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Censored Cooperative Positioning for Dense Wireless Networks

Kallol Das, Henk Wymeersch
Department of Signals and Systems
Chalmers University of Technology, Gothenburg, Sweden
Email: kallol@student.chalmers.se, henkw@chalmers.se

***Abstract*—Cooperative positioning is an emerging topic in wireless sensor networks and navigation. It can improve the positioning accuracy and coverage in GPS-challenged conditions such as inside tunnels, in urban canyons, and indoors. Different algorithms have been proposed relying on iteratively exchanging and updating positional information. For the purpose of computational complexity, network traffic, and latency, it is desirable to minimize the amount of information shared between devices, while still maintaining acceptable performance. We show that information that is not reliable should not be shared, and information that is not informative should not be used. This naturally leads to censoring schemes. We consider different censoring schemes based on the Cramér Rao bound (CRB). We find that by blocking the broadcasts of the nodes that don't have reliable estimates (transmit censoring) and selecting the most usable links after receiving signals from neighbors (receive censoring), complexity, traffic, and latency can be reduced significantly without degrading positioning performance.**

I. INTRODUCTION

Positional information is important in various applications such as wireless sensor networks, navigation, search-and-rescue, etc. [1]. In conventional non-cooperative range-based positioning, accuracy depends on the quality of distance measurements and reference nodes' positions. In certain applications, this type of positioning may fail due to insufficient coverage by the reference nodes (e.g., GPS inside buildings).

The performance of positioning can be dramatically improved by also exchanging positional information *between* agents. Although such a cooperative approach improves positioning accuracy [2], it also increases the computational complexity (as more links have to be taken into account), as well as the network traffic, and positioning latency. Some of the positional information may also be harmful as some devices have poor estimates [3]. Hence, an important question is how to select and discard information. In [4] the authors proposed to use the links which are the closest from the agent in consideration. However, the closest neighbors may not correspond to the best links as positioning also depends on the geometric configuration of the agent and its neighbors. Recently, the Cramér Rao bound (CRB) has been proposed as a criterion to censor ineffective links [3], [5]. Rather than relying on proximity as a criterion, [5] proposed to use a preset number of links that give lowest CRB. Similarly, in the context of GPS, geometric dilution of precision, which is related to the CRB, was used to select the best four satellites among a larger set [6], [7]. In [5] the authors proposed to use the CRB to select those anchors that would give the best positioning accuracy. The mentioned methods are suitable for non-cooperative networks where the question is to select the best anchors. In cooperative positioning, the number of links available may be in the order of 10-30 in a dense network. Using above methods may degrade the positioning quality as only a preset, small number of links will be used for updates. This idea was extended to cooperative networks in [3], where adaptively links are removed without effecting the quality of positioning. All of the methods discussed above can only remove the worst links *after* receiving information from neighbors. Hence, the number of packet exchanges is not proactively reduced.

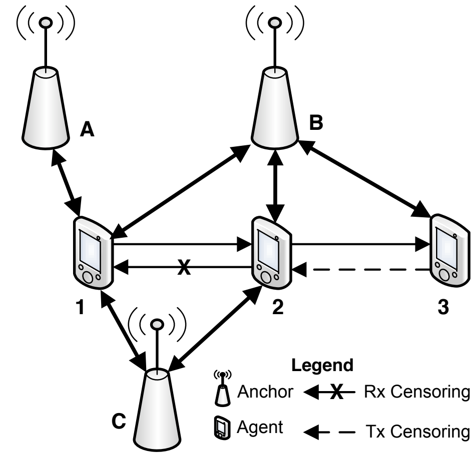

Figure 1. Transmit and receive censoring schemes in a cooperative network, with 3 agents (1, 2, 3) and 3 anchors (A, B, C).

In this paper, we extend previous algorithms with a combination of transmit and receive censoring (see Figure 1). In our proposed method, we adopt a distributed criterion to censor information without hampering the positioning quality. We use CRB as the censoring parameter, not only to remove non-informative links after receiving information from neighbors (receive censoring), but also to block the broadcast of unreliable nodes (transmit censoring).

