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BICM Transmission using Non-Uniform QAM Constellations: Performance Analysis and Design

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***Abstract*—In this paper we study bit-interleaved coded modulation (BICM) transmission using non-uniform (NU) quadrature amplitude modulation (QAM) constellations. For such a NU-QAM-BICM transmission, we develop closed-form approximations for the probability density function of the L-values, and we use them to predict the coded bit error rate (BER) performance of the system in the AWGN channel. We then numerically optimize NU-QAM-BICM based on convolutional codes. Compared to uniform QAM-based BICM transmission, and for a target BER of 10^{-7}, we reach gains up to 1 dB. When the design is applied to turbo codes a decrease in the error floor can be obtained.**

I. Introduction

Bit-interleaved coded modulation (BICM) [1]–[3] is used in most of the existing communication standards (cf. HSPA, IEEE 802.11a/g, IEEE 802.16, DVB-S2, etc.). In BICM the channel encoder and the modulator are separated by a bit-level interleaver, which makes the design simple and flexible, i.e., the code rate and the constellation can be chosen independently. Besides its flexibility, BICM maximizes the code diversity, and therefore, it outperforms trellis coded modulation in fading channels.

When BICM is used with Gray-mapped 4-ary quadrature amplitude modulation (QAM), all the coded bits are equally treated by the modulator. However, the coded performance of the system will change if the modulator introduces unequal error protection (UEP). UEP caused by the binary labeling of the constellation for BICM transmissions was formally analyzed in [4]. In particular, it was shown in [4] that gains can be obtained if the UEP is exploited by properly designing the interleaver or the code. UEP for BICM transmissions can be also obtained/exploited by changing the binary labeling of the constellation, by allowing unequal power allocation for systematic/parity bits, by deleting some bits (puncturing), or by allowing non-equally spaced constellations.

Non-uniform (NU) constellations (also known as hierarchical, embedded, multi-resolution, or asymmetrical constellations) consist of non-uniformly spaced signal points (c.f. [5], [6]). They offer different levels of protection to the transmitted bits in the same symbol, which can be adjusted by changing the relative distances between the symbols. Due to this property, NU constellations have received a great deal of attention for many applications, for example, multimedia transmission [7], multi-resolution image transmission [6], [8], simultaneous voice and multi-class data transmission [9], and superposition of bits from different users in the same carrier of an OFDMA system [10]. NU constellations have also been standardized for digital video broadcasting-terrestrial [11], and in Qualcomm's Media forward link only [12].

NU constellations have been analyzed in the literature in terms of uncoded bit-error-rate (BER). For example, exact expressions for uncoded BER of binary reflected gray coded (BRGC) hierarchical QAM transmission in an additive white Gaussian noise (AWGN) channel were derived in [5]. In the context of BICM transmission, NU constellations can be used in order to improve the systems performance. Recently, in [13]–[15], the selection of NU constellations for BICM has been done based on capacity arguments, which can be related to improving the convergence of turbo-codes, or the performance of low density parity check (LPDC) codes.

In general, all the distances between the symbols in a NU constellation can be different. However, in this paper we restrict our analysis to the set of NU-QAM constellations presented in [5], i.e., constellations that preserve certain symmetry (more details in Sec. II-B). Moreover, we focus on BICM using low complexity encoder/decoders (convolutionally-encoded BICM). Since capacity arguments would probably fail in this scenario, we take a different approach compared to the one in [13]–[15]. We select the NU constellation based on minimizing bounds on the coded BER over the AWGN channel, showing that this could also be used to lower the error floor of turbo codes. More particularly, and based on the Gaussian model for the probability density function (PDF) of the L-values proposed in [16], we develop new closed-form expressions for the NU-QAM-BICM scheme. These new expressions are used to develop union bounds (UB) on the BER, which are then used to numerically optimize its design. Presented numerical examples show that convolutionally-encoded NU-QAM-BICM offers gains over uniform QAM-based transmissions. For the particular codes analyzed in this work, the gains can be up to 1 dB for a BER target of 10^{-7} if the distances between the symbols in the NU constellations are optimized according

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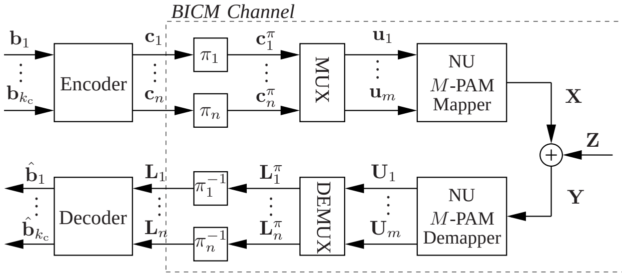

Figure 1. Model of NU-QAM-BICM transmission: a channel encoder followed by the interleavers ($\pi_1, \ldots, \pi_n$), the multiplexing (MUX), the NU M-PAM mapper, the channel, and the inverse processes at the receiver's side.

to the spectrum of the codes. For turbo-encoded NU-QAM-BICM, the gains are visible in the error floor region.

