Spintronics-based mesoscopic heat engine

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We consider a nanowire suspended between two spin-polarized leads and subjected to a nonuniform magnetic field. We show that a temperature drop between the leads can significantly affect the nanowire dynamics. In particular, it is demonstrated that, under certain conditions, the stationary distribution of the mechanical subsystem has a Boltzmann form with an effective temperature, which is lower than the temperature of the "cold" lead; this seems rather counterintuitive. We also find that a change in the direction of the temperature gradient can result in the generation of mechanical vibrations rather than the heating of the mechanical subsystem.

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I. INTRODUCTION

Nanomechanical resonators are devices, which are being employed not only to develop new technological applications, such as ultrasensitive sensors,[1–4] but also to shed light on fundamental questions, such as the transition from the classical to the quantum-mechanical description of macroscopic objects.[5] Investigation of a system where a mechanical degree of freedom controls the properties of a mesoscopic junction between two bulk leads is an important line of research in nanomechanics.[6–8] In such structures, the mechanical part may be considered as a nanoengine whose operation is controlled by the states of two bulk thermodynamic reservoirs. It is well known that a macroscopic mechanical system may be driven into cyclic motion if it is coupled to reservoirs held at different temperatures. A Stirling engine, operating by cyclic compression and expansion of air or other gas and placed between hot and cold spaces, is one example.

By decreasing the size of the heat engine to the nanoscale level, quantum mesoscopic effects come into play and determine the behavior of both the working substance and the mechanical subsystem. This opens new possibilities for the operation of a heat engine, for instance, reduction or even suppression of the mechanical fluctuations—effective cooling of the mechanical subsystem. Recently, it was shown that suppression, leading to ground-state cooling, may be achieved if reservoirs, presented by normal or superconducting metal leads, are held at different electrochemical potentials.[9–12] It was also demonstrated that a temperature drop between reservoirs can also generate this effect if one assumes a very special "three-particle" interaction inside a junction.[13,14] In this paper, we investigate heating, pumping, and cooling of a mechanical mode in a realistic nanojunction where only a "two-particle" interaction between the mechanical degree of freedom and the working subsystem exists. We show that a temperature drop between linked leads can generate cooling or excitation, depending on its direction, of the mechanical subsystem.

To be specific, we consider a carbon nanotube suspended between $La_{1-x}Sr_xMnO_3$ ferromagnets of opposite polarizations (a structure recently realized experimentally[15]) and subject to a nonuniform magnetic field **B** [cf. Fig. 1(b)]. The field can be generated, for example, by a magnetic STM tip in the form of a wedge, which is in proximity to the nanotube [cf. Fig. 1(a)]. If the nanotube is not too long (length $L \lesssim 1\ \mu$m), it can be considered as a quantum dot coupled to the electrodes through tunnel junctions. We assume that inside the nanotube there is only one doubly degenerate, with respect to spin, and spatially quantized electronic level, which participates in electron exchange with the ferromagnetic leads.

If the nanotube is straight and is positioned below the STM sharp end, as shown in Fig. 1(b), the applied magnetic field is directed toward the nanotube (z direction) and Zeeman splits the degenerate electronic level of the nanotube. A two-level system (TLS) is, thus, formed in the latter with levels $\sigma_z = \uparrow, \downarrow$ and energies $\epsilon_{\uparrow,\downarrow}$, respectively. Deflection of the suspended part of the nanotube in the x direction generates interlevel transitions (spin flip) in the TLS. Consequently, the nonuniform magnetic field induces coupling between mechanical and electronic subsystems. In what follows, we will refer to this mechanism of interaction between mechanical and electronic subsystems as *spin-mechanical coupling*. The latter can also be achieved in a different setup by spin-orbit coupling.[16]

We assume that the left and right leads are completely polarized along the z direction (i.e., the density of states are $\nu_{R(L)}^{\uparrow(\downarrow)} = 0$ and $\nu_{R(L)}^{\downarrow(\uparrow)} \equiv \nu_{R(L)} > 0$), and the energy difference between levels of the TLS is $\Delta \equiv \epsilon_\uparrow - \epsilon_\downarrow > 0$. If there is no spin-mechanical coupling and the intrinsic relaxation time τ of the TLS is larger than the dwell time of an electron in the nanotube, then the occupation number of the spin up (down) state inside the nanotube is $n_{\uparrow(\downarrow)} \simeq f_F[(\epsilon_{\uparrow(\downarrow)} - \epsilon_{FL(R)})/T_{L(R)}] \equiv f_{L(R)}$, where f_F is the Fermi-distribution function, $\epsilon_{FL(R)}$ and $T_{L(R)}$ are the Fermi energy and temperature of the left (right) lead, respectively.

