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Optimal and Efficient Power Allocation for OFDM Non-Coherent Cooperative Transmission

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***Abstract*—In this paper, we study the subchannel (SC) power allocation for orthogonal frequency division multiplexing (OFDM) multiple access points (APs) systems with non-coherent cooperative transmission. The objective is to maximize the total capacity under per-AP power constraints. It can be proved that the optimal solution can be obtained by the combination of an optimal SC partition search and the power allocation across SCs for each feasible partition. Existing work exhaustively searched the optimal SC partition and used Lagrange dual method to compute the power allocation across SCs. Since the entire complexity increases exponentially with the number of SCs, the existing method is unsuitable for practical implementation. In this paper, we propose a novel optimal power allocation algorithm for non-coherent cooperative transmission with a much lower complexity. Firstly, a concept of "cut-off SC" is proposed for searching the optimal SC partition. Then, an efficient optimal power allocation algorithm across SCs is proposed for any given cut-off SC. Simulation results demonstrate that the proposed algorithm is optimal with a polynomial complexity, and ends within an acceptable number of iterations.**

I. Introduction

Transmit power allocation across multiple subchannels (SC) is one of the most key techniques for increasing system capacity in orthogonal frequency division multiplexing (OFDM) system. For the single access point (AP) system, the optimal power allocation for maximizing total capacity is well known as the "water-filling" power allocation across SCs [1]-[3], derived via Lagrange dual method [4] with a high complexity. In [5] and [6], sub-optimal schemes were proposed by equally distributing transmit power across all SCs, which has lower complexity at a cost of the total capacity. The optimal and efficient water-filling algorithm for a single AP system was studied in [7], where all the SCs with non-positive power are eliminated at each iteration, and the power on the remaining SCs are recomputed at the next iteration. The iteration runs until all power on the remaining SCs become positive.

Recently, cooperative transmission across multiple APs is proposed to further improve the system performance [8] [9], in which each user can be served by multiple APs simultaneously. An optimal power allocation strategy that maximizes the total capacity of a two-AP system is proposed in [10], considering non-coherent cooperative transmission. The authors in [10] proved that at most one SC is jointly transmitted by the two APs, while the remaining SCs are partitioned into two parts, with each part transmitted by only one of the two APs. However, exhaustive search is used in [10] to find the optimal SC partition with an exponential complexity with respect of the number of SCs. In addition, Lagrange dual method is used to compute the optimal power allocation for each feasible SC partition, leading to an unacceptable high complexity of entire algorithm.

In this paper, we propose a novel and optimal power allocation algorithm with a much lower complexity. The objective is to maximize the total capacity of the two-AP system with non-coherent cooperative transmission. Firstly, based on the derived necessary condition for the optimal SC partition, a concept of "cut-off SC" is introduced. Then, we propose an optimal SC partition search method by checking all possible cut-off SCs, whose complexity is polynomial with respect to the number of SCs. In addition, motivated by the power allocation proposed for the single AP case in [7], we propose an efficient algorithm for the cooperative two-AP system, which simultaneously determines the optimal SC partition and allocates transmit power across SCs for any given cut-off SC. The optimal allocation is finally obtained by selecting the SC partition and the corresponding power allocation with the maximum total capacity. Compared to the algorithm proposed in [10], numerical results show that our algorithm achieves the maximum capacity with a polynomial complexity, and ends in an acceptable number of iterations in most cases.

The remainder of this paper is organized as follows. The system model for two-AP cooperative transmission, and the main work of this paper is described in section II. In section III, the optimal and efficient algorithm is derived with complexity analysis. Simulation results are given in section IV and the paper is concluded in section V.

II. System Model and Problem Description

In this section we first introduce the system model and then, describe the main problem to be solved in this paper.

A. System Model

We consider the downlink of an OFDM two-AP multiuser system with non-coherent cooperative transmission, see Fig. 1. Each SC can be a single or a group of sub-carriers. Ideal backhaul is assumed to connect the two APs to a centralized control unit (CCU). The CCU has the perfect channel state information and users' data of all users, and the transmit power

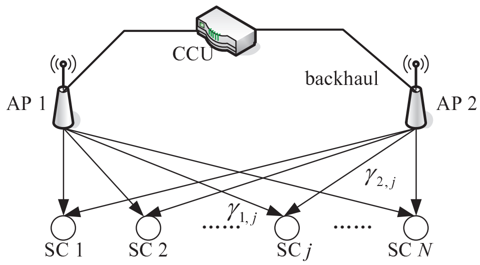

Fig. 1. OFDM downlink cooperative transmission with two APs

on each SC from different APs can be jointly controlled. To focus on the power allocation, we assume the SC allocation has been fixed among users by allocating each SC to at most one user. Therefore, the channel condition on each SC is constant during the power allocation. Based on the Shannon theorem, the capacity on SC j with non-coherent joint transmission is given as

$$r_j(p_{1,j}, p_{2,j}) = B\log_2(1+\sum_{i=1}^{2}\gamma_{i,j}p_{i,j}) \tag{1}$$

where the $\gamma_{i,j}$ denotes the channel-gain-to-noise-ratio (CNR) of AP i on SC j. The $p_{i,j}$ is the transmit power from AP i on SC j and B is the bandwidth of each SC.

