K-over-L Multidimensional Position Modulation

Tobias A. Eriksson, Pontus Johannisson, Benjamin J. Puttnam, Erik Agrell, Peter A. Andrekson, and Magnus Karlsson

***Abstract*—We analyze a family of modulation formats based on multidimensional position modulation (MDPM) with multiple pulses per frame (K-over-L-MDPM) in combination with quadrature phase shift keying (QPSK), polarization multiplexed QPSK (PM-QPSK) and polarization-switched QPSK (PS-QPSK). MDPM is a generalization of pulse position modulation (PPM) where different pulse slots can be realized by time slots, polarization states, frequencies, modes of multimode fibers, cores in multicore fibers or a combination of these. We show that by using K-over-L-MDPM with QPSK, it is possible to simultaneously increase both the spectral efficiency and the asymptotic power efficiency over QPSK. We also identify K-ary inverse-MDPM (KiMDPM-QPSK) as the special case of K-over-($K-1$)-MDPM and show that 4iMDPM-QPSK has a 1.25 dB increased asymptotic power efficiency over QPSK with a maintained spectral efficiency.**

***Index Terms*—Digital modulation, multicore fiber, multidimensional position modulation (MDPM), multiple pulse position modulation (MPPM), optical fiber communication, polarization-switched quadrature phase shift keying (PS-QPSK), power efficiency, pulse position modulation (PPM), quadrature phase shift keying (QPSK), sensitivity, spectral efficiency.**

I. Introduction

COHERENT optical transmission systems have been the topic of a remarkable amount of research, aiming for increased sensitivity and spectral efficiency (SE). Coherent systems enable the use of digital signal processing (DSP) which makes it possible to compensate or mitigate transmission impairments in the digital domain as well as enabling phase tracking which makes coherent receivers with free-running local oscillators (LO) possible [1].

The coherent receiver allows the use of the four-dimensional (4D) signal space that is spanned by the quadratures as well as the two polarizations of an optical field. The polarization states are often used as independent channels where two-dimensional in-phase and quadrature (IQ) modulation formats are transmitted, such as quadrature phase shift keying (QPSK) or rectangular 16-ary quadrature amplitude modulation (16-QAM). Polarization-multiplexed QPSK (PM-QPSK) can be implemented with low-complexity transmitters and offers a reasonable trade-off between SE and asymptotic power efficiency. By considering modulation in the 4D signal space, more power-efficient formats can be found. For instance, polarization-switched QPSK (PS-QPSK) was identified as the most power-efficient modulation format in four dimensions and can be implemented with small modifications to a conventional PM-QPSK transmitter [2].

In 2011, Liu *et al*. demonstrated an optical transmission system with record sensitivity utilizing 16-ary pulse position modulation (PPM) in combination with PM-QPSK [3]. Such high sensitivity could be achieved due to the fact that the dimensionality of the signal space can be effectively increased by using PPM, where each time slot increases the number of dimensions. By using PPM in combination with, e.g. PM-QPSK or PS-QPSK, modulation formats with increased power efficiency can be found at the expense of reduced SE [4]. Other physical properties can also be used to increase the dimensionality of the signal space, i.e. in [5], a combination of PPM-time slots, different frequencies, and the two polarizations is used.

In conventional PPM, data is encoded by transmitting one pulse in a frame with K pulse slots and the position of the pulse determines the transmitted symbol. It is also possible to utilize more than one pulse in each frame where data is encoded in the positions of these pulses. For PPM, this is often called multi-pulse position modulation (MPPM) and has been studied for intensity-modulated communication systems [6], [7]. The specific case of using two pulses per PPM-frame has been investigated for direct detection in uncoded systems [8] and together with convolutional coding and Reed-Solomon (RS) coding [9]. Further, MPPM in combination with different types of error-correction coding has been shown to achieve better sensitivity than coding in combination with conventional PPM. In [10], RS-coded MPPM was shown to be more sensitive than RS-coded PPM and in [11] MPPM with different error-correcting codes was shown to outperform coded PPM with a main focus on free-space optical links. Further, MPPM where the symbols are obtained using the concept of balanced incomplete block design has been shown to be more sensitive compared to conventional MPPM [12]. For intensity-modulated MPPM, performance bounds on power efficiency and SE have been derived [13].

Most of the previous MPPM research has been done for intensity-modulated links. However, MPPM in combination with PS-QPSK has been investigated and it was found that for a fixed power efficiency the SE can be improved over conventional PPM in combination with PS-QPSK [14]. It was also shown that by using MPPM-PS-QPSK, both the SE and the power efficiency can be increased over PS-QPSK when long frames are used [14]. Inverse pulse position modulation (iPPM), where the PPM symbols are encoded in which pulse

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Tobias A. Eriksson, Pontus Johannisson, Peter A. Andrekson, and Magnus Karlsson are with the Photonics Laboratory, Department of Microtechnology and Nanoscience, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden (e-mail: tobias.eriksson@chalmers.se)

Benjamin J. Puttnam is with the Photonic Network Research Institute, National Institute of Information and Communications Technology, Tokyo 184-8795, Japan.

Erik Agrell is with the Communication Systems Group, Department of Signals and Systems, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden.

slot no power is sent, rather than where the power is sent, was introduced in combination with PS-QPSK in [14] and has also been suggested for multicast passive optical networks [15].

In the last few years, multimode [16] and multicore fibers have attracted a lot of research attention due to the possibility to increase the transmission capacity of an optical fiber. Transmission over, e.g., 7-core [17], [18], 12-core [19] and 19-core fibers [20] have been demonstrated. In these studies, the different cores are treated as independent channels. In [21], modulation using different cores or different modes is proposed.