The spin-mechanical coupling generates spin-flip transitions between the energy levels of the TLS. These transitions are inevitably accompanied by absorption or emission of mechanical quanta. The cooling (pumping) process of the mechanical subsystem is the result of an electron transition from the lower (upper) energy level to the upper (lower) energy level of the TLS. In order to effectively cool (pump) the mechanical mode, it is required that $n_\uparrow \ll n_\downarrow$ ($n_\uparrow \gg n_\downarrow$). This can be realized by applying a bias voltage or by subjecting the leads to different temperatures. Below, we consider cooling and pumping of the mechanical subsystem generated only by a temperature gradient between the leads.

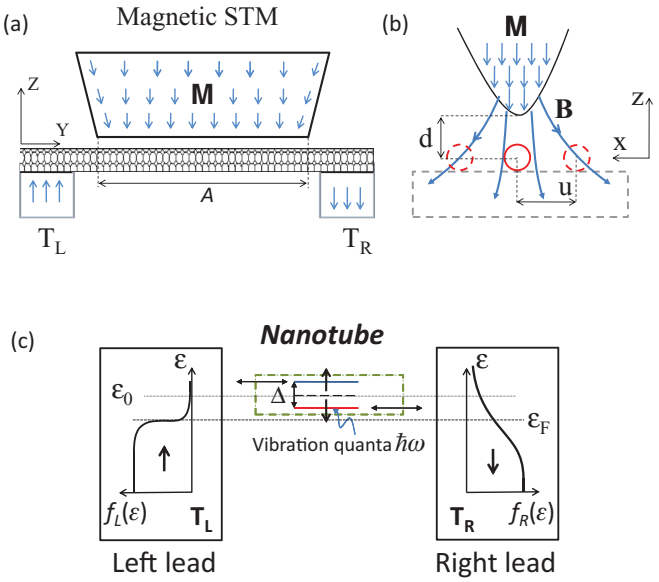

FIG. 1. (Color online) (a) A nanotube suspended between two spin-polarized leads and in proximity to a magnetic scanning tunneling microscopic (STM) tip with magnetization **M**. The leads have opposite polarizations along the z direction. (b) Nonuniform magnetic field **B** created by the magnetic tip. The nanotube (circles) deflection u is in the x direction. (c) A doubly spin-degenerate electronic level exists in the nanotube at energy ϵ_0. The applied magnetic field splits this level into two levels $\sigma_z = \uparrow, \downarrow$ separated by an energy $\Delta \propto B_z \mu_B$. The leads are held at different temperatures $T_{L,R}$ with a zero-bias voltage.

II. MODEL

To perform a quantitative analysis of the system described above, we consider the Hamiltonian,

$$H = H_{nw} + H_l + H_t, \tag{1}$$

$$H_{nw} = \hbar\omega \hat{b}^\dagger \hat{b} + \sum_{\sigma=\uparrow,\downarrow} \epsilon_\sigma \hat{n}_\sigma + g\hat{u}(\hat{d}^\dagger_\uparrow \hat{d}_\downarrow + \hat{d}^\dagger_\downarrow \hat{d}_\uparrow), \tag{2}$$

$$H_l = \sum_k \epsilon_L(k) \hat{a}^\dagger_{k,\uparrow,L} \hat{a}_{k,\uparrow,L} + \epsilon_R(k) \hat{a}^\dagger_{k,\downarrow,R} \hat{a}_{k,\downarrow,R}, \tag{3}$$

$$H_t = \sum_k t_L \hat{a}^\dagger_{k,\uparrow,L} \hat{d}_\uparrow + t_R \hat{a}^\dagger_{k,\downarrow,R} \hat{d}_\downarrow + \text{H.c.}, \tag{4}$$

where $\hat{a}_{k,\sigma,L(R)}(\hat{a}^\dagger_{k,\sigma,L(R)})$ and $\hat{d}_\sigma(\hat{d}^\dagger_\sigma)$ are the annihilation (creation) operators for electrons in the left (right) leads and in the nanotube, respectively, and $\hat{n}_\sigma = \hat{d}^\dagger_\sigma \hat{d}_\sigma$.

