Abstract

The applicability of the wedge splitting test method (WST), for determining fracture properties of fibre-reinforced concrete is discussed. Experimental results, using the WST-method, are compared with results from uniaxial tension tests (UTT) and three-point bending tests (3PBT). Furthermore, for the WST-method, two different specimen sizes have been investigated. Results from this investigation demonstrate the applicability of the WST-method and that the scatter of the test result is lower than for the 3PBT. Moreover, the investigated sizes of the WST-specimens do not seem to have a large influence on the fracture properties or on the specific number of fibres across the fracture surface.

1. Introduction

A fundamental aspect of the mechanical performance of fibre-reinforced concrete (FRC) is the tensile behaviour, which can be related to the debonding and frictional interface mechanisms that develop along the fibre-matrix interface. The tensile fracture toughness can be characterised by the tensile stress versus crack opening response (hereafter abbreviated as the $\sigma$-$w$ relationship), which also is associated with the so-called fictitious crack model originally suggested by Hillerborg; see Hillerborg et al. [1]. In the fictitious crack model the main parameters are: the tensile strength, the fracture energy, and the shape of the $\sigma$-$w$ curve. Since the fictitious crack model was introduced, considerable research has been carried out to determine the most appropriate test method for quantifying the $\sigma$-$w$ curve. For regular concrete, the shape of the $\sigma$-$w$ curve does not vary too much – see Stang [2] and Cornelissen et al. [3] – and for most practical applications it is usually sufficient to determine the fracture energy, $G_f$, and select an appropriate $\sigma$-$w$ relationship. For FRC, on the other hand, the shape of the $\sigma$-$w$ curves varies considerably depending on type and amount of fibres used, quality of the
concrete, etc. Furthermore, the complete fracture energy is of no interest since the stress-free crack opening, $w_c$, occurs at very large crack openings for most fibres, while for most concretes (without fibres) it is seldom larger than 0.3 mm. Hence, there is a need for simple and reliable test methods that can be used for determination of the $\sigma-w$ relationship and which companies can use in their quality control on a routine basis.

The three-point bending test on notched beams is probably the most widespread method for determining the fracture properties; see RILEM TC-50 FMC [4] for conventional concrete and RILEM TC 162-TDF [5] for steel fibre-reinforced concrete. Yet few methods have been developed for a direct determination of the $\sigma-w$ relationship, and it is only with a properly performed uniaxial tension test (UTT) that this can be determined directly. However, the UTT requires sophisticated testing equipment, and it has been shown that the test result is affected by machine specimen interaction; see e.g. Østergaard [6]. The wedge-split test method (WST), originally proposed by Linsbauer & Tschegg [7] and later developed by Brühlwiler & Wittmann [8], is an interesting test method since it does not require sophisticated test equipment; the test is stable and can be run on usual mechanical testing machines with a constant crosshead displacement. Furthermore, a standard cube specimen is used, but the test can also be performed on core-drilled samples. Researchers have used the WST-method extensively, and recently there has been an increased interest in the method. The method has proved to be successful in order to determine fracture properties of ordinary concrete, at early age and later (see Østergaard [6]), and for autoclaved aerated concrete (see Trunk et al. [9]). In addition, the method has been used to study fatigue crack growth in high-strength concrete (see Kim & Kim [10]), and fracture behaviour of polypropylene fibre-reinforced concrete (see Elser et al. [11]). However, not much information is available regarding the WST-method and testing of steel fibre-reinforced concrete.

The purpose of this paper is to demonstrate that the wedge splitting test method can be used for determination of fracture properties of steel fibre-reinforced concrete. Experimental results from three different test methods are compared, namely: the uniaxial tension test, the three-point bending test (3PBT) on notched beam, and the wedge splitting test. Furthermore, for the wedge splitting test, two different specimen sizes have been investigated. Inverse analyses have been used to obtain $\sigma-w$ relationships for the 3PBT and the WST.

2. Test program

2.1 Materials

In total, five different concrete mixes were used in this investigation. The varied parameters were (see Table 1): the volume fraction of fibres, $V_f$; the water binder ratio, $w/b$, of the concrete; and the fibre geometry (the length and diameter of the fibre). Hooked-end steel fibres (type Dramix™) were used: RC 65/60-BN (fibre-length 60 mm, diameter 0.9 mm) and RC 65/35-BN (fibre-length 35 mm, diameter 0.55 mm). The specimens for each test series were made from one batch (220 litres), which was mixed
in a planetary mixer. The concrete was compacted on a table vibrator for about two minutes at 50 Hz. After casting, the specimens were covered with plastic until the next day when they were de-moulded. The specimens were then stored in water, at a temperature of 21±2ºC, until the time of testing which usually took place at 28 days.