In this paper, we introduce multidimensional position modulation (MDPM) for coherent communication systems which is a generalization of PPM where the different dimensions can be realized by for instance different time slots in PPM, different frequencies, different polarizations, different modes in multimode fibers or the different cores in multicore fibers. Using this convention, the formats in [5] where polarization, frequency and time slots (resulting in K possible dimension slots) are used, can be generalized to KMDPM. Here, examples are given for PPM and multicore fibers. PPM is the most common implementation of MDPM and provides intuitive examples which are easy to illustrate and we believe multicore fibers can be one practical implementation since the temporal synchronization between the cores in the same cladding should not be too challenging. In this paper, K-over-L-MDPM ($\binom{K}{L}$-MDPM), which is defined in section IV, is studied in combination with QPSK, PM-QPSK and PS-QPSK in terms of SE and asymptotic power efficiency. We show that by using multiple positions per frame, e.g. multiple pulses per PPM-frame or per set of cores, it is possible to simultaneously increase the power efficiency and the SE over the IQ-modulation format that is used in combination with the $\binom{K}{L}$-MDPM (e.g. QPSK, PM-QPSK or PS-QPSK). We also investigate the special case when $L = K - 1$ in combination with QPSK and PS-QPSK in more detail.

II. Preliminaries and Definitions

In this paper, we study modulation formats in terms of SE and asymptotic power efficiency. It is in most cases desired to have a high SE, which translates into a higher bitrate for a given bandwidth. However, increasing the SE often comes at the cost of decreasing the sensitivity in terms the required signal-to-noise ratio (SNR) for a given error probability. The asymptotic power efficiency is a measure of the sensitivity at asymptotically high SNR and can be computed with low complexity. Therefore, the SE and the asymptotic power efficiency are used to find candidates of modulation formats that are both power and spectrally efficient.

Many other interesting studies can be done such as performance at low SNR, nonlinear transmission characteristics, performance with error-correcting codes and experimental realizations of the formats. However, these kinds of studies require focus on a few selected modulation formats, and to keep this paper general we leave these topics for future studies.

We study the different families of modulation formats assuming a discrete-time memoryless additive white Gaussian

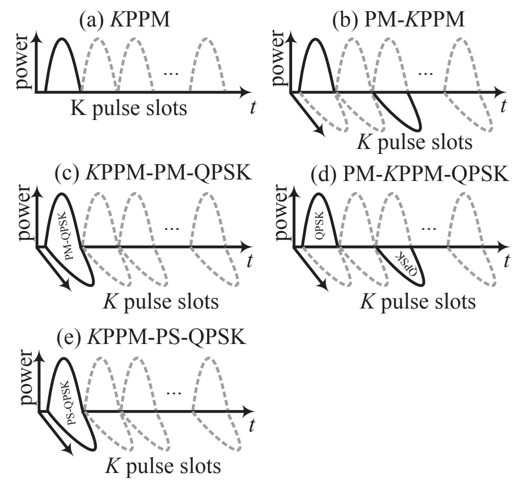


Fig. 1: Different combinations of PPM and IQ-modulation showing (a) conventional KPPM, (b) polarization-multiplexed KPPM, (c) KPPM in combination with PM-QPSK (d) Polarization multiplexed KPPM-QPSK and (e) KPPM in combination with PS-QPSK. Note that (c) and (d) are different. In (c) the PPM slot contains both polarizations and a PM-QPSK symbol is sent. As an example, with 4PPM (c) carries 6 bits and (d) 8 bits per frame.

noise channel where the noise variance per dimension is $N_0/2$. We study all formats in uncoded transmission and assume that all symbols are equally probable. We denote the constellation of a modulation format as $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M\}$ where $\mathbf{c}_m$, $m = 1, 2, \ldots, M$, is an N-dimensional amplitude vector. With this notation, QPSK is described as $\mathcal{C}_{\mathrm{QPSK}} = \{(\pm1, \pm1)\}$, PM-QPSK as $\mathcal{C}_{\mathrm{PM-QPSK}} = \{(\pm1, \pm1, \pm1, \pm1)\}$ and PS-QPSK as $\mathcal{C}_{\mathrm{PS-QPSK}} = \{(\pm1, \pm1, 0, 0), (0, 0, \pm1, \pm1)\}$ which corresponds to sending QPSK in either the x- or the y-polarization [2]. The Euclidean distance between two symbols is $d_{k,j} = \|\mathbf{c}_k - \mathbf{c}_j\|$ and the minimum Euclidean distance for any modulation format is $d_{\min}(\mathcal{C}) = \min_{k\neq j} d_{k,j}$ [22].

To keep the theory general, we do not consider crosstalk between the different dimensions. The amount and properties of the crosstalk will be very different depending on how the formats in this paper are implemented. For instance, the crosstalk between time slots can easily be mitigated with an equalizer whereas the crosstalk between modes or wavelength channels is harder to compensate for.

The asymptotic power efficiency is

$$\gamma = \frac{d_{\min}^2 \log_2 M}{4E_{\mathrm{s}}}, \tag{1}$$

where M is the number of constellation points and E_{s} is the average symbol energy of the N-dimensional modulation format. The factor $1/4$ normalizes the power efficiency to 0 dB for binary phase shift keying (BPSK), QPSK and PM-QPSK [23]. The asymptotic power efficiency equals the sensitivity gain over QPSK at the same bitrate and at asymptotically high SNR. In many cases, a more power efficient modulation format also has less nonlinear distortions since the average power can be decreased.

