HOW MATHEMATICAL IS CONCEPTUAL UNDERSTANDING?

T. ADAWI, Å. INGERMAN AND A-M. PENDRILL
Chalmers University of Technology and Göteborg University, 412 96 Göteborg, Sweden
E-mail: tom.adawi@ckk.chalmers.se, ake.ingerman@mc2.chalmers.se,
Ann-Marie.Pendrill@fy.chalmers.se

Conceptual understanding has widely been suggested as the key first link to gain a solid physics understanding. By the means of empirical data from interviews with first year university students around force and friction problems, we argue that conceptual understanding has to be developed in conjunction with a structural understanding of the potential mathematical solution to master the area more thoroughly. Such structural understanding does neither necessarily seem to follow or precede a conceptual understanding.

Keywords: Conceptual understanding, Newtonian mechanics

1 Introduction

Conceptual understanding of physics has been pointed out as the missing essential in both teaching [1] and student understanding [2]. Concerns have been raised numerous times that students passing their exams do not necessarily have a good understanding of physics.

A widely used tool to give an indication of the level of the students' conceptual understanding of Newtonian mechanics is the Force Concept Inventory, FCI [3]. High scores on the FCI have been associated with success in class and on the exam [4], although the low FCI scores of students often surprise teachers.

For the Chalmers first-year engineering physics students' we observed a more worrying tendency: In spite of high entrance requirements and good FCI results (average ~80% before and after the course) the exam pass rate was poor (~25%). This cannot be explained with unrealistic demands in the exams, which were well on terms with local historical demands. Individual cross-correlation indicates that a good FCI score was a necessary, but not sufficient, condition for passing the exam. We take this to point to that while the students' conceptual understanding was solid, this was not enough to handle the more problem-focused and complex situations considered in the exam, which required clear understanding of the physical situations as well as a structured approach and algebraic/mathematical skill.

2 Empirical investigation

These results led us to start pondering why the students were having difficulties to move from the conceptual problems to the more engineering-oriented quantitative problem situations (even though the mathematics involved is in no way unreasonable in relation to the students' previous courses). This seemed to be related to mathematical aspects of the students' conceptual understanding and we felt prompted to explore the issue from the perspective of the students. We dwelled into empirical investigation, based on 6 student interviews. These touch extensively both on the reasoning the students have in connection with the kind of conceptual questions found in the FCI and when meeting a problem of the kind encountered in the exams, as well as students' reflections on the relationships between the natures of these different problems.

The first of these problems presented the students with the situation of an ox dragging a box. They were asked to discuss what forces acted in the situation. Then, they were asked what might happen if the mass of the box doubled.

The second of the problems concerned a board dragged on the ground using a string attached on the upper side of the board. The students were asked whether the angle between the board and the string mattered, and then to reason what might be an optimal angle to pull, if you wanted to pull as little as possible. In sequence, the students then were asked to discuss what might influence the optimal angle, if a plausible answer could be construed from those elements, how they would go about solving the problem, and finally to solve and evaluate the problem. Through setting up the force balances and solving for the magnitude of the pulling force, F, as a function of the pulling angle, α, the result is

$$F(\alpha) = \mu mg / (\cos\alpha + \mu\sin\alpha), \tag{1}$$

where μ is the coefficient of friction. The optimal angle is then given by the relation

$$\mu = \tan \alpha. \tag{2}$$

All six students were interviewed about two-thirds through their course in mechanics and should therefore be thoroughly familiar with solving mechanics problems around force and friction. The interviews, which took about one hour each to conduct, were audio taped and transcribed verbatim soon after the event.

3 Empirical observations

We found the variation in the ways the students handled the problem situations were fascinating, both within the group and as individuals. Below two case studies are offered in the form of commented narratives for "Emilia" and "Emil" (which are traditional generic names of students at Chalmers University of technology). We have tried to bring out the conceptual struggles they engaged in, the way in which they addressed the problems and their strengths and weaknesses as we as teachers see them. Both of these students passed the exam, while Emil is the stronger student both in terms of exam results and interview reasoning.

3.1 Emilia

In the first problem, when Emilia is asked to identify the forces acting on the box and the ox, she is not able to identify the force from the box that is acting on the ox and she claims that the frictional force on the ox is directed backwards. After much discussion and prompting, she realizes that there is a force from the box on the ox. This also leads her to the conclusion that the frictional force on the ox has to be directed forwards – "otherwise he will just be moving backwards" – which she then immediately relates to the everyday experience of walking. But she also adds: "I am not completely sure, definitely not, but this is the way I see it."

When asked about the relation between the identified forces, Emilia claims that the force pulling on the box has to be *larger* than the frictional force on the box if the box is moving at a *constant* speed. It is interesting to note that, later in the interview, she mentions that if the frictional force on the box "is *larger* than [the pulling force on the box] then it will stand *still*". The naïve laws of motion that she uses therefore depend on the context. She does not notice this conflict during the interview.

In the second problem, when Emilia is asked if the pulling force depends on the angle, she does not understand the nature of the problem because she starts by assuming that the pulling force is *constant* and says that the vertical and horizontal components of the force will depend on the angle, which will make it more or less difficult to pull the board.

