Thesis for The Degree of Licentiate of Engineering

Behavioral Modeling of Radio Frequency Transmitters

Ali Soltani Tehrani

Communication Systems and Information Theory Group
Department of Signals and Systems
Chalmers University of Technology
Göteborg, Sweden, 2009

Behavioral Modeling of
Radio Frequency Transmitters

Ali Soltani Tehrani

Copyright ©2009 Ali Soltani Tehrani
except where otherwise stated.
All rights reserved.

Technical Report No R017/2009
ISSN 1403-266X
Department of Signals and Systems
Communication Systems and Information Theory Group
Chalmers University of Technology
Göteborg, Sweden

This thesis has been prepared using LaTeX.
Printed by Chalmers Reproservice,
Göteborg, Sweden 2009.

To Haideh,
Bahram,
Farideh,
and Sahar

Abstract

The dependence of modern wireless communication systems on the fidelity of the transmitter has increased the importance of modeling these components. Behavioral modeling is a commonly used technique to find the input-output relationship of a system, without the need for the knowledge of the specific components. There have been many models proposed in the literature, which has made it difficult to choose models suitable for the application at hand. By analyzing and categorizing these models depending on the type of distortions they can describe, it becomes possible to understand their distortion handling capabilities. Such an analysis led to the design of two new behavioral models, one customized for power amplifiers and the other for modulators.

Once different models are analyzed, their usefulness is determined in an experimental setup. Different models have different modeling capabilities. While some may be able to model any nonlinear function with high accuracy, others may be capable of modeling at low computational cost. The accuracy/complexity tradeoff for some commonly-used behavioral models is analyzed in this thesis.

In **Paper [A]**, a new behavioral model is proposed for power amplifiers that combines two commonly used modeling techniques. The performance of this model is shown to be better than the two models it is based on, and issues in identification are also discussed.

In **Paper [B]**, a new dual-input model is constructed for modulators. This model has the ability to describe nonlinear imbalance in a modulator. Different variants of this model are also proposed, to reduce the computational complexity.

In **Paper [C]**, a detailed analysis on the accuracy/complexity tradeoff for some commonly-used power amplifier behavioral models is presented. After finding the computational complexity in floating point operations per sample, an experimental setup is used to show that among models studied, the generalized memory polynomial model has the best accuracy/complexity tradeoff.

Keywords

Behavioral modeling, computational complexity, digital predistortion, I/Q imbalance, nonlinear models, power amplifier, transmitter, Volterra series, wireless communications

Acknowledgment

Before diving into the (interesting) technical discussions, I would like to thank many people. A list of people I would like to say thanks to is given here, please bear in mind that this list is not complete, and if I have missed your name you can be sure you were in my thoughts.

First, I would like to thank my supervisors, Associate Prof. Thomas Eriksson and Dr. Christian Fager. I would never have had the opportunity to take on this journey if it wasn't for you. Your knowledge and support has been invaluable. We have had many interesting discussions, that have helped me through rough times.

I would like to give thanks to Profs. Erik Ström and Herbert Zirath, for providing a pleasant working environment in the Communication Systems group and the Microwave Electronics Laboratory. I would also like to thank Prof. Jan Grahn for his tireless efforts in the GigaHertz centre.

I would like to give special thanks to my friends and colleagues. You have made this experience so much more easier. Thanks to Haiying Cao, for helping me find ways to solve problems, and for the many helpful discussions. Thanks to Hossein Mashad Nemati, for helping me learn about the hardware and support in doing measurements. Thanks to Arash Tahmasebi, for helping me fit in. Thanks to Ulf Gustavsson, for always having an exciting research topic to discuss with. Thanks to Guillermo García, for always listening to what I have to say and encouraging me. Everyone at ComSys, thanks.

On the administrative side, I would like to send my gratitude to Agneta Kinnander, Madeline Persson, and Catharina Forssén. I would like to thank Lars Börjesson, Anders Olausson and Jan Andersson for the computer support.

I would also like to say thanks to the Iranian people and the green movement. You have given me much needed hope for the future of my country.

Finally, I would like to say thanks to the most important people in my life, my family. Special thanks to my parents for supporting me through everything. To my mother, you are always in my heart. To my father, thanks for guiding me through so many obstacles and always being there. Thanks to my brothers, you guys rock! And most importantly, I would like to thank my wife, you are my anchor. I would have nothing without you.

****

This research has been carried out in GigaHertz Centre in a joint project financed by the Swedish Governmental Agency for Innovation Systems (VINNOVA), Chalmers University of Technology, Ericsson AB, Infineon Technologies Austria AG, and NXP Semiconductors BV.

List of Publications

Appended papers

This thesis is based on the following papers:

[A] A. Soltani Tehrani, H. Cao, T. Eriksson and C. Fager "Orthonormal-basis power amplifier model reduction,"
Proc. of Integrated Nonlinear Microwave Circuits Workshop, pp. 39-42, Nov. 2008.

[B] H. Cao, A. Soltani Tehrani, C. Fager, T. Eriksson and H. Zirath "Dual-input nonlinear modeling for I/Q modulator distortion,"
Proc. of IEEE Radio and Wireless Symposium, pp. 39-42, Jan. 2009.

[C] A. Soltani Tehrani, H. Cao, S. Afsardoost, M. Isaksson, T. Eriksson and C. Fager "A comparative analysis of the complexity/accuracy tradeoff in power amplifier behavioral models,"
Submitted to *IEEE Trans. on Microwave Theory and Techniques*, 2009.

Other papers

The following papers are published but not appended to the thesis, either due to contents overlapping that of appended papers, or due to contents not related to the thesis.

[a] A. Soltani Tehrani, H. M. Nemati, H. Cao, T. Eriksson and C. Fager "Dynamic load modulation of high power amplifiers with varactor-based matching networks,"
Proc. of IEEE MTT-S International Microwave Symposium Digest, pp. 1537-1540, Jun. 2009.

[b] A. Soltani Tehrani, H. Cao, T. Eriksson and C. Fager "Comparative analysis of the complexity/accuracy tradeoff for power amplifier behavior models,"
GigaHertz Symposium, Mar. 2008.

[c] H. Cao, A. Soltani Tehrani, C. Fager, T. Eriksson and H. Zirath "I/Q imbalance compensation using a nonlinear modeling approach,"
IEEE Trans. on Microwave Theory and Techniques, vol. 57, no. 3, pp. 513-518, Mar. 2009.

[d] H. Cao, H. M. Nemati, A. Soltani Tehrani, T. Eriksson, J. Grahn and C. Fager "Linearization of efficiency optimized dynamic load modulation transmitter architectures,"
Submitted to *IEEE Trans. on Microwave Theory and Techniques*, 2009.

[e] H. Cao, A. Soltani Tehrani, H. M. Nemati, C. Fager, T. Eriksson and H. Zirath "Time alignment in a dynamic load modulation transmitter architecture,"
In Proc. of 39^{th} *European Microwave Conference* , pp. 1211-1214, Oct. 2009.

[f] H. Cao, A. Soltani Tehrani, H. M. Nemati, T. Eriksson and C. Fager "Time mismatch effects in a dymanic load modulation transmitter architecture,"
RF Measurement Technology for State of the Art Production and Design conference, Oct. 2009.

[g] H. Cao, A. Soltani Tehrani, C. Fager, T. Eriksson and H. Zirath "Dual-input nonlinear modeling for I/Q modulator distortion,"
Proc. of Integrated Nonlinear Microwave Circuits Workshop, pp. 131-134, Nov. 2008.

[h] H. Cao, A. Soltani Tehrani, C. Fager, T. Eriksson and H. Zirath "Identification of Distortions in a RF Measurement System,"
GigaHertz Symposium, Mar. 2008.

[i] T. Eriksson, C. Fager, H. Cao, A. Soltani Tehrani, H. M. Nemati and H. Zirath "Modeling of dual-input power amplifiers,"
GigaHertz Symposium, Mar. 2008.

Contents

Chapter 1

Introduction

The general trend for modern wireless data communication systems has been the need for speed. Communication data rate has constantly increased, from 2.4 Kbits per second for Advanced Mobile Phone System (AMPS), to the objective of a nominal data rate of 1 Gbit/s for relatively stationary nodes in the International Mobile Telecommunications-Advanced (IMT-Advanced) standard. This progress has been hampered greatly by the limited frequency resources allocated to such systems, which has in turn increased the importance of spectrum utilization. Coupled with the other major trend in radio frequency (RF) communication – the increase in number of users – it has forced modern digital wireless systems to adopt more complex modulations schemes with varying amplitude. Such signals depend heavily on the fidelity of the transmitter hardware, i.e. any slight distortion created might render the system impractical to use. This thesis attempts to address the latter issue by means of modeling the transmitter, as a necessary basis for any distortion cancelation technique. Such models may also be of interest when evaluating the performance of wireless commination systems in system level simulations.

1.1 Modeling RF transmitters

The RF transmitter architecture has historically been developed in two separate tracks. Communication system engineers have generally been focused on mapping the information into symbols, while RF engineers have been realizing these symbols in hardware. The first group is mainly concerned with the data rate and quality of service, while the second group worries about practical issues in constructing the hardware. As signal processing power increases and costs decrease, these two tracks have begun to somewhat merge. The use of signal processing tools to alleviate hardware problems has become more common, and will be necessary for future generation systems. A block diagram of such modern transmitters can be shown in Fig. 1.1.

From Fig. 1.1 it can be noticed that the modulator is associated with the analog circuitry that produces a real-valued passband modulated signal from the complex-valued baseband signal. The dashed box in the figure represents the entire act of modulating a bit stream into a waveform suitable for the RF channel, as defined by Shannon [1].

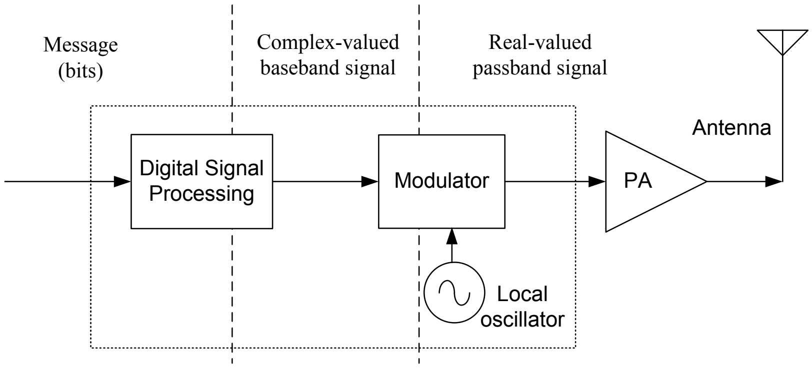

Figure 1.1: Block diagram of a simplified modern transmitter architecture.

Many different approaches can be taken to model the transmitter. Depending on the type of data needed for identification, models can be divided into two main groups: physical and circuit models, and empirical models [2,3]. Physical models give an accurate description of a device based on fundamental physical laws [4]. In circuit models, electrical circuit elements and circuit theory are used to model the system. Such techniques have high precision limited only to the quality of the device models. This precision has a high price in simulation time, limiting the use of these type of models in practice for modeling wireless systems.

Empirical models attempt to model the system with no *a priori* knowledge of the internal circuitry of the devices. They are commonly called *behavioral models*, or *black-box models*, and are constructed from the sampled measured input and output signals. These models typically are used for applications like digital predistortion and system level modeling [2]. The accuracy of these models depend heavily on the model structure and parameter identification procedure. Many different behavioral models have been proposed and developed by researchers [2, 4–6]. By comparing model performance, identifying efficient model structures becomes possible. Such an analysis can help in choosing a suitable model structure for simulations and predistortion, or by providing a platform to construct novel behavioral models. Therefore, the thesis focuses on analyzing transmitter behavioral models and comparing their performance.

1.2 Distortion in RF transmitters

The transmitter architecture shown in Figure 1.1 has three main sources of distortion: the modulator, the power amplifier and the antenna. The thesis is focused on the distortions created by the modulator and PA as the two dominant sources of distortion in the transmitter architecture [7].