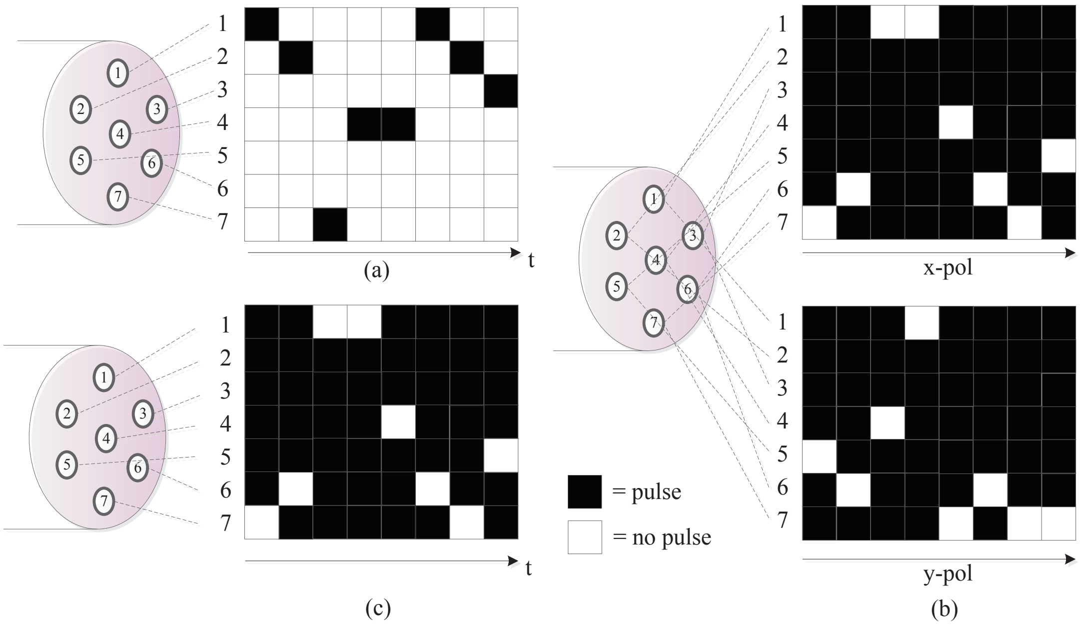


Fig. 2: Examples of how the cores of a multicore fiber can be utilized for MDPM illustrated with a 7-core fiber. (a) KMDPM-QPSK with $K = 7$. (b) Each core has two polarization states which gives a 7-core fiber 14 MDPM slots. (c) A 7-core fiber can be used for PM-7iMDPM-QPSK which is illustrated in (c), but could also be used for e.g. 14iMDPM-QPSK. For clarity, slots with no power are marked with a red cross in (b) and (c).

The SE per dimension pair, i.e. per polarization, is given by

$$\mathrm{SE} = \frac{\log_2 M}{N/2}. \quad (2)$$

The SE of BPSK, QPSK and PM-QPSK is 2 bit/symbol/polarization [23].

III. Multidimensional Position Modulation

In this section we discuss conventional PPM and PPM in combination with IQ-modulation formats followed by the definition and discussion of multidimensional position modulation.

A. Conventional PPM and PPM in Combination with IQ-Modulation

For conventional PPM, data is encoded by sending one pulse in one of the slots in a frame comprised of K consecutive time slots as illustrated in Fig. 1(a). The KPPM format has $N = K$ dimensions and carries $B_\mathrm{F} = \log_2(K)$ bits per PPM frame. The SE for KPPM is $\mathrm{SE} = 2\log_2(K)/K$ and the asymptotic power efficiency is $\gamma = \log_2(K)/2$ [4]. The KPPM signal can be polarization multiplexed as illustrated in Fig. 1(b). It should be noted that with the above definition, PM-KPPM has the same γ and SE as KPPM.

Instead of encoding data only in the selection of the pulse slot, it is possible to modulate the pulse to increase the data rate. If the pulses are modulated with PM-QPSK, as shown in Fig. 1(c) and experimentally demonstrated for 16PPM-PM-QPSK in [3], it is possible to transmit $B_\mathrm{F} = \log_2(K) + 4 = \log_2(16K)$ bits per PPM-frame. The SE and the asymptotic power efficiency for KPPM-PM-QPSK are $\mathrm{SE} = (\log_2 16K)/(2K)$ and $\gamma = \log_2(16K)/4$ respectively. However, this is a somewhat inefficient utilization of the polarization states if we compare to PM-KPPM-QPSK where KPPM-QPSK is polarization multiplexed as illustrated in Fig. 1(d). This yields $B_\mathrm{F} = 2\log_2(K) + 4$ bits per PPM-frame, $\mathrm{SE} = \log_2(4K)/K$ and $\gamma = \log_2(4K)/2$. For any K, this family of formats has both higher SE and γ compared to KPPM-PM-QPSK which was also discussed in [4]. Note that the same SE and γ are obtained for KPPM-QPSK as PM-KPPM-QPSK. A third possibility is to modulate the PPM-pulses with PS-QPSK, as illustrated in Fig. 1(e). This gives $B_\mathrm{F} = \log_2(K) + 3 = \log_2 8K$ bits per PPM-frame, $\mathrm{SE} = \log_2 8K/(2K)$ and $\gamma = \log_2(8K)/(2K)$.

B. Multidimensional Position Modulation

In conventional PPM, symbols are encoded by transmitting power in one out of K dimensions where the dimensions are realized by dividing time into different slots. This can be generalized into *Multidimensional Position Modulation* (MDPM) where the dimensions can be realized by time slots, polarizations, frequencies, modes, cores of a multi-core fiber or a combination of these. For the frequency realization, the modulation is usually called frequency shift keying (FSK) or if the frequency components are orthogonal it is designated orthogonal frequency-division multiplexing (OFDM). Further, if the data on the frequency channels are independent it is called wavelength division multiplexing (WDM). For the polarization the term *polarization shift keying* is often used. For multi-core fibers the different cores can act as new dimensions and we can encode data by transmitting power in one out of K cores. We call this format *core-selection modulation* (CSM). The

TABLE I: Examples of experimentally implemented MDPM formats

Modulation Format	MDPM representation	Reference
QPSK	1MDPM-QPSK	
2PPM	2MDPM	[24]
PS-QPSK	2MDPM-QPSK	[2]
2PPM-QPSK	2MDPM-QPSK	[25]
4FPS-QPSK[1]	4MDPM-QPSK	[26]
PolSK-4FSK[2]	8MDPM	[5]
4PPM-PM-QPSK	4MDPM-PM-QPSK	[5]
8PPM-PS-QPSK	8MDPM-PS-QPSK	[4]
16PPM-PM-QPSK	16MDPM-PM-QPSK	[3]
64PPM-PS-QPSK	64MDPM-PS-QPSK	[27]
256PPM	256MDPM	[24]

[1] 4-ary Frequency and Polarization Switched QPSK
[2] Polarization-shift keying with 4-ary FSK.

equivalent format for multimode fibers, using modes instead of cores, would have the designation *mode-selection modulation*.