Table 1 – Concrete mix compositions.

<table>
<thead>
<tr>
<th>Constituents</th>
<th>Density [kg/m³]</th>
<th>Mix 1 [kg/m³]</th>
<th>Mix 2 [kg/m³]</th>
<th>Mix 3 [kg/m³]</th>
<th>Mix 4 [kg/m³]</th>
<th>Mix 5 [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEM II/A-LL 52.5 R</td>
<td>3100</td>
<td>260</td>
<td>260</td>
<td>360</td>
<td>360</td>
<td>360</td>
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<tr>
<td>Fly ash</td>
<td>2250</td>
<td>-</td>
<td>-</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>150</td>
<td>150</td>
<td>172</td>
<td>172</td>
<td>172</td>
</tr>
<tr>
<td>Eqv. w/b-ratio</td>
<td>-</td>
<td>0.58</td>
<td>0.58</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>SIKA ViscoCrete 34</td>
<td>1090</td>
<td>0.4</td>
<td>0.953</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Aggregates:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>00 – 04 mm</td>
<td>2535</td>
<td>822.6</td>
<td>794.1</td>
<td>745.4</td>
<td>745.4</td>
<td>744.3</td>
</tr>
<tr>
<td>04 – 08 mm</td>
<td>2642</td>
<td>345.8</td>
<td>333.8</td>
<td>313.3</td>
<td>312.9</td>
<td>312.9</td>
</tr>
<tr>
<td>08 – 16 mm</td>
<td>2637</td>
<td>700.7</td>
<td>676.5</td>
<td>635</td>
<td>634</td>
<td>634</td>
</tr>
<tr>
<td>Fibres, V_f</td>
<td>7800</td>
<td>0.5%</td>
<td>1.0%</td>
<td>0.5%</td>
<td>1.0%</td>
<td>1.0%</td>
</tr>
<tr>
<td>(Aspect ratio/Length)</td>
<td></td>
<td>(65/60)</td>
<td>(65/60)</td>
<td>(65/60)</td>
<td>(65/60)</td>
<td>(65/35)</td>
</tr>
<tr>
<td>Measured air content</td>
<td></td>
<td>4.5%</td>
<td>7.0%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

2.2 Uniaxial tension test
The uniaxial tension tests were conducted according to the recommendations of RILEM TC-162 TDF, see [12], in a 250 kN Instron 8502 machine. A special test setup developed at DTU were used, see Østergaard [6]. With the setup a rotationally stiff connection between the testing machine and the specimen is achieved, the machine stiffness has been measured to 251 kNm/rad by Østergaard [6]. For each mix, at least three specimens were tested.

2.3 Three-point bending test
The three-point bending tests were conducted according to the recommendations of RILEM TC-162 TDF, see [5], in a Instron 6025 universal testing machine with 100 kN capacity. The crack mouth opening displacement, CMOD, was measured with an Instron clip gauge, having gauge length 10 mm and maximum travel 5 mm, and the net load point deflection was measured by two LVDT transducers. The tests were performed with CMOD control. For each mix, five specimens were tested.

2.4 Wedge splitting test
For the WST-method, two different specimen sizes have been investigated, see Fig. 1. To avoid any wall effects, the starter notch was sawed; and to ensure a vertical crack propagation, a 25 mm deep guide notch was cut on each side of the specimens (see Fig. 1) to produce a specimen with an effective thickness of 100 mm. For each mix, six specimens were tested. The tests were performed in a Instron 6025 universal testing machine with 100 kN capacity, under CMOD control. The CMOD was measured with a
Instron clip gauge, gauge length 10 mm and maximum travel 5 mm. In the tests, a wedge angle of 15° was used and the roller bearings used were of the double-row-deep-groove type from SKF (designation 4203 ATN9).

3. Test results and discussion

3.1 Fibre content
Since the number of fibres crossing the crack has a great influence on the toughness and the \( \sigma-w \) relationship, the fibres crossing the fracture surface have been counted for all specimens. In Fig. 2(a) the number of fibres per cm\(^2\) is compared to the theoretical number of fibres for a random 3D-orientation for all the specimen types. In Fig. 2(b) the coefficient of variance (COV) for the total number of fibres can be seen.

The tendency is that the UTT-specimens have only about half of the expected number of fibres per area in the fracture surface; this naturally has a large influence on the test
The unsatisfactory fibre distribution was probably caused in the casting and vibration process, which may have oriented the fibres parallel to the notch. For the other specimens, the number of fibres is higher than the theoretical value, for the shorter fibres the difference is smaller. There is no clear trend indicating any difference between the small and the large WST specimens, i.e. there seems to be no direct influence of the specimen size on the relative number of fibres. Moreover, in most cases the beam specimens have fewer fibres (per cm$^2$) than the WST specimens, which probably is due to increased wall effects for the WST specimens.

