Application of ray-tracing through the high resolution numerical weather model HIRLAM applying the Conformal Theory of Refraction

S. Garcia-Espada, R. Haas, F. Colomer

Abstract In space geodetic techniques like VLBI and GPS, accuracy is limited by atmospheric propagation effects by neutral atmosphere in the troposphere. In recent years numerical weather models (NWM) have been applied to improve mapping functions which are used for tropospheric delay modeling in VLBI and GPS data analyses. A troposphere correction model applying ray-tracing through the Limited Area numerical weather prediction (NWP) HIRLAM 3D-VAR model and applying the Conformal Theory of Refraction is developed. The advantages of HIRLAM model are the high spatial resolution (0.2°x0.2°) and the high temporal resolution in prediction mode (every 3 hours). The advantages of the Conformal Theory of Refraction (Moritz, 1967) is that the atmospheric propagation effects are evaluated along the line of sight and the known vacuum elevation angle is used so no iterative calculations are needed. When ray-tracing through HIRLAM profiles and calculating the slant delays using the Conformal Theory of Refraction, we include the effect of an inhomogeneous atmosphere in the slant delays values.

Keywords HIRLAM, NWM, ray-tracing, Conformal Theory of Refraction

1 Introduction

The electromagnetic signals on their way through the earth's atmosphere experience propagation delays. Troposphere delays are one of the main contributors to the total error and are usually taken into account by parametrization of atmosphere delays as unknown parameters which are estimated together with the other parameters of interest in the geodetic VLBI analysis data like gradients, clocks or station positions. The propagation delays relate to the refractivity of the medium which in the so-call neutral atmosphere is influenced by temperature, pressure and humidity.

In recent years, regional-scale NWMs have improved in terms of accuracy and precision. Thus it appears to be reasonable to calculate slant delays by ray-tracing through these NWMs and to apply these slant delays as external information for the analysis of space geodetic data.

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2 HIRLAM Numerical Weather Model

The HIRLAM project[1] has been established in order to provide the best available operational short-range forecasting system for the National Meteorological Services in Denmark, Finland, Iceland, Ireland, Netherlands, Norway, Spain and Sweden. Meteo-France has a research cooperation agreement with HIRLAM. The HIRLAM system is a complete NWP system including data assimilation with analysis of conventional or non-conventional observations and a limited area forecasting model with a comprehensive set of physical parametrization. The forecast model is a limited area model with a boundary relaxation scheme. The model exists both in a grid-point version and in a spectral version. Initial and boundary conditions are taken from European Centre for Medium Range Weather Forecast (ECMWF). The HIRLAM model is a synoptic scale model which means it is displaying conditions simultaneously over a broad area. It is a numerical short-range ($<$ 48 h) weather forecasting system. The advantage of the HIRLAM model are its high spatial resolution (22 km to 5 km horizontally (Fig. (1)), 16 to 60 levels vertically) and high temporal resolution (6 hours assimilation data and analysis and prediction at 00h, 06h, 12h, 18h; 3 hours cycle also available).

3 Conformal Theory of Refraction

The optical path length σ between a fixed origin and a variable point is described by the Eikonal equation, $(grad\sigma)^2 = n^2$ which

[1] Synoptic scale model HIRLAM, http://hirlam.org/

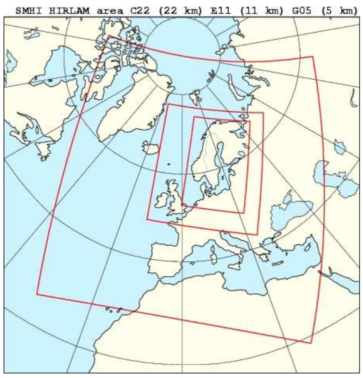

Fig. 1 HIRLAM horizontal grid.