In the rest of this paper we will consider the general case of MDPM. Some already investigated formats and their representation as MDPM are shown in Table I. The main difference between PPM, CSM, FSK and mode-selection modulation is the amount of crosstalk between the dimensions. However, in this paper we do not consider crosstalk with the motivation that in many cases the crosstalk is negligible or can be compensated with an equalizer. We will focus on PPM and CSM, mainly since PPM is the most straightforward example and CSM is an example of utilizing the different cores of a multi-core fiber in another way than as parallel channels, much like PS-QPSK utilizes both polarization states.

The realization of KMDPM-QPSK in the multicore case would be KCSM-QPSK and is illustrated in Fig. 2(a) where one QPSK symbol is sent in either of the cores. The equivalent to conventional PPM can be illustrated by the same figure but leaving the pulses unmodulated.

IV. K-over-L-MDPM in combination with IQ-modulation

Instead of transmitting one pulse per frame it is possible to send L pulses which has been investigated for PPM and intensity modulated systems [6]–[13]. The number of symbols, M, per frame with K pulse slots is then given by the number of possible ways to transmit L pulses in a frame with K slots, i.e. $M = \binom{K}{L}$. It should be noted that for most cases $\log_2 M$ is not an integer number of bits. In this paper however, we study all possible number of symbols, since the symbols of a constellation with a non-integer number of bits per symbol could be used with for instance coded modulation. However, these types of applications are out of the scope of this paper. When K equals a power of two and L is either 1 or $K-1$, $\log_2 M$ is an integer number of bits. Conventional KMPDM is the case when $L = 1$ and when $L = K-1$, no power is denoting the sent MDPM-slot and the rest of the slots are filled with pulses. Hence, we denote this specific case of $\binom{K}{K-1}$-MDPM by K-ary inverse-MDPM (KiMDPM). In intensity-modulated PPM systems, KiMDPM makes little sense since compared to conventional KMDPM, the SE is the same but the power efficiency is reduced due to the increase in E_s. However, when the pulses are modulated, the number of bits per frame can be increased significantly. In the following, we will investigate the performance of $\binom{K}{L}$-MDPM in combination with QPSK, PM-QPSK and PS-QPSK as well as the special case of KiMDPM with QPSK and PS-QPSK.

For $\binom{K}{L}$-MDPM in combination with a single-pulse IQ-modulation format, $\mathcal{C}_\mathrm{mod}$, with b bits per symbol, where each of the L pulses are modulated using this format, the number of bits per frame, B_F, is

$$B_\mathrm{F} = \log_2 M = bL + \log_2 \binom{K}{L}. \tag{3}$$

The minimum Euclidean distance of a general $\binom{K}{L}$-MDPM format is

$$d_\mathrm{min} = \min\left\{d_\mathrm{min}(\mathcal{C}_\mathrm{mod}), \sqrt{2}d_\mathrm{min}(\mathcal{C}_\mathrm{mod} \cup \{\mathbf{0}\})\right\}, \tag{4}$$

where $\mathbf{0} \notin \mathcal{C}_\mathrm{mod}$. The zero-vector cannot be a part of the single-pulse IQ-modulation format $\mathcal{C}_\mathrm{mod}$ since that prevents the use of MDPM-modulation.

As an example, we consider $\binom{4}{2}$-MDPM-QPSK, for which $C_\mathrm{mod} = \{(\pm 1, \pm 1)\}$, and investigate the possible nearest neighbors for the symbol $(1,1,1,1,0,0,0,0)$ where the four ones represent two-out-of-four QPSK symbols. The first possibility is that a QPSK symbol is changed resulting in, say, the symbol $(-1,1,1,1,0,0,0,0)$ and the second is that the MDPM part has changed so that the symbol is, say, $(1,1,0,0,1,1,0,0)$. Both these cases have a Euclidean distance of 2 to the original symbol. The same holds for PS-QPSK and PM-QPSK, although E_s will be different.

The average symbol energy is dependent on the single-pulse IQ-modulation format used and is $E_\mathrm{s} = e_\mathrm{s} L$ where e_s is the average symbol energy for the IQ-modulation format. Using this notation, the average symbol energy for $\binom{K}{L}$-MDPM with IQ-modulation is $E_\mathrm{s} = 2L$ with QPSK, $E_\mathrm{s} = 4L$ with PM-QPSK and $E_\mathrm{s} = 2L$ with PS-QPSK and the minimum Euclidean distances for all these formats are 2. The dimensionality of a $\binom{K}{L}$-MDPM format is $N = 2K$. Using (3) and (1) we get the power efficiency for $\binom{K}{L}$-MDPM in combination with IQ-modulation as

$$\gamma = \frac{d_\mathrm{min}^2}{4e_\mathrm{s}L}\left[bL + \log_2\binom{K}{L}\right]. \tag{5}$$

In the same way, using (3) and (2), the SE for $\binom{K}{L}$-MDPM with modulation is

$$\mathrm{SE} = \frac{2}{nK}\left[bL + \log_2\binom{K}{L}\right], \tag{6}$$

where n is the dimensionality of an MDPM slot which is dependent on the modulation format used. For QPSK n = 2 and for PS-QPSK as well as for PM-QPSK n = 4. The SE as a function of the inverse asymptotic power efficiency in dB for $\binom{K}{L}$-MDPM in combination with QPSK, PM-QPSK and PS-QPSK is shown in Fig. 3. For each format, the different curves represent $L = K-1, K-2, \ldots, K-32$, as indicated in the inset in Fig. 3, and for each curve, K is plotted up to $K = 100$. We first note that $\binom{K}{L}$-MDPM in combination with QPSK has the potential to achieve the highest SE. The performance of $\binom{K}{L}$-MDPM-PM-QPSK is worse than $\binom{K}{L}$-MDPM-QPSK in terms of both SE and γ for any choices of K and L, except when $L = 1$ for $\binom{K}{L}$-MDPM-QPSK. $\binom{K}{L}$-MDPM-PS-QPSK cannot reach the same SE as $\binom{K}{L}$-MDPM-QPSK and using a large K it is possible to achieve similar asymptotic power efficiencies for the two formats. However, for a fixed K, $\binom{K}{L}$-MDPM-PS-QPSK can achieve a better power efficiency. For any $K-L$, the SE and γ approach the values for QPSK when $K \to \infty$ for $\binom{K}{L}$-MDPM-QPSK and $\binom{K}{L}$-MDPM-PM-QPSK and approaches those of PS-QPSK for $\binom{K}{L}$-MDPM-PS-QPSK. Further, the performance for conventional KMDPM without any combined IQ-modulation is plotted as black triangles and dashed line. The points corresponding to QPSK and PS-QPSK are indicated by text labels.