The first term in Eq. (2) describes the nanotube mechanical degrees of freedom, which we restrict to the fundamental flexural mode. This mode is described as a simple harmonic oscillator with vibrational frequency ω and $\hat{b}(\hat{b}^\dagger)$ as the annihilation (creation) operator for an elementary excitation (vibron). The second term in Eq. (2) describes the TLS with energy levels $\epsilon_\sigma = \epsilon_0 \pm \Delta/2$, where Δ is the Zeeman-splitting energy proportional to the z component of the applied magnetic field B_z and ϵ_0 is the zero-field energy, which is measured relative to the lead Fermi energy. The last term in Eq. (2) describes the spin-mechanical coupling. It is proportional to the oscillator displacement $\hat{u} = (\hat{b}^\dagger + \hat{b})/\sqrt{2}$ and to the spin-flip operator. The coupling parameter g is equal to $C\mu_B x_0 \partial_x B_x(\mathbf{0})$, where μ_B is the Bohr magneton, x_0 is the zero-point vibrational amplitude, $\partial_x B_x(\mathbf{0})$ is the field gradient along the x direction, and C is a numerical factor $\sim A/L$, where L is the nanowire length and A is the length of the STM wedge. This factor accounts for the electronic state inside the nanotube being extended over the whole length of the nanotube, whereas, the magnetic field is concentrated only in the region below the STM. The field gradient $\partial_x B_x(\mathbf{0})$, induced by a Fe-based magnetic tip 7.5-nm thick with magnetization $M = 1.75 \times 10^6$ A/m at a distance 7.5 nm, is on the order of 35 mT $\times$ nm^{-1}. For this value of field gradient and for a nanotube with vibrational frequency $\omega = 2\pi \times 100$ MHz, mass 1 ag, and $A/L = 0.1$, g is on the order of $2\pi \times 10^6$ Hz. The interaction term between the mechanical and the electronic subsystems can be formally derived in a way similar to the one given in Ref. 17. The term H_l in Eq. (3) describes the left and right leads. The term H_t in Eq. (4) describes the tunneling of electrons from the nanowire to the leads and vice versa, and $t_{L(R)}$ are tunneling amplitudes.[18]

To analyze the performance of the system, we start from the Liouville-von Neumann equation for the total density operator $\hat{\varrho}$ and then eliminate the lead electronic degrees of freedom.[19] If the temperatures of the leads are much greater than $\hbar\omega/k_B$, the Fermi distributions $f_{L,R}(\epsilon)$ are smooth functions within the energy interval $\hbar\omega$. As a result, one gets the following Lindblad master equation (5) for the reduced density matrix $\rho = \text{Tr}_{R+L}\hat{\varrho}$. The latter describes the mechanical degree of freedom and the electronic state of the TLS of the nanotube,

$$\partial_t \rho = -\frac{i}{\hbar}[H_{nw}, \rho] + \mathcal{L}_L[\rho] + \mathcal{L}_R[\rho], \tag{5}$$

where

$$\mathcal{L}_\alpha[\rho] = \Gamma_\alpha[(1 - f_\alpha)\hat{d}_{\sigma_\alpha}\rho\hat{d}^\dagger_{\sigma_\alpha} + f_\alpha(\hat{d}^\dagger_{\sigma_\alpha}\rho\hat{d}_{\sigma_\alpha}\rho) - (1/2 - f_\alpha)\{\hat{n}_{\sigma_k}, \rho\}]. \tag{6}$$

Here, $\alpha = (L,R)$, $\sigma_{L(R)} = \uparrow (\downarrow)$, and $\Gamma_\alpha = 2\pi|t_\alpha|^2 \nu_\alpha/\hbar$ are the tunneling rates, ν_α is the density of states, and $\{\hat{A}, \hat{B}\}$ denotes an anticommutator. The collision integrals $\mathcal{L}_\alpha[\rho]$ describe the decoherence in the electronic subsystem induced by the bulk electronic reservoirs.

