Fourth order tensor-based diffusion MRI signal modeling

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This abstract describes forth order tensor-based diffusion signal modeling as proposed in [1]. The Stejskal-Tanner equation for dMRI signal attenuation is:

$$S(\mathbf{g}_i) = S_0 \exp(-bd(\mathbf{g}_i)) \tag{1}$$

where $d(\mathbf{g}_i)$ is the diffusivity function, $S(\mathbf{g}_i)$ is the measured signal when the diffusion sensitizing gradient is applied in the direction $\mathbf{g}_i = [x_i \; y_i \; z_i]$, S_0 is the observed signal in the absence of such a gradient, and b is the diffusion weighting taken to be constant over all measurements. The diffusivity function $d(\mathbf{g}_i)$ is modeled using fourth order symmetric positive semi-definite tensors as follows:

$$d(\mathbf{g}_i) = \mathbf{t}^T \mathbf{a}_i \tag{2}$$

where $\mathbf{a}_i = [z_i^4 \; 4y_iz_i^3 \; 6y_i^2z_i^2 \; 4y_i^3z_i \; y_i^4 \; 4x_iz_i^3 \; 12x_iy_iz_i^2 \; 12x_iy_i^2z_i \; 4x_iy_i^3 \; 6x_i^2z_i^2 \; 12x_i^2y_iz_i \; 6x_i^2y_i^2 \; 4x_i^3z_i \; 4x_i^3y_i \; x_i^4]^T$ and $\mathbf{t} \in \mathbb{R}^{15}$ contains distinct entries of the n-th order tensor. Note that both vectors $\mathbf{t}$ and $\mathbf{a}_i$ are vectors in $\mathbb{R}^{15}$ and $d(\mathbf{g}_i,\mathbf{t}) = d(\mathbf{g}_i)$ is used for simplification. Given measurements in $N > 15$ different directions $\mathbf{g}_i$, the tensor estimation problem can then be formulated as:

$$\min_{\mathbf{t}\in\mathbb{R}^{15}} (\mathbf{s} - \mathbf{G}\mathbf{t})^T(\mathbf{s} - \mathbf{G}\mathbf{t}) \tag{3}$$

where $\mathbf{G}$ is an $N \times 15$ matrix defined as $\mathbf{G} = [\mathbf{a}_1 \; \mathbf{a}_2 \cdots \; \mathbf{a}_N]^T$ and $s_i = -b^{-1}\ln(S(\mathbf{g}_i)/S_0)$. The solution is $\hat{\mathbf{t}} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{y}$. The reader is referred to [1], [2] for more detail.

Proposed work:To participate in WMM challenge (ISBI 2015), we extend the model above as follows:

$$S(\mathbf{g}_i) = S_0 \exp(\frac{-TE}{T2}) \exp(-bd(\mathbf{g}_i)) \tag{4}$$

where TE is the echo time and $T2$ is the spin-spin (or transverse) relaxation time of the tissue. To estimate the diffusion signal for a given acquisition protocol with $TE = TE_x$, $b = b_x$ and $\delta = \delta_x$: (i) First, we use two non-diffusion weighted measurements with closest TEs to TE_x (measurements with $\delta = \delta_x$ are considered), to estimate $T2$ and S_0 for each voxel; (ii) Then, we use data form closet shell to b_x (shells with $\delta = \delta_x$ are considered), to estimate the tensor describing underlying structure. We use the tensor estimation method in [3].

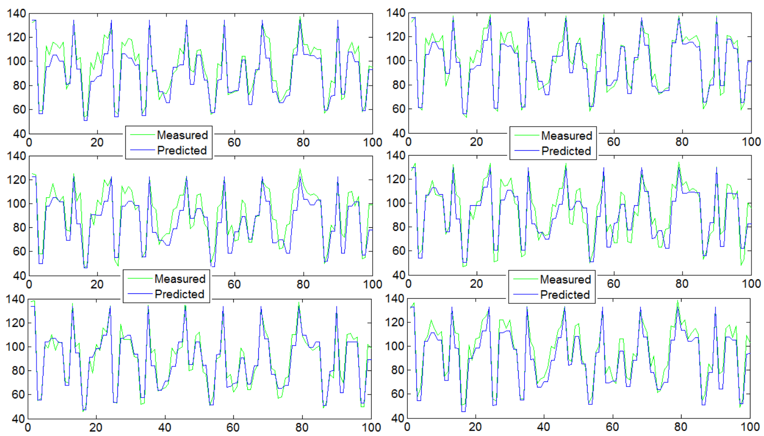

Fig. 1. Results for prediction of the MR signal for six voxels of the genu ROI where $TE_x = 107ms$, $b_x = 500s/mm^2$ and $\delta_x = 3ms$.

References

[1] E. Ozarslan and T. H. Mareci, "Generalized diffusion tensor imaging and analytical relationships between diffusion tensor imaging and high angular resolution diffusion imaging.," *Magn Reson Med*, vol. 50, no. 5, pp. 955–965, Nov 2003.

[2] L. Qi, G. Yu, and E. Wu, "Higher order positive semidefinite diffusion tensor imaging," *SIAM Journal on Imaging Sciences*, vol. 3, no. 3, pp. 416–433, 2010.

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