

Power measurement techniques for non-sinusoidal conditions

The significance of harmonics for the measurement of power and other AC quantities

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Abstract

The increased application of power electronics and other non-linear loads makes it necessary to re-evaluate the measuring techniques used in the power system, and the measuring problems these loads cause. An instrument utilising digital sampling techniques has been built and evaluated at the Swedish National Testing and Research Institute (SP). The Digital Sampling Watt Meter (DSWM) is based on standard laboratory equipment: digital multimeters, voltage dividers, shunt resistors and a PC. The DSWM is versatile and can be used for calibrations of many quantities. The most basic ones are the (total) active power and the amplitude and phase angle of individual harmonics of non-sinusoidal voltages and currents.

The DSWM was first verified for sinusoidal signals. At 120 V and 5 A and power factor one, the DSWM has an estimated uncertainty (2σ) of 60 ppm at 50 Hz and 600 ppm at 20 kHz. The wattmeter has also participated in three international comparisons with satisfactory results. The most important additional feature, the input distortion, has been verified to be less than 800 ppm for all harmonics and lower than 100 ppm for most harmonics.

Some AC quantities, as the reactive power, are not properly defined for non-sinusoidal situations. Efforts are made in this work to understand and explain the problems of extending the reactive power definition to cover non-sinusoidal conditions. The main conclusion is that reactive power is used to obtain information on more than one property of the power transmission mechanism, e. g. phase angle, transmission efficiency and line voltage drop. No single power definition can alone provide information on all these properties in a non-sinusoidal situation. Moreover, instrument designs may not comply with any of the extended definitions and these meters exhibit extra errors due to this non-compliance for non-sinusoidal conditions.

Some conclusions on future demands on energy meters can be drawn, based on the error analysis of these meters and an analysis on how the responsibility for the harmonic currents and voltages in the power system can be determined and shared. One conclusion is that it is not possible to make a precise determination of the responsibility for harmonics based on any power measurement alone.

Key words: sampling wattmeter, harmonics, reactive, power, non-sinusoidal

Preface

The Swedish National Testing and Research Institute at Borås, is appointed the National Laboratory for electrical quantities. Two of these quantities are electrical power and electrical energy. During the last decade the need for measurement and calibration at other frequencies than 50 Hz, and under non-sinusoidal conditions, has increased rapidly. To meet this demand a project was started at SP in 1989 aimed at creating a calibration facility for these conditions. The project had two major parts; the realisation of a reference wattmeter and the study of measuring problems under non-sinusoidal conditions.

In 1994, Elforsk joined the project. The project became a part of a post-graduate study on non-sinusoidal related problems of power measurements. The design and uncertainty analysis of the reference wattmeter, and the analysis of power measuring problems due to distortion of the current and voltage, are part of this study. In 1996, a licentiate degree report was published, presenting some material on this subject[1].

Although this doctoral thesis is a monograph, most of its material is based on papers presented in various conferences or published in transactions. The third chapter is based on two papers, "Power analysers, measurement uncertainty and calibration", and "Power measurement uncertainties in a nonsinusoidal power system" [2, 3], both contributions to conferences. Some editorial changes have been made in these papers as well as in the paper "Preferred measurement and metering methods"[4], on which chapter 4 is based. Chapter 6 and the first part of chapter 7 is based on a journal paper[5] describing the basic function and performance of the reference wattmeter. The second and third part of chapter 7 are two journal papers "A watt meter for the audio frequency range"[6], and "Verification of a calibration system for power quality instruments"[7]. Further, the part of chapter 8 which is named "flicker calibration" is based on a conference paper[8]. In Appendix 1 the result of a bilateral power measuring comparison between SP, Sweden (the DSWM) and Physikalisch-Technische Bundesanstalt, Germany is given. Finally, in Appendix B a journal paper describing a comparison between the DSWM and a dedicated system for power measurement at power factor zero[9] which is also built at SP is presented.

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1 Introduction

Accurate measurement of power and other AC quantities is extremely important at all levels of the electrical power system, and is of value for both for power distributors and power consumers. The design of equipment used so far is based on the assumption that the voltage sources are sinusoidal and the loads are linear, so that the resulting current is also sinusoidal. As demands on accuracy have increased and non-linear loads have become more common, this approximation is often no longer valid. Efforts are made to investigate the influence of non-linear loads on measurements and new instrumentation is developed to cope with non-sinusoidal conditions on the power delivery network. One part of this effort, which is described in this work, is the development and verification of a digital sampling watt meter, for measurements at non-sinusoidal conditions at standard laboratory accuracy level. The work is also aimed at a better understanding of the measuring problems due to non-sinusoidal conditions.

1.1 Power measurements

The verification of a power measuring system, and making precise measurements in non-sinusoidal situations, requires an understanding of the error mechanisms. Therefore effort have been made to understand and explain these mechanisms, both for standard measuring equipment and for instrumentation dedicated to measurements at non-sinusoidal conditions.

A serious problem with measurements at non-sinusoidal conditions is that quite a few of the measured quantities do have more than one possible definition. Even worse, standard measuring equipment such as ampere meters or reactive power meters may use measuring algorithms which do not comply with any valid definition in the non-sinusoidal situation, and may therefore exhibit large errors. These problems are investigated, starting with the definition of the quantities related to power measurement.

1.2 The digital sampling approach

Measurement and calculation of electrical power based on digital sampling of voltage and current have been discussed for some time, e.g. by Clark and Stockton 1982[1]. The advantage of such a system is that it is comparatively easy to calibrate, and that digital multiplication is precise and does not cause linearity problems that could occur in power meters based on analogue multiplication. Furthermore, it enables accurate power measurements in non-sinusoidal situations and makes it possible to calculate the power of the different harmonics.

Not so long ago, the problems of digital sampling were substantial due to the speed and precision required for A-D conversion and speed requirements for the calculation. However, during the last decades sampling techniques and Analogue-to-Digital-Converter (ADC) techniques have advanced rapidly. The precision and the sampling rate of ADCs have now increased to such a degree that even some of the most accurate voltmeters use this technique. Moreover, these instruments are often equipped with standard computer interfaces which solves much of the practical data acquisition problem. The electrical isolation which is needed in this kind of system between the voltage channel, the current channel and the computer is also taken care of by the instruments. Thus, instruments like these are very well suited for accurate and fast digitising, as required for power metering.

Because of this development and the increasing calculating capability of Personal Computers, SP decided to start a project aimed at building a digital sampling watt meter based on commercially available equipment. In this project two Hewlett-Packard 3458A multimeters have been used in conjunction with a PC, thus saving time and money spent on building up the system. Also, the building blocks are already well proven, and verification can be concentrated on checking the properties that are especially important for this application.

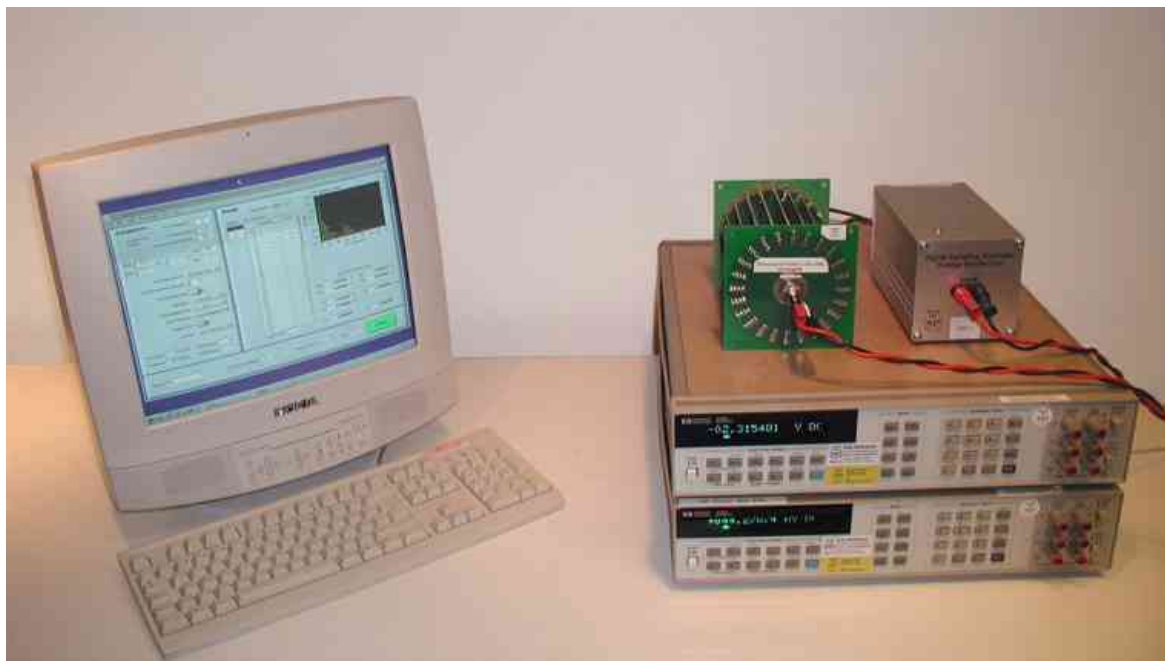


Figure 1. The Digital Sampling Wattmeter (upgraded version)

When a proper set of samples of voltage and current is collected by the multimeters and transferred to the computer a range of parameters can be calculated with different calculation methods. This project, however, has concentrated on two different methods for calculating the power; the discrete Fourier transform (DFT) and the discrete integration method (DI).

1.3 References

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2 Power theory for nonsinusoidal conditions

2.1 Mathematical notations

This survey of proposed definitions of power in case of nonsinusoidal signals covers many papers by authors from different parts of the world and of different times. Therefore, the denominations of various quantities and the mathematical notations differ much from each other. This paper partly uses the notations of the authors but most often more uniform notations are used in order to enhance the readability. Instantaneous values and functions of time are denoted by lower-case letters while rms-values and mean values are denoted by upper-case letters. No distinction is made between scalars and complex numbers.

2.2 General definitions of power

For the general case the active (mean) electrical power is

$$P = \frac{1}{T} \int_T u \cdot i \, dt \quad (1)$$

where T is the time of interest or the observation time, or for periodic signals, the period time.

In an ideal power system, the voltages and currents are (purely) sinusoidal with a frequency of 50 Hz or 60 Hz. However, non-ideal characteristics of real-life power system components and non-ideal loads will cause distortion. Currents and voltages will be nonsinusoidal and will contain harmonics. In most cases, the currents and the voltages will still be (approximately) periodic with a fundamental frequency of 50 Hz or 60 Hz. If the voltage and current both are periodic functions of time with the same period T, the voltage and current can both be expressed as a Fourier series and the power can be defined as

$$P = \sum_n U_n I_n \cos \Phi_n \quad (2)$$

where n is an order for which both the voltage and current harmonics exist, and Φ_n is the phase angle difference between u_n and i_n . Further, for the special case where both the voltage and the current are sinusoidal the active power can be expressed by the familiar equation

$$P = UI \cos \Phi \quad (3)$$

These definitions are based on the physical phenomena of electrical power and energy; this power or energy can be transferred to for instance thermal power and energy, and can be measured as thermal or mechanical properties. Therefore, there are no controversies about Equation (1) to Equation (3), neither in the general case nor in the special cases of sinusoidal signals and nonsinusoidal, periodic signals.

Apparent and reactive powers, on the other hand, are not based on a single, well defined, physical phenomenon as the active power is. They are conventionally defined quantities that are useful in sinusoidal or near-sinusoidal situations. For sinusoidal voltages and currents reactive power is defined as

$$Q = UI \sin \Phi = \sqrt{S^2 - P^2} \quad (4)$$

and apparent power is defined as

$$S = UI = \sqrt{P^2 + Q^2} \quad (5)$$

At nonsinusoidal conditions there is, more or less, a general agreement on using

$$S = UI \quad (6)$$

as the apparent power definition, where U and I are the root-mean-square values of the voltage and current. For a periodic nonsinusoidal signal, the apparent power will then be equal to

$$S = \sqrt{\sum_n U_n^2 \sum_n I_n^2} \quad (7)$$

There are quite a few proposals on how to extend the definition of reactive power to cover nonsinusoidal situations. The definition that is most widely spread, and is also approved of by ANSI/IEEE as standard [1], has been given by Budeanu [2]:

$$Q = \sum_n U_n I_n \sin \Phi_n \quad (8)$$

It is usual to denote this expression of reactive power by Q_B . The power triangle is generally not satisfied by this definition so another quantity D

must be defined to determine the relation between apparent power, reactive power and active power:

$$D^2 = S^2 - P^2 - Q^2 \quad (9)$$

However, the definition according to Budeanu is not considered useful for any practical applications[3, 4]. Furthermore, as stated earlier, reactive power is not a quantity defined by any single physical phenomenon but a mathematically defined quantity that has some very useful characteristics and physical interpretations at *sinusoidal* conditions.

The most important characteristics of reactive power at *sinusoidal* conditions are as follows, compare with[5]:

1. The reactive power is equal to the magnitude (peak value) of the purely bidirectionally pulsating instantaneous power through a point in a power system.
2. The reactive power is proportional to (the mean of) the difference between the electric energy stored in inductors and the energy stored in capacitors.
3. If the reactive power is reduced to zero, the power factor will be unity.
4. The reactive power completes the power triangle, $Q^2 + P^2 = S^2$.
5. The sum of all reactive powers in a node of a power system is zero.
6. Reactive power can be expressed by the terms U, I and $\sin\Phi$.
7. Reactive power can be positive or negative (the sign specifies whether a load is of inductive or capacitive type).
8. Reactive power can be reduced to zero by inserting inductive or capacitive components.
9. The voltage drop of power system transmission lines is approximately proportional to the reactive power.

All these characteristics are tightly related in the sinusoidal case and depend directly on the phase angle between the voltage and the current, being the only degree of freedom given for a specified voltage and power. All the above characteristics are valid for the expression $Q = UI\sin\Phi$, in sinusoidal situations. For nonsinusoidal signals, the Budeanu definition does not always agree with the characteristics 3,4, 8.