B. Problem Description

Assuming the total number of SCs is N, the capacity maximizing power allocation problem under per-AP power constraints is formulated as

$$\begin{aligned} \text{maximize} \quad & R(\mathbf{P}_1,\mathbf{P}_2) = \sum_{j=1}^{N} r_j(p_{1,j}, p_{2,j}) \\ \text{subject to} \quad & \sum_{j=1}^{N} p_{i,j} \le P_i, i=1,2 \\ & p_{i,j} \ge 0, i=1,2, j=1,2,...,N \end{aligned} \tag{2}$$

where $\mathbf{P}_i = [p_{i,1},...,p_{i,j},...,p_{i,N}]$ denotes the power vector on each SC of AP i and P_i is the maximum transmit power of AP i.

Now we revisit the optimal solution formulation of (2) in [10] to describe the main problem to be solved in this paper. Assume that the power allocation from AP 2 is fixed as $\mathbf{P}_2 = [p_{2,1},...,p_{2,j},...,p_{2,N}]$. Then, the optimization problem (2) becomes strictly concave with respect to $p_{1,j}$. Define

$$f_{i,j}(p_{i,j}) = \frac{\partial r_j}{\partial p_{i,j}} = \frac{B\gamma_{i,j}}{(1+\sum_{i=1}^{2}\gamma_{i,j}p_{i,j})\ln 2} \tag{3}$$

as the bias derivative of (1) with respect to $p_{i,j}$. Based on KKT (Karush-Kuhn-Tuker) condition, the optimal $p^*_{1,j}$ satisfies

$$f_{1,j}(p^*_{1,j}) = f_{1,k}(p^*_{1,k}) = \lambda_1, p^*_{1,j}, p^*_{1,k} > 0, \forall j,k \tag{4}$$

where λ_1 is the Lagrange multiplier. Similarly, we have $f_{2,j}(p^*_{2,j}) = f_{2,k}(p^*_{2,k})$ for $p^*_{2,j}, p^*_{2,k} > 0$, where $p^*_{2,j}$ is the optimal solution of $p_{2,j}$. Hence, it can be derived that for $\forall j,k,j \neq k$, if the two APs instantaneously transmit power on both SC j and k, i.e. $p^*_{1,j}, p^*_{2,j}, p^*_{1,k}, p^*_{2,k} > 0$, then

$$f_{1,j}(p^*_{1,j})f_{2,k}(p^*_{2,k}) = f_{1,k}(p^*_{1,k})f_{2,j}(p^*_{2,j}) \tag{5}$$

Define

$$\eta_j = \frac{\gamma_{1,j}}{\gamma_{2,j}} \tag{6}$$

as the ratio of the CNR of AP 1 to AP 2 on SC j. According to (3) and (5), we can derive $\eta_j = \eta_k$.

However, in the practical system, the probability of $\eta_j = \eta_k$ is almost zero due to the large randomness of channel conditions. Hence, it can be concluded that at most one SC should be given power by both the two APs. Then, the SCs can be partitioned into three sets: 1) Φ_1: the SCs only transmitted by AP 1; 2) Φ_2: the SCs only transmitted by AP 2; 3) Φ_c: the SC jointly transmitted by two APs, with $|\Phi_c| = 1$ or $|\Phi_c| = 0$, where $|\Phi_c|$ denotes the size of Φ_c. The power allocated on the SCs within set Φ_1 and Φ_2 take the form of single AP water-filling (SAPWF). For simplicity, we denote a combination of the three sets (Φ_1, Φ_2, Φ_c) as a SC partition. Then, the optimal solution of (2), named as joint-water-filling (JoWF) in [10], for any given SC partition can be rewritten as follows:

$$\begin{cases} p^*_{i,j} = \left[\dfrac{B}{\lambda_i \ln 2} - \dfrac{1}{\gamma_{i,j}}\right]^+, p^*_{i',j} = 0, \forall j \in \Phi_i \\ p^*_{i,j} = \dfrac{B}{\lambda_i \ln 2} - \dfrac{1+\gamma_{i',j}p^*_{i',j}}{\gamma_{i,j}}, j \in \Phi_c \end{cases} \tag{7}$$

where $i \neq i', i, i' = 1,2$. the optimal $\lambda_i, i = 1,2$ can be obtained by the Lagrange dual method.

