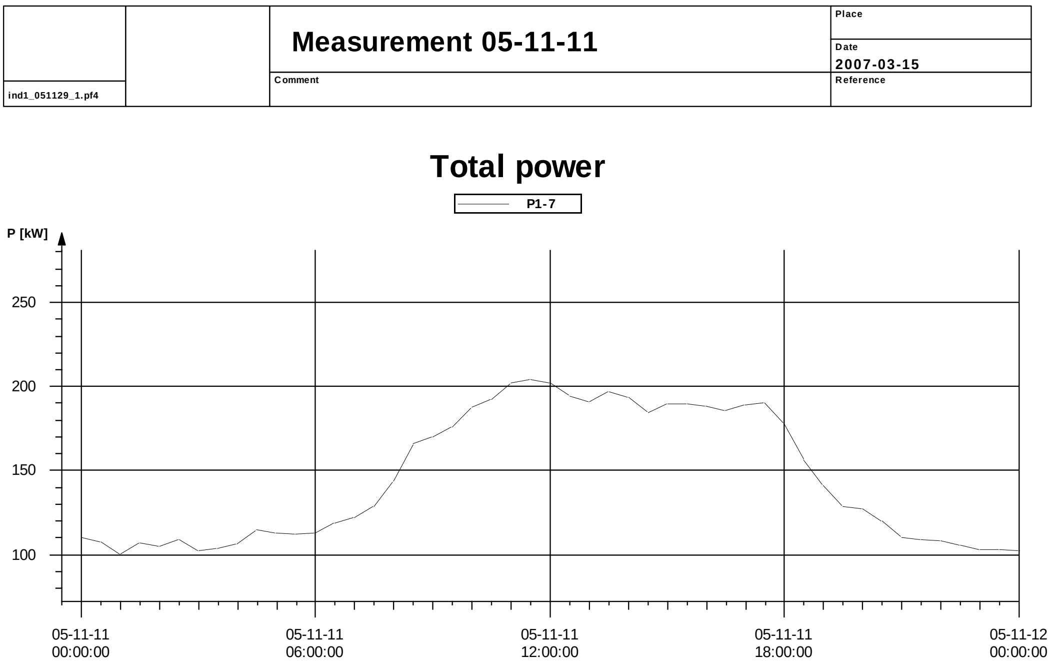

Figure 21 The power consumption 11/11, 00.00 – 12/11, 00.00

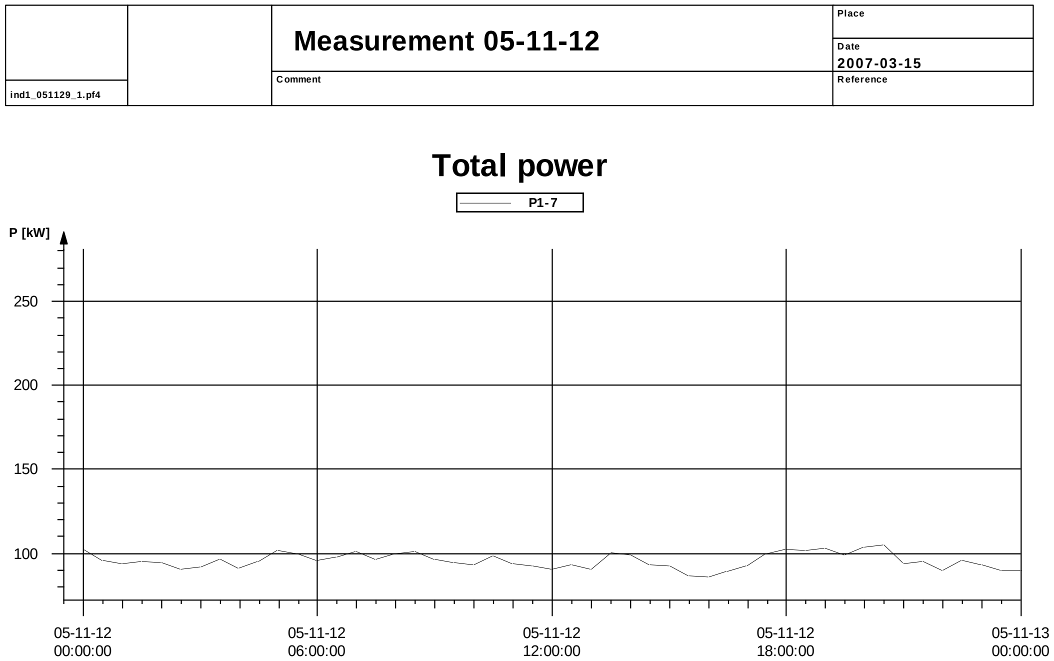

Figure 22 The power consumption 12/11, 00.00 – 13/11, 00.00

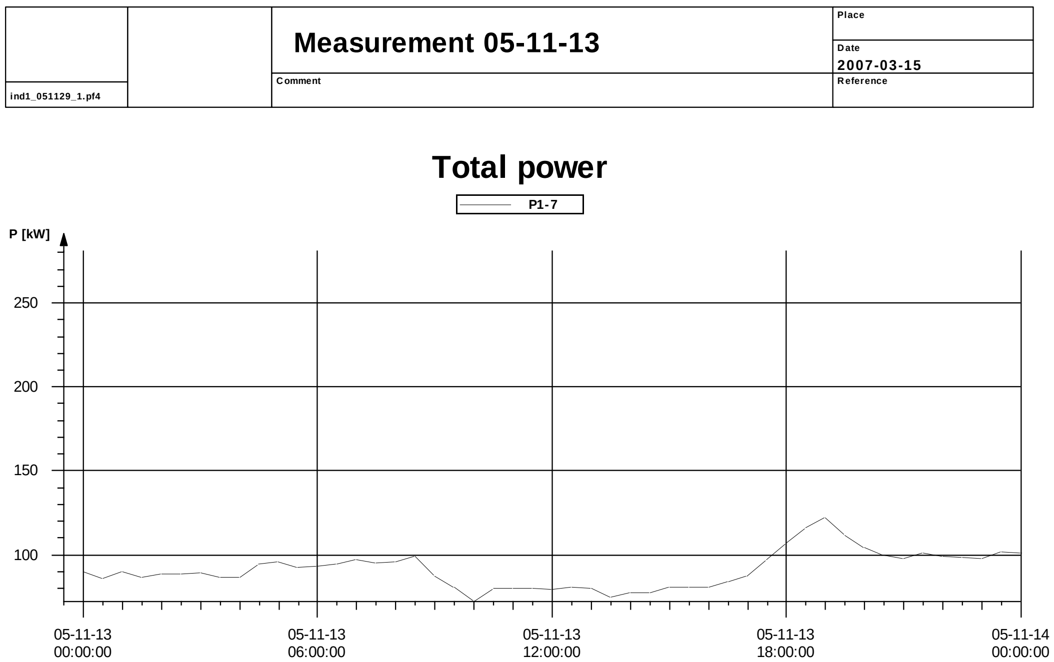

Figure 23 The power consumption 13/11, 00.00 – 14/11, 00.00

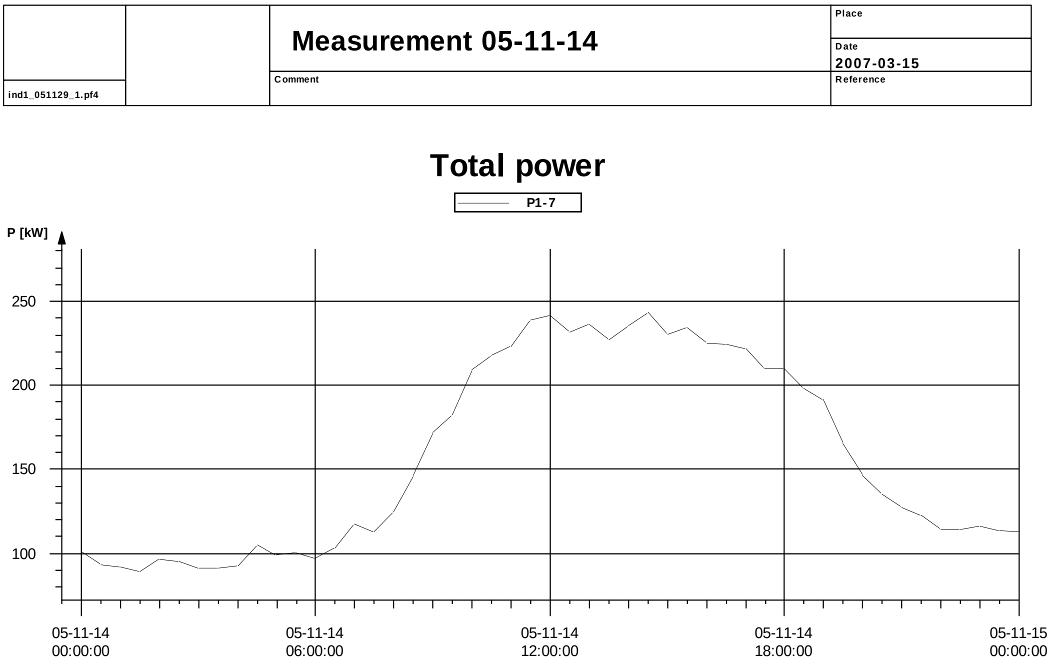

Figure 24 The power consumption 14/11, 00.00 – 15/11, 00.00

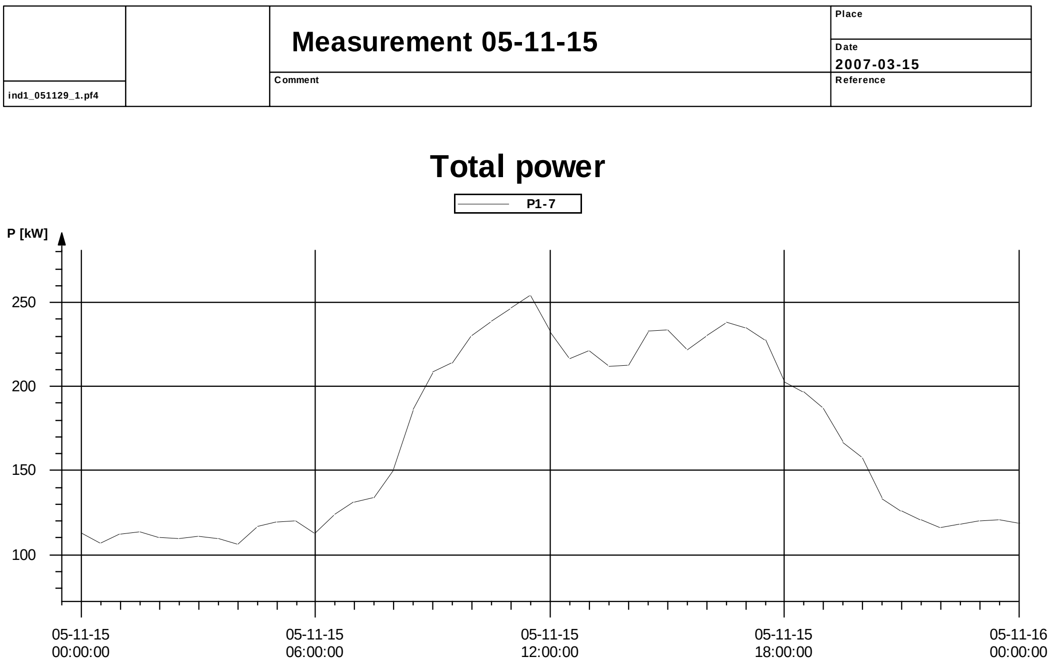

Figure 25 The power consumption 15/11, 00.00 – 16/11, 00.00

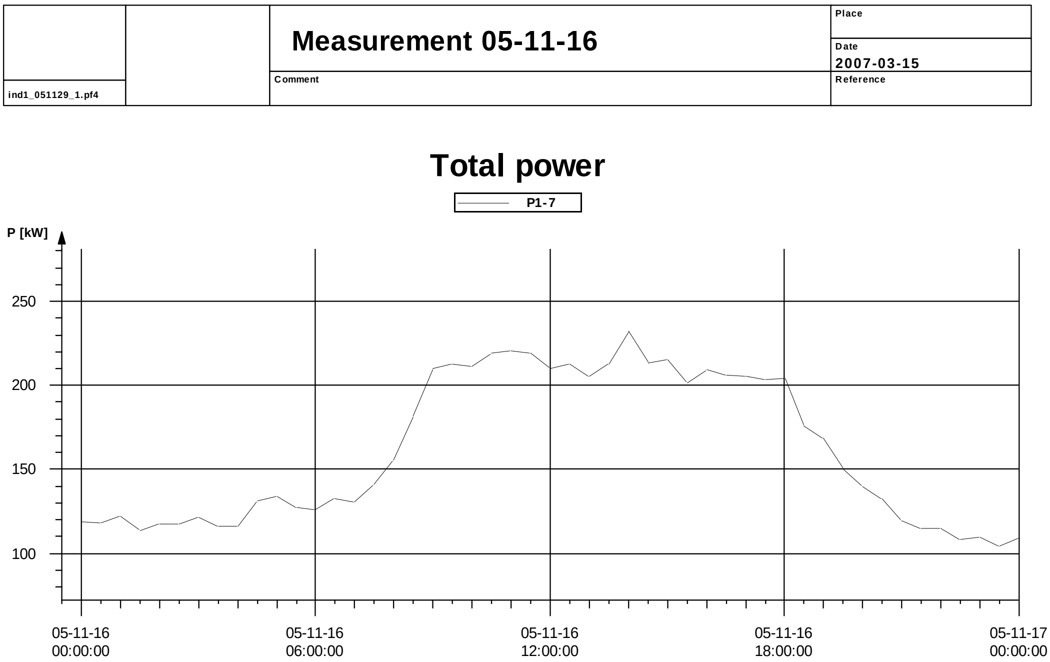

Figure 26 The power consumption 16/11, 00.00 – 17/11, 00.00

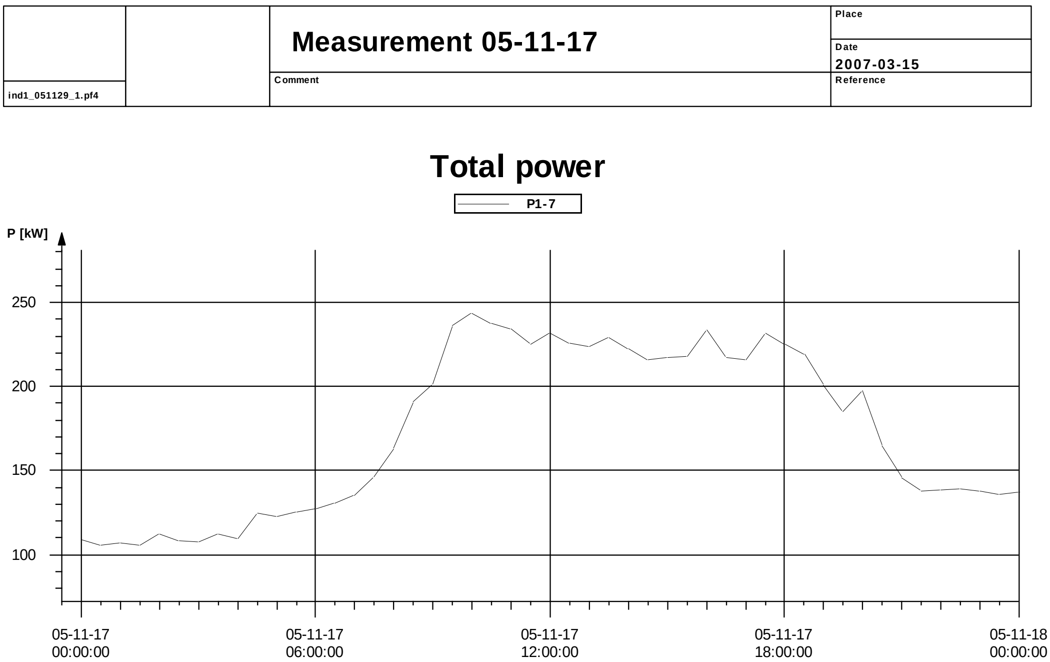

Figure 27 The power consumption 17/11, 00.00 – 18/11, 00.00

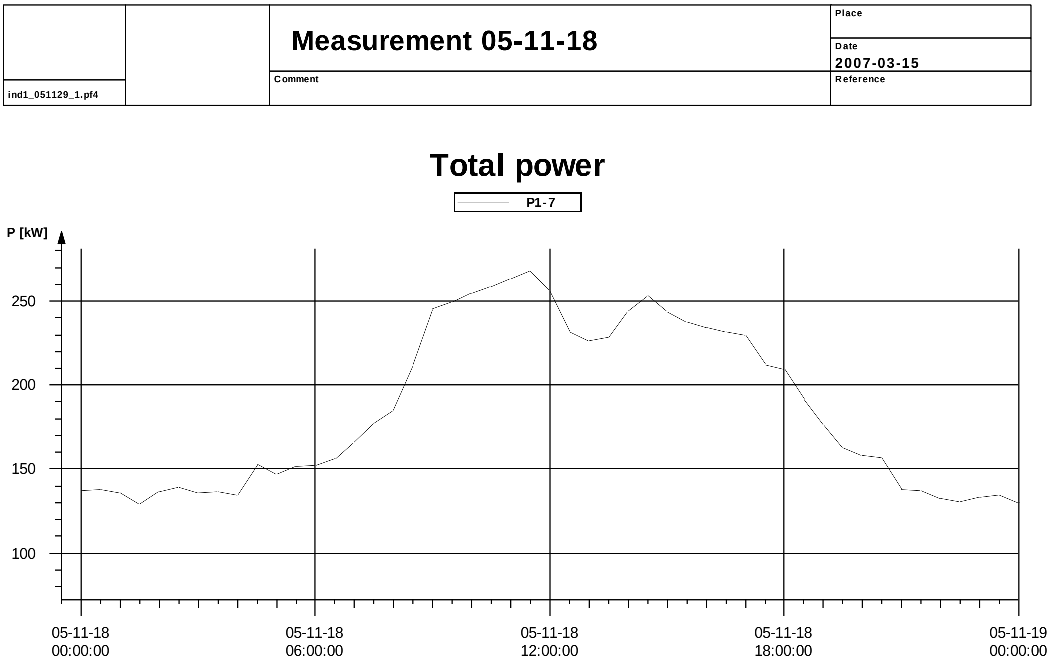

Figure 28 The power consumption 18/11, 00.00 – 19/11, 00.00

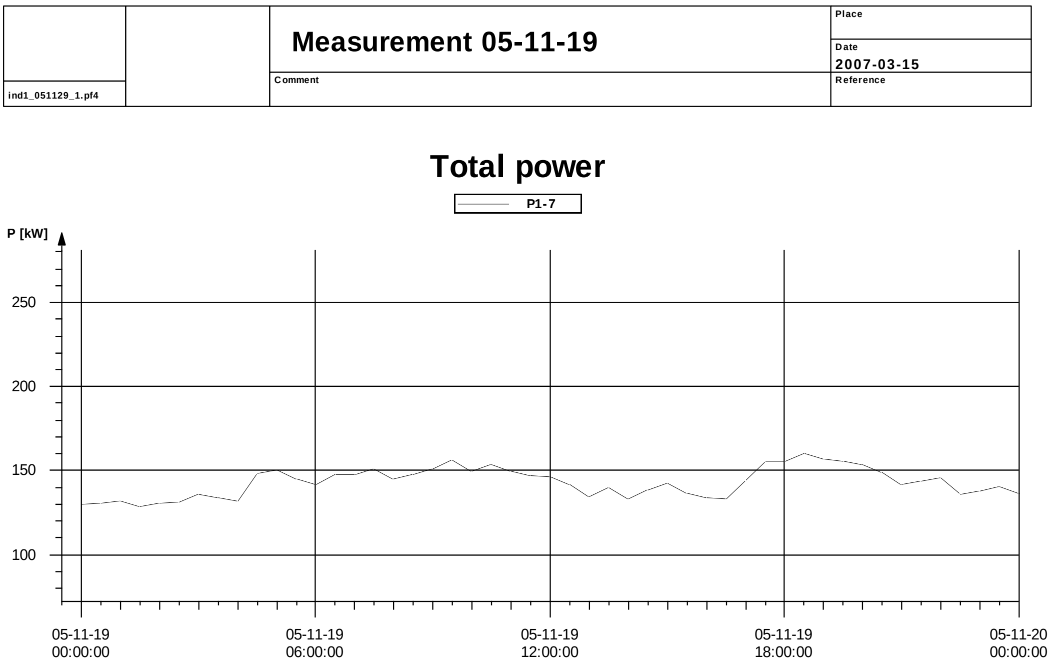

Figure 29 The power consumption 19/11, 00.00 – 20/11, 00.00

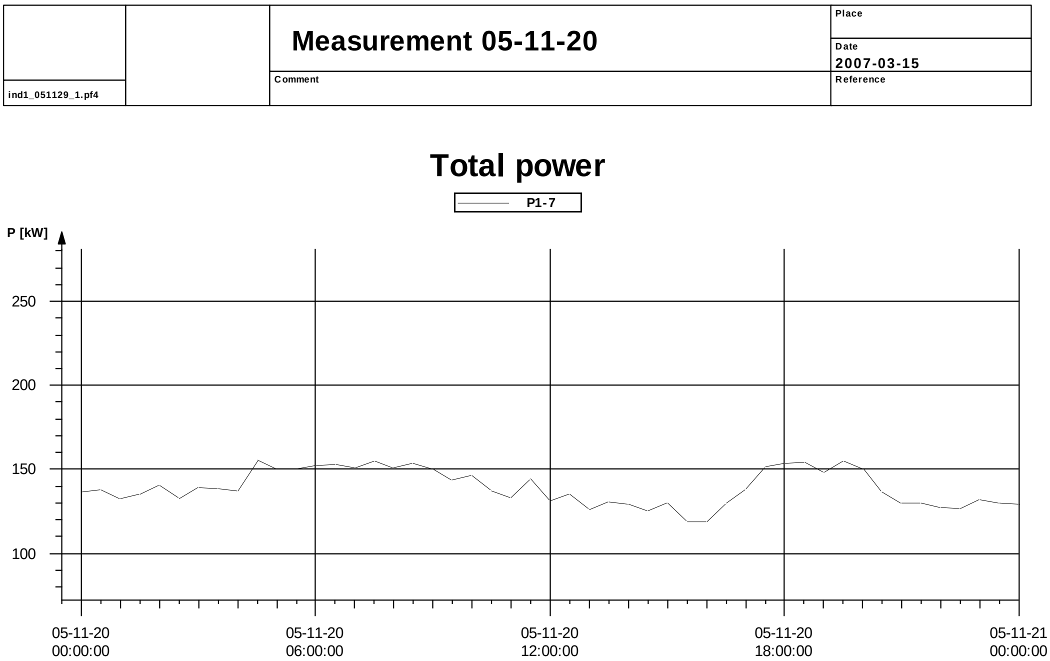