It can be further shown that the apparent power as defined by $U_{\text{rms}} \cdot I_{\text{rms}}$, see Equation (6) and Equation (7), contains cross-products between current and voltage harmonics of different orders, while active power does not. Therefore, the characteristic 4 requires that the reactive power does contain cross-products, which contradicts characteristic 6. Consequently, a

nonsinusoidal definition of reactive power that agrees with both characteristics 4 and 6 is impossible.

Due to the problems of extending the reactive power concept as described above, there is no obvious principle on which to base a general definition of reactive power. From the debate on how to define reactive power it is evident that the different approaches at least partly are due to the application the suggested reactive power is aimed at. In the near future, we either have to try to find and agree upon the most practical of the theoretically acceptable definitions or we will have to live with different concepts aimed at different applications.

2.3 Orthogonality

The term orthogonality is a central issue for most of the reasoning behind the suggested definitions. In this context, it means that if two functions, e.g. two currents i_a and i_b , have a common period time T and are orthogonal, then

$$\frac{1}{T} \int_T i_a i_b dt = 0 \quad (10)$$

Further, if $i=i_a+i_b$ then the square of the rms-value of i will be

$$I^2 = \frac{1}{T} \int_T (i_a + i_b)^2 dt = \frac{1}{T} \int_T i_a^2 dt + \frac{1}{T} \int_T 2i_a i_b dt + \frac{1}{T} \int_T i_b^2 dt = I_a^2 + I_b^2 \quad (11)$$

That is, the rms-value of a sum of two orthogonal currents or voltages contains no cross-products and the squared total rms-value is equal to the sum of the squared rms-values. Thus, when a division in orthogonal current components is made, it is easy to define apparent power components from these currents by multiplying them with the squared rms-voltage:

$$S^2 = U^2 (I_a^2 + I_b^2) = S_a^2 + S_b^2 \quad (12)$$

The Fourier series, and each sine and cosine term in such series, are orthogonal by nature, which is one reason why they are so useful. There are, however, many other possibilities of dividing the voltage and current into orthogonal components, as shown by the different authors in the following.

In a concept as described above the apparent power is derived from a number of rms-values. A quantity defined by the multiplication of rms-values does

not have any sign. As will be seen in this survey, only a few power components will be defined as signed quantities.

2.4 Reactive power definitions

2.4.1 Definition proposed by C Budeanu

The active power in a nonsinusoidal, but periodic environment is defined as

$$P = \sum_n P_n = \sum_n U_n I_n \cos \Phi_n \quad (13)$$

where U_n and I_n are the rms-values of the voltage and current harmonics of order n , and Φ_n is the phase angle difference between them. Therefore, it makes sense to define reactive power by

$$Q = \sum_n Q_n = \sum_n U_n I_n \sin \Phi_n \quad (14)$$

which is proposed by Budeanu [2]. However, this equation does not comply with the power triangle equation $S^2 = P^2 + Q^2$, since the apparent power is defined by the rms-values of the voltage and current according to Equation (6) and then

$$S^2 = \sum_n U_n^2 \cdot \sum_n I_n^2 \geq \left(\sum_n U_n I_n \cos \Phi \right)^2 + \left(\sum_n U_n I_n \sin \Phi \right)^2 \quad (15)$$

Therefore a quantity named distortion power, D , was added by Budeanu according to

$$D^2 = S^2 - P^2 - Q^2 \quad (16)$$

which yields the equation

$$S^2 = P^2 + Q^2 + D^2 \quad (17)$$

The distortion power mainly consists of cross-products of voltage and current harmonics of different orders and will be reduced to zero if the harmonics are reduced to zero, i.e. at sinusoidal conditions.

To distinguish the reactive power Q according to this definition from reactive power according to other definitions it is often denoted by the index B as Q_B . The distortion power is also sometimes referred to as D_B ,

The main advantage of this definition is that it complies with characteristic 5 given above, i.e. that the sum of all reactive powers into a point in a power system is zero. The main disadvantages are that the definition does not comply with characteristic 3 and 8, i.e. it is not sure that the power factor will be unity if the reactive power by this definition is reduced to zero and that the reactive power can be totally compensated by inserting inductive or capacitive components. Further, designing an analogue meter that measures Q_B is virtually impossible since it requires a filter that utilises a phase angle displacement of 90 degrees for all frequencies and at the same time has an amplification factor of unity for all frequencies.

2.4.2 Definition according to S Fryze

The reactive power definition proposed by [6] is based on a time domain analysis. The current is divided into two parts. The first part, i_a , is a current of the same wave-shape and phase angle as the voltage, and has an amplitude such that $I_a \cdot U$ is equal to the active power. The second part of the current is just a residual term named i_r . The two currents will then be determined by the equations

$$i_a = \frac{P}{U^2} \cdot u \quad (18)$$

and

$$i_r = i - i_a \quad (19)$$

The reason for this division is that the current i_a is the current of a purely resistive load that, for the same voltage, would develop the same power as the load measured on. That is, if i_r can be compensated, the source will see a purely resistive load and the power factor will be equal to unity. It can easily be shown that i_a and i_r are orthogonal and then the rms-values can be determined by

$$I^2 = I_a^2 + I_r^2 \quad (20)$$

In fact, (18) gives the only possible amplitude of i_a if it should be orthogonal to the residual term i_r and have the same waveshape as u . The apparent power can then be obtained as the product of the rms current and the rms voltage :

$$S^2 = U^2 I^2 = U^2 (I_a^2 + I_r^2) = P^2 + Q^2 \quad (21)$$

Fryze uses P_b instead of Q in his reactive power definition. In other literature reactive power according his definition is often denoted by Q_F and named "fictitious power".

The advantage of this definition is that it does not introduce any fourth power component. It also complies with characteristics 3, i.e. when the reactive power is reduced to zero the power factor will be unity. Designing an analogue meter measuring reactive power by this definition is also relatively simple and an analogue design that decomposes the current according to this definition has been presented [7]. The main disadvantage is that the defined reactive power does not comply with characteristic 5, i.e. it is not sure that the sum of the reactive powers in a node of a power system is equal to zero, and Q_F can therefore not generally be used in power flow calculations. Further, although the power factor will be unity if the reactive power Q_F is decreased to zero, this cannot generally be accomplished with just capacitors or inductors, and Q_F does not provide the information on how to compensate by passive components. For sinusoidal signals, this definition of reactive power is of course equal to the conventional reactive power.

On the other hand, the residual current i_r is a very good input for an active compensator and is frequently used in for this purpose[8]. However, when the current i_r is compensated, the voltage drop along the source impedance will change. Then the voltage across the load will also change its magnitude and waveshape slightly, which means that the load is *not* fully compensated. Full compensation will need a feedback loop or knowledge of the source impedance. This compensation concept shares this drawback with most other concepts, because the only way to avoid this problem is to measure the load impedance and create a suitable "negative impedance".

2.4.3 Definition proposed by N L Kusters and W J M Moore

This definition of reactive power[9], is again a time domain definition. It expands the definition according to Fryze by a further split of the residual current into two orthogonal components. How this split is made depends on whether the load is predominantly a capacitive or an inductive load. The three currents achieved by this split are then named active current, inductive or capacitive reactive current and the residual reactive current, which results in an apparent power sum:

$$S^2 = P^2 + Q^2 = P^2 + Q_c^2 + Q_{cr}^2 = P^2 + Q_l^2 + Q_{lr}^2 \quad (22)$$

The active current is (as by Fryze) defined by

$$i_p = \frac{P}{U^2} \cdot u = \frac{\frac{1}{T} \int u idt}{U^2} \cdot u \quad (23)$$

and the capacitive reactive current is similarly defined as

$$i_{qc} = u_{der} \cdot \frac{\frac{1}{T} \int u_{der} idt}{U_{der}^2} \quad (24)$$

and the inductive reactive current as

$$i_{ql} = u_{int} \cdot \frac{\frac{1}{T} \int u_{int} idt}{U_{int}^2} \quad (25)$$

where u_{der} and u_{int} are the periodic part of the derivative and integral of the (instantaneous) voltage, respectively, and U_{der} and U_{int} the corresponding rms-values. Both these currents can then be shown orthogonal to the residual current in the same way as i_p , although this is not explicitly stated in the paper. Also, and not explicitly shown in the paper, if a capacitor or inductor is installed that draws $-i_{ql}$ or i_{qr} , the residual current will be minimised as far as possible with one parallel inductor or capacitor. Because of the orthogonality P , Q_c and Q_l can now be determined by the equations:

$$P = UI_p \quad (26)$$

$$Q_c = UI_{qc} = \frac{U}{U_{der}} \cdot \frac{1}{T} \int u_{der} idt \quad (27)$$

$$Q_l = UI_{ql} = \frac{U}{U_{int}} \cdot \frac{1}{T} \int u_{int} idt \quad (28)$$

The reactive powers Q_c and Q_l will then be signed quantities that can be compensated by capacitors or inductors if they are negative. That is, Q_c

follows the sign convention of the reactive power in sinusoidal situations while Q_l will have an opposite sign. The rest terms will be determined by

$$i_{qcr} = i - i_p - i_{qc}, \quad i_{qlr} = i - i_p - i_{ql} \quad (29)$$

and

$$Q_{cr} = \sqrt{S^2 - P^2 - Q_c^2}, \quad Q_{lr} = \sqrt{S^2 - P^2 - Q_l^2} \quad (30)$$

Q_l and Q_c are not equal to the reactive power according to Budeanu, but for sinusoidal signals they will be equal to Q (apart from the sign of Q_l). The rest term will be zero for sinusoidal signals. According to C. H. Page [10] it is possible to make an optimal compensation with a shunt inductor and capacitor by using the expression

$$i_q = a \cdot u_{der} + b \cdot u_{int} + i_r \quad (31)$$

where a and b are constants which are optimised with the least square method.

In most practical cases the result of the optimising will be that one constant, a or b , declines to zero. Then the same current split as by the Kusters-Moore definition will be achieved.

Compared with the Fryze decomposition, the definition by Kusters and Moore has the advantage that it identifies the part of the current that can be compensated with a shunt capacitor or inductor, and that characteristic 7 and 8 are fulfilled for Q_c and Q_l . The value of the reactive compensating component can easily be calculated. This is, however, only valid if the source impedance is negligible, i.e. the voltage change when the compensation is applied must be negligible.

2.4.4 Definition proposed by W Shepherd and P Zakikhani

This definition of reactive power [11] is based on a frequency domain analysis. A nonlinear load connected to an ideal source will result in current harmonics that do not have any corresponding voltage harmonics. In order to handle such nonlinear loads, the current and voltage harmonics are divided into "common" and "non-common" harmonics. For the common harmonic of order n both U_n and I_n are nonzero, while for the noncommon harmonic of order n only one of U_n and I_n is nonzero. Then the apparent power can be expressed as

$$S^2 = \left(\sum_{n \in N} U_n^2 + \sum_{m \in M} U_m^2 \right) \cdot \left(\sum_{n \in N} I_n^2 + \sum_{p \in P} I_p^2 \right) \quad (32)$$

where N is the set of all common harmonic orders and M and P contain all noncommon, nonzero, harmonic orders of the voltage and the current respectively (that is, M is the set of orders for which the voltage harmonics are nonzero while the corresponding current harmonics, due to nonlinearity, are zero). The active power is still of course defined by

$$P = \sum_n U_n I_n \cos \Phi_n \quad (33)$$

Shepherd then suggests a split of apparent power according to

$$S_R^2 = \sum_{n \in N} U_n^2 \sum_{n \in N} I_n^2 \cos^2 \Phi_n \quad (34)$$

$$S_X^2 = \sum_{n \in N} U_n^2 \sum_{n \in N} I_n^2 \sin^2 \Phi_n \quad (35)$$

and a rest term

$$S_D^2 = \sum_{n \in N} U_n^2 \sum_{p \in P} I_p^2 + \sum_{m \in M} U_m^2 \cdot \left(\sum_{n \in N} I_n^2 + \sum_{p \in P} I_p^2 \right) \quad (36)$$

which yields

$$S^2 = S_R^2 + S_X^2 + S_D^2 \quad (37)$$

As all apparent power components are defined by rms-values, none of them have a sign. Shepherd et al consider their definition to be closer to the physical reality, especially for compensation of reactive power for a maximum power factor (with passive components). This is only achieved if S_X^2 is minimised, according to Shepherd et al, since S_D^2 only contains noncommon harmonics that cannot be compensated by passive components. This is only approximately true, since the values of the harmonic current I_n will be affected by a compensation and then the last part of (36) will be somewhat affected by a compensation.

One major disadvantage of this scheme is that S_R^2 is not equal to P^2 , even if it does contain P^2 , which follows directly if the Cauchy-Schwarz inequality is applied on S_R^2 and P^2 . If the voltage (or the current) is purely sinusoidal then $S_R=UI_1\cos\Phi_1=P$, $S_X=Q_B=UI\sin\Phi$ and $S_D=D$. For linear systems $S_D^2=0$ since there are no noncommon harmonics. However, in practical measurement situations the source impedance will always be nonzero and therefore truly noncommon harmonic orders will never exist and S_D will always be zero for practical linear and nonlinear systems. So, even if the voltage is close to sinusoidal, a highly distorted current will result in a large contribution to S_R and S_X , mainly from the cross-products between current harmonics and the fundamental voltage harmonic. That is, S_R may be far from equal to P , and S_X may be far from equal to Q_B even if the voltage is close to sinusoidal. Further, in a measurement situation there would always be input noise and input harmonic distortion so even if a measurement showed some zero amplitude voltage harmonics it would just be a matter of a limited resolution of that instrument, and another instrument with higher resolution would show a non-zero value. Therefore a division into common and noncommon harmonics should only be seen as a tool to handle a theoretical case and it would be dangerous to implement S_D according to this definition in an instrument.