The remainder of this paper is arranged as follows. In Section II, we describe our model and assumptions. In Section III, our proposed methods are explained. Results from simula-

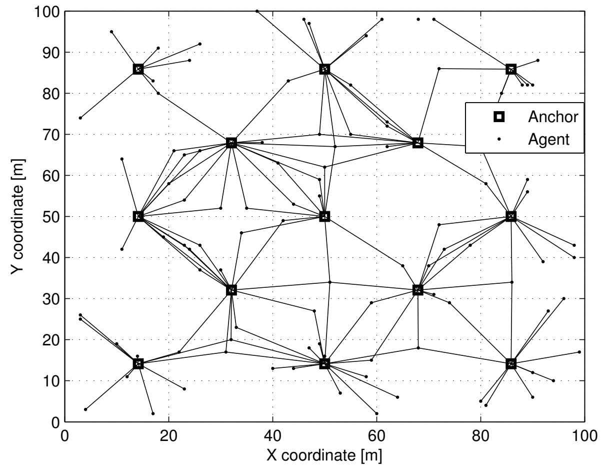

Figure 2. A typical non-cooperative network with 100 agents and 13 anchors; average connectivity = 1.55.

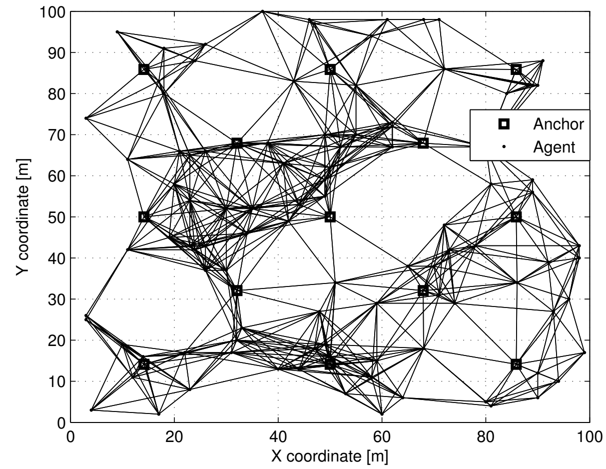

Figure 3. A typical cooperative network with 100 agents and 13 anchors; average connectivity = 13.67.

tions are presented in Section IV. In Section V, we draw our conclusions and mention possible extensions of this work.

II. System model

We consider a wireless network with N nodes. In our model we have two types of nodes: anchors, which know their positions, and agents, which do not know their positions. In cooperative networks, agents iteratively update their position estimates. The update phase of the agents depends on range-measurements between agents and anchors as well as on agent-to-agent measurements.

We denote by $\mathbf{x}_i$ the position of node i in the network and by $\hat{\mathbf{x}}_i$ the corresponding estimated position. $S_{\rightarrow i}$ is the set of nodes from which node i can receive signals. Based on the signal received from node $j \in \mathcal{S}_{\rightarrow i}$, node i can estimate $\hat{d}_{j\rightarrow i} = \|\mathbf{x}_i - \mathbf{x}_j\| + n_{j\rightarrow i}$, where $n_{j\rightarrow i}$ is the measurement noise. We assume $n_{j\rightarrow i} \sim \mathcal{N}\left(0, \sigma^2_{j\rightarrow i}\right)$ [2]. The goal of node i is to estimate its own position. Ideally, the positioning process should require low complexity and communication overhead per node as well as low latency.

A comparison between a cooperative and non-cooperative network is shown in Figures 2–3, respectively. We consider 13 anchors and 100 agents, with a communication range of 20 m. Note the ten-fold increase of usable links in the cooperative network, making cooperative positioning promising, but challenging to implement.

III. Proposed method

A. Censoring: Overview

In a dense network, agents can receive information from many neighbors. Not all of those links are useful and by censoring the bad links, we can achieve reduced complexity and latency, at little or no performance loss. In Figure 1, two censoring schemes are shown. The three agents have connectivity between them and with fixed anchors. Agent 3 is connected to only one anchor, so initially it has limited knowledge about its position. Hence, this agent cannot help other nodes to be localized and should not broadcast its positional information. We define this as *transmit censoring*. Agent 1 can get information from three anchors and also from agent 2. By discarding the information from agent 2, its positioning accuracy may be unaffected. We define this as *receive censoring*.

Clearly, censoring requires a criterion based on which agents decide whether or not to censor. We propose to use the Cramér-Rao bound (CRB), because of its rigorous foundation, and its wide applicability to cooperative positioning algorithms.