To find the maximum achievable SE for a given γ we define $\xi \equiv L/K$ as the fraction of filled MDPM slots per frame. Using (1) and (2) we find the relation between γ and SE

$$\gamma = n\frac{d_{\min}^2}{8\xi e_{\mathrm{s}}}\mathrm{SE}. \tag{7}$$

Using the asymptotic expression for the binomial coefficients [28, Eq. (11.53)]

$$\lim_{K\to\infty}\frac{1}{K}\log_2\binom{K}{\xi K} = h(\xi), \tag{8}$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is the binary entropy function, we can approximate the SE for large K as

$$\lim_{K\to\infty}\mathrm{SE} = \frac{2}{n}\left(b\xi + h(\xi)\right). \tag{9}$$

This SE can be maximized with respect to ξ, giving

$$\xi_{\max} = \frac{2^b}{1+2^b}. \tag{10}$$

The maximum achievable SE for the three formats and the corresponding power efficiencies as well as the percentage of filled MDPM slots to achieve maximum SE are given in Table II. The achievable results if the frame length approaches infinity are plotted plotted as the upper thick lines for the three formats in Fig. 3. Shown as the lower thick lines are KMDPM-QPSK, KMDPM-PM-QPSK and KMDPM-PS-QPSK which is the same as $\binom{K}{1}$-MDPM in combination with the three formats.

V. Inverse-MDPM with QPSK or PS-QPSK

As mentioned, iMDPM is the special case of $\binom{K}{L}$-MDPM where $L = K-1$ and in combination with QPSK or PS-QPSK we denote these formats as KiMDPM-QPSK and KiMDPM-PS-QPSK respectively. These are interesting formats since they can be implemented with relatively low complexity, since K and L are small for the formats with good performance, the

TABLE II: The maximum achievable SE with $K \to \infty$ for $\binom{K}{L}$-MDPM in combination with IQ-modulation

Modulation Format	max SE†	$1/\gamma$‡	ξ_{opt}⋆
$\binom{K}{L}$-MDPM-QPSK	2.32	-1.62 dB	80.0%
$\binom{K}{L}$-MDPM-PM-QPSK	2.04	-0.36 dB	94.1%
$\binom{K}{L}$-MDPM-PS-QPSK	1.59	-2.51 dB	88.9%

† in bit/symbol/polarization.
‡ $1/\gamma$ at the maximum SE.
⋆ the percentage of MDPM slots that are filled with pulses when maximum SE is achieved.

MDPM bits are encoded in a single slot and also since they carry an integer number of bits per symbol when K is a power of 2. Therefore, we will discuss this family of modulation format in more detail. Using (5) and (6) we can simplify the expressions for the power efficiency and SE for KiMDPM in combination with IQ-modulation to

$$\gamma_{\mathrm{iMDPM}} = \frac{b}{2} + \frac{\log_2 K}{2(K-1)}, \tag{11}$$

and

$$\mathrm{SE}_{\mathrm{iMDPM}} = \frac{2}{nK}\Big[b(K-1) + \log_2 K\Big]. \tag{12}$$

The power efficiency and SE for KiMDPM-QPSK and KiMDPM-PS-QPSK with an integer number of bits per symbol are shown in Fig. 4. We first note that 2iMDPM-QPSK is the same format as 2MDPM-QPSK which has the same SE and γ as PS-QPSK. We further observe a few interesting points. For KiMDPM-QPSK we find 4iMDPM-QPSK and 8iMDPM-QPSK which have better performance in terms of SE and power efficiency compared to QPSK. However, as seen in Fig. 4, a further increase of K degrades the performance in terms of both SE and power efficiency.

We also find 4iMDPM-PS-QPSK, 8iMDPM-PS-QPSK and 16iMDPM-PS-QPSK where 4iMDPM-PS-QPSK is a format for which the SE and power efficiency is intermediate that of 2MDPM-PS-QPSK and PS-QPSK. 8iMDPM-PS-QPSK and 16iMDPM-PS-QPSK have an increased performance in terms of SE and power efficiency over PS-QPSK. As seen in Fig. 4, using KiMDPM with QPSK brings more gain over QPSK than what KiMDPM with PS-QPSK brings over PS-QPSK [14]. However, KiMDPM-PS-QPSK is more power efficient than KiMDPM-QPSK.

An example of KiMPDM-QPSK, for $K = 7$, is shown in Fig. 2(c), where CSM in a 7-core multicore fiber using single polarization is illustrated. This format could also be polarization multiplexed as illustrated in Fig. 2(b) or 14iMDPM-QPSK could be used, utilizing iMDPM over all 14 slots realized by the cores and the polarization states.