These quantities are defined in the frequency domain, and can only be measured by FFT-equipment. At the time of this proposal (1972) this was a major disadvantage; it may still be to some extent for cost-sensitive instrumentation like revenue meters.

2.4.5 Definition proposed by Sharon

This definition of reactive power [12] is also based on a frequency domain analysis. It is a slight but important development of the above suggestion of power definition. It starts with the same division into common and noncommon harmonic components.

$$S^2 = \left(\sum_{n \in N} U_n^2 + \sum_{m \in M} U_m^2 \right) \cdot \left(\sum_{n \in N} I_n^2 + \sum_{p \in P} I_p^2 \right) \quad (38)$$

where N is the set of all common harmonic orders and M and P contain all noncommon, nonzero, harmonic orders of the voltage and the current respectively (that is, M is the set of orders for which the voltage harmonics are nonzero while the corresponding current harmonics, due to nonlinearity, are zero). The active power is defined by

$$P = \sum_n U_n I_n \cos \Phi_n \quad (39)$$

Sharon then suggests an apparent power component according to

$$S_Q^2 = U_{rms}^2 \sum_{n \in N} I_n^2 \sin^2 \Phi_n \quad (40)$$

and a rest term

$$S_C^2 = \sum_{m \in M} U_m^2 \cdot \sum_{n \in N} I_n^2 \cos \phi_n + U_{rms}^2 \cdot \sum_{p \in P} I_p^2 + \frac{1}{2} \sum_{\beta \in N} \sum_{\gamma \in N} (U_\beta I_\gamma l \cos \phi_\gamma - U_\gamma I_\beta l \cos \phi_\beta) \quad (41)$$

which yields

$$S^2 = P^2 + S_Q^2 + S_C^2 \quad (42)$$

The author also gives a formula for the optimum parallel compensating capacitor or inductor as

$$C_{opt} = \frac{\frac{1}{\omega} \sum_{n \in N} U_n n I_n \sin \phi_n}{\sum_{n \in N \cup M} n^2 U_n^2} \quad (43)$$

$$L_{opt} = \frac{\frac{1}{\omega} \sum_{n \in N \cup M} \frac{1}{n^2} U_n^2}{\sum_{n \in N} U_n \frac{1}{n} I_n \sin \phi_n} \quad (44)$$

Sharon states that compensation by a parallel inductor or capacitor only affects S_Q . This is true if the source impedance is negligible; P is not affected because the load voltage is unchanged and $I_n \cos \Phi_n$ is constant, S_C is not affected since U_{rms} , U_p , I_p and $I_n \cos \Phi_n$ remain constant.

There are two important differences between this definition and the definition according to Shepherd and Zakikhani. The first is that in the definition by Sharon, P is one of the power components and not separately defined. The second is less obvious and is that S_Q is derived by a multiplication by the total rms voltage and not only the rms voltage of the common harmonic orders. This may seem a minor change but it removes some of the ambiguity due to the difficulty of sorting the noncommon orders from the common in a

measurement situation. The active power is of course not affected by such a sorting. S_Q is not affected by any voltage harmonic sorting problem because all voltage harmonics is already used for the calculation of it. However, a large current harmonic with a very small corresponding voltage harmonic can still cause a rather large uncertainty due to whether it is considered common or noncommon. While this kind of current will affect the measurement, it will not affect the compensation, since it has a negligible effect on C_{opt} and L_{opt} according to (43) and (44).

2.4.6 Definition proposed by L S Czarnecki

This is a frequency domain definition[3]. The author comments on the earlier suggested definitions. He shows that a single shunt reactance can be quite an ineffective compensator even at moderate levels of harmonics (~10%) if the source impedance is not negligible. This would make the definition according to Kusters and Moore less useful. He also points out the weakness of the definition according to Shepherd and Zakikhani, that S_R is not equal to P . He suggests a frequency domain definition, for linear loads, which makes use of the time-domain-defined active current i_a according to Fryze.

The instantaneous value of a periodic voltage can be expressed as a complex Fourier series:

$$u = \sqrt{2} \operatorname{Re} \sum_n U_n e^{jn\omega_1 t} \quad (45)$$

where ω_1 is the fundamental angular frequency, and n is a harmonic order for which U_n is nonzero. In a power system this voltage may be connected to a linear load with the admittance

$$Y_n = G_n + jB_n \quad (46)$$

that is, both G and B can be dependent on the frequency. The current will then be

$$i = \sqrt{2} \operatorname{Re} \sum_n U_n (G_n + jB_n) e^{jn\omega_1 t} \quad (47)$$

Assuming that all power is absorbed by a (frequency invariant) conductance G_e , as in the power definition according to Fryze, this conductance can be determined by

$$G_e = \frac{P}{U^2} \quad (48)$$

When exposed to the voltage U , the current through this conductance will be equal to the active current i_a according to [6]. The residual current can then be calculated by

$$i - i_a = \sqrt{2} \operatorname{Re} \sum_n (G_n - G_e + jB_n) U_n e^{jn\omega_1 t} \quad (49)$$

This current can further be divided into

$$i_s = \sqrt{2} \operatorname{Re} \sum_n (G_n - G_e) U_n e^{jn\omega_1 t} \quad (50)$$

which is called scatter current by Czarnecki and

$$i_r = \sqrt{2} \operatorname{Re} \sum_n jB_n U_n e^{jn\omega_1 t} \quad (51)$$

which is denoted reactive current. All these currents are orthogonal and therefore the rms-values of the currents can be expressed by

$$I^2 = I_a^2 + I_s^2 + I_r^2 = \frac{P^2}{U^2} + \sum_n (G_n - G_e)^2 U_n^2 + \sum_n B_n^2 U_n^2 \quad (52)$$

If this expression is multiplied by U^2 , one will get the (squared) apparent power as

$$S^2 = P^2 + D_s^2 + Q_r^2 \quad (53)$$

The reactive power by this definition, Q_r , can according to Czarnecki be reduced to zero by a one-port shunt reactance, e.g. a multiple pole filter designed such that the filter susceptance B_{cn} is $-B_n$ for all frequencies. Such a compensation would theoretically not be dependent of the load impedance as many other compensating concepts are, but it demands a measurement of B_n . The scatter power, D_s , can only be reduced to zero by active compensation or filtering, since some scattered current harmonics will result in positive active power and some in negative.

Further, this definition of Q_r will be equal to S_x according to [11] for linear loads and both D_s and Q_r are exactly equal to S_C and S_Q of Sharon. The particular with this definition is therefore not the resulting power components

but the concept of dealing with susceptances rather than with voltages, currents and powers. The equality of Q_r and S_Q can be shown by

$$Q_r^2 = U^2 I_r^2 = U^2 \sum_n B_n^2 U_n^2 = U^2 \sum_n I_n^2 \sin^2 \phi_n = S_Q^2 \quad (54)$$

Analogue to the definitions [11, 12], current harmonics with a very low corresponding voltage harmonic will result in a large value and a large uncertainty of B_n due to the uncertainty of the phase angle determination.

2.4.7 Suggestion of an IEEE working group on harmonics

The IEEE working group on "nonsinusoidal situations: Effects on meter performance and definition of power" has suggested "practical definitions for powers", [13]. The main difference between this definition and other definitions is that it separates the fundamental quantities P_1 and Q_1 from the rest of the apparent power components. Focus is also rather put on revenue metering than on compensation. The starting point is a separation of the fundamental voltage and current harmonics from the total rms values as

$$V^2 = V_1^2 + V_H^2 = V_1^2 + \sum_{h \neq 1} V_h^2 \quad (55)$$

and

$$I^2 = I_1^2 + I_H^2 = I_1^2 + \sum_{h \neq 1} I_h^2 \quad (56)$$

By multiplication, the apparent power is formed as

$$S^2 = (VI)^2 = (V_1 I_1)^2 + (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2 \quad (57)$$

where

$$(V_1 I_1)^2 = S_1^2 = P_1^2 + Q_1^2 = (V_1 I_1 \cos \phi_1)^2 + (V_1 I_1 \sin \phi_1)^2 \quad (58)$$

which is called fundamental apparent power, fundamental active power and fundamental reactive power, respectively. The three remaining parts of (57) is called nonfundamental apparent power and is then defined as

$$S_N^2 = (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2 = S^2 - S_1^2 \quad (59)$$

Also, the nonactive power

$$N = \sqrt{S^2 - P^2} \quad (60)$$

is proposed, which is a deviation from the above splitting strategy. Further, the harmonic apparent power is defined and further divided as

$$S_H^2 = (V_H I_H)^2 = P_H^2 + N_H^2 \quad (61)$$

where P_H is the total harmonic power and N_H the total harmonic nonactive power. It is stated that a direction of flow may be assigned to P_1 and Q_1 but may not be assigned to any of the three parts of the nonfundamental apparent power defined by (59). It is further shown that the normalised harmonic apparent power S_H/S_1 is approximately equal to the $\text{THD}_U \cdot \text{THD}_I$ and that the normalised nonfundamental apparent power S_N/S_1 is approximately equal to THD_I .

The definition is further extended to cover unbalanced three-phase systems, but this is beyond the scope of this thesis. Most of the aspects of implementing this power definition system for measurements under balanced condition are extensively discussed in chapter 4.

2.5 National and international standards

The International Electrotechnical Commission (IEC) standard IEC 27-1 and the Swedish national standard SEN 01 01 01 both assign the symbol Q for the general case of reactive power. The SEN further states "for sinusoidal signals $Q=UI\sin\Phi$ ". This might suggest that the term reactive power is valid also in nonsinusoidal situations, but no definition is given for that case. Apparent power, on the other hand is defined in both standards by the expression

$$S^2 = P^2 + Q^2 \quad (62)$$

without any limitation to the sinusoidal case, which implies that

$$Q^2 = S^2 - P^2 \quad (63)$$

However, as long as the apparent power is not also defined by $U_{\text{rms}} \cdot I_{\text{rms}}$, or explicit instructions on how to treat reactive power in nonsinusoidal situations are given, the ambiguity will remain.

A working group, TC 25/ WG 7 was appointed by the IEC around 1975, and although no official document is left from this group reference [14] are frequently given. The agenda of this group was to investigate how to treat "reactive power and distortion power", and this was mainly caused by the use of the quantity "distortion power", D , suggested by Budeanu. The conclusion,

however, was that the distortion power could be included in the (total) reactive power as $Q^2 = Q_B^2 + D^2$, and therefore no new quantity had to be defined in the standards.

The American IEEE standard dictionary of electrical and electronic terms assigns the letter U to apparent power and defines a number of vector quantities. It uses S for "phasor power" which is defined by

$$S = P + jQ_B \quad (64)$$

and uses F for "fictitious power" which is defined by

$$F = jQ_B + kD \quad (65)$$

where j and k are unit vectors along perpendicular axes. This fictitious power is equal to the reactive power according to Fryze. Further, nonactive power N is defined by

$$N = iP + kD. \quad (66)$$

The apparent power is denoted U and is then given by

$$U^2 = P^2 + F^2 = P^2 + Q^2 + D^2 \quad (67)$$

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3 Preferred measurement and metering methods

Digital power and energy meters of today and tomorrow, capable of frequency analysis, offer new possibilities. Old metering concepts can be changed and refined, and ambiguities due to the inability of old metering equipment can be overcome if the measured quantities are defined for the prevailing nonsinusoidal conditions. A discussion is needed to determine the preferred measuring methods and measured quantities, to separate them from the large number of quantities and methods possible. This chapter discusses the advantages and disadvantages of a metering that separates the fundamental power components from the other parts of the apparent power.

3.1 Introduction

Theoretical work on power components under nonsinusoidal conditions has been extensive during the last two decades. Specialised instruments, designed for power quality measurement have become quite common. The cost of meters capable of performing the spectral analysis needed for measurements of (most of) the suggested new quantities has however prevented them from being commonly used for revenue purposes, or for use in other permanent installations. In the near future, the sampling technique will be common in meters for stationary installation. The practical use of nonsinusoidal power theories must therefore be discussed.

In power theory work, the main focus has been on which concept is theoretically correct or at least the best choice from a theoretically point of view[1]. Besides, special power concepts for special situations or applications have been proposed[2]. Most of the suggested definitions of power components start by dividing the current into orthogonal components, where one component is i_a , the active current responsible for the active power if the load had been purely resistive[3].

$$i = i_a + i_n = \frac{P}{U^2} u + i_n \quad (68)$$

The rest term in can be considered as a generalised reactive or, preferably, non-active current. Since i_a and i_n are orthogonal, the apparent power can be calculated from the rms currents I_a and I_n and the rms voltage, and divided into active and non-active power:

$$S^2 = U^2 (I_a^2 + I_n^2) = P^2 + N^2 \quad (69)$$

The non-active current component can be further divided into other orthogonal current components. When these (squared) components are multiplied by the (squared) voltage as in (69) this leads to a long list of multifrequency power components that can be added in the same manner as the sinusoidal active and reactive power to form the (squared)apparent power.

From a power system engineering point of view it is reasonable and often appropriate to divide the voltage and the current and hence the power into the necessary fundamental component and the unwanted harmonic components, thus separating the fundamental power components from the rest. This has been recognised by some authors, and a power definition based on these thoughts has been suggested [4].