Figure 30 The power consumption 20/11, 00.00 – 21/11, 00.00

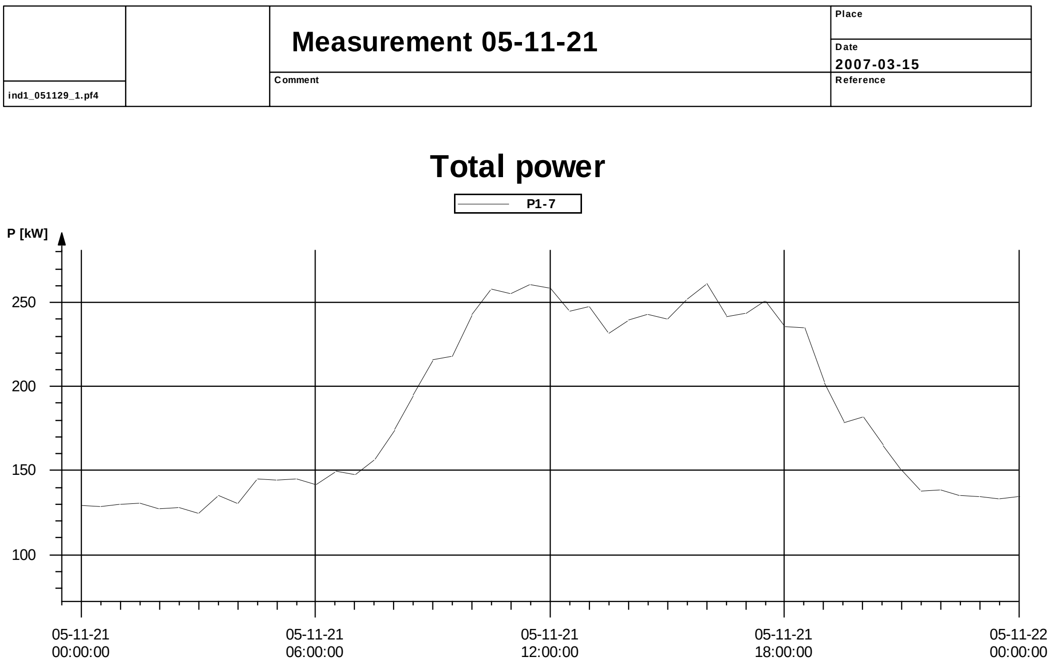

Figure 31 The power consumption 21/11, 00.00 – 22/11, 00.00

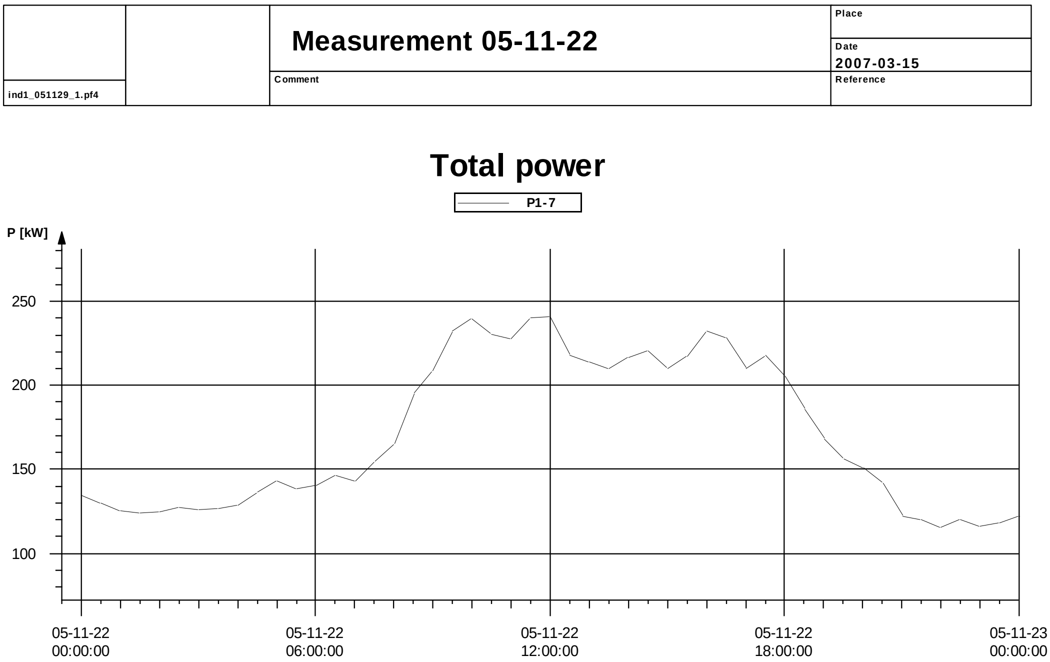

Figure 32 The power consumption 22/11, 00.00 – 23/11, 00.00

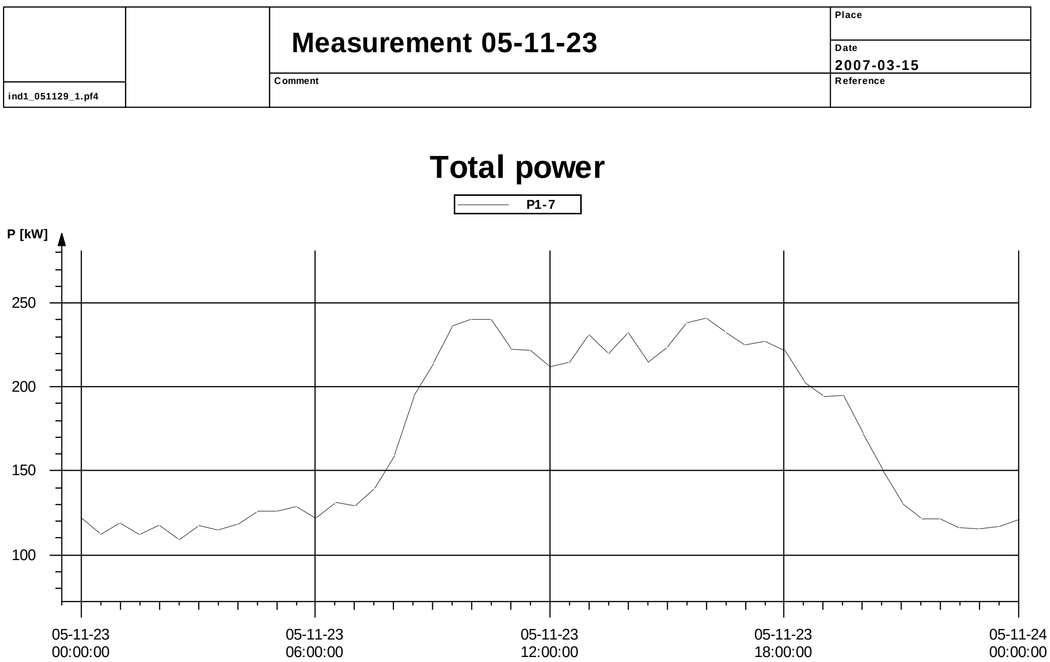

Figure 33 The power consumption 23/11, 00.00 – 24/11, 00.00

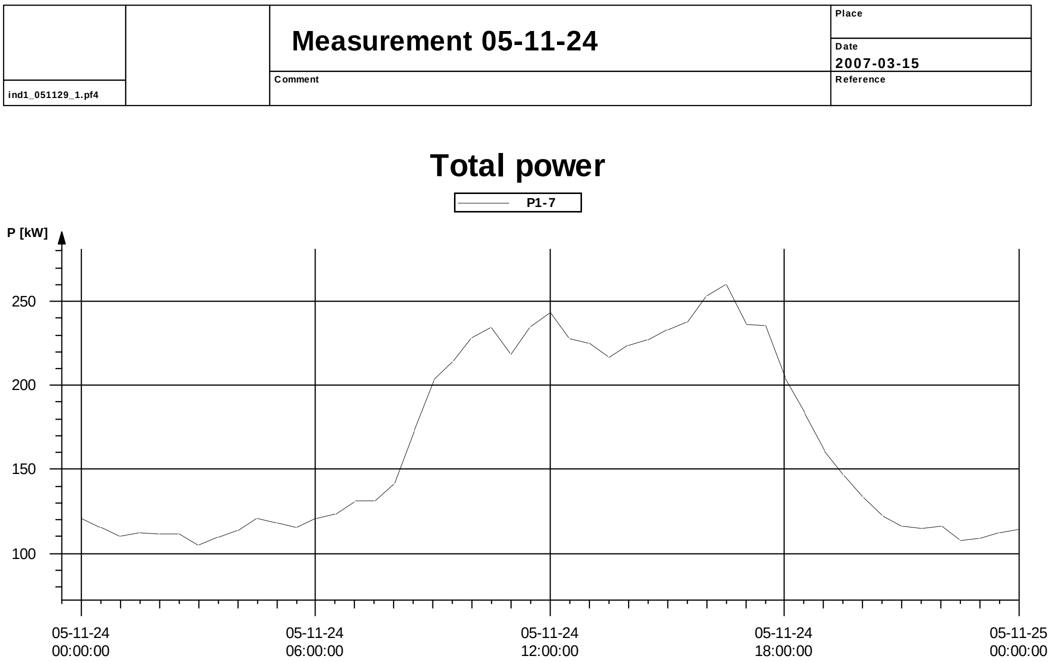

Figure 34 The power consumption 24/11, 00.00 – 25/11, 00.00

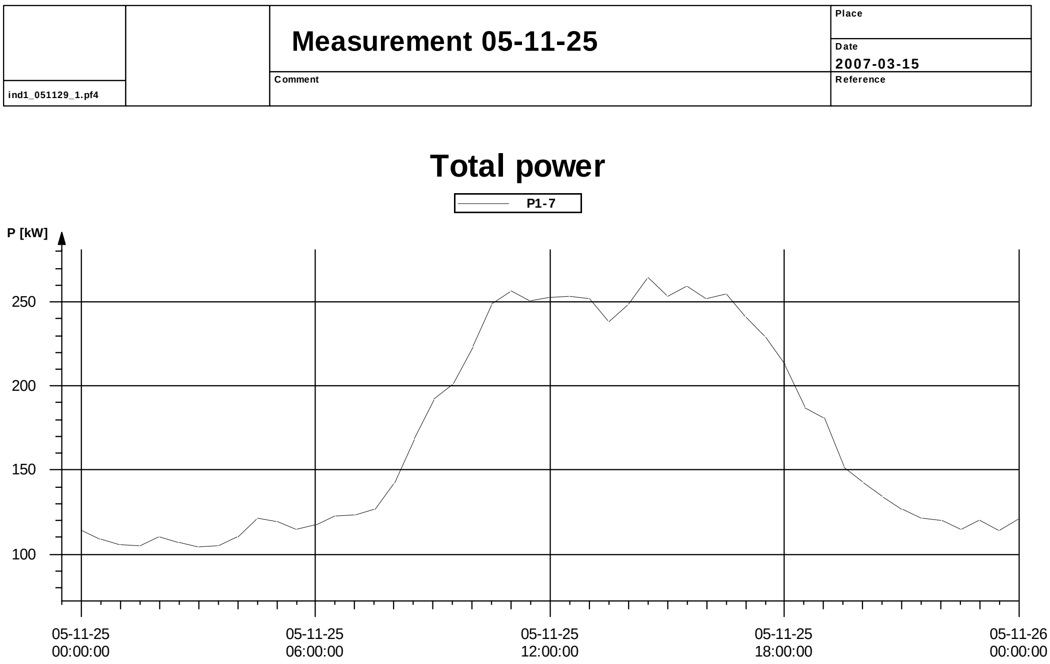

Figure 35 The power consumption 25/11, 00.00 – 26/11, 00.00

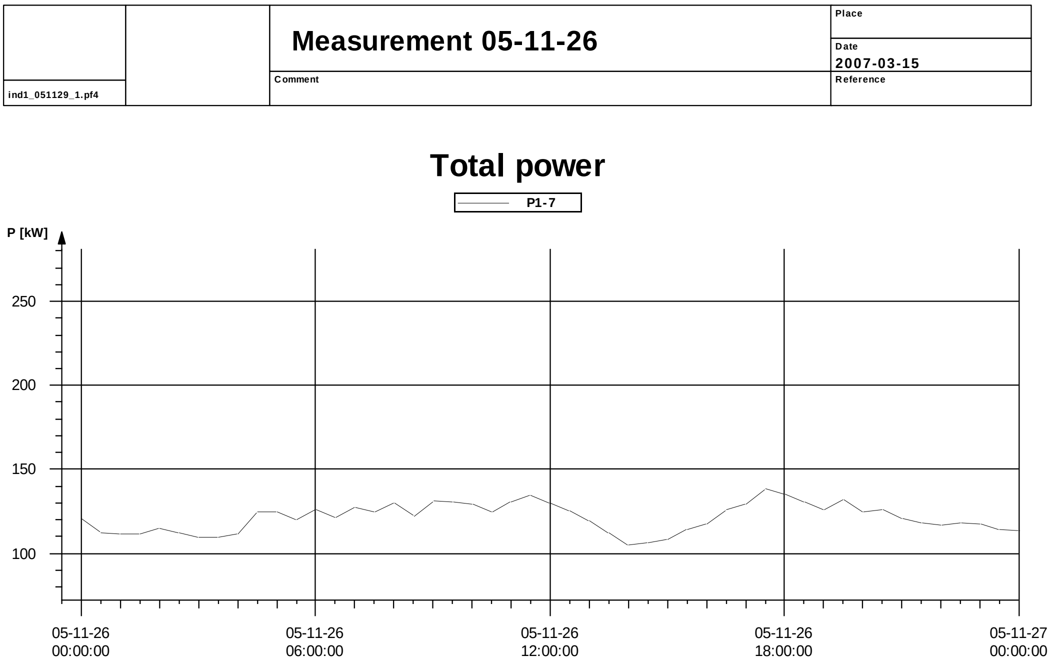

Figure 36 The power consumption 26/11, 00.00 – 27/11, 00.00

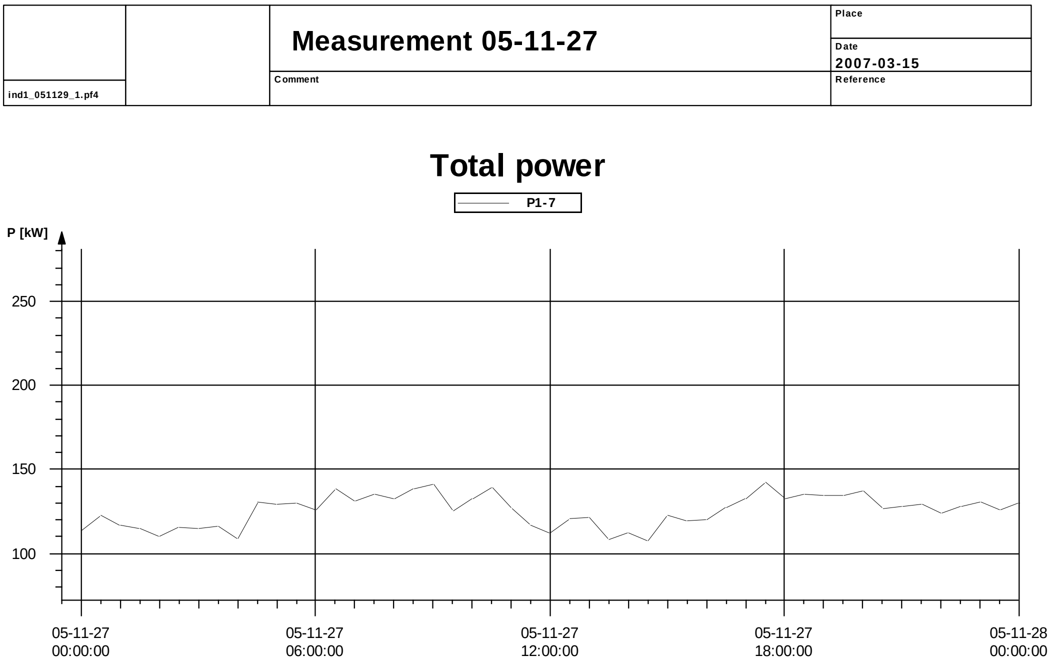

Figure 37 The power consumption 27/11, 00.00 – 28/11, 00.00

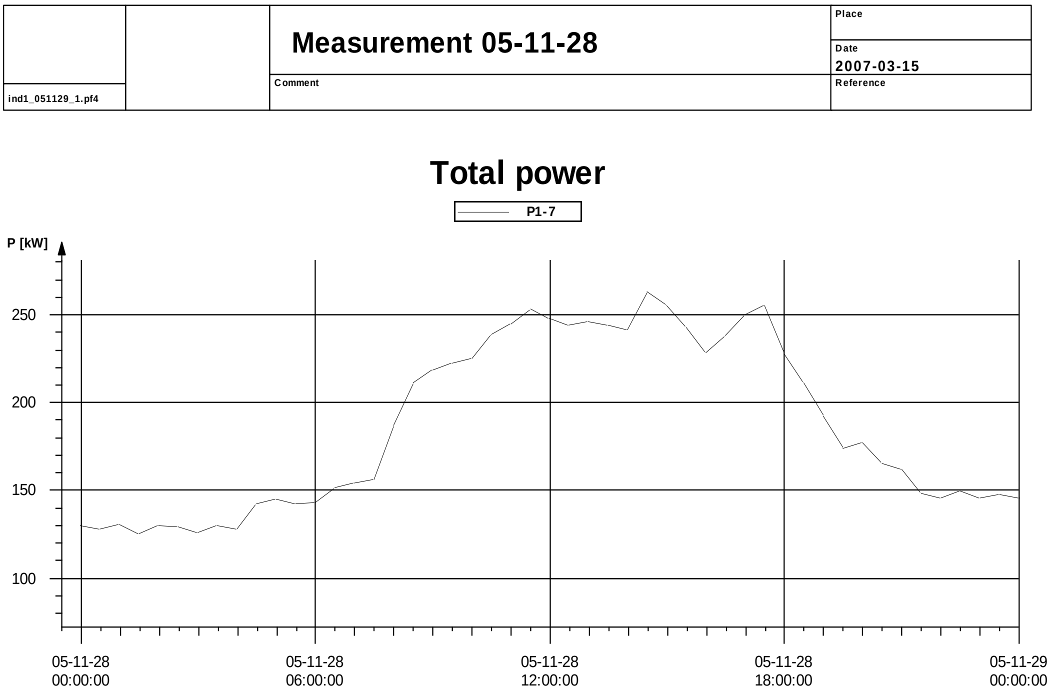

Figure 38 The power consumption 28/11, 00.00 – 29/11, 00.00

3 EVALUATION OF MEASUREMENTS

3.1 Common

The evaluation is focused on parameters to be used in a stochastic model named ”Load_make”. This model is described in point 3.2 and in [1].

3.2 Load_make

"Load_make" is a program function (subroutine) that generates a stochastic model of a grid load. The routine works with 6 separated day time points, TP_0 – TP_5. TP_0 is defined as midnight (0 or 24). TP_1 – TP_5 are defined as input values. The routine also uses 4 primary base power levels, A_prim, B_prim, C_prim and D_prim, that are related to the time points according to:

- A_prim: base power at TP_0
- B_prim: base power at TP_1 to TP_2
- C_prim: base power at TP_3 to TP_4
- D_prim: base power at TP_5

That means that the primary base levels follow the following sequence for a 24 hours cycle:

A_prim(at TP_0) – B_prim (at TP_1) – B_prim (at TP_2) – C_prim(at TP_3) –

- C_prim(at TP_4) – D_prim(at TP_5) – A_prim(back to a new 24 hours cycle, TP_0).

The shifts between the 4 base power levels are performed lineary.

The principle is illustrated in Figure 39.

