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Electronic dispersion compensation by Hadamard transformation

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Abstract: We demonstrate that the Hadamard transform can be applied to look up table (LUT)-based predistorters to simultaneously compress and optimize the LUT for dispersion-limited links.

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1. Introduction

Chromatic dispersion (CD) greatly limits the bandwidth that can be transmitted through a fiber and has to be compensated for in systems with high data rates. Predistortion in the electrical domain has been demonstrated using linear filters [1], lookup table (LUT) based nonlinear filters [2], or both [3]. Real-time systems with both linear and nonlinear filters have been demonstrated in [4], as well as linear finite impulse response (FIR) filters for predistortion in a 1200 SMF link [5]. Another real-time system employing an LUT-based nonlinear filter, which compensates for CD as well as nonlinearities to some degree up to a distance of 800 km, was recently implemented as well [6].

In this paper, we introduce a method of improving LUT-based systems by using the Hadamard transform (HT). The HT, which is a linear invertible transform similar to the discrete Fourier transform (DFT), can reduce the size of the LUT significantly, and for CD compensation making it scale linearly with the number of bits instead of exponentially, which is of significant practical importance [5, 6]. In addition we show that the HT performs an averaging that will optimize the LUT over longer bit sequences than was used in its construction. Finally we point out that the HT also provides a unique way of quantifying how nonlinear the predistortion LUT is, and hence the nonlinearity of the communication link.

The basic idea of precompensation is to predistort the transmitted signal in such a way that, at the receiver, the desired signal without CD is obtained. This precompensator can be implemented by using LUT-based filters as shown in Fig. 1(a) [2]. For every block $\boldsymbol{b}$ of k input bits, the LUT stores predistorted waveforms of duration T, the symbol time. Each such waveform is stored as a complex row vector $\boldsymbol{d}(\boldsymbol{b}) = \boldsymbol{d}_I + j\boldsymbol{d}_Q$ of r samples. The LUT is a $2^k \times r$ complex matrix $\boldsymbol{D}$ whose rows are $\boldsymbol{d}(0\cdots00), \boldsymbol{d}(0\cdots01), \ldots, \boldsymbol{d}(1\cdots11)$. Each bit to be transmitted is shifted into the row vector $\boldsymbol{b}$ and a new waveform of duration T (r samples) is output to the parallel dual-drive Mach–Zehnder modulator (DDMZM) after digital-to-analog (D/A) conversion. Note that this LUT accounts not only for the linear dispersion, but also for the nonlinear transfer characteristic of the DDMZM.

Fig. 1(b) illustrates the principle for an LUT with $k = 4$ input bits. Each row represents the inphase waveform corresponding to a certain k-bit pattern. Amplitudes are color-coded and the horizontal axis denotes the time from the center of bit 2 to the center of bit 3. The quadrature-component waveforms can also be viewed in a similar way. The number of columns in the LUT is determined by the sampling rate of the D/A converter.

2. LUT analysis using the Hadamard transform

The HT is a discrete, linear, orthogonal transfom, like for example the DFT, but its coefficients take values in ± 1 only. Specifically, the HT of a matrix $\boldsymbol{X}$ having $M = 2^k$ rows is given by $\tilde{\boldsymbol{X}} = \boldsymbol{H}\boldsymbol{X}/M$, where element (i, j) of the Hadamard matrix $\boldsymbol{H}$ is given by $\prod_{n=0}^{k-1}(-1)^{b_n(i)b_n(j)}$ and $b_n(l)$ is bit n in the base-2 representation of an integer $0 \leq l < M$. The inverse Hadamard transform is given by $\boldsymbol{X} = \boldsymbol{H}\tilde{\boldsymbol{X}}$.