B. Cramér-Rao bound

The Cramér-Rao bound (CRB) is a lower bound on the performance of any unbiased estimator. It is calculated by taking inverse of the Fisher Information matrix (FIM) [8], [9]. Considering the position $\mathbf{x}_i$ of agent i, then the FIM is given by

$$\mathbf{F}\left(\mathbf{x}_i\right) = -\mathbb{E}\left\{\frac{\partial^2 \Lambda(\mathbf{x}_i)}{\partial \mathbf{x}_i^2}\right\}, \qquad (1)$$

where $\Lambda(\mathbf{x}_i)$ is the log-likelihood function and the expectation occurs over the ranging noise. It can be shown that the FIM of $\mathbf{x}_i$ will be of the form [10]

$$\mathbf{F}(\mathbf{x}_i) = \sum_{j \in S_{\rightarrow i}} \frac{1}{\sigma^2_{j\rightarrow i}} \mathbf{q}_{ij}\mathbf{q}_{ij}^T, \qquad (2)$$

where

$$\mathbf{q}_{ij} = \frac{\mathbf{x}_i - \mathbf{x}_j}{\|\mathbf{x}_i - \mathbf{x}_j\|}.$$

Finally, the CRB can be calculated as

$$\mathrm{CRB}(\mathbf{x}_i) = \text{trace}\left(\left[\mathbf{F}(\mathbf{x}_i)\right]^{-1}\right). \qquad (3)$$

Algorithm 1 Cooperative positioning with censoring (at an arbitrary iteration)

1: **nodes** $i = 1$ to N **in parallel**
2: calculate $\mathrm{CRB}\left(\hat{\mathbf{x}}_i\right)$ {only for agents}
3: set $\mathrm{CRB}\left(\hat{\mathbf{x}}_i\right) = 0$ for anchors
4: **if** $\mathrm{CRB}\left(\hat{\mathbf{x}}_i\right) < \gamma_{\mathrm{TX}}$ **then**
5: broadcast current positional information
6: **end if**
7: receive positional information from neighbors, $j \in \mathcal{S}_{\rightarrow i}$
8: **while** $|\mathcal{S}_{\rightarrow i}| > 3$ **do**
9: determine the worst link $\hat{k} \in \mathcal{S}_{\rightarrow i}$
10: **if** $\mathrm{CRB}_{\hat{k}}\left(\hat{\mathbf{x}}_i\right) < \gamma_{\mathrm{RX}}$ **then**
11: remove link $\hat{k}$ from $\mathcal{S}_{\rightarrow i}$
12: **else**
13: break
14: **end if**
15: **end while**
16: update positional information using $\mathcal{S}_{\rightarrow i}$
17: **end parallel**

C. Transmit Censoring

In transmit censoring, an agent will decide to broadcast or censor its positional information based on its CRB. Since the true position of the agent, nor of its neighbors, is known, the CRB will be calculated by using the *estimated* positions. Hence, an agent will transmit-censor when

$$\mathrm{CRB}\left(\hat{\mathbf{x}}_i\right) = \mathrm{trace}\left(\left[\tilde{\mathbf{F}}\left(\hat{\mathbf{x}}_i\right)\right]^{-1}\right) > \gamma_{\mathrm{TX}}, \tag{4}$$

where

$$\tilde{\mathbf{F}}\left(\hat{\mathbf{x}}_i\right) = \sum_{j \in S_{\rightarrow i}} \frac{1}{\sigma_j^2} \tilde{\mathbf{q}}_{ij} \tilde{\mathbf{q}}_{ij}^T, \tag{5}$$

in which

$$\tilde{\mathbf{q}}_{ij} = \frac{\left(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_j\right)}{\|\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_j\|},$$

and γ_{TX} is a threshold. The value of this threshold depends on the ranging model and the performance requirements. Initially, the set of neighbors $S_{\rightarrow i}$ contains very few elements (e.g., anchors within range). After some iterations, the number of elements in the set of neighbors $S_{\rightarrow i}$ will increase. This helps the agent to attain lower CRB and meet the bound of sharing information, at which point the agent will start broadcasting.

