Bounds and Algorithms for Multiple Frequency Offset Estimation in Cooperative Networks

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***Abstract*—The distributed nature of cooperative networks may result in multiple carrier frequency offsets (CFOs), which make the channels time varying and overshadow the diversity gains promised by collaborative communications. This paper seeks to address multiple CFO estimation using training sequences in space-division multiple access (SDMA) cooperative networks. The system model and CFO estimation problem for cases of both decode-and-forward (DF) and amplify-and-forward (AF) relaying are formulated and new closed-form expressions for the Cramer-Rao lower bound (CRLB) for both protocols are derived. The CRLBs are then applied in a novel way to formulate training sequence design guidelines and determine the effect of network protocol and topology on CFO estimation. Next, two computationally efficient iterative estimators are proposed that determine the CFOs from multiple simultaneously relaying nodes. The proposed algorithms reduce multiple CFO estimation complexity without sacrificing bandwidth and training performance. Unlike existing multiple CFO estimators, the proposed estimators are also accurate for both large and small CFO values. Numerical results show that the new methods outperform existing algorithms and reach or approach the CRLB at mid-to-high signal-to-noise ratio (SNR). When applied to system compensation, simulation results show that the proposed estimators significantly reduce average-bit-error-rate (ABER).**

***Index Terms*—Cooperative communications, synchronization, carrier frequency offset estimation, Cramer-Rao Lower Bound (CRLB), MUltiple SIgnal Characterization (MUSIC).**

I. Introduction

COOPERATIVE multiplexing and diversity, which are achieved when multiple terminals collaborate through distributed transmissions, are shown to increase capacity and reliability in wireless networks [1]–[5]. However, the majority of the analysis in the area of cooperative communications is focused on improving capacity and reliability while assuming perfect frequency synchronization [1]–[5].

The presence of multiple *carrier frequency offsets (CFOs)* in distributed cooperative networks arises due to simultaneous transmissions from spatially separated nodes with different oscillators and Doppler shifts. The CFOs result in the rotation of the signal constellation causing *signal to noise ratio (SNR)* loss. The amount of SNR loss and channel estimation accuracy are highly dependent on CFO estimation precision at the receiver [6]. Thus, accurate CFO estimation is key to successful deployments of cooperative networks.

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In [7]–[9] and references therein, space time coding techniques are proposed that provide full spatial diversity in the presence of CFOs. However, the schemes outlined in [7]–[9] require CFOs to be estimated and equalized at the destination and do not address CFO estimation.

Previously proposed multiple CFO estimation methods for frequency-flat *multiple-input-multiple-output (MIMO)* systems include [10]–[13]. In [10], a *maximum-likelihood estimator (MLE)* is presented that requires exhaustive search and performs poorly when the CFOs are close to one another. In [11], a *correlation-based estimator (CBE)* is proposed using orthogonal training sequences transmitted from different antennas. However, the CBE suffers from an error floor, requires the use of correlators at the receiver, and performs very poorly when normalized CFO values are larger than 0.05. In [12] and [13], iterative schemes are proposed to eliminate the CBE's error floor. However, since CBE is used as the initial estimator, the estimators in [12] and [13] also perform poorly at large CFO values. While the assumption of small CFO values in [11]–[13] might hold for point-to-point MIMO systems, it is not justifiable for cooperative systems with distributed nodes and independent oscillators. In addition, the estimators in [10]–[14] cannot be directly applied to the case of *amplify-and-forward (AF)* relaying networks due to the different training signal model.

In [15] a *maximum a posteriori (MAP)* CFO estimator for single-relay frequency-flat 3-terminal *decode-and-forward (DF)* networks is presented. However, the approach in [15] is limited to the case of DF relaying and suffers from the same shortcomings as in [10]. While a multiple CFO estimator for DF relaying cooperative networks is proposed in [16], no specific performance analysis is provided. In [17], CFO estimation in two-way AF relaying networks is investigated. However, the system model consists of a single relay only and the effect of Doppler shift is ignored. In [18], CFO estimation in multi-relay *orthogonal frequency division multiple access (OFDMA)*-based cooperative networks is addressed. However, to simplify the CFO estimation problem, it is assumed in [18] that at any given time only a single relay transmits its signal to the receiver. Finally, CFO estimation in AF relaying single-relay *orthogonal frequency division multiplexing (OFDM)*-based cooperative networks has been analyzed in [19], where similar to [18], it is assumed that the received signal is affected by only a single CFO.

The *Cramer-Rao lower bound (CRLB)* [20] has been used as a quantitative performance measure for CFO estimators and can be applied to determine the effects of network protocol and topology on CFO estimation accuracy in cooperative systems. The CRLB for CFO estimation in MIMO point-to-point systems is derived in [10]. In [15], the CRLB for 3-terminal DF cooperative networks is

presented. However, the analysis in [15] is limited to single-relay DF networks and the results are based on an assumed Gaussian distribution for the CFO, which is not realistic, given that the sources of CFO do not undergo significant changes, as shown in [10]–[13], [21], [22]. To the best of the authors' knowledge there are no CRLB results in the literature for joint estimation of multiple CFOs and channel gains in a multi-relay cooperative network.

The *MUltiple SIgnal Characterization (MUSIC)* algorithm is a spectral estimation method that has been applied to the estimation of parameters of a received signal, including CFO and direction-of-arrival in point-to-point systems [23]. The application of the MUSIC algorithm to CFO estimation in multi-relay or multi-user networks is difficult, however, due to the following shortcomings: i) estimating closely-spaced CFO values, and ii) no method of assigning each CFO to its source. In the case of OFDMA-based systems, the MUSIC algorithm has been proposed as a suitable method of estimating each user's CFO [24], [25]. However, to address the shortcomings of MUSIC, the algorithms in [24], [25] are based on the assumptions that in OFDMA systems, the users' carrier frequencies are well spaced and each user has a specific set of subcarriers assigned to it. Both of these assumptions are not applicable to *space-division multiple access (SDMA)* networks, where multiple relays simultaneously transmit their signals over the same frequency band [4], [5], [9].

Under the consideration of frequency-flat fading channels, this paper seeks to extend the results in [26] so as to derive more general expressions of the CRLB, provide a more comprehensive investigation of the performance of the proposed estimators, and gain insight on the effect of CFO estimation accuracy on the performance of cooperative networks. The contributions and organization of this paper can be summarized as follows:

- In Section II, a system model for CFO estimation in DF and AF relaying networks is outlined. Flat-fading channels are considered, which is motivated by pioneering work in the area of multiple CFO and channel gain estimation for MIMO and cooperative networks [10]–[13], [15], [16]. In addition, consideration of frequency-selective channels for multi-relay cooperative networks would require estimation of very large numbers of parameters and is beyond the scope of this paper.
- In Section III, new closed-form CRLB expressions for CFO estimation for AF and DF multi-relay cooperative systems are derived. In addition to serving as a benchmark for assessing the performance of CFO estimators, the CRLBs are used in a novel way to quantitatively determine the effect of network protocol and number of relays on CFO estimation accuracy.
- Section IV proposes an algorithm that uses linearly independent training sequences transmitted from each relay to address certain shortcomings of MUSIC and accurately estimate and assign each CFO to its corresponding relay. Unlike the algorithms in [10]–[13] the proposed estimators have accuracies that are maintained over the full range of possible CFO values. Moreover, it is shown that the proposed CFO estimators are also applicable to AF relaying networks. Finally, a complexity analysis for both estimators is presented.
- In Section V, numerical results are presented showing that the proposed estimators either reach or approach the CRLB at mid-to-high SNR. By combining the proposed CFO estimation technique with the CFO compensation method in [27], it is also shown that frequency synchronization and significant performance gains in cooperative networks may be achieved.

Notation: italic letters (x) are scalars, bold lower case letters ($\mathbf{x}$) are vectors, bold upper case letters ($\mathbf{X}$) are matrices, $\mathbf{X_{k,m}}$ represents the kth row and mth column element of $\mathbf{X}$, $\mathbf{I}$ denotes the identity matrix, $\odot$ stands for Schur (element-wise) product, and $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and Tr$(\cdot)$ denote conjugate, transpose, conjugate transpose (hermitian), and trace, respectively.

II. System Model

A half-duplex SDMA cooperative network consisting of a source and destination pair and a cluster of R relay nodes is considered, where the relays are assumed to be distributed throughout the network as shown in Fig. 1. Multiple CFO estimation using a *training sequence (TS)* is analyzed, where during the training interval, the CFOs and channel gains corresponding to R relay nodes are estimated. These estimates can be applied in the data transmission interval to improve system performance. Throughout this paper, the following set of assumptions and system design parameters are considered:

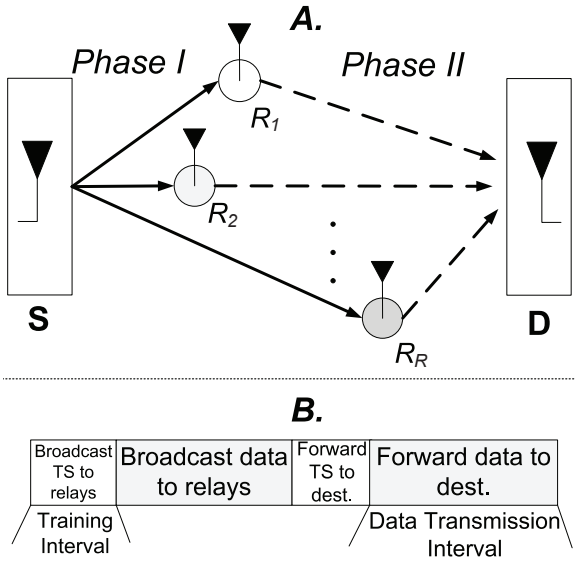

Fig. 1. The system model for the cooperative network and scheduling diagram for training and data transmission intervals.

1) In *Phase I* the source broadcasts its TS to the relays and in *Phase II* the relays transmit R *linearly independent and orthonormal* TSs *simultaneously* to the destination, as in Fig. 1.
2) Without loss of generality, it is assumed that unit amplitude *phase-shift keying (PSK)* TSs are transmitted.
3) Quasi-static and flat-fading channels are considered, where the channel gains are assumed not to change over the length of a frame of symbols but to change from frame to frame.
4) CFOs are modeled as *unknown* non-random parameters.
5) Similar to most CFO and channel estimation methods, it is assumed that nodes within the network are synchronized

in time [10]–[13]. In addition, it is noted in [10], [21] that timing offset estimation can be decoupled from CFO estimation.