If the resonant condition $\Delta = \hbar\omega$ is fulfilled and $\omega \gg g/\hbar, \Gamma_L$, the rotating-wave approximation can be used to obtain the following rate equations:

$$\begin{aligned}
\dot{P}_m(n) &= \tilde{g}[P^i_{\uparrow\downarrow}(n) - P^i_{\uparrow\downarrow}(n+1)], \\
\dot{P}_\uparrow(n) &= -\tilde{g}P^i_{\uparrow\downarrow}(n+1) - \Gamma_R f_R P_\uparrow(n) + \Gamma_L f_L P_0(n) \\
&\quad - \Gamma_L(1 - f_L)P_\uparrow(n) + \Gamma_R(1 - f_R)P_2(n), \\
\dot{P}_\downarrow(n) &= \tilde{g}P^i_{\uparrow\downarrow}(n) + \Gamma_R f_R P_0(n) - \Gamma_L f_L P_\downarrow(n) \\
&\quad + \Gamma_L(1 - f_L)P_2(n) - \Gamma_R(1 - f_R)P_\downarrow(n), \\
\dot{P}_2(n) &= -\Gamma_L(1 - f_L)P_2(n) - \Gamma_R(1 - f_R)P_2(n) \\
&\quad + \Gamma_L f_L P_\downarrow(n) + \Gamma_R f_R P_\uparrow(n), \\
2\dot{P}^i_{\uparrow\downarrow}(n) &= \tilde{g}n[P_\uparrow(n-1) - P_\downarrow(n)] - (\Gamma_L + \Gamma_R)P^i_{\uparrow\downarrow}(n),
\end{aligned} \tag{7}$$

where $\tilde{g} = g\sqrt{2}/\hbar$. Here, $P_\sigma(n)$, $P_0(n)$, and $P_2(n)$ are the joint probabilities to find a vibrational mode in a Fock state with n vibronic quanta and one electron on the nanotube with spin σ, empty nanotube, and two electrons on the nanotube, respectively. Therefore, the total probability to find n

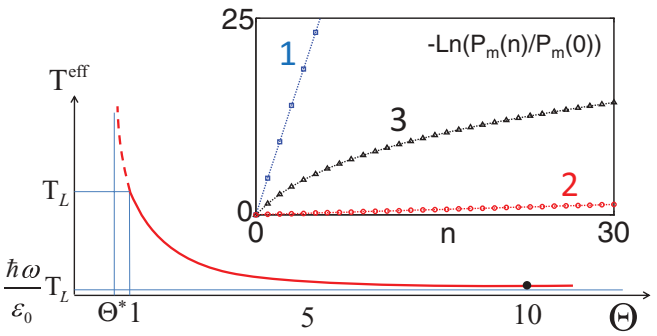

FIG. 2. (Color online) Effective temperature $T^{\rm eff}$ of the vibrational mode as a function of the ratio $\Theta = T_R/T_L$. For $\Theta > 1$, $T^{\rm eff} < \min\{T_L, T_R\}$, for $\Theta^* < \Theta < 1$, $T^{\rm eff} > \max\{T_L, T_R\}$, and for $\Theta < \Theta^*$, no stationary distribution exits unless additional dissipation mechanisms are included. Inset: Vibron stationary distribution on a logarithmic scale when the mechanical subsystem interacts only with ferromagnetic leads (curve 1), only with a bosonic bath at temperature T_b and coupling parameter γ (curve 2), and both the fermionic and bosonic baths (curve 3). We use $\omega/2\pi = 100$ MHz, $T_L = 0.02$ K, $\Theta = 10$, $T_b = (T_L + T_R)/2$, $\epsilon_0 = k_B T_R/2$, $\Gamma_L = \sqrt{2}g/\hbar$, $\Gamma_R = 1.62g/\hbar$, and $\gamma = 0.002g/\hbar$.

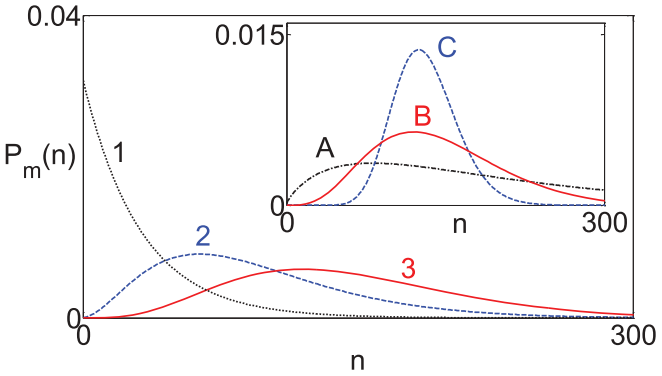

FIG. 3. (Color online) Vibron stationary distribution $P_m(n)$ when $\Theta < \Theta^*$. We include a bosonic bath with temperature T_b and coupling parameter γ. The stationary distribution exhibits a peak at $n_{\rm max}$, which increases as γ gets smaller. We use $n_b\gamma\hbar/g \gg 1$ (curve 1), $n_b\gamma\hbar/g = 0.021$ (curve 2), and $n_b\gamma\hbar/g = 0.01$ (curve 3). Inset: The width of the distribution $P_m(n)$ scales inversely with T_b. $T_b(A) = 1.5$ K, $T_b(B) = 0.3$ K, $T_b(C) = 0.06$ K, and $n_b\gamma\hbar/g = 0.01$ for all curves A–C.

