Analytical Modelling of Short-term Response of Ground Heat Exchangers in Ground Source Heat Pump Systems

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SUMMARY

Short-term variations in the response of ground heat exchangers (GHEs) have significant effects on the overall performance of ground source heat pump (GSHP) systems. In this paper, a review of existing analytical solutions to determine the short-term response of the GHE is presented. The strengths and limitations of the noteworthy analytical solutions are highlighted and the transient heat transfer as considered by these solutions is critically analyzed. The short-term response predicted by the existing analytical solutions is presented and compared to the response predicted by a new comprehensive analytical solution.

INTRODUCTION

Determining the long-term response of GHEs has traditionally been the major focus of the GHE related modelling. Several analytical and numerical solutions have been developed to model the thermal response of GHEs after 10-25 years of their operation. The long-term thermal response is imperative to calculate the required length of the GHE and to determine the performance deterioration of the GHE over time. The thermal interactions between multiple boreholes of a GHE have also been studied [1] using the long-term thermal response.

However, in recent years the focus of the GHE related modelling has shifted from long-term to short-term response calculations. One of the major reasons behind this shift is the fact that heating and cooling demands of buildings have changed considerably. Today, many commercial and office buildings have a cooling demand during the day, even in a cold climate such as the Swedish, and a heating demand during the night. Other commercial buildings, like shopping centres and supermarkets, have simultaneous heating and cooling demands. The changing heating and cooling demands of the buildings have shifted the optimization focus of the GSHP systems from long-term seasonal storage to hourly, daily and weekly balancing of the GHE loads. Nowadays, the short-term response of the GHE is regarded as a key input parameter when optimizing the overall performance of the GSHP systems.

The short-term response of the GHE is also significant when conducting thermal response tests (TRTs) to estimate the ground thermal properties. A typical TRT measures the response of the ground during heat-flux build-up stages. The measured response is then analyzed using a heat transfer model to estimate properties like ground thermal conductivity and thermal resistance of the GHE. The long-term response solutions have often been utilized to analyze TRTs but using these solutions result in longer and more expensive tests as these solutions require a steady-state heat-flux situation. The short-term response solutions are much better suited to evaluate TRTs because of their inherent ability to model the transient heat transfer during the heat-flux build-up stages.

In this paper, we firstly present a general overview of the existing short-term response solutions. Secondly, we critically analyze three noteworthy analytical solutions which have been recently presented, in literature, to determine the short-term response of GHEs. Finally, we introduce a new analytical approach to evaluate the short-term response and compare it to the existing analytical solutions.

EXISTING SOLUTIONS

The classical methods to determine the thermal response of a GHE include the line source (LS) and the cylindrical source (CS) solutions. The LS solution [2] treats the radial heat transfer in a plane perpendicular to the GHE. The GHE is assumed to be a line source of constant heat output and of infinite length surrounded by an infinite homogeneous ground. On the other hand, the CS solution [3] models the GHE as a cylinder surrounded by homogeneous ground and having constant heat-flux across its outer boundary. Both the LS and CS solutions are inaccurate for times smaller than 10-15 hours as both these solutions oversimplify the geometry of the GHE and disregard its internal heat transfer. Nevertheless, these solutions have been extensively used to determine the short-term response of GHEs when evaluating TRTs.

A number of numerical solutions have been developed to overcome the shortcomings of the LS and CS solutions when evaluating TRTs. Among others, Shonder and Beck [4] and Austin [5] have developed solutions which numerically solve the heat transfer in GHEs. Shonder and Beck solve the 1-D radial heat transfer problem using a finite difference approach. They model the U-tube as an equivalent-diameter pipe surrounded by a thin film layer in the grout. The film layer accounts for the thermal capacity and the convective heat transfer of the fluid, the thermal capacity of the pipe and the thermal resistance of the grout. Austin [5], on the other hand, uses a 2D finite volume approach. His solution approximates the cross-sections of the two legs of the U-tube as pie-sectors with constant heat-flux entering the numerical domain for each time step. Yavuzturk and Spitler [6] extended Austin's work and developed the so-called short time-step g-functions, which are essentially an extension to Eskilsons' g-functions [7] from the superposition borehole model. The short time-step g-functions are non-dimensional temperature response functions to a unit step heat pulse. These two numerical solutions are very attractive for parametric analysis and to obtain precise solutions, but as all other numerical solutions, they too have limited flexibility. The fact that these and other similar numerical solutions cannot be directly incorporated into building energy simulation software has vastly limited their use for optimization of GSHP systems.