Note that Assumptions 2, 3, and 4 are in line with previous CFO estimation analyses in [10]–[13], [21] and are also intuitively justifiable, since the main sources of CFO are oscillator mismatch and Doppler shift. In addition, oscillator properties, Doppler shifts, and channel gains are assumed not to change significantly during a transmission block consisting of TS and data.

A. Training Signal Model for DF Relaying Cooperative Networks

1) Training Signal Model at the Relays: For DF relaying, the signal at the relays is down-converted to baseband, matched-filtered, and decoded [1], [2], [9], [28]. Thus, the CFO from the source to the kth relay, $\nu_k^{[\mathrm{sr}]}$, for $k = 1, 2, \cdots, R$, needs to be estimated at each relay, similar to that of a *single-input-single-output (SISO)* system. The baseband received training signal, $r_k(n)$ at the kth relay node at time n, for $n = 1, \cdots, L$ and $k = 1, \cdots, R$, is given by

$$r_k(n) = h_k e^{j2\pi n \nu_k^{[\mathrm{sr}]}} t^{[s]}(n) + v_k(n), \tag{1}$$

where:

- L denotes the length of the TS,
- $\mathbf{t}^{[s]} \triangleq \left[t^{[s]}(1), \cdots, t^{[s]}(L)\right]^T$ is the *known* TS broadcast from the source to the relay nodes,
- $\nu_k^{[\mathrm{sr}]} \triangleq \Delta\nu_k^{[\mathrm{sr}]} T$ is the normalized CFO from the source to the kth relay with T as the symbol duration,
- h_k represents the *unknown* channel gain from the source to the kth relay, and
- $v_k(n)$ is the zero-mean *additive white Gaussian noise (AWGN)* at the kth relay with variance $\sigma_{v_k}^2$ and denoted by $\mathcal{CN}(0, \sigma_{v_k}^2)$.

Given that CFO estimation in SISO systems has been extensively addressed in the literature, estimation of $\boldsymbol{\nu}^{[\mathrm{sr}]} \triangleq \left[\nu_1^{[\mathrm{sr}]}, \cdots, \nu_R^{[\mathrm{sr}]}\right]^T$ is not discussed further. Instead the reader is referred to [6].

2) Training Signal Model at the Destination: The baseband received training signal model at the destination, y, for a DF cooperative network consisting of R relay nodes is given by

$$y(n) = \sum_{k=1}^{R} g_k e^{j2\pi n \nu_k^{[\mathrm{rd}]}} t_k^{[r]}(n) + w(n), \quad n = 1, \cdots, L \tag{2}$$

where:

- $\mathbf{t}_k^{[r]} \triangleq \left[t_k^{[r]}(1), \cdots, t_k^{[r]}(L)\right]^T$ is the distinct and *known* TS transmitted to the kth relay,
- g_k denotes the *unknown* channel gain from the kth relay to the destination,
- $\nu_k^{[\mathrm{rd}]} \triangleq \Delta\nu_k^{[\mathrm{rd}]} T$ is the normalized CFO from the kth relay to the destination, and
- $w(n)$ is the AWGN at the destination with $\mathcal{CN}(0, \sigma_w^2)$.

According to (2), the CFOs $\boldsymbol{\nu}^{[\mathrm{rd}]} \triangleq \left[\nu_1^{[\mathrm{rd}]}, \cdots, \nu_R^{[\mathrm{rd}]}\right]^T$, and channel gains, $\mathbf{g} \triangleq [g_1, \cdots, g_R]^T$, need to be jointly estimated at the destination.

B. Training Signal Model for AF Relaying Cooperative Networks

1) Training Signal Model at the Relays: Since signals traveling through different channels experience different Doppler shifts and different oscillator offsets, the received signal at the destination is affected by multiple CFOs even if the relay does not convert the signal to baseband. Hence, to achieve frequency synchronization in an AF relaying network, we propose the baseband processing structure in Fig. 2 at each relay. In practice, AF relaying networks typically require signals at the relays to be converted to baseband [1], [4]. We remark that the proposed baseband processing structure of Fig. 2 does not increase hardware complexity at the relays and is significantly simpler than that of DF networks, which require relays to be equipped with a matched filter, decoder, detector, and pulse shaping filter for retransmission.

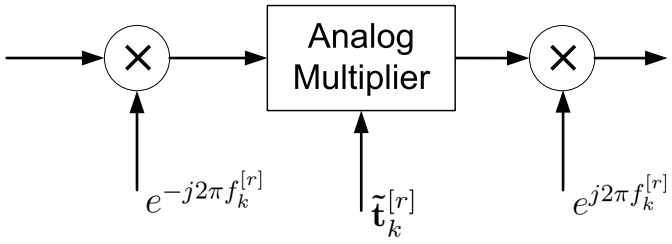

Fig. 2. The baseband processing structure at the relays in the case of AF relaying multi-relay networks ($f_k^{[r]}$ denotes the carrier frequency at the kth relay).

2) Training Signal Model at the Destination: For AF relaying, the signal model is given by

$$y(n) = \underbrace{\sum_{k=1}^{R} \zeta_k g_k h_k e^{j2\pi n \nu_k^{[\mathrm{sum}]}} \tilde{t}_k^{[r]}(n) t^{[s]}(n)}_{\textbf{desired signal}} + \underbrace{\sum_{k=1}^{R} \zeta_k g_k e^{j2\pi n \nu_k^{[\mathrm{rd}]}} \tilde{t}_k^{[r]}(n) v_k(n) + w(n)}_{\textbf{overall noise}}, \tag{3}$$

where ζ_k is a scaling factor that is used to satisfy the kth relay's power constraint, $\nu_k^{[\mathrm{sum}]} \triangleq \nu_k^{[\mathrm{rd}]} + \nu_k^{[\mathrm{sr}]}$, $\tilde{t}_k^{[r]}(n)$ is used to modulate the received TS, $t^{[s]}(n)$ to ensure the kth relay has a specific TS.

Eq. (3) follows from the fact that the received signal, $r_k(n)$, is amplified and forwarded without being decoded. Note that in (3), $\tilde{\mathbf{t}}_k^{[r]} = \left[\tilde{t}_k^{[r]}(1), \cdots, \tilde{t}_k^{[r]}(L)\right]^T$ does not change the statistical properties of the noise, v_k, assuming unit-amplitude PSK training symbols.

According to (3), $2R$ quantities containing CFOs are present in the signal model: 1) $\boldsymbol{\nu}^{[\mathrm{sum}]} \triangleq \left[\nu_1^{[\mathrm{sum}]}, \cdots, \nu_R^{[\mathrm{sum}]}\right]^T$, which result in rotation of the signal constellation and 2) $\boldsymbol{\nu}^{[\mathrm{rd}]} \triangleq \left[\nu_1^{[\mathrm{rd}]}, \cdots, \nu_R^{[\mathrm{rd}]}\right]^T$, which affect the noise at the relays. Since components of $\boldsymbol{\nu}^{[\mathrm{rd}]}$ only phase-shift the noise in (3) and do not affect signal detection, the terms $\boldsymbol{\nu}^{[\mathrm{sum}]}$ are the only CFO-related quantities that influence system performance. From the above, we conclude that it suffices to estimate $\boldsymbol{\nu}^{[\mathrm{sum}]}$ at the destination.

Note that it has been shown in [29] that for effective signal detection at the destination in the case of AF relaying coopera-

tive networks, only the overall channel gains from source-relay-destination links, $g_k h_k$, for $k = 1, \cdots, R$, need to be estimated at the destination, whereas the channel gains $\mathbf{h} \triangleq [h_1, \cdots, h_R]^T$ and $\mathbf{g} \triangleq [g_1, \cdots, g_R]^T$ do not need to be estimated separately.

III. Cramer-Rao Lower Bound

In this section, the CRLBs for joint CFO and channel estimation in multi-relay networks are derived.

A. Amplify-and-Forward Cooperative Networks

The signal model in (3) can be rewritten as

$$y(n) = \sum_{k=1}^{R} \left(\zeta_k \alpha_k e^{j2\pi n \nu_k^{[\text{sum}]}} c_k(n) + \beta_k e^{j2\pi n \nu_k^{[\text{rd}]}} \tilde{v}_k(n) \right) + w(n), \tag{4}$$

where $\alpha_k \triangleq g_k h_k$, $\beta_k \triangleq \zeta_k g_k$, $c_k(n) \triangleq \tilde{t}_k^{[r]}(n) t^{[s]}(n)$, and $\tilde{v}_k \triangleq \tilde{t}_k^{[r]}(n) v_k(n)$. Note that due to the assumption of unit-amplitude PSK TSs, $\tilde{v}_k$ has the same statistical properties as v_k.

Based on the signal model in Section IIB, the vector of parameters of interest, $\boldsymbol{\lambda}^{[\text{AF}]}$, is given by

$$\boldsymbol{\lambda}^{[\text{AF}]} \triangleq \left[\text{Re}\{\boldsymbol{\alpha}\}^T, \text{Im}\{\boldsymbol{\alpha}\}^T, \left(\boldsymbol{\nu}^{[\text{sum}]}\right)^T \right]^T, \tag{5}$$

where $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_R]^T$, and $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote real and imaginary parts, respectively.

Throughout this section AWGN at the relays and destination is considered, where $\mathbf{v}_k = [v_k(1), \cdots, v_k(L)]^T$, for $k = 1, 2, \cdots, R$, and $\mathbf{w} = [w(1), \cdots, w(L)]^T$ are distributed according to $\mathcal{CN}\left(0, \sigma_{v_k}^2 \mathbf{I}\right)$ and $\mathcal{CN}\left(0, \sigma_w^2 \mathbf{I}\right)$, respectively. Moreover, $\mathbf{v}_k$, $\mathbf{v}_m$, $\forall k \neq m$, and $\mathbf{w}$, are assumed to be mutually independent.