II. System Model

The NU-QAM-BICM system model under consideration is shown in Fig. 1. In what follows, we describe functionalities of various blocks of such transmission scheme, where we closely follow the system model from [4].

A. Encoder, Interleaving, and Multiplexing

The k_c vectors of information bits $\mathbf{b}_l$ are encoded by a rate $R = k_\mathrm{c}/n$ channel encoder, where $l = 1, \ldots, k_\mathrm{c}$. The vectors of coded bits $\mathbf{c}_1, \ldots, \mathbf{c}_n$ are then fed to n parallel interleavers, which are assumed to be infinite and independent (ideal), yielding randomly permuted sequences of the coded bits $\mathbf{c}_p^\pi = \pi_p\{\mathbf{c}_p\}$, $p = 1, 2, \ldots, n$. The multiplexing unit (MUX) assigns the coded and interleaved bits to the different bit positions in the NU M^2-QAM constellation, and it is defined using a matrix $\mathbb{K}_{n\times m} \equiv \mathbb{K}$ of dimensions $n \times m$, whose elements, $0 \le \kappa_{p,k} \le 1$, denote the fraction of bits $\mathbf{c}_p^\pi$ that will be assigned to the kth bit position $\mathbf{u}_k$. For example, if $n = m$ and $\mathbb{K}$ is chosen to be an identity matrix, all the bits from the first encoder's output will be transmitted in the first bit position, the bits from the second encoder's output through the second bit position, and so on.

B. Non-uniform M-PAM Constellations

In this paper we consider NU M^2-QAM constellations, where the binary labeling is the so-called BRGC [17]. Therefore, each symbol is a superposition of independently modulated real/imaginary parts, which allows us to focus on the equivalent NU M-PAM constellation, where $M = 2^m$ (cf. Fig. 1). At any time instant t, the coded and interleaved bits $[u_1(t), \ldots, u_m(t)]$ are mapped to a NU M-PAM symbol $x(t) \in \mathcal{X} = \{x_0, \ldots, x_{M-1}\}$ using a binary memoryless mapping $\mathcal{M} : \{0,1\}^m \to \mathcal{X}$. Since the mapper is memoryless, from now on we drop the time index t. We consider general NU M-PAM constellations labeled with the BRGC as the one shown in Fig. 2 ($M = 8$). In this figure, the M constellation points are shown with black circles, where the white squares/triangles are "virtual" symbols that help to understand the construction of the NU constellation as explained below. We use $k = 1, 2, \ldots, m$ to denote the bit

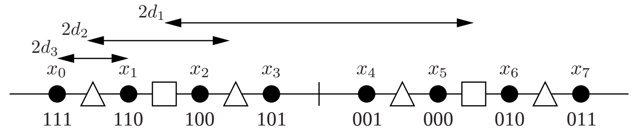

Figure 2. NU 8-PAM constellation with BRGC.

position of the binary labeling, where $k = 1$ represents the left most bit position. The bit value of the most significant position ($k = 1$) selects one of the two squares in Fig. 2. Similarly, for a given value of the first bit, the bit value for the next position ($k = 2$) selects one of the two triangles that surround the previously selected square. Finally, given the bit values for $k = 1$ and $k = 2$, the bit value of bit position $k = 3$ selects one of the two black symbols that surround the previously selected triangle. This selected black symbol is finally transmitted by the modulator. Since the NU 8-PAM constellation can be seen as superposition of virtual NU 2-PAM and 4-PAM constellations, it is referred to as hierarchical 2/4/8-PAM constellations in [5].