Notice that (7) is calculated based on a given SC partition. Hence, in order to find the optimal solution of (2), the optimal partition (Φ_1, Φ_2, Φ_c) needs to be determined. In [10], exhaustive search is used to find the optimal SC partition with a highly exponential complexity with respect of the number of SCs N. Hence, the main work in this paper is to design an optimal algorithm to solve the (2) with much less complexity.

III. Optimal and Efficient Algorithm

In this section, a concept of "cut-off SC" is proposed to find the optimal SC partition with a polynomial complexity. Then, motivated by [7], we propose an efficient power allocation algorithm for any given cut-off SC. Finally, the entire algorithm is described with complexity analysis.

A. Optimal SC Partition Search with Cut-off SC

Based on the water-filling theorem, we prove that the optimal power allocation must satisfy:

Lemma 1: For any $j \neq k$, if the SC j is transmitted power by AP i with $p^*_{i,j} > 0$, while AP i does not transmit power on SC k, i.e. $p^*_{i,k} = 0$, then the bias derivative of r_j defined in (1) with respect to $p_{i,j}$ is no less than the derivative of r_k with respect to $p_{i,k}$, i.e. $f_{i,j}(p^*_{i,j}) \ge f_{i,k}(p^*_{i,k}), i = 1,2$.

The proof is given in Appendix A. Then, consider arbitrary SC $j \in \Phi_1$ and SC $k \in \Phi_2$, with $p^*_{1,j}, p^*_{2,k} > 0$ and $p^*_{2,j}, p^*_{1,k} = 0$, according to *Lemma 1* we have

$$f_{1,j}(p^*_{1,j})f_{2,k}(p^*_{2,k}) \ge f_{2,j}(p^*_{2,j})f_{1,k}(p^*_{1,k}) \;\Rightarrow\; \eta_j \ge \eta_k \tag{8}$$

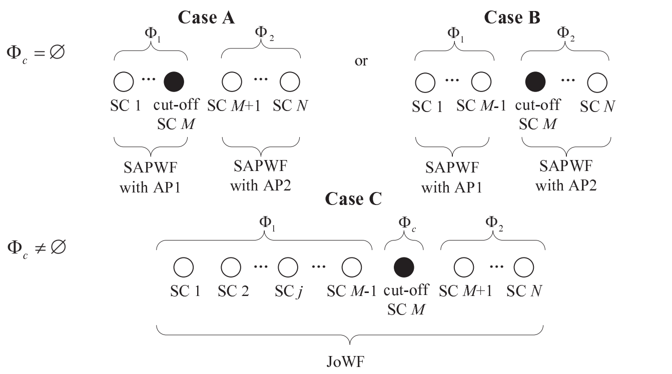

Fig. 2. Necessary condition for the optimal SC partition with the cut-off SC

Based on (8), a necessary condition for the optimal SC partition is derived as follows:

***Condition** 1:* Assuming all SCs are sorted in a descending order of η_j, i.e. $\eta_j \geq \eta_k$ for $\forall j \leq k$, there exits a "cut-off SC" M with $1 \leq M \leq N$ that, $j \in \Phi_1, \forall j < M$ and $j \in \Phi_2, \forall j > M$. And if $\Phi_c = \emptyset$, SC M belongs to either Φ_1 or Φ_2, otherwise, $\Phi_1 = \{j|j < M\}$, $\Phi_2 = \{j|j > M\}$ and $\Phi_c = \{M\}$. See Fig. 2.

As illustrated in Fig. 2, if the optimal cut-off SC is found, the optimal SC partition must be one of the Case A, Case B and Case C. Hence, a method for the optimal SC partition search is proposed in **Algorithm 1**. By checking each SC as the cut-off SC, the corresponding SC partition satisfying *Condition 1* and the optimal power allocation across SCs for each given cut-off SC are determined. The optimal SC partition is obtained according to the partition of the cut-off SC with the maximum total capacity.

Algorithm 1 Optimal SC partition search

```
1: Sort the SCs in descending order of η_j ;
2: For j=1 : N do
3:   Set the SC m = j as the cut-off SC;
4:   Decide the SC partition (Φ_1^m, Φ_2^m, Φ_c^m) satisfying
     Condition 1 and the power allocation P_{i=1,2}^m for m;
5:   Compute total capacity R_m = R(P_1^m, P_2^m) according
     to (2);
6: End for
7: Set m* = arg max_m R_m, the optimal SC partition
   and the optimal power allocation are selected as
   (Φ_1^{m*}, Φ_2^{m*}, Φ_c^{m*}) and P_i^{m*}, i = 1, 2.
```

Note that the Algorithm 1 only needs search over all N possible cut-off SCs to obtain the optimal partition with a polynomial complexity with N, which outperforms the exponential complexity of the exhaustive search used in [10].