Based on the above set of assumptions, the vector of received training signal, $\mathbf{y} = [y(1), \cdots, y(L)]^T$, is distributed as $\mathcal{CN}\left(\boldsymbol{\mu}_\mathbf{y}, \boldsymbol{\Sigma}_\mathbf{y}\right)$, where $\boldsymbol{\mu}_\mathbf{y}$ and $\boldsymbol{\Sigma}_\mathbf{y}$ are given by

$$\boldsymbol{\mu}_\mathbf{y} = \sum_{k=1}^{R} \alpha_k \mathbf{e}_k^{[\text{sum}]}, \quad \text{and} \tag{6}$$

$$\begin{aligned}
[\boldsymbol{\Sigma}_\mathbf{y}]_{l,i} =& E\left[\left(\sum_{k=1}^{R} \beta_k e^{j2\pi l \nu_k^{[\text{rd}]}} \tilde{t}_k^{[r]}(l) v_k(l) + w(l) \right) \right. \\
& \left. \times \left(\sum_{m=1}^{R} \beta_m e^{j2\pi i \nu_m^{[\text{rd}]}} \tilde{t}_m^{[r]}(i) v_m(i) + w(i) \right)^H \right] && \text{(7a)} \\
=& \sum_{k=1}^{R} \sum_{m=1}^{R} \beta_k \beta_m^* E\left[e^{j2\pi l \nu_k^{[\text{rd}]}} \tilde{t}_k^{[r]}(l) \left(e^{j2\pi i \nu_m^{[\text{rd}]}} \tilde{t}_m^{[r]}(i) \right)^* \right] \\
& \times E[v_k(l) v_m^*(i)] + E[w(l) w^*(i)] && \text{(7b)} \\
=& \begin{cases} 0 & i \neq l \\ \sum_{k=1}^{R} |\beta_k|^2 \sigma_{v_k}^2 + \sigma_w^2 & i = l \end{cases} && \text{(7c)}
\end{aligned}$$

where $\mathbf{e}_k^{[\text{sum}]} \triangleq \zeta_k \left[c_k(1) e^{j2\pi \nu_k^{[\text{sum}]}}, \cdots, c_k(L) e^{j2\pi L \nu_k^{[\text{sum}]}} \right]^T$. Note that (7b) follows from the fact that the noise at the destination and kth relay, $\mathbf{w}$ and $\mathbf{v}_k$, respectively, are mutually independent, $\forall k$, and (7c) follows the AWGN assumption, where $E[v(i)v(l)] = E[w(i)w(l)] = 0$ for $i \neq l$. Thus, $\boldsymbol{\Sigma}_\mathbf{y}$ is given by

$$\boldsymbol{\Sigma}_\mathbf{y} = \left(\sum_{k=1}^{R} |\beta_k|^2 \sigma_{v_k}^2 + \sigma_w^2 \right) \mathbf{I}. \tag{8}$$

To determine the CRLB, the $3R \times 3R$ *Fisher's Information Matrix (FIM)* needs to be determined. For parameter estimation from a complex Gaussian observation sequence, the FIM entries are given by [20]

$$\begin{aligned}
\mathbf{FIM}(\boldsymbol{\lambda})_{k,m} =& 2\text{Re}\left\{ \frac{\partial \boldsymbol{\mu}_\mathbf{y}^H}{\partial \lambda_k} \boldsymbol{\Sigma}_\mathbf{y}^{-1} \frac{\partial \boldsymbol{\mu}_\mathbf{y}}{\partial \lambda_m} \right\} \\
& + \text{Tr}\left(\boldsymbol{\Sigma}_\mathbf{y}^{-1} \frac{\partial \boldsymbol{\Sigma}_\mathbf{y}}{\partial \lambda_k} \boldsymbol{\Sigma}_\mathbf{y}^{-1} \frac{\partial \boldsymbol{\Sigma}_\mathbf{y}}{\partial \lambda_m} \right),
\end{aligned} \tag{9}$$

where $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{[\text{AF}]}$ is defined in (5). The corresponding components of the CRLB are computed as

$$\frac{\partial \boldsymbol{\mu}_\mathbf{y}}{\partial \nu_k^{[\text{sum}]}} = j\alpha_k \mathbf{D}_\mathbf{L} \mathbf{e}_k^{[\text{sum}]}, \quad \frac{\partial \boldsymbol{\mu}_\mathbf{y}}{\partial \text{Re}\{\alpha_k\}} = -j \frac{\partial \boldsymbol{\mu}_\mathbf{y}}{\partial \text{Im}\{\alpha_k\}} = \mathbf{e}_k^{[\text{sum}]}, \tag{10}$$

where $[\mathbf{D}_\mathbf{L}]_{L \times L} \triangleq \text{diag}\left(2\pi, 4\pi, \cdots, 2L\pi\right)$,

$$\frac{\partial \boldsymbol{\Sigma}_\mathbf{y}}{\partial \text{Re}\{\alpha_k\}} = \frac{\alpha_k^* + \alpha_k}{|h_k|^2} \zeta_k^2 \sigma_{v_k}^2 \mathbf{I} = \frac{2\text{Re}\{\alpha_k\}}{|h_k|^2} \zeta_k^2 \sigma_{v_k}^2 \mathbf{I}, \tag{11a}$$

$$\frac{\partial \boldsymbol{\Sigma}_\mathbf{y}}{\partial \text{Im}\{\alpha_k\}} = j\frac{\alpha_k^* - \alpha_k}{|h_k|^2} \zeta_k^2 \sigma_{v_k}^2 \mathbf{I} = \frac{2\text{Im}\{\alpha_k\}}{|h_k|^2} \zeta_k^2 \sigma_{v_k}^2 \mathbf{I}. \tag{11b}$$

After substituting the derivatives in (10), (11a), and (11b) into (9) and after carrying out straightforward algebraic manipulations for $k, m = 1, \cdots R$, we arrive at

$$\begin{aligned}
\mathbf{FIM}_{k,m} =& \frac{2}{\sigma_n^2} \text{Re}\left\{ \left(\mathbf{e}_k^{[\text{sum}]} \right)^H \mathbf{e}_m^{[\text{sum}]} \right\} \\
& + 2L \frac{\text{Re}\{\alpha_k\}\text{Re}\{\alpha_m\}}{|h_k|^2 |h_m|^2} \frac{\sigma_{v_k}^2 \sigma_{v_m}^2 \zeta_k^2 \zeta_m^2}{\sigma_n^4},
\end{aligned} \tag{12}$$

$$\begin{aligned}
\mathbf{FIM}_{k,R+m} =& -\frac{2}{\sigma_n^2} \text{Im}\left\{ \left(\mathbf{e}_k^{[\text{sum}]} \right)^H \mathbf{e}_m^{[\text{sum}]} \right\} \\
& + 2L \frac{\text{Re}\{\alpha_k\}\text{Im}\{\alpha_m\}}{|h_k|^2 |h_m|^2} \frac{\sigma_{v_k}^2 \sigma_{v_m}^2 \zeta_k^2 \zeta_m^2}{\sigma_n^4},
\end{aligned} \tag{13}$$

$$\mathbf{FIM}_{k,2R+m} = -\frac{2}{\sigma_n^2} \text{Im}\left\{ \alpha_m \left(\mathbf{e}_k^{[\text{sum}]} \right)^H \mathbf{D}_\mathbf{L} \mathbf{e}_m^{[\text{sum}]} \right\}, \tag{14}$$

$$\begin{aligned}
\mathbf{FIM}_{R+k,R+m} =& \frac{2}{\sigma_n^2} \text{Re}\left\{ \left(\mathbf{e}_k^{[\text{sum}]} \right)^H \mathbf{e}_m^{[\text{sum}]} \right\} \\
& + 2L \frac{\text{Im}\{\alpha_k\}\text{Im}\{\alpha_m\}}{|h_k|^2 |h_m|^2} \frac{\sigma_{v_k}^2 \sigma_{v_m}^2 \zeta_k^2 \zeta_m^2}{\sigma_n^4},
\end{aligned} \tag{15}$$

$$\mathbf{FIM}_{R+k,2R+m} = \frac{2}{\sigma_n^2} \text{Re}\left\{ \alpha_m \left(\mathbf{e}_k^{[\text{sum}]} \right)^H \mathbf{D}_\mathbf{L} \mathbf{e}_m^{[\text{sum}]} \right\}, \tag{16}$$

$$\mathbf{FIM}_{2R+k,2R+m} = \frac{2}{\sigma_n^2} \text{Re}\left\{ \alpha_k^* \alpha_m \left(\mathbf{e}_k^{[\text{sum}]} \right)^H \mathbf{D}_\mathbf{L}^2 \mathbf{e}_m^{[\text{sum}]} \right\}, \tag{17}$$

Note that (11a) and (11b) follow (8) due to the fact that $\frac{\partial a}{\partial (ab)} = \frac{1}{b}$.

where $\sigma_n^2 \triangleq \sum_{k=1}^{R} |\beta_k|^2 \sigma_{v_k}^2 + \sigma_w^2$. Rewriting (12)-(17), the FIM can be rewritten in compact form as shown in (18) at the bottom of this page. In (18),

- $[\mathbf{D}_{\boldsymbol{\alpha}}]_{R\times R} \triangleq \text{diag}(\alpha_1, \cdots, \alpha_R)$,
- $[\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}]_{L\times R} \triangleq [\mathbf{e}_1^{[\text{sum}]}, \cdots, \mathbf{e}_R^{[\text{sum}]}]$, and
- $\boldsymbol{\rho} = \sqrt{\frac{L}{\sigma_n^2}} \left[\frac{\alpha_1 \zeta_1^2 \sigma_{v_1}^2}{|h_1|^2}, \frac{\alpha_2 \zeta_2^2 \sigma_{v_2}^2}{|h_2|^2}, \cdots, \frac{\alpha_R \zeta_R^2 \sigma_{v_R}^2}{|h_R|^2} \right]^T$.

Next, using partitioned matrix inverse in a similar manner to that in [10], [30], a closed-form expression for the CRLB for CFO estimation in the case of AF relaying cooperative networks can be derived as shown in (19) at the bottom of this page. In (19),

- $[\boldsymbol{\Upsilon}]_{R\times R} \triangleq \mathbf{D}_{\boldsymbol{\alpha}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}}^{\mathbf{2}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}$,
- $[\boldsymbol{\Pi}]_{R\times R} \triangleq \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}$, and
- $\mathbf{F}_{\mathbf{2R}\times\mathbf{2R}}$ is the first $2R$ rows and $2R$ columns of the $\mathbf{FIM}$ in (18).