As shown in Fig 2, the distances between the symbols evolve in a hierarchical fashion, i.e., $2d_1$ represents the distance between the points in the virtual BPSK constellation formed by the two squares, $2d_2$ represents the distance between the virtual BPSK constellations formed by the two pairs of triangles centered around the squares. Finally, $2d_3$ represents the distance between points in the virtual BPSK constellation centered around the triangles. Then for a generalized NU M-PAM constellation, $2d_1$, $2d_2$,..., $2d_m$ represent respectively the distances between points in the first, second, ..., mth levels of hierarchy. In order to keep the BGRC of the constellation, the distance $d_k \ge \sum_{i=1}^{k-1} d_i$. The NU M-PAM constellation is defined then by the elements in $\mathcal{X}$ which can be expressed in terms of the distances d_k as

$$x_i = \sum_{k=1}^{m} (-1)^{1+b_k(i,m)} d_k, \tag{1}$$

where $b_k(i,m)$ denotes the kth bit of the length-m binary representation of the integer number i.

We define the so-called priority parameters as $\alpha_k = \frac{d_k}{d_m}$ with $k = 1, \ldots, m-1$. Changing them, the uncoded BER performance of the different bit positions can be varied, cf. [5]. We will show that these parameters control the coded BER performance of BICM transmission as well. Using (1), it is possible to write the average symbol energy assuming equally likely symbols $E_\mathrm{s} = \sum_{k=1}^{m} d_k^2$. Throughout this paper, we consider that the constellation is normalized to have unit energy and it translates into the following relation between d_m and the priority parameters [5] $d_m = \frac{1}{\sqrt{\alpha_1^2+\alpha_2^2+\ldots+\alpha_{m-1}^2+1}}$.

C. Demultiplexing, Deinterleaving and Decoding

The result of the transmission of a symbol is given by $Y = X + Z$, where $X \in \mathcal{X}$, and $Z \in \mathbb{R}$ are samples of zero-mean independent Gaussian random variables with variance $N_0/2$. The signal-to-noise ratio (SNR) per symbol is given

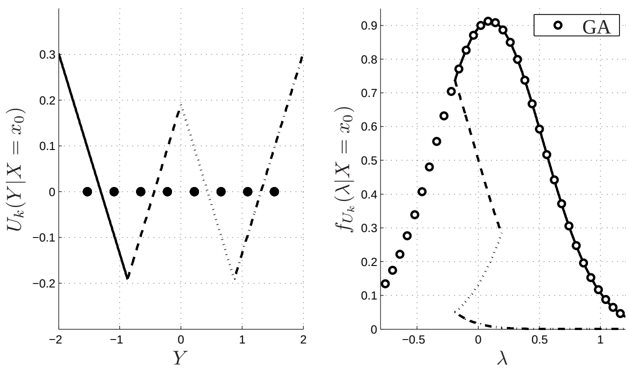

Figure 3. Left: Pice-wise linear relationship (2), conditioned on $X = x_0$, $\gamma = -3$ dB, 8-PAM, and $k = 3$. The black circles represent the uniform 8-PAM constellation. Right: Piece-wise Gaussian functions that, added up, form the PDF of the L-values, where each Gaussian piece corresponds to a linear piece in the left figure. The GA is shown with white circles.

by $\gamma = \frac{1}{N_0}$. At the receiver's side, logarithmic likelihood ratios (L-values) are calculated for each bit in the transmitted symbol (U_k). These L-values are then demultiplexed (L_k^{π}), deinterleaved (L_k) and then passed to a channel encoder which produces an estimate of the transmitted bits.

III. PDF of L-values and Equivalent Channel Model

In order to predict the coded BER performance of the system using union bounding techniques, finding the PDF of the L-values passed to the channel decoder is crucial. We use $f_{U_k}(\lambda | X = x_i)$ to denote the PDF of the L-value for bit position k, conditioned on the transmitted symbol x_i. The L-value for kth bit position for received signal Y and given the transmitted symbol x_i can be written as [1], [2]

$$U_k(Y|X = x_i) = \gamma \left[\min_{x \in \mathcal{X}_{k,0}} \left\{ (Y - x)^2 \right\} - \min_{x \in \mathcal{X}_{k,1}} \left\{ (Y - x)^2 \right\} \right], \quad (2)$$

where $\mathcal{X}_{k,b}$ is the set of symbols labeled with the kth bit equal to b, and where we have used the so-called max-log approximation. This approximation is often used in practical implementations, and it is known to have small impact on the receivers's performance when Gray-mapped constellations are used.

