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Constellation optimization for coherent optical transmission systems

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Abstract: We present and discuss optimized four-dimensional modulation formats that have better sensitivities than established formats such as, e.g., binary or quarternary phase-shift keying.

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1. Introduction

Today's coherent transmission technologies, with some remarkable experimental progress [1, 2] are using communication systems modulating all four quadratures of the electromagnetic wave [3–5], so that a four-dimensional (4-d) constellation space needed to represent the signals. A popular modulation format such as dual-polarization quadrature phase-shift keying (DP-QPSK) [1–3, 6] is, for example, using the 16 vertices of the 4d cube as constellation points. Such a format is essentially four parallel binary phase-shift keying (BPSK) lines, so its performance in terms of bit error rate is the same as that for BPSK.

However, by taking full advantage of the 4d modulation space, one may find modulation formats that have better sensitivities than DP-QPSK. In this paper we will present a few such formats.

2. Sphere packing optimization

We consider the transmission of digital source symbols $\vec{s}_i$, taken equiprobably from an $N-$dimensional constellation $\mathcal{C} = \{\vec{c}_1, \ldots, \vec{c}_M\}$ of M symbols, over an additive white Gaussian noise (AWGN) channel. In the high-SNR limit, the symbol error rate (SER) is determined by the minimum distance, $d_{\min}$, between two constellation points, according to $SER \approx (M_{\min}/M)\,\mathrm{erfc}(d_{\min}/(2\sqrt{N_0}))$, where $N_0/2$ is the noise variance per dimension and $M_{\min}$ is the number of symbol pairs with separation $d_{\min}$. If we fix the symbol separation to be $d_{\min}$ (and thereby approximately fix the SER), the average transmitted symbol energy $E_s = E[\|\vec{s}_i\|^2] = E[\|\vec{c}_k\|^2]$ can be minimized by a judicious selection of the constellation $\mathcal{C}$. This is equivalent to finding the constellation (packing) of M N-dimensional spheres, with diameter $d_{\min}$, that minimizes the average square distance from the origin. Such constellations, here denoted by $\mathcal{C}_{N,M}$, have been reported elsewhere [3, 4, 7]. They were found as the best cases from simulations of thousands of random constellations of hard spheres that are contracted under suitable attractive, gravity-like forces to form densely packed clusters of spheres. Quite often such constellations have interesting symmetries that are relevant also in other branches of physics such as, e.g., the clustering of nanoparticles [8], although then limited to physical dimensions, e.g., $N \leq 3$. In communications, however, higher-dimensional constellations are often of interest.

3. Results and discussion

A common way of comparing modulation formats is to plot them in a chart with spectral efficiency vs. sensitivity [9, Sec. 5.9]. The spectral efficiency is defined as the number of bits per symbol per polarization (i.e., per dimension pair), $SE = (\log_2 M)/(N/2)$, and the sensitivity is the SNR E_b/N_0 required to achieve a certain SER, where $E_b = E_s/\log_2 M$ is the average bit energy. In Fig. 1 (a) we have thus charted the constellations $\mathcal{C}_{N,M}$ for $N = 2$ and 4 dimensions, and $M = 2$ up to 16 and 32 levels respectively. Fig. 1 (b) shows the 4-d constellations $\mathcal{C}_{4,M}$ for $M = 2, 3, \ldots, 32$ levels, at different SERs as indicated by the curve labels. The constellations were taken from [7] and their SER performance was estimated using the union bound [9, Sec. 4.3.2] (for $SER \leq 10^{-3}$) and by Monte Carlo integration (for $SER \geq 10^{-3}$). The contours are drawn as continuous lines for visibility, although they connect a discrete set of constellations. The AWGN channel capacity $E_b/N_0 \geq (2^{SE} - 1)/SE$, which lowerbounds the SNR for reliable coded communication [9, Sec. 3.3.4, 5.4.4], is included as a reference.