The suggested definition starts by dividing the rms voltage and the rms current into fundamental and harmonic parts:

$$U^2 = U_1^2 + U_H^2 = U_1^2 + \sum_{n>1} U_n^2 \quad (70)$$

$$I^2 = I_1^2 + I_H^2 = I_1^2 + \sum_{n>1} I_n^2 \quad (71)$$

The voltage and current are then multiplied to form the apparent power, and the power components (for balanced circuits) are suggested as:

$$S^2 = S_1^2 + S_N^2 = P_1^2 + Q_1^2 + P_H^2 + N_H^2 + (U_1 I_H)^2 + (U_H I_1)^2 \quad (72)$$

where

$$S_1^2 = P_1^2 + Q_1^2 = (U_1 I_1 \cos \phi_1)^2 + (U_1 I_1 \sin \phi_1)^2 \quad (73)$$

$$P_H = \sum_{n>1} U_n I_n \cos \phi_n \quad (74)$$

$$N_H^2 = S_H^2 - P_H^2 = \sum_{n>1} U_n^2 \cdot \sum_{n>1} I_n^2 - P_H^2 \quad (75)$$

$$(U_1 I_H)^2 = U_1^2 \sum_{n>1} I_n^2, \quad (U_H I_1)^2 = I_1^2 \sum_{n>1} U_n^2 \quad (76)$$

N_H is a harmonic rest term and (76) describes the cross-products between fundamental and harmonic parts. The difference between the apparent power division described above and other suggested power definitions may seem

subtle but is quite substantial. The most obvious difference, is that the (total) active power no longer is one of the power components. P_1 is one of the components, and P_H is one, and the active power is $P=P_1+P_H$, but $P^2 \neq P_1^2 + P_H^2$ and can not be put into (72). However, as a supplement to the apparent power split above the nonactive power is given as:

$$N = \sqrt{S^2 - P^2} \quad (77)$$

The proposed concept of separating fundamental components from harmonic ones raises some basic questions: 1) Should evaluation of the effects of harmonics on the power system be included in the reactive power measurements? 2) Should both the active and reactive power metering be restricted to the fundamental harmonic only? 3) If the effect of harmonics on the power system should be measured separately, what measuring methods and which quantities should be preferred? 4) In case of harmonics generated by a load: does the measured quantity reflect the costs and problems for the power distributor and for the neighbours? 5) Will the result of such a measurement be fair, considering the power quality responsibility shared between the consumer(s) and the distributor of electric power? 6) Can reasonable accuracy be achieved with the measuring methods available today or in the near future?

The analysis given here is concentrated on measurement for billing purposes and on responsibility sharing of power quality issues especially regarding harmonics, and it is based on the situation in the Nordic countries, more particularly Sweden.

Most phenomena such as losses and equipment ratings can be related to current and voltages separately rather than by any power component. Therefore, the following distinctions are made:

- a) The fundamental voltage is one of the basic control parameters in the power distribution system and for each voltage level it should ideally be constant in both the long and the short time frame and equal in all parts of the system.
- b) The harmonic voltage is an unwanted effect of nonlinear components in the system, mainly nonlinear loads that draw harmonic current, which in its turn causes voltage harmonics. The voltage harmonics cause unusable or even harmful harmonic currents in for instance motors and power-factor correction capacitors. They might increase (but will most often decrease) the peak voltage causing an increased stress on insulation.
- c) The fundamental current is often subdivided in the in-phase current, which constitutes the main contribution to active power, and the quadrature current

that causes reactive power in the classical sense. The quadrature current causes losses, but the main concern is often that if it is not compensated locally it causes voltage drops making it difficult to maintain the voltage level equal throughout the power system.

d) The harmonic current is also an unwanted effect of nonlinear components, mainly loads. Harmonic current causes losses and it causes voltage harmonics when acting on system impedance. Further, it increases the risk for resonance phenomena at harmonic frequencies. The voltage drop (of the total rms voltage) due to harmonic current is, however, negligible.

3.2 Load and system characteristics

For most purposes related to metering, the load characteristics at the fundamental harmonic can be defined by its resistance and reactance. These can be calculated from the measured fundamental active and reactive power and the voltage, given that the impedance of the power system is much lower than the load impedance. Many loads are self-regulating or controlled to be constant-power loads rather than constant-impedance loads, at least in a longer time frame. In that case, they are preferably defined by their (fundamental) active and reactive power consumption.

Loads that are nonlinear (in the sub-period time frame) are the major source of harmonic current. These loads can, in most cases, be modelled as multi-frequency current sources in parallel with frequency dependent impedance see Figure 2a. The system impedance, on the other hand, is predominantly inductive and generally increases with frequency. However, shunt capacitors may locally cause the system impedance to decrease with frequency and sometimes inductors and capacitors form resonant circuits.

Different sources of harmonics are generally not in phase with each other and their load currents will interact in a rather unpredictable way, see Figure 3b and Figure 2a. The harmonic current interaction and resonance phenomena as demonstrated in Figure 2b, cause the power system harmonic impedance to vary quite a lot both with time and place. The impedance, or more exactly $\text{Re}(U_n/I_n)$, seen by a harmonic source can even be negative for some frequencies in some points of the power system. In such a point even a rather large current harmonic source could still produce zero or negative harmonic power. Therefore, as opposed to the case of fundamental harmonic voltage and current, harmonic power measurement alone (or measurement of the harmonic voltage, current and phase angles), is not sufficient to characterise either the load or the system impedance or the load and system harmonic sources. At best, the measurement will only show the effect of the harmonic current on the power system in each moment. A complete assessment of the characteristics of

the nonlinear load involves one or more known changes of the state of the system, for example a step change of the voltage[5, 6]. However, even then we will only find the values of a linearised model of the load, not the true nonlinear behaviour of the load.

Harmonic voltage affects loads in different ways. Resistive types of loads can most often benefit from the harmonic power resulting from the harmonic voltage. Inductive motors are retarded and heated by the symmetrical part of negative sequence harmonics like the 5th and the 11th. Electronic loads like adjustable speed drives are not directly affected but the control circuits can be upset if the voltage distortion is too high.

3.3 The responsibility problem

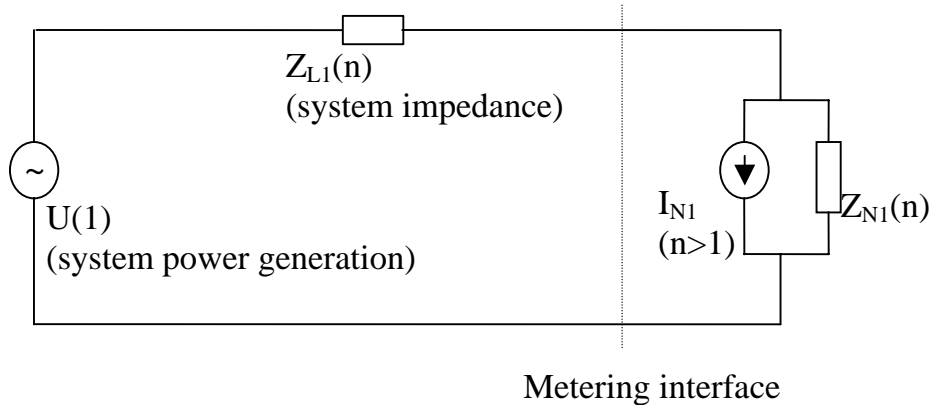
A simple and much used manner of dividing responsibility and costs of power quality problems, including reactive power, is to state that voltage is the responsibility of the power distributor and current is the responsibility of the consumer[7-9]. This works well as long as the voltage and current characteristics do not interact too much, so that cause and effect can be separated.

The magnitude of the fundamental quadrature current depends almost entirely on the load characteristics, which can be measured as reactive power in its classical sense. Therefore, the responsibility for quadrature fundamental current can be rather easily divided amongst the consumers of power. There is one situation where a quadrature fundamental current billing based on classical reactive metering can be unfair; if there are two nearby customers, one with an inductive load and one with a capacitive load. Then one or both might have to pay a penalty for the reactive power that in reality cancels and causes no problem for the distributor. However, large capacitive loads are scarce so this can be handled as an exceptional situation.

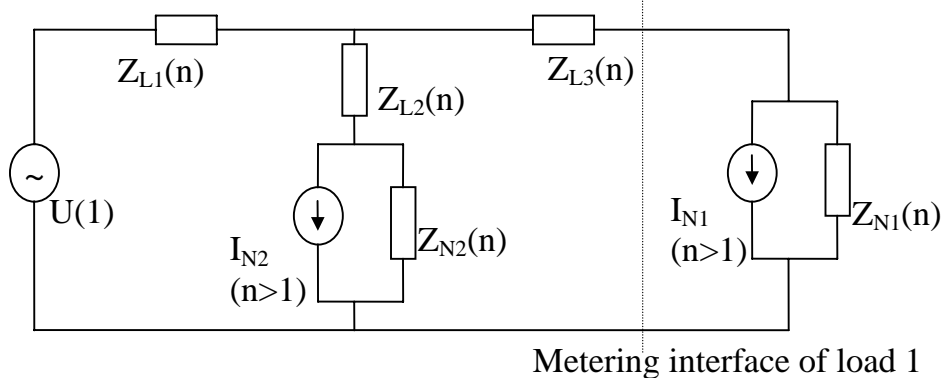
Harmonic current is mainly a distribution problem and not so much a transmission problem. Most problems are of local nature since load interaction cause current cancellation, reducing the relative harmonic current level from clusters of loads. Figure 2 and Figure 3 demonstrates the metering problems due to the interactions between loads, especially between load currents. From Figure 3 it is evident that the relative sizes of the impedance of the lines and/or the impedance of transformers, Z_{L1} , Z_{L2} and Z_{L3} , will very much determine the current level in the metering point of a load. The distribution transformers often represent a major part of the impedance seen by the harmonic current sources, and will to some extent act as a harmonic current barrier. Therefore, consumers who share a distribution transformer generally will experience more current harmonic interaction and therefore higher harmonic current

amplitudes in their metering points than consumers not sharing a transformer will. The current cancellation as well as the interaction is most evident for harmonics of order >10 , where the natural spread of phase angles are high.

In Figure 3a, a nearby load generates current of an opposite sign. The result in the metering point is a high harmonic current level, but since this current is driven by both loads and does not pass any significant impedance, the measured harmonic voltage and harmonic (active) power are negligible. Further, the harmonic current through the system impedance will also be negligible, so even if the harmonic current is high in the metering point, this current will not cause problems for either the distributor or other loads. However, if the load currents would have been of the same sign, their currents would instead have approximately been added and been forced through the system impedance, leading to a higher active harmonic power generation in both the studied load and the nearby load. The harmonic voltage and the (active) harmonic power would then be high, about twice as high as if the metered load had been the only nonlinear load in the neighbourhood. The active harmonic power generated of a nonlinear load is therefore highly dependent on both the system impedance and neighbouring nonlinear loads.

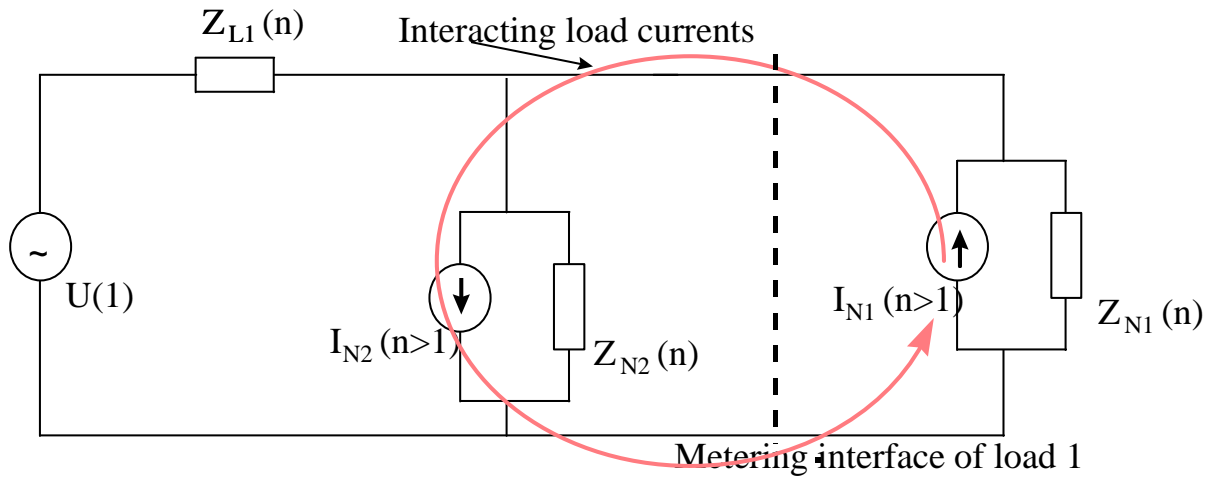


a) simple load-source model for the case of one large non-linear load

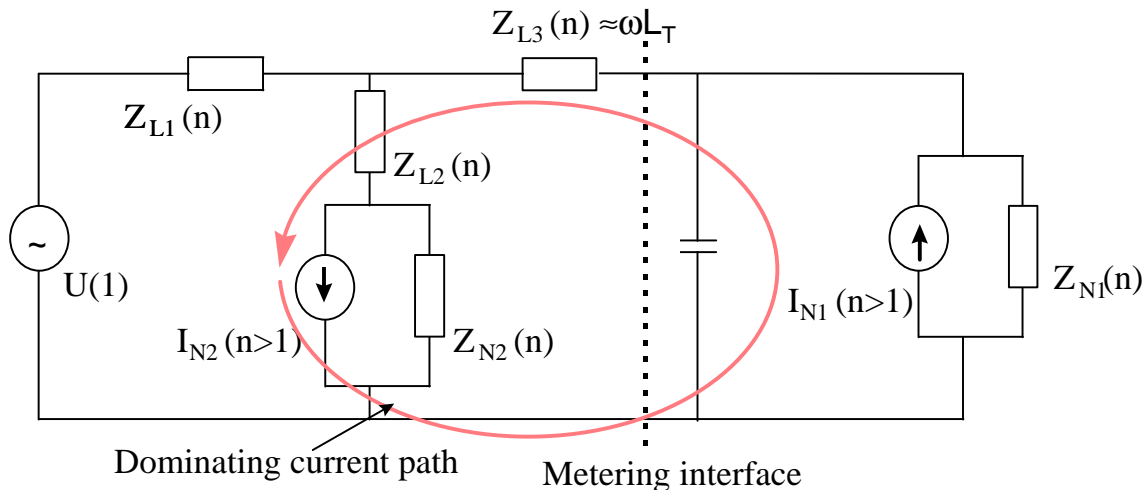


b) Load-source model, load 1 is studied, all other loads are lumped together into one extra load, and the system impedance divided into three parts. Normal case: $Z_{L3}(n) > Z_{L2}(n) > Z_{L1}(n)$.