The distortion created by the analog front end of modulators in amplitude and phase is commonly referred to as in-phase/quadrature imbalance – IQ imbalance [8]. These impairments are generally created in the conversion of the complex-valued baseband signal to the real-valued analog signal, and can result in both linear and nonlinear distortion. When dealing with these distortions,

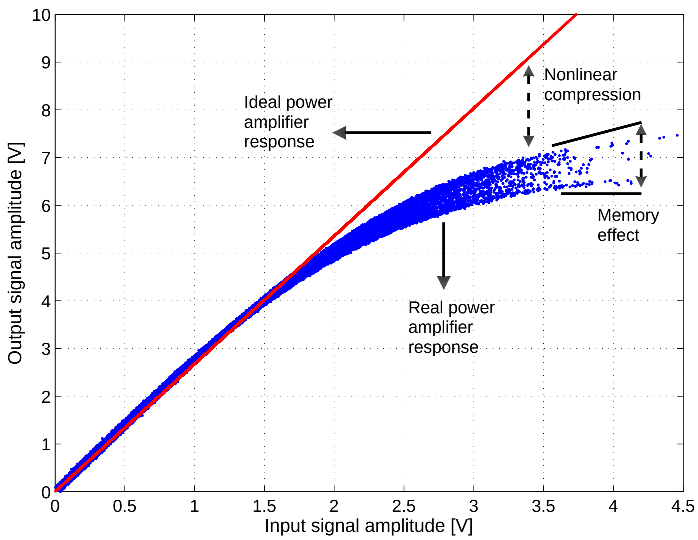

Figure 1.2: Input-output characteristics of an ideal and a practical power amplifier.

special care has to be taken into consideration regarding the nonlinear order terms, as discussed in section 2.1.1.

Power amplifiers are one of the main power consuming devices in the transmitter architecture [9], and have a severe impact on the communication system. Power amplifiers are ideally designed to linearly amplify the communication signal to the required output power level to overcome channel losses. In practice, these devices tend to have nonlinear behaviors, especially when driven close to saturation. As wireless standards adopt spectrally-efficient amplitude-varying modulated signals, the requirements on power amplifier linearity becomes more stringent. A typical input-output amplitude characteristic for a power amplifier is shown in Figure 1.2.

From Figure 1.2, two of the main distorting effects of the power amplifier can be noticed. The ideal input-output amplitude relationship should be a linear line, as seen in the figure. However, in a practical amplifier, as the amplitude increases, the output becomes saturated and the power amplifier gain diminishes. This is the dominant effect behind what is known as the power amplifier *nonlinear distortion*. For communication signals in standards such as Global System for Mobile communications (GSM), where the amplitude is constant and has no information, this is not an important factor, i.e., the gain will be constant for a constant amplitude. This does not hold for signals that have varying amplitude, and the amplification of the communication samples will no longer be constant. This will result in a loss of information and hence, an increase in bit-error-rate (BER), if not dealt with properly.

Another power amplifier distortion that is visible from Figure 1.2, is what is known as the power amplifier *memory effect*. The output of a power amplifier not only depends on the communication sample at time t, but also on samples

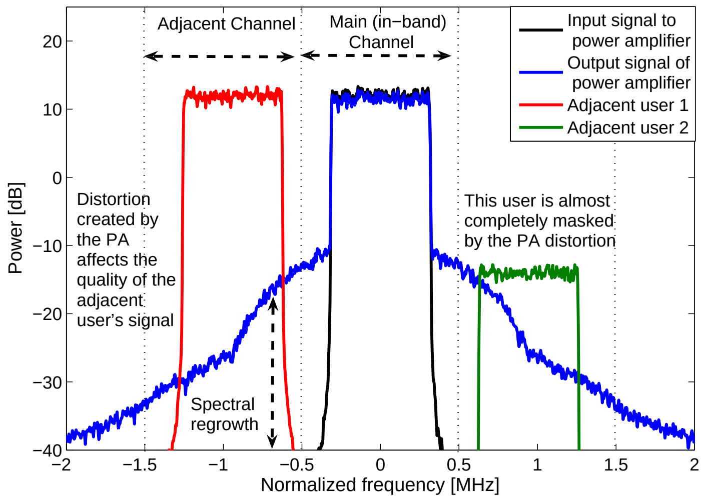

Figure 1.3: Adverse effects of power amplifier distortion on adjacent users.

that have passed before this time. Hence, the input-output relationship is no longer a one-to-one function, and the same input sample may result in a range of output samples depending on the signal history. This is mainly due to electrical and thermal dispersion effects [10], and shows itself in the figure as the blurring when the amplitude increases.

The adverse effects of power amplifier distortion mentioned were seen in the time domain in Figure 1.2. These effects will distort the signal constellation and introduce noise, before the signal even leaves the transmitter. The unwanted noise will then result in a loss of signal-to-noise ratio (SNR) and an increase in the error vector magnitude (EVM) at the receiver. However, these distortions also have an impact on the frequency response of the communication system, resulting in what is known as *spectral regrowth*. This corresponds to the spectral leakage of power into adjacent channels of the frequency spectrum. Figure 1.3 shows the effects of the distortion of a power amplifier in the frequency domain. It can be observed that spectral regrowth results in out-of-band leakage that may not satisfy the requirements set by frequency regularization organizations [11]. For example in this figure, the out-of-band distortion is so strong that it partially masks an adjacent user and distorts its communication.

In order to reduce and remove these transmitter impairments, identifying the distorting effects becomes critical. Therefore, behavioral modeling can help to identify these effects and in the design of digital predistortion techniques. Digital predistortion (DPD) is a linearization technique where the signal is passed through an expanding filter to reverse the compressing effect of the power amplifier. DPD has been shown to reduce the size and cost for linearization compared to other linearization methods [12], and has the added benefit of being independent of the operating frequency.

1.3 Thesis outline

The thesis is organized as follows. In Chapter 2, a theoretical mathematical background for behavioral modeling of transmitters is established. Considerations on the requirements of power amplifier behavioral model structure is explained. Some commonly-used power amplifier behavioral models are categorized, allowing for a better understanding of the differences between models. A novel power amplifier model based on combining techniques used in the literature is also presented. Behavioral modeling of modulators is briefly discussed and a dual-input model is presented. Finally, parameter estimation for behavioral models is analyzed and some important issues are reviewed. In Chapter 3, some important models are evaluated with respect to both accuracy and complexity. First, some common accuracy measures are defined and discussed. Then behavioral modeling complexity is presented. Finally a comparative analysis on the accuracy/complexity tradeoff for power amplifier behavioral models is analyzed. A summary of the appended paper is presented in Chapter 4 and future work is presented in Chapter 5.

Chapter 2

Behavioral Modeling of Transmitters

As discussed in the previous chapter, modeling the transmitter is an important pre-requisite for modern wireless communication system design. As the main source of distortion in the transmitter, power amplifiers have received considerable attention in the literature, and many power amplifier models have been proposed. The nonlinear models developed for power amplifiers can also be used to model modulators and in fact complete transmitters [13], with certain considerations. Therefore, while in this chapter the focus is mainly on power amplifier behavioral modeling, it can be applicable to modulator and transmitter modeling as well.

By classifying the models into groups, models can be differentiated based on the type of distortions they describe, which can help when choosing a model for an application. This can also help to better understand the important terms in a behavioral model, which can result in constructing new models that are better equipped to handle distortions, e.g., papers [A] and [B]. Therefore, an overview of models proposed in the literature is presented in this chapter, and important characteristics of each is briefly discussed.

As there are many models in the literature, it is a tedious task to list all. Therefore, in this work, some models which are representative of most models are chosen, and some of these models are analyzed further in Chapter 3. Before analyzing behavioral models, some necessary theoretical background is discussed, to layout a framework for the behavioral models. Finally, parameter identification in behavioral models is analyzed, and some important issues are discussed.

2.1 Theoretical background

Power amplifiers are *passband* devices, and discrete passband signals should be used for behavioral modeling and identification. However, by assuming that the input signal to the power amplifier is band-limited, computationally efficient techniques can be constructed to represent the power amplifier with discrete baseband models [14]. This greatly reduces the computational complexity, and is a necessary step for digital algorithms. This section focuses on the theoretical issues for constructing baseband behavioral models.

2.1.1 Baseband representation of passband signals

For a real-valued passband signal $\tilde{x}(t)$, the Fourier transform $\tilde{X}(f)$ shows a symmetry around zero frequency. Specifically, the real part of $\tilde{X}(f)$ is an even function and the imaginary part is odd. This means that it is possible to reconstruct $\tilde{x}(t)$ uniquely from $\tilde{X}(f)$ using only $f > 0$ [15]. The signal containing only positive frequencies is called the *analytical signal*, $x_{\text{analytical}}(t)$, and can be constructed by [16]

$$X_{\text{analytical}}(f) = 2u(f)\tilde{X}(f), \tag{2.1}$$

where $\tilde{X}(f)$ is the Fourier transform of $\tilde{x}(t)$ and $u(f)$ is the step function. The time-domain representation of the analytical function can be constructed by taking the inverse Fourier transform of (2.1) and can be written as

$$x_{\text{analytical}}(t) = \tilde{x}(t) + j\hat{x}(t), \tag{2.2}$$

where $\hat{x}(t)$ is called the *Hilbert transform* of $\tilde{x}(t)$ and is defined as

$$\hat{x}(t) \stackrel{\text{def}}{=} \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\tilde{x}(\theta)}{t-\theta} d\theta. \tag{2.3}$$

From (2.2), the signal $\tilde{x}(t)$ can be recovered by the output of a system with transfer function $2u(f)$ by simply taking the real value of the analytical signal [15],

$$\tilde{x}(t) = \Re\left[x_{\text{analytical}}(t)\right]. \tag{2.4}$$

In passband signals, when the relative bandwidth of a signal is smaller compared to the center frequency f_0 – a *narrowband signal* [17], the analytical signal can be written in the form [15]

$$x_{\text{analytical}}(t) = x(t)e^{j2\pi f_0 t}, \tag{2.5}$$

where f_0 is the center frequency and $x(t)$ is a (generally) complex signal called the *complex envelope* of $\tilde{x}(t)$. $x(t)$ has a spectrum that is concentrated around the origin of the frequency axis, commonly referred to as a baseband signal. Thus, a real-valued narrowband passband signal has been effectively represented by a complex-valued baseband signal.

Combining equations (2.4) and (2.5), the narrowband signal $\tilde{x}(t)$ can be written as [15]

$$\tilde{x}(t) = x_{\text{I}}(t)\cos(2\pi f_0 t) - x_{\text{Q}}(t)\sin(2\pi f_0 t), \tag{2.6}$$

where

$$x_{\text{I}}(t) \stackrel{\text{def}}{=} \Re[x(t)] = \tilde{x}(t)\cos(2\pi f_0 t) + \hat{x}(t)\sin(2\pi f_0 t), \tag{2.7}$$

and

$$x_{\text{Q}} \stackrel{\text{def}}{=} \Im[x(t)] = \hat{x}(t)\cos(2\pi f_0 t) - \tilde{x}(t)\sin(2\pi f_0 t), \tag{2.8}$$

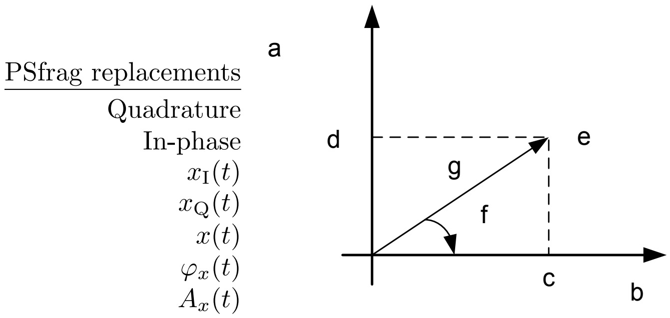

Figure 2.1: Vector representation of complex-baseband signal $\tilde{x}(t)$.

or alternatively [16]

$$\tilde{x}(t) = \frac{e^{j2\pi f_0 t} x(t) + e^{-j2\pi f_0 t} x^*(t)}{2}, \tag{2.9}$$

where $x^*(t)$ is the complex conjugate of $x(t)$. $x_{\mathrm{I}}(t)$ is commonly called the in-phase component of $x(t)$ and $x_{\mathrm{Q}}(t)$ the quadrature, and this representation is called *cartesian representation*.

