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A Modified CMA for PS-QPSK

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Abstract: A modified constant modulus algorithm (CMA) is presented that allows polarization demultiplexing of polarization-switched QPSK. The suggested algorithm has been found to work well on both numerical and experimental data.
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1. Introduction

By viewing a polarization-multiplexed (PM) signal as a four-dimensional (4D) signal, a number of suggestions for new modulation formats have been found [1,2]. Among these, an 8-point constellation, named polarization-switched QPSK (PS-QPSK), is very interesting due to its optimal power efficiency. This format can be described in the following way: Starting from a single-polarization QPSK signal we can, for every transmitted symbol, choose the launch polarization. Comparing with PM-QPSK, a (4D) PS-QPSK symbol carries 3 bits instead of 4 bits, but only half the power. The sensitivity improvement is 1.76 dB at high SNR and 0.97 dB at a bit-error rate (BER) of 10^{-3}.

We have performed transmission experiments comparing the performance of PS-QPSK and PM-QPSK. During this work, it was found that the constant-modulus algorithm (CMA) cannot be used for PS-QPSK. Investigating this problem, we have found a modified CMA that allows polarization demultiplexing and equalization of PS-QPSK. We will refer to the traditional CMA as *PM-CMA* and to the new algorithm as *PS-CMA*.

2. Description of the PS-CMA

The cost function for PM-CMA is

$$J_{\text{PM-CMA}} = \mathbb{E}\left[\left(|E_x|^2 - P_0\right)^2 + \left(|E_y|^2 - P_0\right)^2\right] = \mathbb{E}\left[\frac{(s_0 - 2P_0)^2}{2} + \frac{s_1^2}{2}\right], \tag{1}$$

where E_x (E_y) is the received electric field in the x (y) polarization, P_0 is the mean power of each polarization, and $\mathbb{E}$ denotes the expectation operator. The cost function has been rewritten in terms of the received Stokes parameters, which shows that PM-CMA makes the total power equal to $2P_0$ and s_1^2 as small as possible. The latter agrees with the fact that $s_1 = 0$ for all symbols in PM-QPSK. However, for PS-QPSK the symbols are mapped to only two points on the Poincaré sphere according to $(s_1, s_2, s_3) = (\pm 1, 0, 0)$. Thus, s_1^2 should be maximized on the Poincaré sphere and we use $s_0^2 = s_1^2 + s_2^2 + s_3^2$ and minimize $s_2^2 + s_3^2 = s_0^2 - s_1^2$. Replacing s_1^2 with $Q(s_2^2 + s_3^2)$, where Q is a constant, we get the cost function

$$J_{\text{PS-CMA}} = \mathbb{E}\left[\frac{(s_0 - P)^2}{2} + Q\frac{s_2^2 + s_3^2}{2}\right] = \mathbb{E}\left[\frac{(|E_x|^2 + |E_y|^2 - P)^2}{2} + 2Q|E_x|^2|E_y|^2\right], \tag{2}$$

where the constant P is the total mean power, i.e., the sum of the mean power is the two polarizations.

We denote the signal after equalization by $\mathbf{y} = [y_1, y_2]^{\mathrm{T}}$ and the sampled signal by the column vectors $\mathbf{x}_1$ and $\mathbf{x}_2$. These are related by the filter column vectors according to $y_1 = \mathbf{h}_{11}^{\mathrm{T}}\mathbf{x}_1 + \mathbf{h}_{12}^{\mathrm{T}}\mathbf{x}_2$ and $y_2 = \mathbf{h}_{21}^{\mathrm{T}}\mathbf{x}_1 + \mathbf{h}_{22}^{\mathrm{T}}\mathbf{x}_2$. Performing the differentiation for PS-CMA, we find the update rules