B. Efficient Power Allocation for a given Cut-off SC

It can be seen from Algorithm 1 that, in order to obtain the optimal solution, the SC partition $(\Phi_1^m, \Phi_2^m, \Phi_c^m)$ satisfying *Condition 1* and the corresponding optimal power allocation $\mathbf{P}_i^m, i = 1, 2$ for a given cut-off SC m need to be determined.

Algorithm 2 Power allocation algorithm for $m \notin \Phi_c^m$

```
1: Initialization: Φ_1^m = {j|j < m}, Φ_2^m = {j|j > m};
Check the Case A:
2: SAPWF for Φ_1^m ∪ {m} with AP 1, obtain p_{1,j}^m(A)
   for j ≤ m, set P_1^m(A) = {p_{1,1}^m(A), .., p_{1,m}^m(A), 0, .., 0};
3: SAPWF for Φ_2^m with AP 2, obtain p_{2,j}^m(A) for
   j > m, set P_2^m(A) = {0, .., 0, p_{2,m+1}^m(A), .., p_{2,N}^m(A)};
Check the Case B:
4: SAPWF for Φ_2^m ∪ {m} with AP 2, obtain p_{2,j}^m(B)
   for j ≥ m, set P_2^m(B) = {0, .., 0, p_{2,m}^m(B), .., p_{2,N}^m(B)};
5: SAPWF for Φ_1^m with AP 1, obtain p_{1,j}^m(B) for
   j < m, set P_1^m(B) = {p_{1,1}^m(B), .., p_{1,m-1}^m(B), 0, .., 0};
Compare the Case A and Case B:
6: Compute the capacity R_m(l) = R(P_1^m(l), P_2^m(l))
   for l = A, B according to (2);
7: Set l* = arg max_l R_m(l), then the P_i^m(l*), i = 1, 2
   is the desired power allocation for the cut-off SC m.
```

According to *Condition 1*, deciding the SC partition for a given m equivalents to deciding which set, Φ_1^m, Φ_2^m or Φ_c^m, should m belong to. As shown in Fig. 2, if $\Phi_c^m = \emptyset$, i.e. $m \in \Phi_1^m$ (Case A) or $m \in \Phi_2^m$ (Case B), the corresponding power allocation $\mathbf{P}_i^m, i = 1, 2$ is equivalent to two separate SAPWF for Φ_1^m and Φ_2^m respectively. Then, with the fast SAPWF algorithm in [7], the power allocation for $m \in \Phi_1^m$ and $m \in \Phi_2^m$ are obtained respectively. Comparing the total capacities between the two cases, the SC partition and $\mathbf{P}_i^m, i = 1, 2$ are derived by choosing the one with the larger capacity. The power allocation for $m \notin \Phi_c^m$ is given in **Algorithm 2**.

Hence, the remaining question of deciding the SC partition is how to decide whether the $m \in \Phi_c^m$ (Case C), which needs to calculate the optimal power allocation for $m \in \Phi_c^m$, i.e. the JoWF power allocation of Case C. However, since the SAPWF method in [7] can't be directly used in JoWF, efficient method to allocate power across SCs for $m \in \Phi_c^m$ becomes difficult to design.

To determine the affiliation of the cut-off SC m and calculate the JoWF for $m \in \Phi_c^m$, we first prove that the SAPWF method in [7] can be extended to the calculation of JoWF by assuming $m \in \Phi_c^m$. Then, an efficient iterative procedure to obtain the JoWF power allocation is proposed, based on which the question that whether the $m \in \Phi_c^m$ can be answered.

Assuming $m \in \Phi_c^m$, the power on SCs $j \neq m$ can be rewritten as follows according to (7)

$$p_{1,j\in\Phi_1^m}^m = \frac{W^m + H_{1,j}^m}{|\Phi_1^m| + |\Phi_2^m| + 1}, p_{2,j\in\Phi_2^m}^m = \frac{\eta_m W^m + H_{2,j}^m}{|\Phi_1^m| + |\Phi_2^m| + 1} \tag{9}$$

where

$$W^m = P_1 + \frac{1}{\eta_m} P_2$$

Algorithm 3 Efficient JoWF algorithm for $m \in \Phi_c^m$

1: Initialization: $t = 0$, $\mathbf{P}_i^m(t) = \{p_{i,1}^m(t), ..., p_{i,N}^m(t)\} = \mathbf{0}, i = 1, 2$, $\Phi_1^m = \{j | j < m\}$, $\Phi_2^m = \{j | j > m\}$;