Since the relationship between the L-values and the channel outcome Y is non-linear, cf. (2), it was proposed in [16] to divide the observation space of Y into adjacent regions for a given bit position. Therefore, in a given region, the L-value in (2) becomes linear respect to Y. This led to closed-form expressions for the PDF of the L-values for uniform QAM constellations. For illustration, in the left part of Fig. 3 we show this piece-wise linear relationship for the particular case of uniform 8-PAM, bit position $k = 3$ and transmitted symbol $X = x_0$. It was shown in [16, eq. (22)] that the exact PDF of the L-values is a sum of piece-wise Gaussian functions defined over each region in which linear relationship holds. In the right side of Fig. 3 we show four (corresponding to each linear relationship in the left figure) piece-wise Gaussian functions for 8-PAM and $k = 3$, where the exact PDF of the L-values is simply the sum of all those functions (lines).

Table I
Values of δ_k and $(x_i - \rho_k)$ for NU 8-PAM defining the parameters in (3).

k	x_i	δ_k	$(x_i - \rho_k)$
1	x_3	$-2(d_1 - d_2 - d_3)$	$-(d_1 - d_2 - d_3)$
	x_2	$-2(d_1 - d_2 - d_3)$	$-(d_1 - d_2 + d_3)$
	x_1	$-2(d_1 - d_2 - d_3)$	$-(d_1 + d_2 - d_3)$
	x_0	$-2(d_1 - d_2 - d_3)$	$-(d_1 + d_2 + d_3)$
2	x_1	$-2(d_2 - d_3)$	$-(d_2 - d_3)$
	x_0	$-2(d_2 - d_3)$	$-(d_2 + d_3)$
3	x_0	$-2d_3$	$-d_3$

A. Gaussian Approximation for the PDF of the L-values

Because of the mathematical simplicity, a purely Gaussian model was further presented in [16], which allows for simple analysis of the system with a negligible impact on the coded BER performance. This Gaussian approximation (GA) simply replaces the exact PDF of the L-values (sum of piece-wise Gaussian functions defined over different intervals in λ) by one single Gaussian function defined over $\lambda \in \mathbb{R}$. We adopt the so-called zero crossing model (ZCMod), which selects one of the Gaussian functions around $\lambda = 0$. In what follows, we briefly review it here while more details can be found in [16, Sec. II-C].

For a given bit position k, it can be shown that there are 2^{k-1} regions which contain two symbols with opposite bit value for that particular bit. In these regions the value of λ crosses zero and they are referred to as zero crossing regions [16, Sec. II-C]. For example, for $M = 8$ and $k = 3$ in Fig. 3, there are four zero crossing regions on the left side of the figure, and therefore, there are four Gaussian pieces on the right side. Among those 2^{k-1} zero-crossing regions (2^{k-1} Gaussian functions around $\lambda = 0$), the ZCMod only considers the one which is the closest to the transmitted symbol (Fig. 3, since $X = x_0$, the considered region is the one with solid line). According to the ZCMod, the mean and variance of the Gaussian function for a transmitted symbol x_i are written as

$$\mu_k(x_i) = 2\gamma\delta_k \left[x_i - \rho_k \right], \qquad \sigma_k^2 = 2\gamma\delta_k^2, \quad (3)$$

where $\delta_k \triangleq (\bar{a}_{k,1} - \bar{a}_{k,0})$, $\rho_k \triangleq \frac{1}{2}(\bar{a}_{k,1} + \bar{a}_{k,0})$, and where $\bar{a}_{k,b}$ denotes the symbol labeled bit value b at position k in a zero-crossing region closest to the transmitted symbol x_i.

Our objective now is to find expressions for the mean values and variances in (3) which are valid for NU M-PAM constellations. In Table I, we present the values δ_k and $(x_i - \rho_k)$ as a function of the constellation distances d_k. In this table, and for a given bit position, we only consider transmission of those symbols that correspond to a bit “1”[1].

[1]This is simply because the computation the UB requires considering sequences of either ones or zeros.

Moreover, we present only the symbols in the constellation that yield a different mean value. This is because of the symmetry of the BRGC, and for a given bit position, there can be different symbols labeled with a bit "1" which have identical mean.