Another common representation is the instantaneous amplitude $A_x(t)$ and the instantaneous phase $\varphi_x(t)$ defined as

$$A_x(t) \stackrel{\text{def}}{=} |\tilde{x}(t)| = \sqrt{x_{\mathrm{I}}(t)^2 + x_{\mathrm{Q}}(t)^2} \tag{2.10}$$

$$\varphi_x(t) \stackrel{\text{def}}{=} \arg\left[\tilde{x}(t)\right] = \tan^{-1} \frac{x_{\mathrm{Q}}(t)}{x_{\mathrm{I}}(t)}. \tag{2.11}$$

This representation is commonly called *polar representation*. The relationship between these representations of the narrowband signal is shown in Figure 2.1.

2.1.2 Baseband model structure

In the previous section the baseband representation for passband signals was presented. However, when dealing with behavioral models, it is possible to find certain characteristics in the baseband model structure. This section analyzes these properties.

In the passband, a memoryless power amplifier can be thought of as a mapping of a real-valued input signal to a real-valued output signal [18]. Approximating this nonlinearity by a power series – under a range of general conditions for the power amplifier like stability, continuity, fading memory, etc. [2] – the output can be written as

$$\tilde{y}(t) = \sum_{k=1}^{K} \tilde{b}_k \tilde{x}^k(t), \tag{2.12}$$

where $\tilde{x}(t)$ is the passband power amplifier input, $\tilde{b}_k$ are real-valued coefficients, and $\tilde{y}(t)$ is the passband output. From [17], since the output of a power amplifier is normally passed through a passband filter centered at $\pm f_0$, only terms that are centered around this frequency will contribute to the output signal. Therefore, (2.12) can be constructed in baseband form as [15][p.69]

$$y(t) = \sum_{\substack{k=1 \\ k \text{ odd}}}^{K} b_k x(t) \left| x \right|^{k-1}, \tag{2.13}$$

where

$$b_k = \frac{1}{2^{k-1}} \binom{k}{\frac{k-1}{2}} \tilde{b}_k. \tag{2.14}$$

Two important observations can be made from these equations. First, in (2.13), only odd order power terms exist, as the even order terms are far from the carrier frequency. Secondly, that in (2.14), since $\tilde{b}_k$ is real-valued, b_k are also real-valued. Therefore, only amplitude-amplitude distortions (AM/AM) are generated by a memoryless power amplifier. By allowing the $\tilde{b}_k$ to be complex-valued, quasi-memoryless models can be constructed, which can also account for some amplitude-phase distortions (AM/PM). These kinds of complex baseband power series form the basis for most of the power amplifier models presented in subsequent sections.

By considering the baseband representation, behavioral models can be considerably simplified and many parameters can be reduced. Therefore, all models presented in this thesis will be in baseband form and the reductions discussed in this section are applied.

Model structure categories

Categorizing power amplifier behavioral models can help in understanding their differences and in comparing their model structures. It can also help in choosing a model based on the type of distortions we are interested in. In wideband wireless systems, power amplifier distortion is mainly attributed to three types of physical phenomena, [19]

- Static and memoryless nonlinearities from device characteristics.
- Linear memory effects, which may be attributed to time delays, or phase shifts, in the matching networks and the device and circuit elements used.
- Nonlinear memory effects, that may be caused by non-ideal bias networks, trapping effects, temperature dependence and other sources.

Correspondingly, power amplifier models have been compared with respect to what kind of memory effects they can describe [2, 5, 19]. Therefore it has become common to classify single-input single-output power amplifier nonlinear behavioral models in three main categories: memoryless models, models with linear memory and models with nonlinear memory. A similar classification is used in this chapter.

2.2 Memoryless models and models with linear memory

As described in the previous section, a complex power series can be used for power amplifier modeling. As the input-output relationship only depends on the instantaneous sample, this type of model is commonly called a *static* or *memoryless* model. In this section, first an overview of these type of models is presented, then some models that considered linear memory effects are presented.

2.2.1 Memoryless models

Historically, traveling wave tubes (TWTs) were the first widely deployed power amplifiers. Since these devices are inherently wideband and have a near-constant delay, memoryless models were able to effectively predict the intermodulation distortion created by these devices [5].

A commonly used memoryless model is the complex power series model (2.13). In this model, the input output relationship is expressed as

$$y(t) = \sum_{\substack{p=1 \\ p \text{ odd}}}^{P} a_p x(t)|x(t)|^p, \tag{2.15}$$

where a_p are the coefficients for each nonlinear order.

Other popular memoryless models are the Saleh models [20], both the original model and the modified version, and the Rapp model [21]. The original Saleh model – in Cartesian form – can be written as

$$\Re[y(t)] = \frac{\alpha_{\mathrm{I}}|x(t)|}{1+\beta_{\mathrm{I}}|x(t)|^2}, \tag{2.16}$$

$$\Im[y(t)] = \frac{\alpha_{\mathrm{Q}}|x(t)|^3}{\left(1+\beta_{\mathrm{Q}}|x(t)|^2\right)^2}, \tag{2.17}$$

where α_{I}, β_{I}, α_{Q}, and β_{I} are the fitting parameters. An interesting observation of Saleh's model is that

$$\Im[y(t)] = \left.\frac{\partial \Re[y(t)]}{\partial \beta_{\mathrm{I}}}\right|_{\alpha_{\mathrm{I}} \to \alpha_{\mathrm{Q}}; \beta_{\mathrm{I}} \to \beta_{\mathrm{Q}}} \tag{2.18}$$

In the modified Saleh model two extra parameters are added to model the phase shift and a varying exponent. This model is analyzed further in [5] and [20]. The Rapp model uses two parameters to model power amplifiers, one representing the smoothness factor and the other the saturation level. More memoryless models can be found in [5], like the Fourier series models, Bessel-Fourier models, Hetrakul and Taylor models and etc.

2.2.2 Models with linear memory

The memoryless models have acceptable performance for narrowband systems. As the signal bandwidth increases, memory effects become more apparent.

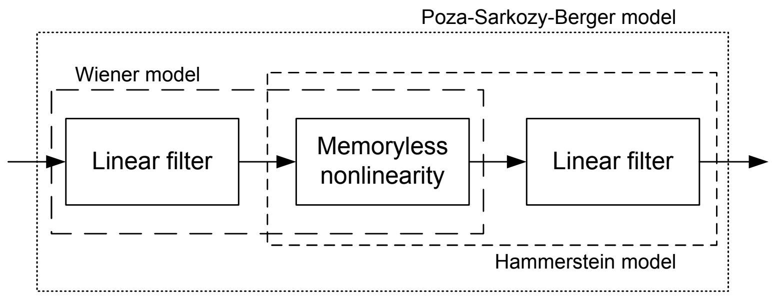

Figure 2.2: The block diagram of the two-box and three-box models.

The models presented in this section were the first models that attempted to address these effects, by using linear memory.

In the simplest case, authors have suggested that the memory effects and the nonlinearities can be separated. This has resulted in a class of commonly called two-box models. In this class, a nonlinearity is followed by a linear filter – known as the Hammerstein model – or a filter is followed by a nonlinear function – the Wiener model [22] as depicted in Figure 2.2. These models can be written as

$$y(t) = \sum_{p=1}^{P} a_p \left(\sum_{m=0}^{M} h(m)x(t-m) \right)^p \tag{2.19}$$

for the Wiener model and

$$y(t) = \sum_{m=0}^{M} h(m) \sum_{p=1}^{P} a_p x^p(t-m) \tag{2.20}$$

for the Hammerstein model.

An interesting observation for these two models is that the Hammerstein model is linear in the parameters, while the Wiener model is not. This will later prove important when parameter identification is discussed.

In order to better represent linear memory, a three-box model has also been used in the literature. These models tend to have a linear filter – memoryless nonlinearity – linear filter model structure, and are also called three-box Wiener-Hammerstein models. Examples of such models are the frequency dependent Saleh model [20], and the Poza-Sarkozy-Berger model [23]. It is important to note that even after adding an additional filter, these models are still not able to account for nonlinear memory effects [5]. A simple block diagram of the three classes of models explained is shown in Figure 2.2.

Another class of models that deal with linear memory are parallel-cascade models. In these models, parallel branches are constructed to enhance the modeling capabilities. Two important models in this class are the polyspectral model [24–26] and the Abuelma'aati model [27].

2.3 Models with nonlinear memory

As communication signals become more wideband in modern warless communication systems, the need for advanced models that can describe nonlinear memory effects becomes evident. Many mathematical tools have been suggested for such modeling purposes, such as polynomial-based functions and neural networks among others. The focus of this thesis is mainly on polynomial based models, due to interesting properties in identification – like being linear in the parameters – and ease of use.

2.3.1 Volterra series model

The Volterra series is a widely used mathematical tool for modeling any nonlinear function including memory. The Volterra series and the Volterra theory was developed by Vito Volterra in the late 19$^{\text{th}}$ century [28]. It has been commonly described as a generalized Taylor series with memory, and the discrete time passband Volterra series can be written as [29]

$$\begin{aligned}\tilde{y}_{\text{Volterra}}[n] = \sum_{p=1}^{P} \sum_{m_1=0}^{M} \sum_{m_2=0}^{M} \cdots \sum_{m_P}^{M} \\ h_{p,m_1,m_2,\cdots,m_p} \prod_{k=1}^{p} \tilde{x}[n-m_k],\end{aligned} \tag{2.21}$$

where P is the nonlinear order and M is the memory depth of the model. Following the considerations in section 2.1.1, in order to represent this model in baseband terms, even order powers will disappear. However, another consideration must also be taken. Substituting (2.9) instead of $\prod_{k=1}^{P} \tilde{x}[n-m_k]$, and dropping the even order terms leads to

$$\begin{aligned}y_{\text{Volterra}}[n] = \sum_{\substack{p=1\\p\text{ odd}}}^{P} \sum_{m_1=0}^{M} \sum_{m_2=0}^{M} \cdots \sum_{m_P}^{M} \\ h_{p,m_1,m_2,\cdots,m_p} \left[e^{j2\pi f_0 t} x[n] + e^{-j2\pi f_0 t} x^*[n]\right] \\ \prod_{k=1}^{p} \left[e^{j2\pi f_0 t} x[n-m_k] + e^{-j2\pi f_0 t} x^*[n-m_k]\right].\end{aligned} \tag{2.22}$$

From (2.22), it can be observed that all terms where the number of $x[n]$ terms differ by anything other than one from the number of $x^*[n]$ terms, are not centered at the center frequency f_0 and hence, do not contribute to the output signal [17].

Using the discussions above, the discrete baseband representation of the Volterra series can be formulated as

$$\begin{aligned}y_{\text{Volterra}}[n] = \sum_{\substack{p=1\\p\text{ odd}}}^{P} \sum_{m_1=0}^{M} \sum_{m_2=m_1}^{M} \cdots \sum_{m_{(p+1)/2}=m_{(p-1)/2}}^{M} \\ \times \sum_{m_{(p+3)/2}=0}^{M} \cdots \sum_{m_p=m_{p-1}}^{M} h_{p,m_1,m_2,\cdots,m_p} \\ \times \prod_{i=1}^{(p+1)/2} x[n-m_i] \prod_{k=(p+3)/2}^{p} x^*[n-m_k].\end{aligned} \tag{2.23}$$

The series can be re-written in matrix form as

$$\mathbf{y}_{\text{Volterra}} = \mathbf{X}\mathbf{h}, \tag{2.24}$$

where $\mathbf{h}$ is a vector containing all the coefficients $h_{p,m_1,m_2,\cdots,m_p}$, $\mathbf{X}$ is a matrix containing all the permutations of $x[n]$ from (2.23):

$$\mathbf{X}(n,j) = \prod x[n-m_i] \prod x^*[n-m_k], \tag{2.25}$$

where $\mathbf{X}(n,j)$ is the n^{th} row and j^{th} column entry. It is common to call (2.25) the *kernels* of the Volterra series.

In this representation for the Volterra series, the issues discussed in section 2.1.1 are applied to construct the baseband model. However, for a modulator, because the passband criterion is no longer valid, such a simplification can not be done. In paper [B], a dual-input Volterra series is constructed to model the modulator. This model enables the identification and compensation of nonlinear distortions between the I and Q branches, which is not possible with a normal baseband-derived Volterra series.