Iteratively compute the JoWF assuming $m \in \Phi_c^m$:

2: Compute $p_{i,j \neq m}^m(t), j \in \Phi_i^m$ by (9), and $p_{i,m}^m(t) = P_i - \sum_{j \in \Phi_i^m} p_{i,j}^m(t)$;

3: If $p_{i,j \neq m}^m(t) \leq 0$, $0 \rightarrow p_{i,j \neq m}^m(t)$, $\Phi_i^m / \{j\} \rightarrow \Phi_i^m$;

4: $t = t + 1$;

5: Repeat 2-4 until all $p_{i,j \neq m}^m(t) \geq 0$ or $\Phi_1^m, \Phi_2^m = \emptyset$;

Check the assumption of $m \in \Phi_c^m$:

6: If the last $p_{i,m}^m(t) > 0, i = 1, 2$, set the $(\Phi_1^m, \Phi_2^m, \Phi_c^m)$ and $\mathbf{P}_i^m(t), i = 1, 2$ as the desired SC partition and JoWF power allocation for the cut-off SC m;

7: otherwise, the SC m belongs to either Φ_1^m or Φ_2^m, and $\Phi_c^m = \emptyset$.

$$H_{1,j}^m = \sum_{\substack{k \in \Phi_1^m \cup \{m\} \\ k \neq j}} \left(\frac{1}{\gamma_{1,k}} - \frac{1}{\gamma_{1,j}}\right) + \frac{1}{\eta_m} \sum_{k \in \Phi_2^m} \left(\frac{1}{\gamma_{2,k}} - \eta_m \frac{1}{\gamma_{1,j}}\right)$$

$$H_{2,j}^m = \sum_{\substack{k \in \Phi_2^m \cup \{m\} \\ k \neq j}} \left(\frac{1}{\gamma_{2,k}} - \frac{1}{\gamma_{2,j}}\right) + \eta_m \sum_{k \in \Phi_1^m} \left(\frac{1}{\gamma_{1,k}} - \frac{1}{\eta_m} \frac{1}{\gamma_{2,j}}\right)$$

and the power on SC m is given as

$$p_{i,m}^m = P_i - \sum_{j \in \Phi_i^m} p_{i,j}^m, i = 1, 2 \tag{10}$$

The values of (9) and (10) may be non-positive for some j because the non-negativity constraint on $p_{i,j}^m$ has not been reflected yet. To extend the method in [7], we propose

***Theorem** 1:* Consider SCs $j_1, j_2 \neq m$ with non-positive power computed by (9). Assume the SC j_1 is eliminated from its corresponding $\Phi_i^m, i = 1, 2$. Then if recompute the power on the remaining SCs according to (9) and (10), the reassigned power on SC j_2 will still be non-positive.

The proof is given in Appendix B. The theorem implies that all the SCs $j \neq m$ with non-positive power can be eliminated at the same time. Hence, an iterative procedure to calculate the JoWF is proposed as follows:

In each iteration, the power on each SC is computed by (9) and (10), and all the SCs $j \neq m$ with non-positive power are assigned zero power and eliminated from corresponding $\Phi_i^m, i = 1, 2$. Then the power on remaining SCs are recomputed at the next iteration. The iteration runs until all power on remaining SCs become non-negative, or all SCs $j \neq m$ have been eliminated.

Then, according to the results obtained by the iterative procedure, it is reasonable that if the assumption of $m \in \Phi_c^m$ is true, the power on the SC m from the two APs $p_{i,m}^m, i = 1, 2$ are both positive, then the obtained power allocation is exact the desired JoWF of Case C for the cut-off SC m; otherwise, the SC m belongs to either Φ_1^m or Φ_2^m, and $\Phi_c^m = \emptyset$, and the corresponding SC partition and the power allocation can be obtained following Algorithm 2.

