THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

On the optimization of opportunistic maintenance activities

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Abstract

Maintenance is a source of large costs; in the EU the maintenance costs amount to between 4% and 8% of the total sales turnover. Opportunistic maintenance is an attempt to lower the maintenance cost by considering the failure of one component as an opportunity to replace yet non-failed components in order to prevent future failures. At the time of failure of one component, a decision is to be made on which additional components to replace in order to minimize the expected maintenance cost over a planning period.

This thesis continues the work of Dickman et. al. [9] and Andréasson [1] on the opportunistic replacement problem. In Paper I, we show that the problem with time-dependent costs is NP-hard and present a mixed integer linear programming model for the problem. We apply the model to problems with deterministic and stochastic component lives with data originating from the aviation and wind power industry. The model is applied in a stochastic setting by employing the expected values of component lives. In Paper II, a first step towards a stochastic programming model that considers components with uncertain lives is taken by extending the problem to allow non-identical lives for component individuals. This problem is shown to be NP-hard even with time-independent costs. We present a mixed integer linear programming model of the problem. The solution time of the model is substantially reduced compared to the model presented in [1]. In Paper III, we then study the opportunistic replacement problem with uncertain component lives and present a two-stage stochastic programming approach. We present a deterministic equivalent model and develop a decomposition method. Numerical studies on the same data as in Paper I from the aviation and wind power industry show that the stochastic programming approach produces maintenance decisions that are on average less costly than decisions obtained from simple maintenance policies and the approach used in Paper I. The decomposition method requires less CPU-time than solving the deterministic equivalent on three out of four problems.

Keywords: mixed integer linear programming; complexity theory; stochastic programming

Appended papers:

Paper I: *The opportunistic replacement problem: analysis and case studies*, Submitted to Naval Research Logistics (with Torgny Almgren, Niclas Andréasson, Michael Patriksson, and Ann-Brith Strömberg).

Paper II: *Models and complexity analysis of the opportunistic replacement problem with individual component lives*, To be submitted to Annals of Operations Research (with Michael Patriksson and Ann-Brith Strömberg).

Paper III: *The stochastic opportunistic replacement problem: a two-stage solution approach*, To be submitted to Annals of Operations Research (with Michael Patriksson and Ann-Brith Strömberg).

Other publications:

A method for simulation based optimization using radial basis functions, to appear in *Engineering and Optimization*, published online 19 June 2009 (with Stefan Jakobsson, Michael Patriksson and Johan Rudholm).

An opportunistic maintenance optimization model for shaft seals in feed-water pump systems in nuclear power plants, Proceedings of 2009 IEEE Bucharest PowerTech Conference, June 28th – July 2nd, Bucharest, Romania, pp. 2962-2969 (with Julia Nilsson, Michael Patriksson, Ann-Brith Strömberg, and Lina Bertling).

An optimization framework for opportunistic maintenance of offshore wind power system, Proceedings of 2009 IEEE Bucharest PowerTech Conference, June 28th – July 2nd, Bucharest, Romania, pp. 2970-2976 (with François Besnard, Michael Patriksson, Ann-Brith Strömberg, and Lina Bertling).

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Contents

1 Introduction

1.1 Maintenance optimization

The purpose of this thesis is to develop and solve *mixed integer linear programming* (MILP, see [14] for more about integer programming) and *stochastic programming* (SP, see Section 1.2) models for maintenance decision support problems. Applications of the models developed here derive from the aviation industry (through a cooperation with Volvo Aero Corporation on the maintenance of aircraft engines) and from the energy production sector (on the maintenance of wind turbines and of feed water pumps in nuclear power plants). We begin with a short overview of the field of maintenance optimization (for more comprehensive overviews of maintenance optimization see e.g. [15] and [17]).

The research on mathematical models of and methods for maintenance decision problems began during WWII at the military RAND institute at Santa Monica, CA, USA. Richard Bellman, who was a member of RAND, invented dynamic programming which was the first efficient method for maintenance optimization (see [3]). The focus of the work up to the 1960s was on simple maintenance or replacement policies for systems consisting of few components (often just one). The classical book by Barlow and Proschan [2] covers these types of methods. The focus of the research since the 1960s has mainly been on extending the simple policies towards more complex systems with more components and more complex system features (such as several states of deterioration and different types of repairs). The field of maintenance today is rather vast, and a characterization of a maintenance problem is therefore necessary.

Firstly, maintenance problems can be characterized as either single- or multi-component problems. Much of the early research on maintenance problems was concerned with single-component systems and this area does still pose some interesting challenges. Here, however, we focus on multi-component systems, in which the dependencies between components are important. The dependencies may be either economic, stochastic or structural (or a combination of these). We consider positive economic dependencies, which here means that the cost of simultaneous replacement of several components is lower than the sum of the costs of individual component replacements.

A second important characterization of a maintenance problem is the type of maintenance actions considered. An early categorization of maintenance actions was into corrective maintenance (CM) or preventive maintenance (PM). CM is performed upon failure in order to restore the system to a functioning state. PM, on the other hand, is performed on functioning systems in order to avoid future failures. The scheduling of PM is often periodic and fixed over a long period of time. More frequent PM implies that less CM is required and that fewer inconveniences connected to system failures arise (such as system downtime etc.). Too much PM will, however, increase maintenance costs and enforce system downtime due to maintenance stops. Opportunistic maintenance (OM) is an approach that intends to combine CM and PM. The idea is to consider the failure of a component as an opportunity to perform PM on other components. Instead of planning the PM activities beforehand, a policy is used that decides which PM to perform at a given state of the system. Condition based maintenance (CBM) is maintenance performed based on information obtained from a condition monitoring system (CMS), which performs system measurements and calculates the risk of future failures or provides estimates of remaining useful component lives. The maintenance type considered in this thesis is mainly OM.

A third important property of a maintenance problem is the types of component lives considered; it is normally distinguished between stochastic and deterministic lives. Deterministic lives are often assigned to critical components in such a way that the probability of failure is very low before the assigned life is reached (this is common in the aircraft industry, see [1, Ch. 2]). Other components, however, are replaced at failure or when some critical indication level is reached. The lives of such components are considered to be stochastic. In this thesis, we begin by studying a problem comprising components with deterministic lives and then generalize it to allow components to possess stochastic lives.

Stochastic maintenance problems, that is, problems that include components with uncertain lives, can be approached by stationary or dynamic models. Stationary models are solved once for a given maintenance problem and the solution provided (such as a maintenance policy, or a frequency for PM) is then utilized for all maintenance decisions. Stationary models often consider an infinite horizon. The models can not incorporate dynamic information about the state of the system (such as information from a CMS). Dynamic models, on the other hand, are solved each time new information is available. The models often consider a finite planning horizon and can incorporate dynamic information. The previous research on dynamic problems mostly cover pure scheduling problems (such as in [6] and [13]), or problems where the risk of failure (see [16]) or the deviation from an infinite horizon solution (see [8] and [20]) is penalized. To our knowledge, the only research done on dynamic models which includes the cost of future failure is [7]; however, only the maintenance costs until the next failure time was considered in that work.

Problems with deterministic component lives are not very well studied. An opportunistic maintenance problem with deterministic component lives is the topic of [9]. This pioneering work develops a MILP model for the problem which is further studied and improved in [1]. In this thesis, we continue the work on the problem with deterministic component lives and extend it to include stochastic component lives. We then create a dynamic model for the problem by extending the original MILP model to a stochastic programming model.