It has been shown that a wide class of nonlinearities can be represented at good precision with a Volterra filter [30, 31]. It is also interesting to note that while the Volterra series is a nonlinear model, it is linear in the parameters which greatly simplifies the identification process.

It can be noticed further from (2.23), that as the nonlinear order P or memory depth M increases, the number of parameters grows rapidly. This has rendered the Volterra series useful for only mildly nonlinear systems, and much research has been done to find ways to reduce the number of parameters and terms in the Volterra model. Two techniques have been utilized to alleviate this problem, either pruning parameters from the Volterra series, or finding recursive descriptions to decrease the needed memory depth and nonlinear order. The two techniques and models based on them are described in the next sections.

2.3.2 Reduced Volterra series based models

A popular technique to obtain behavioral models from the Volterra series is to identify and construct a model based on the most important terms in the series. Therefore in practice, these models are often named reduced Volterra series models or pruned Volterra series models. An overview od some of the most widely-used models are presented in this section.

Memory Polynomial

An important and widely used model is the memory polynomial (MP) model [32, 33]. This model can be described as both an extension of the normal polynomial model to include memory, or as a reduction of the Volterra model to only include diagonal terms. This model has also been called the parallel Hammerstein model in literature [6]. The model can be written as

$$y_{\text{MP}}[n] = \sum_{p=1}^{P} \sum_{m=0}^{M} h_{p,m} x[n-m] \left| x[n-m] \right|^{p-1}. \tag{2.26}$$

It can be noticed that the memory polynomial model is also linear in the parameters.

The memory polynomial model, when originally purposed, included both odd and even order power terms. However, because the amplitude operation $|x[n-m]|$ is an even function, as long as $x[n]$ is an odd function, the considerations of section 2.1.1 are met, and the corresponding terms will all be odd functions, and contribute to the output signal [18, 34, 35].

For modulators, using a similar approach, it is possible replace the dual-input Volterra series in paper [B], with a dual-input memory polynomial model. This will greatly reduce the number of parameters, and can help obtain better performance.

Parallel Wiener

Similar to the parallelization of the Hammerstein model in MP, the Wiener model can be parallelized as well. This model, known as the parallel-cascade Wiener model [36], can also model nonlinear memory effects in a power amplifier. However, it suffers from the same complex identification as the Wiener model, as it is no longer linear in the parameters.

Generalized Memory Polynomial

Morgan *et. al* in [17] proposed a new model that generalized the memory polynomial model by including leading and lagging terms. This model can be written as

$$\begin{aligned} y_{\mathrm{GMP}}[n] = & \sum_{p=0}^{P}\sum_{m=0}^{M} h_{p,m,0}x[n-m]|x[n-m]|^{p} \\ + & \sum_{p=1}^{P}\sum_{m=0}^{M}\sum_{g=1}^{G} a_{p,m,g}x[n-m]|x[n-m-g]|^{p} \\ & +b_{p,m,g}x[n-m]|x[n-m+g]|^{p}, \end{aligned} \tag{2.27}$$

and is called the generalized memory polynomial (GMP) model. Compared to the memory polynomial model, there is an extra degree of freedom in terms of choosing the leading or lagging delay. By setting $G = 0$, it is observed that this model becomes the memory polynomial model, i.e., the memory polynomial model is a special case of the GMP model.

Baseband-derived Volterra

The authors in [35] derive a base-band Volterra series for power amplifier modeling. By rewriting the Volterra series terms with amplitude and phase as inputs, they generalize the memory polynomial model. The model structure is somewhat similar to a "generalized" GMP structure. They also discuss the difference of odd order power terms, and odd order functions.

Dynamic Volterra Series

In order to find ways to rewrite the Volterra series, in [37] a new mathematical model for power amplifiers is presented based on modeling the static and dynamic parts separately. This work was constructed into the behavioral model format in [38] and [39]. Further work was done in [40] and [41]. This model, known as the Volterra model with dynamic deviation reduction (Volterra DDR), or the dynamic Volterra series representation, adds a parameter R by which the number of dynamics involved in the modeling are limited. In this way, the number of parameters in the Volterra series may be reduced. The baseband equivalent model expanded from [42] can be written as

$$\begin{aligned} y_{\mathrm{DDR}}[n] = & \sum_{\substack{p=1 \\ p \text{ odd}}}^{P} h_{p,0} \underbrace{x[n]|x[n]|^{p-1}}_{\text{zero order dynamic}} \\ + & \sum_{\substack{p=1 \\ p \text{ odd}}}^{P} \sum_{m_1=1}^{M} h_{p,m_1} \underbrace{x[n-m_1]|x[n]|^{p-1}}_{1^{st} \text{ order dynamics path 1}} \\ + & \sum_{\substack{p=3 \\ p \text{ odd}}}^{P} \sum_{m_2=1}^{M} h_{p,m_2} \underbrace{x^*[n-m_2]x^2[n]|x[n]|^{p-3}}_{1^{st} \text{ order dynamics path 2}} \end{aligned} \tag{2.28}$$

where up to 1^{st} order dynamics are shown.

Sliding kernels dynamic Volterra series

An interesting modeling approach was proposed in [43], where the Volterra series was modified by considering the Fourier integral in place of the convolution, and the sliding kernels dynamic Volterra series model was constructed. This model was further developed in [44], to describe longer-term memory effects.

Switching models

Models which partition the input signal into regions and find different models for these regions have also been analyzed in the literature. **Piecewise Volterra filters** were derived in [45], and it is shown that parameter estimation remains a linear problem when the regions are partitioned. In [46], a **piecewise Hammerstein structure** is used to construct the piecewise model. In [47], **spline functions** are used to switch between different regions. In [48], a model is proposed for envelope tracking applications based on **vector threshold decomposition**. Normally for these types of models, each output sample is involved with many models. Therefore, computational complexity becomes a limiting factor in these type of model structures.

2.3.3 Generalized Volterra series based models

A second track of Volterra based behavioral modeling has been to find ways to further exploit the recursive nature of a power amplifier. This can help reduce the amount of memory depth needed when modeling. However, these models tend to no longer be linear in the parameters, which complicates the identification process.

IIR models

The Volterra series is a natural expansion from a linear finite impulse response (FIR) model [19]. In this format, it is assumed that the output signal can be modeled with the input signal only. In practice, power amplifiers tend to have long-term memory effects, that cannot be easily modeled by only using the input signal. These effects originate from inherent feedback in the power amplifier circuit and biasing networks. One solution was therefore, the use of infinite impulse response filters (IIRs) in [49], but due to the recursive nature, stability was a major problem.

Volterra model with Laguerre functions

In order to model longer-term memory effects and avoid the stability issues, authors have proposed the use of orthonormal basis functions instead. In [50], Zhu *et. al.* proposed the use of Laguerre functions as the basis for the Volterra expansion, replacing the Dirac impulses of the Volterra FIR filter with a fixed-pole orthonormal Laguerre function. This function decays exponentially to zero at a controlled rate, and has a similar structure to an IIR filter with a pre-decided pole to alleviate the stability issues.

Volterra model with Kautz functions

In [51], the Kautz function was suggested as an orthonormal basis. This model is similar to the Laguerre-based model, except that in the Laguerre-based model the orthonormal-basis poles are chosen to be real, while in the Kautz-based model the poles are allowed to be complex as well.

Reduced-orthonormal model

In paper [A], we propose a new model that can combine both modeling techniques. This model has a similar model structure to the memory polynomial model, but adds an additional IIR-like filter using Kautz-functions to capture longer term memory effects. Therefore, this model can exploit longer term memory effects with the correct choice of pole, like the Kautz-Volterra model, but also has a much reduced number of parameters, like the memory polynomial model. The block diagram of this model is shown in Figure 2.3.

2.3.4 Other models

For sake of completeness, some other important models which either could not be easily classified into the three groups above or were not the main focus of the thesis are discussed in this section.

Look-up tables (LUT)

A widely used technique to model and predistort power amplifiers, are lookup tables. In this technique, the AM/AM and AM/PM characteristics of a PA are used to construct tables for modeling and inverting the PA. In [52], multiple lookup tables for different power levels are used to model and predistort the power amplifier, enabling a faster response to changes in the PA characteristics.

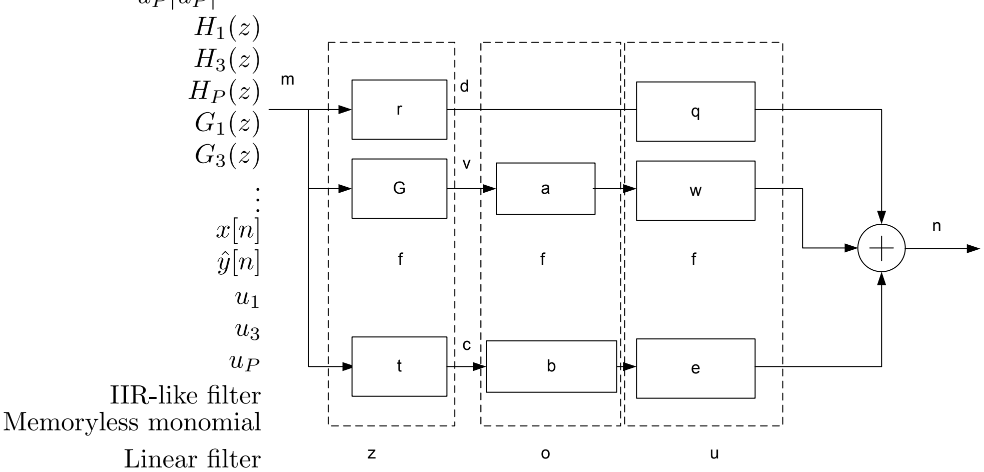

Figure 2.3: Block diagram of the reduced orthonormal model.

Artificial neural networks

Artificial neural networks (ANNs) have also gained recognition in recent years [53] for PA modeling. It has been shown that the single-hidden-layer multilayer perceptron (MLP) ANN have universal approximation capabilities [54, 55]. Two main approaches have been taken in ANN design for behavioral modeling, MLP and time-delayed neural networks (TDNN) [56–60], and radial-basis function neural networks (RBFNNs) [61,62]. For TDNNs, it was shown in [2], that the network is similar to the memory polynomial model in structure, with a different nonlinear function. Different approaches have been taken to model the power amplifier using TDNNs, the common approach has been to use two real-valued TDNNs for the I and Q signals and then combine the output. Another approach has been the use of a complex valued neural network [63]. In [56], one real-valued neural network is used with both I and Q as the input. RBFNNs consist of three layers, an input layer, a hidden layer, and an output layer. The input layer to the hidden layer space has a nonlinear transformation using Green's function [64], while the hidden layer to output layer has a linear transformation.

NARMA models

The nonlinear autoregressive moving-average (NARMA) model has been used to model power amplifiers [5,65]. These models could also be treated as models with nonlinear memory. In this model, a nonlinear feedback path is added to enable the modeling of IIR terms. However NARMA models generally suffer from the same stability issues as in [49]. Some studies on the stability and the stability criterion for this model can be found in [66].

State-space models

Another type of behavioral model that has been used are state-space models [67]. These models may include linear memory terms, or nonlinear terms, based on the formulation. The main advantage with these models is the ability to model the power amplifier behavior as a full two port device, and not as a single-input single output system. Such a model is proposed in [68], where power amplifiers are modeled as nonlinear two-port RF networks. In [69], a dual-input Volterra series model is proposed that takes both RF input and supply power as inputs, to remove voltage ripple of the power supply.

Gray-box models

All models discussed in this thesis were black-box models. However, it is important to also mention *gray box* models. In these type of models, some knowledge of the internal circuitry is used to find good behavioral models. Identification of such models is discussed in [70], and in [71], the circuit structure is used to find the relationship between parameter terms and physical phenomenon.

2.4 Model parameter identification

An important issue in behavioral modeling is parameter identification. It is well-known that black-box models suffer from uncertainty in modeling [2], and hence, the parameter estimation process has to be analyzed carefully.