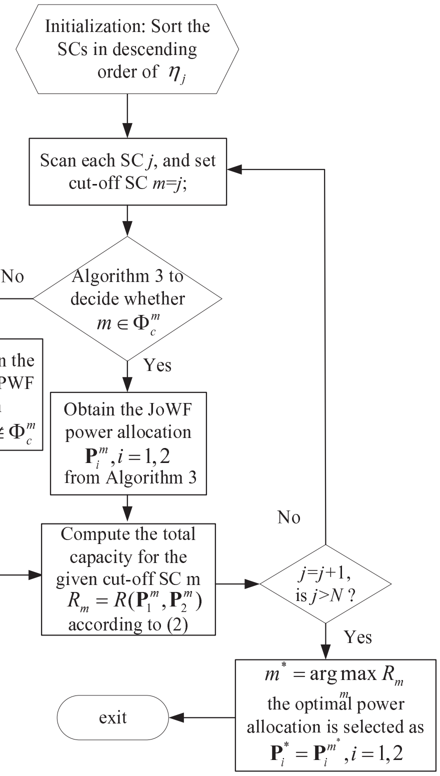

Fig. 3. Optimal and efficient power allocation algorithm

As a result, an efficient algorithm for $m \in \Phi_c^m$ is proposed in **Algorithm 3**, where the steps 1-5 iteratively calculate the JoWF solution by assuming $m \in \Phi_c^m$, and the steps 6 and 7 determine whether the assumption is true. If $m \in \Phi_c^m$ is determined, the result of steps 1-5 is chosen as the JoWF power allocation for the given cut-off SC m. Hence, with Algorithm 2 and 3, the SC partition and the corresponding power allocation for each given cut-off SC are obtained.

C. Entire Algorithm Description and Complexity Analysis

Based on the Algorithm 1-3, the entire proposed algorithm is described in Fig. 3. For the proposed algorithm, the complexity of SCs sorting is $O(N \log_2 N)$ and it needs to scan all N SCs to find the optimal cut-off SC. For each scanned SC, the four SAPWF in Algorithm 2 and the one JoWF in Algorithm 3 totaly need at most $3N$ iterations, and each iteration needs a search in order of $O(N)$ to find all those SCs with non-positive power, resulting a complexity of $O(3N^2)$ for Algorithm 2 and 3. Therefore, the total complexity of entire algorithm is $O(N \log_2 N) + O(N \times 3N^2) = O(3N^3)$, i.e. the complexity of entire proposed algorithm increases polynomially with the number of SC N.

IV. NUMERICAL RESULTS

In this section, the performance and complexity of the proposed algorithm is verified. Without loss of generality, we assume the SC bandwidth is normalized, and the maximum transmit power on each AP is identical denoted as P. The

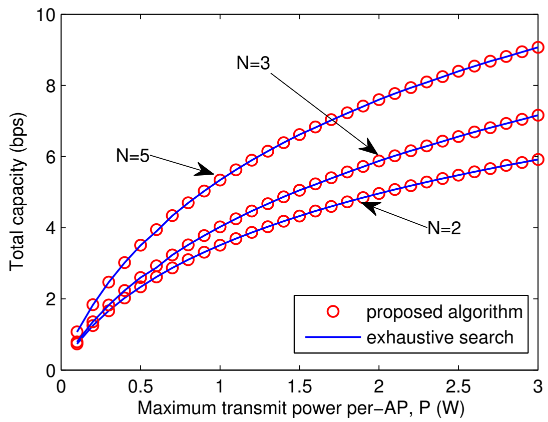

Fig. 4. Total capacity vs. maximum transmit power of AP

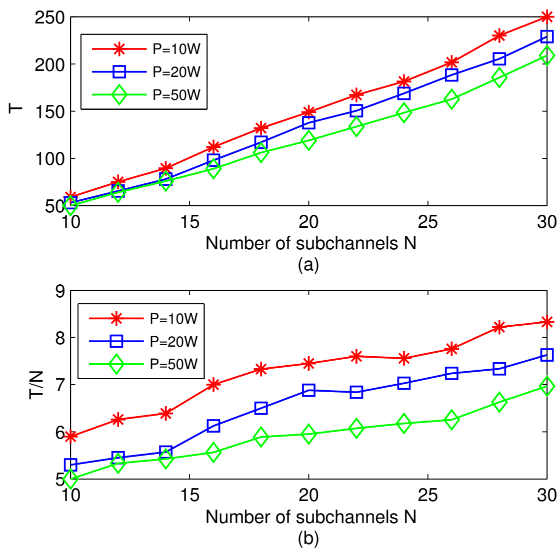

Fig. 5. (a): Total iteration number T vs subchannel number N; (b): The sum of iteration number of Algorithm 2 and 3, i.e. the ratio of iteration number to subchannel number T/N

channel conditions $\gamma_{i,j}$ are randomly selected. We also denote T as the total number of iterations of proposed algorithm.

In Fig. 4, the proposed algorithm is compared with the optimal exhausted search with respect to total capacity under different per-AP power constraints. Due to the high complexity of exhausted search, we limit the number of SCs $N \leq 5$. It can be seen that the results of proposed algorithm are always identical to the optimal exhausted search. This verifies the optimality of proposed algorithm.