1.2 Stochastic Programing

Stochastic Programming (SP), sometimes also denoted as optimization under uncertainty, is concerned with decision making when data is random with a known probability distribution. We will here give a short overview of the field with the intention to introduce some key concepts used in the SP models of maintenance problems in Sections 2.3 and 3.1. For a more comprehensive introduction to SP see for instance

[5] or [12].

Consider first a standard linear programming (LP) problem

$$\begin{array}{ll} \text{minimize} & c^T x, \\ \text{subject to} & Ax = b, \\ & x \geq 0. \end{array}$$

The data consists of the vectors c and b, and the matrix A. If the data is deterministic we obtain the optimal solution by solving the LP. However, if our model includes phenomena such as future outcomes of financial markets, weather forecasts, or uncertain future demand and price of a product, we may at best obtain probability distributions for the data. A simple approach is to use the expected values of c, b and A, and solve the corresponding deterministic LP. The resulting solution is denoted the *expected value* solution. It often leads to suboptimal decisions, as it only considers the expected value of the data and does not take the whole probability distribution into account (for instance, for some events, which occur with a low probability, it may yield very bad results).

In order to create a model which takes stochasticity into account, we need to distinguish between two types of decisions. The *first stage* decisions are those taken before the realization of the uncertainty (such as to decide on the production plan for a product before knowing its demand and price). The second stage decisions are taken once the uncertainty has been realized (such as to decide to which customers the product should be sold given the realized demand and price). Let x and y denote the first and second stage variables, ω a possible realization or *scenario* of the uncertain parameters, and Ω the probability space of all possible realizations ω. A standard *two-stage* stochastic linear program is formulated as that to

$$\underset{x}{\text{minimize}} \quad c^T x + \mathbb{E}_{\omega \in \Omega}[Q(x, \omega)], \tag{1a}$$
$$\text{subject to} \quad Ax = b, \tag{1b}$$
$$x \geq 0, \tag{1c}$$

where

$$Q(x, \omega) = \underset{y}{\text{minimum}} \quad q(\omega)^T y, \tag{1d}$$
$$\text{subject to} \quad W(\omega)y = h(\omega) - T(\omega)x, \tag{1e}$$

and $q(\omega)$, $W(\omega)$, $h(\omega)$ and $T(\omega)$ denote the stochastic parameters of the problem. The function $\mathcal{Q} : \mathbb{R}^n \to \mathbb{R}$ defined by $\mathcal{Q}(x) = \mathbb{E}_{\omega \in \Omega}[Q(x, \omega)]$, is called the *recourse* function and the problem (1d)–(1e) is denoted the subproblem.

A common assumption in stochastic programs is that the probability space Ω is finite. For a continuous or finite but large probability space an approximation of the same is obtained by sampling a finite number of scenarios. This is denoted the *sample average approximation*. For finite probability spaces we can formulate a (large) linear program, denoted the *deterministic equivalent*, the solution of which is equivalent to the solution to the stochastic program. Let $p(\omega)$ denote the probability of the scenario $\omega \in \Omega$. The deterministic equivalent of program (1) is to

$$\begin{aligned} \underset{x;y(\omega),\omega\in\Omega}{\text{minimize}} \quad & c^T x + \sum_{\omega\in\Omega} p(\omega) q(\omega)^T y(\omega), \\ \text{subject to} \quad & Ax = b, \\ & W(\omega)y(\omega) + T(\omega)x = h(\omega), \ \forall \omega \in \Omega, \\ & x \geq 0. \end{aligned}$$

An equivalent formulation of this problem is to

$$\underset{x(\omega),y(\omega),\omega\in\Omega}{\text{minimize}} \quad \sum_{\omega\in\Omega} p(\omega)(c^T x(\omega) + q(\omega)^T y(\omega)), \tag{2a}$$

$$\text{subject to} \quad Ax(\omega) = b(\omega), \quad \omega \in \Omega, \tag{2b}$$

$$W(\omega)y(\omega) + T(\omega)x = h(\omega), \quad \omega \in \Omega, \tag{2c}$$

$$x(\omega_1) = x(\omega_2), \ \omega_1, \omega_2 \in \Omega, \tag{2d}$$

$$x(\omega) \geq 0, \qquad \omega \in \Omega. \tag{2e}$$

The constraints (2d) are denoted the *non-anticipativity* constraints. If these are relaxed, the problem separates into solving one smaller problem for each scenario $\omega \in \Omega$. This means that we would anticipate the realization of the stochastic process before determining the optimal value of the first stage variables or, equivalently, that we have information about the realization of the stochastic process beforehand. The solution obtained from solving the program (2a)–(2c),(2e) is denoted the expected *perfect information* solution.

In order to solve instances of stochastic programs for which the deterministic equivalent becomes to large to solve, decomposition techniques have been proven useful. The most common technique is called the *L-shaped* (see [19]) method which is an application of *Benders' decomposition* (see [4]).

We are going to consider stochastic integer programs, where integrality constraints are present for both the first and second stage variables. In such a setting, the L-shaped method is not applicable, as it is based on the fact that strong duality is satisfied for the subproblem, which is in general not fulfilled for subproblems with integrality constraints. Decomposition techniques for integer stochastic programs exist (see for instance [18]), but they are neither as general nor as efficient as the L-shaped method for linear stochastic programs. In Paper III we present a decomposition method adapted to the opportunistic replacement problem studied in this thesis.

In many applications, including the maintenance problems studied in this thesis, a sequence of decisions are to be taken. First is a first stage decision taken (for instance, to buy or sell a stock), then outcomes of a stochastic process is realized (e.g., the price of the stock is altered), followed by a second stage decision (e.g., we deciding whether or not to sell or buy more of the stock), and again the realization of a stochastic process (the price is again altered), followed again by a third stage decision (a new decision on buying or selling the stock), and so on. Such problems are denoted *multistage* problems, and are generally much more difficult to solve than two-stage problems. The mathematical formulation of a multistage problem includes a recourse function in the objective of the subproblem (1d)–(1e). The recourse function is evaluated by solving one subproblem for each scenario, which then again contains a recourse function, and so on. We can formulate a deterministic equivalent for multistage problems; the number of variables, however, tend to become very large. Multistage problems can be solved by using *nested decomposition* approaches, which means that the subproblems in the decomposition are themselves solved by a decomposition, and so on (see [5, Ch. 7] for more on multistage programs and nested decomposition). In the main part of this thesis (except in Section 3.1) we ignore the multistage structure of the problem and approximate it by either an expected value model or a two-stage model.

1.3 Problem overview

The goal of the research track, which this thesis is part of, is to analyze and solve maintenance decision problems. We consider a system (for instance a wind power turbine or a jet engine). Any maintenance stop for such a system generates a cost, independent of the type of maintenance performed. We assume that a maintenance stop is enforced at the *current time* because the system requires CM or scheduled PM, or because of indications from a CMS. The maintenance stop is an occasion to perform more PM than what is required and thus avoid costly maintenance stops in the near future. However, performing PM also generates costs. We wish to take a decision on which maintenance actions to perform at the current time[1] in order to minimize the expected cost (or some alternative objective) over the remaining planing horizon or contract period. This is denoted as the *current problem*. The vision is to create a decision support system that, given the system state and a failure, returns an optimal (wrt. expected costs) maintenance decision at the current time by solving the current problem (see Figure 1).