In order to derive the CRLB for the estimation of the combined real and imaginary parts of channel gains, $\boldsymbol{\alpha}$, the set of parameters of interest, $\boldsymbol{\lambda}^{[\text{AF}]}$, is modified as

$$\overline{\boldsymbol{\lambda}}^{[\text{AF}]} \triangleq \underbrace{\begin{bmatrix} \mathbf{I} & j\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{\triangleq \mathbf{J}} \boldsymbol{\lambda}^{[\text{AF}]}, \tag{20}$$

where $\overline{\boldsymbol{\lambda}}^{[\text{AF}]} = \left[\boldsymbol{\alpha}^T, \left(\boldsymbol{\nu}^{[\text{sum}]}\right)^T \right]$. The CRLB for the estimation of $\overline{\boldsymbol{\lambda}}^{[\text{AF}]}$ is given by [20]

$$\mathbf{CRLB}\left(\overline{\boldsymbol{\lambda}}^{[\text{AF}]}\right) = \mathbf{JCRLB}\left(\boldsymbol{\lambda}^{[\text{AF}]}\right) \mathbf{J}^H. \tag{21}$$

Using (18) and (21) the CRLB for the estimation of channel gains is given in (22) at the bottom of this page. In (22), $\overline{\mathbf{J}} \triangleq [\mathbf{I} \quad j\mathbf{I}]$ and $\boldsymbol{\Phi}$ is defined in (19).

B. Decode-and-Forward Cooperative Networks

According to the received signal model in (2), the CFOs, $\boldsymbol{\nu}^{[\text{rd}]}$, and channel gains, $\mathbf{g} = [g_1, \cdots, g_R]$, need to be jointly estimated at the destination. Therefore, the parameter vector of interest is

$$\boldsymbol{\lambda}^{[\text{DF}]} \triangleq \left[\text{Re}\{\mathbf{g}\}^T, \text{Im}\{\mathbf{g}^T\}, \left(\boldsymbol{\nu}^{[\text{rd}]}\right)^T \right]^T. \tag{23}$$

In addition, under the assumption of additive Gaussian noise the received signal vector, $\mathbf{y} = [y_1, \cdots, y_L]^T$ is distributed as $\mathcal{CN}(\boldsymbol{\mu}_{\mathbf{y}}, \boldsymbol{\Sigma}_{\mathbf{y}})$, where $\boldsymbol{\mu}_{\mathbf{y}} = \sum_{k=1}^{R} g_k \mathbf{e}_k^{[\text{rd}]}$, $\boldsymbol{\Sigma}_{\mathbf{y}} = \sigma_w^2 \mathbf{I}$, and $\mathbf{e}_k^{[\text{rd}]} \triangleq \left[t_k^{[r]}(1) e^{j2\pi\nu_k^{[\text{rd}]}}, \cdots, t_k^{[r]}(L) e^{j2\pi L\nu_k^{[\text{rd}]}} \right]^T$.

Using (9) and similar steps as in Section III A., the CRLBs for the estimation of the CFOs, $\boldsymbol{\nu}^{[\text{rd}]}$, and channel gains, $\mathbf{g}$, in DF relaying cooperative networks are given by

$$\mathbf{CRLB}\left(\boldsymbol{\nu}^{[\text{rd}]}\right) = \frac{\sigma_w^2}{2} \underbrace{\left(\text{Re}\left\{ \mathbf{D}_{\mathbf{g}}{}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H \mathbf{D}_{\mathbf{L}} \boldsymbol{\Xi}_{\boldsymbol{\nu}^{[\text{rd}]}} \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \mathbf{D}_{\mathbf{g}} \right\} \right)^{-1}}_{\triangleq \boldsymbol{\Theta}}, \tag{24}$$

and (25) at the bottom of this page, respectively. In (24) and (25), $[\mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}]_{L\times R} \triangleq [\mathbf{e}_1^{[\text{rd}]}, \cdots, \mathbf{e}_R^{[\text{rd}]}]$, $[\mathbf{D}_{\mathbf{g}}]_{R\times R} \triangleq \text{diag}(g_1, g_2, \cdots, g_R)$, $[\boldsymbol{\Xi}_{\boldsymbol{\nu}^{[\text{rd}]}}]_{L\times L} \triangleq \mathbf{I} - \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \left(\mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \right)^{-1} \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H$, and $\left[\overline{\boldsymbol{\Pi}}\right]_{R\times R} \triangleq \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \mathbf{D}_{\mathbf{g}}$.

The following remarks are in order:

Remark 1: Fig. 3 presents numerical evaluations of the CRLBs for DF and AF cooperative networks based on (24) and (19), respectively, where $\mathbf{h} = [0.2790 - 0.9603i, 0.8837 + 0.4681i]^T$ and $\mathbf{g} = [0.7820 + 0.6233i, 0.9474 - 0.3203i]^T$, $\boldsymbol{\nu}^{[\text{rd}]} = \boldsymbol{\nu}^{[\text{sum}]} = [0.1, 0.2, 0.3, 0.4]$, $R = 4$, $L = 24$, and Walsh-Hadamard codes are used as the TSs. Numerical results in Fig. 3 show that in order to reach the same CFO estimation accuracy compared to DF, an AF relaying network requires link SNRs to be at least 5dB higher.

Remark 2: When the CFO values corresponding to source-relay-destination links are identical, e.g., in the absence of Doppler shift, since it is assumed that linearly independent and orthonormal TSs are transmitted from the relays, it can be shown through straightforward algebraic manipulations that the matrices

$$\mathbf{FIM} = \frac{2}{\sigma_n^2} \begin{bmatrix} \text{Re}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}\right\} + \text{Re}\{\boldsymbol{\rho}\}\,\text{Re}\{\boldsymbol{\rho}\}^T & -\text{Im}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}\right\} + \text{Re}\{\boldsymbol{\rho}\}\,\text{Im}\{\boldsymbol{\rho}\}^T & -\text{Im}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}\right\} \\ \text{Im}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}\right\} + \text{Im}\{\boldsymbol{\rho}\}\,\text{Re}\{\boldsymbol{\rho}\}^T & \text{Re}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}\right\} + \text{Im}\{\boldsymbol{\rho}\}\,\text{Im}\{\boldsymbol{\rho}\}^T & \text{Re}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}\right\} \\ -\text{Im}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}\right\}^T & \text{Re}\left\{\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}\right\}^T & \text{Re}\left\{\mathbf{D}_{\boldsymbol{\alpha}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}^H \mathbf{D}_{\mathbf{L}}^{\mathbf{2}} \mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}} \mathbf{D}_{\boldsymbol{\alpha}}\right\} \end{bmatrix} \tag{18}$$

$$\mathbf{CRLB}\left(\boldsymbol{\nu}^{[\text{sum}]}\right) = \frac{\sigma_n^2}{2} \underbrace{\left(\text{Re}\{\boldsymbol{\Upsilon}\} + \begin{bmatrix} \text{Im}\{\boldsymbol{\Pi}\}^T & -\text{Re}\{\boldsymbol{\Pi}\}^T \end{bmatrix} \mathbf{F}_{\mathbf{2R}\times\mathbf{2R}}^{-1} \begin{bmatrix} -\text{Im}\{\boldsymbol{\Pi}\} \\ \text{Re}\{\boldsymbol{\Pi}\} \end{bmatrix} \right)^{-1}}_{\triangleq \boldsymbol{\Phi}} \tag{19}$$

$$\mathbf{CRLB}\left(\boldsymbol{\alpha}^{[\text{AF}]}\right) = \frac{\sigma_n^2}{2} \left[\overline{\mathbf{J}} \mathbf{F}_{\mathbf{2R}\times\mathbf{2R}}^{-1} \overline{\mathbf{J}}^H + \overline{\mathbf{J}} \mathbf{F}_{\mathbf{2R}\times\mathbf{2R}}^{-1} \begin{bmatrix} -\text{Im}\{\boldsymbol{\Pi}\} \\ \text{Re}\{\boldsymbol{\Pi}\} \end{bmatrix} \boldsymbol{\Phi} \begin{bmatrix} \text{Im}\{\boldsymbol{\Pi}\}^T & -\text{Re}\{\boldsymbol{\Pi}\}^T \end{bmatrix} \mathbf{F}_{\mathbf{2R}\times\mathbf{2R}}^{-1} \overline{\mathbf{J}}^H \right] \tag{22}$$

$$\mathbf{CRLB}\left(\mathbf{g}\right) = \frac{\sigma_w^2}{2} \left(2 \left(\mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \right)^{-1} + \left(\mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \right)^{-1} \overline{\boldsymbol{\Pi}} \boldsymbol{\Theta} \overline{\boldsymbol{\Pi}}^H \left(\mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}^H \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}} \right)^{-1} \right) \tag{25}$$

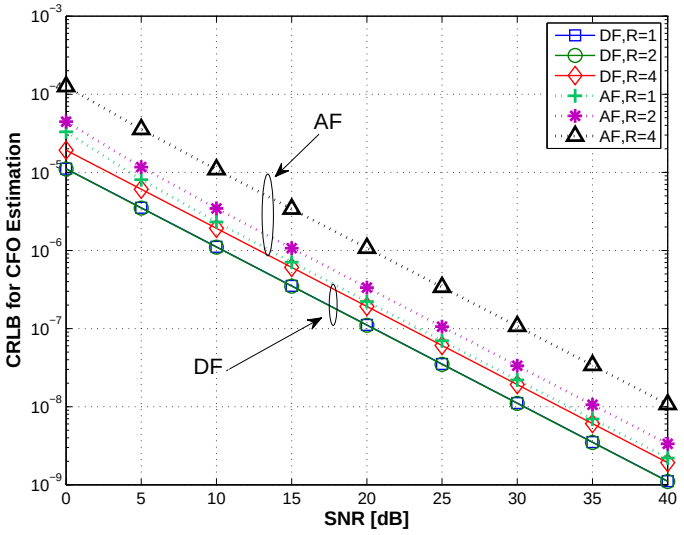

Fig. 3. CRLB for the estimation of ν_{rd} and ν_{sum} in DF and AF relaying cooperative networks, respectively. $\boldsymbol{\nu}^{[\text{rd}]} = \boldsymbol{\nu}^{[\text{sum}]} = \{0.1, 0.2, 0.3, 0.4\}$, and $L = 24$.