The total iteration number of the proposed algorithm excluding SC sorting, T, is given in Fig. 5(a), and the ratio of iteration number to the SC number, T/N, is shown in Fig. 5(b). This ratio denotes the sum iteration number of the four SAPWF's and one JoWF in Algorithm 2 and 3, which at most is $3N$. It can be seen from Fig. 5(b) that, though the ratio is unpredictable since it strongly depends on the channel conditions, it is much smaller than $3N$ in most cases. This makes the entire algorithm stops in a very limited iterations number, which is nearly linear to N, as shown in Fig. 5(a). So the total complexity can be $O(TN) \approx O(KN^2)$ in our simulation, where $K \ll N$. It can be also found that the total iteration number decreases as the maximum transmit power of each AP, P, increases. This is because larger the power constraint is, the less SCs with non-positive power in iterative computation of JoWF and SAPWF, resulting in a less iteration number to remove them.

In Table. I, the complexity of proposed algorithm is compared with the algorithm in [10], in which the optimal SCs partition is obtained by exhaustive searching over $3 \times N \times 2^{N-1}$ possible combinations. Hence, suppose the identical complexity of the power allocation calculation for each SC partition is $O(N^2)$, the entire complexity of the algorithm in [10] is $O(3 \times 2^{N-1} \times N^3)$. The proposed algorithm has a much lower complexity even at the worst case, and has a even lower complexity of nearly $O(KN^2)$, where $K \ll N$, in most cases. Hence, the proposed algorithm is both optimal and efficient.

TABLE I
COMPLEXITY COMPARISON

Algorithms	Complexity
Algorithm in Ref. [10]	$O(3 \times 2^{N-1} \times N^3)$
Proposed algorithm at the worst case	$O(3N^3)$
Simulation results of proposed algorithm	nearly $O(KN^2)$, $K \ll N$

V. CONCLUSION

In this paper, a novel optimal power allocation algorithm with low complexity is proposed to maximize the total capacity of a two-AP system with non-coherent cooperative transmission. Firstly, a concept of "cut-off SC" is proposed for the optimal subchannel partition search method. Then, we propose an efficient optimal power allocation across subchannels is for any given cut-off SC. The optimal allocation is finally obtained by selecting the SC partition and corresponding power allocation with the maximum total capacity. Simulation results demonstrate that the proposed algorithm is optimal with polynomial complexity, and stops within a small number of iterations. Algorithm for more than two APs and coherent cooperative transmission are still open issues for future studies.

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APPENDIX A
PROOF OF LEMMA 1

First, we assume the power of AP 2 is fixed and there is a integer $K < N$ satisfying that $p^*_{1,j} > 0, \forall j \leq K$ and $p^*_{1,k} = 0, \forall k > K$. According to (5), there is $f_{1,1}(p^*_{1,1}) =$

$f_{1,2}(p^*_{1,2}) = ... = f_{1,K}(p^*_{1,K}) = \lambda$. Assuming there is a k that $f_{1,k}(p^*_{1,k} = 0) > \lambda$, note that $f_{1,j}(p_{i,j})$ is a monotonically decreasing function, then it exists a series of positive value $p'_{1,j}, j = 1, ..., K, k$ that

$$f_{1,1}(p'_{1,1}) = ... = f_{1,K}(p'_{1,K}) = f_{1,k}(p'_{1,k}) = \lambda' \tag{11}$$

where $\lambda < \lambda' < f_{1,k}(0)$, $\sum_{j=1}^{K} p'_{1,j} + p'_{1,k} = P_1$ and $p'_{1,j} < p^*_{1,j}, j \leq K$. Then with fixed $p_{2,j}$, the different of total capacity of SCs $1, 2, ..., K$ and k between allocation power $p^*_{1,j}$ and $p'_{1,j}$ is given as

$$\begin{aligned}
&\sum_{j=1}^{K} r_j(p^*_{1,j}) - \sum_{j=1}^{K} r_j(p'_{1,j}) - r_k(p'_{1,k}) \\
&= \sum_{j=1}^{K} \int_{p'_{1,j}}^{p^*_{1,j}} f_{1,j}(p_{1,j}) dp_{1,j} - \int_{0}^{p'_{1,k}} f_{1,k}(p_{1,k}) dp_{1,k} \\
&= \sum_{j=1}^{K} (p^*_{1,j} - p'_{1,j}) f_{1,j}(\xi_j) - p'_{1,k} f_{1,k}(\xi_k)
\end{aligned} \tag{12}$$

where $p'_{1,j} < \xi_j < p^*_{1,j}$ and $0 < \xi_k < p'_{1,k}$. It is obvious that $f_{1,j}(\xi_j) < \lambda'$ and $f_{1,k}(\xi_k) > \lambda'$, therefore (12) is less than

$$\sum_{j=1}^{K} (p^*_{1,j} - p'_{1,j})\lambda' - p'_{1,k}\lambda' = (P_1 - \sum_{j=1}^{K} p'_{1,j})\lambda' - p'_{1,k}\lambda' = 0 \tag{13}$$

i.e. $\sum_{j=1}^{K} r_j(p^*_{1,j}) < \sum_{j=1}^{K} r_j(p'_{1,j}) + r_k(p'_{1,k})$, which is contradict to the assumption that $p^*_{1,j}$ is optimal. Hence such k doesn't exist. Applying same procedure for $p^*_{2,j}$, *Lemma 1* is proved.