Figure 2. Load-source models describing the metering of a non-linear load.



a) Load-source model, loads electrically close, $Z_{L3}(n), Z_{L2}(n) \ll Z_{L1}(n)$, resulting in high harmonic current and (in this case) low harmonic voltage and low harmonic active power.



b) Load-source model, at transformer-compensator resonance conditions.

Figure 3. General load-system circuit diagrams for nonsinusoidal conditions showing the problems of the harmonic metering of a load for responsibility sharing purposes.

Resonance risks, as demonstrated by Figure 3b, will further complicate the matter. The compensation of the fundamental reactive power by capacitors generally reduces most of the harmonic voltage levels in the system since it offers a low-impedance path for the current harmonics. However, in case of resonance the harmonic current and/or the harmonic voltage will increase instead, at least locally. A power consumer with a large part of motor load and compensating shunt capacitors can find himself part of a resonance circuit for some harmonic currents emanating from another consumer. He will then get a high harmonic current level and a high harmonic voltage level even though he does not have any significant nonlinear load. Generally, (active) harmonic power meters will indicate a positive harmonic power in this situation. Much

of the (harmonic) losses will occur in the transformer nearest the load, therefore the size of the measured harmonic power will depend on which side of the transformer the metering is placed. A further metering problem of the same kind is that the (symmetrical part of the) third harmonic and other harmonics of zero-sequence character are usually trapped by the distribution transformer. The measured harmonic current will therefore depend on which side of the transformer the measurement is made.

The conclusion is that harmonic current meter will give an indication of whether the load generates harmonics or not, but is unreliable. Active harmonic power metering gives an indication of whether the harmonic current of the load is predominantly caused by the load or not. Neither of them gives a complete picture of what is going on in the load or in the rest of the power system

The responsibility problem for harmonics can be treated in two ways. The first is to state that the current responsibility of the consumer is always more strict than the voltage responsibility of the distributor. Then a resonance or a heavy interaction is to be regarded as part of consumer responsibility, and the harmonic current can always be billed for and/or set limits to (this can also be inverted so if the harmonic voltage level is above the limit it is strictly the distributor's problem and cannot be billed for and has to be solved by the distributor). This is the simplest solution but as demonstrated above it will sometimes be unfair. The second possibility is to use a measuring method that distinguishes between imported and exported current problems and, better, makes a full load and system characterisation. An accurate measurement of (active) power harmonics will distinguish between import and export but cannot alone give a full characterisation[10].

3.4 Costs

The harmonic current and the fundamental quadrature current can be (geometrically) added into one current component, approximately equal to the non-active current component as long as $P_H \ll P_1$. There are, basically, two consequences of non-active currents implying problems and costs, increased losses[11, 12] and voltage drops[13]. The voltage drop is mainly due to the quadrature fundamental current while the extra losses are due to the whole non-active current. Counter-measures against voltage drop will cause infrastructure costs, and losses due to harmonics will cause both direct costs and infrastructure costs because of increased equipment ratings.

The cost of losses is approximately proportional to the square of the currents. If the non-active current is 30% of the active current then its part of the total

losses is less than 10%. If the total transmission and distribution losses are 10% of the active power for high power factors, and the energy price is 2 cents/kWh, the extra cost due to non-active current is $0.1 \cdot 0.1 \cdot 0.02 = 2 \cdot 10^{-4}$ \$ per kWh transferred, or 1% of the active energy cost (infrastructure costs disregarded). By similar calculations, a trend line for the cost of losses can be derived as in Figure 4. The cost is calculated relative the non-active current in percent of the active current, which is equal to current THD if the nonactive current is purely harmonic current, and equal to $\tan\Phi_1$ if the nonactive current is purely a fundamental quadrature current.

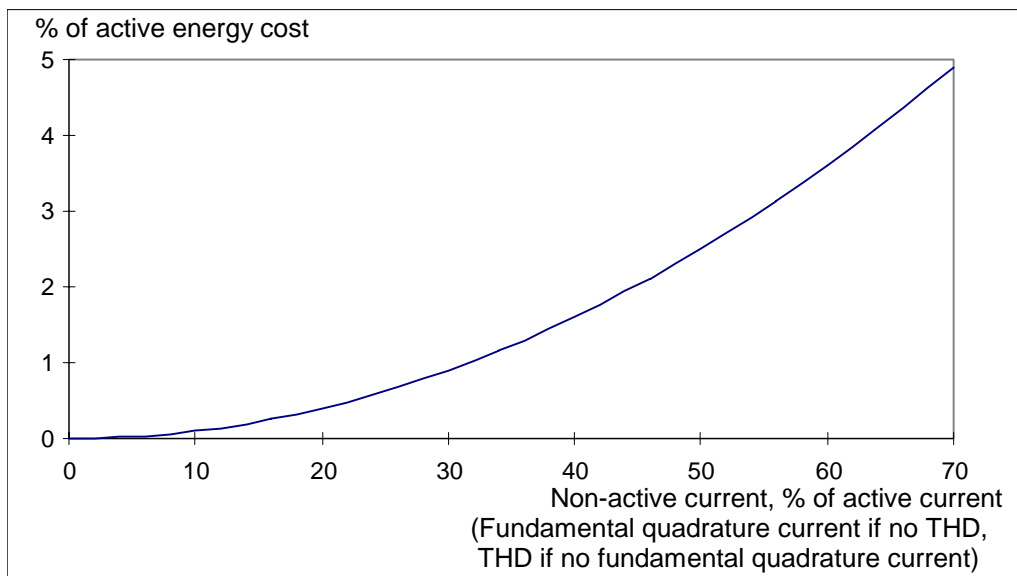


Figure 4. Trend line for the relative cost of losses for non-active current.

The installation of shunt capacitors for the compensation of quadrature fundamental current causes an infrastructure cost. In order to compare the cost of losses to cost of compensation one must assume a certain time of operation per year, 50% may be a reasonable level of operation. According to [13] the cost of shunt capacitors is about 5 \$/kVar at high voltage levels, other sources indicate twice this or more for smaller units at lower voltage levels. One can assume the higher price and that they are in operation half the time, and assume a cost of 10% per year (5% interest rate and a lifetime of twenty years). The compensation cost will then be about $10 \cdot 0.1 / 4000 \approx 2 \cdot 10^{-4}$ \$ per kVarh. Then, for example, if the fundamental reactive power is 30% of the active power, the compensating cost will be $0.6 \cdot 10^{-4}$ \$ per kWh transferred, or 0.4% of the active energy cost. By similar calculations, the cost trend for the compensation or the fundamental quadrature current of Figure 5 has been determined.

The voltage drop due to fundamental quadrature current is basically proportional to the fundamental reactive power and hence proportional to the quadrature rms current since the voltage is approximately constant. The loss

due to fundamental quadrature current is proportional to the square of the rms current or approximately to $(Q/V)^2$.

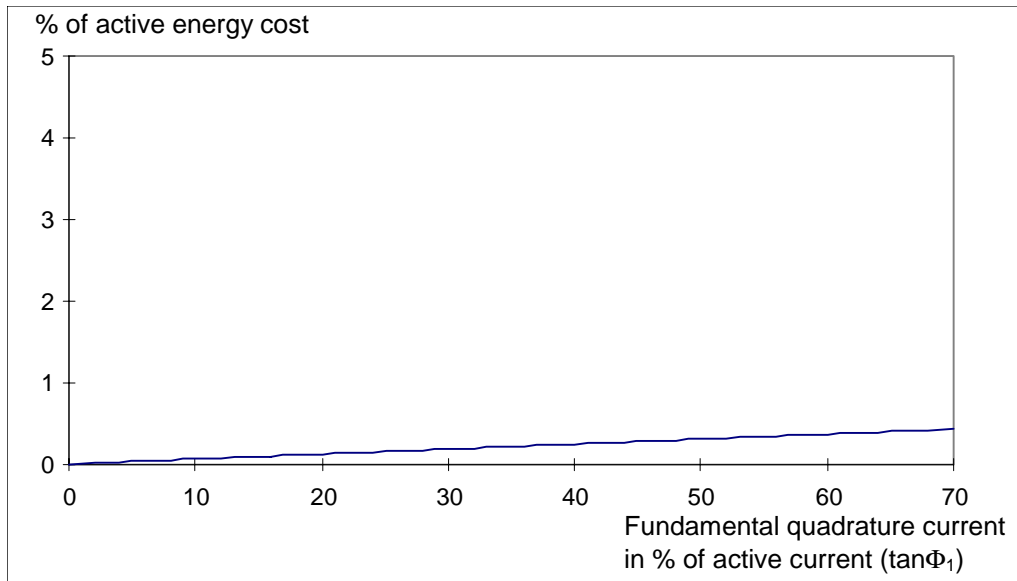


Figure 5. Trend line for the cost of fundamental quadrature current compensation.

In Sweden, customers are generally billed for maximum reactive power during each period of time. This clearly is a billing for an infrastructure, compensation, cost and not for extra losses, since most of the investment costs of compensation is proportional to the maximum values of the quadrature current (and Q_1) while cost of losses are proportional to the integrated squared value of the nonactive current (and Q_1^2).

The loss due to harmonic current is also basically proportional to the square of the rms current. This might, however, be a slight underestimation because the resistance will increase with the frequency due to proximity and skin effects of conductors and, further, the eddy current losses in transformers will increase with frequency. On the other hand, the tendency of current harmonic cancellation will decrease the relative harmonic current content fast when currents from different loads are added. These two effects, increased resistance and current cancellation will counteract, and for overall loss calculation they may cancel each other.

The distribution transformer and cables will, however, still have to be designed with extra margins, which will add to the infrastructure cost. In addition, resonance most often leads to large losses locally, predominantly in capacitors or transformers, even when in other parts of the network no particular increase of the harmonic power can be observed.

3.5 Accuracy

Classic electromechanical active power meters or energy meters generally have a poor frequency response and exhibit an uncertainty of the same order as the harmonic power P_H [14]. New meters generally have a good frequency response but in combination with instrument transformers they might still exhibit uncertainties in the same order as the harmonic power due to the usually unknown frequency response of the instrument transformers and possibly due to a low power factor of the power harmonics.

Ordinary reactive meters that are based on some kind of phase-shifted active meter, do not measure according to any of the suggested reactive power definitions for nonsinusoidal conditions [15]. If they are considered to measure fundamental reactive power, Q_1 , the additional uncertainty due to harmonics is less than a few percent of the apparent power, as long as the current distortion is less than 30%. Where this is not acceptable, a meter based on sampling techniques and DFT can provide more accurate measurements, provided that the frequency response of all parts of metering equipment is good. Generally, the accuracy of reactive metering is not as important as the accuracy of active power metering because the cost of losses or compensation is at least an order of magnitude lower than the cost of active power, as shown by Figure 4 and Figure 5.

Current harmonics and voltage harmonics can be measured with good accuracy with sampling meters if the amplitude frequency response is good enough [16]. The nonactive power S_N according to [4] can then also be measured with a good accuracy under the same conditions. If voltage and/or current transformers are used, they most often represent the major source of uncertainty from a harmonic measurement point of view. It has been indicated [17] that less than 5% amplitude error can be expected up to at least 1 kHz for most instrument transformers on distribution voltage levels. If low amplitude voltage harmonics are to be measured the linearity of the transformers and/or the meter is also vital. Problems can also arise when measuring on fast varying harmonics, or if a signal contains subharmonics or interharmonics, according to [17].

Accurate current and voltage measurements are necessary for accurate power measurements, but there are also other factors complicating the matter. The harmonic active power, P_H , and the harmonic apparent power, S_H , are small on a per unit (of apparent power) base, since they are the result of multiplication of numbers smaller than one. It has been argued [4], maybe because of this, that they are less useful for metering purposes. However the relative uncertainty of P_H and S_H due to current and voltage amplitude uncertainties

will be less than 1.4 times the largest of these two, just a marginal increase. For individual harmonics, the main reason for increased relative active-power uncertainty is instead the phase angle uncertainty. The phase angle uncertainty can be up to 5 degrees for harmonics of orders 10 - 20 at distribution levels, mainly due to the frequency response of instrument transformers according to [17]. The power factor of individual harmonics might further be very low and then the relative uncertainty of the active harmonic power can approach infinity. Further, when the active power of individual harmonics are summed to form the active harmonic power, P_H , they may be of different sign; this decreases the value of P_H and further increases the relative uncertainty. If harmonic active power or energy is used for metering purposes, it must always be compared with the harmonic apparent power and/or the apparent power.

3.6 Discussion

As earlier observed the fundamental power P_1 does not contain the whole (active) power. If the voltage is a good approximation of a sinewave, the difference will still be low, most often less than 1 percent at a metering point[15]. However in the case of a nonsinusoidal voltage source, for example a squarewave, very large differences will occur. Therefore, the division of apparent power in components with P_1 as the active power component must generally be restricted for use under quasi-sinusoidal voltage conditions only.

Under quasi-sinusoidal conditions a load that causes much current harmonics will most probably export harmonic power, P_H , which lowers the reading of an ordinary power or energy meter measuring P_1+P_H . That might be a valid argument to exclude the harmonic part of the power [18]. However, the harmonic source is basically a current source, not a power source, and the exported harmonic power does not only depend on the load characteristics but also on the source impedance and if there are other nearby current sources. Further, the exported harmonic power is “first” imported as fundamental power and “then” exported as harmonic power. It is a zero-power-process and this is best reflected if the whole active power is measured.

The conclusion is that there are arguments for both the measurement of P_1 and the total (active) power P . Most often the difference is negligible. When old meters with bad frequency response are replaced, they can in most cases be replaced either with P_1 meters or $(P_1+ P_H)$ meters. In cases where high harmonic currents and voltages are suspected, both meters and instrument transformers should have a verified and good frequency response.