The main part of the research presented in this thesis is on a fairly simple maintenance problem, although the intention is to generalize results for this problem to more general settings in the future. The problem studied can be described as follows. A system consists of the components $\mathcal{N} = \{1, \ldots, n\}$ with known failure distributions. We assume that every component must be replaced at failure. Moreover, each maintenance stop generates the maintenance occasion cost d and the replacement of a component $i \in \mathcal{N}$ generates the replacement cost c_i. We wish to minimize the expected maintenance cost over the planning period $[0, S]$. The problem is denoted the opportunistic replacement problem (ORP) or the stochastic opportunistic replacement problem (SORP) depending on if component lives are deterministic or stochastic respectively. Example 1 contains a small example instance of the SORP.

Example 1 (SORP). *Consider a system consisting of two components. Each component has a stochastic life with a known probability distribution. The replacement costs are c_1, c_2 and the maintenance occasion cost is d. Minimize the expected maintenance cost over the time period $[0, 10]$.*

[1] The current time can be a period that stretches from a couple of hours to weeks, months or years. It is a period for which the decision to perform maintenance or not can not be postponed.

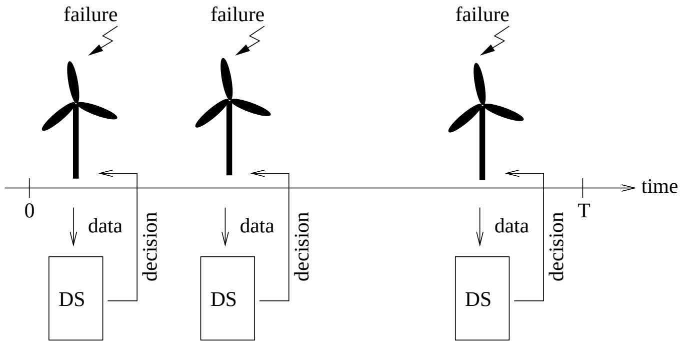


Figure 1: Illustration of a decision support system (DS) for a wind power turbine. At failure, the system data is sent to the DS, which solves the current problem and returns a decision on the type of maintenance action to perform at the current time.

For systems with deterministic lives, a minimal cost maintenance schedule for the entire planning period exists. For systems with stochastic component lives we can not create such a maintenance schedule, since the maintenance decisions depend on the state of the system (for instance, it is always optimal not to perform any maintenance on a system in which all components are working). Instead, we wish to solve the current problem. Example 2 presents a current problem for the SORP instance introduced in Example 1.

Example 2 (SORP cont.)**.** *Assume that component 1 failed at time* 4*. The current problem is then defined as: Should component 2 be replaced or not in order to minimize the expected total maintenance cost until time* 10*?*

In general, the current problem for the SORP is described as follows. Given the failure of one or more components at the current time $s \in [0, S]$, we wish to decide which (if any) additional (non-failed) components to replace in order to minimize the expected maintenance cost over the remaining planning period $[s, S]$.

In MILP and SP models, it is common to discretize time. For models of the current problem, a time discretization δ is introduced such that failures and maintenance decisions are assumed to occur at time steps $\{s, s+\delta, s+2\delta, \ldots, s+T\delta\}$, where $T = \lceil \frac{S-s}{\delta} \rceil$. These time steps will be denoted as $\{0, 1, \ldots, T\}$. For models of the ORP we set $s = 0$ and proceed similarly.

The remainder of this thesis is organized as follows. Section 2 contains a summary of and an introduction to the appended papers. The topic of Section 2.1 is Paper I, which considers the ORP. The goal is to find an optimal maintenance schedule for the system over the whole planning period. The topic of Section 2.2 is Paper II, which is a first step towards solving problems with uncertain component lives. We study an extension of the ORP in which the lives are deterministic but may differ between individuals of each component. The problem is denoted as the opportunistic replacement problem with individual lives (ORPIL). The topic of Section 2.3 is

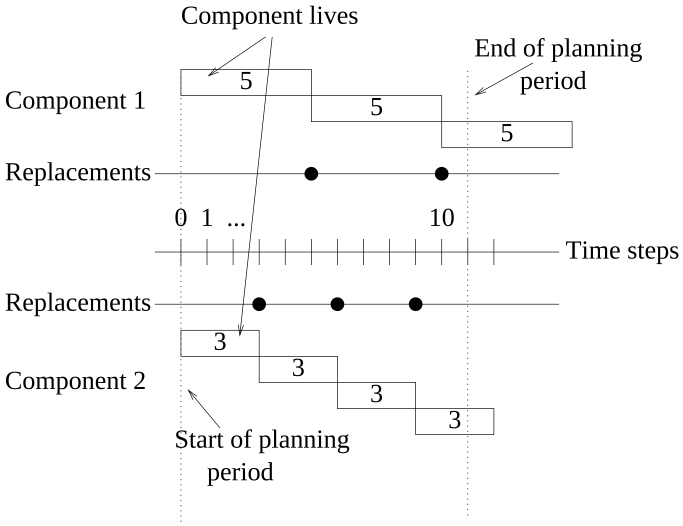


Figure 2: An illustration of a non-opportunistic maintenance schedule for the ORP defined in Example 3.

Paper III, which presents a two-stage approximation for the current problem of the SORP. Section 3 contains additional research on maintenance optimization not included in the appended papers. In Section 3.1 we present a model that takes the multistage structure of the current problem of the SORP into account but is computationally demanding to solve. In Section 3.2 we present a method for assigning the components a value at the end of the planning period and including it into the objective. Finally, Section 4 summarizes the contribution of the thesis and outlines future research possibilities.

2 Summary of the appended papers

2.1 Paper I: The opportunistic replacement problem

In this paper we study the opportunistic replacement problem (ORP), first introduced in [9] and further studied in [1]. We begin by presenting an example of an instance of the ORP.

Example 3 (ORP). *Consider a system consisting of two components having deterministic lives. Each component has to be replaced at the latest at failure. Assume that the replacement cost of component 1 is c_1, the replacement cost of component 2 is c_2, the life of component 1 is 5 time steps, the life of component 2 is 3 time steps, and that the maintenance occasion cost is d. We wish to find a minimum cost maintenance schedule over a time period defined by the time steps* $1, \ldots, 10$.

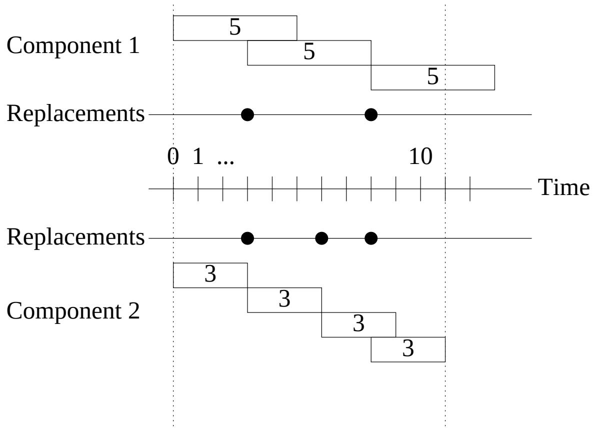


Figure 3: An illustration of the optimal maintenance schedule for the ORP defined in Example 3.