D. Receive Censoring

In receive censoring, an agent will receive positional information from its neighbors and it can calculate its present Cramér Rao bound, $\mathrm{CRB}\left(\hat{\mathbf{x}}_i\right)$. The agent will then remove links, as long as $\mathrm{CRB}\left(\hat{\mathbf{x}}_i\right) < \gamma_{\mathrm{RX}}$. Here, the threshold γ_{RX} again depends on the model and the desired performance. In particular, the agent can discard the worst links according to following greedy algorithm:

1) Let $S^k_{\rightarrow i}$ be the set of neighbors obtained by removing the kth element from $S_{\rightarrow i}$. Calculate

$$\mathrm{CRB}_k\left(\hat{\mathbf{x}}_i\right) = \mathrm{trace}\left(\left[\tilde{\mathbf{F}}_k\left(\hat{\mathbf{x}}_i\right)\right]^{-1}\right), \tag{6}$$

where

$$\tilde{\mathbf{F}}_k\left(\hat{\mathbf{x}}_i\right) = \sum_{j \in S^k_{\rightarrow i}} \frac{1}{\sigma_j^2} \tilde{\mathbf{q}}_{ij} \tilde{\mathbf{q}}_{ij}^T. \tag{7}$$

2) Select the worst link:

$$\hat{k} = \arg\min_k \mathrm{CRB}_k\left(\hat{\mathbf{x}}_i\right). \tag{8}$$

3) If $\mathrm{CRB}_{\hat{k}}\left(\hat{\mathbf{x}}_i\right) < \gamma_{\mathrm{RX}}$, set $S_{\rightarrow i}$ to $S^{\hat{k}}_{\rightarrow i}$ and go to step 1, otherwise STOP.

E. Combination of Transmit and Receive Censoring

We can merge both transmit and receive censoring schemes as shown in Algorithm 1. In line number 4-6 of Algorithm 1 the transmit censoring scheme is applied, while lines 8-16 implement the receive censoring scheme. This algorithm can remove harmful links and also select the best links for update.

IV. Numerical Results

A. Simulation Setup

In our simulation, we consider a wireless sensor network with 100 randomly placed agents with 20 m communication limit, and 13 anchors placed in a organized way in a 100 m $\times$ 100 m map (see Figure 3). The standard deviation of range measurement noise is 10 cm (standard for indoor UWB measurements) [2]. To test our proposed method we use a cooperative least square (LS) estimator [2], [3]. We briefly describe the cooperative LS positioning algorithm. The LS estimator minimizes the following cost function with respect to $\mathbf{x} = \left[\mathbf{x}_1, \ldots, \mathbf{x}_N\right]$,

$$\mathcal{C}_{\mathrm{LS}}(\mathbf{x}) = \sum_{i=1}^{N} \sum_{j \in \mathcal{S}_{\rightarrow i}} \left\| \hat{d}_{j\rightarrow i} - \|\mathbf{x}_i - \mathbf{x}_j\| \right\|^2.$$

The update phase of cooperative LS algorithm becomes (for a detailed derivation, see [2])

$$\hat{\mathbf{x}}_i^{(l)} = \hat{\mathbf{x}}_i^{(l-1)} + \delta_i^{(l)} \sum_{j \in S_{\rightarrow i}} \left(\hat{d}_{j\rightarrow i} - \tilde{d}^{(l-1)}_{j\rightarrow i}\right) \tilde{\mathbf{q}}_{ij}^{(l-1)}, \tag{9}$$

where l is the iteration index, and $0 < \delta_i^{(l)} \ll 1$ is the step size corresponding to node i at iteration l,

$$\tilde{d}^{(l-1)}_{j\rightarrow i} = \left\| \hat{\mathbf{x}}_i^{(l-1)} - \hat{\mathbf{x}}_j^{(l-1)} \right\|,$$

and

$$\tilde{\mathbf{q}}_{ij}^{(l-1)} = \frac{\left(\hat{\mathbf{x}}_i^{(l-1)} - \hat{\mathbf{x}}_j^{(l-1)}\right)}{\left\|\hat{\mathbf{x}}_i^{(l-1)} - \hat{\mathbf{x}}_j^{(l-1)}\right\|}. \tag{10}$$

We fix the value of $\delta_i^{(l)} = 0.075 \; \forall\, i, l$ as it gives a good positional accuracy and convergence trade-off. The second

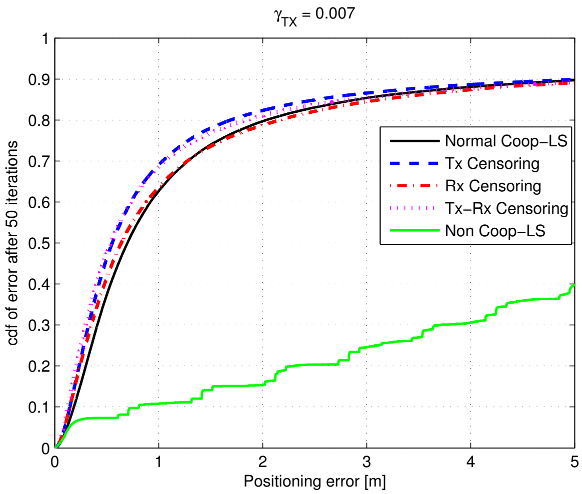

Figure 4. Performance comparison of different censoring schemes in conservative approach.