### 3.2 Stress-crack opening ($\sigma$-$w$) relationships

In order to completely characterize the mechanical behaviour of fibre-reinforced concrete in tension, it is necessary to determine the $\sigma$-$w$ relationship, which is required for design based on non-linear fracture mechanics (see RILEM TC-162 TDF [13]). The $\sigma$-$w$ relationship can be determined either directly, by an uniaxial tension test (UTT), or indirectly by performing an inverse analysis. In an inverse analysis the output from a test (typically a load-CMOD curve) is used as an input for the analysis and the material parameters are then changed until the analysis result fits the test result. The inverse analysis was conducted using a Matlab program, developed at DTU by Østergaard [6], which is based on the cracked hinge model by Olesen [14]. In the cracked hinge model it was assumed that the $\sigma$-$w$ relationship could be approximated by a bi-linear function.

For the inverse analysis, each specimen was analysed separately and the average $\sigma$-$w$ relationships are presented in Figs. 3 to 5, together with the test results from the uniaxial tension tests. Since the fibre orientation was not representative for the UTT specimens, a direct comparison does not apply. However, the test result can be used to evaluate the tensile strength (referred to as the cracking stress), which can then be compared with the result from the other test methods. Furthermore, as the number of fibres per cm$^2$ for the UTT-specimens from Mix 4 approximately correspond to the theoretical value for Mix 3 this may serve as a comparison for the $\sigma$-$w$ relationships; thus, the result for the UTT in Fig. 4(b) are comparable with the results in Fig. 4(a). Table 2 shows the cracking stress and it can be seen that the crack stress is higher for the mixes with the lower w/c (mixes 3 to 5). For Mix 5 the UTT resulted in a lower cracking stress than for Mix 3 and Mix 4. Generally, the 3PB T gives the highest value for the tensile strength; between 5% and 25% higher than for the UTT. For the WST method it varies between 5% lower and 24% higher compared to the UTT. The variation between the two WST specimen sizes is small, less than 5%.

| Table 2 – Cracking stress (in MPa) determined with the different methods. |
|----------------|-----|-----|-----|-----|-----|
| Test method    | Mix 1 | Mix 2 | Mix 3 | Mix 4 | Mix 5 |
| UTT            | 2.24  | 1.99  | 3.25  | 3.05  | 2.85  |
| 3PB T          | 2.62  | 2.31  | 3.40  | 3.66  | 3.55  |
| WST-S          | 2.42  | 2.18  | 3.07  | 3.43  | 3.46  |
| WST-L          | 2.39  | 2.29  | 3.13  | 3.30  | 3.52  |
Fig. 3 – $\sigma$-w relationships from UTT and inverse analyses: (a) for Mix 1; and (b) for Mix 2.

Fig. 4 – $\sigma$-w relationships from UTT and inverse analyses: (a) for Mix 3; and (b) for Mix 4.

Fig. 5 – (a) $\sigma$-w relationships, from UTT and inverse analyses, for Mix 5. (b) Diffused cracking in beam specimen.

For mixes 1 and 2, the inverse analyses give quite good agreement between the 3PBT and the WST, and between the two WST specimen sizes (see Fig. 3). For the other mixes
the discrepancy is larger. Figures 3 and 4 clearly demonstrates the effect of an increased fibre addition (Mix 1 compared to Mix 2, respectively Mix 3 compared to Mix 4), i.e. the stress after cracking (the residual stress) increases. The effect of the fibre length (the short fibre in Mix 5) can be seen in Fig. 5(a), the slope of the second part of the stress-crack opening relationship is steeper indicating that it drops to zero faster. For design purposes the stress (residual stress) at a specific crack opening, $w$, may be of interest. For $w = 0.5$ mm the difference in residual stress is between 7% and 33%; while it for $w = 2.0$ mm is between 8% and 29%. The small WST-specimens generally give lower values than the large specimens. Moreover, for the mixes with 1% fibres the 3PBT gives consistently higher stresses, while the difference is small for the mixes with 0.5% fibres. This is thought to be caused by multiple cracking in some of the 3PBT specimens, see Fig. 5(b), which is not possible to capture in the inverse analysis. For the WST-specimens, the guide notch prevents this type of cracking and the crack is forced to propagate vertically. Some of the differences in the determined $\sigma-w$ relationships can most likely be related to the variation in fibre content (see Fig. 2). Figure 6 show a comparison between the test results and the inverse analyses, as can be seen the general agreement is good. However, for the specimens with 1% fibres it is difficult to get as good agreement with the inverse analysis. It is likely that a better agreement could be achieved by using a tri-linear function to describe the $\sigma-w$ relationship.