Figure 2 illustrates a non-opportunistic maintenance schedule for the system, that is, each component is replaced at failure. The resulting maintenance cost is $2c_1 + 3c_2 + 5d$. *Figure 3 illustrates an optimal maintenance schedule for the system with the resulting maintenance cost* $2c_1 + 3c_2 + 3d$.

We now present a general problem definition, where we allow the replacement and maintenance occasion costs to depend on time. Given a set of components $\mathcal{N} = \{1, \dots, n\}$ and a time period defined by the set $\mathcal{T} = \{1, \dots, T\}$ the problem is formally be defined as follows.[2]

Definition 1 (opportunistic replacement problem). *Let* d_t *be a fixed cost for a maintenance occasion,* c_{it} *be the cost for replacing a component* $i \in \mathcal{N}$ *at time* $t \in \mathcal{T}$*, and let* T_i *time steps be the life of component* $i \in \mathcal{N}$*. Find a maintenance schedule over the time period defined by* $\mathcal{T}$ *that minimizes the total maintenance cost and such that each component* $i \in \mathcal{N}$ *is replaced at least once every* T_i *time steps.* □

A major contribution of the paper is the result that the *set covering* problem is polynomially reducible to the ORP given in Definition 1 which implies that the ORP is NP-hard (see [10] for more on complexity theory). The reduction relies on the fact that the costs are allowed to be time dependent. The complexity of the ORP with *time independent* costs is still unknown.

[2]The time period begins at time 0 and ends before time $T + 1$. These times are not included into $\mathcal{T}$ since an optimal maintenance schedule without replacements at time 0 and time $T + 1$ always exists.

We present an integer programming model for the ORP. Let

$$z_t = \begin{cases} 1, & \text{if maintenance shall occur at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad t \in \mathcal{T},$$

and

$$x_{it} = \begin{cases} 1, & \text{if component } i \text{ shall be replaced at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad i \in \mathcal{N}, \quad t \in \mathcal{T}.$$

The ORP optimization model is then to

$$\begin{aligned}
\underset{(x,z)}{\text{minimize}} \quad & \sum_{t \in \mathcal{T}} \Big(\sum_{i \in \mathcal{N}} c_{it} x_{it} + d_t z_t \Big), && && \text{(3a)} \\
\text{subject to} \quad & \sum_{t=\ell+1}^{\ell+T_i} x_{it} \geq 1, && \ell = 0, \ldots, T - T_i, \quad i \in \mathcal{N}, && \text{(3b)} \\
& x_{it} \leq z_t, && t \in \mathcal{T}, \quad i \in \mathcal{N}, && \text{(3c)} \\
& x_{it} \geq 0, && t \in \mathcal{T}, \quad i \in \mathcal{N}, && \text{(3d)} \\
& z_t \leq 1, && t \in \mathcal{T}, && \text{(3e)} \\
& x_{it} \in \{0,1\}, && t \in \mathcal{T}, \quad i \in \mathcal{N}, && \text{(3f)} \\
& z_t \in \{0,1\}, && t \in \mathcal{T}. && \text{(3g)}
\end{aligned}$$

In Paper I, all non-superfluous constraints, out of the constraints (3b)–(3e), are shown to describe facets of the convex hull of the set of feasible solutions to the ORP. Furthermore, it is shown that the integrality requirements on x_{it} for all $i \in \mathcal{N}$ and $t \in \mathcal{T}$ can be relaxed.

The paper contains numerical tests on problem instances from the aircraft and wind power industries; these include both stochastic and deterministic ORP:s. The results for the deterministic problems indicate that the use of the model (3) can reduce costs by up to 40% compared to those gained by simple maintenance policies. In the stochastic ORP (that is, the SORP) the component lives are uncertain; we thus obtain a maintenance decision at the time of failure (i.e., a solution to the current problem) by solving an instance of the ORP in which the expected values of the component lives are employed. Using this expected value solution for the current problem of the SORP does also show good results, although if the maintenance occasion cost d is low or the standard deviations of the component lives are high, using non-opportunistic maintenance yields lower costs on average. To improve these results, a stochastic programming model for the current problem that take the uncertainty explicitly into account is developed in Paper II and Paper III.

2.2 Paper II: Models and complexity analysis of the opportunistic replacement problem with individual component lives

The purpose of this paper is to study an extension of the ORP (studied in Paper I) which allows different lives for different individuals of the same component. This

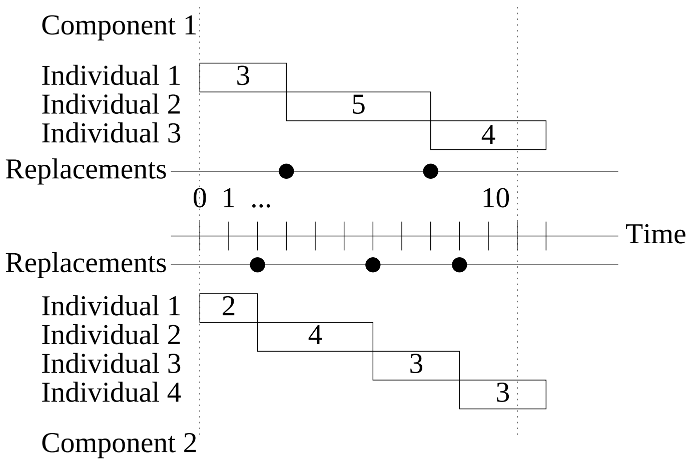


Figure 4: An illustration of a non-opportunistic maintenance schedule for the ORPIL defined in Example 4.

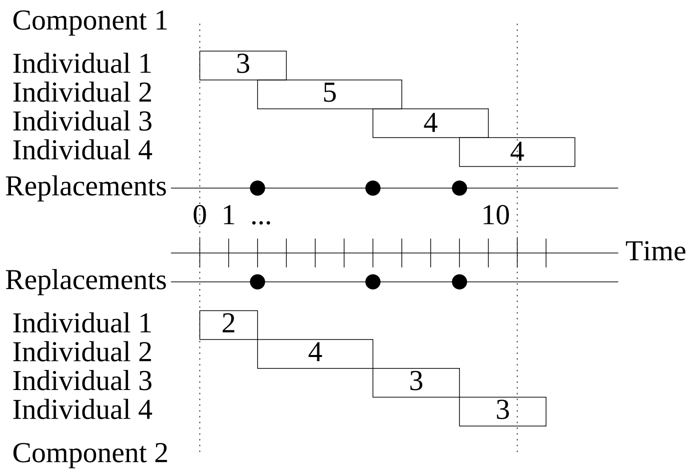


Figure 5: An illustration of an opportunistic maintenance schedule for the ORPIL defined in Example 4.

problem is called the opportunistic replacement problem with individual lives (ORPIL). The motivation for studying the ORPIL is that solving a SORP (an ORP with uncertain component lives, see Section 1.3) with perfect information (see Section 1.2) about individual component lives results in solving an ORPIL. Furthermore, as shown in Paper III, a model for the ORPIL is the basis for a model of the current problem for the SORP. We start by presenting an example.