2.4.1 Identification data signal properties

A power amplifier behavioral model will depend on the type of data that is used in the identification procedure. Therefore, it is advantageous to identify the behavioral model with the same communication signal that will be used in the system, e.g. if the system is designed for a WCDMA signal, WCDMA-like signals should be used for identification. This will guarantee that the input signals are persistently exciting for the communication system [72], i.e., that the power amplifier is excited with the correct frequency and amplitude range.

2.4.2 Identification algorithms

Different models will have different parameter identification strategies. All models that are linear in parameters for example, may be identified by the least-squares estimate (LSE) algorithm. This is an important advantage for such models, as the least-squares algorithm guarantees global convergence [73]. The LSE solution can generally be written as [73]

$$\hat{h} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^H y, \tag{2.29}$$

where X is a matrix containing all permutations of the input signal of a model structure (for example from 2.23-2.24), $\hat{h}$ is the estimated parameters, and y is the output signal.

For models that are not linear in parameters, iterative procedures have to be used for parameter identification. There is no guarantee for global convergence for these models, and in some cases local minima may hinder the identification process. In this work, for the models discussed in section 2.3.3 a full search of poles for each nonlinear order as in [51] is used. In this technique, after finding the optimum poles, the problem becomes a normal least squares estimation.

2.4.3 Bias and variance in parameter identification

When identifying a power amplifier behavioral model, it is important to be aware of the two types of errors in parameter estimation; bias and variance. In general, the expected value of the quadratic error in parameter estimation can be approximated as [72] [p500-504]

$$E\left[\left|y_{\text{model}}(Z^N) - y_{\text{model}}(\hat{h}_k, Z^N)\right|^2\right] \approx \underbrace{V_N(\hat{h}_k, Z^N)}_{\text{Bias}} + \underbrace{2\lambda\frac{k}{N}}_{\text{Variance}}, \tag{2.30}$$

where V_N is the sum of the squared errors, $\hat{h}_k$ are the estimated k parameters, Z^N is the input and output data vector, λ is a scaling for the expectation of the square of error, and N is the size of the data set. As the number of parameters grow, the first term, the bias, becomes smaller since more parameters can reduce the quadratic error, but due to the uncertainty in parameter estimation, the second term, the variance, grows.

The implications of this fact is that in order to be able to identify the parameters properly, the data set size has to be large enough to avoid *overfitting*. Overfitting occurs when the number of parameters becomes comparable to the data set size. This phenomenon can be observed in the two tests in Figure 2.4. In the first test, labeled closed test, memory polynomial models (2.26) with different number of parameters are identified, and the same data set that was used for identification is also used for validation. In the second experiment, labeled open test, the model is identified with one set of data, and another independent set of similarly generated data is used for validation. In these experiments the data length size is kept fixed.

It can be seen that as the number of parameters increase, in the closed test, the model performance consistently improves. However, this is misleading, since in the open test the performance diminishes as the number of parameters increase. This is because the uncertainty in the parameter estimation grows as $\frac{k}{N}$ grows, and overfitting occurs. Overfitting can be avoided by using different, but statistically similar, data sets for identification and validation. All model evaluation results presented in this thesis use different data sets for the model identification and validation, respectively.

2.5 Discussion

Due to the importance of behavioral modeling, the detailed analysis of the important issues in modeling of this chapter was necessary. It helped in understanding the sources of distortion and in constructing new models. The parameter identification issues discussed were also important, for behavioral modeling applications.

However, in practice, when modeling a power amplifier, we are faced with many choices. As there are many models in the literature, it is not obvious which one can be suitable for our application. By just looking at different

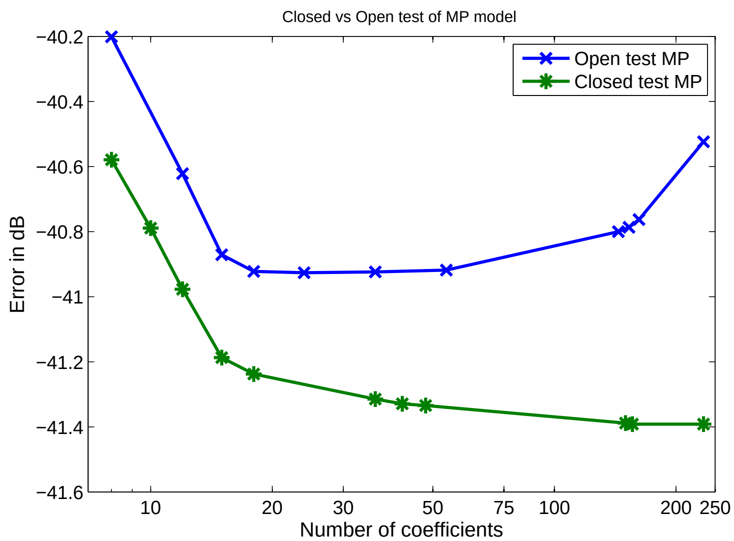

Figure 2.4: Overfitting in power amplifier behavioral modeling identification.

model structures, we might be able to narrow down our model choice, but there are still many models to choose from. Which model to use for a specific application depends on many factors. Therefore, the focus of the next chapter is a detailed comparison between different behavioral models. Some of the methods to evaluate models are discussed, which can provide some guidance on which models are more suitable than others.

Chapter 3

Evaluation of Behavioral Models

In the previous chapter, a number of power amplifier behavioral models were presented. While the model structure may sometimes help in choosing a behavioral model, in most cases we are mainly interested in how well the model can predict a power amplifier performance. In practice, in order to choose a behavioral model over another, modeling accuracy is the most important factor. This chapter deals with comparing different behavioral models performance, and provides criteria to be able to choose behavioral models suitable for the application at hand.

The first efforts to compare power amplifier behavioral models was done in [74], which included neural networks, the Volterra series and Hammerstein models. Isaksson *et. al.* extended this work in [6] with more behavioral models, input signals with different characteristics, and cross-validation. Signal bandwidth effect was also analyzed.

In order to compare different behavioral models, two criteria have to be established. The first and most obvious is the accuracy of the model. This will measure how well the model is able to represent a power amplifier output signal. However, this measure is not able to completely compare different models, as it is a well-known fact that given high orders, the Volterra series will be able to model any nonlinear function, and hence have negligible error. Thus, a second criterion is the computational effort needed to construct the behavioral model. Section 3.1 deals with the first measure and section 3.2 deals with the second.

3.1 Modeling accuracy

The most important criterion in behavioral model performance, is how accurate the model prediction is compared to the true output signal of a power amplifier. In literature, many measures exist for this comparison. A simple measure has been to compare the AM/AM and AM/PM plots of the model output vs. an ideal power amplifier output. Another technique is to plot the time-domain signal or frequency response of the output of a model and a power amplifier and compare. These techniques do not yield a quantitative result, and may be subject to the human interpretation.

In literature, many figures of merit (FOM) for comparison have been used. A summary and comparison is presented in [75]. Some of the most important measures are the normalized mean square error (NMSE), the error vector magnitude (EVM), the adjacent channel power ratio (ACPR), the adjacent channel power error measure (ACEPR), the weighted error-to-signal power ratio (WESPR), the memory effect ratio (MER), and the memory effect modeling ratio (MEMR).

It has been pointed out that NMSE, for all practical cases, will mainly be affected by in-band errors [75]. This is because it is written as a power measure, and the sum of squared errors. Since most of the power is in-band (Figure 1.3), in-band errors will dominate. In some applications this may be the important measure and NMSE and EVM will be of interest. In some instances, due to spectral regulations, the out-of-band performance of power amplifiers is of more importance. Since ACPR, ACEPR, and WESPR measure out-of-band performance more effectively, they are more suited for such applications. The MER and MEMR measures represent the memory effect, and have also been used to compare behavioral models.

3.1.1 In-band measures

An important metric that is mainly affected by in-band performance is the EVM. The EVM is written as [5]

$$\mathrm{EVM} = \sqrt{\frac{\sum_n |y_{\mathrm{meas}}[n] - y_{\mathrm{model}}[n]|^2}{\sum_n |y_{\mathrm{meas}}[n]|^2}}. \tag{3.1}$$

EVM is also used in communication systems to measure the performance of the entire system at the receiver. EVM is closely related to NMSE, which is commonly defined as [6]

$$\mathrm{NMSE} = \frac{\sum_n |y_{\mathrm{meas}}[n] - y_{\mathrm{model}}[n]|^2}{\sum_n |y_{\mathrm{meas}}[n]|^2}, \tag{3.2}$$

where y_{meas} is the time domain sampled version of the measured output signal of the power amplifier, and y_{model} is the output of the behavioral model. In this formulation, the measured signal and the model must be time aligned to minimize the mean-square error. It has also been defined more generally as [5]

$$\mathrm{NMSE} = \frac{\mathrm{cov}\left(y_{\mathrm{meas}} - y_{\mathrm{model}}, y_{\mathrm{meas}} - y_{\mathrm{model}}\right)}{\mathrm{cov}\left(y_{\mathrm{meas}}, y_{\mathrm{meas}}\right)}, \tag{3.3}$$

where $\mathrm{cov}(\cdot,\cdot)$ is the covariance function. The NMSE is statistically equivalent to the EVM in the zero-mean case.

The variance accounted for (VAF) is another measure used to compare in-band model performance and can be written as [76]

$$\mathrm{VAF} = \frac{\mathrm{cov}\left(y_{\mathrm{meas}}, y_{\mathrm{meas}}\right) - \mathrm{cov}\left(y_{\mathrm{meas}} - y_{\mathrm{model}}, y_{\mathrm{meas}} - y_{\mathrm{model}}\right)}{\mathrm{cov}\left(y_{\mathrm{meas}}, y_{\mathrm{meas}}\right)} = 1 - \mathrm{NMSE}. \tag{3.4}$$

The MER can be defined as [77]

$$\mathrm{MER} = \sqrt{\frac{\sum_n |y_{\mathrm{meas}}[n] - y^0_{\mathrm{model}}[n]|^2}{\sum_n |y_{\mathrm{meas}}[n]|^2}}, \tag{3.5}$$

where $y^0_{\mathrm{model}}[n]$ is the model output by setting memory order to zero.

3.1.2 Out-of-band measures

As mentioned previously, NMSE and EVM are normally dominated by in-band error. In cases where the out-of-band performance is also of interest, other measures have been suggested. In wideband code division multiple access schemes (WCDMA), an entity is defined called the adjacent channel leakage ration (ACLR). Similarly, the ACPR is defined as

$$\mathrm{ACPR} = \frac{\int\limits_{\mathrm{adj}} |Y(f)|^2}{\int\limits_{\mathrm{ch}} |Y(f)|^2}, \tag{3.6}$$

where $Y(f)$ is the Fourier transform of the output signal, the integration in the denominator is over the in-band channel signal bandwidth and the integration in the numerator is over both adjacent channels to the signal channel with the same bandwidth. It can be seen that for behavioral model purposes, this measure does not include the model, and hence cannot predict model performance. However, the difference in dB of this ratio for the model and for the measured power amplifier has been used as a comparative measure

$$\Delta\mathrm{ACPR} = \mathrm{ACPR}_{\mathrm{meas}}/\mathrm{ACPR}_{\mathrm{model}}. \tag{3.7}$$

ACEPR was defined in [6] as

$$\mathrm{ACEPR} = \max_{\mathrm{m}=1,2}\left[\frac{\int\limits_{(\mathrm{adj})_\mathrm{m}} |\mathrm{Y}_{\mathrm{meas}}(\mathrm{f}) - \mathrm{Y}_{\mathrm{mod}}(\mathrm{f})|^2}{\int\limits_{\mathrm{ch}} |\mathrm{Y}_{\mathrm{meas}}(\mathrm{f})|^2}\right], \tag{3.8}$$

where $Y_{\mathrm{mod}}(f)$ is the Fourier transform of the model data, $Y_{\mathrm{meas}}(f)$ is the Fourier transform of the measurement data. This merit can effectively measure the out-of-band performance of a behavioral model. In [5], it was found that ACEPR was the best low-complex measure to identify nonlinear mismatches.