$\mathbf{E}^H_{\boldsymbol{\nu}^{[\text{sum}]}}\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}$, $\mathbf{E}^H_{\boldsymbol{\nu}^{[\text{sum}]}}$, $\mathbf{D}_{\mathbf{L}}\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}$, and $\mathbf{E}^H_{\boldsymbol{\nu}^{[\text{sum}]}}\mathbf{D}^2_{\mathbf{L}}\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}$ in (18) are diagonal. Therefore, the FIM is not singular and the CRLBs for both channel and CFO estimation can easily be determined. On the other hand, if linearly dependent training sequences are transmitted the matrices $\mathbf{D}^H_{\boldsymbol{\alpha}}\mathbf{E}^H_{\boldsymbol{\nu}^{[\text{sum}]}}\mathbf{D}^2_{\mathbf{L}}\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}\mathbf{D}_{\boldsymbol{\alpha}}$ and $\mathbf{E}^H_{\boldsymbol{\nu}^{[\text{sum}]}}\mathbf{D}_{\mathbf{L}}\mathbf{E}_{\boldsymbol{\nu}^{[\text{sum}]}}\mathbf{D}_{\boldsymbol{\alpha}}$ in the FIM in (18) will have linearly dependent rows resulting in a singular FIM. According to [31] a singular FIM, indicates that an unbiased estimator does not exist that can jointly estimate $\boldsymbol{\nu}^{[\text{sum}]}$. This motivates the choice of orthogonal TSs for transmission and the baseband processing structure in Fig. 2.

IV. Proposed CFO Estimators

In this section we propose two estimators based on multiple signal characterization (MUSIC), namely, *iterative-MUSIC (I-MUSIC)* and *iterative correlation-based-MUSIC (I-C-MUSIC)* and highlight their novelty.

A. I-MUSIC for DF Networks

For notational clarity $\boldsymbol{\nu}^{[\text{rd}]}$ is denoted by $\boldsymbol{\nu}$ throughout this subsection. Let us partition the TS, $\mathbf{t}_k^{[r]}$, of length L symbols into M blocks of length N symbols ($M = L/N$). Under the assumptions of constant channels gains over the length of each block and narrow band transmitted signals, the temporal covariance matrix of the received signal, $\mathbf{y}$, in (2) is given by

$$\mathbf{Q}_{\mathbf{y}} = E\left[\mathbf{y}(m)\mathbf{y}^H(m)\right] = \boldsymbol{\Gamma}(\boldsymbol{\nu})\mathbf{S}\boldsymbol{\Gamma}^H(\boldsymbol{\nu}) + \sigma_w\mathbf{I}, \tag{26}$$

where

- $[\boldsymbol{\Gamma}(\boldsymbol{\nu})]_{N\times R} \triangleq [\boldsymbol{\gamma}(\nu_1), \cdots, \boldsymbol{\gamma}(\nu_R)]$ with $\boldsymbol{\gamma}(\nu_k) = \left[e^{j2\pi(m+1)\nu_k}, \cdots, e^{j2\pi(m+N)N\nu_k}\right]^T$,
- $\mathbf{y}(m) \triangleq [y(m+1), \cdots, y(m+N)]^T$,
- $\mathbf{S} = E\left[\mathbf{s}(m)\mathbf{s}^H(m)\right]$, and
- $\mathbf{s}(m) \triangleq [s_1(m), \cdots, s_R(m)]^T$, with kth element given by $s_k(m) \triangleq g_k t_k^{[r]}(m)$.

Let $\varsigma_1 \geq \varsigma_2, \cdots, \geq \varsigma_N$ denote eigenvalues of $\mathbf{Q}_{\mathbf{y}}$. If the CFO values are distinct, rank $(\boldsymbol{\Gamma}(\boldsymbol{\nu})\,\mathbf{S}\boldsymbol{\Gamma}^H(\boldsymbol{\nu})) = R$ and it follows that $\varsigma_k > \sigma_w$ for $k = 1, \cdots, R$ and $\varsigma_k = \sigma_w$ for $k = R+1, \cdots, N$. Denote the unit-eigenvectors corresponding to $\varsigma_{R+1}, \cdots, \varsigma_N$ as $\left[\boldsymbol{\Psi}^{[N]}\right]_{N\times(N-R)} \triangleq [\boldsymbol{\psi}_{R+1}, \cdots, \boldsymbol{\psi}_N]$. Using steps in [26], the MUSIC estimate of ν_k, $\hat{\nu}_k$, for $k = 1, \cdots, R$, is given by

$$\hat{\nu}_k = arg \max_{\nu_k \in [-\epsilon:\Delta\nu_k:\epsilon)} \left(\boldsymbol{\gamma}^H(\nu_k)\boldsymbol{\Psi}^{[N]}\left(\boldsymbol{\Psi}^{[N]}\right)^H\boldsymbol{\gamma}^H(\nu_k)\right)^{-1}, \tag{27}$$

where $[-\epsilon, \epsilon)$ and $\Delta\nu_k$ represent the range and step size of the 1-dimensional maximization for the kth relay, respectively. In (27) the matrix multiplication $\boldsymbol{\Psi}^{[N]}\left(\boldsymbol{\Psi}^{[N]}\right)^H$ needs to be calculated only once, since $\boldsymbol{\Psi}^{[N]}$ is not a function of ν_k. The temporal covariance matrix $\mathbf{Q}_{\mathbf{y}}$ can be estimated by time averaging over M_F blocks of *training and data* symbols

$$\hat{\mathbf{Q}}_{\mathbf{y}} = \frac{1}{M_F}\sum_{m=1}^{M_F}\mathbf{y}(m)\mathbf{y}^H(m), \tag{28}$$

where $M_F = L_F/N$ and L_F is the number of symbols in a frame. Though accurate, the above MUSIC-based CFO estimator, similar to MLE, cannot distinguish among close-spaced CFO values [26], [32] and does not associate estimated CFOs to corresponding relays, which is necessary for equalization and detection at the destination. These shortcomings are addressed below by utilizing the linearly independence of the TSs transmitted from each relay.

1) Initialization of I-MUSIC: Let q denote the number of distinct CFOs present in $\mathbf{y}$, where q can be estimated using the algorithms in [33]–[35]. The following two possible scenarios are considered:

Scenario 1) q=R: Given that $\mathbf{y}$ is distributed as $\mathcal{CN}(\boldsymbol{\mu}_{\mathbf{y}}, \sigma_w^2\mathbf{I})$, the negative *log likelihood function (LLF)* of the CFO vector and the channel gains is proportional to

$$\delta(\boldsymbol{\nu}, \mathbf{g}) = \|\mathbf{y} - \mathbf{E}_{\boldsymbol{\nu}}\mathbf{g}\|^2, \tag{29}$$

where for a given $\boldsymbol{\nu}$, the minimizer of (29) and the ML estimates of the channel gains, $\hat{\mathbf{g}}$, are given by

$$\hat{\mathbf{g}} = (\mathbf{E}^H_{\boldsymbol{\nu}}\mathbf{E}_{\boldsymbol{\nu}})^{-1}\mathbf{E}^H_{\boldsymbol{\nu}}\mathbf{y}, \tag{30}$$

where $\hat{\mathbf{g}} \triangleq [\hat{g}_1, \cdots, \hat{g}_R]^T$ and $\mathbf{E}_{\boldsymbol{\nu}} \triangleq \mathbf{E}_{\boldsymbol{\nu}^{[\text{rd}]}}$.

To estimate the CFOs, the MUSIC algorithm is used, where in (27) the search is performed for R maxima. Next, using (30) and the first $M-1$ blocks of the received training signal, $\mathbf{y}^{[\text{c}]} \triangleq [y(1), \cdots, y((m-1)N)]^T$, the channel gain corresponding to each CFO is determined. Given that linearly independent TSs are transmitted from each relay node, the Mth block of received training signals, $\mathbf{y}^{[\text{n-c}]} \triangleq [y((m-1)N+1), \cdots, y(mN)]^T$ and the LLF in (29) are used to assign the pairs $\hat{\boldsymbol{\nu}}$ and $\hat{\mathbf{g}}$ to specific relays by carrying out the minimization

$$\hat{\boldsymbol{\nu}}^{[\text{A}]}, \hat{\mathbf{g}}^{[\text{A}]} = arg \min_{\hat{\boldsymbol{\nu}}, \hat{\mathbf{g}}} \delta(\boldsymbol{\nu}, \mathbf{g}), \tag{31}$$

where $\hat{\boldsymbol{\nu}}^{[\text{A}]}$ and $\hat{\mathbf{g}}^{[\text{A}]}$ denote the sets of estimated CFOs and channel gains corresponding to each relay node, respectively. Note that in this case (31) needs to be carried out $R!$ times.

Scenario 2) $q<R$: The set of q estimated CFOs in combination with the orthonormal TSs are used to estimate and assign the CFOs and channel gains to each path. See the algorithm in Table I.

TABLE I
INITIALIZATION STEPS FOR I-MUSIC AND I-C-MUSIC

Step 1)	**Initialization** Using the method in [33]–[35], determine q. Using (27), determine the set of q distinct CFOs, $\hat{\boldsymbol{\nu}}^{[\mathrm{q}]}$.
Step 2)	**CFO and Channel Gain Assignment** For $o = 1, 2, \cdots, \binom{R}{q}$ • Construct $(\hat{\boldsymbol{\nu}})^{[\mathrm{o}]} = \hat{\boldsymbol{\nu}}^{[\mathrm{q}]} \bigcup \left(\hat{\boldsymbol{\nu}}^{[\mathrm{R\text{-}q}]}\right)^{[\mathrm{o}]}$, where $\hat{\boldsymbol{\nu}}^{[\mathrm{R\text{-}q}]}$ is a set of frequencies selected from $\hat{\boldsymbol{\nu}}^{[\mathrm{q}]}$. • Using (30), determine $(\hat{\mathbf{g}})^{[\mathrm{o}]}$ corresponding to $(\hat{\boldsymbol{\nu}})^{[\mathrm{o}]}$. • Using (31), determine $\left(\hat{\boldsymbol{\nu}}^{[\mathrm{A}]}\right)^{[\mathrm{o}]}$ and $\left(\hat{\mathbf{g}}^{[\mathrm{A}]}\right)^{[\mathrm{o}]}$. Select $\left(\hat{\boldsymbol{\nu}}^{[\mathrm{A}]}\right)^{[\mathrm{o}]}$ and $\left(\hat{\mathbf{g}}^{[\mathrm{A}]}\right)^{[\mathrm{o}]}$ that result in the smallest LLF value, $\delta\left(\left(\hat{\boldsymbol{\nu}}^{[\mathrm{A}]}\right)^{[\mathrm{o}]}, \left(\hat{\mathbf{g}}^{[\mathrm{A}]}\right)^{[\mathrm{o}]}\right)$ as $\hat{\boldsymbol{\nu}}^{[\mathrm{A}]}$ and $\hat{\mathbf{g}}^{[\mathrm{A}]}$, the set of estimated CFOs and channel gains corresponding to each relay node, respectively.