While the DFT and its variants are natural choices for transforming waveforms in the time domain (which we do not consider in this paper), the binary coefficients of the HT makes it especially suitable for transforming an LUT along the codeword axis (vertically in Fig. 1(b)). As fas as we know, this application of the HT has not been previously reported. We may now obtain the HT of the LUT $\tilde{\boldsymbol{D}}$ as $\tilde{\boldsymbol{D}} = \boldsymbol{H}\boldsymbol{D}/M$. One row in $\tilde{\boldsymbol{D}}$ represents an offset (the average of all waveforms in $\boldsymbol{D}$), k other rows represent waveforms that depend linearly on a certain bit, and the remaining

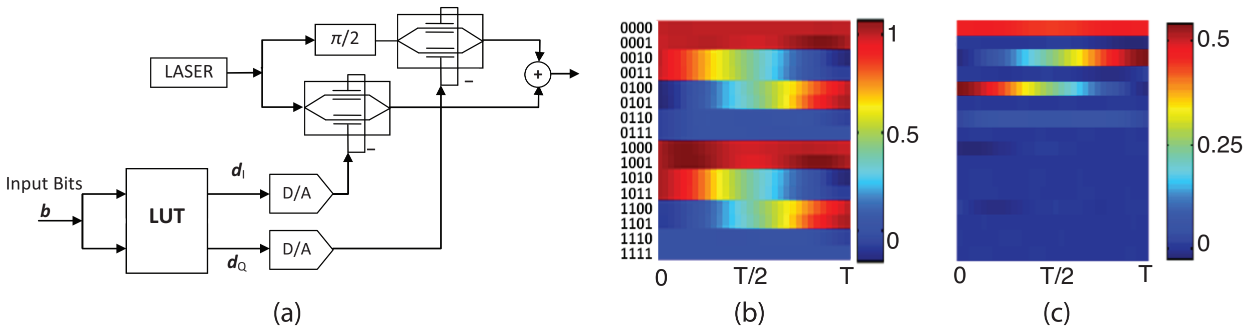

Fig. 1. (a) Block diagram of an LUT-based transmitter. (b) Example of a 4-bit LUT, $\boldsymbol{D}$, with color-coded waveform amplitude. An input bit 0 corresponds to a high drive signal, which in turn corresponds to a low optical intensity because of the DDMZM cosine transfer characteristic. (c) Hadamard-transformed LUT $\tilde{\boldsymbol{D}}$ of the table in (b).

$M-k-1$ rows represent various classes of nonlinearities. The HT is the unique transform with this property. Hence, measuring the energy in the rows of $\tilde{\boldsymbol{D}}$ provides a simple means to quantify the amount of nonlinearity in a system.

3. LUT compression

Representing the LUT by $\boldsymbol{D}$ or $\tilde{\boldsymbol{D}}$ is equivalent in terms of memory and performance, since the transform is lossless. As indicated in the previous section, however, many of the rows of $\tilde{\boldsymbol{D}}$ (but not $\boldsymbol{D}$) are close to zero if the waveforms in $\boldsymbol{D}$ represent an essentially linear system. Such rows do not need to be stored in memory, since the full LUT $\boldsymbol{D}$ can be recovered from the nonzero rows only. This gives an opportunity to compress the LUT considerably without sacrificing performance. We define a compressed lookup table (CLUT) as the complex $(k+1)\times r$ matrix $\boldsymbol{L}$ consisting of rows $0, 2^0, 2^1, \ldots, 2^{k-1}$ of $\tilde{\boldsymbol{D}}$. If the LUT was computed for a linear system, linear combinations of the rows of the CLUT can reproduce all entries of the LUT.

Fig. 1(c) shows the Hadamard transform $\tilde{\boldsymbol{D}}$ of the LUT in Fig. 1(b). It can be observed that most of the rows are close to zero. The offset and linear components, which are represented by rows 0, 1, 2, 4, and 8, dominate the table. A few of the other rows also provide some noticeable variation. These deviations from the all-zero blue color are caused by the nonlinear transfer characteristic of the DDMZM (which is a fully modulated cosine) and by the influence of bits before and after the considered window of four input bits.