vibronic quanta is $P_m(n) \equiv P_0(n) + P_\uparrow(n) + P_\downarrow(n) + P_2(n)$. The off-diagonal elements of the density matrix $P^i_{\uparrow\downarrow}(n) = {\rm Im}\langle 0|\hat{d}_\uparrow b^{(n-1)}\rho(\hat{b}^\dagger)^n\hat{d}^\dagger_\downarrow|0\rangle/(n-1)!$ describe the quantum entanglement between the electronic and the mechanical subsystems generated by the correlation between the spin flip of an electron and a change in the number of vibronic quanta.

III. EFFECTIVE TEMPERATURE OF THE MECHANICAL SUBSYSTEM

Equation (7) always has a stationary solution where $P^{st}_m(n)$ has a Boltzmann form

$$P^{st}_m(n) = Z^{-1}\exp(-\hbar\omega n/k_B T^{\rm eff}), \tag{8}$$

where $Z = [1 - \exp(-\hbar\omega/k_B T^{\rm eff})]^{-1}$ and the effective temperature is given by

$$T^{\rm eff} = T_L\frac{\hbar\omega}{\epsilon_0}\left[1 - \Theta^{-1} + \frac{\hbar\omega}{2\epsilon_0}(1+\Theta^{-1})\right]^{-1}, \tag{9}$$

where $\Theta = T_R/T_L$. This solution has physical meaning only if $T^{\rm eff} > 0$. Negative effective temperature indicates that there is a permanent energy pump into the mechanical subsystem. In this case, in order to achieve a stationary regime, one has to introduce additional external sources of dissipation.

From Eq. (9), one can see that, for $\epsilon_\downarrow > 0$ and $T_R > T_L$, the vibrational mode is effectively cooled to a final temperature, which is smaller than the temperature of the cold lead ($T^{\rm eff} < \min\{T_L, T_R\} = T_L$)—cooling regime. From this equation also follows that, in the interval $1 > \Theta > \Theta^* \equiv (1 - \hbar\omega/2\epsilon_0)/(1 + \hbar\omega/2\epsilon_0)$, the effective temperature is greater than the temperature of the hot lead ($T^{\rm eff} > \max\{T_L, T_R\} = T_L$)—heating regime [cf. Fig. 2].

Note that when a temperature difference is used to reduce mechanical fluctuations of the nanotube, the minimum effective temperature of the mechanical subsystem is always greater than $T^{\rm eff}_{\rm min} = 2T_L/(1 + 2\epsilon_0/\hbar\omega)$, approaching this value as $\Theta \to \infty$. As a result, the average vibron number $\langle n\rangle = \sum_n nP_m(n) \sim \exp(-\epsilon_0/k_B T_L)$ is finite but exponentially small. This is a consequence of the finite temperature of the cold lead, which leads to infrequent pumping processes and does not allow for achieving absolute ground-state cooling.

From Eq. (7) with T_L and T_R such that $T^{\rm eff} < 0$ ($\Theta < \Theta^*$), it follows that $d\langle n\rangle/dt > 0$, and the average number of vibrons increases with time. However, in a real physical situation, the mechanical subsystem is also coupled to the phononic thermal baths of the leads. To account for the dissipation due to this coupling, to the left side of Eq. (5), we add a Lindblad operator $\gamma\mathcal{L}_\gamma$.[20] Here, the temperature of the phononic bath is $T_b \sim (T_L + T_R)/2$, and $\gamma = \omega/Q$ is the coupling parameter characterized by the resonator quality factor Q, which can be very large, $Q \simeq 10^5$, at low temperatures.[21] We properly modify the system of equations (7) and solve it numerically. The resulting shape of $P^{st}_m(n)$ is depicted in Fig. 3. From this figure, one can see that, at small n, the probabilities $P^{st}_m(n)$ increase with n. This is because, for small $n < \Gamma/g$ and $\gamma < g$, the pumping rate ($\propto gn$) is larger than the dissipation rate ($\propto \gamma n$). Then, the pumping rate eventually saturates to $\Gamma \simeq \min\Gamma_{L(R)}$ at $n = n_s \sim \Gamma/g$, whereas, the dissipation rate governed by the interaction with phonon reservoirs continues to increase linearly with n and finally overcomes the pumping rate. As a result, the distribution function reaches a maximum at $n = n_{\rm max} \sim \Gamma/\gamma$ and then exponentially decays for larger $n \gg n_{\rm max}$.