The need for robust, yet rapid, solutions for parameterized design and optimization of GSHP systems has resulted in the development of various analytical and semi-analytical solutions. A detailed review of these solutions can be found in [8]. Gu and Neal [9] developed an analytical solution assuming a cylindrical source in an infinite composite region. They solve the GHE's transient heat transfer problem using the generalized orthogonal expansion technique which requires calculation of multiple eigenvalues. Young [10] modified the classical buried electric cable solution [3] to develop his borehole fluid thermal mass solution. Lamarche and Beauchamp [11] extended the classical CS solution and developed short-term solutions for two cases of constant heat transfer rate and constant fluid temperature. More recently, Bandyopadhyay et al. [12] adapted the classical Blackwell solution [13] in their 'virtual solid' solution. They also developed a semi-analytical solution [14] in which they first solve the GHE's heat transfer problem in the Laplace domain and then use a numerical inversion to obtain the time domain solution.

ANALYSIS OF THREE EXISTING ANALYTICAL SOLUTIONS

As seen from the previous section, all the analytical and numerical solutions make simplifying assumptions regarding the geometry and the heat transfer problem of the GHE. Figure 1 illustrates the actual GHE geometry and also shows the thermal interactions of the GHE. The heat transfer inside the GHE depends on the thermal properties and interactions of its elements (i.e. the circulating fluid, the U-tube pipe and the grouting material). However, the conductive heat transfer across the GHE boundary also involves the thermal properties of the neighbouring ground. Assumptions regarding the GHE geometry and the thermal properties and the interactions of the GHE elements can significantly affect the short-term response calculations.

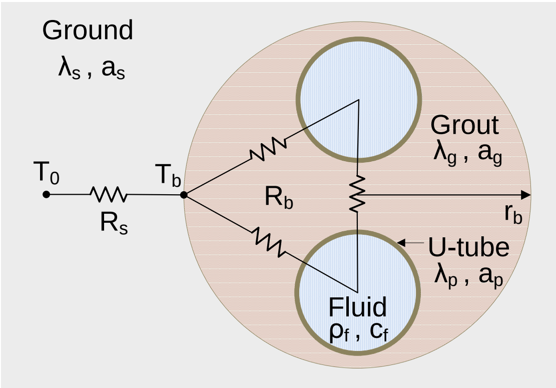

Figure 1. The actual geometry and the thermal interactions of a GHE

In most practical cases, the GHE's heat transfer is assumed to be radial only. This requires the U-tube to be approximated as a single equivalent-diameter pipe. Various methods [15, 16, 17, 18] have been suggested to approximate the equivalent radius (r_e) of the equivalent-diameter pipe. The resulting radial heat transfer problem is shown in Figure 2a. Some researchers have also developed solutions which eliminate the grout and study the heat transfer from the equivalent-diameter pipe directly to the surrounding ground (Figure 2b). Such solutions are useful for cases when the GHE has been backfilled with borehole cuttings.

Borehole Fluid Thermal Mass (BFTM) and Buried Electric Cable (BEC) Solutions

Young [10] and Lamarche and Beauchamp [11], among others, solved the radial heat transfer problem of the grouted GHE. Young, in his BFTM solution, compared a buried electric cable to a vertical GHE by relating the core, the insulation and the sheath of the electric cable to the equivalent-diameter pipe, the thermal resistance and the grout of the vertical GHE. This enabled Young to apply the following classical BEC solution [3] to the vertical GHEs.

$$T - T_0 = \frac{\dot{q}_b}{\lambda_s} \cdot \frac{2 \cdot \alpha_1^2 \cdot \alpha_2^2}{\pi^3} \int_0^{\infty} \frac{1 - e^{-u^2 \cdot Fo}}{u^3 \cdot \Delta(u)} du \qquad (1)$$

where

$$\Delta(u) = \left(u(\alpha_1 + \alpha_2 - h \cdot u^2)\, J_0(u) - \alpha_2(\alpha_1 - h \cdot u^2)\, J_1(u)\right)^2 + (u(\alpha_1 + \alpha_2 - h \cdot u^2)\, Y_0(u) - \alpha_2(\alpha_1 - h \cdot u^2)\, Y_1(u))^2.$$

$$\alpha_1 = \frac{2 \cdot \pi \cdot r_b^2 \cdot \rho_s \cdot c_s}{S_f}, \qquad \alpha_2 = \frac{2 \cdot \pi \cdot r_b^2 \cdot \rho_s \cdot c_s}{S_g}, \qquad \text{and} \qquad h = 2 \cdot \pi \cdot \lambda_s \cdot R_b.$$

The BFTM solution is primarily a variation of the grouted GHE case shown in Figure 2a. In the actual BEC solution [3], the core and the sheath of the electric cable are both assumed to be metallic conductors and thus have lumped thermal capacities and temperatures. However, this assumption is not very accurate for vertical GHEs. Therefore, Young uses a grout allocation factor to assign a portion of the thermal capacity of the grout to the equivalent-diameter pipe. This improves the accuracy of the BFTM solution. Additionally, Young also suggests using a logarithmic extrapolation to get better precision. However, in reality the grout allocation factor varies from case to case and is extremely ambiguous to calculate. This makes the practical implementation of the BFTM solution quite challenging. Hence, the BFTM solution has rarely been implemented in its true essence.

