Towards Predictive Yaw Stability Control

Mohammad Ali*,** Claes Olsson* Jonas Sjöberg**

* Volvo Car Corporation
Vehicle Dynamics & Active Safety

** Chalmers University of Technology
Dept. of Signals and Systems

***Abstract*— In this paper the possibility to predict vehicle control loss using information about the host vehicles states and the road ahead is investigated. An introduction to conventional yaw stability control is presented and a threat assessment algorithm is proposed that can be used in an active safety system to e.g either issue earlier yaw control interventions or completely autonomous maneuvers in order to keep the vehicle on the road. In addition an experimental assessment in which a vehicle equipped with yaw stability control is driven on a test track is presented. It is shown that it is possible to predict powerful understeer situations if the future geometrical path of the vehicle is known.**

I. Introduction

It is well known that a huge amount of people are killed in traffic, according to e.g. the study presented in [1] approximately 40 000 people are [illegible] 3 [illegible] are injured each year in the US alone. Studies also show that unintentional roadw[illegible] share of traffic related fatalities, in e.g. [2] it is stated that in motorized countries about half of all fatal traffic accidents are single vehicle crashes.

During the years the automotive industry has developed active safety systems that aim to prevent or mitigate accidents. One example is the yaw stability control systems that assists the driver in regaining control over the vehicle.

In addition a new category of active safety systems is emerging and has started to appear on the market. These systems are often categorized with the name Collision Avoidance and Driver Support (CADS). CADS often use sensor data with information about the host vehicle position and surroundings, like road geometry or other vehicles position and velocity.

As CADS systems are introduced in vehicles, information about the vehicles surroundings will be available to be used by other systems as well. Current yaw control systems, however, are not ready to take advantage of preview capabilities envisioned to be a standard functionality in vehicles equipped with CADS. Given information about the geometry of the oncoming road, an intelligent yaw control system might steer the vehicle towards the actual direction of the road, rather than towards the direction of the front wheels. However such an autonomous intervention can be both intrusive and dangerous if issued at the wrong time. Before such an autonomous intervention can be issued, it therefore needs to be justified with a reliable threat assessment that can predict vehicle control loss within a future time horizon.

Today there is commercially available curve overspeed warning systems that is successful in warning for upcoming curves. However in e.g. the study presented in [3] no statistically significant change in driver behaviour due to these systems has been seen. In addition, a common requirement cited by drivers using these systems is that the amount of false alarms need to be reduced. The threat assessment used in these systems is relatively simple and suitable for issuing early warnings or velocity control in curves for e.g. an adaptive cruise control.

In section II, the term control loss is introduced and an explanation is given of how it should be interpreted in this paper, section III gives an introduction to state of the art yaw control and some of the limitations that is addressed in this paper. Further, a novel threat assessment approach is presented in section IV and experimental results in section V.

II. Loss of Control

In which situations a driver feels that he has lost control of his vehicle is of course highly dependant on the skills of the driver. An experienced driver in a race might e.g. intentionally create a skid and immediately correct the skid once the vehicle has performed the intended maneuver.

In general however it can be stated that maneuverability of a vehicle is dramatically decreased when the vehicle is driven close to the limit of adhesion between tyre and road [4]. In such situations the relation between the drivers input and the forces generated at the tyres contact patch is highly nonlinear making the vehicles response difficult for a driver to predict [5]. In addition, the weight of a vehicle is redistributed when a vehicle undertakes a powerful maneuver. When a vehicle e.g. brakes, a portion of the vehicles weight is shifted to the front and consequently the friction force at the front wheels is increased while the force at the rear wheels decreases. Likewise weight is redistributed laterally during cornering causing the tyre forces at one side to increase while they are decreased at the other.

All this affects the vehicles behaviour and the vehicle might either get into under- or oversteer. From a normal drivers point of view, the vehicle is perceived to turn less than the driver intended with his steering input in an understeer situation and more than intended in an oversteer situation. For a thorough explanation of the terms see e.g. [6].

Whether a vehicle tends to go into over- or understeer is sometimes discussed as a vehicle property. One parameter that has influence on a vehicles behaviour is e.g. whether the vehicle is driven by the front or the rear wheels. It is e.g. well known that with throttle on, a front wheel driven car is more likely to end up in understeer [6].