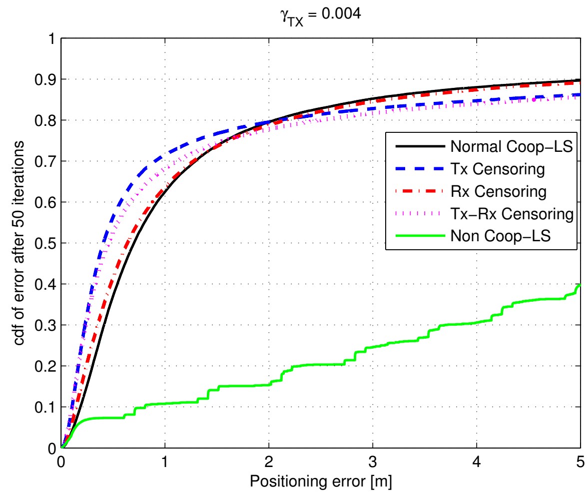

Figure 5. Performance comparison of different censoring schemes in aggressive approach.

term in the right hand side of (9) is the *correction* from all of the neighbors. When the correction falls below a threshold (depending on the ranging model and quality requirements) we can stop updating the positional information of that agent. We call the threshold *stoplimit*.

B. Simulation Parameters

We first set several parameters: the stoplimit, γ_{TX} and γ_{RX}. For the stoplimit, we consider the number of iterations after which 90% of the agents have converged in the normal cooperative LS without censoring. When we relax our expectation of positioning accuracy (i.e., increase the stoplimit) most of the agents will converge after few iterations. On the other hand, if we tighten our accuracy requirement (i.e., reduce the stoplimit), cooperative LS may need more iterations to converge. For rest of the simulations we fix the stoplimit such that agents converge in less than 100 iterations, leading to a stoplimit 0.015 m.

We now fix the censoring thresholds γ_{TX} and γ_{RX}. Our goal is to maintain a performance similar to normal cooperative LS, with reduced packet exchanges and complexity. Censoring can be conservative (i.e., less censoring) or aggressive (i.e., more censoring). The smaller γ_{TX}, the more aggressive we perform transmit censoring. Similarly, the larger γ_{RX}, the more aggressive we perform receive censoring. We set the thresholds based on the percentiles of the expected CRB. We consider aggressive receive censoring by setting $\gamma_{\mathrm{RX}} = (8\mathrm{cm})^2$, which is the 90th percentile of the CRB. This means that roughly 90% of the agents will perform receive censoring. For transmit censoring, we consider on two types of censoring: conservative and aggressive, corresponding to the 60th and 90th percentile of the CRB, respectively. This leads to, respectively, $\gamma_{\mathrm{TX}} = (6\,\mathrm{cm})^2$ (conservative) and $\gamma_{\mathrm{TX}} = (8\mathrm{cm})^2$ (aggressive). This means that roughly 40% of the agents will perform transmit censoring under the aggressive approach.

C. Simulation Discussion

We now investigate the performance of the different schemes in previously mentioned censoring approaches, showing results after 50 iterations, after which the algorithms have converged most of the time. The cumulative distribution function (cdf) of the positioning error is shown in Figures 4-5, corresponding conservative and aggressive transmit censoring, respectively. In addition to the normal cooperative and non-cooperative LS, we show the performance of transmit (Tx) censoring, receive (Rx) censoring, and combined (Tx-Rx) censoring.