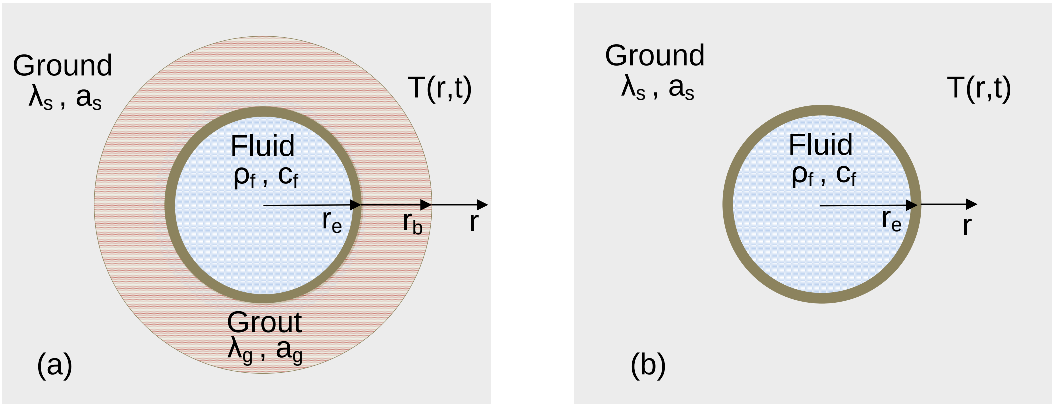

Figure 2. Simplified GHE geometries

Lamarche and Beauchamp's (L&B) Solution

Lamarche and Beauchamp [11] also solved the radial heat transfer problem for grouted GHEs. In their solution, they assumed the equivalent-diameter pipe of Figure 2a as a hollow cylinder with no thermal capacity. This solution is essentially an extension of the classical CS solution [3] as it solves the heat transfer problem assuming a steady heat-flux condition across the hollow cylinder boundary. However, unlike the CS solution, this solution incorporates the thermal properties of the grout. This is done by taking the steady heat-flux condition across the hollow cylinder instead of the GHE boundary as considered by the CS solution.

$$T - T_0 = \frac{\dot{q}_b}{\lambda_s} \cdot \frac{8\,\tilde{\lambda}}{\pi^5 \cdot \delta^2} \int_0^\infty \frac{(1 - e^{-u^2 \cdot Fo})}{u^5(\phi^2 + \psi^2)} du \tag{2}$$

where

$$\phi(u) = Y_1(u)\left[Y_0(u\delta\gamma)J_1(u\delta) - Y_1(u\delta\gamma)J_0(u\delta)\tilde{\lambda}\gamma\right] - J_1(u)\left[Y_0(u\delta\gamma)Y_1(u\delta) - Y_1(u\delta\gamma)Y_0(u\delta)\tilde{\lambda}\gamma\right],$$

& $$\psi(u) = J_1(u)\left[J_0(u\delta\gamma)Y_1(u\delta) - J_1(u\delta\gamma)Y_0(u\delta)\tilde{\lambda}\gamma\right] - Y_1(u)\left[J_0(u\delta\gamma)J_1(u\delta) - J_1(u\delta\gamma)J_0(u\delta)\tilde{\lambda}\gamma\right].$$

The heat transfer from the equivalent-diameter pipe to the grout depends on the thermal heat capacity and the convective heat transfer of the fluid and the physical and the thermal properties of the pipe. The temperature increase of the fluid is dampened by the presence of the fluid thermal capacity. In its absence, the fluid temperature will rapidly increase for short times before converging to the long-term response. Hence, the response calculated using the L&B solution is not accurate for short times.