2) Iterative Step for I-MUSIC: Since the unit amplitude PSK TSs are known, the effect of data modulation corresponding to the ith node can be eliminated according to

$$\begin{aligned}\tilde{y}_i(n) =& y(n)\left(t_i^{[r]}(n)\right)^* \\ =& g_i e^{j2\pi n\nu_i^{[\mathrm{rd}]}} t_i^{[r]}(n)\left(t_i^{[r]}(n)\right)^* \\ &+ \left(\sum_{k=1,k\neq i}^{R} g_k e^{j2\pi n\nu_k^{[\mathrm{rd}]}} t_k^{[r]}(n) + w(n)\right)\left(t_i^{[r]}(n)\right)^* \\ =& \underbrace{g_i e^{j2\pi n\nu_i^{[\mathrm{rd}]}}}_{\textbf{desired term}} + \underbrace{\sum_{k=1,k\neq i}^{R} g_k e^{j2\pi n\nu_k^{[\mathrm{rd}]}} t_{k,i}^{[d]}(n)}_{\textbf{interference}} + \underbrace{\tilde{w}_i(n)}_{\textbf{noise}},\end{aligned} \tag{32}$$

where $t_{k,i}^{[d]}(n) = t_k^{[r]}(n)\left(t_i^{[r]}(n)\right)^*$ and $\tilde{w}_i(n) = w(n)\left(t_i^{[r]}(n)\right)^*$. Note that $\tilde{w}_i(n)$ has the same statistical properties as $w(n)$, since multiplication by $\left(t_i^{[r]}(n)\right)^*$ only results in a phase shift of the noise.

The initial estimates of $\boldsymbol{\nu}^{[\mathrm{rd}]}$ and $\mathbf{g}$, $\left(\hat{\boldsymbol{\nu}}^{[\mathrm{rd}]}\right)^{[1]}$ and $(\hat{\mathbf{g}})^{[1]}$, respectively, are used to reduce the interference term in (32) according to

$$f_i(n) = \tilde{y}_i(n) - \sum_{k=1,k\neq i}^{R} \hat{g}_k e^{j2\pi n\hat{\nu}_k^{[\mathrm{rd}]}} t_{k,i}^{[d]}(n), \tag{33}$$

where $\mathbf{f_i} = \{f_i(1), \cdots, f_i(L)\}$ is applied in the next iteration to estimate the CFO corresponding to the ith node using (27). This approach also transforms the joint CFO estimation problem into multiple single-parameter estimation problems. In addition, for closely-spaced CFO values the MLE in (30) does not perform well since the term $\mathbf{E}_{\boldsymbol{\nu}}^H\mathbf{E}_{\boldsymbol{\nu}}$ in (30) becomes nearly singular. To address this shortcoming, at each iteration the ith relay's channel gain is estimated via

$$\hat{g}_i = \frac{1}{L}\sum_{n=1}^{L} \frac{f_i(n)}{e^{j2\pi n\hat{\nu}_i^{[\mathrm{rd}]}}}, \tag{34}$$

which is based on the *expectation conditional maximization (ECM)* algorithm [36]. The iteration stops when the absolute difference between the LLF of two iterations is smaller than a threshold value χ

$$\left| \|\mathbf{y} - \mathbf{E}_{(\hat{\boldsymbol{\nu}})^{[\mathrm{o+1}]}}(\hat{\mathbf{g}})^{[\mathrm{o+1}]}\|^2 - \|\mathbf{y} - \mathbf{E}_{(\hat{\boldsymbol{\nu}})^{[\mathrm{o}]}}(\hat{\mathbf{g}})^{[\mathrm{o}]}\|^2 \right| \leq \chi, \tag{35}$$

where $(\hat{\boldsymbol{\nu}})^{[\mathrm{o}]}$ and $(\hat{\mathbf{g}})^{[\mathrm{o}]}$ denote CFO and channel gain estimates corresponding to the oth iteration.

B. I-C-MUSIC for DF Networks

By transforming the problem from R-dimensional to one-dimensional estimation, a variety of CFO estimation methods suitable for different scenarios may be applied to improve upon the proposed algorithm [6], [37]. Here we apply the estimator in [37], which consists of estimating the ith node's CFO as

$$2\pi\hat{\nu}_i^{[\mathrm{rd}]} = \sum_{n=1}^{L-1} \varpi(n)\mathrm{angle}\{f_i^*(n)f_i(n+1)\}, \tag{36}$$

where $\varpi(n)$ is a window designed to reduce the estimator's variance (see [37] for details). Similar steps as outlined in Section IV A may be used to determine the CFO values, where (36) is used instead of (27) in the iterative step (see [26] for details).

C. CFO Estimation in AF Networks

To apply the MUSIC algorithm for CFO estimation: 1) the length of each block, N, needs to be larger than the number of relays, 2) the additive noise needs to be zero-mean, and 3) the additive noise needs to be white [23]. By simply choosing N to be larger than R the first condition can be satisfied. According to the signal model in Eq. (3) and based on the assumption of zero-mean AWGN at the relays, the mean of the additive noise at the destination can be shown to be

$$\begin{aligned}&E\left[\sum_{k=1}^{R} \beta_k e^{j2\pi n\nu_k^{[\mathrm{rd}]}} \tilde{t}_k^{[r]}(n)v_k(n)\right] + E[w(n)] = \\ &\sum_{k=1}^{R}\left(\beta_k E\left[e^{j2\pi n\nu_k^{[\mathrm{rd}]}}\tilde{t}_k^{[r]}(n)\right]E[v_k(n)]\right) = 0,\end{aligned} \tag{37}$$

which satisfies the second condition. Finally, in (8), it is shown that the overall noise at the destination is white, satisfying the third condition. Thus, the proposed I-MUSIC and I-C-MUSIC algorithms can also be applied to AF relaying cooperative networks.

D. Complexity of I-MUSIC and I-C-MUSIC

Throughout this section it is assumed that the step size in (27) for all relays are the same, i.e., $\Delta\nu = \Delta\nu_1 = \cdots = \Delta\nu_R$, and computational complexity is defined as the number of additions plus multiplications. Accordingly, the computational complexity of the initialization step for I-MUSIC and I-C-MUSIC, denoted

by C_I, is calculated as shown in (38) at the bottom of this page. In (38), $\vartheta = \frac{2\epsilon}{\Delta\nu}$, where ϵ is defined in (27).

Using (38), the computational complexity of I-MUSIC and I-C-MUSIC can be determined as

$$C_{\text{I-MUSIC}} = C_I + \kappa R \Big[\underbrace{N^2(N-R) + \vartheta\left(2N^2+1\right)}_{\textbf{(27)}} + \underbrace{2MN+1}_{\textbf{(34)}}\Big], \text{ and} \tag{39}$$

$$C_{\text{I-C-MUSIC}} = C_I + \kappa R \Big[\underbrace{2MN-2}_{\textbf{(36)}} + \underbrace{2MN+1}_{\textbf{(34)}}\Big], \tag{40}$$

where κ represents the number of iterations. (39) and (40) demonstrate that the computational complexity of I-MUSIC is considerably higher than that of I-C-MUSIC for the same κ. A numerical comparison of number of iterations versus performance is provided in Section V.

The computational complexity of the MLE based on [10, Eq.(22)] and implemented using the iterative alternating projection method [38] is calculated as

$$C_{\text{MLE}} = \kappa_{\text{MLE}} R\vartheta\left[R^3 + 2R^2L + (R+1)L^2 + L\right] + R^3 + 2R^2L + RL^2, \tag{41}$$

where κ_{MLE} represents the number of iterations required by the alternating projection method, which is between 3-4 [38]. Table II represents a quantitative comparison between the computational complexity of I-MUSIC, I-C-MUSIC, and MLE in [10]. As shown in Table II, compared to MLE in [10], I-MUSIC and I-C-MUSIC are on average 30 and 1800 times less computationally intensive, respectively. In addition, the algorithm in [13] requires correlating the received signal with each TS, which can be computationally intensive and due to its poor performance when the CFOs are far apart may not be applicable to the case of cooperative networks. Therefore, a quantitative comparison between the computational complexity of the algorithm in [13] and I-MUSIC and I-C-MUSIC is beyond the scope of this paper.

TABLE II
NUMBER OF ADDITIONS AND MULTIPLICATION FOR I-MUSIC, I-C-MUSIC, AND MLE [10] $\times 10^7$

$\Delta\nu = 10^{-5}$, $\kappa = 10$, $\kappa_{\text{MLE}} = 4$ [38], $q = R$, and $\epsilon = 0.5$.			
	MLE [10]	**I-MUSIC**	**I-C-MUSIC**
$L = 24$, $R = 2$, $N = 8$	156	27.1	1.3
$L = 64$, $R = 2$, $N = 16$	1030	108	5.1
$L = 64$, $R = 4$, $N = 16$	3620	210	5.1
$L = 64$, $R = 8$, $N = 16$	14600	416	5.8
$L = 128$, $R = 8$, $N = 32$	52600	1660	21.8

V. NUMERICAL RESULTS AND DISCUSSIONS

Throughout this section the propagation loss is modeled as [6], $\beta = (d/d_0)^{-m}$, where d is the distance between the transmitter and receiver, d_0 is the reference distance, and m is the path loss exponent. The following results are based on $d_0 = 1$km and $m = 2.7$, which corresponds to urban area cellular networks. $\epsilon = 0.5$ in (27). Without loss of generality, Walsh-Hadamard codes are used as the training sequences. *Binary phase-shift keying (BPSK)* modulation is used for transmission of the training sequences. Finally, $\sigma^2_{v_1} = \cdots = \sigma^2_{v_k} = \sigma^2_w = 1/\text{SNR}$.