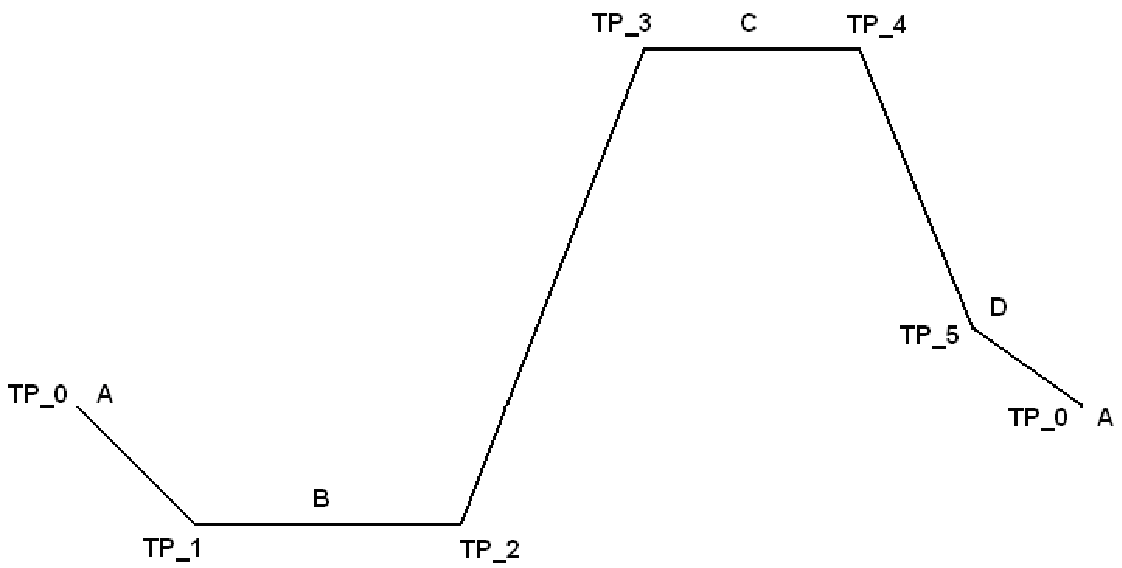

Figure 39 An illustration of the base power shifts between the different time points during a 24 hours cycle

The 4 primary base power levels undergo a stochastic variation, according a normal process. This stochastic variation are performed once per 24 hours cycle. This variation is named “the low frequency” noise”. The statistic parameters are named *“L_My_L”* and *“L_Sigma_L”* (the first L stands for Load and the second stands for Low), representing the mean value respectively the standard deviation in the Normal distribution. The so called “*Load_factor*” that is achieved by the Normal distribution and that is generated once per 24 hours cycle, is used as a factor for all base levels. This means that the base levels are updated every new 24 hours cycle according to Equation 1.

Equation 1. (1 – 4):

1. $A = A\_prim \cdot Load\_factor$

2. $B = B\_prim \cdot Load\_factor$

3. $C = C\_prim \cdot Load\_factor$

4. $D = D\_prim \cdot Load\_factor$

The values of A, B, C and D are the real (used) base values during the present 24 hours cycle. The next coming 24 hours cycle will result in a new set of base levels (from the original levels of A_prim, B_prim, C_prim and D_prim) and so on.

For each simulation step a “Normal distributed noise” is added to the present level (that is achieved by the base level in question). This “high frequency noise” has the statistic parameters *“L_My_H”* and *“L_Sigma_H”* (the L stands for Load and the H stands for High), representing the mean value respectively the standard deviation in the Normal distribution.

An example of this “Power Noise” effect is illustrated in Figure 40 (no “High frequency Noise added) and Figure 41 (“High frequency Noise” added).

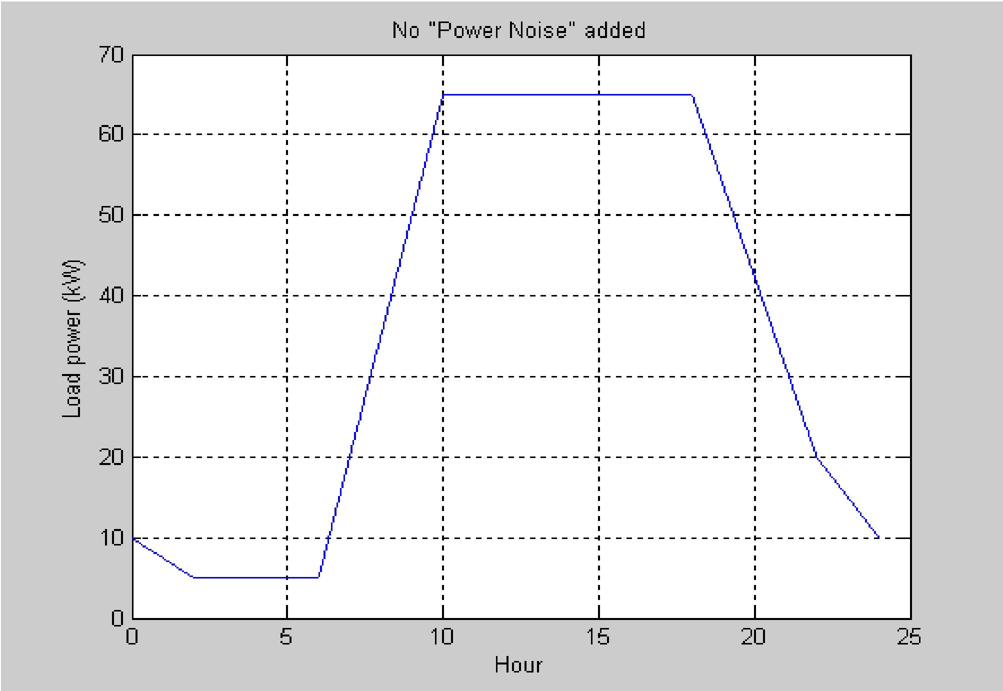

Figure 40 An example with **no** high frequency noise added to the base levels

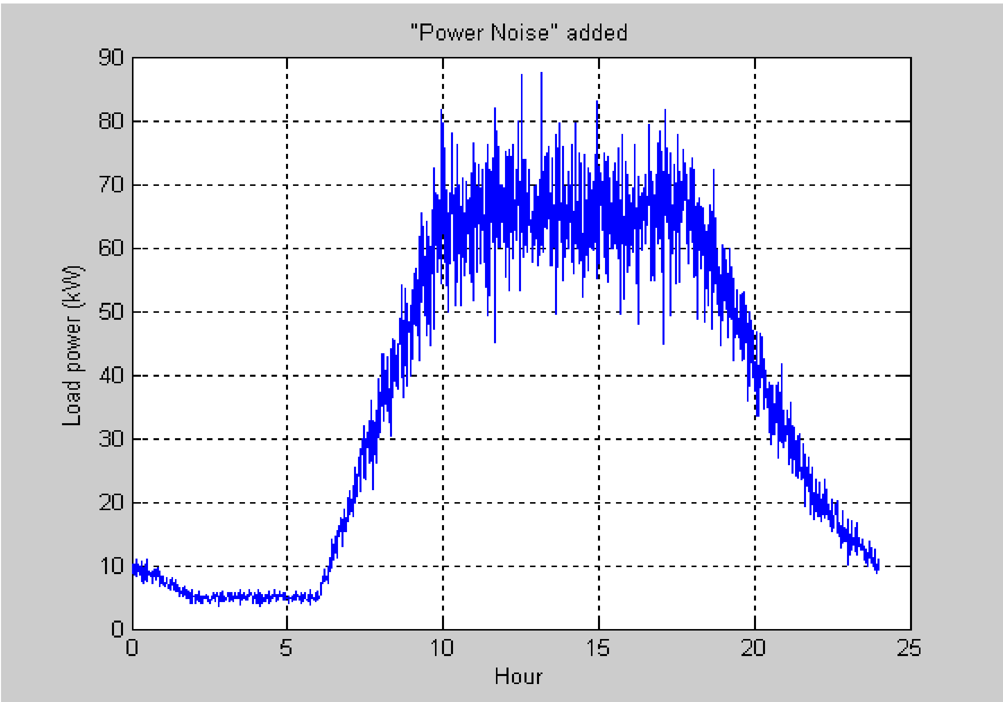

Figure 41 An example **with** high frequency noise added to the base levels. The standard deviation *"L_sigma_H"* (in this example) is 10 % of present base level. The noise mean value *"L_My_H"* is zero.

The load model works with 2 categories of loads:

- Category a
- Category b

The simulation starts with category a. This category is modeled corresponding to a specified number of days, "*a_limit*". When these days are completed, simulation of category b will be continued. This category is modeled corresponding to a specified number of days, "b_limit". This sequence, category a to category b ans so on, is repeated as long as the total simulation continues. The dividing into 2 separated categories is useful when modelling for example the power consumption in e.g. an industry area. In this case it could be a large difference between the power profile during the working week compared with the weekend.

The effect of this separating into 2 categories is illustrated in Figure 42. The figure shows the load during 10 days. The first 5 days correspond to a working week. Then follow 2 days corresponding to the weekend, and so on.

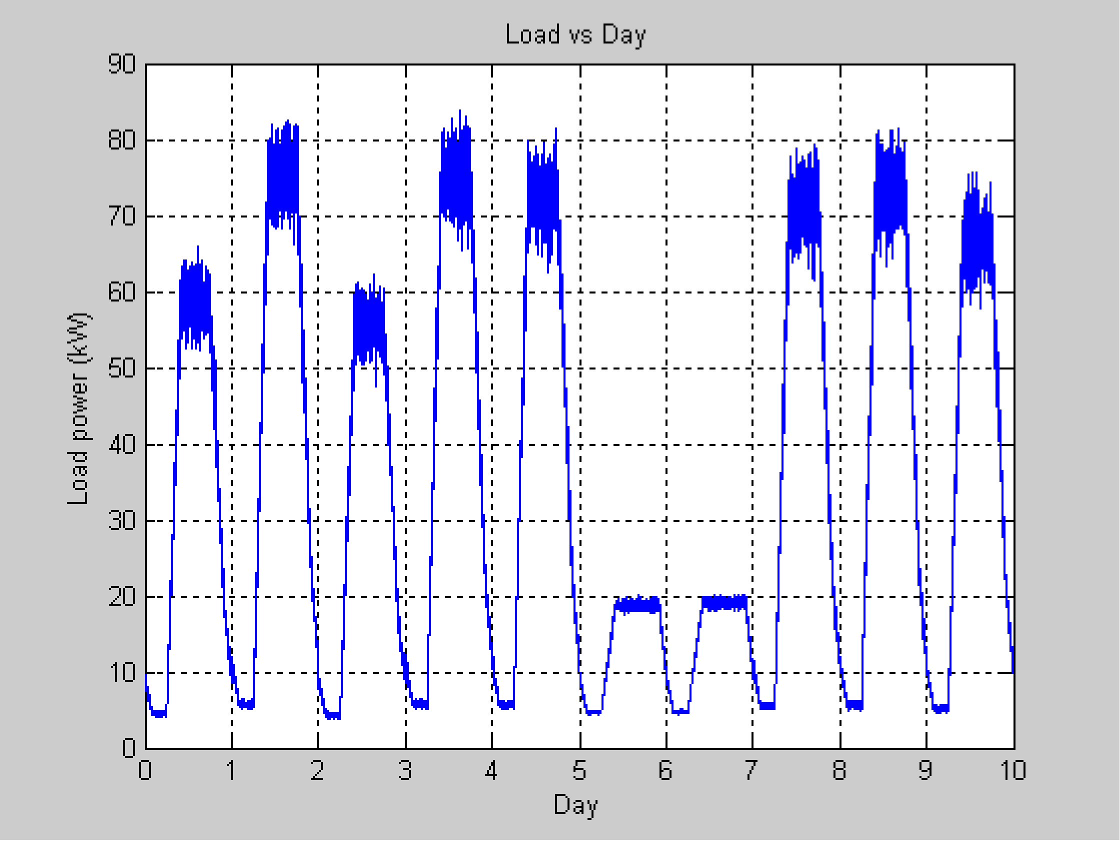

Figure 42 A simulation sequence corresponding to 5 working days followed by 2 days of weekend

3.3 The measured power consumption profile

To calculate a representative mean value regarding the daily consumption profile of the 24 working days respectively the 10 weekend days during the measurement period the measure samples are organized in 2 matrixes. One matrix for the working days and one matrix for the weekend days. The raws corresponds to the specific days and the columns to the specific time points in question. I.e. index (n,m) stands for day number n (working day number n respectively weekend day number n) and time point number m. The matrixes $\overline{\overline{M}}^{W}$ and $\overline{\overline{M}}^{WE}$ are defined in Equation 2 and Equation 3.

The mean values of the columns in each matrix results in vectors $\overline{P^{W}}$ and $\overline{P^{WE}}$ according to Equation 4 to Equation 9. These vectors represent the mean values of the daily consumption profile for the working day respectively the weekend day. See Figure 43 and Figure 44. The corresponding profiles of the standard deviations are illustrated in Figure 45 and Figure 46.

Equation 2:

$$\overline{\overline{M^{W}}} = \left[\begin{array}{cccc:cc} M^{W}{}_{1,1} & M^{W}{}_{1,2} & & M^{W}{}_{1,n} & M^{W}{}_{1,n+1} & M^{W}{}_{1,48} \\ M^{W}{}_{2,1} & M^{W}{}_{2,2} & & M^{W}{}_{2,n} & M^{W}{}_{2,n+1} & M^{W}{}_{2,48} \\ & & & & & \\ M^{W}{}_{m,1} & M^{W}{}_{m,2} & & M^{W}{}_{m,n} & M^{W}{}_{m,n+1} & M^{W}{}_{m,48} \\ \hdashline M^{W}{}_{m+1,1} & M^{W}{}_{m+1,2} & & M^{W}{}_{m+1,n} & M^{W}{}_{m+1,n+1} & M^{W}{}_{m+1,48} \\ & & & & & \\ M^{W}{}_{24,1} & M^{W}{}_{24,2} & & M^{W}{}_{24,n} & M^{W}{}_{24,n+1} & M^{W}{}_{24,48} \end{array}\right]$$

The measure samples for the working days are organised in a matrix $\overline{\overline{M^{W}}}$ with 24rows(corresponding to the number of working days) and 48 columns (corresponding to the number of samples per day). Element $M^{W}{}_{m,n}$ corresponds to the measure sample for working day *m* and time point *n*.

