

Some Problems on Iterated Maps
and Kinetic equations
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Abstract

The thesis consists of one paper about iterated maps on the interval, which also was my licentiate thesis (1989), and two papers about the linear Boltzmann equation.

The paper on iterated maps considers one-parameter families of S-unimodal maps on the unit interval. These are studied with the attention on certain sequences of parameters, corresponding to superstable periodic orbits within the so called chaotic region of the parameter interval. Convergence rates for last superstable parameters for the 2^k -th iterates of the maps in the family are derived. The idea of renormalization is utilized in the proof.

The second paper, written in collaboration with Rolf Pettersson and Tommy Gustafsson, deals with the time dependent linear Boltzmann equation in a bounded and convex domain with diffuse boundary reflection. Growth rates for the solution and its moments are estimated, in L^1 for both soft and hard interactions, in L^p ($1 < p \leq \infty$) for moderately soft interactions. Central for the proof are geometric ideas about the domain and certain estimates of the collision term.

The third paper considers the stationary linear Boltzmann equation in a bounded, convex domain. At the boundary, a combination of given indata and diffuse reflection is assumed. The first result establishes the existence of a unique solution in weak L^1 , together with bounds for its higher moments. The main idea of proof is to estimate the mass for low velocities, using more easily proved estimates for high velocities. This is achieved by using the interplay between pre- and post-collisional velocities in the collisions. The second result handles the case with pure diffuse reflection at the boundary, proving the existence of a solution in weak L^1 sense. Here, the result depends upon weak* compactness for measure-valued limit solutions, appearing as the given indata vanishes.

Key words: Unimodal map, renormalization, superstable orbit, full family, linear Boltzmann equation, growth rates in L^p , stationary Boltzmann equation, existence and uniqueness, diffuse boundary reflection, convex boundary, measure-valued solution.

This thesis consists of an introduction and the following papers

- **Paper I:** *On Superstable Parameters for Iterated Maps of an Interval*
- **Paper II:** *On L^p Solutions to the Linear Boltzmann Equation with Diffuse Boundary Reflections*
- **Paper III:** *Existence of Solutions to the Stationary Linear Boltzmann Equation*