Even if the vehicle properties plays a big role, also the drivers behaviour has influence on whether the vehicle goes into under- or oversteer. Consider a driver in a front wheel driven car entering a curve in high speed. If the driver when realizing that he is driving too fast panics, he might e.g suddenly release the gas pedal and make a powerful turn. This will most likely put the vehicle in oversteer.

Over and understeer situations is a result of vehicle states, vehicle properties and driver behaviour. When the over or understeer becomes large enough, drivers are normally disturbed. As the over or understeer grows and becomes more evident a normal driver will feel that he is not able to control his vehicle. If the vehicle is equipped with a yaw stability control system it will issue an intervention that assists the driver and helps him regain control. These situations occur when the vehicle is operated in the region where the tyre forces nonlinear characteristics become evident. Such situations, in which maneuverability of the vehicle is reduced are referred to in this paper as situations where the driver has lost control. This means that in a situation where the vehicle is driven in the nonlinear region of the tyres, the driver is considered to have lost control, even if the driver is skilled and has intentionally provoked the situation.

III. Conventional Yaw Stability Control

Yaw stability control systems have been commercially available since the 1990s, [7]. The main idea is that the nonlinear behaviour of a vehicle in highly dynamic situations is too difficult for a driver to handle or understand and that he therefore needs some kind of assistance in such situations.

A. Underlying Idea

Loss of control can be identified by e.g. considering the vehicle slip angle β. The slip angle is illustrated in figure 1 and is defined as the angle of the velocity vector in the vehicles coordinate system. If β is large, turning the steering wheel will create little or no yaw moment on the vehicle, [8][9]. The possibility to control the vehicle through the steering wheel will then be limited. One of the main tasks of a yaw stability control system is thus to make sure that the slip angle remains low. How low it needs to be depends on available friction, in general it can be said that a higher slip angle can be allowed if much friction is available. Unfortunately it is not possible to measure the slip angle with sensors available in conventional vehicles. Estimation algorithms can be good in special conditions like e.g. during full braking, however in the general case, estimation of the slip angle can be quite uncertain [8]. Another measure is therefore introduced that considers the vehicles yaw rate to identify when the driver has lost control and needs assistance [8][9].

This measure, or the threat assessment and control principle that is based on it can be viewed in different ways. In e.g. [10] the threat assessment is explained as a comparison between the vehicles actual trajectory and an interpretation of the trajectory that the driver intends to follow. If the difference between the drivers intentions and the vehicles actual movement becomes too large the system decides to assist the driver in following the intended trajectory. Interpretation of the drivers intentions is done by feeding the drivers input i.e. steering angle through a simplified vehicle model with the assumption that it corresponds to the drivers perception of a vehicles behaviour. With this view, one can say that the control system aims at making the car follow the drivers intentions.

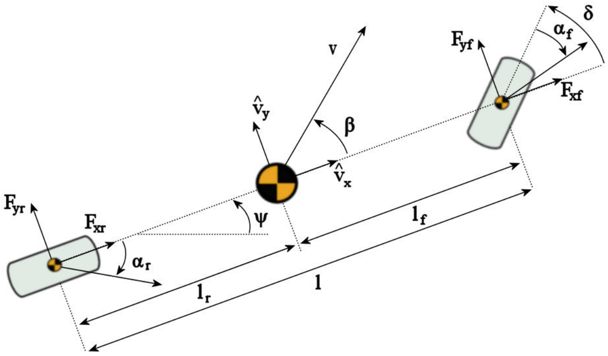

Fig. 1. Notation for the single track model, forces in the picture are expressed in the vehicles coordinate system.

Another perspective of the same procedure is presented in e.g. [11]. The assumption is again that the complex and nonlinear nature of a vehicle is difficult for a driver to handle. In extreme situations, a driver will therefore be unable to predict how the vehicle will respond to his inputs. By making the vehicle act according to the simplified vehicle model, it is assumed that the vehicles simplified behaviour will make the driver find it easier to predict how the vehicle will respond to his inputs. With this view one can say that the control system makes the vehicle easier to maneuver and reduces the risk that the vehicle runs of the road due to loss of control.