Example 4 (ORPIL)**.** *Consider a system consisting of two components. The first failure of component* 1 *occurs at time* 3*, that is, the life of individual* 1 *of component* 1 *is* 3 *time steps. Individual* 1 *is replaced by individual* 2*, which fails after* 5 *time steps, and is then replaced by individual* 3*, which fails after* 4 *time steps. All individuals* r *with* $r \geq 3$ *have lives of* 4 *time steps. For component* 2*, individual* 1 *has a life of* 2 *time steps, individual* 2 *has a life of* 4 *time steps and individual* r *such that* $r \geq 3$ *has a life of* 3 *time steps. Let* c_1 *and* c_2 *denote the replacement cost of components* 1 *and* 2 *respectively, and let* d *denote the maintenance occasion cost. We wish to find a minimum cost maintenance schedule over the period defined by the time steps* $0, \ldots, 10$*.*

Figure 4 illustrates a non-opportunistic maintenance schedule for the system. The maintenance cost is $2c_1 + 3c_2 + 5d$*. Figure 5 illustrates the most opportunistic maintenance schedule for the system: at failure of one component both components are replaced. The maintenance cost is* $3c_1 + 3c_2 + 3d$*. If* $2d \geq c_1$*, this opportunistic schedule is optimal. If* $2d < c_1$ *a schedule which is similar to the non-opportunistic schedule in Figure 4, but in which the first replacement of component* 1 *is made at time* 2*, is optimal.*

We now present the general definition of the ORPIL. In order to obtain problems that are less computationally difficult to solve, we only allow the first q individuals non-identical lives. Let again $\mathcal{N} = \{1, \ldots, n\}$ be the set of components and $\mathcal{T} = \{0, \ldots, T\}$ the time period[3]. To simplify the presentation, we consider a problem with time independent costs, i.e. $c_{it} = c_i$ and $d_t = d$ for all $i \in \mathcal{N}$ and $t \in \mathcal{T}$.

Definition 2 (opportunistic replacement problem with individual component lives (ORPIL))**.** *Let* d *be the fixed cost for a maintenance occasion,* c_i *the cost for replacing a component* $i \in \mathcal{N}$*, and* T_{ir} *the life of individual* $r \in \mathbb{N}$ *of component* $i \in \mathcal{N}$*, and assume that* $T_{ir} = T_i$ *for* $r > q$*. Find a maintenance schedule over the time period defined by* $\mathcal{T}$ *that minimizes the maintenance cost, and such that each individual* $r \in \mathbb{N}$ *of component* $i \in \mathcal{N}$ *is used in the system no more than* T_{ir} *time steps.*

In Example 4 we have $q = 2$, $T_{11} = 3$, $T_{12} = 5$, and $T_1 = 4$; $T_{21} = 2$, $T_{22} = 4$, and $T_2 = 3$. Note that for $q = 0$ the ORPIL reduces to the ORP with time independent costs and for $q = T$ we obtain a problem in which all individuals may posses non-identical lives. The ORPIL problem for the cases $q = 1$ and $q = T$ was briefly studied in [1].

A major contribution of Paper II is the result that the ORPIL is NP-hard by reduction from the *vertex cover* problem. This problem reduction and the problem reduction performed in Paper I utilize different properties of the problems analyzed.

[3]We include time 0, as the model is intended for solving the current problem and hence a failure of one of the components at time 0 may enforce the replacement of such a component.

Here, we utilize the property that lives of the first two individuals may differ from the remaining individuals. In Paper I, we utilize the property that the costs of the component replacement may be time dependent. Hence the complexity of the ORPIL with $q \in \{0, 1\}$ can not be determined by the analysis in Paper I and Paper II.

In the paper, we introduce two ILP models for the ORPIL — model I and model II — and show that in model I the integer requirements on most of the variables may be relaxed. We also show that in model I all the non-superfluous constraints define facets of the convex hull of the feasible points of the model. Furthermore, we demonstrate that relaxing the integer requirements in model II and Andréasson's model (the model initially studied in [1]) results in fractional optimal solutions. Numerical studies confirm that the solution time of model I is significantly shorter than that of model II and Andréasson's model.

2.3 Paper III: The stochastic opportunistic replacement problem: a two-stage solution approach

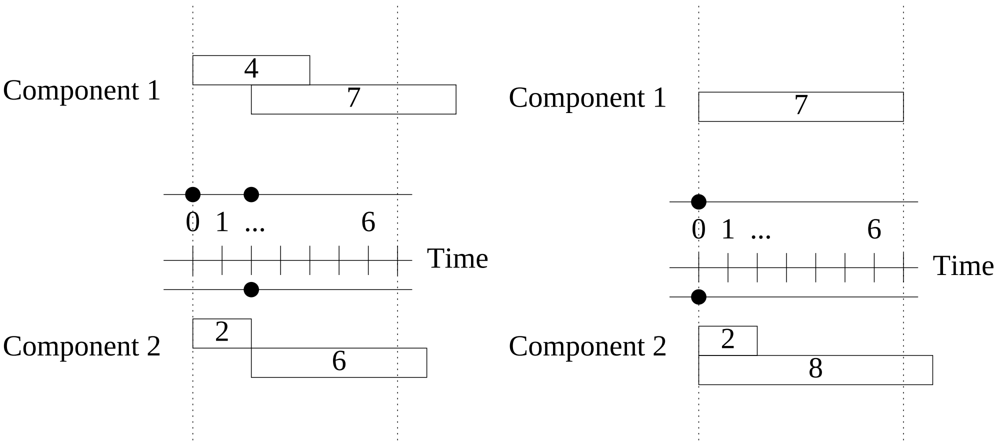
In this paper we develop and study a two-stage stochastic programming approach for the current problem of the SORP, that is, the ORP with uncertain component lives. The SORP is common in applications since future failure times of components are seldom known. We can, however, estimate failure distributions from historical data.

As noted in Section 1.3, for systems with stochastic component lives we can not create a maintenance schedule for the entire planning period. Instead, we wish to solve the current problem. Given is the system consisting of the components $\mathcal{N} = \{1, \ldots, n\}$, with replacement costs c_i for each $i \in \mathcal{N}$. Let v denote the state of the system at the current time, which contains the age of each component and the remaining planning period. Let $\xi_i = 1$ if component $i \in \mathcal{N}$ has failed and $\xi_i = 0$ otherwise, and $x_i = 1$ if we decide to replace component $i \in \mathcal{N}$ at the current time and $x_i = 0$ otherwise. The current problem is formally to

$$\begin{array}{ll} \text{minimize} & c^T x + \mathcal{Q}_v(x) \\ \text{subject to} & x_i \geq \xi_i, \qquad i \in \mathcal{N}, \\ & x_i \in \{0, 1\}, i \in \mathcal{N}, \end{array}$$

where $\mathcal{Q}_v : \mathbb{B}^{|\mathcal{N}|} \to \mathbb{R}$ is the recourse function (see Section 1.2) such that $\mathcal{Q}_v(x)$ gives the minimal expected future maintenance cost over the remaining planning period given the current decision x.

The difficulty in solving the current problem stems from the fact that the recourse function is hard to evaluate. Consider the discretized current problem of a system with components that have failure rates which increase with the components' age. In Paper III we show that an evaluation of the recourse function for such a problem, given a replacement decision at the current time, provides a lower bound on the recourse function of every other replacement decision. The bounds can be interpreted as that by replacing less components we can not lower the recourse function value, and by replacing an additional component j, we can at most lower the recourse



(a) scenario 1 (b) scenario 2

Figure 6: Illustration of optimal replacement schedules for the ORPIL scenarios in Example 6.