The block diagram of the system for regenerating predistorted waveforms is shown in Fig. 2(a). The structure is similar to an FIR filter, but our CLUT system is more general in the sense that it also includes oversampling, pulse shaping, and an explicit (nonadaptive) method to generate the coefficients. The first entry of the CLUT corresponds to the offset component. For each input bit pattern $\boldsymbol{b}\in\{0,1\}^k$, we generate a predistorted waveform $\boldsymbol{d}'(\boldsymbol{b}) = [1 \quad \boldsymbol{c}]\boldsymbol{L}$, where $\boldsymbol{c} = \boldsymbol{1} - 2\boldsymbol{b} \in \{-1,1\}^k$. If $\boldsymbol{L}$ includes all nonzero rows of $\tilde{\boldsymbol{D}}$, then $\boldsymbol{d}'(\boldsymbol{b}) = \boldsymbol{d}(\boldsymbol{b})$ and the CLUT reproduces all LUT entries perfectly. If also some of the excluded rows of $\tilde{\boldsymbol{D}}$ are nonzero, then $\boldsymbol{d}'(\boldsymbol{b})$ is a weighted average over $\boldsymbol{d}(\boldsymbol{b}')$ for several bit patterns $\boldsymbol{b}'$. This averaging will deteriorate the performance in nonlinear systems. If, however, the LUT waveforms depend on anything besides the considered k input bits $\boldsymbol{b}$, then the performance is improved by averaging over several similar bit sequences. This happens if the LUT waveform generation includes noise or if the channel impulse response is longer than the transmission time for $k/2$ bits. The latter case will be exemplified in the next section.

4. Performance

For the uncompressed LUT approach, the memory requirement is $2\cdot 2^k rs$ bits, where the factor of 2 represents inphase and quadrature drive signals, k is the number of input bits considered, r is sampling rate (or LUT width), and s is the D/A resolution. The LUT memory requirement scales exponentially with the number of input bits, which is a strong limiting factor in the design of such systems over long distances. In the CLUT system, the memory requirement is linear, only $2(k+1)rs$ bits, a reduction by a factor of $(k+1)/2^k$.

The runtime complexity of the CLUT is slightly increased, since $2(k+1)r$ additions are required to generate the predistorted waveforms corresponding to one symbol. However, no multiplications or other arithmetical operations need to be implemented besides the additions, as the multipliers in Fig. 2(a) just imply a change of sign.

In order to verify the proposed system we carried out numerical simulations of a direct-detection on–off keying