Virtual Solid (VS) Solution

Bandyopadhyay et al. [12] recently presented their VS solution to determine the short-term response of the GHE backfilled with borehole cuttings (Figure 2b). The solution, which was originally developed by Blackwell [13], models the circulating fluid as a virtual solid whereas the injected heat is assumed to be generated uniformly over the length of the virtual solid. The solution accounts for the thermal capacity of the circulating fluid through the S/S_e ratio which is the ratio of the thermal capacity of an equivalent volume to the thermal capacity of the equivalent-diameter pipe. The solution also considers the flow related convective heat transfer by using the Biot number. The VS solution also has limited practical application as most of the GHEs are backfilled with a material quite different from their surrounding ground.

$$T - T_0 = \frac{\dot{q}_b}{\lambda_s} \frac{8}{\pi^3} \left(\frac{S}{S_e}\right)^2 \int_0^\infty \frac{1 - e^{-u^2 \cdot Fo}}{u^3 (O^2 + P^2)} du \tag{3}$$

where

$$O = u\, Y_0(u) + 2\left(\frac{u^2}{Bi} - \frac{S}{S_e}\right) Y_1(u) \qquad \text{and} \qquad P = u\, J_0(u) + 2\left(\frac{u^2}{Bi} - \frac{S}{S_e}\right) J_1(u).$$

NEW ANALYTICAL APPROACH

It is obvious from the above analysis of the three analytical solutions that even the state-of-the-art analytical solutions have used unrealistic assumptions when determining the short-term response of GHEs. These solutions either overlook the thermal properties of some GHE elements or oversimplify the GHE geometry and the thermal interactions between GHE elements. Hence, the short-term response evaluated by these solutions is inaccurate. There exists a genuine need of a new analytical approach capable of simulating short-term response of the GHE considering all the significant heat transfer processes and without distorting the geometry of the GHE. To fill this need, a new analytical solution has been developed to model the complete *radial* heat transfer problem in the GHEs.

In this section, we only indicate the fundamental background of the new approach. The detailed background, the mathematical formulation and the validation of the solution is presented in [19]. In the new analytical solution, the GHE's heat transfer and the related boundary conditions have been studied in the Laplace domain. A set of equations of the Laplace transforms for the boundary temperatures and heat-fluxes are obtained. These equations are represented by a thermal network. The use of the thermal network enables swift and precise evaluation of any thermal or physical setting of the GHE. Finally, very concise formulas of the inversion integrals have been developed to obtain the time dependent solutions. The new analytical solution considers the thermal capacities, the thermal resistances and the thermal properties of all the GHE elements and provides an exact solution to the radial heat transfer problem in vertical GHEs.

COMPARISON OF ANALYTICAL SOLUTIONS

In this section we compare the new analytical solution to the previously discussed solutions (1), (2) and (3). Three different cases, shown in Table 1, are considered for the comparison. For all three cases, the increase in the fluid temperature for a unit heat injection is simulated for a GHE of 110 mm diameter.

Table 1. Grout and ground properties of three simulated cases

Case	Grout Properties	Ground Properties
1. GHE filled with groundwater	$\lambda_g = 0.57$ W/(m·K) $\rho_g = 1000$ kg/m^3 $c_g = 4180$ J/(kg·K)	$\lambda_s = 3$ W/(m·K) $\rho_s = 2500$ kg/m^3 $c_s = 750$ J/(kg·K)
2. GHE backfilled with TEG (Thermally enhanced grout)	$\lambda_g = 1.5$ W/(m·K) $\rho_g = 1550$ kg/m^3 $c_g = 2000$ J/(kg·K)	
3. GHE backfilled with borehole cuttings	$\lambda_g = 3$ W/(m·K) $\rho_g = 2500$ kg/m^3 $c_g = 750$ J/(kg·K)	

Figure 3a presents the comparison of the analytical solutions for a groundwater-filled GHE. Such installations are common in all Scandinavian countries and particularly in Sweden. In this study, we have not considered the effects of the convective heat transfer in the groundwater. As seen, the BFTM [10] and VS [12] solutions both underestimate the short-term fluid temperature increase. This is because both these solutions do not consider the actual thermal properties of the groundwater. The VS solution takes the thermal properties of the groundwater as those of the surrounding ground while the BFTM solution uses a lumped capacity and temperature for the groundwater. On the other hand, the L&B solution [11] initially overestimates the short-term increase of the fluid temperature because it ignores the thermal capacity of the fluid. Hence, the predicted increase in the fluid temperature is higher than that predicted by the new analytical solution. However, the increase in the fluid temperature predicted by these two solutions converge with time.