The major reason for extracting the fundamental reactive power, Q_1 from the rest of the non-active power is its compensation by capacitors. The

compensation by capacitors takes care of both voltage drops and losses due to the fundamental reactive power. Major overcompensation may occur if the current is heavily distorted and other metering methods than the measurement of Q_1 are used, for example if reactive meters measuring the total nonactive power is used. The total impact, including harmonics, of the insertion of a capacitor cannot be calculated from the measurement of Q_1 . However, if the whole impact of a shunt capacitor is to be calculated, a characterisation of both the load and the system must be made at all frequencies of interest. This cannot be made by any standard metering equipment today or in the near future. The metering of the fundamental reactive power may therefore serve as the most cost-effective alternative in simple compensation control.

Table 1. Summary of possible differences between some power quantities in a metering point.

Quantities	Difference
$P-P_1$	<1%
Q_B-Q_1	<1%
$N(Q_F \text{ or } Q_{tot})-Q_B,$	0-50%
$S_N/S_1-\text{THD}_I$	<2%*

* percent units

The interest in the splitting of apparent power into power components is a consequence of the fruitful tradition of splitting the apparent power into active and reactive power in sinusoidal situations. This theory or rather mathematical tool is well known and useful in many applications. However, apparent power has no definite physical interpretation and is therefore a poor basic quantity for a formulation of a generalised power theory[19]. Beside the measurement of active power and (fundamental) reactive power, there are not many good reasons for making measurements of other power components. The harmonic problem is rather a load current and a system impedance problem than a power problem. From a power distributor point of view, the harmonic problem is mainly a problem of keeping the system impedance low enough in all parts of the system so that a reasonable harmonic current can be tolerated.

The distributor might still want the consumer to pay for losses due to harmonic currents. However, as seen in the load characterising section and the section about responsibility a complete picture of the cause and effect includes more elaborate measurements than any standard power meter can give. Measurement of voltage and current harmonics can give a good picture of the situation. Power harmonics give some information on the responsibility,

provided that the frequency response of both instrument transformers and meters are verified. As long as we have little knowledge of the phase angle accuracy of the metering, we have to rely only on current and voltage harmonic measurement.

In [4] an energy metering is suggested based on the component ($U_1 I_H$) or rather S_N as a way of monitoring I_H . The reason given is that for near-sinusoidal cases S_N is close to $U_1 I_H$ and therefore S_N/S_1 is close to the total harmonic distortion of the current, THD_I . However, a meter capable of measuring S_N and S_N/S_1 can also measure the rms current directly, integrating it over time or measuring the true rms for each hour, whatever is preferred. Further, the absolute quantity I_H is preferred over the relative quantity THD because a large THD value might well be related to a non-interesting low-load situation.

Further, [4] states that S_N is a better indicator of the level of harmonic “pollution” than P_H because an increased harmonic current will increase S_N but not necessarily P_H . This statement is true when the measurement is for indication purposes, but if one is interested in the responsibility for that pollution, P_H is the only power component that will give some guidance, because generally P_H is precisely the system (including other loads) power loss due to harmonics from the particular load that is monitored.

The concept of measuring a quantity in terms of “energy” or “power” such as S_N might be tempting because it could be easier for a customer to accept it. However, this alone is not a valid argument.

3.7 Conclusions

The division of voltage and current in fundamental and harmonic components and the subsequent division of apparent power in fundamental components, harmonic components and cross-products works well only as long as the voltage is near-sinusoidal. For nonsinusoidal sources, the concept may lead to large measurement errors or uncertainties. Therefore, this division should be seen as a practical approximation for the near-sinusoidal voltage case rather than a power definition. Of the suggested power components Q_1 and (for some conditions) P_1 have good metering applications.

The question of measuring the total active power P or only the fundamental active power P_1 , has no immediate answer. The difference is small in near-sinusoidal situations, so most often it does not matter. In the case of heavily distorted voltages and currents a total metering solution should be decided on, including both the harmonic power and the harmonic voltage and current.

The concept of measuring Q_1 has its value as a basis of power flow calculations, and the very common application of reactive power compensation by shunt capacitors. In these cases, a measurement of reactive power with meters measuring non-active power N according (2) might lead to a greatly overestimated need for reactive power compensation and would give the wrong input values for power flow calculations.

For loss calculations, the non-active current I_n according to (1) is generally of most use, when integrated over an appropriate time. If loss due the fundamental quadrature power is already considered by the measurement of Q_1 , the harmonic current I_H is preferably metered. In this case the proposed use of the non fundamental power S_N according to[4] is most often not wrong, but an unnecessary approximation. The use of power components such as S_N for the measurements of harmonics is an unnecessary approximation and there is no intrinsic value in the use of a quantity with the dimension kWh or kVA.

For harmonic responsibility questions, the best choice is to measure the harmonic impedance of the load and the system as well as the harmonics generated by the load and the system. As this might be impossible in practice, the second best is to measure voltage harmonics, current harmonics and the power harmonics. Power harmonic measurements are generally useful only if the current and voltage instrument transformers have a known frequency response. Monitoring harmonic voltage and harmonic current might still be useful for indication purposes.

In the long term, all instrument transformers should have a verified frequency response.

3.8 References

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4 Power measurement uncertainties under non-sinusoidal conditions^{1,2}

4.1 Power system metering uncertainties

4.1.1 Introduction

The presence of harmonics in the power system put higher demands on both power system components and on the measurement equipment. For components this has mostly been dealt with by derating exposed components such as transformers and capacitors. In power system measurements the problems caused by harmonics have been incorporated in increased uncertainty figures, or worse, have just been ignored. However, the increasing levels of harmonics and the economic value of an optimised power system as well as the possibility of a larger stability margin makes a more thorough study of the influence of harmonics on measurements well worth while. This chapter will summarise what is known about the influence of harmonics on measurements made in power systems, and give an estimate of the magnitude of the additional errors that can be expected due to harmonics.

A variety of quantities are measured on different locations in the power system. Which quantity is measured and what uncertainty level is needed depend on the purpose of the measurement.

4.1.2 Purpose of Measurement

4.1.2.1 Billing

In Sweden, domestic power consumers are billed for active energy and maximum apparent power demand. The maximum apparent power demand is not measured but is derived from the rating of the fuses applied and is therefore more precisely a maximum possible current billing.

Large power consumers are often billed for active energy and reactive energy, and sometimes for active power demand. The reactive energy metering is used to bill for the reduced power transportation capability caused by a load current that is larger than needed for the consumed active power and/or the cost of power factor correction.

¹ Power system metering uncertainties: main part is published in Stockholm Power Tech, International symposium on electric power engineering, proceedings, paper PS-20-04-0359, pp 617-622, Stockholm, Sweden, June 18-22, 1995.

² Power analyser uncertainties: main part is published in "Nordiska konferansen Måleteknik og kalibrering, proceedings, paper 22, Halmstad, Sweden, Nov. 1995,

In some countries the billing for reactive energy is substituted with billing for apparent power, and active power demand with apparent power demand.

4.1.2.2 Control system

Transducers for voltage, current and active and reactive power are used on the Swedish national grid, in addition to the active and reactive energy metering. The values acquired from the transducers can be compared with the energy meters and adjusted to decrease measurement uncertainty. The output of the transducers is used as inputs for the control system and can be used for load flow analysis. On regional levels there are similar systems.

4.1.2.3 Problem solving

Much of the problems associated with power quality are due to (fast or slow) voltage fluctuations or high levels of harmonics. Many different all-round instruments such as voltmeters, oscilloscopes and wattmeters are used in these situations. Specialised instruments, such as power analysers, combine the functions of all these instruments and are increasingly being used to determine the power quality. The measured quantities are mainly voltage, current and power spectra, but active and reactive power and total harmonic distortion, THD, can also be of interest in a problem-solving situation. Flicker meters that evaluate the stability of the voltage in the short time perspective are also increasingly being used, and are sometimes incorporated in power analysers.

4.1.2.4 Active Filtering

Efforts have been made to solve problems with harmonics on the power system with active filtering or combinations of active and passive filtering. Special measuring methods, measuring quantities such as instantaneous reactive power, have been developed for this purpose.

4.1.2.5 Relay Protection

Although not often regarded as measuring instruments, protective relay circuits contain current and voltage measuring modules, which may be affected by the non-sinusoidal waveforms, mainly current harmonics.

4.1.3 Measurement errors due to harmonics

As seen from the survey of measurements made in different parts of the power system, the term "power measurements" should also, in this context, include voltage and current measurements. Total Harmonic Distortion, or distortion factor (denominated THD, d or h), is often used to quantify the level of harmonics. This quantity is a good example of how different definitions can cause confusion and possible errors. The THD for a non-sinusoidal signal is defined either as the total rms value of the harmonics

compared to the value of the fundamental[1] or compared to the total rms value[2]. For example, THD for voltage is defined as

$$\text{THD} = \sqrt{\frac{\sum_{n>1} U_n^2}{U_1^2}} \quad \text{or} \quad \text{THD} = \sqrt{\frac{\sum_{n>1} U_n^2}{U_{\text{rms}}^2}} \quad (78)$$

Below approximately 10% THD, the difference between the two definitions is negligible. The former definition seems to gain more and more support in standardisation organisations and is used in this work since some of the calculations will be easier with this definition. For practical reasons the number of harmonics is limited when THD is calculated; this limit is most often in the range $n=30$ to $n=50$. To separate voltage THD and current THD the abbreviations THD_U and THD_I are sometimes used in this work.

Although a lot of case studies have been made in situations where problems due to harmonics have occurred[3, 4], there are no general rules of what can be expected in terms of current and voltage THD for different power consumers. Individual smaller one-phase loads including computers and energy-saving lamps can cause current THD of about 100%, voltage-stiff three-phase converters will also cause current distortion at this level[5]. A current-stiff three-phase 6-pulse converter without filters will cause a maximum current THD of 20-30%. Most electronic equipment and fluorescent lighting will cause current THD from 20% to 100%. However, current harmonics with different phase angles will partially cancel each other even at the level of individual power consumers. Thus, even if the whole load consists of small high-THD loads the THD of the total current will be substantially lower than 100%. Further, the current THD level for the total current will not reach its maximum at high-load conditions because climate-regulating equipment with a low THD are then a substantial part of the load; this is valid at least in the Nordic countries.

In the vicinity of larger non-linear loads the voltage THD is mainly determined by the combined effects of the source impedance and the current THD level. The voltage THD will therefore often be a factor 5 - 15 lower than the current THD for larger loads. For smaller loads there is no such rule because the voltage THD will then also depend on other factors, such as other nearby non-linear loads.

In this work 60% current THD and 6% voltage THD are considered as maximum values (the case of measurements on an individual high distortion load is disregarded). Reasonably high values can be 20-30 % current THD

and 3-5% voltage THD. According to a large survey made by the Swedish utilities [6] a current THD of 10-15% for offices and 5-10% for industries at the point of common coupling can be considered as high, in urban areas. The voltage THD will then usually be approximately 2%.

The additional errors due to the influence of harmonics on the measurements can be divided into three groups. The first group is the common errors caused by non-linearities in the input or measuring circuitry and a limited frequency response. The second group consists of systematic errors caused by instruments using measurement principles that do not comply with the extended definitions of non-sinusoidal situations. The third group of errors is due to the use of different competing definitions of some quantities, giving different results in non-sinusoidal situations. Additionally, if the measured quantities are used in further calculations these errors will affect the results of the calculation in a way that is determined by the calculation algorithm.

4.1.4 Strategy and assumptions for the error analysis

It is of interest to know if the above errors can be directly derived from the harmonic level or if the additional errors due to harmonics must be calculated from measurements of other quantities. As a starting point the rather common situation on low voltage levels with mainly electronic loads is considered, where the fundamental phase angle Φ_1 is close to zero but the current THD is high. A worst case example will be used: a non-linear load with 60% current THD which on a linear power system with a certain impedance causes 6% voltage THD. To simplify calculations and make the problem evident we further assume that the phase angles between the corresponding voltage and current harmonics are $\Phi_1 = 0$, $\Phi_n = \pi$. The calculations are made in per unit, the fundamental voltage $U_1 =$ the fundamental current $I_1 = 1$ pu.

4.1.4.1 Error analysis for voltage and current

Normally, voltage and current meters indicate Root-Mean-Square (rms) values which for a periodic signal is given by

$$U_{rms} = \sqrt{\frac{1}{T} \int_T u^2 dt} . \quad (79)$$

In the following the index rms is used only when needed for clarity, otherwise rms-values are presumed when capital letters are used. Very often meter that measures rms values are called "true rms" meters to separate them from other common meters indicating rms values while really measuring something else, like the rectified mean value. Although rather an awkward designation, "true rms" is adopted in this work for simplicity. The additional errors of true rms meters due to harmonics will mainly be caused by higher

demands on bandwidth and linearity for the non-sinusoidal situations. The above definition can be applied on a Fourier series representation of a signal, and will then become

$$U_{rms} = \sqrt{\sum_n U_n^2} = \sqrt{U_1^2 \left(1 + \frac{\sum_{n>1} U_n^2}{U_1^2} \right)} = \sqrt{1 + THD_U^2} \approx 1 + \frac{THD_U^2}{2} \quad (80)$$

when U_1 is set to 1 pu. Similarly, the relative rms current will be

$$I_{rms} = I_1 \sqrt{1 + THD_I^2} = \sqrt{1 + THD_I^2} \approx 1 + \frac{THD_I^2}{2} \quad (81)$$

That is, an instrument with a severely limited frequency response would show an error due to this limitation that would be approximately $THD_U^2/2 = 0.06^2/2 = 0.18\%$ for our example of 6% voltage THD. For current the same equation applies and the worst case additional error would be about $THD_I^2/2 = 0.6^2/2 = 18\%$. However, true rms meters are usually designed with non-sinusoidal measurements in mind and have a rather good frequency response. They will most probably show small additional errors due to harmonics for reasonable THD values and crest factors, the expected error is just a fraction of the worst case error calculated above.