Equation 3:

$$\overline{\overline{M^{WE}}} = \left[\begin{array}{cccc|cc} M^{WE}{}_{1,1} & M^{WE}{}_{1,2} & & M^{WE}{}_{1,n} & M^{WE}{}_{1,n+1} & M^{WE}{}_{1,48} \\ M^{WE}{}_{2,1} & M^{WE}{}_{2,2} & & M^{WE}{}_{2,n} & M^{WE}{}_{2,n+1} & M^{WE}{}_{2,48} \\ & & & & & \\ M^{WE}{}_{m,1} & M^{WE}{}_{m,2} & & M^{WE}{}_{m,n} & M^{WE}{}_{m,n+1} & M^{WE}{}_{m,48} \\ \hline M^{WE}{}_{m+1,1} & M^{WE}{}_{m+1,2} & & M^{WE}{}_{m+1,n} & M^{WE}{}_{m+1,n+1} & M^{WE}{}_{m+1,48} \\ & & & & & \\ M^{WE}{}_{10,1} & M^{WE}{}_{10,2} & & M^{WE}{}_{10,n} & M^{WE}{}_{10,n+1} & M^{WE}{}_{10,48} \end{array}\right]$$

The measure samples for the weekend days are organised in a matrix $\overline{\overline{M^{WE}}}$ with 10 rows (corresponding to the number of weekend days) and 48 columns (corresponding to the number of samples per day). Element $M^{WE}{}_{m,n}$ corresponds to the measure sample for weekend day *m* and time point *n*.

Equation 4:

$$P_n^W = \frac{\sum_{m=1}^{24} M_{m,n}^W}{24}$$ n = 1 to 48

Equation 5:

$$P_{49}^W = P_1^W$$ comment: time 0 equivalent with time 24

Equation 6:

$$P_n^{WE} = \frac{\sum_{m=1}^{10} M_{m,n}^{WE}}{10}$$ n = 1 to 48

Equation 7:

$$P_{49}^{WE} = P_1^{WE}$$ comment: time 0 equivalent with time 24

Equation 8:

$$\overline{P^W} = \begin{bmatrix} P_1^W & P_2^W & P_3^W & & P_n^W & \vdots & P_{n+1}^W & & P_{49}^W \end{bmatrix}$$

The vector $\overline{P^W}$ is built up by the mean values of the columns in the matrix $\overline{\overline{M^W}}$. The vector represent the mean value of the daily consumption profile for the working day. See Figure 43.

Equation 9:

$$\overline{P^{WE}} = \left[P_1^{WE} \quad P_2^{WE} \quad P_3^{WE} \qquad\qquad P_n^{WE} \;\vdots\; P_{n+1}^{WE} \qquad P_{49}^{WE} \right]$$

The vector $\overline{P^{WE}}$ is built up by the mean values of the columns in the matrix $\overline{\overline{M^{WE}}}$. The vector represent the mean value of the daily consumption profile for the weekend day. See Figure 44

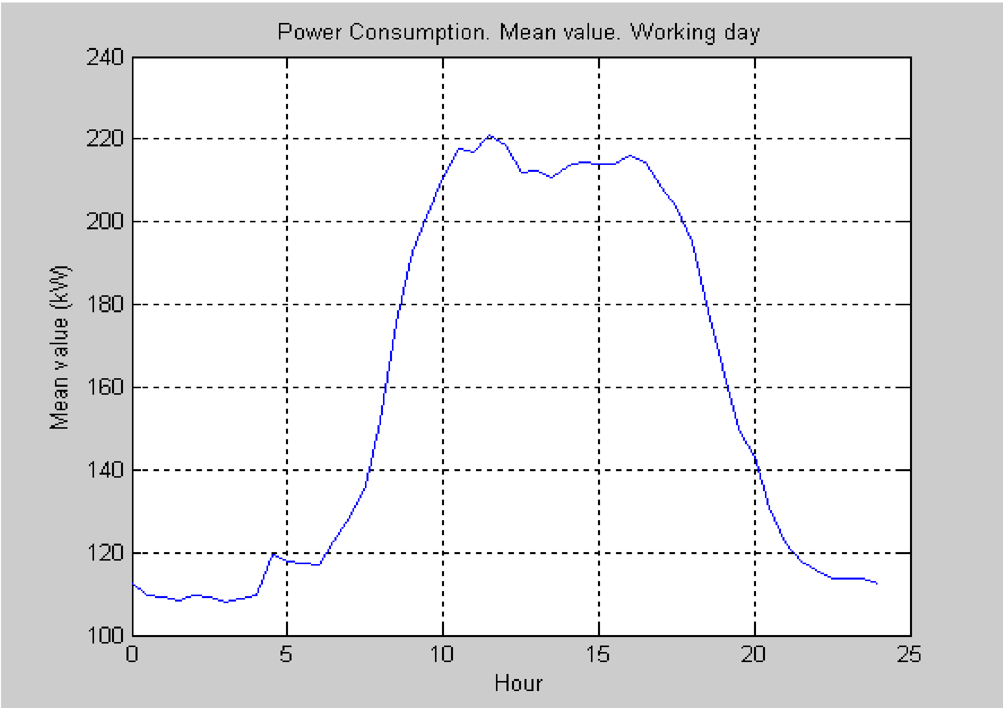

Figure 43 The power consumption mean value profile for a working day

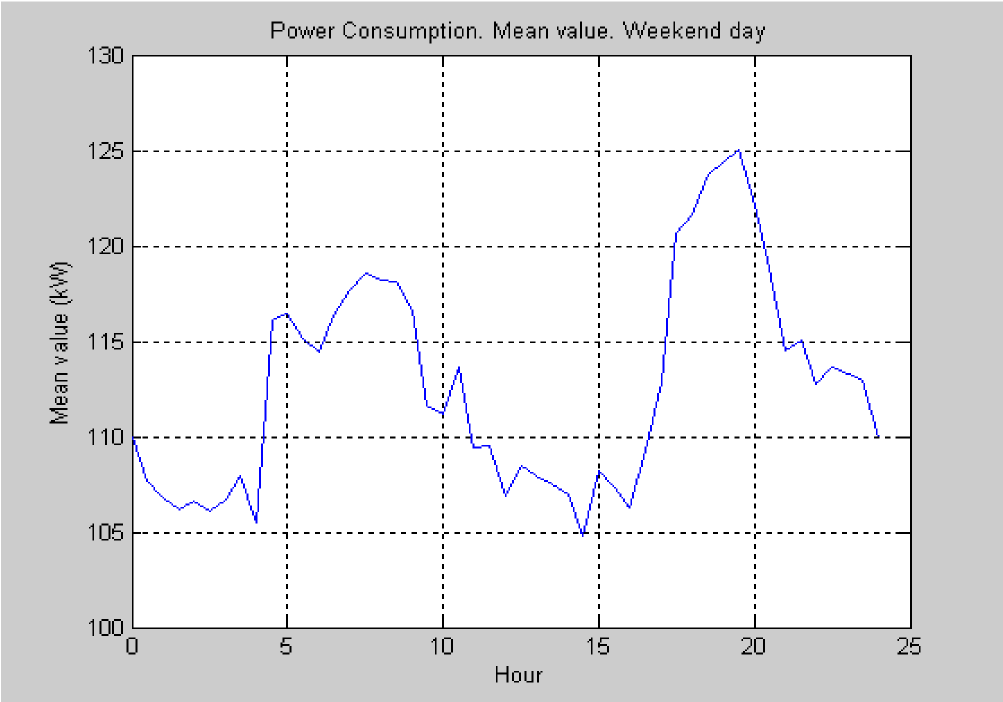

Figure 44 The power consumption mean value profile for a weekend day

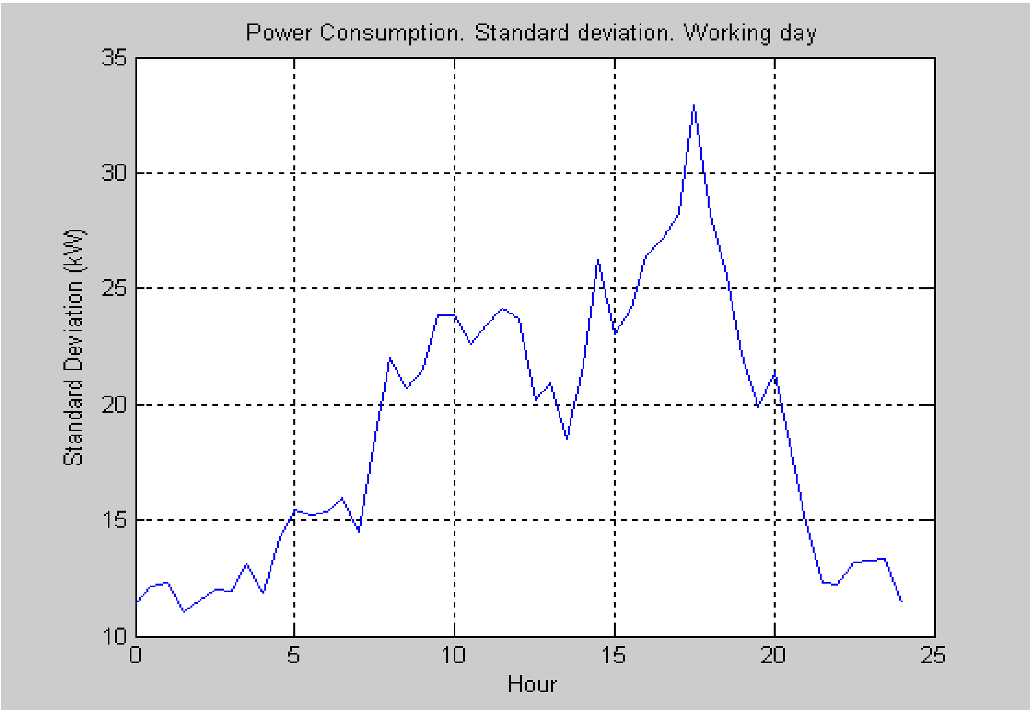

Figure 45 The power consumption standard deviation profile for a working day

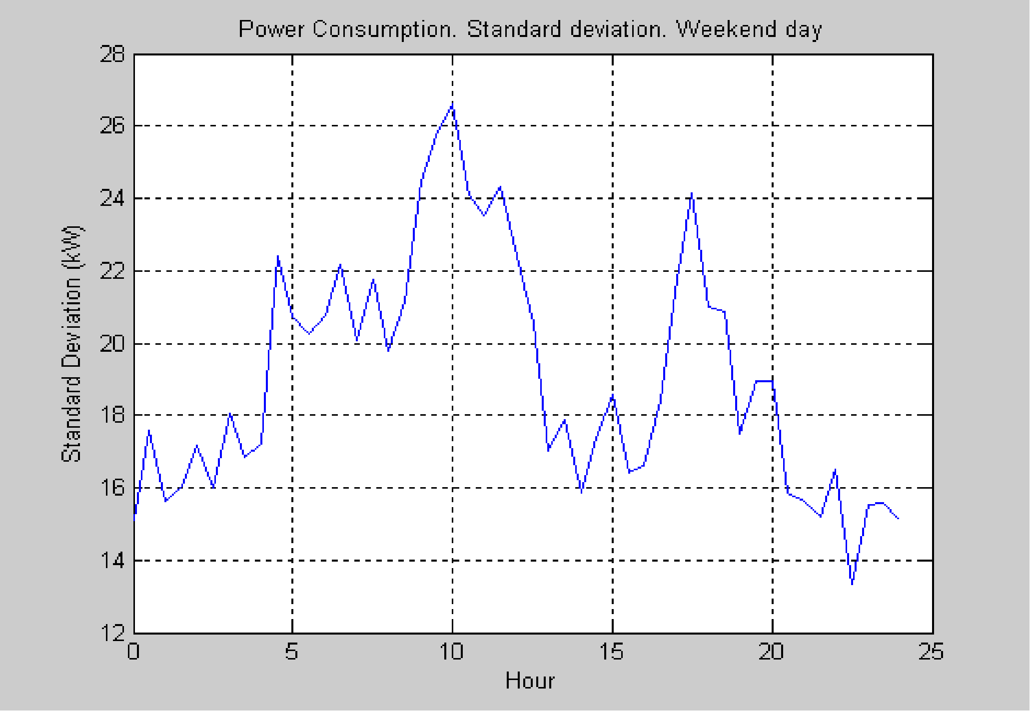

Figure 46 The power consumption standard deviation profile for a weekend day

3.4 Model parameters

As a result of the measurements the following model parameters will be estimated (see also point 3.2 and [1]):

- TP_1 – TP_5 (TP_0 is by definition assigned as midnight)
- A_prim, B_prim, C_prim and D_prim
- L_My_L and L_Sigma_L
- L_My_H and L_Sigma_H

As can be established by Figure 43 and Figure 44 there are quite different model parameters for a working day compared with a weekend day.

3.4.1 Working Days

3.4.1.1 TP_1 to TP_5

From the mean value result according to Figure 43 the time points TP_1 to TP_5 are (roughly) estimated as:

TP_1: 1

TP_2: 6

TP_3: 10

TP_4: 17

TP_5: 21

3.4.1.2 A_prim, B_prim, C_prim and D_prim

The used principle to calculate A_prim, B_prim, C_prim and D_prim follows according to Equation 10 to Equation 24:

Equation 10:

$$C\_prim = \mu P_{34}$$

Where:

μP_{34}: power mean value in the time region TP_3 to TP_4 (Equation 11)

Equation 11:

$$\mu P_{34} = \frac{\sum_{n=pos\_TP\_3}^{pos\_TP\_4} P_n^W}{pos\_TP\_4 - pos\_TP\_3 + 1}$$

Where:

P_n^W : element *n* in power vector $\overline{P^W}$ (Equation 8)

pos_TP_3, pos_TP_4: the vector positions (n-values) corresponding to the time points TP_3 respectively TP_4 in vector $\overline{P^W}$

Equation 12:

$$D\_prim = 2 \cdot \mu P_{45} - C\_prim$$

Where:

μP_{45}: power mean value in the time region TP_4 to TP_5 (Equation 13)

The principle of Equation 12 is discribed in Figure 47.