It is only after identifying all the forces acting on the board and writing down the equations of motion that Emilia realizes that there could be an optimal angle “since the friction also depends on the force upwards, since it affects the normal force.” But then she gets stuck and do not know how to proceed: “It [the equation of motion] does not mean anything to me. I have difficulties seeing what I am looking for, what I must do to get an angle”. (It should be pointed out that she is working with “inequalities of motion” since, for her, constant motion requires an imbalance in forces. This makes the whole thing much more difficult for her.)

After a while and much confusion, Emilia solves for the force, gets the correct function of the angle in the denominator, and realizes that the smallest pulling force is obtained for the largest value of the function. In order to find the largest value of this simple trigonometric function, she starts to try different angles, but also realizes that “there is perhaps a better way of doing it, finding the maximum. It is perhaps simple but I cannot think of it right now.” However, when asked how she would determine the maximum/minimum value of a graph she immediately replies: “Then I would use derivatives.” Her direct answer to the question why she is not using derivatives in this case is: "Because I can not make the connection between mathematics and mechanics.”

Interestingly, when Emilia is asked what would happen to the optimal angle if the mass of the board is doubled, she goes back to the beginning of the problem to look at how each force changes before she concludes that the angle would not change.

At the end of the interview, when asked to comment on the two problems, Emilia says that although she realizes that the second problem is mathematically more complex, she personally finds the second problem (board) “much, much easier” than the first one (box and ox), but otherwise she does not think that there is a big difference between the problems.

3.2 Emil

Emil does not have any difficulties with the first problem: he quickly identifies all forces acting on the box and on the ox and is able to write down the relevant relations between the forces. He also comments on the frictional force acting on the ox: “It is a tricky thing … like with moving cars … the ox pulls it legs backwards so there must be a force forwards.”

Interestingly, when asked to discuss what happens if the mass of the box is doubled and still moving with constant velocity, Emil first maintains that it is impossible to say anything about how the frictional force on the box changes since “there is a limit to how large it can become and then the frictional force does not increase anymore.” He seems to confuse the frictional force on an object before it starts to move (which cannot increase indefinitely) with the frictional force on a moving object (which is proportional to the normal force).

In the second problem, when Emil is asked if the pulling force depends on the angle, he is able to see that an upward force makes the frictional force smaller but he thinks that this effect is negligible, and hence decides that it is best to pull parallel to the board.

Emil easily writes down the equations of motion for the board and quickly solves for the pulling force. He also realizes that he needs to maximize the function in the denominator to get the best pulling angle, but finds it very difficult and gives up saying: “I am worthless at trigonometric formulas.” On the other hand, when he is asked how he would determine the maximum/minimum value of a function he immediately replies: “derivatives”. He then easily differentiates the trigonometric function, puts it equal to zero, and gets the optimal angle as a function of the coefficient of friction. He also instantly concludes that the optimal angle does not change if the mass of the board is doubled.

At the end of the interview, when asked to comment on the two problems, Emil says that he finds the problems equally difficult and he does not think that there is a big difference between the problems.

4 Conclusions

From the empirical data we conclude that conceptual understanding is not the only sensible way for students to make sense of mechanics problems. Emilia is conceptually weak, and lacks some basic conceptual understanding of Newton's laws, which does not seem to come to her throughout the interview. But perhaps surprisingly she can still offer quite complex and insightful reasoning around the two problems, with respect to what we would call the mathematical structure of the problems, even though she struggles also with that side. Emilia uses a phenomenological understanding (i.e. intuition and everyday experience) to help her understand the physics, but, although she is quite capable/confident in using mathematics, the link of mathematics to physics – and intuition – is very weak. Emil is stronger, both conceptually and structurally, but the link between mathematics and physics sometimes eludes him.

Putting the two empirical illustrations in relation to the exam results that were the puzzling starting point of the investigation, the conclusion is that conceptual understanding is not the only necessary part, but that there is also a *structural* aspect to grasp, before being able to put them together to a solid understanding. Neither conceptual nor structural understanding is necessarily the first step, and they do not follow from each other. The students, having a well-developed conceptual understanding, did not get the possibility to develop the structural aspects to handle the difficulties in the exam.

Next time around, we will thus as teachers strive to emphasize things as being able to see the structure of possible solutions and answers before actually solving the problem and what it means to have a system of equations for several unknowns, out of which some are interesting and some not.

Acknowledgements

We thank the students for sharing their time and experiences. We gratefully acknowledge fruitful discussions with Cedric Linder. Support for this work was provided by the committee for educational science (UVK) of the Swedish Research Council (VR), the Centre for Digital Media and Higher Education, Chalmers (CKK), and by Chalmers Strategic Effort in Learning and Teaching (CSELT).

References

[1] P. Hewitt, The missing essential: A conceptual understanding of physics, American Journal of Physics **51** (1983) 305-311

[2] L. C. McDermott, Physics education research – the key to student learning, American Journal of Physics **69** (2001) 1127-1137.

[3] D. Hestenes, M. Wells and G. Swackhammer, Force concept inventory, The Physics Teacher **30** (1992) 141-158.

[4] C. Henderson, Common concerns about the force concept inventory, The Physics Teacher **40** (2002) 542-547.