Low-cost voltage and current meters are, however, often really measuring the rectified mean value and then indicating rms values using the scale (form) factor 1.1107, valid for sinusoidal waveform only. Some simple transducers even respond to the peak value while indicating a rms value. Instruments responding to the peak value and rectified-mean responding meters will have an additional error due to harmonics that does not primarily depend on the distortion but on the waveform. As an example of this, a fundamental voltage of 1 pu with an added third harmonic of 0.3 pu, is measured by an rectified-mean responding meter. If the phase angle (α_3) of the third harmonic is changed the waveform changes which results in different errors compared to the true rms value, see Figure 1.

The maximum theoretically positive error of rectified-mean responding meters will be +11,07%, for a square-wave, while the relative negative error can be infinitely large for signals with a high crest factor.

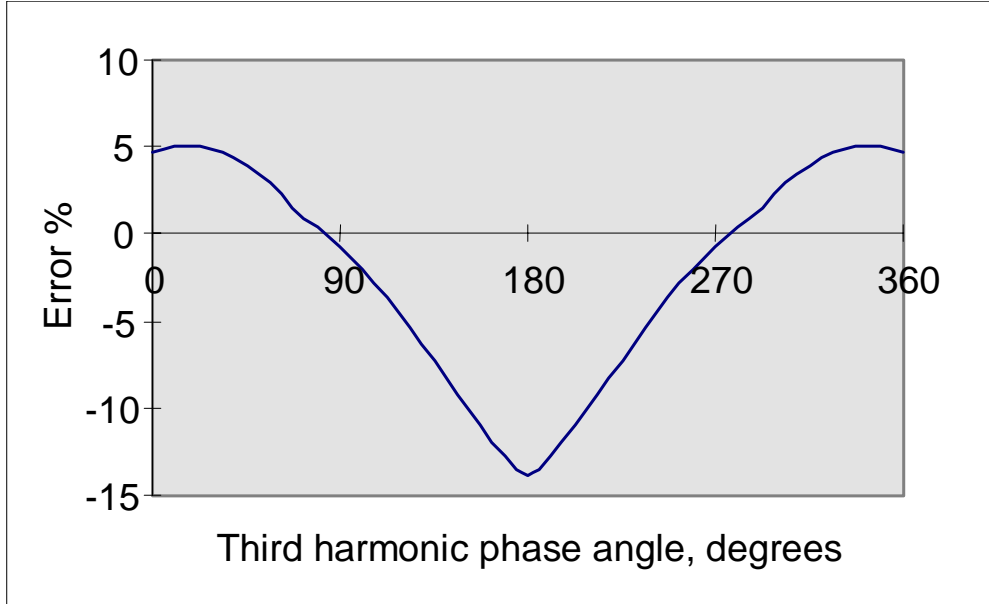


Figure 6. The phase angle dependence of the errors of a rectified-mean-measuring meter when measuring a signal with 30% third harmonic signal.

The voltage waveform often resembles a top-flattened sinusoidal signal in power system measurement situations. Rectified-mean responding voltage measuring instruments will then have a maximum additional error about $\pm 1\%$, if the THD_U is below 8%.

Current measurement errors will be larger than voltage measurement errors because the current usually contains much higher levels of harmonics, and the waveform can be more unfavourable. The maximum error will be in the range -20% to +7% for reasonable wave-shapes. Rectified-mean (and peak) responding meters should be avoided whenever harmonics are present and accurate measurements of rms values are needed.

4.1.4.2 Error analysis for apparent power

Apparent power is seldom measured in Sweden. It can be calculated from the current and voltage measurement. Using Equation (80) and (81) for both current and voltage and the above assumption of $THD_U = 0.06$ and $THD_I = 0.6$, the relative apparent power will be ($U_1 = I_1 = 1$)

$$\begin{aligned}
 S &= U_{rms} I_{rms} = U_1 \sqrt{1 + THD_U^2} \cdot I_1 \sqrt{1 + THD_I^2} = \\
 &= \sqrt{1 + 0.06^2} \cdot \sqrt{1 + 0.6^2} = 1.17
 \end{aligned} \tag{82}$$

That is, if the frequency response were severely limited the error of the measurement would be approximately 17%. A further approximation will give

$$S = U_1 I_1 \sqrt{1 + THD_I^2} \cdot \sqrt{1 + THD_U^2} \approx \sqrt{1 + THD_I^2 + THD_U^2} \approx 1 + \frac{THD_I^2}{2} + \frac{THD_U^2}{2}$$

For distortions higher than 10 - 30 %, depending on the needed uncertainty, the above approximations may not be valid.

4.1.4.3 Active power

The active power definition at the presence of harmonics is:

$$P = \frac{1}{T} \int_T u(t)i(t)dt = \sum_n U_n I_n \cos\Phi_n = P_1 + \sum_{n>1} P_n \quad (83)$$

Additional errors of active power meters due to harmonics will mainly be caused by limitations of the frequency response, especially for older analogue meters. As the fundamental phase angle is assumed to be zero in our example and all other angles are assumed to be π , the active power will be

$$\begin{aligned} P &= \sum_n U_n I_n \cos\Phi_n = U_1 I_1 - \sum_{n>1} U_n I_n = \\ &= U_1 I_1 \left(1 - \frac{\sum_{n>1} U_n I_n}{U_1 I_1} \right) = 1 - \frac{\sum_{n>1} U_n U_n}{R_s U_1 I_1} \end{aligned} \quad (84)$$

where R_s is the total source impedance as Φ_1 is assumed to be 0. R_s is then assumed frequency independent, and can therefore be calculated from the THDs as

$$\frac{THD_U}{THD_I} = \sqrt{\frac{I_1^2 \sum_{n>1} U_n^2}{U_1^2 \sum_{n>1} I_n^2}} = \sqrt{\frac{I_1^2 R_s^2 \sum_{n>1} I_n^2}{U_1^2 \sum_{n>1} I_n^2}} = \frac{I_1}{U_1} R_s \quad (85)$$

and then the active power can be calculated:

$$P = 1 - \frac{\sum_{n>1} U_n U_n}{R_s U_1 I_1} = 1 - \frac{THD_U^2}{THD_U / THD_I} = 1 - THD_U \cdot THD_I \approx 0.96 \quad (86)$$

That is, the total active power P is maximum approximately 4 % less than the fundamental active power P_1 . Hence, the error of a meter with very limited bandwidth would be about 4 % in this situation. The choice of phase angles made here is then the worst case. As the level of harmonics of the current drawn by domestic power consumers is relatively low, the active harmonic power components, P_n , are most often negligible from an active power measuring point of view. Therefore the additional errors also most often will be negligible for this consumer category. Power consumers as office buildings and industries can generate more substantial harmonic power components, but as most of the power is contained in the lower harmonics which even the meters with a bad frequency response measures, the additional error caused by harmonics still typically is less than 1%.

At higher voltage levels, the magnitude of the harmonics is lower because harmonics with different phase angles will partially cancel each other. The additional active power error caused by harmonics will then be well below 1%. Exceptions can occur close to very large non-linear loads.

It has been questioned if it is advisable to measure active power according to the definition; it could be in the interest of the power suppliers to exclude all power harmonics from the measurements [7]. To the author's knowledge, however, no energy meter that measures only fundamental frequency power is yet on the market, so it is still a purely theoretical question.

4.1.4.4 Error Analysis for reactive power

The main cause of uncertainty due to harmonics at reactive power measurements is that there is no generally accepted reactive power definition when harmonics are present. There are a number of suggestions made how to define this quantity. The two most well-known are the Budeanu [8] reactive power definition Q_B and the reactive power definition, Q_F , according to Fryze [9]:

$$Q_B = \sum_n Q_n = \sum_n U_n I_n \sin \phi_n ; Q_F = \sqrt{S^2 - P^2} \quad (87)$$

In the presence of harmonics the two definitions will give different results, the difference is named distortion power (D_B) and calculated by

$$D_B = \sqrt{S^2 - P^2 - Q_B^2} = \sqrt{Q_F^2 - Q_B^2} \quad (88)$$

With no harmonics present both these, and most other proposed definitions, will be identical to the sinusoidal reactive power definition

$$Q = UI\sin\Phi \quad (89)$$

and $D_B = 0$. The major reason for the confusion about the reactive power definition is the fact that the mathematically defined quantity reactive power can, in sinusoidal situations, be assigned some very useful physical properties. When extended to the non-sinusoidal situation no definition can maintain all these properties [10-12]. It has also been shown that the Budeanu definition, although widely recognised, is of little use in practical situations. For more details see chapter 2.

If different reactive power definitions are used the deviation between measurements can become very large. With the above example, $\text{THD}_U = 0.06$ and $\text{THD}_I = 0.6$, and $\Phi_1 = 0$, $\Phi_n = \pi$, the reactive power according to Budeanu will be

$$Q_B = U_1 I_1 \sin 0 + \sum_{n>1} U_n I_n \sin \pi = 0 \quad (90)$$

and the reactive power according to Fryze will be

$$Q_F = \sqrt{Q_B^2 + D_B^2} = D_B = \sqrt{S^2 - P^2} \approx \sqrt{1.17^2 - 0.96^2} \approx 0.67 \quad (91)$$

This means a difference in reactive power of > 50% of the total apparent power due to different definitions! The magnitudes of harmonics are lower at higher voltage levels, and the difference will be less, but it will still be considerable.

To avoid the whole question of harmonics and reactive power definitions the idea has therefore been put forward to reserve the term reactive power for the fundamental frequency component Q_1 [12, 13] which results in the definition:

$$Q_{(1)} = U_1 I_1 \sin \Phi_1 \quad (92)$$

This definition has some advantages, especially in power flow analysis.

Some solid-state meters and many power analysers use the reactive power definition according to Fryze. However, most analogue meters measure something that does not comply with any of the above definitions. They are most often based on active power meters with a phase-shifting filter added that causes a phase angle displacement of $\pi/2$ for the fundamental frequency. There are, however, no filters capable of maintaining both the amplitude and

the phase angle shift for varying frequencies. The phase angle shift can be obtained by a time delay, which may be easily utilised in a solid state meter based on sampling technique. The amplitude will then be constant but the phase shift for a harmonic of order n will be $n \cdot \pi/2$ and the meter will measure

$$Q = Q_1 - P_2 - Q_3 + P_4 + Q_5 + \dots \quad (93)$$

where Q_n are the reactive harmonic components $Q_n = U_n I_n \sin \Phi_n$ and P_n are the active harmonic power components. If the filter is designed to obtain a constant phase angle shift the amplitude will be frequency dependent. If the filter is of the differentiating type, the meter will measure reactive power as

$$Q = \sum_n \frac{1}{n} Q_n \quad (94)$$

Three-phase reactive power meters may use the phase-to-phase voltage of two phases together with the current of the third phase to get the desired phase angle displacement of $\pi/2$. This results in a phase angle shift for harmonics similar to the phase shift for the time delay filter above. For a symmetrical load the measured quantity will then be

$$Q = Q_1 - Q_2 + 0 \cdot Q_3 + Q_4 - Q_5 + \dots \quad (95)$$

The maximum deviations of these sums from Q_B are in the same range as the maximum deviation of the harmonic active power, therefore the deviation will seldom be above a few percent. The deviation from Q_F is on the other hand considerable, as shown in Equation (87) to Equation (91), deviations higher than 10% must be considered.

The confusion over the definition of reactive power, and the large differences in measurements based on different definitions, makes it impossible to disregard the definition problem if any measurement accuracy is to be achieved. A basic solution is to consider what information is really wanted from the measurements and then choose the measuring method accordingly. A load flow analysis calculates the load flow of the fundamental frequency power. Therefore the fundamental frequency reactive power, Q_1 , or simply the phase angle Φ_1 should be the desired quantity. For billing purposes the desired information is the unnecessary current drawn by the load, therefore the reactive power Q_F or the apparent power or simply the rms current could be a natural choice. See further the discussion in chapter 3.

When making problem-solving measurements, it is most important to know exactly what your instrument measures so you do not get contradictory results or make the wrong decisions. Most power analysers measure Q_F , and as a result large differences could exist when comparing these results with other meters, because of definition problems or problems caused by meters not measuring according to any definition in the non-sinusoidal situation.

For other purposes, such as power factor correction, other definitions may be a better choice. It is the author's opinion that all these purposes can not be satisfactorily served by any single reactive power definition, neither now nor in the future.

4.1.4.5 Miscellaneous

As earlier stated the apparent power is defined by $S = V_{\text{rms}} \cdot I_{\text{rms}}$ in the single-phase case. For an unbalanced three-phase system there are some different definitions proposed [13, 14]. The most commonly utilised definition in meters, to the author's knowledge, is also the most straightforward:

$$S = S_1 + S_2 + S_3 \quad (96)$$

This might however give an unreasonably low value for unbalanced loads [14] and a more correct definition might be

$$\begin{aligned} S &= 3U_e I_e = 3\sqrt{\frac{(U_1^2 + U_2^2 + U_3^2)}{3}} \cdot \sqrt{\frac{(I_1^2 + I_2^2 + I_3^2)}{3}} = \\ &= 3\sqrt{\frac{(U_1^2 + U_2^2 + U_3^2) \cdot (I_1^2 + I_2^2 + I_3^2)}{9}} \end{aligned}$$

where U_e and I_e are the equivalent voltage and current, and the phase voltages U_1 to U_3 are measured with reference to a virtual zero if the system is a three-conductor system. There will be a difference between the two above algorithms if the load is unbalanced. The evaluation of this is beyond the scope of this work. Normally, a choice of either of these summation formulas will cause insignificant additional errors in the presence of harmonics, because these errors are of second-order type.

Some special power quantities have been developed for active filters and have been suggested useful for other measurement purposes [15]. They are, however, not commonly used outside their specific areas and therefore disregarded in this paper.