B. The Control Error

The simplified vehicle model that is used to compute the intended- or reference trajectory is a single track model and is illustrated in figure 1, [8][10][9]. The dynamic equations of motion for the single track model are

$$\ddot{\psi} = \frac{1}{J_z}(F_{yf}l_f - F_{yr}l_r) \tag{1}$$

$$\dot{v}_y = \frac{1}{m}(F_{yf} + F_{yr} + \dot{\psi}v_x) \tag{2}$$

where J_z is the vehicles moment of inertia in the yaw direction, m is the vehicle mass and the rest of the parameters are defined in figure 1.

The lateral tyre forces at each tyre are approximated to be linearly related to the tyre slip angle according to

$$F_{yi} = K_{yi}\alpha_i \qquad i = f, r \tag{3}$$

where K_{yi} is the cornering stiffness at each tyre [10]. The tyre slip angles are estimated and assuming small angles they can be approximated as

$$\alpha_f = \frac{v_y + l_f \dot{\psi}}{v_x} - \delta \tag{4}$$

$$\alpha_r = \frac{v_y + l_r \dot{\psi}}{v_x} \tag{5}$$

By integrating the dynamic equations of motion (1) and (2), a reference yaw rate $\dot{\psi}_{ref}$ is acquired. The calculation of the yaw rate reference might however vary, depending on the manufacturer of the yaw control system, in e.g. [9] the yaw rate reference $\dot{\psi}_{ref}$ is suggested as the steady state yaw rate instead i.e. the dynamics are neglected. In addition $\dot{\psi}_{ref}$ is usually upper bounded to take into account the physical limitation of available friction force [8][10][9]. In order for $\dot{\psi}_{ref}$ to vary smoothly, it is usually also low-pass filtered in some way, see e.g. [10]. This is particularly useful when friction is low and the vehicle reacts slowly to the drivers inputs [8].

Once $\dot{\psi}_{ref}$ has been calculated, the vehicles measured yaw rate is subtracted from it and the control error $\Delta\dot{\psi}$ is formed. In certain situations, where estimation of the slip angle is satisfactory, a corresponding $\Delta\beta$ is also calculated and the control error then becomes a weighted combination of $\Delta\dot{\psi}$ and $\Delta\beta$ [9]. [illegible] certain deadband th[illegible] vehicle is considered to respond well to the drivers inputs. H[illegible]wever when [illegible] sufficiently large, th[illegible] yaw stability [illegible]uce it by controlling the [illegible]gine torque and applying brake torque on individual wheels in order to generate additional yaw moment in the appropriate direction.

C. Limitations

Yaw stability control systems have been proven to be very efficient in reducing the amount of fatalities in traffic. In the study presented in [7], it is stated that these systems reduces the amount of fatal single vehicle crashes by 30-50% for cars and 50-70% for SUVs.

One limitation with conventional yaw stability systems is however that they only utilize measurements of the own vehicles states. No information about the vehicles surroundings is utilized and loss of control is not detected before it actually happens. In e.g. a powerful understeer situation however it might be beneficial if the control loss can be identified and attended to earlier. In understeer, a yaw control system could typically brake the inner back wheel in order to generate additional yaw moment. Such a situation is illustrated in figure 2, as can be seen available friction force in such a situation is low at the inner back wheel. This is due to that in a curve situation, much of the vehicles weight is redistributed to the outer side. The influence of the brake intervention is thus limited and if the understeer is severe, the brake force might not be sufficient to keep the vehicle on the road. If the brake intervention can be issued earlier, before available friction is reduced it will have a more significant effect and thus increasing the possibility for the vehicle to stay on the road.

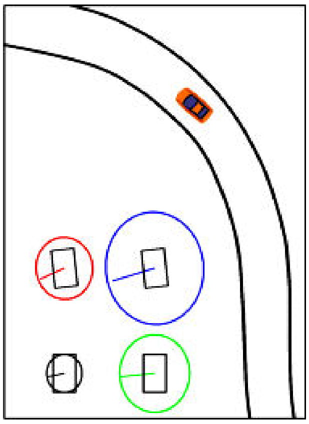

Fig. 2. Illustration of a vehicles load distribution in a curve situation. The ellipses represent available friction at each wheel, the forces produced at the tyres are constrained to lye within the ellipses.