A. *4iMDPM-PS-QPSK, 8iMDPM-PS-QPSK and 16iMDPM-PS-QPSK*

From Fig. 4 it can be seen that the SE and power efficiency of 4iMDPM-PS-QPSK are 1.375 bit/symbol/polarization and -2.63 dB respectively. This format could be an alternative

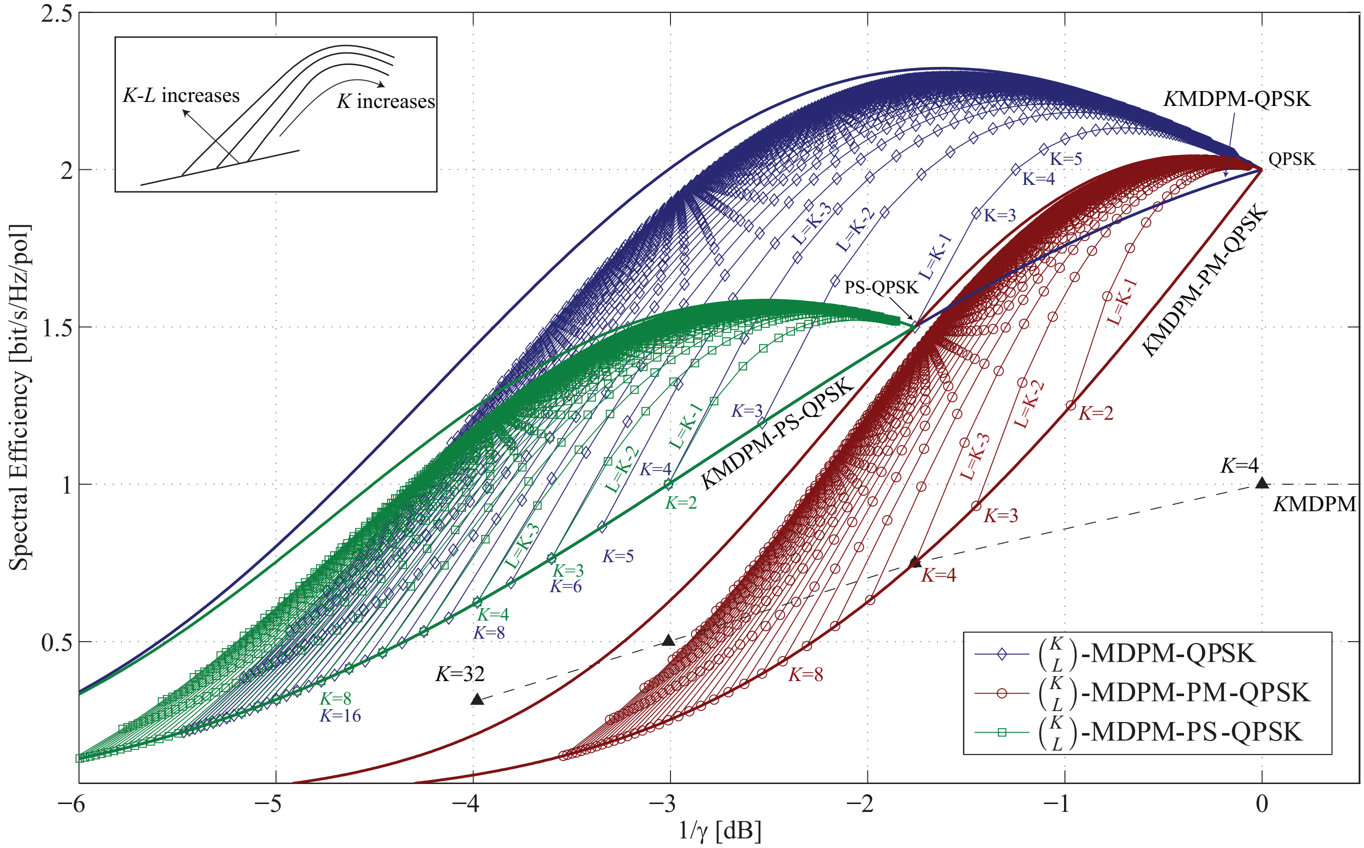


Fig. 3: The SE as a function of $1/\gamma$ for $\binom{K}{L}$-MDPM-QPSK (blue diamonds), $\binom{K}{L}$-MDPM-PM-QPSK (red circles), and $\binom{K}{L}$-MDPM-PS-QPSK (green squares). The curves are plotted for $K \leq 100$ and $L = K-1, K-2, \ldots, K-32$. The solid thick upper lines show the maximum achievable SE as a function of γ for the three families of formats and the lower thick solid lines show KMDPM-QPSK, KMDPM-PM-QPSK and KMDPM-PS-QPSK which is the same as $\binom{K}{1}$-MDPM in combination with QPSK (blue), PM-QPSK (red) and PS-QPSK (green). Note that the plot for KMDPM-QPSK partially overlaps with the plot for KMDPM-PS-QPSK. Also shown is conventional KMDPM (black triangles and dashed line) which is often implemented as KPPM.

when a more power efficient modulation format than PS-QPSK is preferred but when the loss in SE of going to 2MDPM-PS-QPSK or 4MDPM-QPSK is too high. 8iMDPM-PS-QPSK has the same SE as PS-QPSK but has 0.58 dB better power efficiency. Finally, using 16iMDPM-PS-QPSK the SE and power efficiency can be increased over PS-QPSK with 0.03 bit/symbol/polarization and 0.37 dB respectively. This format were suggested in [14].

B. 4iMDPM-QPSK and 8iMDPM-QPSK

As seen in Fig. 4, 4iMDPM-QPSK has a 1.25 dB increased power efficiency over QPSK while having the same SE of 2 bit/symbol/polarization as QPSK. The dimensionality for this format is 8 and it can be implemented in a PPM system as either PM-4iPPM, as shown in Fig. 5(a), or by combining two time slots and two polarizations to get the 4 slots as shown in Fig. 5(b). For multicore fibers the format could be implemented by using two cores and the two polarization states to achieve 4 slots. This format is of relatively low complexity, since K and L are small and it carries an integer number of bits per frame. Also included in Fig. 4 is the 4D format $C_{4,16}$, which has been found by sphere packing [29], and has the same SE as QPSK and 1.11 dB better power efficiency.