A. Estimation Performance

Throughout this subsection, TS length, $L = 24$, block length, $N = 8$, and $R = 2$ relays are considered. Without loss of generality, only CFO estimation performance for the first node is presented. The estimators' performances are investigated for both far-apart and closely-spaced CFO values.

1) DF cooperative networks: Fig. 4 compares the performance of I-MUSIC and I-C-MUSIC for the estimation of $\boldsymbol{\nu}^{[\text{rd}]}$ in DF relaying networks against the CRLB in Eq. (24), the MLE in [10], and the estimator in [13]. In (35), the threshold is set to $\chi = 0.001$, which corresponds to approximately $6-20$ iterations. Similar to [10] and [13], the channel gains, $\mathbf{h}$ are drawn from *independent and identically distributed (i.i.d)* zero-mean complex Gaussian processes with unit variance. For our particular channels $\mathbf{h} = [0.2790 - 0.9603i, 0.8837 + 0.4681i]^T$. Two sets of CFO values are selected, $\boldsymbol{\nu}^{[\text{rd}]} = \{0.1, 0.2\}$ and $\boldsymbol{\nu}^{[\text{rd}]} = \{0.2, 0.205\}$, which in Fig. 4 are represented by solid lines and dotted lines, respectively.

For the case of $\boldsymbol{\nu}^{[\text{rd}]} = \{0.1, 0.2\}$, simulation results reveal that I-MUSIC is close to but does not reach the CRLB. This is due to the inherent shortcoming of the MUSIC algorithm [32]. However, I-C-MUSIC reaches the CRLB at mid-to-high SNR but exhibits poorer performance at low SNR. Fig. 4 also shows that both algorithms outperform the MLE and the estimator in [13] at mid-to-high SNR. The MLE, on the other hand, requires that only one node transmits its training sequence at a time. Therefore, for a fair comparison, in the case of MLE a training sequence length of L/R is used, resulting in higher *mean-square error* (MSE). As expected, the estimator in [13] fails since the initial CFO estimates are extremely poor whenever the CFOs are larger than 0.05 (the results in [13] are based on normalized CFO values of 0.01 and 0.015). For the case of closely-spaced CFO values $\left(\boldsymbol{\nu}^{[\text{rd}]} = \{0.2, 0.205\}\right)$, Fig. 4 illustrates that I-MUSIC and I-C-MUSIC approach the CRLB but do not reach it.

Fig. 5 compares the number of iterations for I-MUSIC, I-C-MUSIC, and the estimator in [13]. Note that both I-MUSIC and I-C-MUSIC algorithms require very few iterations to reach or approach the CRLB. As illustrated in Fig. 5 as the CFO values get

$$C_I = \begin{cases} \underbrace{N^2(N-R) + \vartheta\left(2N^2+1\right)}_{\textbf{(27)}} + \underbrace{M_F N^2 + 1}_{\textbf{(28)}} + R! \underbrace{(R+2)N}_{\textbf{(29)}} + \underbrace{R^3 + R^2MN + M^2N^2}_{\textbf{(30)}} & q = R \\ \underbrace{N^2(N-R) + \vartheta\left(2N^2+1\right)}_{\textbf{(27)}} + \underbrace{M_F N^2 + 1}_{\textbf{(28)}} + \binom{R}{q} \underbrace{\left[(R+2)N + R^3 + R^2MN + M^2N^2\right]}_{\textbf{(29)}+\textbf{(30)}} & q < R \end{cases} \tag{38}$$

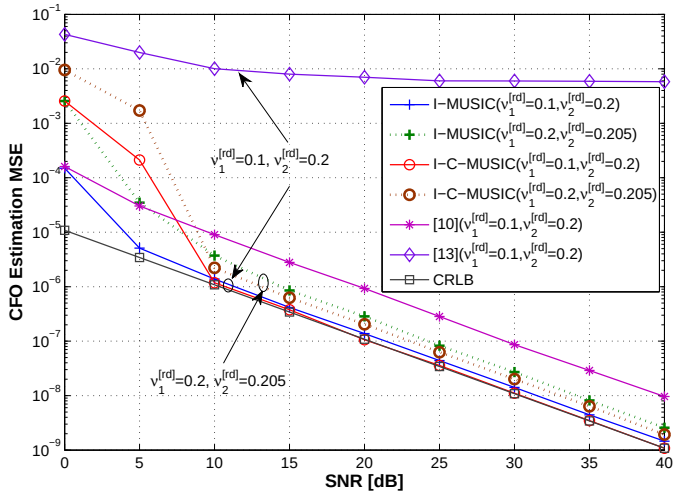

Fig. 4. The MSE of I-MUSIC and I-C-MUSIC for the estimation of $\nu_1^{[rd]}$ for DF networks vs. the algorithms in [10] and [13] and the CRLB in Eq. (24) with $L = 24$ and $L_F = 512$.

close, I-MUSIC and I-C-MUSIC both require more iterations to approach the CRLB, due to the rough initial estimates. However, even for closely-spaced CFO values, both algorithms require considerably fewer iterations and overhead compared to [13] at all SNR values.

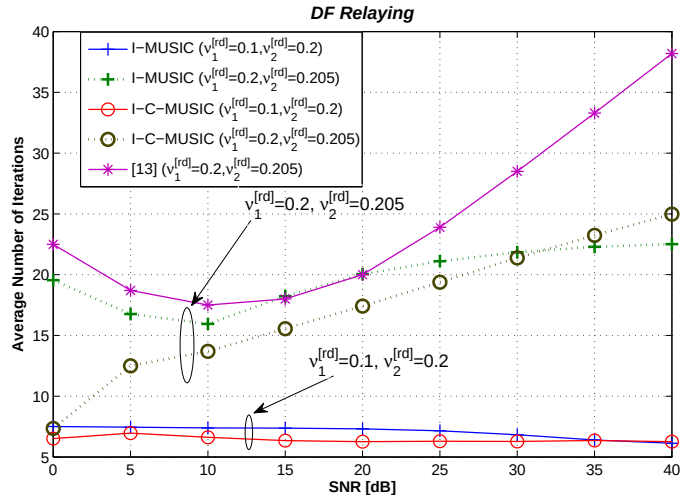

Fig. 5. Average number of iterations for I-MUSIC, I-C-MUSIC, and the algorithm in [13] for the estimation of $\nu_1^{[rd]}$ for DF networks with $L = 24$ and $L_F = 512$.

2) AF cooperative networks: Fig. 6 compares the performance of I-MUSIC and I-C-MUSIC for the estimation of $\boldsymbol{\nu}^{[\text{sum}]}$ in AF networks against the CRLB in (19). Again, the threshold in (35), $\chi = 0.001$. The channel gains are $\mathbf{h} = [0.2790 - 0.9603i, 0.8837 + 0.4681i]^T$ and $\mathbf{g} = [0.7820 + 0.6233i, 0.9474 - 0.3203i]^T$. The normalized CFOs are $\boldsymbol{\nu}^{[\text{sum}]}\{0.1, 0.2\}$ and $\boldsymbol{\nu}^{[\text{sum}]} = \{0.21, 0.2\}$, which in Fig. 6 are represented by solid lines and dotted lines, respectively.

In the case of AF relaying, I-C-MUSIC reaches the CRLB while I-MUSIC is very close to the CRLB and demonstrates better performance at low SNR values. Also, for the case of closely-spaced CFO values, the performance gap between I-MUSIC and I-C-MUSIC and the CRLB is larger for the case of AF compared to that of DF. This can be explained by the fact that the noise introduced by the relay nodes, which is amplified and forwarded to the destination, cannot be removed by the iterative algorithm.

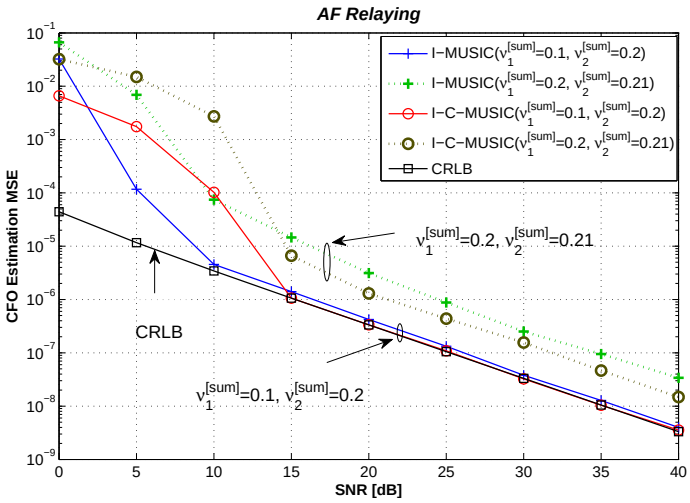

Fig. 6. The MSE of I-MUSIC and I-C-MUSIC for the estimation of $\nu_1^{[\text{sum}]}$ for AF networks vs. the CRLB in Eq. (19) with $L = 24$ and $L_F = 512$.

Fig. 7 compares the number of iterations of I-MUSIC and I-C-MUSIC for AF cooperative networks, where it is illustrated that both algorithms require fewer iterations to reach the CRLB when the CFOs are not very close. I-C-MUSIC requires fewer iterations to reach or approach the CRLB at low-to-mid SNR compared to I-MUSIC, while this advantage is reversed as the SNR increases. In Fig. 7, when the CFO values are close to one another (dashed + marked and ∘ marked plots) at low SNR, the average number of iterations required by I-MUSIC and I-C-MUSIC is small, since the initial CFO and channel estimates are quite poor and further iterations do not result in better estimates. However, at high-SNR due to the shortcoming of the MUSIC algorithm in distinguishing between closely-spaced CFO values, the MSE of the initial CFO and channel estimates moves further away from the CRLB as the SNR increases and more iterations are required to reach the CRLB. Finally, in the case of AF, both algorithms require considerably more iterations to approach the CRLB for closely-spaced CFO values compared to the case of DF. These results demonstrate that in addition to requiring higher SNRs between the nodes, AF networks also require more time and overhead to achieve frequency synchronization.