![Figure 6](https://via.placeholder.com/150)

*Fig. 6 – Comparison of test results and inverse analyses (thick lines represent the test result): (a) For the small WST-specimens; and (b) For the 3PBT.*

### 3.3 Variation of the test results for the 3PBT and the WST-method

By evaluating the energy dissipated during fracture, $G_i$, at different CMODs the scatter in the test results can be compared without any errors introduced when interpreting the test data, as is the case when performing an inverse analysis. The work of fracture, $W_F$, can be calculated from the area under the load deflection diagram (or the area under the splitting force CMOD diagram). The dissipated energy, $G_i$, is the work of fracture divided by the ligament area, $A_{lig}$, which is the projected area on a plane parallel to the ideal crack direction. However, it should be pointed out that the evaluated fracture energy is not suitable as a material parameter for design. Furthermore, it is not possible to directly compare the dissipated energy between the two test methods, as the measured
CMOD corresponds to different crack openings at the tip of the notch depending on the geometry of the specimen. On the other hand, the dissipated energy may be used as a qualitative indicator when comparing different FRC compositions. The results from the 3PBT are presented in Fig. 7. Due to a malfunctioning transducer, the deflection for specimens of Mix 2 has been calculated on the basis of a relationship between CMOD and deflection according to RILEM TC-162 TDF; see [5]. In Fig. 7(a) it can be seen that the energy consumed during fracture increases with improved matrix quality and increased fibre volume; the length of the fibre does not seem to have any influence. Fig. 7(b) shows the coefficient of variance (COV) which varies between 2% and 35%, having the lowest value for Mix 5 with the short fibre. The corresponding results from the WST are presented in Figs. 8 and 9. In Fig. 8 it can be seen that the results from the WST show the same tendency as for the 3PBT. Moreover, there is no great difference between the two specimen sizes; it is only for mixes 4 and 5 that the small specimens show a different result compared to the large ones. However, this is probably related to the high number of fibres in the small specimens for Mix 4 (1.5 times the theoretical, see Fig. 2). Figure 9 shows the coefficient of variance (COV), which varies between 6% and 20% for the small specimens and between 4% and 21% for the large specimens.

Fig. 7 – (a) Dissipated energy, $G_f$, versus CMOD. (b) COV for $G_f$ at different CMODs.

Fig. 8 – Dissipated energy, $G_f$, versus CMOD: (a) for small specimens; and (b) for large specimen.
4. Conclusions

The results from this study suggest that the wedge splitting test method could very well be used as a fracture test for steel fibre-reinforced concrete. The scatter is, on the whole, smaller for the WST than for the 3PBT (see Figs. 7(b) and 9). For the two WST specimen sizes, there is no apparent difference neither in the specific number of fibres (per unit area of fracture surface) nor in the fracture properties. Furthermore, the inverse analyses indicate no systematic differences in the determined parameters between the 3PBT and the two WST specimens (see Fig. 3 to 5(b)). It was not possible to compare the \( \sigma-w \) relationships from the inverse analysis with the results from the UTT because of a poor fibre distribution in the UTT-specimens. The major contributing factor to the scatter in the test results is believed to be related to the variation in the number of fibres across the fracture plane; the coefficient of variance is up to 30%. Regarding the scatter in the test results, similar values have been reported by others, e.g.: Kooiman [15], COV from 10% to 30% for tests on notched beams in three-point bending; and Lee & Barr [16], COV in the order of 20% for tests on notched beams in three-point bending. The relatively shallow notch depth for the 3PBT causes diffuse cracking (multiple cracks) for the test series with a fibre volume of 1%. For the WST-method, the deep notch and the use of a guide notch successfully prevent any diffused cracking and seems to be a good solution for mixes with high fibre volumes (1% in this study), but might not be necessary for mixes with lower fibre volumes.

In conclusion, the benefits of the wedge splitting test method are that it does not require any sophisticated testing equipment and that less concrete is needed to perform a test since the specimen is smaller. With inverse analysis it is possible to determine a stress-crack opening relationship but it is believed that a better agreement could be achieved by using a tri-linear function representing the \( \sigma-w \) relationship. Regarding the size of the specimen, it is recommended that for fibres with a length less than 60 mm a 200 mm specimen should be used while a 150 mm specimen could be used for shorter fibres.
References