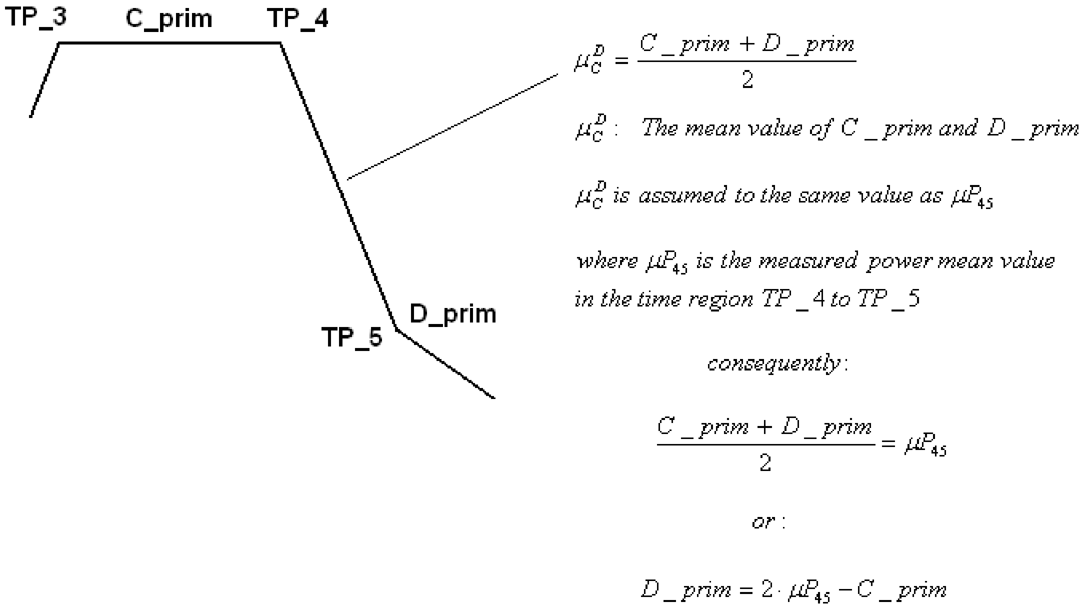

Figure 47 The principle of Equation 12

Equation 13:

$$\mu P_{45} = \frac{\sum_{n=pos\_TP\_4}^{pos\_TP\_5} P_n^W}{pos\_TP\_5 - pos\_TP\_4 + 1}$$

Where:

P_n^W :	element *n* in power vector $\overline{P^W}$ (Equation 8)
pos_TP_4, pos_TP_5:	the vector positions (n-values) corresponding to the time points TP_4 respectively TP_5 in vector $\overline{P^W}$

Equation 14:

$$A\_prim\_1 = P_{49}^{W}$$

Where:

A_prim_1: a first calculated value of A_prim

P_{49}^{W}: element *49* in power vector $\overline{P^{W}}$ (Equation 8)

Equation 15:

$$A\_prim\_2 = 2 \cdot \mu P_{56} - D\_prim$$

Where:

A_prim_2: a second calculated value of A_prim

μP_{56}: the power mean value in the time region TP_5 to TP_6 (Equation 16)

The principle of Equation 15 corresponds to the principle of Equation 12 (see Figure 47).

Equation 16:

$$\mu P_{56} = \frac{\sum_{n=pos\_TP\_5}^{pos\_TP\_6} P_n^{W}}{pos\_TP\_6 - pos\_TP\_5 + 1}$$

Where:

P_n^{W}: element *n* in power vector $\overline{P^{W}}$ (Equation 8) *pos_TP_5, pos_TP_6*: the vector positions (n-values) corresponding to the time points TP_5 respectively TP_6 in vector $\overline{P^W}$

Equation 17:

$$A\_prim = \frac{(A\_prim\_1 + A\_prim\_2)}{2}$$

Where:

A_prim: the final calculated value of A_prim

Equation 18:

$$B\_prim\_1 = 2 \cdot \mu P_{23} - C\_prim$$

Where:

B_prim_1: a first calculated value of B_prim

μP_{23}: the power mean value in the time region TP_2 to TP_3 (Equation 19)

The principle of Equation 18 corresponds to the principle of Equation 12 (see Figure 47).

Equation 19:

$$\mu P_{23} = \frac{\sum_{n=pos\_TP\_2}^{pos\_TP\_3} P_n^W}{pos\_TP\_3 - pos\_TP\_2 + 1}$$

Where:

P_n^W: element *n* in power vector $\overline{P^W}$ (Equation 8)

pos_TP_2, pos_TP_3: the vector positions (n-values) corresponding to the time points TP_2 respectively TP_3 in vector $\overline{P^W}$

Equation 20:

$$B\_prim\_2 = 2 \cdot \mu P_{01} - A\_prim\_2$$

Where:

B_prim_2: a second calculated value of B_prim

μP_{01}: the power mean value in the time region TP_0 to TP_1 (Equation 21)

The principle of Equation 20 corresponds to the principle of Equation 12 (see Figure 47).

Equation 21:

$$\mu P_{01} = \frac{\sum_{n=pos\_TP\_0}^{pos\_TP\_1} P_n^W}{pos\_TP\_1 - pos\_TP\_0 + 1}$$

Where:

P_n^W: element *n* in power vector $\overline{P^W}$ (Equation 8)

pos_TP_0, pos_TP_1: the vector positions (n-values) corresponding to the time points TP_0 respectively TP_1 in vector $\overline{P^W}$

Equation 22:

$$B\_prim\_3 = \mu P_{12}$$

Where:

B_prim_3: a third calculated value of B_prim

μP_{12}: the power mean value in the time region TP_1 to TP_2 (Equation 23)

Equation 23:

$$\mu P_{12} = \frac{\sum_{n=pos\_TP\_1}^{pos\_TP\_2} P_n^W}{pos\_TP\_2 - pos\_TP\_1 + 1}$$

Where:

P_n^W: element *n* in power vector $\overline{P^W}$ (Equation 8)

pos_TP_1, *pos_TP_2*: the vector positions (n-values) corresponding to the time points TP_1 respectively TP_2 in vector $\overline{P^W}$

Equation 24:

$$B\_prim = \frac{(B\_prim\_1 + B\_prim\_2 + 3 \cdot B\_prim\_3)}{5}$$

Where:

B_prim: the final calculated value of B_prim

The weighting coefficient for *B_prim_1*, *B_prim_2* and *B_prim_3* is a result of adaption to get a good resulting modeled mean value of the total power during a 24 hours period compared with the measured mean value.

Equation 10 to Equation 24 used together with the suggested values of TP_1 to TP_5 accoding to 3.4.1.1 result in the following values of A_prim, B_prim, C_prim and D_prim:

A_prim: 112.76 kW

B_prim: 109.92 kW

C_prim: 214.30 kW

D_prim: 118.20 kW

Figure 48 illustrates the measured (mean value of 24 working days) power consumption profile (dashed) together with the model profile (solid) if TP_1 to TP_5 according to 3.4.1.1 and A_prim, B_prim, C_prim and D_prim according to the values above are used. To this model profile statistical variations (noise) shall be added. These parameters are treated in 3.4.1.3 (low frequency noise) and 3.4.1.4 (high frequency noise).

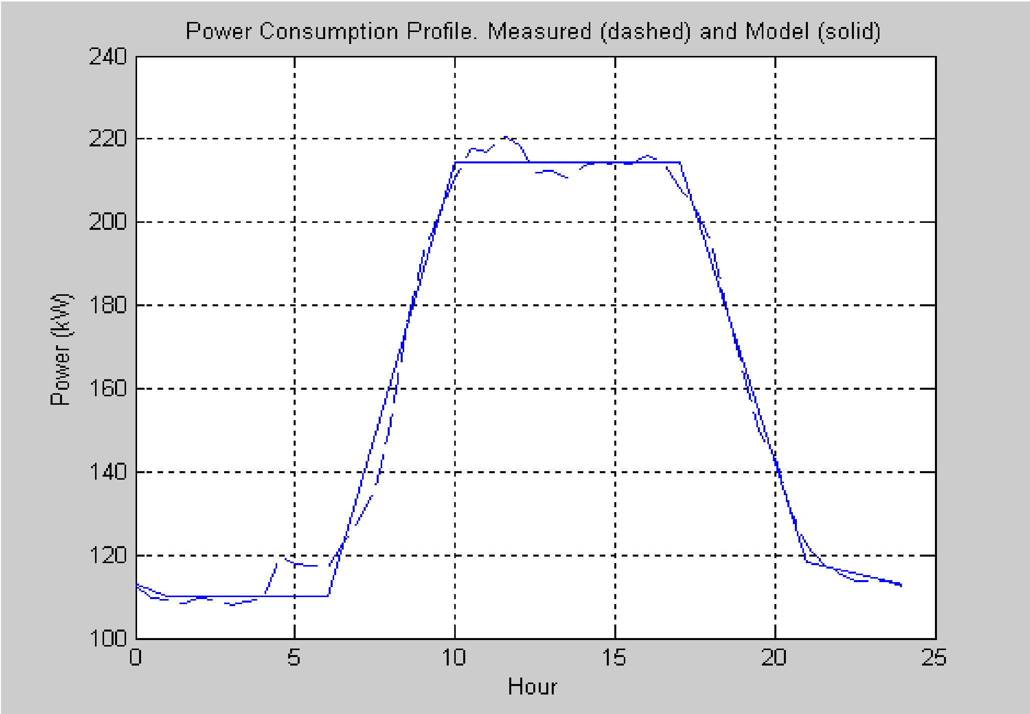

Figure 48 Measured power consumption profile (dashed) together with the model profile (solid) if TP_1 to TP_5 according to 3.4.1.1 and A_prim, B_prim, C_prim and D_prim according to the values above are used

The mean power of the curves (24 hours) in Figure 48 is:

Measured power: 159.2 kW

Modelled power: 159.2 kW

3.4.1.3 L_My_L and L_Sigma_L

The used principle to calculate L_My_L and L_Sigma_L follows according to Equation 25 to Equation 29:

Equation 25:

$$L\_My\_L = 1$$

This is the standard value of this parameter as the stochastic variation is assumed as a relative variation around the mean value.

Equation 26:

$$L\_Sigma\_L = \frac{\sigma(\overline{E^W})}{\mu(\overline{E^W})}$$

Where:

$\sigma(\overline{E^W})$:	the standard deviation of vector $\overline{E^W}$ (Equation 27)
$\mu(\overline{E^W})$:	the mean value of vector $\overline{E^W}$ (Equation 28)
$\overline{E^W}$:	vector where the elements are defined according to Equation 29

Equation 27:

$$\sigma(\overline{E^W}) = \sqrt{\frac{\sum_{m=1}^{24}\left(E_m^W - \mu(\overline{E^W})\right)^2}{23}}$$

Equation 28:

$$\mu(\overline{E^W}) = \frac{\sum_{m=1}^{24} E_m^W}{24}$$

Equation 29:

$$E_m^W = \sum_{n=1}^{48} M_{m,n}^W \qquad m = 1 \text{ to } 24$$

Where:

$M_{m,n}^W$: elements in matrix $\overline{\overline{M^W}}$ (Equation 2)

According to Equation 26, *L_Sigma_L* will be the relative standard deviation of the daily energy consumption.

Equation 26 results in:

L_Sigma_L = 0.0957

3.4.1.4 L_My_H and L_Sigma_H

The used principle to calculate L_My_L and L_Sigma_L follows according to Equation 30 and Equation 31:

Equation 30:

$$L\_My\_H = 0$$

This is the standard value of this parameter as the stochastic variation (power noise) is assumed to be symmetric around zero.

Equation 31:

$$L\_Sigma\_H = \mu(\overline{C^W})$$

Where:

$\mu(\overline{C^W})$: the mean value of vector $\overline{C^W}$ according to Equation 32. The elements C_m^W in vector $\overline{C^W}$ are defined according to Equation 33 and Equation 34. This vector contains the relative standarddeviation (the standard deviation relative to the mean value) of the power in the time interval TP_3 to TP_4 for each working day (1 to 24). Consequently L_Sigma_H is a measure of the mean value of the relative standarddeviation for the power consumption.

Equation 32:

$$\mu(\overline{C^W}) = \frac{\sum_{m=1}^{24} C_m^W}{24}$$

Equation 33:

$$C_m^W = \frac{\sqrt{\frac{\sum_{n=pos\_TP\_3}^{pos\_TP\_4} \left(M_{m,n}^W - M^W\mu(m,3\_4)\right)^2}{pos\_TP\_4 - pos\_TP\_3 - 1}}}{M^W\mu(m,3\_4)} \qquad m = 1 \text{ to } 24$$

Where:

$M_{m,n}^W$: element I matrix $\overline{\overline{M^W}}$ (Equation 2)

$M^W\mu(m,3\_4)$: see Equation 34

Equation 34:

$$M^{W}\mu(m,3\_4)=\frac{\sum_{n=pos\_TP\_3}^{pos\_TP\_4} M^{W}_{m,n}}{pos\_TP\_4-pos\_TP\_3}$$

Where:

Pos_TP_3, Pos_TP_4: the time positions (n-values) corresponding to the time points TP_3 respectively TP_4 in matrix $\overline{\overline{M}}^{W}$

Equation 31 results in:

L_Sigma_H = 0.0458

3.4.2 Weekend days

3.4.2.1 TP_1 to TP_5

From the mean value result according to Figure 43 the time points TP_1 to TP_5 are (roughly) estimated as:

TP_1:	1
TP_2:	4
TP_3:	6
TP_4:	21
TP_5:	23

3.4.2.2 A_prim, B_prim, C_prim and D_prim

To calculate A_prim, B_prim, C_prim and D_prim the corresponding principle as described in 3.4.1.2 is used.

As the power vector $\overline{P^W}$ shall be replaced by the power vector $\overline{P^{WE}}$, the following changes must be done regarding some equations according to:

Equation	Changes
Equation 11	$P_n^W \Rightarrow P_n^{WE}$
Equation 13	-"-
Equation 14	$P_{49}^W \Rightarrow P_{49}^{WE}$
Equation 16	$P_n^W \Rightarrow P_n^{WE}$
Equation 19	-"-
Equation 21	-"-
Equation 23	-"-

The calculations result in:

A_prim: 110.14 kW

B_prim: 107.73 kW

C_prim: 113.96 kW

D_prim: 113.75 kW

Figure 49 illustrates the measured (mean value of 10 weekend days) power consumption profile (dashed) together with the model profile (solid) if TP_1 to TP_5 according to 3.4.2.1 and A_prim, B_prim, C_prim and D_prim according to the values above are used. To the model profile in this figure statistical variations are added. These parameters are treated in 3.4.2.3 (low frequency variations) and 3.4.2.4 (high frequency variations).