Authors in [78] have proposed a measure to simultaneously consider both in-band and out-of-band performance called the weighted error spectrum power ratio (WESPR). In this measure, the signal is weighted using different weighting functions. The model can be written as

$$\mathrm{WESPR} = \frac{\int\limits_{f\in I} |W(f)\left[Y_{\mathrm{meas}}(f) - Y_{\mathrm{model}}(f)\right]|^2}{\int\limits_{f\in J} |\mathrm{Y}_{\mathrm{meas}}(f)|^2}, \tag{3.9}$$

where the integrations are performed over suitable frequency ranges using the weighting function $W(f)$.

3.1.3 Memory measures

The capability of a behavioral model to describe memory effects of a power amplifier has been measured by MEMR, which can be defined as [77]

$$\mathrm{MEMR}_m = \sqrt{\frac{\sum_n |y_{\mathrm{meas}}[n] - y^m_{\mathrm{model}}[n]|^2}{\sum_n |y_{\mathrm{meas}}[n] - y^0_{\mathrm{model}}[n]|^2}}, \tag{3.10}$$

where $y^m_{\mathrm{model}}[n]$ is the model output when the memory depth of is set to m. Different models can be compared with this measure, to show how well each is able to model memory effects.

3.1.4 Summary

In order to capture both in-band and out-of-band behavior of the power amplifier, either measures such as WESPR should be used, or alternatively both the NMSE and ACEPR could be used. In instances where we are interested in comparing both in-band and out-of-band performances with one metric, the WESPR FOM could be of more use. However, WESPR will depend on the window that is used, and with different windows different values can be obtained. When possible, it could be more desirable to consider both NMSE and ACEPR to compare behavioral models, so both in-band and out-of-band performance are analyzed separately. This has the drawback that in some cases, NMSE for one model might be better than another, while the ACEPR is worse.

While the model accuracy using any of the metrics above gives a measure to compare models, it is still not obvious how to assess which of the models in Chapter 2 are more suitable to use. This is because given high enough orders, most models can obtain a relatively similar performance. In other words, if the modeling accuracy is the y axis in the comparison plot, we are still missing an x axis. Complexity may be able to fill that role, and is the focus of the next section.

3.2 Modeling complexity

A common issue that is noticed in the literature, is how to compare behavioral model performance [6, 74]. Obviously, by disregarding model order, the Volterra series will theoretically yield the best performance, since it can model any nonlinear function accurately. In practice however, as the number of parameters grow, the performance is restricted by the uncertainties in parameter identification. Thus, for this reason, and because with a large number of parameters computer processing becomes impossible, the Volterra series has to be truncated.

Different models are truncated differently, making fair comparisons between them difficult. For example, it is often seen that the performance of a Volterra with nonlinear order 3 and memory depth of 2 is compared with a memory polynomial model with nonlinear order 5 and memory depth 4. Such a comparison may not be comprehendible or fair.

Therefore, it becomes apparent that a common fair basis has to be established to which behavioral models can be compared to. Once this has been established, behavioral models can be evaluated correctly. A suggestion for such a basis, is model complexity, i.e., how much computational effort is needed to obtain a certain performance. The main focus of paper [C] is this analysis.

3.2.1 Metrics used for complexity

In literature, complexity has been notated by different measures [79]. Often it is measured in orders denoted by the Landau symbol $O(\cdot)$, which represents the algorithm complexity. Unfortunately, for behavioral model analysis, this representation is not precise enough for practical considerations [80].

A simple approach is to record the running time of the different behavioral models in a software package. This is severely dependent on the hardware setup and the algorithm utilized. In order to have a fair comparison in this case, the algorithms must be optimized for the different behavioral models and for the hardware where they are tested.

In the area of behavioral modeling, it is common to compare models based on the number of parameters. This can determine the memory size needed for a behavioral model. However, this representation may not always be an appropriate measure. For example, the number of parameters for a neural network may not correctly represent the computational complexity of this model, as the main source of complexity stems from the operations needed per sample, and not necessarily the number of parameters.

The number of floating point operations or FLOPs is another widely used measure for complexity. In most DSP hardware, the computational effort is mainly spent on additions, subtractions and multiplications, which is precisely what FLOPs count. FLOPs are also the relevant entity when implementing behavioral models on chip.

3.2.2 Different types of complexity

Another important issue in behavioral model complexity is where the complexity originates from. The computational complexity can be classified into *identification complexity*, *adaptation complexity*, and *running complexity*.

Identification complexity The identification procedure differs for behavioral models, as discussed in section 2.4. Due to statistical properties of measured signals, most Volterra-based models can be identified with a least squares estimate, while other models may need iterative procedures. Since the identification of the behavioral model is typically done offline, this complexity is normally not a major issue.

Adaptation complexity In practical systems, due to slight changes in the power amplifier such as temperature change or different mismatching effects, behavioral models might need to be updated at time intervals. These time intervals can normally be much larger than the symbol period. The adaptation of the behavioral model to these changes is considered adaptation complexity.

In many instances where the variations are slow, this complexity may be of less importance.

Running complexity Running complexity is the number of calculations that is done on each sample when the model is utilized. This complexity severely limits the system due to the fact that it is a real-time problem. Depending on the application the maximum acceptable complexity varies. For a base station, there might be room for more complex algorithms and behavioral models while for mobile hand-held devices requirements are stricter. Since one of the main justifications for DPD linearization techniques is to have more power-efficient transmitters, it is essential that the power saved is not all spent on processing the DPD algorithm.

3.2.3 Complexity of Volterra-based behavioral models

Depending on the implementation of the behavioral model algorithm, the complexity for the Volterra-based behavioral models will differ. In [14] a general algorithm for implementing the Volterra series as a behavioral model is proposed. Here it is simplified and given in two steps:

Step i) Construct *basis functions* – matrix $\mathbf{X}$ from (2.23).

Step ii) Filter the basis with *kernels* ($\mathbf{Xh}$).

The second step is directly related to the number of kernels, since each kernel will be multiplied by the according basis function and then summed with the remaining results. Thus, it is solely dependent on the number of coefficients. Behavioral models will, however, differ in the construction of the basis functions. The complexity C can be written as the sum of the complexity of each part, or

$$C = C_{\text{basis}} + C_{\text{filter}}, \tag{3.11}$$

where C represents the total complexity, C_{basis} represents the basis construction complexity from Step i, and C_{filter} represents the filtering complexity from Step ii.

An important issue in efficient algorithm design, is to avoid regeneration of already available data. For instance, while multiplying two signal values may require a certain number of FLOPs, delaying a signal does not. Therefore it is necessary to fully utilize all available permutations in the behavioral model algorithms. For example, $x[n-1]x[n-2]x^*[n-3]$ can easily be constructed from $x[n]x[n-1]x^*[n-2]$ by a simple delay.

Another issue is that terms that will be used in different combinations should be generated beforehand. For example, when constructing $|x[n]|^4$, if $|x[n]|^2$ is already available, using it will result in much lower complexity than constructing it from scratch.

Table 3.1 shows the operation-FLOP conversion used in this work. The number of FLOPs for the square root operations varies depending on the algorithm, but can generally be considered to be between 6 and 8 FLOPs [81].

Figure 3.1 shows the number of FLOPs needed per coefficient for different models. These results have been obtained using complexity minimized implementations of the models tested. This is representative of the C_{basis} measure,

Table 3.1: Number of FLOPs for different operations.

Operation	**Number of FLOPs**
Conjugate	0
Delay	0
Real addition	1
Real multiplication	1
Complex addition	2
Complex-real multiplication	2
$\lvert . \rvert^2$	3
Complex - complex multiplication	6
Square-root	$6 \sim 8$

since for the same number of parameters C_{filter} is equal between models. It can be noticed that the memory polynomial model has the least complexity per coefficient.

It can be noticed from Figure 3.1 that the basis construction complexity, C_{basis}, generally differs from model to model. Therefore, any comparative analysis of the performance of behavioral models with respect to complexity should include this complexity. This is the main focus of paper [C]. Previous comparative analysis works neglected this complexity, and by including this complexity in the analysis, a more complete description can be obtained.

Finally, in order to compare the different behavioral models, all model combinations – that were computationally possible – have to be analyzed, and the best combinations should be chosen. This is important since many combinations may result in the same number of parameters or occasionally the same number of FLOPs. This is done in paper [C] by constructing a scatter plot, where all combinations are plotted. Fig. 3.2 is the result, where the NMSE is plotted as a function of the complexity in FLOPs for the entire range of parameters. Then, the convex hull of the configurations are chosen to represent the best possible parameter choices. It should be noted that the all FLOP-NMSE pairs on the convex hull are not necessarily realizable, but they represent the approximate performance of the model.

3.3 Discussion

In paper [C], a detailed analysis of different behavioral model performance is done. A summary of the in-band results of the analysis can be shown in Figures 3.3-3.4. It can be noticed that by increasing the model orders, all models obtain relatively similar results. However, it is important to note that some models can obtain a certain accuracy performance at much lower complexity, and the accuracy/complexity tradeoff for different models can be compared. This can help when deciding on a behavioral model for predistortion or simulations. Another interesting comparison is between Figure 3.3 and Figure 3.4. It can be noticed that, for example, the GMP model can obtain -51 dB NMSE at half the number of FLOPs compared to the nearest competing model, Volterra

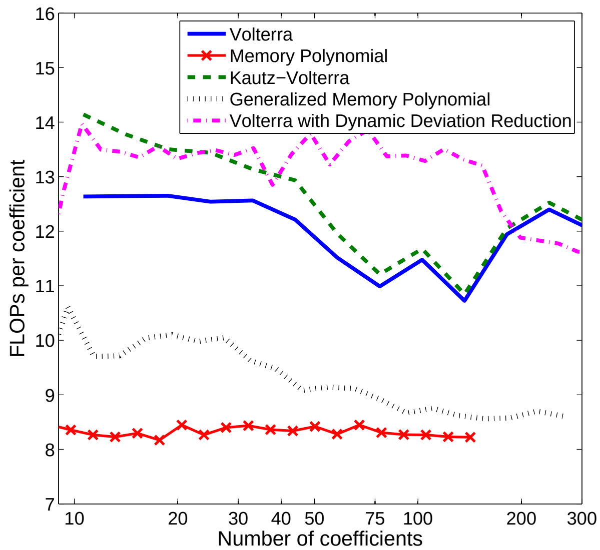

Figure 3.1: Number of FLOPs per coefficient vs number of coefficients for different models.

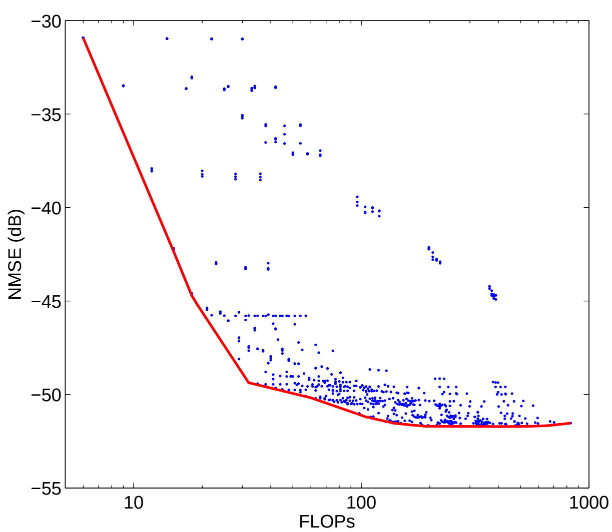

Figure 3.2: Scatter plot of the performance of a behavioral model vs the number of FLOPs. The dots represent the different configurations of parameters. The solid red line shows the convex hull of the configurations that resulted in best performance.

DDR. This conclusion would not have been possible if only presented with Figure 3.3, proving the usefulness of FLOPs in behavioral modeling.

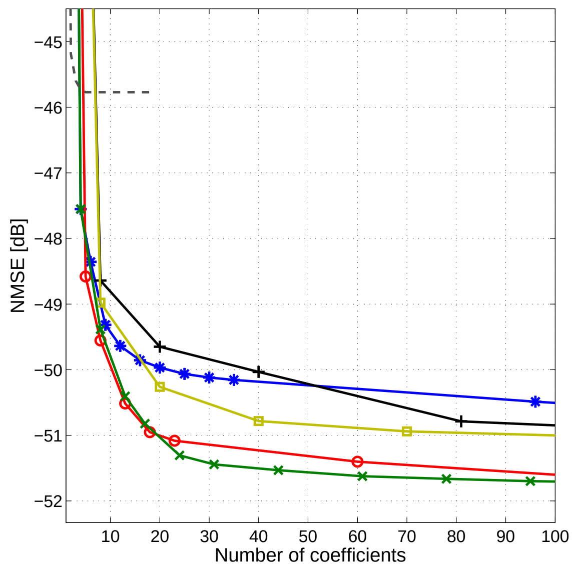

Figure 3.3: NMSE vs number of parameters.