From Figure 4 (conservative approach), we can distinguish three error regimes: the low error regime (errors less than 1 meter), the medium error regime (errors between 1 and 5 meters), and the high error regime (errors above 5 meters). Note that the high error regime occurs for around 10% of the agents (60% for non-cooperative LS), while above 60% of the agents are in the low error regime (10% for non-cooperative LS). In this latter regime, we see that all censoring schemes outperform normal cooperative LS. This observation is congruent with our expectation, since the CRB criterion is most meaningful in the low error regime. We observe that transmit censoring (with or without receive censoring) yields the best performance, while receive censoring by itself is the least effective of all the censoring schemes. In the medium error regime, only transmit censoring is beneficial, while receive censoring leads to slightly worse performance compared to normal cooperative LS. Transmit censoring is beneficial in this regime, since agents with poor positional information can censor themselves and not mislead their neighbors. Receive censoring is detrimental since agents with poor position estimates should not discard information from neighbors. The performance of aggressive transmit censoring is shown in Figure 5. We observe that this approach can achieve better performance in the low error regime but not

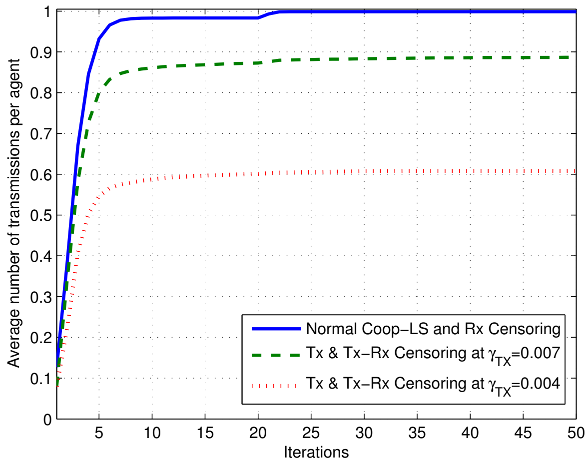

Figure 6. Normalized number of packet transmissions as a function of iterations for different censoring schemes.

Table I
COMPARISON OF AVERAGE LINKS USED FOR UPDATE PHASE

	Normal Coop-LS	Tx Censoring	Rx Censoring	Tx-Rx Censoring
conservative Tx	11.76	11.14	6.00	5.57
aggressive Tx	11.76	9.91	6.00	4.89

in medium error regime.

In addition to the positioning performance, it is also important to evaluate other aspects of censoring algorithms, such as the complexity and number of packets exchanged. Figure 6 shows the average number of packets transmitted per node per iteration, as a function of the iteration index. Initially, only agents that can communicate with at least three anchors have an initial estimate, so those are the only ones that will broadcast their position estimates. Hence, the number of packets per agent is very low. As iterations progress, more of the agents acquire estimates through cooperation and will start broadcasting. When transmit censoring is activated, we achieve approximately 10% and 40% reduction in total data traffic with the conservative and aggressive approaches, respectively. These values are directly related to the transmit censoring threshold γ_{TX}. In receive censoring, the number of packets is the same as that of normal cooperative LS.

Finally, in Table I, we compare the average number of used links per agent during the update of LS positioning. We observe that less than half of the links are used in receive censoring compared to normal cooperative LS. Even fewer links are used in combined transmit-receive censoring, in particular with aggressive transmit censoring. The overall reduction in used links makes little impact on the computational complexity for LS positioning, but will be important when more sophisticated algorithms are considered such as [2], which exhibit a complexity that scales super-linearly with the number of links used.

V. CONCLUSIONS AND FUTURE WORK

In this paper we have evaluated different censoring schemes for cooperative positioning in dense networks, in order to reduce complexity, latency, and packet transmissions. All censoring decisions are distributed and based on a CRB criterion. We have applied these censoring schemes to a standard cooperative least squares positioning algorithm, and found that: (i) receive censoring, which was proposed previously, can improve positioning performance, while at the same time considering less information from neighbors; (ii) two new censoring schemes (based on transmit censoring) can improve positioning performance even further, with drastic reduction in network traffic. These advantages of censoring schemes (distributed nature, improved positioning, reduced complexity and network traffic) make them promising for large-scale dense networks.

Future work includes extending the proposed censoring schemes to account for the uncertainty of neighbors, not only their position estimates. We will modify transmit censoring to consider the modified Cramér Rao bound (MCRB) [11] as a censoring criterion to account for uncertainty of neighbors. Hence, links will be discarded when they are not useful geometrically *or* when they originate from neighbors with a great deal of uncertainty. As a next step, we will consider positioning algorithms that are based on the exchange of full statistical information, instead of merely point estimates [2]. As such algorithms suffer from large computational loads, we expect censoring to be beneficial.

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