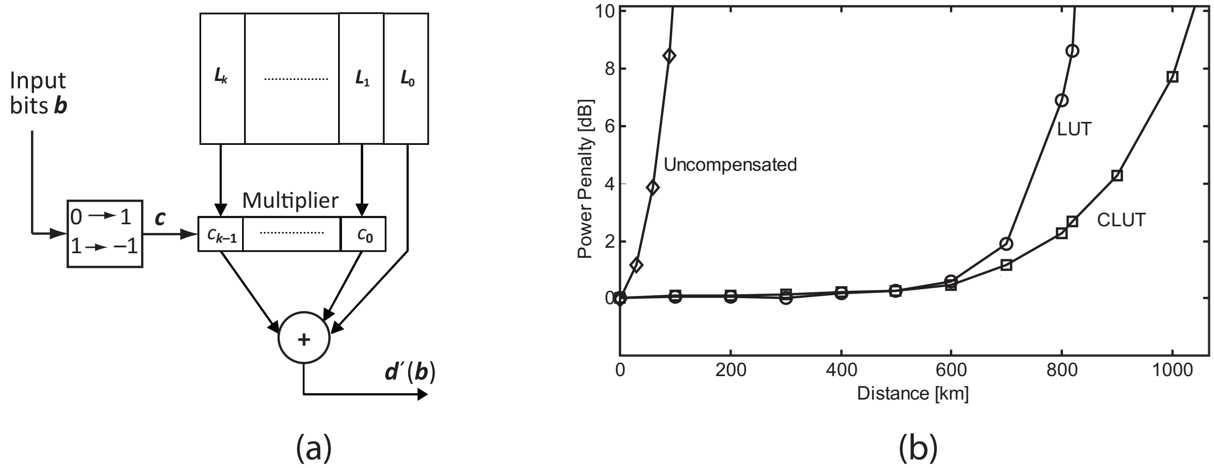

Fig. 2. (a) System for generating the complex predistorted waveform $\boldsymbol{d}'(\boldsymbol{z})$. $L_0, L_1, \ldots, L_k$ represent the $k+1$ rows of the matrix $\boldsymbol{L}$. (b) Simulated power penalty of a 10 Gb/s OOK system with and without compensation at a BER of 10^{-5}.

(OOK) system at 10 Gb/s. Our simulator was developed in Matlab and C++, using (virtually) infinite D/A and A/D precision, since we wish to demonstrate the concept rather than evaluating the detailed performance. For the same reason we used a high sampling rate of 80 GHz ($r = 8$), which can be lowered but will result in a loss of performance unrelated to the LUT/CLUT comparison [2]. An SMF with dispersion parameter $D = 17\,\mathrm{ps}/(\mathrm{km}\cdot\mathrm{nm})$ was used with a carrier wavelength of 1550 nm. The DDMZM nonlinearity (cosine) was considered but no fiber nonlinearities. Optical amplifier-induced noise was modeled as a zero-mean additive white Gaussian noise source with single-sided power spectral density $N_0 = GFh\nu/2$, where G is the gain of the amplifier, F represents the noise figure, and $h\nu$ is the photon energy. The gain G was 30 dB while the noise figure F was 5 dB. The receiver employed a 35 GHz optical bandpass filter, and an electrical low-pass Bessel filter with a 3 dB bandwidth of 7 GHz. The length of the input bit sequence of the LUT was $k = 13$ bits. The number of LUT entries was $2 \cdot 2^{13}$, which was reduced to $2 \cdot 14$ for the CLUT system. The LUT was generated from a training sequence of length 2^{15} containing all possible 13-bit sequences at least once.

From the BER curves for both (LUT/CLUT) OOK systems for different fiber lengths, the EDFA input power required to attain a BER of 10^{-5} was calculated. Using the back-to-back as reference, the corresponding power penalties are shown in Fig. 2(b). At low dispersion lengths (up to 500 km), the CLUT system has the same performance as the uncompressed LUT system. At longer dispersion lengths, however, it can be observed that the CLUT system performs better. For example at a fiber length of 800 km, the EDFA input power required to achieve a BER of 10^{-5} for the CLUT system is about 4.5 dB less than for the uncompressed LUT system. The performance improvement comes from the fact that at longer distances, an optimal precompensator would need to consider a longer window than k bits. In the absence of a longer window, the averaging effect in the generation of $\boldsymbol{d}'(\boldsymbol{z})$ provides a performance gain, as explained in the previous section.

5. Conclusions

We have demonstrated that applying the Hadamard transform to look-up-table-based systems gives a number of unique benefits. It can: (i) significantly compress the LUT for linear systems and dramatically reduce the memory requirements, (ii) average waveforms to enable longer reach than the original LUT, (iii) identify nonlinearities in the LUT, and (iv) provide an explicit way of generating filter coefficients.

References

[1] J. McNicol et al., in *Proc. Optical Fiber Communication Conference 2005,* paper OThJ3, 2005.
[2] R. I. Killey et al., *IEEE Photonics Technology Letters,* vol. 17, no. 3, pp. 714–716, Mar. 2005.
[3] R. I. Killey et al., in *Proc. Optical Fiber Communication Conference 2006,* paper OWB3, 2006.
[4] K. Roberts et al., *IEEE Photonics Technology Letters,* vol. 18, no. 2, pp. 403–405, Jan. 2006.
[5] P. M. Watts et al., *Optics Express,* vol. 16, no. 16, pp. 12171–12180, Aug. 2008.
[6] R. Waegemans et al., *Optics Express,* vol. 17, no. 10, pp. 8630–8640, May 2009.