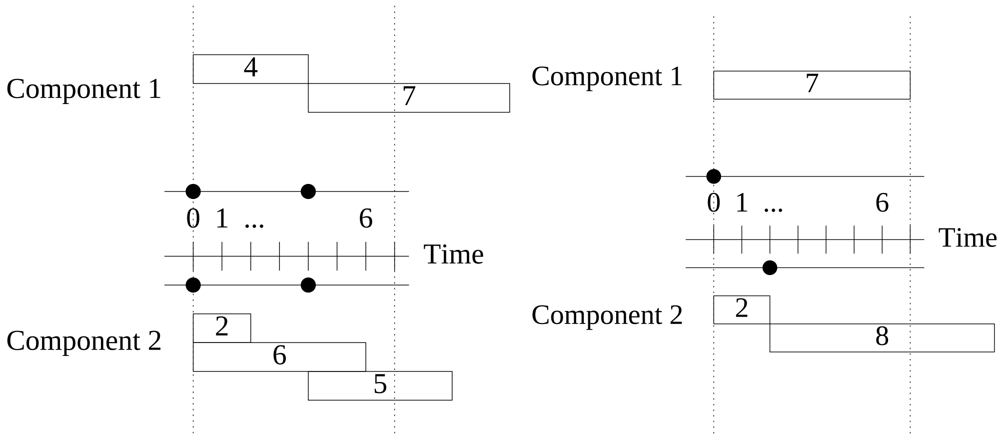
function value by $c_j + d$. Example 5 illustrates the bounds on the current problem from Example 2 in Section 1.3.

Example 5 (SORP cont.). *Assume that the recourse function value for replacing both components at the current time is $\alpha = \mathcal{Q}(1,1)$. Then the recourse function value of only replacing component 1 is bounded from below by $\mathcal{Q}(1,0) \geq \alpha$. If, on the other hand, the recourse function value of replacing only component 1 is $\beta = \mathcal{Q}(1,0)$, then then the recourse function value of replacing both components is bounded from below by $\mathcal{Q}(1,1) \geq \beta - d - c_2$.*

To solve the current problem, we generate a large number of scenarios for the component lives. However, we can not solve each scenario individually, since there must be only one decision at the current time; this decision is common for all the scenarios. The following example illustrates that the solution of ORPIL:s corresponding to two different scenarios yield different suggestions for the current decision.

Example 6 (SORP cont.). *Consider two following possible scenarios for the current problem. Using notation from Section 2.2 we can describe the scenarios by the life of each individual of every component. In scenario 1, we have $T_{11} = 4$, $T_{12} = 7$, T_1=5, $T_{21} = 2$, $T_{22} = 6$, $T_2 = 4$. In scenario 2, we have $T_{11} = 7$, $T_{12} = 6$, $T_1 = 5$ $T_{21} = 2$, $T_{22} = 8$, $T_2 = 4$. In both scenarios, $T = 6$. Figure 6 illustrates the optimal maintenance schedules for the two scenarios (obtained by solving the ORPIL). The maintenance cost in scenarios 1 and 2 becomes $2c_1 + c_2 + 2d$ and $c_1 + c_2 + d$ respectively. For scenario 1, the optimal current decision is to replace component 1 only, whereas in scenario 2 both components should be replaced.*

In order to impose a common current decision for the two different scenarios, we formulate one ORPIL for each scenario and force the decisions at time 0 (i.e., the



(a) scenario 1 - replace both components at time 0 (b) scenario 2 - replace component 1 only at time 0

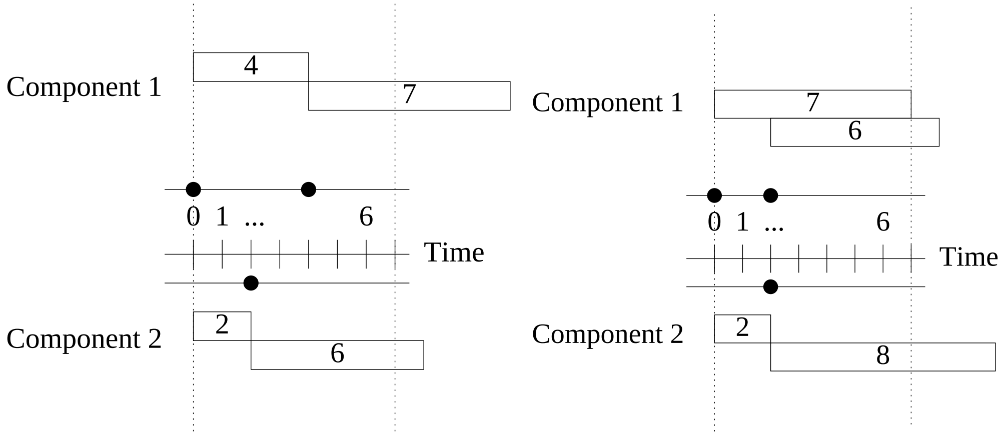
Figure 7: Illustration of the optimal second stage decisions for scenario 1 and 2 if we replace both components or component one only respectively.

current time) to be equal (i.e., we impose non-anticipativity at time 0). The decisions at time 0 constitute the first stage variables, and all other decisions constitute the second stage variables.

Example 7 (SORP cont.). *After imposing non-anticipativity at time* 0*, we can either replace component* 1 *only (Figures 6(a) and 7(b)) or both components (Figures 7(a) and 6(b)) in both scenarios. Let* $p(\omega)$ *denote the probability of scenario* $\omega = 1, 2$*. The minimal expected cost in the two-stage model is* $p(1)(2d + 2c_1 + c_2) + p(2)(2d + c_1 + c_2)$ *if we replace component* 1 *only, and* $p(1)(2d + 2c_1 + 2c_2) + p(2)(d + c_1 + c_2)$ *if we replace both components.*

In paper III we present a two-stage deterministic equivalent model and a decomposition method for the discretized current problem. The deterministic equivalent is based on the ORPIL model. The decomposition is based on the lower bounds on the recourse function and the sub problems are instances of the ORPIL.

Numerical experiments on problems from the aviation engine and wind power industries are performed (the applications formulated in Paper I) and on two smaller test problems. By using the stochastic programming approach for solving the current problem a lower average total maintenance cost is obtained compared to the use of simple policies or the expected value approach (used in Paper I). The experiments also show that the decomposition method requires a shorter solution time compared to solving the deterministic equivalent on three out of four problems considered. The decomposition method reduces the solution time by up to 80 % and is most efficient on problems requiring a long solution time, whereas the deterministic equivalent is most efficient on problems with a short solution time.



(a) scenario 1 - replace component 2 (b) scenario 2 - replace both components

Figure 8: Illustration of the optimal multistage solutions for scenario 1 and 2 if we replace component 2 only or both components respectively.

3 Additional contributions

This section contains work on the ORP and the SORP which was not included in the appended papers. Since it contains new research, the style of this section is more technical than that of Section 2.

3.1 A multistage stochastic opportunistic replacement model

In Paper III we present a two-stage model for the current problem of the SORP. The current problem is, however, a multistage problem since each time step should be considered as a stage. We continue Example 7 in Section 2.3.