In addition conventional yaw stability systems rely on the drivers actions in order to generate the reference trajectory. This means that if the driver does not behave well, due to e.g. panic, the vehicle might still leave the road. According to [8], it is common that vehicle motion reaches the limit of adhesion due to the panic reactions of the driver. Also in the study presented in [12] it was found that human factors are the defini[illegible] cause in approximately 70% of all crashes [illegible]nd that drive[illegible] where totally nonresponsible in only 2%. A reference, or i[illegible]licator that can utilize e.g. information about the road ahea[illegible]nd is less dependant on the drivers skills can [illegible]eneficial.

IV. Predicting Loss of Control

A. Conceptual Idea

In order to find a measure that is less dependant on the driver, one can use information about the geometry of the oncoming road and predict whether the vehicle will lose control within a future time horizon. This can be done by simulating the vehicles motion along its future path and evaluating key indicators in order to assess the vehicles yaw stability.

An indicator that is interesting to evaluate is e.g. the predicted $\Delta\dot{\psi}$. In particular, the maximum $\Delta\dot{\psi}$ of the simulation, $\Delta\dot{\psi}_{max}$ can be used as an indicator. In addition a corresponding β_{max} or $\Delta\beta_{max}$ can be used as indicator as well. The indicators can be used to either issue interventions by a separate predictive yaw control system or as a weighted part in the control error of a conventional yaw stability control system.

One problem that arises is however that even if full measurements of the vehicles states and the geometry of the road would be given, there is still an uncertainty about the drivers future behaviour. In a situation where a vehicle is e.g. approaching a curve in high speed it is difficult for an active safety system to know whether the driver intends to slow down before entering the curve. If the vehicles future motion is to be simulated, certain assumptions about the drivers future behaviour therefore has to be made. The assumptions can be such that, the driver is skilled and if there is a way for the driver to continue along the road without leaving it or losing control, he does so.

The drivers assumed or predicted future inputs and consequently the predicted states of the vehicle can then be obtained by solving an optimization problem. The steering angle and brake torque that is applied will then be chosen to minimize a cost function that is based on assumptions of the drivers future behaviour, over a predefined prediction horizon. In the formulation of the optimization problem, keeping the vehicle on the road and avoiding loss of control can be incorporated as soft constraints i.e. a higher cost is acquired if these requirements are not met.

The proposed indicators will then be based on a reference, assuming that the driver will try to keep the vehicle on the road while maintaining control. A control system that actively keeps these indicators low thus aims at maintaining vehicle controllability while keeping the vehicle on the road.

B. *Benefit*

Assuming a skilled driver leads to quite conservative simulations that will predict loss of control only once it has become inevitable. The conservative approach is however necessary if the simulations are to be used in order to justify interventions that can be perceived as intrusive by a driver. The cost of avoiding false alarms will thus be that the simulations will fail to predict loss of control in cases where the control loss could have been avoided by the driver.

The thresholds for when a system based on the proposed indicators should intervene is off course a parameter that can be subject to tuning in the classical balancing of not failing to predict loss of control when it occurs, while avoiding false alarms. Several thresholds can however be defined in which the severity of the interventions issued by such a system is gradually increased. On a first level when loss of control is predicted but with uncertainty the system could e.g. prepare the brakes for an intervention, while actual braking or steering can be issued later when loss of control is inevitable.

The benefit of predicting loss of control is greatest when the control loss is powerful. In particular, in a powerful understeer situation the benefit lies [illegible] intervention can be issued before available friction [illegible] is reduced i.e the actuator is used when it can deliver more. In such critical situat[illegible] by conventional yaw stability control, like maintaining the vehicles velocity is less important. On the contrary it is necessary to reduce the vehicles speed in order to help the driver maintain control. Using the brakes to reduce velocity in a critical situation can however be problematic since braking will reduce the side force, which in turn might be needed to stay on the road. A system that issues an early intervention can then be beneficial since it might enable the possibility to reduce speed before the side force is needed.

Powerful understeer situations, that might be caused by e.g. entering a curve with excessive speed on a slippery surface are easier to predict with certainty than situations where the understeer is less severe. It can therefore be argued that the situations that benefit the most of the predictive approach are less likely to be missed, while the benefit in situations that is difficult to predict is that the brakes are prepared when the thresholds for conventional yaw control are reached.

V. Experimental Results

As a preliminary assessment of the possibility to predict control loss, experimental testing has been conducted on a test track.