8iMDPM-QPSK has 0.84 dB better power efficiency than QPSK while also the SE is increased to 2.13 bit/symbol/polarization. The dimensionality of this format is 16 and two straightforward PPM implementations are illustrated in Fig. 5. It can be implemented as PM-8iPPM-QPSK as shown in Fig. 5(c), where 8 slots are generated in 8 consecutive time slots and then the signal can be polarization multiplexed. Another possible implementation is to combine 4 time slots and the two polarization states to get 8 slots which is illustrated in Fig. 5(d). These two implementations are similar, but in terms of equalization and polarization demultiplexing there are some differences. For instance, in the first case there is the possibility that a time slot has no power in any of the polarization states whereas for the second case this can never happen. For a multicore fiber two possible implementations of this format would be to utilize the polarization states of 4 cores to achieve 8 slots or to use the two polarization states of two cores in combination

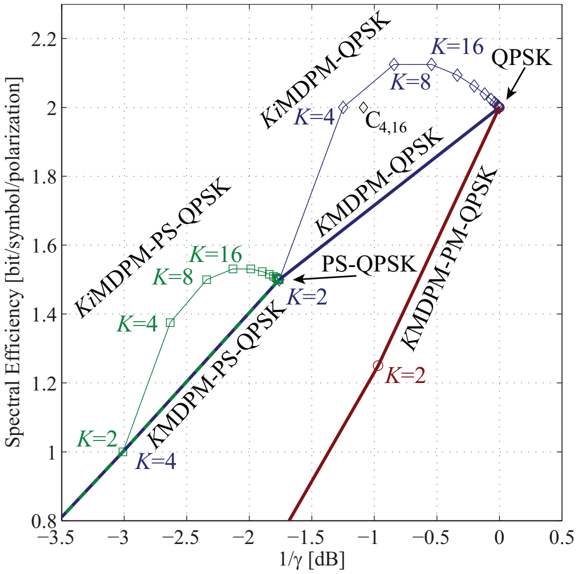


Fig. 4: SE as a function of $1/\gamma$ for the specific cases of KiMDPM-QPSK (blue diamonds and solid line) and KiMDPM-PS-QPSK (green squares and solid line). Also shown are KMDPM-QPSK (blue thick solid line), KMDPM-PM-QPSK (red circles and solid line) and KMDPM-PS-QPSK (green thick dashed line). Also indicated are QPSK and PS-QPSK as well as $C_{4,16}$ (black diamond).

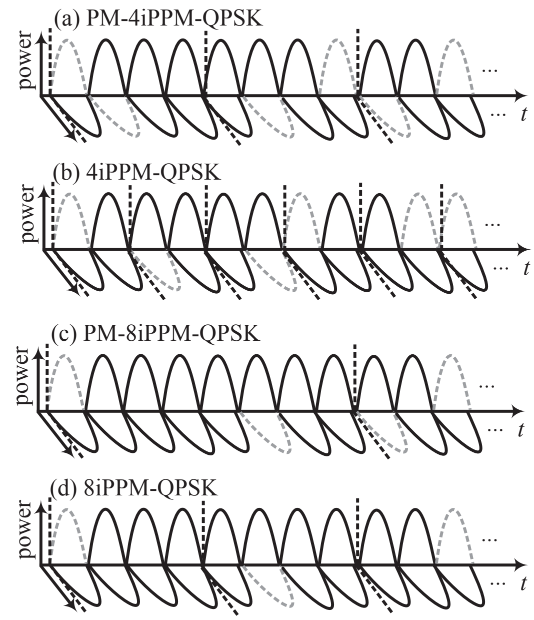


Fig. 5: Two possible PPM-implementations of 8iMDPM-QPSK (a) and (b) and of 4iMDPM-QPSK (c) and (d).

with two PPM-slots. Compared to 8-ary QAM (8-QAM) or 16-QAM, which have SE of 3 bit/symbol/polarization and 4 bit/symbol/polarization respectively, the gain in SE of 8iMDPM-QPSK is modest. However, it should be noted that going to 8-QAM or 16-QAM reduces the asymptotic power efficiency considerably to 3.01 dB and 3.98 dB worse than QPSK, respectively.

C. 4iMDPM-QPSK and 8iMDPM-QPSK in the low SNR regime

The asymptotic power efficiency gives the sensitivity gain at asymptotically high SNR. However, many coherent communication systems operate in a low SNR regime utilizing forward error correction coding to handle the higher error-probabilities. To investigate the performance of 4iMDPM-QPSK and 8iMDPM-QPSK in the low SNR regime we carried out Monte Carlo simulations in combinations with the nearest neighbor approximation of the union bound [30, p. 195] where the latter gives the symbol error probability as

$$\text{SER} \approx A_{\min} Q\left(\sqrt{\frac{d_{\min}^2 E_b}{N_0}}\right), \tag{13}$$

where $A_{\min}$ is the average number of symbol pairs at distance $d_{\min}$. The Monte Carlo simulations were performed in the low SNR regime with additive white Gaussian noise as the only impairment and minimum Euclidean distance decoding. 4iMDPM-QPSK has $A_{\min} = 18$ and 8iMDPM-QPSK $A_{\min} = 42$. The optimal bit-to-symbol mapping for these two constellations are not known. However, since $\log_2(M) < A_{\min}$, it is clear that the constellation cannot be Gray coded. We can find the lower bound of the bit error probability for 4iMDPM-QPSK and 8iMDPM-QPSK as