From Table I, it is possible to infer that for a given k, there are $M_k \triangleq M/2^k$ different values of $(x_i - \rho_k)$. From the definition of the mean value in (3), and since δ_k does not depend on the transmitted symbol, there will be a total of M_k mean values. From the evolution of $(x_i - \rho_k)$ and δ_k in Table I in terms of the distances d_k, and using (3), we can write a generic closed-form expression for mean values and variances of the Gaussian functions for NU M-PAM constellations as

$$\mu_{k,j} = 4\gamma\bigg[d_k - \sum_{i=k+1}^{m} d_i\bigg]\bigg[d_k + \sum_{i=k+1}^{m}(-1)^{1+b_i(j-1,m-k)}d_i\bigg] \tag{4}$$

$$\sigma_k^2 = 8\gamma\bigg[d_k - \sum_{i=k+1}^{m} d_i\bigg]^2, \tag{5}$$

where $j = 1, 2, \ldots, M_k$ and $k = 1, 2, \ldots, m$.

B. Equivalent Channel Model

Using the GA results presented in the above section, it is possible to build an equivalent model for the M^2-*QAM BICM channel* as done in [4]. In this model each bit u_k after the MUX can be seen as being sent over a *virtual channel* whose output L-value U_k has a distribution that depends on k and the symbol sent. In [4], because of uniform distance between the constellation symbols, the variances for all bit positions were same and only the mean values were dependent on transmitted symbols. On the other hand, for NU transmission, the variance depends on the bit position k whereas the mean depends on both bit position as well as the sent symbol. Therefore, in case of NU constellation-based BICM transmission, the virtual channel representation is quite different than uniform constellation-based transmission. Specifically, instead of one index as used in [4], we use two indexes: one index is k for bit position and the other index is j that depends on the symbol out of M_k possible symbols. Using these notations, all possible virtual channels can be denoted by $\Theta_{k,j}$ with $k = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, M_k$. There will be a total of $T = 2^m - 1$ virtual channels whose mean and variances are given by (4) and (5).

The probability that a given bit from pth encoder output is transmitted through virtual channel $\Theta_{k,j}$, $\xi_{p,k,j}$ depends on the probability of assigning pth encoder output to kth bit position i.e., $\kappa_{p,k}$ and the probability of sending symbol from the constellations. $\xi_{p,k,j}$ corresponds to the product of these two probabilities and for equiprobable symbol transmission, it can be written as $\xi_{p,k,j} = \frac{\kappa_{p,k}}{M_k}$.

For illustration purpose, the equivalent BICM channel for kth bit position with equiprobable symbol transmission is shown in Fig. 4. Once pth encoder output is assigned to kth bit position with probability $\kappa_{p,k}$, the bit sequence $c_{p|k}$ is passed through one of the M_k channels with probability $1/M_k$.

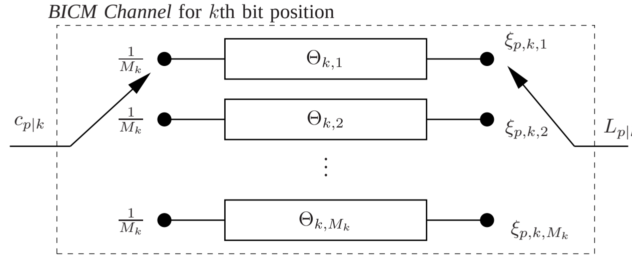

Figure 4. Equivalent channel model for the kth bit position of the "BICM channel" in Fig. 1.

Therefore, the PDF of L-value given that the pth output bit is transmitted in the kth bit position $L_{p|k}$, is summation of all possible M_k Gaussian function weighted by the corresponding probability $\xi_{p,k,j}$. With this defined probability and virtual channel model, the pth output $L_p \in \mathbb{R}$ of this channel is associated with the pth binary input c_p, where L_p is a Gaussian mixture with density given by

$$f_{L_p}(\lambda) = \sum_{k=1}^{m}\sum_{j=1}^{M_k} \xi_{p,k,j}\,\Phi(\lambda; \mu_{k,j}, \sigma_k^2), \tag{6}$$

where $\Phi(\lambda; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-\frac{(\lambda-\mu)^2}{2\sigma^2}}$.

IV. Performance Analysis

Using the generalized weight distribution spectrum (GWDS) of the code, the union bound (UB) on the BER for a BICM system using a linear code is given by [4]

$$\mathrm{BER} \leq \mathrm{UB} = \sum_{l=w_{\mathrm{free}}}^{\infty} \sum_{\mathbf{w}\in\mathcal{W}_n(l)} \beta(\mathbf{w})\mathrm{PEP}(\mathbf{w}), \tag{7}$$

where w_{free} is the free distance of the code, $\beta(\mathbf{w})$ is the GWDS of the code, $\mathrm{PEP}(\mathbf{w})$ is the pairwise error probability which represents the probability of detecting a codeword with generalized weight $\mathbf{w}$ instead of the transmitted all-one codeword. We also define the set $\mathcal{W}_i(l)$ as all the combinations of i nonnegative integers whose sum equals l, i.e., $\mathcal{W}_i(l) \triangleq \{(w_1, \ldots, w_i) \in (\mathbb{Z}^+)^i : w_1 + \ldots + w_i = l\}$.