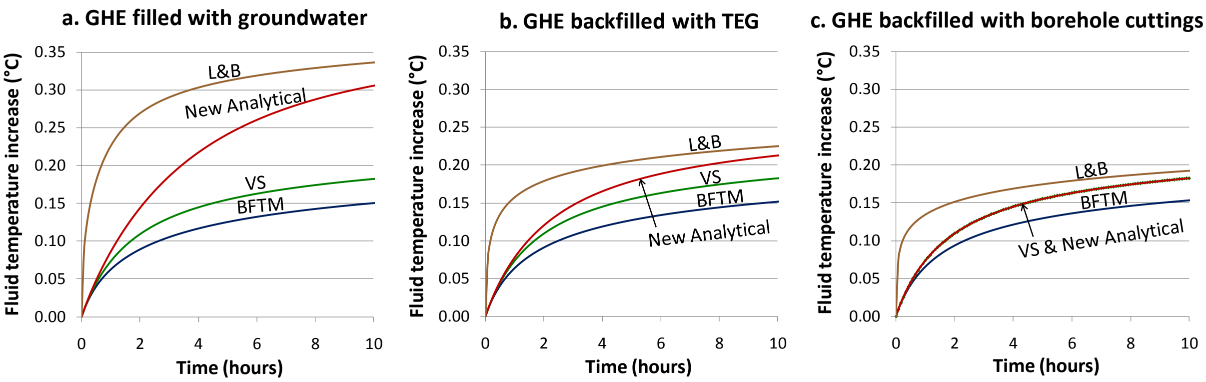

Figure 3. Fluid temperatures using different solutions

As indicated by Figure 3b, the different predictions of the increase in the fluid temperature deviate less for a GHE backfilled with TEG than a groundwater-filled GHE. This is because the thermal properties of the TEG are closer to the thermal properties of the ground than are those of water. Consequently, the short-term fluid temperature increase of GHE backfilled with TEG, as predicted by the new analytical [19] and the L&B [11] solutions, is lower than that of a groundwater-filled GHE. This reduces the differences between the solutions in general as the fluid temperature predicted by the VS [12] and the BFTM [10] solutions remain virtually unchanged.

For the GHE backfilled with borehole cuttings (Figure 3c), the new analytical [19] and the VS [12] solutions predict the same short-time fluid temperature increase. Like the first two cases, the L&B [11] solution overestimates the increase in the fluid temperature for short times but converge to the results from the new analytical and VS solutions in the long run.

The results clearly indicate the significance of unrealistic assumptions regarding the geometry and the elements of the GHE. Thermal properties of the fluid and the grout are both critical to the short-term response modelling. Unreasonable assumptions, like not accounting for the fluid thermal capacity or not considering the actual grout properties, lead to unrealistic results. It is also interesting to see the influence of different grout properties on the predicted fluid temperature. Increase in the thermal conductivity of the grout and decrease in its thermal capacity have considerable influence on the short-term response of the GHE as indicated by the results of the new analytical solution for the three cases. As seen from Figure 3, the predicted increase in the fluid temperature varies around 75% between the groundwater-filled GHE (i.e. the lowest grout thermal conductivity and the highest grout thermal capacity) and the GHE backfilled with borehole cuttings (i.e. the highest grout thermal conductivity and the lowest grout thermal capacity). For all the three cases, the BFTM [10] solution tends to be the most inaccurate. To improve the BFTM-solution accuracy, Young recommends the use of a grout allocation factor and a logarithmic extrapolation procedure. This, however, was not done in this implementation of the BFTM solution.

CONCLUSION

A review of the existing analytical solutions to predict the short-time response of the GHE was carried out. It was shown that the existing analytical solutions are based on over-simplifying assumptions. A new exact analytical solution was introduced. A comparison between the existing analytical solutions and the new analytical solution was done for three different types of GHE. The comparison revealed the limitations of the existing analytical solutions.

NOMENCLATURE

a	ground thermal diffusivity ($m^2\ s^{-1}$)	***Greek Symbols***	
Bi	Biot number $= 2hr_e/\lambda_s$	γ	$(a_g/a_s)^{1/2}$
c	specific heat (J $kg^{-1}\ K^{-1}$)	δ	r_b/r_e
Fo	Fourier number $= at/r^2$	λ	thermal conductivity (W $m^{-1}K^{-1}$)
J_x	xth − order Bessel function of the first kind	$\tilde{\lambda}$	λ_s/λ_g
$\dot{q}$	heat flow per unit length (W m^{-1})	ρ	density (kg m^{-3})
R	thermal resistance (K m W^{-1})		
r	radius (m)	***Subscripts***	
S	thermal capacity (J K^{-1} m^{-1})	b	borehole
t	time (s)	e	equivalent
T	temperature (K)	f	fluid
u	integral parameter	g	grout
Y_x	xth − order Bessel function of the second kind	p	pipe
		s	soil / ground
		0	undisturbed

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