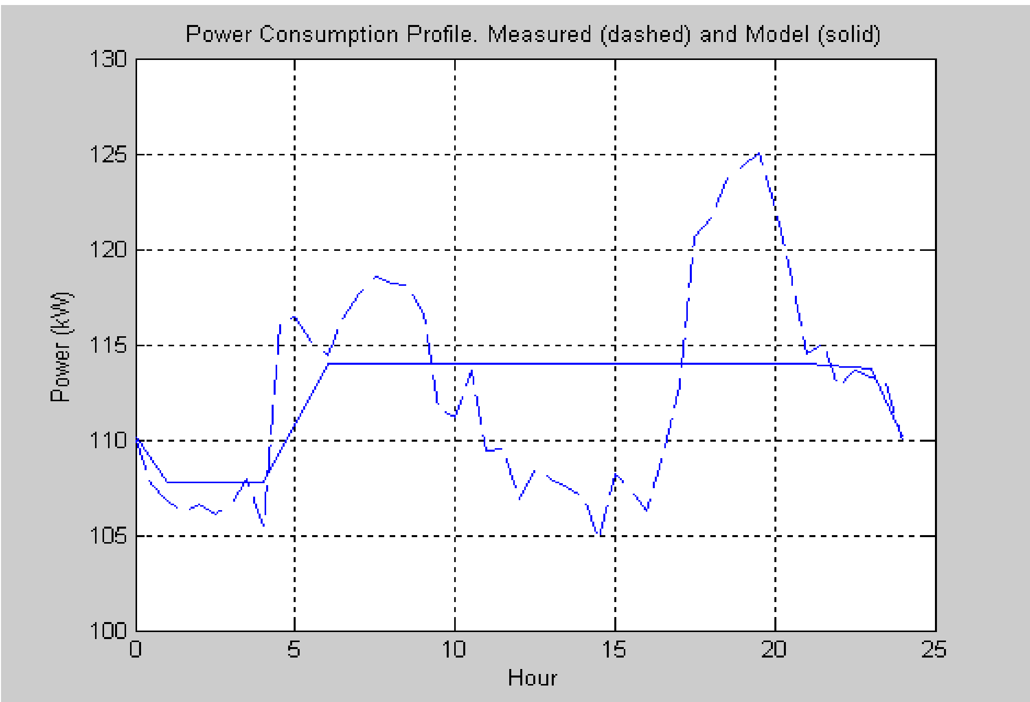

Figure 49 Measured power consumption profile (dashed) together with the model profile (solid) if TP_1 to TP_5 according to 3.4.2.1 and A_prim, B_prim, C_prim and D_prim according to the values above are used.

The mean power of the curves (24 hours) in Figure 49 is:

Measured power: 112.7 kW

Modelled power: 112.6 kW

3.4.2.3 L_My_L and L_Sigma_L

To calculate L_My_L and L_Sigma_L the corresponding principle as described in 3.4.1.3 is used.

The following index changes shall bee done regarding the equations:

Equation	Changes
Equation 26	$\overline{E^{W}} \Rightarrow \overline{E^{WE}}$
Equation 27	$\overline{E^{W}} \Rightarrow \overline{E^{WE}}$, $E_m^{W} \Rightarrow E_m^{WE}$
Equation 28	-“- -“-
Equation 29	$E_m^{W} \Rightarrow E_m^{WE}$, $M_{m,n}^{W} \Rightarrow M_{m,n}^{WE}$

The calculations results in:

L_My_L = 1

L_Sigma_L = 0.1632

3.4.2.4 L_My_H and L_Sigma_H

To calculate L_My_H and L_Sigma_H the corresponding principle as described in 3.4.1.4 is used.

The following index changes shall bee done regarding the equations:

Equation	Changes
Equation 31	$\overline{C^{W}} \Rightarrow \overline{C^{WE}}$
Equation 32	$\overline{C^{W}} \Rightarrow \overline{C^{WE}}$, $C_m^W \Rightarrow C_m^{WE}$
Equation 33	$C_m^W \Rightarrow C_m^{WE}$, $M_{m,n}^W \Rightarrow M_{m,n}^{WE}$, $M^W \mu(m,3\_4) \Rightarrow M^{WE} \mu(m,3\_4)$
Equation 34	$M^W \mu(m,3\_4) \Rightarrow M^{WE} \mu(m,3\_4)$, $M_{m,n}^W \Rightarrow M_{m,n}^{WE}$

The calculations result in:

L_My_H = 0

L_Sigma_H = 0.0755

3.4.3 Model parameters. Conclusion

Table 2 concludes the resulted statistical model parameters for the analysed electrical load.

Model Parameter	Working day	Weekend day
TP_1	1	1
TP_2	6	4
TP_3	10	6
TP_4	17	21
TP_5	21	23
A_prim	112.76 kW	110.14 kW
B_prim	109.92 kW	107.73 kW
C_prim	214.30 kW	113.96 kW
D_prim	118.20 kW	113.75 kW
L_My_L	1	1
L_Sigma_L	0.0957	0.1632
L_My_H	0	0
L_Sigma_H	0.0458	0.0755

Table 2 Statistical model parameters for the analysed electrical load

4 SIMULATIONS

Simulations have been performed based on the resulted parameters according to Table 2.

To illustrate the effect of the noise parameters (L_Sigma_L and L_Sigma_H) the simulations have been performed with a gradual change (increasing of the noise) according to Table 3.

Figure 55 gives the result with three in sequence coming weekend days while the simulations according to Figure 56 and Figure 57 have been performed on the condition that a week consists of 5 working days followed by 2 weekend days (i.e. a normal week).

It could be interesting to compare the power profile according to Figure 58 (measured profile) with Figure 57 (simulated profile).

Figure	Day *)	Number of days	L_Sigma_L **)	L_Sigma_H **)	Comment
Figure 50	W	1	0	0	No noise
Figure 51	W	1	0	M	"High frequency noise" included
Figure 52	W	3	M	M	"High- and low frequency noise" included
Figure 53	WE	1	0	0	No noise
Figure 54	WE	1	0	M	"High frequency noise" included
Figure 55	WE	3	M	M	"High- and low frequency noise" included
Figure 56	W WE	10	M	M	"High- and low frequency noise" included
Figure 57	W WE	34	M	M	"High- and low frequency noise" included

Table 3 Figure 50 to Figure 57 illustrate the simulated power profile with a gradual change (increasing) of the noise parameters