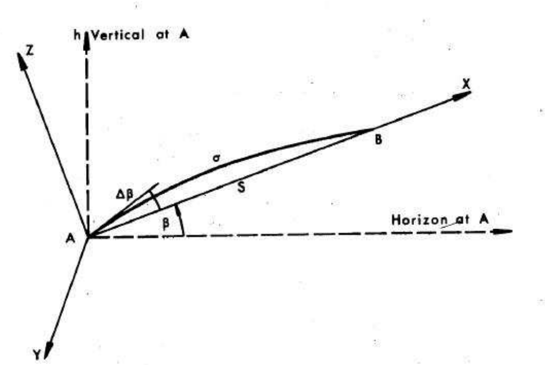

Fig. 2 Local system defined in the Conformal Theory of Refraction showing the effects of wave propagation through the atmosphere (Moritz, 1967).

is a first order partial differential equation for the optical distance σ, and corresponds to the well known principle of Fermat which states that light or other electromagnetic waves will follow the path between two points which involves the minimum travel time.

The Conformal Theory of Refraction was derived as an approximate solution of the calculation of the optical path length, and the vertical and lateral angles of refraction which include the solution of the Eikonal equation in the equations (Moritz, 1967). Figure (2) shows the local system defined by Moritz (1967) to develop the Conformal Theory of Refraction. The extra path delay can be written:

$$\Delta S = 10^{-6}\int_0^S N dX - \frac{1}{2}10^{-12}\int_0^S [(\int_0^S \frac{dN}{dY}\varepsilon d\varepsilon)^2 + (\int_0^S \frac{dN}{dZ}\varepsilon d\varepsilon)^2]\frac{dX}{X^2}$$

where ε denotes only an integration variable along the chord (Moritz, 1967).

If we neglect the small effect of the curvature due to lateral refraction caused by the gradient $\frac{dN}{dY}$ assuming it is approximately 0, and if we replace the gradient $\frac{dN}{dZ}$ perpendicular to the chord AB, with sufficient accuracy, by the vertical gradient of refractivity like:

$$\frac{dN}{dZ} = cos\beta(\frac{dN}{dh})$$

where β is the vacuum elevation angle. Then both simplifications lead finally to the practical approximation:

$$\Delta S = 10^{-6}\int_0^S N dX - \frac{cos^2\beta}{2}10^{-12}\int_0^S (\int_0^S (\frac{dN}{dh})\varepsilon d\varepsilon)^2 \frac{dX}{X^2}$$

where $\frac{dN}{dh}$ stands for the vertical gradient of the refractivity N, β is the vacuum elevation angle and ε denotes only an integration variable along the chord AB (Brunner and Angus-Leppan, 1976). The main advantage is that atmospheric propagation effects are evaluated along the known chord line AB and not along the unknown wave path. The second advantage is that the vacuum elevation angle β is used so no iterative calculations are needed.

4 Application

We calculated the slant delay caused by the neutral atmosphere via ray-tracing through the numerical weather model HIRLAM applying the Conformal Theory of Refraction equation for the analysis of 15 geodetic European VLBI experiments from EURO75 (22^{nd} March 2005) to EURO89 (3^{rd} September 2007). Twelve stations were involved: Crimea in Ukraine, Dss65a in Spain; Matera, Noto, and Medicina in Italy; Metsähovi in Finland; Ny-Ålesund in Norway; Onsala in Sweden; Svetloe and Zelenchukskaya in Russia and Wettzell and Effelsberg in Germany. Badary station in Russia was also involved in EURO87 but it is not included in HIRLAM grid so we did not included it in the calculations.

We used HIRLAM files with 22 km horizontal resolution, 40 vertical levels and 6 hours time resolution (00h, 06h, 12h, 18h). We did interpolation in time for each scan of the mentioned geodetic VLBI experiments between the nearest time files. We did horizontal interpolation in the 40 km horizontal grid between the four nearest points profile around the station. We did interpolation in the vertical, we refined from 40 vertical levels to approximately 1000 layers, where the step size between them depends on the atmosphere height at each step. The atmosphere height was extrapolated to 136 km. The ray-tracing algorithm goes through the necessary number of HIRLAM vertical profiles depending on the ray (scan) elevation angle β until it crosses the complete atmosphere. For each step through the atmosphere, there is a horizontal and vertical interpolation of the ray point going through the atmosphere. Station heights were calculated over *WGS84* ellipsoid and undulations were calculated using the potential coefficient model *EGM86*. Stations heights were introduced to the HIRLAM vertical profile, in some cases it was necessary to interpolate or extrapole them in the vertical profiles, which means that the HIRLAM topography models differently the terrain topography depending on the location of the station.