Example 8 (SORP cont.)**.** *Assume that we decide to replace only component 1 at time 0 (that is, we fix a first stage decision). The optimal two-stage solution is then shown i Figures 6(a) and 7(b). Note that the decision at time 2 differs between scenarios 1 and 2. If we consider the state of the system at time 2, in both scenarios component 2 has failed and component 1 is working and has the age 2. The decision to replace both components in scenario 1 but component 2 only in scenario 2 is based on the future lives of the components and not on the state of the system, and thus violates non-anticipativity. In a multistage model, we either replace both components or component 2 only in both scenarios at time 2.*

If we replace component 2 only at time 2 (as shown in Figures 8(a) and 7(b)), the component will fail at time 4 in scenario 1 but continue to function in scenario 2. We may, thereafter, take different decisions in the two scenarios. If, on the other hand, we replace both components at time 2 (as shown in Figures 6(a) and 8(b)), the decisions in both scenarios must be equal during the whole planning period.

In [1, Sec 5.7] Andréasson writes that "every attempt to formulate a stochastic multistage model resulted in a non-linear model". As illustrated in Example 8, the difficulty in constructing a multistage model for the current problem lies in the fact that the outcome of the uncertainty (i.e., whether a failure occurs or not) depends on the decisions. In standard multistage problems, decisions made in two scenarios must be equal up to some time t, after which the decisions in the scenarios may differ. This time t is known a priori. We may thus construct equality constraints to handle the non-anticipativity. Here, the non-anticipativity constraints at time $t \in \mathcal{T}$ depend on maintenance decisions made earlier than time t. Stochastic programming models for problems with decision dependent uncertainty are not very well studied. The model presented here is based on an approach similar to that in [11].

Before presenting the multistage deterministic equivalent model, we need to introduce some notation. As in Paper III, in order to reduce the computational effort, scenarios contain non-identical lives for component individuals $r \in \mathcal{R} = \{1, \ldots, q\}$. Component individuals $q+1, q+1, \ldots$ are assigned the expected component lives. We define a scenario ω for the current problem as an instance of the ORPIL, i.e., by assigning values to T_{ir}^{ω} for all $r \in \mathcal{R}$ and $i \in \mathcal{N}$, and to T_i for all $i \in \mathcal{N}$. Let Ω be the probability space of all scenarios, $p(\omega)$ the probability of scenario ω, and $\xi_i = 1$ if component $i \in \mathcal{N}$ has failed at the current time and $\xi_i = 0$ otherwise. Observe that the decisions in two scenarios $\omega_1, \omega_2 \in \Omega$ of the current problem are allowed to be non-identical at time $t \in \mathcal{T}$ if, at a time $s \leq t$, a failure of an individual r of component i has occurred in scenario ω_1 and the same individual has *not* failed in scenario ω_2 (because then the scenarios can be distinguished). Define the following set of triples

$$S_{\omega_1 \omega_2} = \left\{ (\omega_-, i, r) \in \{\omega_1, \omega_2\} \times \mathcal{N} \times \mathcal{R} \mid T_{ir}^{\omega_-} < T_{ir}^{\omega_+} \text{ for } \omega_+ \in \{\omega_1, \omega_2\} \backslash \{\omega_-\} \right\}.$$

Consider a pair of scenarios $(\omega_1, \omega_2) \in \Omega \times \Omega$. If $(\omega_-, i, r) \in S_{\omega_1, \omega_2}$, then, for each time $s \in \mathcal{T}$, a failure of individual r of component i in scenario ω_- at time s implies that the scenarios ω_1 and ω_2 are possible to distinguish between at times $t \geq s$.

We are now ready to present the multistage model. Define the variables

$$\tilde{x}_{it}^{r\omega} = \begin{cases} 1, & \text{if individual } r \text{ of component } i \text{ in scenario } \omega \\ & \text{shall be replaced at or before time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad i \in \mathcal{N}, t \in \mathcal{T}, r \in \mathcal{R}, \omega \in \Omega,$$

$$v_{it}^{r\omega} = \begin{cases} 1, & \text{if individual } r \text{ of component } i \text{ in scenario } \omega \\ & \text{fails at or before time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad i \in \mathcal{N}, t \in \mathcal{T}, r \in \mathcal{R}, \omega \in \Omega,$$

$$x_{it}^{\omega} = \begin{cases} 1, & \text{if an identical life individual of component } i \\ & \text{in scenario } \omega \text{ shall be replaced at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad i \in \mathcal{N}, t \in \mathcal{T}, \omega \in \Omega,$$

$$z_{t}^{\omega} = \begin{cases} 1, & \text{if maintenance shall occur at time } t \text{ in scenario } \omega, \\ 0, & \text{otherwise,} \end{cases} \qquad t \in \mathcal{T}, \omega \in \Omega.$$

The model is then to

$$\text{minimize} \quad \sum_{\omega \in \Omega} p(\omega) \left(\sum_{i \in \mathcal{N}} \left(\sum_{r \in \mathcal{R}} c_i \tilde{x}_{iT}^{r\omega} + \sum_{t \in \mathcal{T}} c_i x_{it}^{\omega} \right) + \sum_{t \in \mathcal{T}} d z_t^{\omega} \right), \tag{4a}$$

$$\text{subject to} \quad \tilde{x}_{it}^{r\omega} \leq \tilde{x}_{i,t+1}^{r\omega}, \qquad i \in \mathcal{N}, t \in \mathcal{T} \setminus \{T\}, \; r \in \mathcal{R}, \omega \in \Omega, \tag{4b}$$

$$\tilde{x}_{i,t+1}^{r+1,\omega} \leq \tilde{x}_{it}^{r\omega}, \qquad i \in \mathcal{N}, t \in \mathcal{T} \setminus \{T\}, \; r \in \mathcal{R} \setminus \{q\}, \omega \in \Omega, \tag{4c}$$

$$x_{it}^{\omega} + \sum_{r \in \mathcal{R}} \tilde{x}_{it}^{r\omega} - \tilde{x}_{i,t-1}^{r\omega} \leq z_t^{\omega}, \qquad i \in \mathcal{N}, t \in \mathcal{T} \setminus \{0\}, \; \omega \in \Omega, \tag{4d}$$

$$x_{i0}^{\omega} + \tilde{x}_{i0}^{1\omega} \leq z_0^{\omega}, \qquad i \in \mathcal{N}, \omega \in \Omega, \tag{4e}$$

$$\tilde{x}_{it}^{r\omega} \leq \tilde{x}_{i,t+T_{i,r+1}^{\omega}}^{r+1,\omega}, \qquad i \in \mathcal{N}, r \in \mathcal{R} \setminus \{q\}, \; \omega \in \Omega, \; t \in \{0, \ldots, T - T_{i,r+1}^{\omega}\}, \tag{4f}$$

$$\sum_{t=l+1}^{l+T_i} x_{it}^{\omega} \geq \tilde{x}_{il}^{q\omega}, \qquad i \in \mathcal{N}, \omega \in \Omega, \; l \in \{0, \ldots, T - T_i\}, \tag{4g}$$

$$x_{it}^{\omega} \leq \tilde{x}_{i,t-1}^{q\omega} \qquad i \in \mathcal{N}, t \in \mathcal{T} \setminus \{0\}, \; \omega \in \Omega, \tag{4h}$$