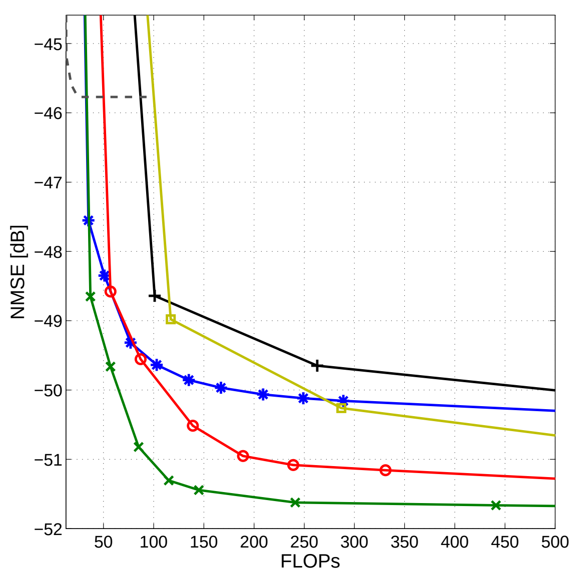

Figure 3.4: NMSE vs FLOPs.

Chapter 4

Summary of appended papers

4.1 Paper A

By combining two techniques of pruning Volterra series and using IIR filters, a new model is constructed in paper [A]. In this model, for each nonlinear order p, a filter $G_p(z)$ with pole ξ_p is followed by a simple memoryless monomial $u|u|^{p-1}$ and a linear filter $H_p(z)$.

It was noticed that new model is able to achieve better performance with less number of parameters than both the memory polynomial and the Kautz-Volterra model in the parameter range of $5 - 20$.

4.2 Paper B

In paper [B], we construct a new model that has the ability to model all kinds of distortion created by the IQ modulator. In this model, we separate the input signal into the real and imaginary parts, and then construct a dual-input nonlinear model to consider all nonlinear and memory effects.

As the model is dual input, the number of parameters will grow even more severely compared to a single input model. Therefore techniques similar to the reduced Volterra models are applied and new models are proposed in paper [B], including the dual-input memory polynomial model and the dual-input dynamic deviation reduction model.

4.3 Paper C

In this paper, complexity in power amplifier behavioral modeling is discussed, and FLOPs are suggested as a suitable measure to compare power amplifier behavioral models. After finding the complexity for different behavioral models, experiments are done to find the accuracy for different models.

Finally, some representative models are used to find the accuracy/complexity tradeoff between power amplifier behavioral models. It was shown that the generalized memory polynomial model could obtain the best performance at low computational cost.

Chapter 5

Summary and Future work

5.1 Summary

Behavioral modeling of RF transmitter architectures were studied in this thesis. Important issues in power amplifier behavioral model design were discussed, including simplifications in model structure and identification issues.

Some representative power amplifier behavioral models in the literature were analyzed, and the models were organized into three groups; memoryless models, models with linear memory and models with nonlinear memory. Model structures were analyzed. Because of the trend in increasing signal bandwidths in modern communication signals, the focus was on nonlinear models with nonlinear memory. By analyzing model structures, a new power amplifier behavioral models was proposed that combined both parameter and memory reduction techniques. The modeling technique can be applied to other behavioral models as well, to achieve improvement. Issues in identification of behavioral modeling was also discussed.

A dual-input model for IQ modulators was introduced. As the complexity of this model is a vital issue, model reduction techniques were applied and reduced dual-input models were constructed. The reduced dual-input models were able to cope with wideband signals, since they could model higher memory depths.

Finally, a comparative analysis of the tradeoff between complexity and accuracy for some representative models were presented. It was shown that accuracy by itself can not completely represent a behavioral model performance. Complexity in FLOPs was found for different models, and it was shown that the generalized memory polynomial model could achieve good performance with around half the number of FLOPs as the nearest competing behavioral model. Such results could not have been achieved by using other measures like number of parameters.

5.2 Future Work

Some interesting topics that the author will focus future research on.

Behavioral model design The analysis done in this work for behavioral modeling paves the way for inventing new behavioral models. By identifying important terms in the modeling process, it becomes possible to construct models that are suited for certain applications. For example, in mobile handset devices, it is vital that as little as power as possible is spent on the DPD linearization. This requires behavioral models that have adequate performance with very low complexity. Other models such as switching models may also be constructed from the knowledge gained though this work.

Digital predistortion and adaptation Simultaneously with the work presented in the thesis, digital predistortion algorithms have been analyzed as well. An issue that must be addressed, is on modeling and compensating for long-term memory effects. For example the temperature change from day to night in a base station power amplifier. More advanced adaptation algorithms are also of interest, with faster response to changes in the power amplifier characteristics. Model-based adaptation may provide a means to achieve a more proactive algorithm, compared to the reactive algorithms in use today.

Receiver estimation of power amplifier distortions Another interesting topic, is to identify and compensate for distortions created by the power amplifier at the receiver. This may be done with, or without, the knowledge of the PA behavioral model. If PA behavioral model knowledge is available, it can be used to identify how the PA has distorted the original signal. If not, the PA behavioral model has to first be estimated, and then it can be used to compensate for the distortions.

Efficiency enhancement techniques The linearity of a power amplifier is crucial for low-error transmission. However, another important factor is power amplifier power efficiency. In fact, there is a tradeoff between the linearity of a power amplifier and the power efficiency. It is of interest to find transmitter architecture and algorithms that can achieve high power efficiency for the transmitter while maintaining the linearity.

Bibliography

[1] C. E. Shannon, "A Mathematical Theory of Communication," *Bell System Technical Journal*, vol. 27, pp. 379–423 and 623–656, Jul. and Oct. 1948.

[2] J. C. Pedro and S. A. Maas, "A comparative overview of microwave and wireless power-amplifier behavioral modeling approaches," *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 4, pp. 1150–1163, 2005.

[3] C. J. Clark, C. P. Silva, A. A. Moulthrop, and M. S. Muha, "Power-amplifier characterization using a two-tone measurement technique," *IEEE Trans. Microw. Theory Tech.*, vol. 50, no. 6, pp. 1590–1602, 2002.

[4] H. Ku, "Behavioral Modeling of Nonlinear RF Power Amplifiers for Digital Wireless Communication Systems with Implications for Predistortion Linearization Systems," Ph.D. dissertation, Georgia Institute of Technology, 2003.

[5] D. Schreurs, M. O'borma, A. A.Goacher, and M. Gadringer, *RF power amplifier behavioral modeling.* Cambridge University Press, 2008.

[6] M. Isaksson, D. Wisell, and D. Ronnow, "A comparative analysis of behavioral models for RF power amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 1, pp. 348–359, 2006.

[7] L. Anttila, M. Valkama, and M. Renfors, "Frequency-selective I/Q mismatch calibration of wideband direct-conversion transmitters," *IEEE Trans. Circuits Syst.-II: Express Briefs*, vol. 55, no. 4, pp. 359-363, Apr. 2008.

[8] M. Windisch and G. Ferrweis, "Standard-independent I/Q imbalance compensation in OFDM direct-conversion receivers," in *Proc. 9th International OFDM Workshop*, 2004, pp. 57-61.

[9] S. Cripps, *RF power amplifiers for wireless communications*, 2nd ed. Boston: Artech House, 2006.

[10] J. Vuolevi, *Distortion in RF Power Amplifiers.* Norwood, MA: Artech House, 2003.

[11] H. Takaqi and B. H. Walke, *Spectrum Requirement Planning in Wireless Communications: Model and Methodology for IMT - Advanced.* New York: Wiley, 2008.

[12] P. B. Kenington, "Linearized transmitters: an enabling technology for software defined radio," *IEEE Commun. Mag.*, vol. 40, no. 2, pp. 156–162, 2002.

[13] H. Cao, A. Soltani Tehrani, C. Fager, T. Eriksson, and H. Zirath, "I/Q imbalance compensation using a nonlinear modeling approach," *IEEE Trans. Microw. Theory Tech.*, vol. 57, no. 3, pp. 513–518, 2009.

[14] G. M. Raz and B. D. van Veen, "Baseband Volterra filters for implementing carrier based nonlinearities," *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 103–114, 1998.

[15] S. Benedetto and E. Biglieri, *Principles of Digital Transmission: With Wireless Applications*, 2nd ed. New York: Kluwer Academic/Plenum Publishers, 1999.

[16] M. Isaksson, *Behavioral modeling of Radio Frequency Power Amplifiers: An evaluation of Some Block Structure and Neural Network Models.* Licentiate Thesis, Department of Engineering Sciences, Uppsala University, 2005.

[17] D. R. Morgan, Z. Ma, J. Kim, M. G. Zierdt, and J. Pastalan, "A generalized memory polynomial model for digital predistortion of RF power amplifiers," *IEEE Trans. Signal Process.*, vol. 54, no. 10, pp. 3852–3860, 2006.

[18] L. Ding, "Digital Predistortion of Power Amplifiers for Wireless Applications," Ph.D. dissertation, Georgia Institute of Technology, 2004.

[19] A. Zhu and T. J. Brazil, "An overview of Volterra series based behavioral modeling of RF/microwave power amplifiers," in *Proc. Wirel. Micro. Techn. Conf.*, 2006, pp. 1–5.

[20] A. Saleh, "Frequency-Independent and Frequency-Dependent Nonlinear Models of TWT Amplifiers," *Communications, IEEE Transactions on*, vol. 29, no. 11, pp. 1715–1720, 1981.

[21] C. Rapp, "Effects of HPA-nonlinearity on a 4-DPSK/OFDM-signal for a digital sound broadcasting system," in *the Second European Conference on Satellite Communications*, 1991, pp. 179–184.

[22] D. Westwick and R. Kearney, "Identification of physiological systems: a robust method for non-parametric impulse response estimation," *Medical and Biological Engineering and Computing*, vol. 35, no. 2, pp. 83–90, 1997.

[23] H. B. Poza, H. L. Berger, and D. M. Bernstein, "A wideband data link computer simulation model," *Computers and Electrical Engineering*, vol. 5, no. 2, pp. 135–149, 1978.

[24] C. P. Silva, "Time-domain measurement and modeling techniques for wideband communication components and systems," *Int. J. RF Microwave Computer-Aided Eng.*, vol. 13, no. 1, pp. 5–31, 2003.

[25] C. P. Silva, A. A. Moulthrop, and M. S. Muha, "Introduction to polyspectral modeling and compensation techniques for wideband communications systems," in *ARFTG Conference Digest-Fall, 58th*, vol. 40, 2001, pp. 1–15.

[26] C. P. Silva, C. J. Clark, A. A. Moulthrop, and M. S. Muha, "Optimal-filter approach for nonlinear power amplifier modeling and equalization," in *Microwave Symposium Digest., 2000 IEEE MTT-S International*, vol. 1, 2000, pp. 437–440 vol.1.

[27] M. Abuelma'atti, "Frequency-Dependent Nonlinear Quadrature Model for TWT Amplifiers," *Communications, IEEE Transactions on*, vol. 32, no. 8, pp. 982–986, 1984.

[28] M. Schetzen, *The Volterra and Wienenr Theories of Nonlinear Systems.* New York: Wiley, 1980.

[29] V. J. Mathews and G. L. Sicuranza, *Polynomial Signal Processing.* New York: Wiley, 2000.

[30] I. W. Sanderg, "Uniform approximation with doubly finite Volterra series," *IEEE Trans. Signal Process.*, vol. 40, no. 6, 1992.

[31] R. D. Nowak and B. D. Van Veen, "Tensor product basis approximations for Volterra filters," *IEEE Trans. Signal Process.*, vol. 44, no. 1, pp. 36–50, 1996.

