Abstract

For any quasi-projective irreducible variety $U \subset \mathbb{P}^n_K$, defined over a number field K, one may consider a counting function

$$N(U,B) = \#\{P \in U(K) \;:\; H_K(P) \leq B\},$$

where $H_K : \mathbb{P}^n_K(K) \to \mathbb{R}_{>0}$ is the standard multiplicative height relative to K. In this thesis we are concerned with the problem of finding good upper bounds for $N(U,B)$. The thesis consists of four papers.

In the first paper we study points on non-singular cubic surfaces $X \subset \mathbb{P}^3_K$ with three coplanar lines defined over K. We show that

$$N(U,B) = O_{X,\varepsilon}(B^{4/3+\varepsilon})$$

for every $\varepsilon > 0$, where $U \subset X$ is the complement of the lines on X. This result was proved by Heath-Brown in the case $K = \mathbb{Q}$.

In the second paper we generalise another result of Heath-Brown. Given an irreducible variety $X \subset \mathbb{P}^n_K$ we find a hypersurface $H \subset \mathbb{P}^n_K$ of degree bounded in terms of invariants of X and the number $B \geq 1$ such that X is not contained in H but the points of $X(K)$ of height at most B is contained in $X(K) \cap H(K)$. We use this result to find uniform estimates of $N(U,B)$, where $U \subset X$ is the complement of the lines on a non-singular surface $X \subset \mathbb{P}^4_{\mathbb{Q}}$.

In the third paper we prove a conjecture of Serre in some special cases. For finite dominant morphisms $f : X \to \mathbb{P}^2_{\mathbb{Q}}$ of degree more than two we obtain the bound

$$\#\{P \in f(X(\mathbb{Q})) \;:\; H_{\mathbb{Q}}(P) \leq B\} = O_{f,\varepsilon}(B^{2+\varepsilon}).$$

In the final paper we estimate the number of points of bounded height on curves in weighted projective planes.

Keywords: rational point, bounded height, counting function, Batyrev-Manin conjecture, thin set

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