To calculate the PEP we need to calculate the probability that the decoder selects a codeword with generalized weight $\mathbf{w}$ instead of the transmitted all-one codeword. To this end, we note that the decision is made based on the sum of $w_1 + \ldots + w_n$ L-values in the divergent path. Let D be the decision variable where

$$D = \sum_{i=1}^{w_1} L_1^{(i)} + \ldots + \sum_{i=1}^{w_n} L_n^{(i)} = \sum_{p=1}^{n}\sum_{i=1}^{w_p} L_p^{(i)}, \tag{8}$$

i.e., a sum of l independent random variables, where the random variable associated with the ith output is a sum of i.i.d. Gaussian mixtures given by (6). Consequently, for a given value of $\mathbf{w}$, the PEP can be calculated as the tail integral of the PDF of D.

Using the similar methodology as in [4], we can approximate the UB of BER for BICM transmission using NU-QAM constellations as follows:

$$\mathrm{UB} \approx \sum_{l=w_{\mathrm{free}}}^{l_{\max}} \sum_{\mathbf{w}\in\mathcal{W}_n(l)} \beta(\mathbf{w}) \sum_{\mathbf{r}_1,\ldots,\mathbf{r}_n} g(\mathbf{r}_1,\ldots,\mathbf{r}_n)\cdot \mathrm{Q}\left(h(\mathbf{r}_1,\ldots,\mathbf{r}_n)\right), \quad (9)$$

$$g(\mathbf{r}_1,\ldots,\mathbf{r}_n) = \prod_{p=1}^{n}\left[\binom{w_p}{\mathbf{r}_p}\prod_{k=1}^{m}\prod_{j=1}^{M_k}\xi_{p,k,j}^{r_{p,k,j}}\right], \quad (10)$$

$$h(\mathbf{r}_1,\ldots,\mathbf{r}_n) = \frac{\sum_{p=1}^{n}\sum_{k=1}^{m}\sum_{j=1}^{M_k} r_{p,k,j}\mu_{k,j}}{\sqrt{\sum_{p=1}^{n}\sum_{k=1}^{m}\sigma_k^2\sum_{j=1}^{M_k} r_{p,k,j}}}, \quad (11)$$

where $\mu_{k,j}$ and σ_k^2 are defined in (4) and (5), respectively, $\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \mathrm{e}^{-t^2/2}\,\mathrm{d}t$, $\mathbf{r}_p = [r_{p,1,1}\ldots r_{p,1,M_1}\ldots r_{p,m,1}\ldots r_{p,m,M_m}] \in \mathcal{W}_T(w_p)$ for $p = 1,\ldots,n$, and the multinomial coefficients are defined as $\binom{w_p}{\mathbf{r}} \triangleq \frac{w_p!}{\Pi_{k=1}^{m}\Pi_{j=1}^{M_k} r_{p,k,j}}$. For practical reasons, the outer sum in the UB approximation in (10) is limited to $l_{\max}$.

Analyzing the expression in (9), it is possible to see that it is composed of three terms: $\beta(\mathbf{w})$ which depends only on the code, $\mathrm{Q}\left(h(\mathbf{r}_1,\ldots,\mathbf{r}_n)\right)$ which depends only on the channel and constellations distance parameters, and $g(\mathbf{r}_1,\ldots,\mathbf{r}_n)$ which depends on the interleaver. In the next section we show numerical examples of a system where the interleaver and the constellation parameters are optimized.

V. Numerical Results

We now present numerical examples in order to quantify the potential gains when NU-QAM constellations are used. In particular, we analyze the simplest case where $n = m = 2$, i.e., a NU 4-PAM constellation and a rate-1/2 convolutionally-encoded or turbo-encoded BICM system. Moreover, we consider two multiplexing matrices: $\mathbb{K}' = \mathbb{I}_2$ ($\mathbb{I}_2$ being the identity matrix), and $\mathbb{K}''$ which is simply $\mathbb{K}'$ with its columns permuted. These two interleaver designs correspond then to Zehavi's design [1, Fig. 7] where all the bits from the same encoder's output are assigned to the same modulator's input (the first encoder output is assigned to the first bit position for $\mathbb{K}'$, and vice-versa for $\mathbb{K}''$).