IV. MECHANICAL GROUND-STATE COOLING

We discuss the conditions for ground-state cooling of the mechanical subsystem; i.e., a stationary regime with a final vibron number $\langle n\rangle \lesssim 1$. Thus far, we have considered the situation of completely polarized leads. However, in order to analyze the conditions for ground-state cooling, it is necessary to estimate the effect of partial spin polarization in the leads. To quantify the degree of polarizability of the right lead, we introduce a parameter $\eta_{L(R)} = 1 - \nu^{\downarrow(\uparrow)}_{L(R)}/\nu^{\uparrow(\downarrow)}_{L(R)}$ and find that, for a symmetric case ($\Gamma_R = \Gamma_L$), $1 - \eta_R \ll 1$, $T_L \ll T_R$, and

$\epsilon_0 \sim T_R k_B \gg \hbar\omega$, the average number of vibrons is

$$\langle n\rangle \approx \langle n\rangle_{\eta=0} + (1-f_R)(1-\eta_R) + O[(1-\eta_R)^2]. \quad (10)$$

Hence, for a 90% spin-polarized right lead, the average vibron number is increased only by ≈ 0.1.

A necessary condition to achieve ground-state cooling comes from Eq. (9), which suggests that, in order to achieve the minimum effective temperature, one needs $T_R \gg T_L$ and $\epsilon_0 \gg \hbar\omega$. The sufficient condition comes from the requirement that the cooling rate κ should be larger than the damping rate γ due to a bosonic bath. The largest cooling rate for a given temperature gradient is achieved when $\Gamma_L \gtrsim g/\hbar \gtrsim \Gamma_R(1-f_R)$. Thus, the conditions for ground-state cooling of the mechanical mode in the presence of a bosonic bath are $T_R \gg T_L$, $\epsilon_0 \gg \hbar\omega$, and $Q \gg \hbar\omega n_b/g f_R$, where $n_b = [\exp(\hbar\omega/k_B T_b) - 1]^{-1}$ (T_b is the temperature of the bosonic bath). From the last inequality, it follows that the best cooling regime is achieved when $k_B T_R \approx \epsilon_0$. Our analysis shows that, for a nanotube with frequency $\omega = 2\pi \times 100$ MHz ($\hbar\omega/k_B \approx 6$ mK), realistic coupling parameter $g \approx 2\pi \times 10^6$ Hz (see above), and quality factor $Q = 10^5$, the average vibronic number can be reduced to $\langle n\rangle = 0.44$ for $T_R = 200$ mK and $T_L = 20$ mK.

V. CONCLUSIONS

In conclusion, we have studied heating, pumping, and cooling of the mechanical vibrations of a nanotube suspended between two highly polarized magnetic leads. We have shown that spin-mechanical coupling between the mechanical and the electronic subsystems generated by a nonuniform magnetic field may result in suppression or generation of mechanical vibrations when the leads are held at different temperatures. In particular, it was demonstrated that, under certain conditions, the stationary distribution of the mechanical subsystem has a Boltzmann form with an effective temperature, which is smaller than the temperature of the cold lead. This counterintuitive result is a consequence of the Fermionic nature of the baths coupled to the mechanical subsystem and, in the case of fully spin-polarized leads, coupling of one level of the TLS to only one lead. Notice that coupling to bosonic baths results in an effective temperature of the vibrational mode equal to the mean value of the baths' temperature. Also, changing direction of the temperature gradient results in generation of mechanical vibrations rather than heating of the mechanical subsystem. Finally, for partial spin polarization in the leads, ground-state cooling of the mechanical vibration can be achieved at realistic physical parameters if the leads have $\gtrsim 50\%$ spin polarization.

ACKNOWLEDGMENTS

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