$$\mathbf{h}_{11}^{(k+1)} = \mathbf{h}_{11}^{(k)} - \mu_{\text{PS}}\left[|y_1|^2 + (1+2Q)|y_2|^2 - P\right] y_1 \mathbf{x}_1^*, \quad \mathbf{h}_{12}^{(k+1)} = \mathbf{h}_{12}^{(k)} - \mu_{\text{PS}}\left[|y_1|^2 + (1+2Q)|y_2|^2 - P\right] y_1 \mathbf{x}_2^*, \tag{3}$$

$$\mathbf{h}_{21}^{(k+1)} = \mathbf{h}_{21}^{(k)} - \mu_{\text{PS}}\left[(1+2Q)|y_1|^2 + |y_2|^2 - P\right] y_2 \mathbf{x}_1^*, \quad \mathbf{h}_{22}^{(k+1)} = \mathbf{h}_{22}^{(k)} - \mu_{\text{PS}}\left[(1+2Q)|y_1|^2 + |y_2|^2 - P\right] y_2 \mathbf{x}_2^*, \tag{4}$$

where μ_{PS} is the step size and k is the iteration number. These expressions are similar to the PM-CMA update rules and switching between PS-CMA and PM-CMA can be done by changing the numerical parameters, which is convenient if PS-QPSK serves as a fall-back for PM-QPSK [3]. We set $Q = 1/2$ for PS-CMA and $Q = -1/2$ for PM-CMA.

3. Polarization tracking capabilities of PS-CMA and PM-CMA

The PS-CMA has been found to work well on experimental data and is capable of performing polarization demultiplexing and equalization in all cases. We have also investigated the polarization tracking capabilities numerically for the 1-tap filter case. For this study, the fiber matrix was modeled as a polarization rotation matrix with a rotation angle that was evolving linearly with time. A symbol sequence was generated and complex white Gaussian noise (AWGN) was added. Running the algorithms on long sequences of symbols, we used the fiber matrix and the demultiplexing matrix to find an averaged value of the SNR penalty. Plotting this penalty as a function of the angular frequency of ϕ for a selection of step sizes, we obtain a quantitative measurement of the algorithm tracking capability.

The result of the tracking comparison is seen in Figure 1. Simulations have been run using three different step sizes which are indicated in the figure. The blue and the red curves show the results for PS-QPSK and PM-QPSK, respectively, when noise has been added to make the BER $= 10^{-3}$. The black curve shows the case for PS-QPSK with equal SNR as for PM-QPSK. In general, a larger step size corresponds to better tracking capability but worse steady-state performance. Comparing the different cases, we find that the tracking capability of PS-CMA well matches that of PM-CMA. The blue curves, which correspond to a lower SNR, shows a somewhat larger steady-state penalty.

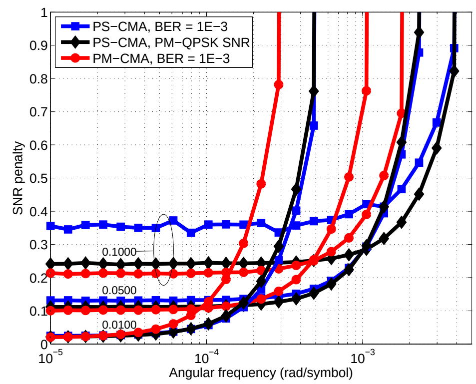

Fig. 1. The SNR penalty (in dB) as a function of the angular drift frequency of the fiber matrix. As an example, 10^{-3} rad/symbol corresponds to 10 Mrad/s at 10 Gbaud, which is a very high polarization rotation rate. The step sizes have been indicated in the figure.

In summary, we have found that the PS-CMA shares many properties with PM-CMA. The update expressions are similar and the convergence and tracking performance have been found to be similar. Unfortunately, the PS-CMA also shows a problem similar to the *singularity problem* of PM-CMA. The PS-CMA is therefore in many ways a natural replacement for the PM-CMA when PS-QPSK is used.

4. Conclusion

We have presented a modified CMA that can perform polarization demultiplexing and equalization when using PS-QPSK. The algorithm has been found to work well on experimental data and numerical simulations have shown similar performance for PS-CMA compared to PM-CMA.

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