[32] J. Kim and K. Konstantinou, "Digital predistortion of wideband signals based on power amplifier model with memory," *Electron. Lett.*, vol. 37, no. 23, pp. 1417–1418, 2001.

[33] H. Ku and J. S. Kenney, "Behavioral modeling of nonlinear RF power amplifiers considering memory effects," *IEEE Trans. Microw. Theory Tech.*, vol. 51, no. 12, pp. 2495–2504, 2003.

[34] L. Ding and G. T. Zhou, "Effects of even-order nonlinear terms on power amplifier modeling and predistortion linearization," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 156–162, 2004.

[35] E. G. Lima, T. R. Cunha, H. M. Teixeira, M. Pirola, and J. C. Pedro, "Base-band derived volterra series for power amplifier modeling," in *Microwave Symposium Digest, 2009. MTT '09. IEEE MTT-S International*, 2009, pp. 1361–1364.

[36] H. Ku, M. D. McKinley, and J. S. Kenney, "Quantifying memory effects in rf power amplifiers," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 50, no. 12, pp. 2843–2849, 2002.

[37] F. Filicori and G. Vannini, "Mathematical approach to large-signal modelling of electron devices," *Electron. Lett.*, vol. 27, no. 4, pp. 357–359, 1991.

[38] D. Mirri, G. Iuculano, F. Filicori, G. A. V. G. Vannini, G. A. P. G. Pasini, and G. A. P. G. Pellegrini, "A modified Volterra series approach for the characterization of non-linear dynamic systems," in *Proc. IEEE Instr. Meas. Tech. Conf.*, vol. 1, 1996, pp. 710–715 vol.1.

[39] D. Mirri, F. Filicori, G. Iuculano, and G. Pasini, "A nonlinear dynamic model for performance analysis of large-signal amplifiers in communication systems," *IEEE Trans. Instrum. Meas.*, vol. 53, no. 2, pp. 341–350, 2004.

[40] A. Zhu, J. Dooley, and T. J. Brazil, "Simplified Volterra series based behavioral modeling of rf power amplifiers using deviation-reduction," in *Proc. IEEE MTT-S Int. Micro. Symp. Dig.*, 2006, pp. 1113–1116.

[41] A. Zhu, J. C. Pedro, and T. J. Brazil, "Dynamic deviation reduction-based Volterra behavioral modeling of RF power amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 12, pp. 4323–4332, 2006.

[42] A. Zhu, P. J. Draxler, J. J. Yan, T. J. Brazil, D. F. Kimball, and P. M. Asbeck, "Open-loop digital predistorter for RF power amplifiers using dynamic deviation reduction-based Volterra series," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 7, pp. 1524–1534, 2008, 0018-9480.

[43] E. Ngoya, N. Le Gallou, J. M. Nebus, H. Buret, and P. Reig, "Accurate rf and microwave system level modeling of wideband nonlinear circuits," in *Microwave Symposium Digest., 2000 IEEE MTT-S International*, vol. 1, 2000, pp. 79–82 vol.1.

[44] A. Soury, E. Ngoya, and J. M. Nebus, "A new behavioral model taking into account nonlinear memory effects and transient behaviors in wideband sspas," in *Microwave Symposium Digest, 2002 IEEE MTT-S International*, vol. 2, 2002, pp. 853–856.

[45] E. A. Heredia and G. R. Arce, "Piecewise volterra filters based on the threshold decomposition operator," in *Acoustics, Speech, and Signal Processing, 1996. ICASSP-96. Conference Proceedings., 1996 IEEE International Conference on*, vol. 3, 1996, pp. 1593–1596 vol. 3.

[46] K. Wan-Jong, C. Kyoung-Joon, S. P. Stapleton, and K. Jong-Heon, "Piecewise pre-equalized linearization of the wireless transmitter with a Doherty amplifier," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 9, pp. 3469–3478, 2006.

[47] N. Safari, "Linearization and Efficiency Enhancement of Power Amplifiers Using Digital Predistortion," Ph.D. dissertation, Norwegian University of Science and Technology, 2008.

[48] A. Zhu, P. J. Draxler, H. Chin, T. J. Brazil, D. F. Kimball, and P. M. Asbeck, "Digital predistortion for envelope-tracking power amplifiers using decomposed piecewise Volterra series," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 10, pp. 2237–2247, 2008.

[49] J. Dooley, B. O'Brien, and T. J. Brazil, "Behavioral modeling of RF power amplifiers using adaptive recursive polynomial functions," in *Proc. IEEE MTT-S Int. Micro. Symp. Dig.*, 2006, pp. 852–855.

[50] A. Zhu and T. J. Brazil, "RF power amplifier behavioral modeling using Volterra expansion with Laguerre functions," in *Proc. IEEE MTT-S Int. Micro. Symp. Dig.*, 2005, p. 4 pp.

[51] M. Isaksson and D. Ronnow, "A Kautz-Volterra behavioral model for RF power amplifiers," in *Proc. IEEE MTT-S Int. Micro. Symp. Dig.*, 2006, pp. 485–488.

[52] W. J. Jung, W. R. Kim, K. M. Kim, and K. B. Lee, "Digital predistorter using multiple lookup tables," *Electronics Letters*, vol. 39, no. 19, pp. 1386–1388, 2003.

[53] Q. Zhang, K. C. Gupta, and V. K. Devabhaktuni, "Artificial neural networks for rf and microwave design - from theory to practice," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 51, no. 4, pp. 1339–1350, 2003.

[54] G. Cybenko, "Approximation by superpositions of a sigmoidal function," *Mathematics of Control, Signals, and Systems (MCSS)*, vol. 2, no. 4, pp. 303–314, 1989.

[55] K. Hornick, M. Stinchcombe, and H. White, "Multilayer feedforward networks are universal approximators." *Neural Networks*, vol. 2, no. 5, pp. 359–366, 1989.

[56] T. Liu, S. Boumaiza, and F. M. Ghannouchi, "Dynamic behavioral modeling of 3g power amplifiers using real-valued time-delay neural networks," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 52, no. 3, pp. 1025–1033, 2004.

[57] E. Srinidhi, A. Ahmed, and G. Kompa, "Power amplifier behavioral modeling strategies using neural network and memory polynomial models," *Microwave Review Journal*, vol. 12, no. 1, pp. 15–20, 2006.

[58] J. Xu, M. C. E. Yagoub, D. Runtao, and Z. Qi-Jun, "Neural-based dynamic modeling of nonlinear microwave circuits," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 50, no. 12, pp. 2769–2780, 2002, 0018-9480.

[59] J. Wood, D. E. Root, and N. B. Tufillaro, "A behavioral modeling approach to nonlinear model-order reduction for rf/microwave ics and systems," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 52, no. 9, pp. 2274–2284, 2004, 0018-9480.

[60] H. Hwangbo, S. C. Jung, Y. Yang, C. S. Park, B. S. Kim, and W. Nah, "Power amplifier linearization using an indirect-learning-based inverse TDNN model," *Microw. J.*, vol. 49, no. 11, pp. 76–88, 2006.

[61] J. Xu, L. Mingyu, and X. Youbai, "Radial basis function neural network models for power-amplifier design," in *Communications, Circuits and Systems, 2004. ICCCAS 2004. 2004 International Conference on*, vol. 2, 2004, pp. 1066–1070 Vol.2.

[62] M. Isaksson, D. Wisell, and D. Ronnow, "Nonlinear behavioral modeling of power amplifiers using radial-basis function neural networks," in *Microwave Symposium Digest, 2005 IEEE MTT-S International*, 2005, p. 4 pp.

[63] M. Ibnkahla, "Applications of neural networks to digital communications - a survey," *Signal Process.*, vol. 80, no. 7, pp. 1185–12, 2000.

[64] S. Haykin, *Neural Networks*, 2nd ed. Upper Saddle River, New Jersey: Prentice-Hall, 1999.

[65] P. L. Gilabert, G. Montoro, and A. Cesari, "A recursive digital predistorter for linearizing rf power amplifiers with memory effects," in *Microwave Conference, 2006. APMC 2006. Asia-Pacific*, 2006, pp. 1040–1043.

[66] C. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. Academic Press, Inc., 1975.

[67] D. Schreurs, K. A. Remley, M. Myslinski, and R. Vandersmissen, "State-space modelling of slow-memory effects based on multisine vector measurements," in *ARFTG Microwave Measurements Conference*, 2003, pp. 81–87.

[68] D. D. Weiner and G. H. Naditch, "A scattering variable approach to the volterra analysis of nonlinear systems," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 24, no. 7, pp. 422–433, 1976.

[69] J. T. Stauth and S. R. Sanders, "Power supply rejection for rf amplifiers: Theory and measurements," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 55, no. 10, pp. 2043–2052, 2007.

[70] R. K. Pearson and M. Pottmann, "Gray-box identification of block-oriented nonlinear models," *Journal of Process Control*, vol. 10, no. 4, pp. 301–315, 2000.

[71] J. C. Pedro, N. B. Carvalho, and P. M. Lavrador, "Modeling nonlinear behavior of band-pass memoryless and dynamic systems," in *Microwave Symposium Digest, 2003 IEEE MTT-S International*, vol. 3, 2003, pp. 2133–2136 vol.3.

[72] L. Ljung, *System Identification: Theory for The User*. NJ: Prentice-Hall, 1986.

[73] S. M. Kay, *Fundamentals of Statistical Processing, Volume I: Estimation Theory*, ser. Prentice Hall Signal Processing Series. Prentice Hall Professional Technical Reference, 1993.

[74] I. Santamaria, J. Ibanez, M. Lazaro, C. Pantaleon, and L. Vielvo, "Modeling nonlinear power amplifiers in OFDM systems from subsampled data: a comparative study using real measurements," *EURASIP J. Appl. Signal Process.*, vol. 2003, no. 1, pp. 1219–1228, 2003, 1283369.

[75] P. Landin, M. Isaksson, and P. Handel, "Comparison of evaluation criteria for power amplifier behavioral modeling," in *Proc. IEEE MTT-S Int. Micro. Symp. Dig.*, 2008, pp. 1441–1444.

[76] D. Westwick and R. Kearney, *Identification of Nonlinear Physiological Systems.* John Wiley and Sons, 2003.

[77] H. Ku and J. S. Kenney, "Behavioral modeling of nonlinear rf power amplifiers considering memory effects," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 51, no. 12, pp. 2495–2504, 2003.

[78] D. Wisell, M. Isaksson, and N. Keskitalo, "A general evaluation criteria for behavioral PA models," in *Proc. 69th ARFTG Conf. Dig.*, 2007, pp. 251–255.

[79] S. LLoyd, *Programming the Universe: A Quantum Computer Scientist Takes on the Cosmos.* Alfred A. Knopf, 2006.

[80] D. de Ridder, E. Pekalska, and R. P. W. Duin, "The economics of classification: Error vs. complexity," in *Intern. Conf. Patt. Recog.*, vol. p.20244, 2002.

[81] P. Soderquist and M. Leeser, "Division and square root: Choosing the right implementation," *IEEE Micro*, vol. 17, no. 4, pp. 56–66, 1997.

Notice

All paper are available separately in Chalmers Publication Library

http://publications.lib.chalmers.se/cpl/

Paper A

Orthonormal-basis power amplifier model reduction

A. Soltani Tehrani, H. Cao, T. Eriksson and C. Fager

***Proc. of Integrated Nonlinear Microwave Circuits Workshop*, pp. 39-42, Nov. 2008.**

©IEEE 2008
Layout has been revised

Paper B

Dual-input Nonlinear Modeling for I/Q Modulator Distortion

H. Cao, A. Soltani, C. Fager, T. Eriksson and H. Zirath

***Proc. of IEEE Radio and Wireless Symposium*, pp. 39-42, Jan. 2009.**

©IEEE 2009
Layout has been revised

Paper C

A comparative analysis of the complexity/accuracy trade-off in power amplifier behavioral models

A. Soltani Tehrani, H. Cao, S. Afsardoost, M. Isaksson, T. Eriksson and C. Fager

Submitted to *IEEE Trans. on Microwave Theory and Techniques*, 2009.

©IEEE 2009
Layout has been revised