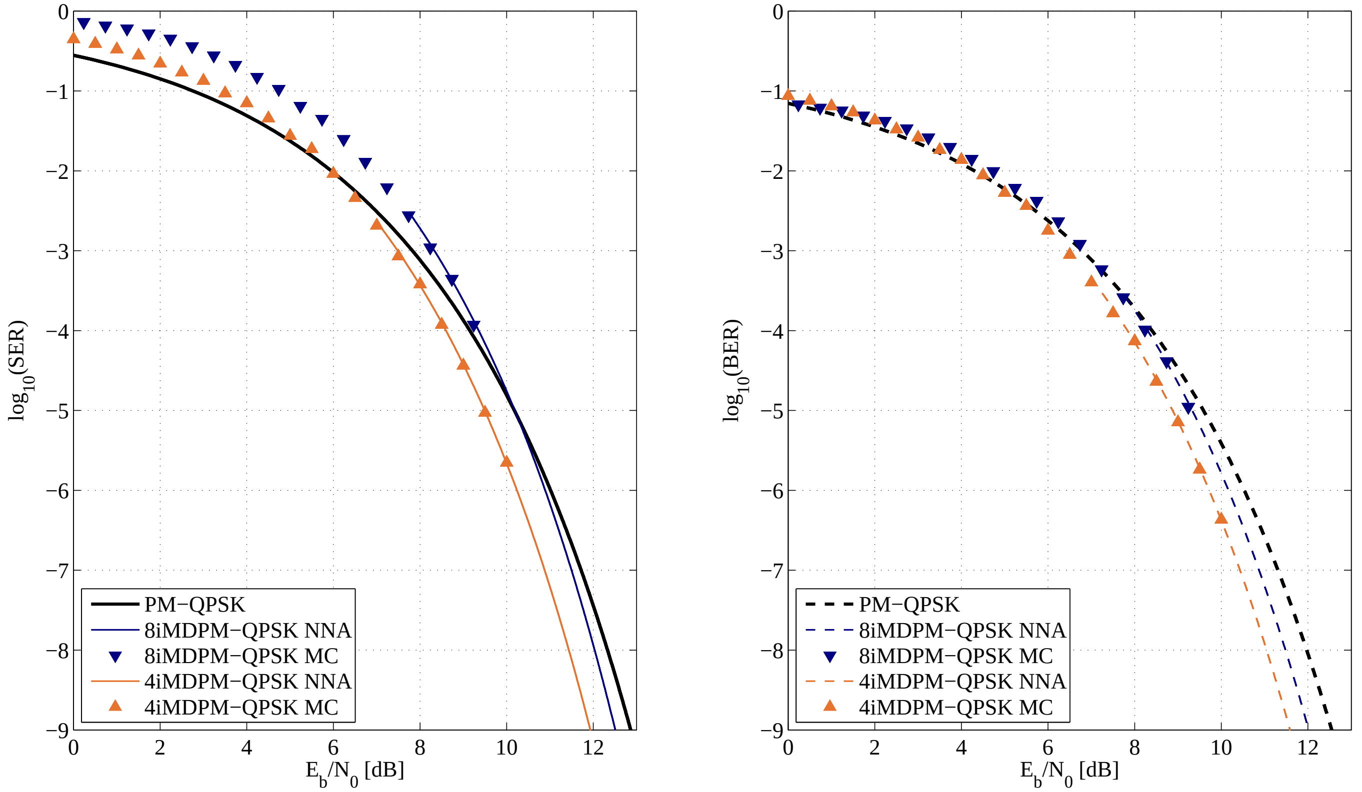
$$\text{BER} \geq \frac{1}{B_\text{F}}\left(\frac{B_\text{F}}{A_{\min}}1 + \frac{A_{\min} - B_\text{F}}{A_{\min}}2\right)\text{SER}, \tag{14}$$

where it is assumed that the bit-to-symbol mapping can be done such that for each symbol, an error to B_F number of neighboring symbols causes 1 bit error and an error to the remaining $(A_{\min} - B_\text{F})$ neighboring symbols results in 2 bit errors. For QPSK we use the exact expression for the symbol error probability [30, p. 183].

The SER for 4iMDPM-QPSK, 8iMDPM-QPSK and PM-QPSK is plotted in Fig. 6a. For $E_b/N_0 < 10$ dB the Monte Carlo simulations are shown and for higher values the nearest neighbor expression is shown as solid lines. The lower bound from (14) for 4iMDPM-QPSK and 8iMDPM-QPSK together with the exact expression for PM-QPSK are shown in Fig. 6b. As seen, for low E_b/N_0 PM-QPSK has the best performance of the three formats. However, at ≈ 4.5 dB and higher where the BER $= 8.3 \times 10^{-3}$ for both PM-QPSK and 4iMDPM-QPSK, the latter format has the best performance. 8iMDPM-QPSK has worse performance compared to PM-QPSK for E_b/N_0 lower than ≈ 7.3 dB where the BER $= 5.0 \times 10^{-4}$ for both formats. For asymptotically high E_b/N_0 the sensitivity difference of the format approaches the values expected from the asymptotic power efficiency.

VI. Conclusions

We have analyzed $\binom{K}{L}$-MDPM in combination with QPSK, PM-QPSK and PS-QPSK which is a generalization of PPM where different multidimensional slots can be achieved by PPM time slots, polarization states, frequencies, different



(a) (b)

Fig. 6: (a) SER for PM-QPSK (black solid lines), 4iMDPM-QPSK (orange upwards triangles and solid lines) and 8iMDPM-QPSK (blue downwards triangles and solid lines). Markers show results from Monte Carlo (MC) simulations and solid lines from the nearest neighbor approximation (NNA) in equation (13). (b) The BER performance of PM-QPSK (dashed black line) and the lower bound from equation (14) for 4iMDPM-QPSK (orange upwards triangles and dashed lines) and 8iMDPM-QPSK (blue downwards triangles and dashed lines). The BER values have been derived from the SER in (a).

modes of a multimode fiber or the cores of multicore fibers. We have given general expressions for the SE and the asymptotic power efficiency for $\binom{K}{L}$-MDPM in combination with any single-pulse IQ-modulation format.

$\binom{K}{L}$-MDPM-QPSK is shown to have the possibility to simultaneously increase both SE and power efficiency over conventional QPSK and $\binom{K}{L}$-MDPM-PS-QPSK over that of PS-QPSK. The performance limit in terms of SE and power efficiency is approximated for $\binom{K}{L}$-MDPM in combination with QPSK, PM-QPSK and PS-QPSK and we show that $\binom{K}{L}$-MDPM-QPSK has the theoretical possibility to achieve SE = 2.32 bit/symbol/polarization.

We also propose KiMDPM in combination with QPSK and PS-QPSK as interesting modulation formats with reasonable complexity in terms of frame lengths. These formats carry an integer number of bits per frame when K is a power of 2. For instance, we have shown that 4iPPM-QPSK has the same SE as QPSK but with an increased asymptotic power efficiency of 1.25 dB. By Monte Carlo simulations we found that the sensitivity of 4iMDPM-QPSK is better compared to QPSK for BER $\leq 8.3 \times 10^{-3}$.

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Authors biographies not included at authors request due to space constraints.