*) W: working day, WE: weekend day

**) M: the parameter is assigned to the suggested model value according to Table 2

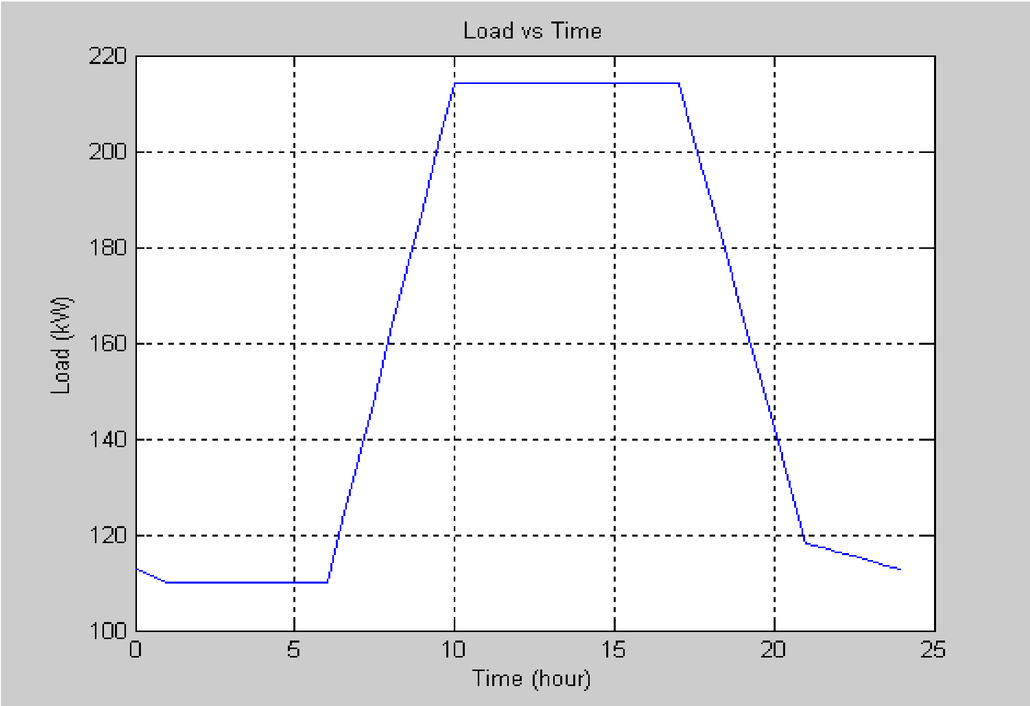

Figure 50 Simulated power profil. 1 working day. No noise

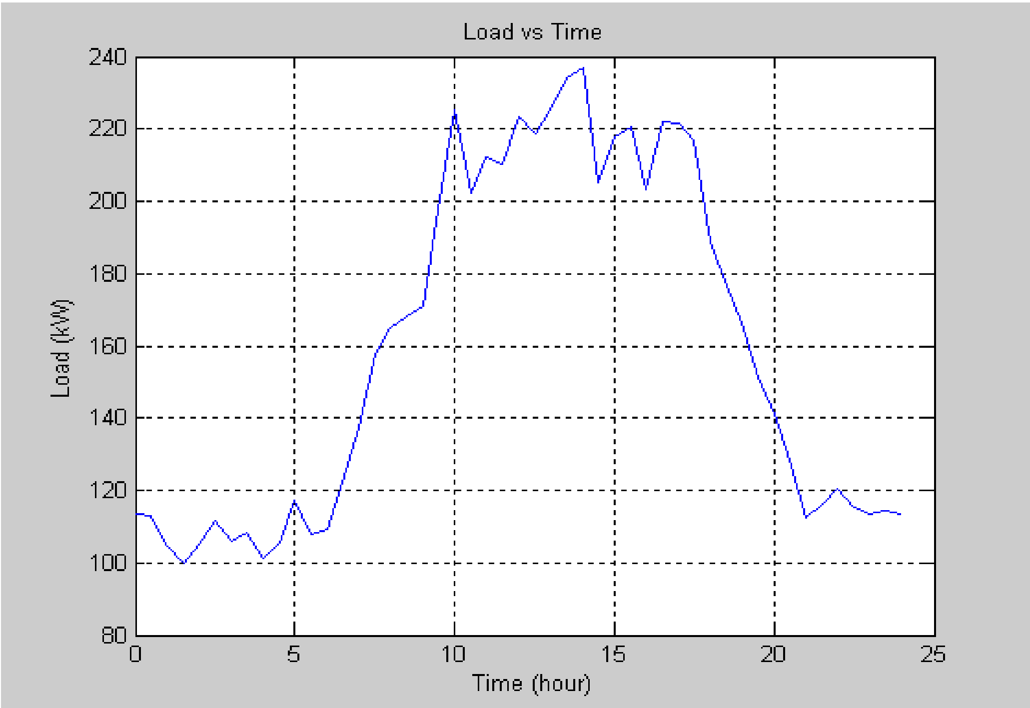

Figure 51 Simulated power profil. 1 working day. High frequency noise

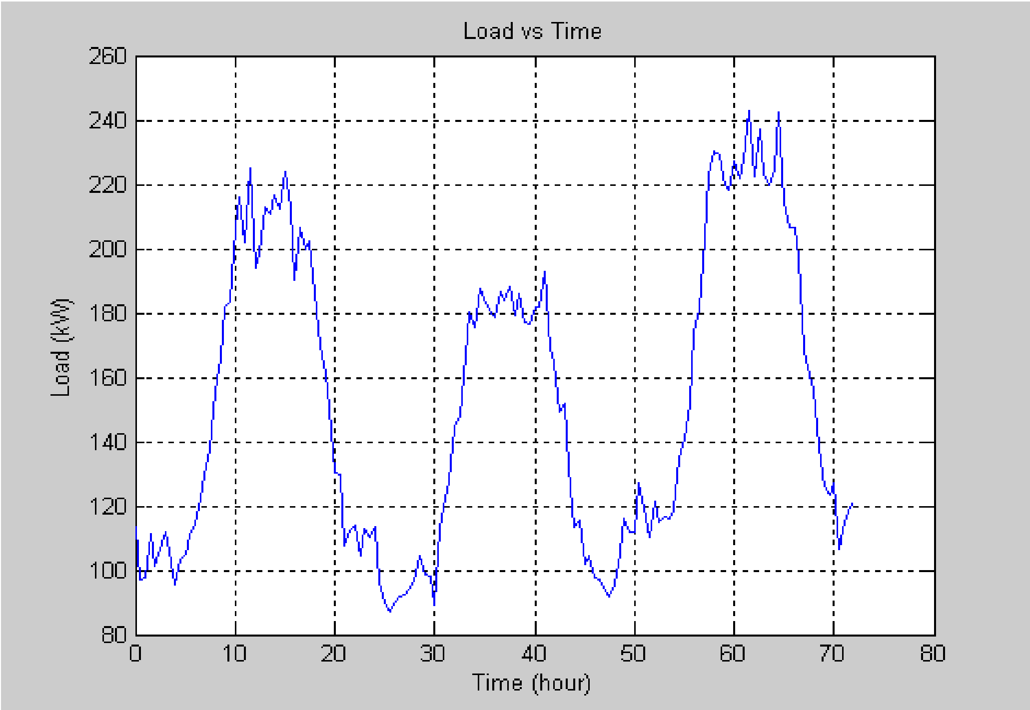

Figure 52 Simulated power profile. 3 working days. High- and low frequency noise

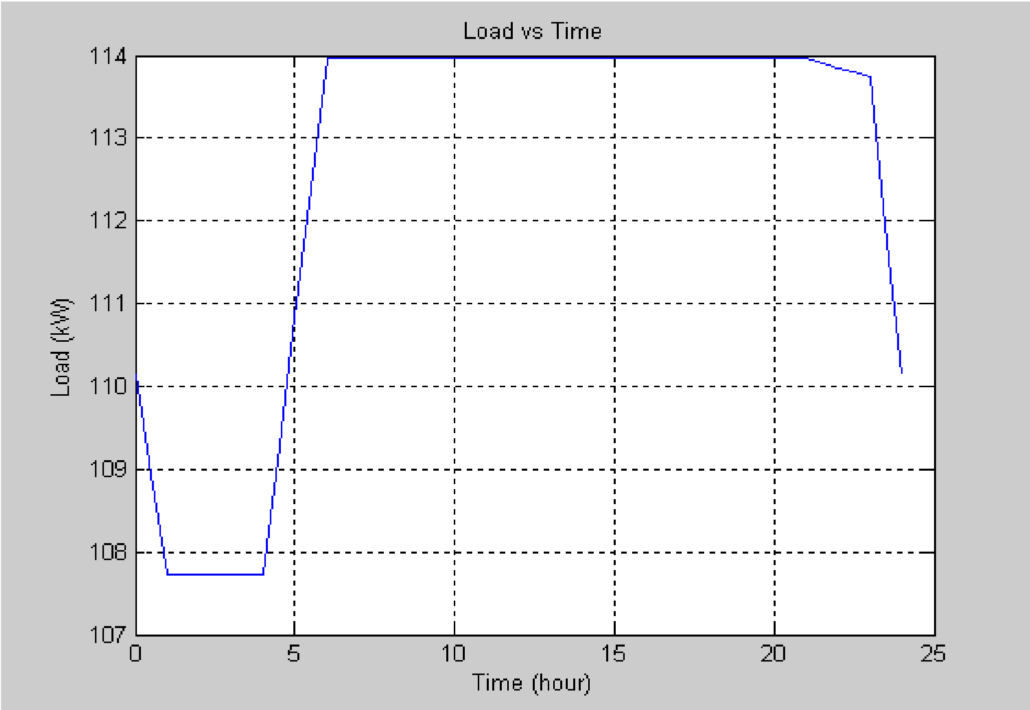

Figure 53 Simulated power profile.1 weekend day. No noise

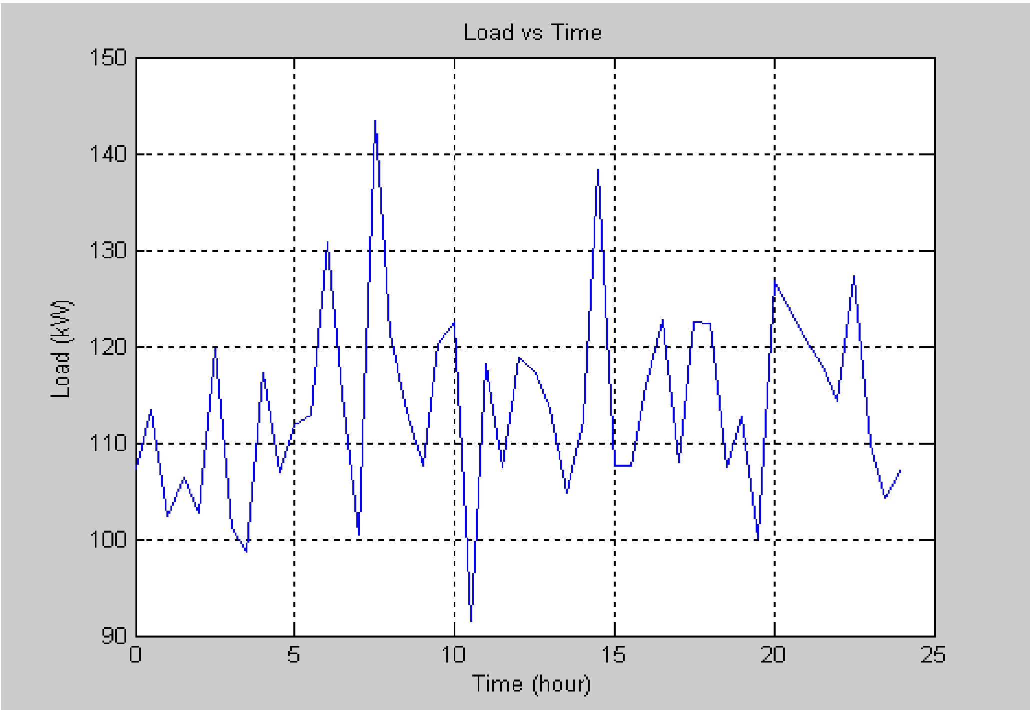

Figure 54 Simulated power profile.1 weekend day. High frequency noise

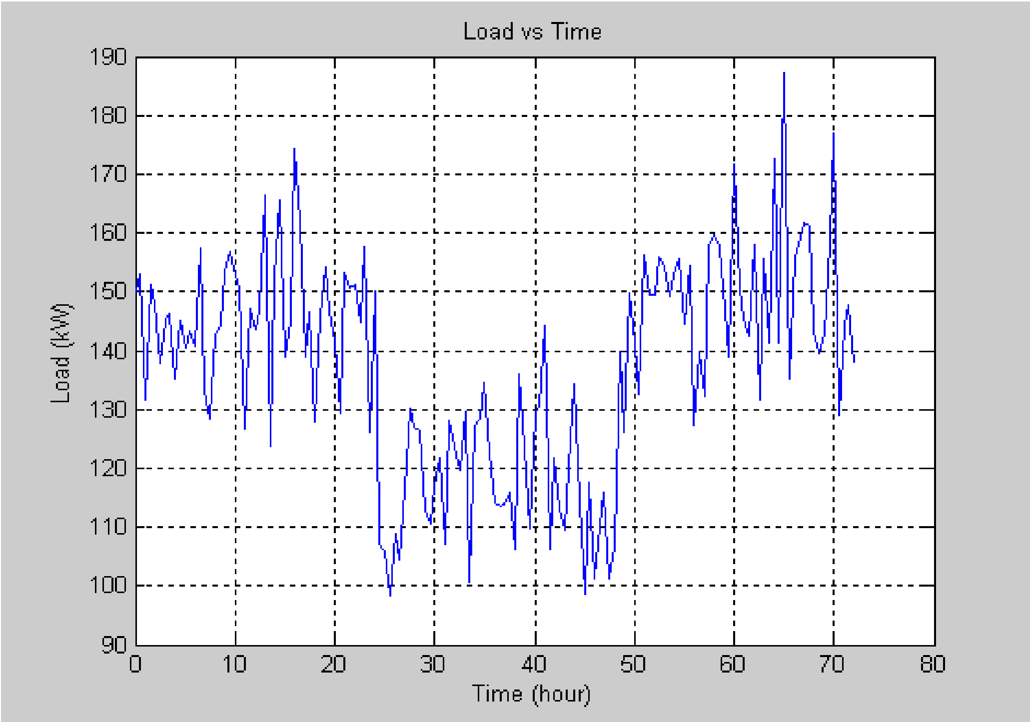

Figure 55 Simulated power profile.3 weekend days. High- and low frequency noise

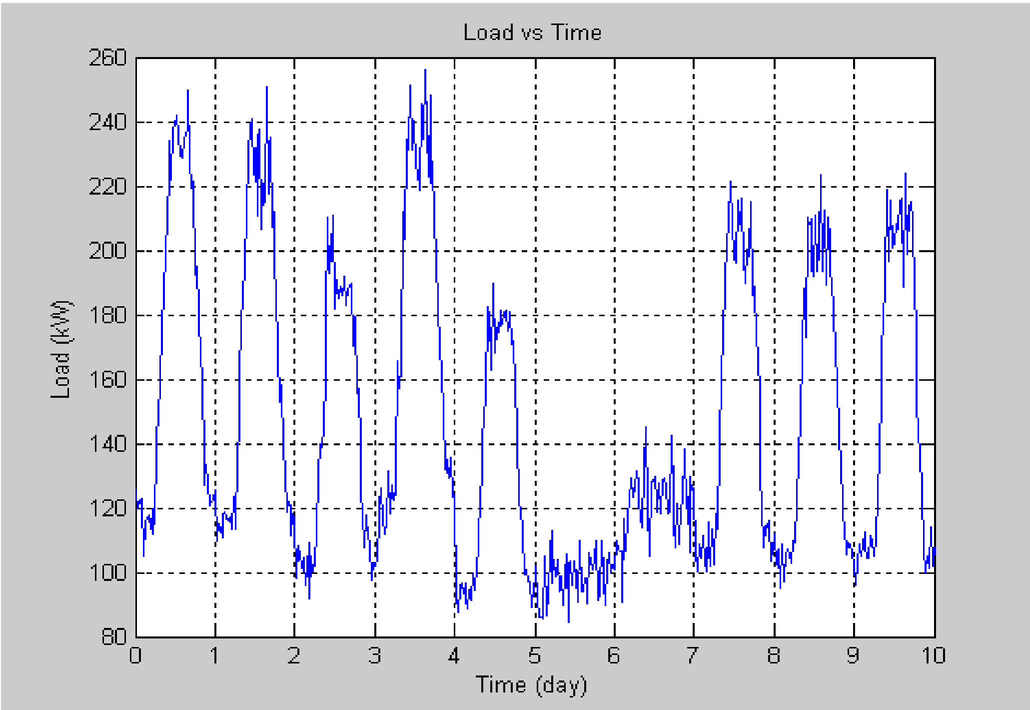

Figure 56 Simulated power profile.10 days with a week consisting of 5 working days and 2 weekend days. High- and low frequency noise

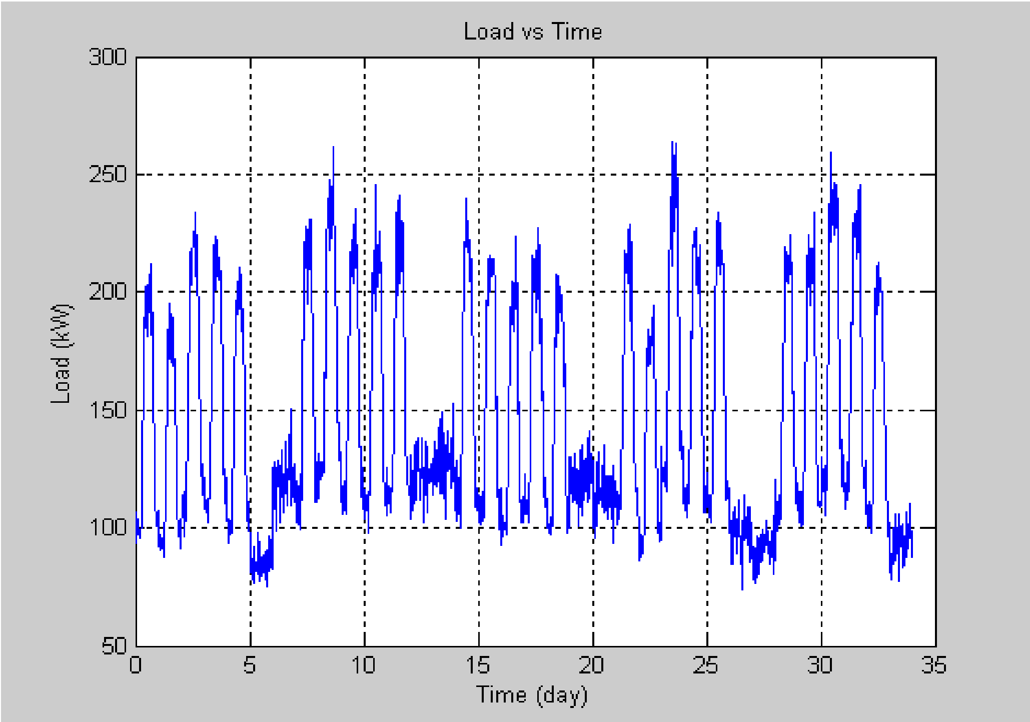

Figure 57 Simulated power profile.34 days with a week consisting of 5 working days and 2 weekend days. High- and low frequency noise

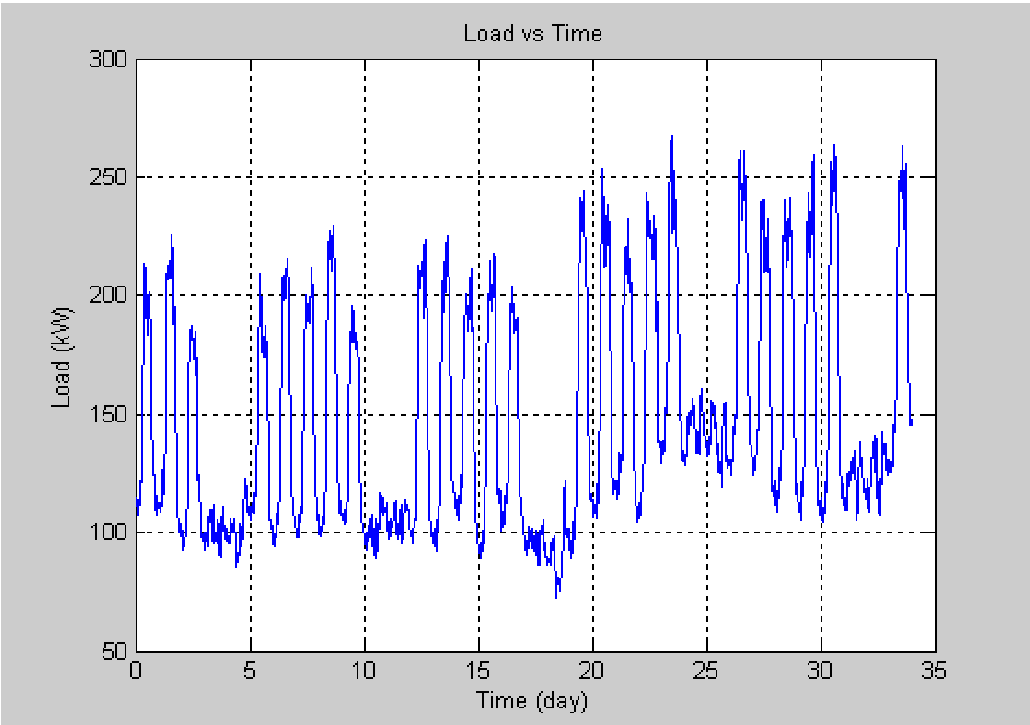

Figure 58 Measured power profile. 34 days of the measuring period have been analysed. The weeks consisted of 5 working days and 2 weekend days.

5 FUTURE WORK

The present analysis has been performed with an object where the electric energy consumption is quite small (about 1.3 GWh / year). In a future study it would be interesting to analyse objects according to the following list:

- larger industrial areas
- cities (including industrial areas)
- regions of cities

6 REFERENCES

[1] Analysis of *Combined Power Systems*
General description of software.
Chalmers University of Technology, 2006
Ingemar Mathiasson