Fig. 8 plots the MSE for the estimation of the overall channel gains from source-relay-destination links, $\boldsymbol{\alpha}$, for both I-MUSIC and I-C-MUSIC in the case of AF relaying cooperative networks versus the CRLB in (22). At mid-to-high SNR, both estimators reach the CRLB for channel estimation. Since similar results are obtained for the estimation of channel gains from relay-destination links, $\mathbf{g}$, in the case of DF relaying, they are not presented here.

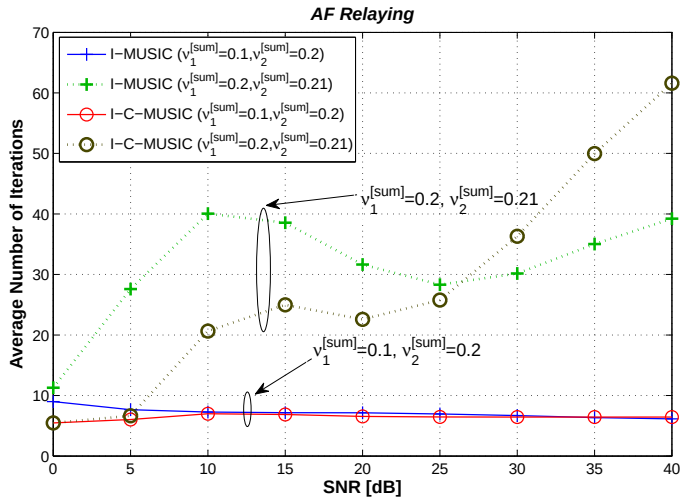

Fig. 7. Average number of iterations for I-MUSIC and I-C-MUSIC for the estimation of $\nu_1^{[\text{sum}]}$ for AF networks with $L = 24$ and $L_F = 512$.

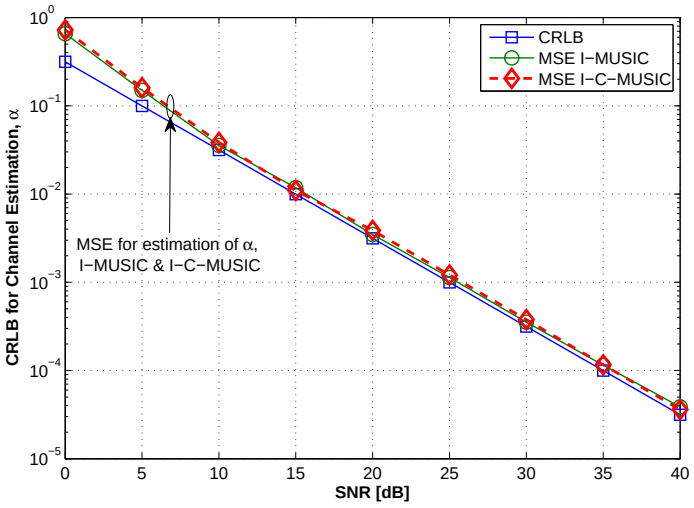

Fig. 8. The MSE for estimation of channel gains, α_1, for AF networks vs. the CRLB in Eq. (22) with $L = 24$.

B. Cooperative Network Performance

During the data transmission interval BPSK modulation is used with a frame length, L_F, of 512 symbols and a synchronization overhead of 12%. In the case of DF relaying an *orthogonal space-time block code (OSTBC)* with a code rate of $^3/_4$, [39], is used for the transmission of the signals from $R = 4$ relays. In the case of AF relaying, since application of OSTBC is not straightforward [40], the algorithm in [4] is used to investigate the performance of the cooperative networks. Quasi-static fading channels are considered, where new channel gains are generated from frame to frame (channel coefficients are complex Gaussian random variables with mean zero and unit variance). For DF relaying it is assumed that only relays that correctly decode the received signal are selected for retransmitting the signal. Finally, relays are uniformly distributed throughout the network such that $d^{[\text{sr}]} \leq 1\text{km}$ and $d^{[\text{rd}]} = \left(1 - d^{[\text{sr}]}\right)\text{km}$.

For the DF relaying scenario, the time varying channel matrix $\mathbf{G}_{L_F \times R} \triangleq \left[g_1 \bar{\mathbf{e}}_1^{[\text{rd}]}, \cdots, g_R \bar{\mathbf{e}}_R^{[\text{rd}]}\right]$, where $\bar{\mathbf{e}}_k^{[\text{rd}]} \triangleq \left[e^{j2\pi\nu_k^{[rd]}}, \cdots, e^{j2L_F\pi\nu_k^{[rd]}}\right]^T$ in combination with the decoder in [39] may be used to mitigate CFOs for decoding the received signal at the destination. In this paper, the computationally simpler method in [27] as referenced in Section V.B is used instead. Unfortunately, lack of space precludes providing the details of the CFO compensation method, which is available in [27] and references therein. On the other hand, for AF relaying to mitigate the effect of CFOs the time varying channel matrix $\boldsymbol{\Gamma}_{L_F \times R} \triangleq \left[\alpha_1 \bar{\mathbf{e}}_1^{[\text{sum}]}, \cdots, \alpha_R \bar{\mathbf{e}}_R^{[\text{sum}]}\right]$ where $\bar{\mathbf{e}}_k^{[\text{sum}]} \triangleq \left[e^{j2\pi\nu_k^{[\text{sum}]}}, \cdots, e^{jL_F 2\pi\nu_k^{[\text{sum}]}}\right]^T$ in combination with *successive interference cancelation (SIC)* as described in [4] is used.

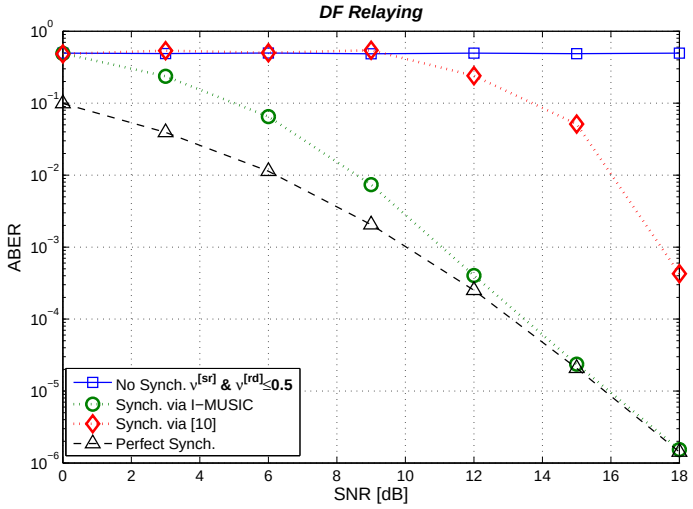

Fig. 9. ABER plots for perfectly synchronized, estimated/imperfectly synchronized via I-MUSIC and the MLE in [10], and unsynchronized systems with normalized CFO in the range $[-0.5, 0.5)$ per node for $R = 4$ relays, $L_F = 512$.

Figs. 9 and 10 illustrate the *average-bit-error-rate (ABER)* of DF and AF relaying SISO multi-relay cooperative networks, respectively. Here, I-MUSIC is used to acquire the completely unknown CFOs and channel gains. This result is compared to an unsynchronized system with normalized CFOs uniformly distributed in the range $[-0.5, 0.5)$ per node, as well as to perfectly synchronized systems. Figs. 9 and 10 reveal a significant performance gap between ABER performances of practical cooperative networks that estimate and compensate multiple CFOs and idealized systems that assume perfect synchronization at low-to-mid SNR. Figs. 9 and 10 also demonstrate that compared to DF, the performance of AF relaying networks is more significantly impacted by CFOs. This outcome is anticipated, since CFO estimation in the case of DF relaying can be performed more accurately as predicted by the CRLB analysis in Fig. 3. Unlike DF relaying, the ABER of an AF relaying network synchronized via I-MUSIC does not reach that of a perfectly synchronized system at high SNR due to this difference in estimation performance.

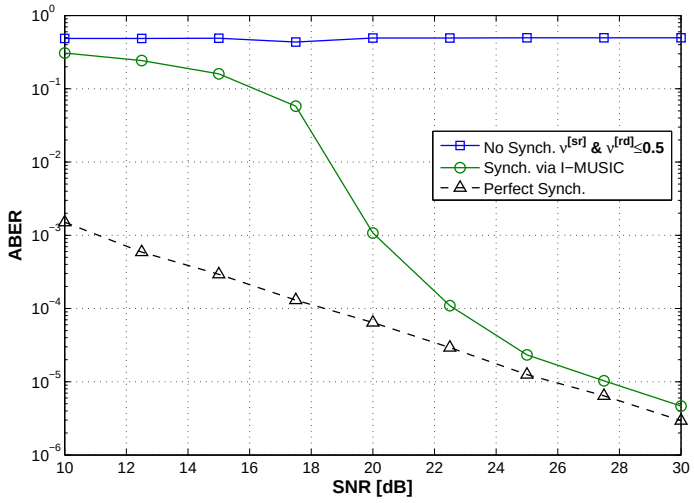

Fig. 10. ABER plots for perfectly synchronized, estimated/imperfectly synchronized via I-MUSIC, and unsynchronized systems with normalized CFO in the range $[-0.5, 0.5)$ per node for $R = 4$ relays, $L_F = 512$.

VI. Conclusion

In this paper we have addressed the topic of CFO estimation in multi-relay cooperative networks. The system model for DF and AF relaying networks in the presence of multiple CFOs has been presented and new CRLB expressions are derived, in closed-form. Two novel multiple CFO estimators are outlined. Numerical analyses demonstrate that the proposed estimators' performances reach or approach the CRLB at mid-to-high SNR and outperform the existing algorithms. The performances of DF and AF relaying cooperative networks in the presence of multiple CFOs has been investigated showing that the application of the proposed estimators result in significant performance gains. In addition, the results in Section V reveal that up to an SNR of 12 and 15dB for DF and AF relaying, respectively, there is a large performance gap between the ABER of idealized cooperative systems that assume perfect frequency synchronization and actual cooperative systems that require the CFOs to be estimated and compensated for at the receiver. Thus, it is important to consider the effect of imperfect CFO estimation when assessing the performance of cooperation methods, e.g., distributed beamforming and distributed space-time coding.

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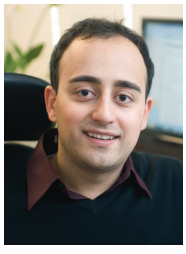

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