To calculate the slant delay ΔS applying the Conformal Theory of Refraction (Moritz, 1967), we integrated through the elevation angle β of each observation starting at the station height.

We neglected the effect of curvature due to lateral refraction caused by $\frac{dN}{dY}$ assuming it is approximately 0. The size and number of the integration steps along the chord line of sight depend on the vacuum elevation angle β and the current height of the ray point in the atmosphere. We needed to find a compromise between the computing time and the number of integration steps, so we used approximately 250 integration steps for each ray. At each point we recalculated the angle β due to the *WGS84* ellipsoid. The calculated slant delay is azimuth angle dependence, it is the 'path delay' through the 3D inhomogeneous atmosphere through the chord line.

5 Preliminary results

We estimated zenith wet delays (ZWD) for all the stations involved in the CONT08 experiments to check that ray-tracing through HIRLAM is consistent with other techniques (Teke et al., 2010). In that case forecast and analysis HIRLAM profiles were combined, so we had a time resolution of 3 hours.

In order to check the Conformal Theory of Refraction we created a schedule in which all the scans had the same time epoch as previous estimated ZWD in CONT08 experiments and which observation elevation angle was 90° in all cases. Then, we calculated the slant delays for all scans at 90°, which is equivalent to calculate estimated ZWD. We can compare estimated ZWD through HIRLAM to the calculation of slant delays using the Conformal Theory of Refraction with an elevation angle of 90°.

Figure (3) shows time series of the differences of estimated ZWD through HIRLAM minus the calculated slant delays using the Conformal Theory of Refraction with elevation angles of 90°, for Wettzell station during CONT08 experiments. For this example, at Wettzell station the mean of the differences is 0.83 mm and the standard deviation of the mean is 0.02 mm. This differences are due to an improvement in the software. The interpolation in space was for estimated ZWD through HIRLAM assumed to be over an sphere and for the calculation of the slant delays at 90° the interpolation in space was calculated over the ellipsoid *WGS84*. For all the other stations in CONT08 we obtain values in the same order of magnitude.

In order to compare an homogeneous atmosphere versus an inhomogeneous atmosphere, we did a comparison of slant delays calculated using ray-tracing through HIRLAM model applying Raytrace software (Davis et al., 1987-1989) as presented in Garcia-Espada et al. (2010) and slant delays calculated using ray-tracing through HIRLAM model applying the Conformal Theory of Refraction explained in this paper. For the Raytrace software only one HIRLAM vertical profile is assumed as an homogeneous atmosphere around the station position. For the Conformal Theory of Refraction different vertical profiles are considered around the station position and the ray-scan crosses a different number of them depending on the corresponding elevation angle β. Figure (4) is a time series comparison between calculated slant delays for an homogeneous atmosphere versus an inhomogeneous atmosphere during EURO75 experiment for Effelsberg station. In this example, the maximum difference is 24.70 cm and the minimum differences is 5.58 cm.

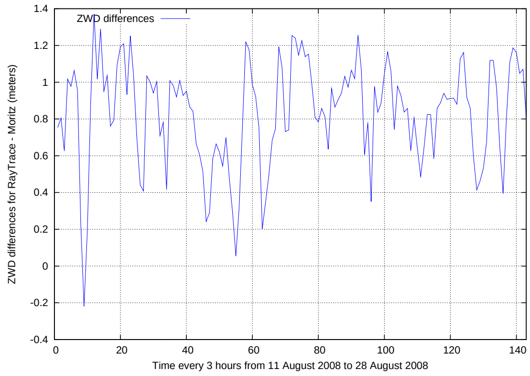

Fig. 3 Differences between estimated ZWD through HIRLAM minus calculated slant delays for scans with elevation angle 90° using the Conformal Theory of Refraction during CONT08 experiments at Wettzell station.