A. *Experimental setup*

A vehicle equipped with yaw stability control was driven around a test track by a professional test driver and control loss (mostly understeer) was provoked in several sections of the track. The vehicle was equipped with a differential gps and a high precision gyro. Measurement data from the equipment was logged together with measurements and other data from standard equipment available in the vehicle. The logged data gives accurate information about the vehicles position and movement. In addition the data contains information about if and when active systems like the yaw stability control system issues interventions. Given data about the vehicles position and movement at one point, the challenge is to evaluate whether the vehicle will lose control further on.

1) Prediction method and assumptions: The idea of simulating the vehicles future motion in order to assess whether the vehicle will lose control presented in section IV is adopted. The problem is however reduced and the future geometrical path of the vehicle is assumed to be known. The assumption of a skilled driver is maintained, without necessarily having a driver that is optimal with respect to some cost function.

In the simulation, the drivers control inputs are the wheel angle at the front wheels, δ and applied wheel torque, T. The brake torque, T is distributed with a fix ratio between the front and the back wheels. The driver is assumed to try to reduce speed by braking so that the longitudinal slip is kept at a desired value at the wheel where the absolute value of the slip is largest. The braking behaviour of the driver is modeled as a PI controller with torque, T as the control [illegible] desired longitudinal slip, κ as reference.

In alignment with the assumption that the driver is skilled, the reference longitudinal slip value is chosen so that it is high enough for the velocity to be reduced but not so high that the braking has a too significant impact on the acquired side force. The choice of longitudinal slip reference is thus the result of a balancing between reducing velocity and maintaining side force. The optimal choice is off course different, depending on the situation. In this experiment however, one balanced value is chosen for all situations.

The steering behaviour of the driver on the other hand is modeled as a PID controller, with front wheel angle, δ as control signal and the future geometrical path as reference. The reference path is available through the logged data.

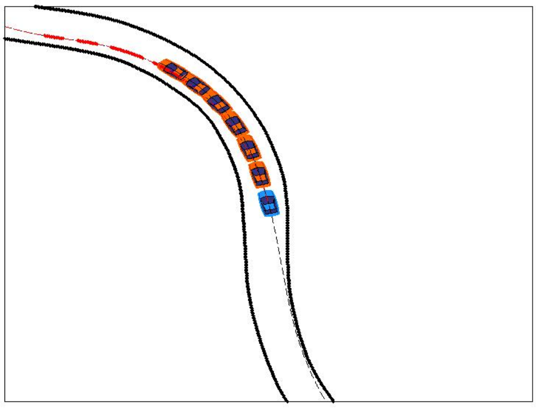

Fig. 3. An illustration. The blue car represents the host vehicle while the red cars show the simulated future motion of the blue car. The simulation gives an estimate of the possibility that the car will lose control further on in the curve. The simulation is repeated with a short interval as the car moves along the road.

With the assumptions above the vehicles future motion is simulated at each time sample and repeated with a predefined time interval. An illustration of a simulation is shown in figure 3. Each simulation is initiated with the measured states of the vehicle at that specific time instant.

As part of each simulation, a simulated $\dot{\psi}_{ref}$ is calculated as described in section III, the calculated $\dot{\psi}_{ref}$ is neither lowpass filtered nor upper bounded. A $\Delta\psi_{max}$ is then calculated and compared to the vehicles [illegible] Δ. A corresponding comparison is also conducted between the predicted β_{max} and the vehicles actual slip angle β.

2) Vehicle model: The level of [illegible] vehicle dynamics, in the simulations needs to be high enough to capture relevant information about the stability of the vehicle. The model used in this experiment is a double track vehicle model, with static load transfer. The dynamic equations of motion for a double track model are

$$\ddot{\psi} = \frac{1}{J_z}[(-F_{x1} + F_{x2} - F_{x3} + F_{x4})\frac{w}{2} + (F_{y1} + F_{y2})l_f - (F_{y3} + F_{y4})l_r] \quad (6)$$

$$\dot{v}_x = (F_{x1} + F_{x2} + F_{x3} + F_{x4})\frac{1}{m} + \dot{\psi} v_y \quad (7)$$

$$\dot{v}_y = (F_{y1} + F_{y2} + F_{y3} + F_{y4})\frac{1}{m} - \dot{\psi} v_x \quad (8)$$

$$\dot{\omega}_i = (T_i - f_{xi} r)\frac{1}{J_w} \qquad i = 1, 2, 3, 4 \quad (9)$$

where J_w is the wheel inertia and the rest of the notation is defined according to figure 4. The forces denoted f are expressed in the tyres coordinate system while they are denoted F when expressed in the vehicle frame. The forces are calculated in the tyres coordinate system and a coordinate transformation is applied when they are expressed in the vehicle frame. In the equations above the self aligning torque is neglected as is often done, see[13].