For the convolutional code (CC), we use rate $R = 1/2$ optimum distance spectrum (ODS) convolutional codes with constraint length $K = 3, 4, 5$ [18]. The decoding is based on the soft-input Viterbi algorithm without memory truncation, and the block length used for simulation is 10.000 information bits. The turbo code (TC) we use for simulation is formed by a parallel concatenation of two recursive systematic convolutional encoders of rate 1/2 with polynomial generators $(1, 5/7)_8$. Alternate puncturing of the parity bits yield $R = 1/2$. The block length is 1.000 information bits. The decoder perform 20 iterations and uses the MaxLogMAP algorithm with a scaling factor of 0.7 applied the extrinsic information.

We first investigate the behavior of the UB approximation given by (9) for a given SNR and different codes with $l_{\max} = 100$. For this particular case ($n = m = 2$) there is only one

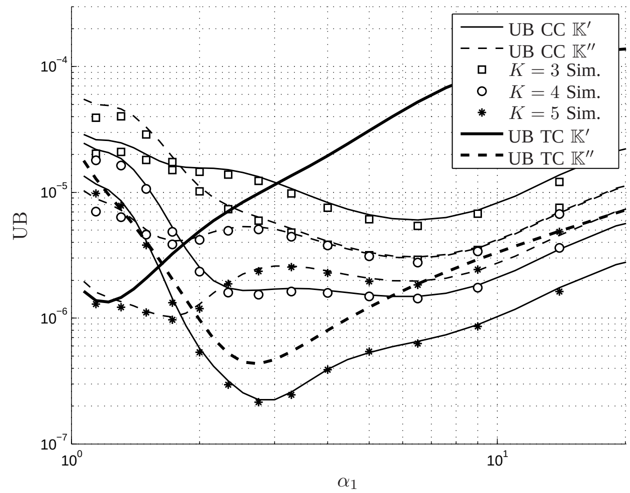

Figure 5. UB using (9) (lines) for different values of α_1, different interleaver designs, different ODS convolutional codes ($K = 3, 4, 5$), and $\gamma = 9$ dB. The results obtained by numerical simulations are shown with markers. The thick lines are the analytical UB for TC for interleaver designs and $\gamma = 6$ dB

constellation priority parameter, i.e., $\alpha_1 = d_1/d_2$. Note that three values of α_1 are of particular interest. If $\alpha_1 = 2$ the uniform 4-PAM constellation is obtained. If $\alpha_1 = 1$ a three-point constellation where the two constellation points x_1 and x_2 are located at zero is obtained. If $\alpha_1 \to \infty$ the NU 4-PAM constellation becomes a 2-PAM constellation ($x_0 = x_1 = -1$ and $x_2 = x_3 = 1$). The results obtained for different values of α_1 are shown in Fig. 5 for CCs and the TC and also for both interleaver designs. If we analyze the results for the CCs and $\alpha_1 = 2$, we can see that there is a performance gain that can be obtained solely by changing the interleaver, which has already been shown in [4]. If the value of α_1 is modified, additional gains can be obtained. Thus, in general, one could choose the optimum $\mathbb{K}$ (denoted by $\mathbb{K}^*$) *and* the optimum α_1 (denoted by α_1^*) simply by minimizing the UB. From this figure it is clear that the optimum pair $(\mathbb{K}^*, \alpha_1^*)$ depends on the code. For the TC similar conclusions can be drawn.

In general the optimization of α_1 and $\mathbb{K}$ can be done for each SNR, however, and for simplicity, we have plotted the results in Fig. 5 for two particular SNR values ($\gamma = 9$ dB for the CC and $\gamma = 6$ dB for the TC) which result in a BER of interest (between 10^{-7} and 10^{-4}). The optimal $(\alpha_1^*, \mathbb{K}^*)$ obtained for the four different codes in Fig. 5 are as follows. For CCs: $(\alpha_1^*, \mathbb{K}^*)\big|_{K=3} = (6.5, \mathbb{K}'')$, $(\alpha_1^*, \mathbb{K}^*)\big|_{K=4} = (6.5, \mathbb{K}')$, and $(\alpha_1^*, \mathbb{K}^*)\big|_{K=5} = (2.75, \mathbb{K}'')$, and for the TC $(\alpha_1^*, \mathbb{K}^*) = (2.75, \mathbb{K}'')$.