Appendix B
Proof of Theorem 1

It can be found that the proof for $j_1 \in \Phi^m_1, j_2 \in \Phi^m_1$ is very similar to the SAPWF case in [7], and we first give out the proof when $j_1 \in \Phi^m_1, j_2 \in \Phi^m_2$. According to (9), after eliminating the SC j_1 at iteration t, the numerator of p^m_{2,j_2} of $j_2 \in \Phi^m_2$ at iteration $t+1$ becomes

$$\begin{aligned}
&\eta_m W^m + \sum_{\substack{k\in\Phi^m_2\cup\{m\}\\ k\neq j_2}} (\frac{1}{\gamma_{2,k}} - \frac{1}{\gamma_{2,j_2}}) \\
&+\eta_m \sum_{k\in\Phi^m_1} (\frac{1}{\gamma_{1,k}} - \frac{1}{\eta_m}\frac{1}{\gamma_{2,j_2}}) - \eta_m(\frac{1}{\gamma_{1,j_1}} - \frac{1}{\eta_m}\frac{1}{\gamma_{2,j_2}})
\end{aligned} \tag{14}$$

First assume $\gamma_{1,j_1}\gamma_{2,m} \leq \gamma_{1,m}\gamma_{2,j_2}$. The first three terms of (14) are the numerator of p^m_{2,j_2}, at t which is non-positive.The last term is also non-positive by the assumption. So p^m_{2,j_2} is still non-positive at $t+1$ in this case.

Now assume $\gamma_{1,j_1}\gamma_{2,m} > \gamma_{1,m}\gamma_{2,j_2}$. The numerator of p^m_{1,j_1} at t is given as

$$\begin{aligned}
&W^m + \sum_{\substack{k\in\Phi^m_1\cup\{m\}\\ k\neq j_1}} (\frac{1}{\gamma_{1,k}} - \frac{1}{\gamma_{1,j_1}}) \\
&+\frac{1}{\eta_m} \sum_{\substack{k\in\Phi^m_2\\ k\neq j_2}} (\frac{1}{\gamma_{2,k}} - \eta_m\frac{1}{\gamma_{1,j_1}}) + \frac{1}{\eta_m}(\frac{1}{\gamma_{2,j_2}} - \eta_m\frac{1}{\gamma_{1,j_1}})
\end{aligned} \tag{15}$$

With the assumption, the last term of (15) is positive, so the first three terms must be negative which can be rewritten as

$$\begin{aligned}
&\frac{1}{\eta_m}[\eta_m W^m + \eta_m \sum_{\substack{k\in\Phi^m_1\cup\Phi^m_c\\ k\neq j_1}} (\frac{1}{\gamma_{1,k}} - \frac{1}{\gamma_{1,j_1}}) \\
&\qquad + \sum_{\substack{k\in\Phi^m_2\\ k\neq j_2}} (\frac{1}{\gamma_{2,k}} - \eta_m\frac{1}{\gamma_{1,j_1}})]
\end{aligned} \tag{16}$$

Since $\gamma_{1,j_1}\gamma_{2,m} > \gamma_{1,j_c}\gamma_{2,m}$, the terms in square bracket of (16) are larger than

$$\begin{aligned}
&\eta_m W^m + \eta_m \sum_{\substack{k\in\Phi^m_1\\ k\neq j_1}} (\frac{1}{\gamma_{1,k}} - \frac{1}{\eta_m}\frac{1}{\gamma_{2,j_2}}) \\
&\qquad + \sum_{\substack{k\in\Phi^m_2\cup\Phi^m_c\\ k\neq j_2}} (\frac{1}{\gamma_{2,k}} - \frac{1}{\gamma_{2,j_2}})
\end{aligned} \tag{17}$$

Therefore (17) is negative. It can be seen that (17) is the numerator of p^m_{2,j_2} after eliminating SC j_1 at t, so p^m_{2,j_2} is also non-positive at iteration $t+1$ in this case. Then with reciprocity, the above results also hold for $j_1 \in \Phi^m_2, j_2 \in \Phi^m_2$ and $j_1 \in \Phi^m_2, j_2 \in \Phi^m_1$. These establish the theorem.

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