$$x_{i0}^{\omega} = 0, \qquad i \in \mathcal{N}, \omega \in \Omega, \tag{4i}$$

$$\tilde{x}_{iT_{i1}^{\omega}}^{1\omega} = 1, \qquad \omega \in \Omega, \; i \in \{j \in \mathcal{N} | T_{j1}^{\omega} \leq T\}, \tag{4j}$$

$$\tilde{x}_{i0}^{r\omega} = 0, \qquad i \in \mathcal{N}, r \in \mathcal{R}\backslash\{1\}, \omega \in \Omega, \tag{4k}$$

$$\sum_{s=0}^{t-1}(\tilde{x}_{is}^{r-1,\omega} - \tilde{x}_{is}^{r\omega}) \geq T_{ir}^{\omega} v_{it}^{r\omega}, \qquad i \in \mathcal{N}, t \in \mathcal{T}\backslash\{0\}, r \in \mathcal{R}\backslash\{1\}, \omega \in \Omega, \tag{4l}$$

$$\sum_{s=0}^{t-1}(1 - \tilde{x}_{is}^{1\omega}) \geq T_{i1}^{\omega} v_{it}^{1\omega}, \qquad i \in \mathcal{N}, t \in \mathcal{T}\backslash\{0\}, \omega \in \Omega, \tag{4m}$$

$$v_{i0}^{r\omega} = 0, \qquad i \in \mathcal{N}, r \in \mathcal{R}, \omega \in \Omega, \tag{4n}$$

$$\tilde{x}_{it}^{r\omega_1} - \tilde{x}_{it}^{r\omega_2} \leq \sum_{(\omega_-, \hat{i}, \hat{r}) \in S_{\omega_1,\omega_2}} v_{\hat{i}t}^{\hat{r}\omega_-}, \qquad \omega_1, \omega_2 \in \Omega, i \in \mathcal{N}, r \in \mathcal{R}, t \in \mathcal{T}, \tag{4o}$$

$$\tilde{x}_{i0}^{1\omega} \geq \xi_i, \qquad i \in \mathcal{N}, \omega \in \Omega, \tag{4p}$$

$$\tilde{x}_{it}^{r\omega} \in \{0,1\}, \qquad i \in \mathcal{N}, t \in \mathcal{T}, r \in \mathcal{R}, \omega \in \Omega, \tag{4q}$$

$$v_{it}^{r\omega} \in \{0,1\}, \qquad i \in \mathcal{N}, t \in \mathcal{T}, r \in \mathcal{R}, \omega \in \Omega, \tag{4r}$$

$$x_{it}^{\omega} \in \{0,1\}, \qquad i \in \mathcal{N}, t \in \mathcal{T}, \omega \in \Omega, \tag{4s}$$

$$z_t^{\omega} \in \{0,1\}, \qquad t \in \mathcal{T}, \omega \in \Omega. \tag{4t}$$

The constraints (4b)–(4k) originate from the ORPIL model (model I in Paper II). The constraints (4l) and (4m) assure that if the age of the individual r and 1, respectively, of component i in scenario ω is less than the its life, then the individual has not failed yet. The constraint (4n) states that no individual has failed at time 0. The constraint (4o) assures that two scenarios have equal decisions at time t, if no failure that can distinguish the scenarios has occurred. The constraint (4p) implies that components which are failed at the current time must be replaced.

The model presented is, unfortunately, computationally intractable already for a small number of components and a small number of scenarios. In order to solve the multistage problem for the current problem a different approach is necessary (such as a nested decomposition or dynamic programming formulation).

3.2 Assigning values to the components at the end of the planning period

In many applications, a value of the system state at the end of the planning period is included into the objective. The value may be a price obtained by selling the equipment or it may simply be included into the contract. In other applications, we wish to use a rolling horizon and thus prevent end-of-horizon effects. These effects refer to schedules which are optimal in the finite horizon setting, but result in systems with many components close to failure at the end of the planning period. For instance, adding the value $\frac{a-T_i}{T_i}c_i$ for each component $i \in \mathcal{N}$, where a is the age of the component individual in the system at the end of the planning period and T_i is the component life, into the objective counteracts end-of-horizon effects.

The ORP model (paper I) can comprise a value $0 \leq v_{ia} \leq c_i$ of component $i \in \mathcal{N}$ of age $a \in \{1, \ldots, T_i\}$ at the end of the planning period into the objective function. Consider an instance of the ORP, and observe that if $c_{it} > 0$ for all $t \in \mathcal{T}$ the optimal maintenance schedule contains at most one replacement of component $i \in \mathcal{N}$ in the time period defined by the time steps $T-T_i+1, \ldots, T$. This replacement thus decides the age of the component i at time $T+1$, that is, at the end of the planning period. The model that includes the value of the component at the end of the planning period into the objective is thus given by

$$\begin{aligned}
\text{minimize} \quad & \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} x_{it}c_{it} + \sum_{t\in\mathcal{T}} d_t z_t - \sum_{i\in\mathcal{N}}\sum_{a=1}^{T_i} v_{ia}x_{i,T-a+1} \\
\text{subject to} \quad & \sum_{t=l+1}^{l+T_i} x_{it} \geq 1, \qquad i \in \mathcal{N}, l = 0, \ldots, T-T_i, \\
& x_{it} \leq z_t, \qquad i \in \mathcal{N}, t \in \mathcal{T}, \\
& x_{it} \in \{0,1\}, i \in \mathcal{N}, t \in \mathcal{T} \\
& z_t \in \{0,1\}, t \in \mathcal{T}.
\end{aligned}$$

Note that this is an ORP with a special cost structure.

4 Summary of the contributions and future research

The focus of this thesis is to study the opportunistic replacement problem (ORP) and extensions of this problem allowing uncertain component lives. The main contributions are the following. I have investigated the complexity of the ORP and shown that it is NP-hard (Paper I). I have also shown that the ORP with non-identical individual lives (ORPIL) and time independent costs is NP-hard (Paper II). Furthermore, I have developed mixed integer linear programming (MILP) models of the ORPIL which are both computationally and theoretically superior to the models introduced in [1] (Paper II). Finally, I have developed a decomposition method for the stochastic ORP (SORP), that is the ORP with uncertain component lives (Paper III). I have also suggested a multistage deterministic equivalent MILP model for the SORP, although the model is computationally intractable. The work resulting in this thesis has improved our understanding of the important properties of maintenance problems studied, and enables the solution of larger problem instances and the production of solutions of improved quality for problems with uncertain component lives.

There are several open paths for the future of this project. One path is to work closer with the industry, study more complex problems and extend the current results to these problems. The applications we currently consider originate from the aviation industry, energy production industry, and the railway sector. Examples of extensions are to include production or staff planning into the problem, systems with redundancy, and systems with several levels of deterioration. A second path is to attempt to solve the multistage SORP by a nested decomposition or dynamic programming technique. This would enable maintenance decisions which additionally reduce the expected maintenance cost. A third possibility is to investigate the facial structure of the ORP and utilize this in a branch-and-cut algorithm in order to be able to solve the ORP more efficiently. These research paths may, of course, also be combined. The goal of the listed paths is to continue the work towards the computational solution of larger and more complex maintenance problems.

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