In Fig. 6 we present the BER obtained using $(\alpha_1^*, \mathbb{K}^*)$ for CCs with $K = 3$ and $K = 5$, and we compare them against a scheme where only the interleaver is optimized, i.e., $\alpha_1 \equiv 2$. This figure confirms that a joint optimization of the interleaver and the constellation outperforms the system where only the interleaver is optimized. It also confirms the tightness of the UB approximation. Note that even if the optimization was performed for $\gamma = 9$ dB, the performance improved for any

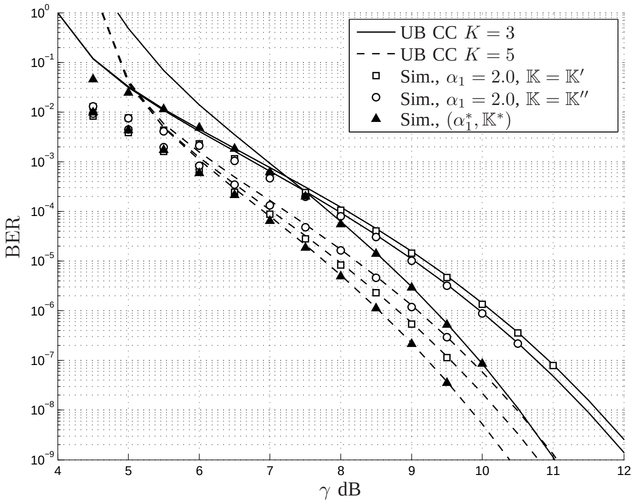

Figure 6. UB using (9) (lines) for different ODS CCs ($K = 3, 5$) using a uniform 4-PAM constellation, and a NU 4-PAM with $(\alpha_1^*, \mathbb{K}^*)$ from Fig. 5. The results obtained by numerical simulations are shown with markers.

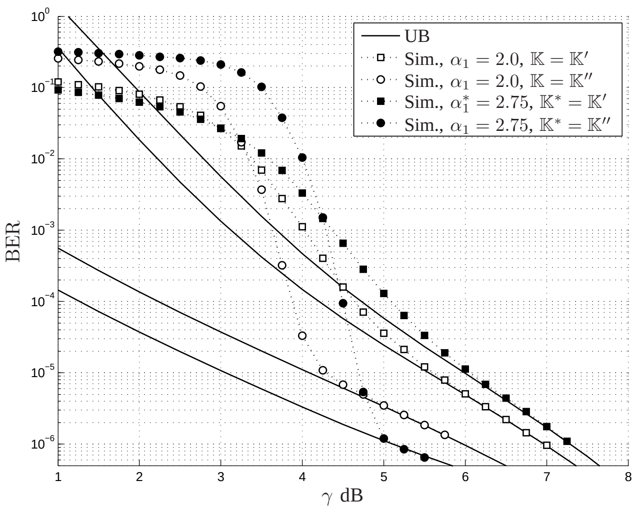

Figure 7. UB using (9) (lines) for the TC using a uniform 4-PAM constellation, and a NU 4-PAM with $(\alpha_1^*, \mathbb{K}^*)$. The results obtained by numerical simulations are shown with markers.

BER below 10^{-3}. Particularly, for a BER target of 10^{-7}, the joint optimization results in gains of up to 1 dB (for $K = 3$).

Finally, in Fig. 7 we present the BER obtained using the optimized system for the TC. From this figure it is clear that the use of NU 4-PAM lowers the error floor of the TC, and that the UB developed in Sec. IV perfectly predicts the error floor of the code. In order to illustrate the fact that the use of NU constellations must be combined with the interleaver design, we show the results for the TC using $\mathbb{K}'$ and $\alpha_1^* = 2.75$ (which is optimal for $\mathbb{K}''$). These results are even worse than those obtained with a uniform 4-PAM case ($\alpha_1 = 2$) and arbitrary interleaver $\mathbb{K}'$ or $\mathbb{K}''$.

VI. Conclusions

In this paper we studied UEP for BICM transmission using NU-QAM constellations. We developed closed-form expressions for the PDF of the L-values for NU constellations, and we used those for computing union bounds. We then used our model to improve the design of BICM transmissions obtaining gains for both convolutionally and turbo encoded BICM.

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