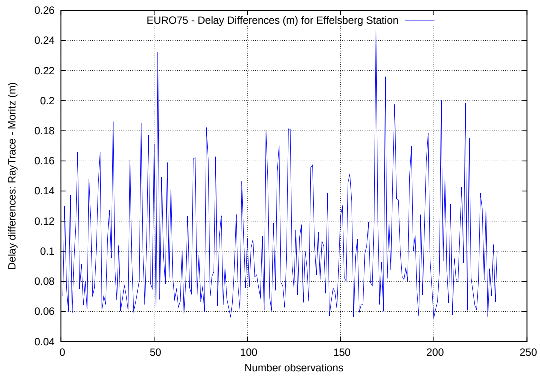

Fig. 4 Slant delays differences between Raytrace software (homogeneous atmosphere) and the Conformal Theory of Refraction (inhomogeneous atmosphere) ray-tracing through HIRLAM.

Station	# observations EURO75	Max diff (m)	Min diff (m)
Effelsberg	234	0.25	0.06
Medicina	263	0.37	0.05
Onsala60	214	0.33	0.05
Nyales20	190	0.35	0.05
Wettzell	225	0.37	0.06
Noto	164	0.29	-0.18

Table 1 Maximum and minimum differences between calculated slant delays with Raytrace software minus calculated slant delays with the Conformal Theory of Refraction for all stations involved in EURO75.

Table (1) shows as an example for EURO75, the maximum and minimum differences between calculated slant delays with Raytrace software minus calculated slant delays with the Conformal Theory of Refraction for the 6 stations involved in the experiment: Effelsberg and Wettzell in Germany, Medicina and Noto in Italy, Onsala60 in Sweden and Nyales20 in Norway.

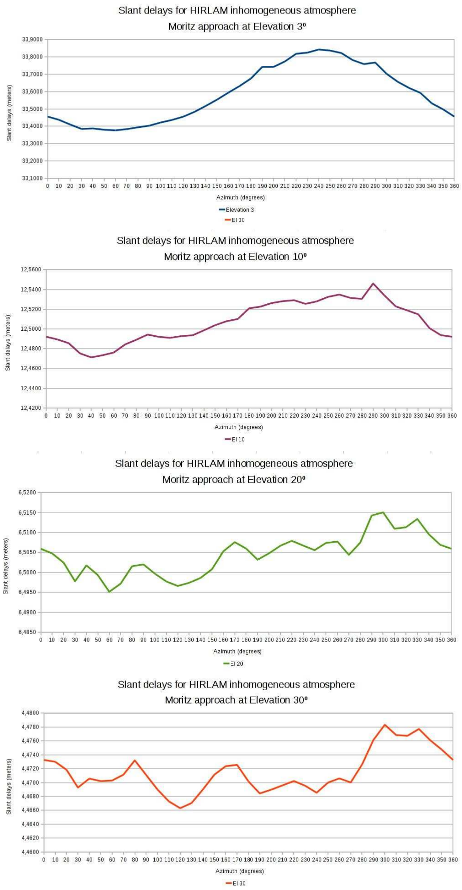

Fig. 5 Calculated slant delays for elevation angles 3°, 10°, 20° and 30° in an inhomogeneous atmosphere using the Conformal Theory of Refraction through HIRLAM for Effelsberg station (25^{th} March 2005 at 12:00 - start time for EURO75).