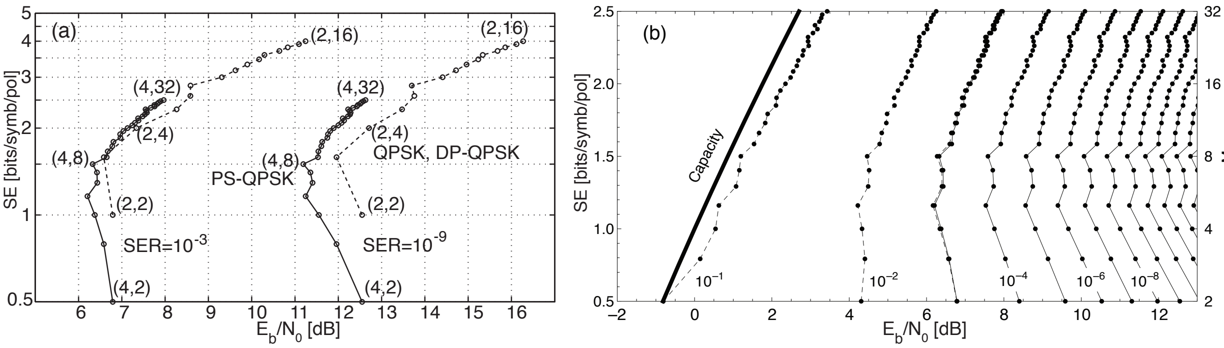

Fig. 1. **(a)** Chart over the sensitivity vs. spectral efficiency of the optimum M-ary, N-dimensional constellations $\mathcal{C}_{N,M}$ at an SER of 10^{-3} (left curves) and 10^{-9} (right curves). The boundary cases and some selected formats are indicated with (N, M), and the lines connecting the points (solid for $N = 4$, dashed for $N = 2$) should be regarded as a guide to the eye. **(b)** Spectral efficiency SE vs. sensitivity E_b/N_0 of the best known 4-d M-ary constellations $\mathcal{C}_{4,M}$ at various SERs, separated by one order of magnitude, as indicated by the labels. The performance was estimated by Monte Carlo integration (dashed) and the union bound (solid). The AWGN channel capacity is included for reference (thick).

We emphasize that the constellations $\mathcal{C}_{N,M}$ are optimized for asymptotically high SNR (and low SER) and in the low-SNR regime, the constellations have not yet reached their asymptotic performance. Thus, the overall most power-efficent 4-d modulation format is $\mathcal{C}_{4,2}$ for $E_b/N_0 \gtrsim 10^{-2}$, $\mathcal{C}_{4,5}$ for $10^{-2} \lesssim E_b/N_0 \lesssim 3 \cdot 10^{-6}$, and $\mathcal{C}_{4,8}$ for $E_b/N_0 \lesssim 3 \cdot 10^{-6}$, which is also asymptotically optimal (in terms of sphere packing) [3, 4]. A more difficult (and much less studied) problem is to optimize constellations for a finite, known SNR, which is beyond our present scope.

The 2-d optimized constellations $\mathcal{C}_{2,M}$ generally consists of subsets of the hexagonal lattice, with the asymptotically most power-efficient constellation $\mathcal{C}_{2,3}$ being 3-PSK. The constellation $\mathcal{C}_{2,4}$ can be thought of as four coins in any constellation where each coin touches at least two others; it is thus not unique, and QPSK has the same asymptotic performance as adding a neighboring point to 3-PSK from the hexagonal lattice [10].

The optimized 4-d constellations $\mathcal{C}_{4,M}$ have more varying forms, which only in some specific cases are subsets of lattices. $\mathcal{C}_{4,5}$ is the simplex, and $\mathcal{C}_{4,6}$ is a tetrahedron sandwiched between two points along one coordinate axis. $\mathcal{C}_{4,8}$ is the cross-polytope, with 3/2 = 1.76 dB asymptotic gain over BPSK. It can be seen as QPSK transmitted in either of two polarization states, and we call it polarization switched QPSK (PS-QPSK). The constellation $\mathcal{C}_{4,16}$ has a 1.11 dB asymptotic SNR gain over its 16-point counterpart DP-QPSK. It consists of a single point, a 3-d cube, a 3-d octahedron, and another single point, all layered along one coordinate axis in 4-d. The $\mathcal{C}_{4,25}$ cluster consists of 24 points symmetrically located around a sphere at the origin. The locii of the surrounding points are give by the union of the 4-d cube and an upscaled version of the cross-polytope $\mathcal{C}_{4,8}$; a geometric figure known as the 24-cell. The corresponding 24-level format is called 6P-QPSK, since it transmits QPSK in either of six different polarization states. It has 0.51 dB of asymptotic gain over BPSK, and a way to map bits to its 24 levels was discussed in [3].

These constellations and others, including optimization with respect to maximum, rather than average, symbol energy are discussed in more detail in [11].

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