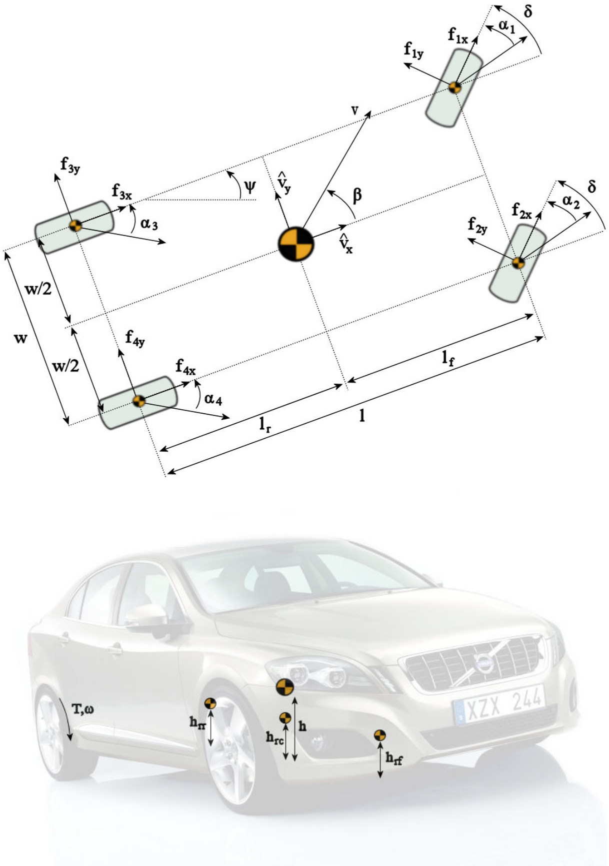

Fig. 4. Notation for the double track model, forces in the picture are expressed in the tyres coordinate system.

The magic tyre formula is used to calculate the tyre forces. In its general form the formula can be expressed

$$Y(x) = Dsin[Carctan\{Bx - E(Bx - arctan(Bx))\}] \quad (10)$$

with Y as either longitudinal or lateral tyre force and x as either longitudinal or lateral slip. The formula is a curve fitting and B,C,D and E are non dimensional parameters that depend on the vertical load. Using(10) the forces are calculated for pure slip conditions i.e. the interaction of lateral and longitudinal force is neglected. The combined slip effects are therefore taken into account according to

$$f_x = f_{x0} G_{x\alpha}(\alpha, \kappa, F_z) \quad (11)$$

$$f_y = f_{y0} G_{x\kappa}(\alpha, \kappa, F_z) + S_{Vy\kappa} \quad (12)$$

with f_{x0}, f_{y0} as the tyre forces under pure slip conditions, $G_{x\alpha}$, $G_{x\kappa}$ as weighting functions, $S_{Vy\kappa}$ the κ-induced side force and f_x, f_y as the tyre forces under combined slip conditions. The influence of the camber angle is not taken into account. A thorough explanation of the magic tyre formula can be found in e.g. [5].

The vertical load or normal force at each tyre is calculated according to

$$F_{z1} = \frac{mgl_r}{2l} - \frac{\Delta F_{z_{long}}}{2} - \frac{\Delta F_{zf_{lat}}}{2} \tag{13}$$

$$F_{z2} = \frac{mgl_r}{2l} - \frac{\Delta F_{z_{long}}}{2} + \frac{\Delta F_{zf_{lat}}}{2} \tag{14}$$

$$F_{z3} = \frac{mgl_f}{2l} + \frac{\Delta F_{z_{long}}}{2} - \frac{\Delta F_{zr_{lat}}}{2} \tag{15}$$

$$F_{z4} = \frac{mgl_f}{2l} + \frac{\Delta F_{z_{long}}}{2} + \frac{\Delta F_{zr_{lat}}}{2} \tag{16}$$