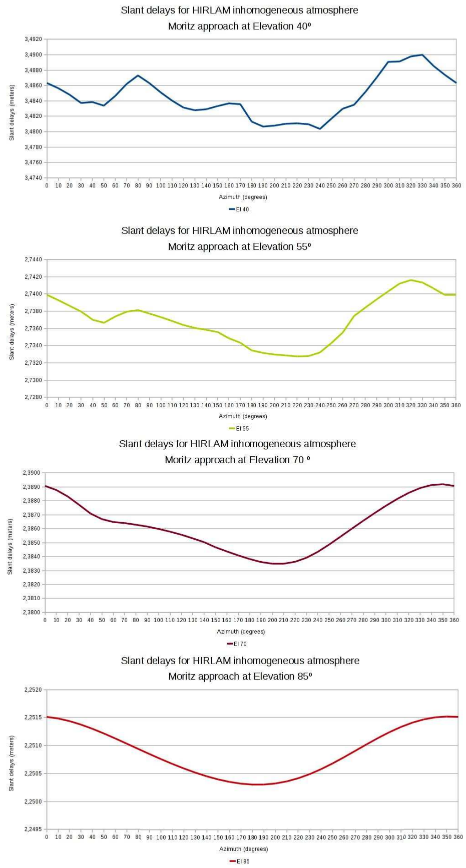

Fig. 6 Calculated slant delays for elevation angles 40°, 55°, 70° and 85° in an inhomogeneous atmosphere using the Conformal Theory of Refraction through HIRLAM for Effelsberg station (25^{th} March 2005 at 12:00 - start time for EURO75).

First preliminary results show differences between the calculated slant delays in an homogeneous and in an inhomogeneous atmosphere. The maximum differences correspond to lower elevations while minimum differences correspond to higher elevations.

Figures (5) and (6) show calculated slant delays using the Conformal Theory of Refraction ray-tracing through HIRLAM for different elevations angles for Effelsberg station during 25^{th} March 2005 at 12:00 (start time for EURO75).

6 Conclusions

We have calculated slant delays modelling an homogeneous and inhomogeneous atmosphere. While using Raytrace software approach simplifies to an homogeneous atmosphere, the Conformal Theory of Refraction approach includes the effect of an inhomogeneous atmosphere in the slant delay calculations. The differences between approaches are mainly the inhomogeneous atmosphere contributions. Comparisons of estimated ZWDs and calculated slant delays with elevation 90° using the Conformal Theory of Refraction is in the order of 1 mm level due to improvements in the interpolation software. We calculate more precisely and accurately slant delays using the Conformal Theory of Refraction. We will continue to compare calculated slant delays using the Conformal Theory of Refraction ray-tracing through HIRLAM to other NWM e.g. ECMWF and other approaches e.g KARAT. We will analyze the 15 geodetic VLBI European data using the calculated slant delays as a priori.

References

J. Böhm, H. Schuh, Vienna Mapping Functions in VLBI Analyses, *IVS 2004 General Meeting Proceedings*, p.277-281

F. K. Brunner and P.V. Angus-Leppan, On the significance of meteorological parameters for terrestrial refraction. *Unisurv G*, 25, 95-108

J. L. Davis, T.A.H. Herring and A.E. Niell, The Davis/Herring/Niell Raytrace program, 1987-1989

S. Garcia-Espada, R. Haas, and F. Colomer, Application of raytracing through the high resolution numerical weather model HIRLAM for the analysis of European VLBI, *6th IVS General Meeting Proceedings*, Hobart, Australia, 2010

T. A. Herring, Modelling atmospheric delay in the analysis of space geodetic data, *Proceedings of Symposium on Refaction of Transatmospheric Signals in Geodesy*, J.C. de Munck and T.A.Th. Spoelstra (eds.), Netherlands Geodetic Commission, Publications on Geodesy, No. 36, New Series, 157-164, 1992

T. Hobiger, R. Ichikawa, Y. Koyama, and T. Kondo, Fast and accurate ray-tracing algorithms for real-time space geodetic applications using numerical weather models, *Journal of Geophysical Research*, VOL. 113, D20302, doi:10.1029/2008JD010503, 2008

H. Moritz Application of the Conformal Theory of Refraction. *Österr. ZfV*, Sonderband 25, pages 323-334, 1967

A. E. Niell Preliminary evaluation of atmospheric mapping functions based on numerical weather models. *Physics and chemistry of the Earth*, 26, 475-480, 2001

B. Stoyanov, R. Haas, L. Gradinarsky, Calculating Mapping Functions from the HIRLAM Numerical Weather Prediction Model, In: In: IVS 2004 General Meeting Proceedings, p.471-475

K. Teke, J. Boehm, T. Nilsson, H. Schuc, P. Steigenberger, R. Dach, R. Heinkelmann, P. Willis, R. Haas, S. Garcia-Espada, T. Hobiger, R. Ichhikawa, S. Shimizu, 2010, Multi-technique comparison of troposphere zenith delays and gradients during CONT08. *Journal of Geodesy* In press