Target control of mechanical systems: A posteriori error estimation and weak formulation of inequality constraints

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ABSTRACT

In this contribution a class of optimal control problems concerning the steering of motion of a mechanical system from an initial state to a target state (target or trajectory control) is considered. The motion of the system depends on forces acting as controls via a set of ordinary differential equations. By considering the equations of motion and the relevant kinematic and control limitations, a constrained optimization problem can be formulated where the control forces are sought to minimize a chosen objective function, such as the energy consumption or the deviation from desired trajectory, while reaching the defined target. Two examples will be considered: the musculoskeletal motion of a human body considered as a discrete mechanical system [3] and the search for an optimal brake-turning strategy for a vehicle manoeuver [4].

For the numerical solution of the optimal control problem, a discretization of finite element type in time is introduced, whereby approximations for the state (coordinates and velocities) and control (external forces) variables are introduced. The optimality conditions are expressed in weak form, in particular, the inequality constraints are enforced weakly, resulting in a nonlinear system of the sought nodal values determining the discrete solution for the state and control. The subject of the present work is to determine the error in the approximate solution compared to the exact solution, in particular with respect to how well the discrete solution satisfies inequality constraints and target conditions.

A feature of the proposed scheme is that the control and state variables are discretized separately. Form previous work on a posteriori error estimates based on the pertinent dual problem (from linearization of the weak form), discretization errors in both state and control variables can be estimated. The sources of errors can be traced to specific regions of the state and control time-meshes, which can be used in an adaptive mesh-refinement procedure. Earlier work on a posteriori error estimation for optimal control problems have been based on the ”optimal control” approach [1], whereas the present contribution will use our previous work in error control for parameter identification problems based on a tangent form of the dual problem [2].

To illustrate the problem setting, consider the example taken from [3] of a very simple dynamical system (a double pendulum) representing the movement of the upper arm in the sagittal plane. The arm is to be lifted from vertical hanging to a horizontal straight position using minimal energy (or related measure) without violating anthropomorphic constraints (the opening angle of the elbow must be between 0 and 135 degrees) and control constraints (the control variables are restricted by maximum and minimum values). The solution algorithm is based on a nested format with a relaxation of the constraints.

The numerical example indicate that a discretization error in the control variable arises in order to ”compensate” for discretization errors in the solution of the equations of motion in order to reach the target state.

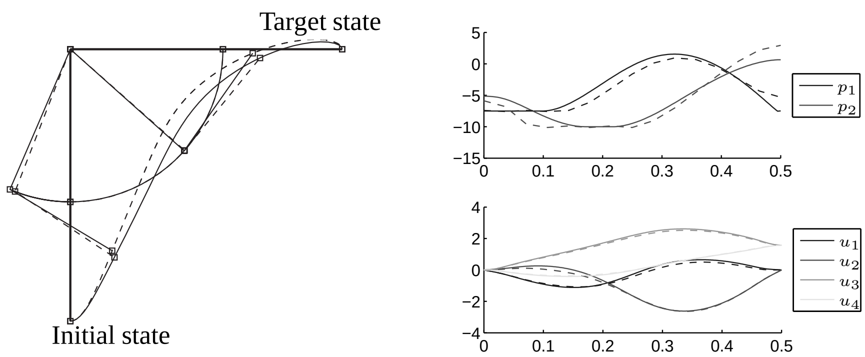

Figure 1: Left: Plot of the arm from vertical hanging to a horizontal straight position. Right: Comparison of the solved state (u_1-u_4) and control (p_1-p_2) variables using fine (dashed line) and coarse discretization (solid line).

References

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