where g denotes gravitational acceleration, $\Delta F_{z_{long}}$ denotes longitudinal load transfer and $\Delta F_{zf_{lat}}$, $\Delta F_{zr_{lat}}$ denotes lateral load transfer. The load transfer is calculated using a static relation as

$$\Delta F_{z_{long}} = \frac{(T_1+T_2+T_3+T_4)h}{rl} \tag{17}$$

$$\Delta F_{zf_{lat}} = \frac{\dot{\psi} v_x m}{w}\left(\frac{h_{rf} l_r}{l} + R_{sf}(h - h_{rc})\right) \tag{18}$$

$$\Delta F_{zr_{lat}} = \frac{\dot{\psi} v_x m}{w}\left(\frac{h_{rr} l_f}{l} + R_{sr}(h - h_{rc})\right) \tag{19}$$

with R_{sf} and R_{sr} as the roll stiffness distribution at the front and rear axles and the rest of the notation according to figure 4. The tyre loads are calculated assuming a fix position of the roll axis, a constant roll stiffness distribution and an infinitely stiff chassis. A thorough derivation of the calculation of the tyre loads is provided in [14].

The lateral slip angles a[illegible]

$$\alpha_1 = \frac{v_y + l_f \dot{\psi}}{v_x - \frac{w}{2}\dot{\psi}} - \delta \tag{20}$$

$$\alpha_2 = \frac{[illegible]}{v_x + \frac{w}{2}\dot{\psi}} - \delta \tag{21}$$

$$\alpha_3 = \frac{v_y - l_r \dot{\psi}}{v_x - \frac{w}{2}\dot{\psi}} \tag{22}$$

$$\alpha_4 = \frac{v_y - l_r \dot{\psi}}{v_x + \frac{w}{2}\dot{\psi}} \tag{23}$$

and finally the longitudinal slip ratios can be expressed

$$\kappa_i = -\left(1 - \frac{\omega_i r}{v_x + \frac{w}{2}\dot{\psi}}\right) \qquad i = 1, 2, 3, 4 \tag{24}$$

B. Results

The conducted experiment showed promising results. The vehicle was driven around the test track with a quite aggressive driving style in order to test positive performance i.e. that loss of control is predicted before it happens and also with a normal driving style in order to test negative performance i.e. false alarms of loss of control are not issued. All understeer situations was predicted within the prediction horizon of two seconds. In addition no false alarms where issued. In two cases the vehicle got into oversteer and the yaw controller intervened, these situations where not identified by the predictions. In figure 5 some of the results from the testing is shown.

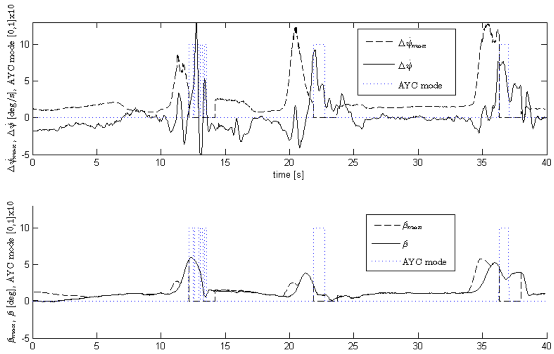

Fig. 5. Make sure to change this text.

VI. Conclusions and Future Work

A conceptual idea of a predictive approach to identify loss of control has been proposed. It has also been shown that powerful understeer can be predicted, using relatively simple assumptions about a drivers future behaviour if the future geometrical path of the vehicle is known.

In practice however the geometrical path is not known a priori and solving a nonlinear optimization problem online as proposed in the conceptual idea is computationally demanding. In an active safety system, assumptions can be made instead about the drivers intended future geometrical path. In order for these assumptions to be in line with the [illegible] approach o[illegible] assuming a skilled driver, a study is being conducted in which the nonlinear optimization problem is solved off line and the optimal solutions are studied. Based [illegible] study, an application can be designed to assume the future geometrical path of a skilled driver.

The friction coefficient, μ is a very important parameter, specially since it is on slippery road the proposed indicators are envisioned to have the greatest benefit. In the experiment presented in this paper, friction was known. In an active safety system however a friction estimator needs to be available allowing for the friction to be known at least in the point where the simulation is started. A sensitivity analysis on how much uncertainty in the friction estimate and other sensor information like the geometry of the road that can be allowed also needs to be conducted.

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