Protective relay circuits as impedance- and differential relays are affected by harmonics. Only a few papers have been published on theoretical work in this area [16]. One reason for this is the wide variety of measurement principles used in this field. Sometimes the problems depend on the waveform rather than on the THD level. The matter is further complicated by the fact that the harmonic power flow often, although not always, has the opposite direction compared to the fundamental power. Uncertainties of the same order as distortion levels should be considered.

At medium and high voltage levels, the frequency response of the voltage and current instrument transformers will contribute to the errors due to harmonics. The errors of higher harmonics can be substantial, but the error in the total rms voltage and the total active power will typically be small for reasonable THD levels. One exception of this rule is the Capacitor Voltage Transformer (CVT). It is prone to resonance, which causes errors for single harmonics in the range of 100% or higher. This problem is treated in more detail later in this chapter

4.1.5 Conclusions

The above calculations show that the errors due to harmonics can be expressed in terms of voltage and current harmonic distortion. Maximum errors, due to the lack of frequency response and the definition problems discussed above, are presented in Table 2 for some different harmonic distortion levels.

Table 2. Maximum errors due to harmonics at different distortion levels.

Dist. Level		Maximum error					
U %	I %	U %	I %	S %	P %	$Q_{Ft}-Q_B$ % of $U_1 \cdot I_1$	Q_B-Q_1 %
6	60	0.18	16.6	18.2	3.60	66.0	3.60
2	10	0.02	0.50	0.52	0.20	12.0	0.20
1.5	5	0.01	0.12	0.14	0.08	6.50	0.08

Uncertainties in the order of 1 % will be sufficient for many applications. In that case, it will be sufficient to evaluate the errors of rms current and reactive power measurements. Most important is then to check that the instrument is measuring according to the expected definition.

However, if sub-one-percent accuracy is desired, the additional errors due to harmonics must be evaluated for all quantities. This is made by checking the actual harmonic level as well as the waveforms, and then evaluating the instrument's ability to measure under these conditions. This evaluation can be

made with considerable better accuracy than 1% with calibration systems adopted for non-sinusoidal situations [17] [18] [19] [20].

4.2 Power analyser uncertainties

Instruments designed to measure power system harmonics and other power quality parameters are usually called power analysers. They have become increasingly popular. The frequency dependence and linearity of these instruments is of prime importance for the accuracy. These limitations and other possible power analyser measurement problems are described and analysed in this part of the work.

4.2.1 General

A new generation of instruments for measuring distorted power-frequency voltages and currents has emerged as a result of advances in digital techniques. These instruments can calculate and display the amplitude and phase angle of each harmonic by the use of sampling techniques and "FFT-analysis", more correctly discrete Fourier analysis (DFT) by use of the fast Fourier transform algorithm.

The unique capability of power analysers is the ability to measure each harmonic of a distorted signal. What makes them so useful, however, is their versatility, which is more or less a spin-off effect from the sampling technique. Most other quantities of interest can be calculated, if the graphical capability of the display allows it, they may also be used as oscilloscopes or waveform recorders.

However, the versatility and the many non-standardised measurements that can be made with these instruments make them difficult to specify and to calibrate. Also, the results of a measurement can easily be misinterpreted.

Most mechanisms that lead to uncertainty in measurements with power analysers are frequency dependent, and some of them are due to non-linearity. A verification or calibration at 50 Hz sinusoidal voltages and current is therefore not sufficient, and a calibration system for non-sinusoidal calibrations is needed. This section describes power analysers, and discusses the possible measurement problems that can affect the accuracy of measurements made with power analysers.

4.2.2 Quantities measured by power analysers

There are few rules or standards defining which quantity should be measured and on how to measure in case of situations where harmonics are suspected to cause problems. Therefore, the available instruments show differences in calculation algorithms and in the degree of complexity. The simplest type of instrument may be a voltmeter displaying only the rms-value and a few harmonics. The most complex instruments can handle 50 or more channels of voltage and current measured in different points as e.g. in a large switchyard and measuring all kinds of quantities.

The measured quantities can be divided into two groups: quantities measured and presented for each harmonic, i. e. frequency domain quantities, and quantities measured and presented for the total signal. From an uncertainty point of view it is also interesting to divide the quantities in two other groups, quantities calculated with or without using the phase angle relation between voltage and current.

4.2.2.1 Quantities measured for each harmonic

The harmonics can be presented with their phase angle and magnitude for each harmonic. The magnitude can be expressed as the peak value or the rms value, but often the magnitudes of the harmonics are presented as a percentage of the fundamental harmonic. In addition, the active power of each harmonic can be presented in watt or as a percentage of the fundamental harmonic power. The phase angle difference between current and voltage, ϕ_n , is also often presented.

4.2.2.2 Quantities measured on the entire signal

For each signal, voltage or current, the following quantities often are calculated:

- rms-value
- total (harmonic) distortion

Using corresponding voltage and current value, the following quantities may be calculated:

- active power
- reactive power
- apparent power
- power factor
- the power factor corresponding to the fundamental harmonic phase angle difference

If the instrument is designed for three-phase measurements, the above quantities most often are available for the total three-phase system as well.

4.2.3 The general power analyser

A power analyser is designed much in the same way as a multichannel data acquisition system, see Figure 7. The input level is often about 100 mV if the inputs are designed to be interchangeable. Amplifiers are then needed to get into the 1 - 10 V range of the A-D converters (ADC). If there are separate voltage and current inputs, the voltage inputs may have higher voltage input levels. Voltage inputs may have an auto-range function, this is rare for the current input.

As current transducer, transformers or current clamps are generally used in combination with a shunt resistor. Some sort of resistive divider system (DIV) is generally used for the voltage input.

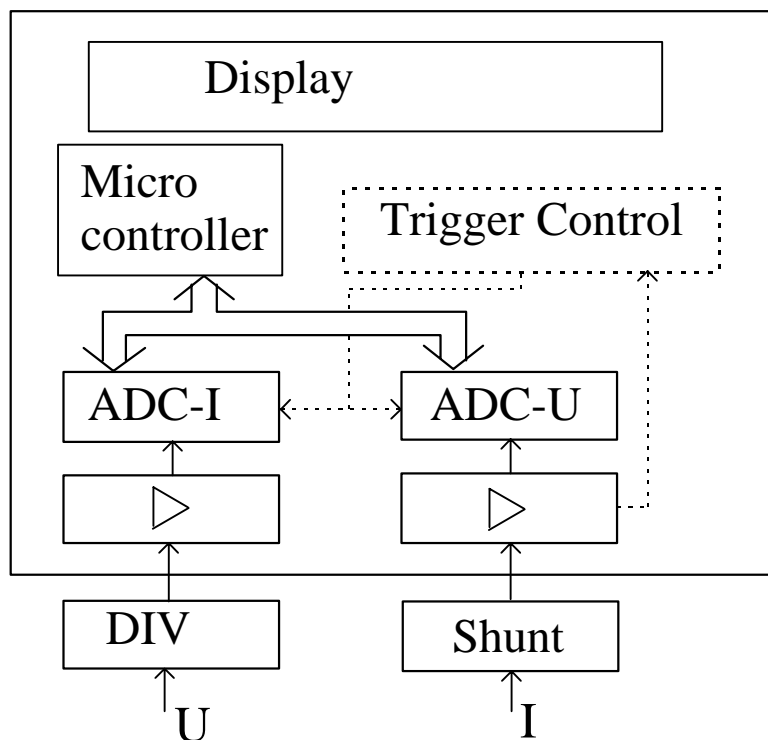


Figure 7. Block diagram for a general power analyser.

To achieve the best possible accuracy, synchronisation of the sampling rate and the fundamental frequency of voltage and current is important, because most of the calculation algorithms are based on measuring the signal for an (exact) integer number of periods.

The sampling rate may be synchronised to the voltage signal by a phase-locking trigger control circuit as in Figure 7, but sometimes it is done by the software and sometimes it is disregarded. Quite often one A-D converter

serves all input channels by means of a multiplexing and sample-and-hold circuitry, instead of using one A-D converter for each channel. The voltage and current waveforms sampled by the A-D converter(s) are then fetched and processed by a microcontroller.

The Fourier transform is theoretically correct only in the periodical case or if the measurement is made over an infinite time. In the general case, the signal and the harmonic content of the signal will fluctuate and the signal might contain inter- and subharmonics. The (window) length of each sampled sequence and the number of samples taken will then affect the measurement. The window length T_w will determine the frequency resolution of the measurement:

$$\Delta f = \frac{1}{T_w}$$

The number of samples taken in each period will determine the number of harmonics that can be measured; the number of harmonics that can be measured is half the number of samples that is taken for each period.

When the phase-locking circuitry does not track the frequency well or when the phase-lock is disregarded or when interharmonics are present the measuring window might not exactly coincide with the signal periods. The start of the sequence will then not exactly match the end and the DFT algorithm will see this as a transient in the signal that causes an erroneous contribution to most of the measured harmonics. Windowing functions that forces the measured sequence down to zero in both the start and the end of the sequence might then greatly enhance the harmonic amplitude measurements [1, 21]. A side effect of most window functions is higher uncertainty for "nice" signals. Also, most window function characteristics are such that a sample in the middle of the window has more weight than samples close to the beginning or the end, which will compromise the repeatability of a measurement on a fluctuating signal. Guidance on window times and window functions can be found in [1], and some more information can be found in chapter 5 of this work.

Anti-alias low-pass filters are needed to avoid that harmonics whose frequencies are higher $f_s/2$ show up in the result as harmonics below $f_s/2$. The quality of these filters is an important factor for the accuracy of measurements of harmonics.

4.2.4 Uncertainty sources of power analysers

Disregarding the possible software contributions (including the choice of algorithms) to the uncertainty, there are generally three major sources contributing to the uncertainty of the power analysers:

- the A/D-converter(s)
- the voltage divider
- the current transducer

The contributions to the uncertainty can further be divided into three groups:

- amplitude errors, which can be frequency dependent
- phase angle errors, which can be frequency dependent
- linearity errors

4.2.4.1 A-D converters

The amplitude error and its frequency dependence can be verified relatively easy. Therefore, this part of the instrument can be expected to have small frequency dependence and to be within the specification for the whole frequency range with respect to amplitude and phase angle of the A-D converters. The linearity can also be expected to be good, with negligible or minor impact on the accuracy.

The number of bits of the A-D converter will not, as it would in a DC-measurement, cause a limited resolution but its main effect is that it introduces a noise that will affect the measurement stability, see Figure 8. Typically the A/D-converter is a 12-bit converter and the noise will then be about 0.02 % of the range. This quantisation noise is negligible compared to other uncertainty sources as long as a reasonably large part of the input range is used.

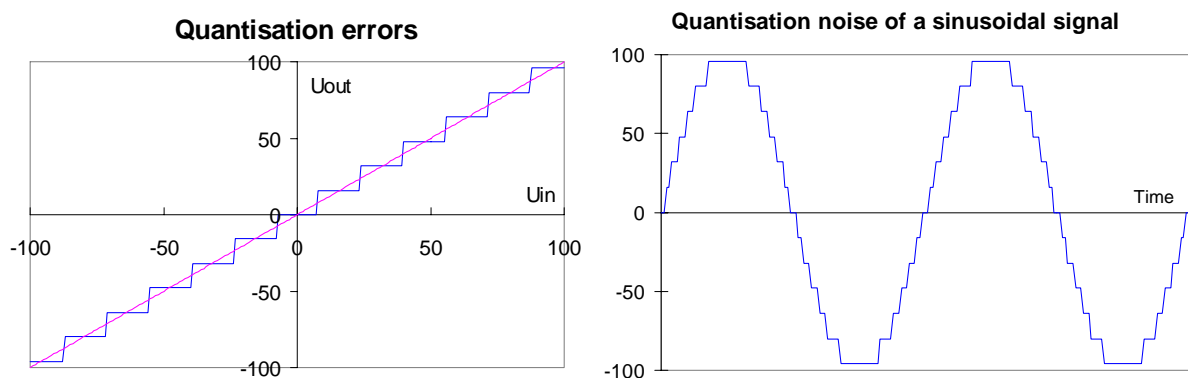


Figure 8. Demonstration of the quantisation error.

4.2.4.2 Voltage dividers

The input signal levels for the A-D converters are often set to about 100mV, so that every channel can be used as either a voltage or a current input. High isolation demands are then put on these channels to achieve this freedom of connection. For lower voltages (<600V) the divider is usually resistive with some compensation for the inductance of the cables and resistors, or resistive with an active buffer circuit.

For higher voltages (instrument) transformers must be used. Most often, the transformers installed for energy metering are used. They are not designed for frequencies above 50 Hz, but for medium voltages some types may be expected to be within 5% and 5°, at least up to 500 - 1000 Hz. This can not, however, be taken for granted [1, 22-24], see further the 4.3 that describes this problem more in detail.

The frequency dependence of voltage transformers often does increase at high voltage levels. For inductive transformers, this is mainly due to stray capacitance in the windings and between the windings that increases when the size of the transformer increases, due to the higher insulation demands. For capacitive voltage transformers, CVTs, there may be resonances even as low as at a few hundred Hz, which creates large amplitude and phase errors[25].

Linearity of voltage dividers is discussed together with current transducers below.

4.2.4.3 Current Transducers

The current transducer is most often a current clamp probe combined with a shunt resistor, but regular current transformers can also often be used in combination with shunt resistors. Sometimes a 5 A current clamp is used to measure on the secondary of current transformers. If a current clamp probe is used it is most often the dominant source of the uncertainty. The uncertainties are usually in the range 0.5 - 3 % for ratio and 1 - 3 ° for phase angle, and varies with the current range, the diameter of the clamp, and the general design.

If possible, regular current transformers should be used if one wishes to achieve a high accuracy. As for the voltage transformers, the current transformers are not designed or specified for anything but 50 Hz. However, for frequencies below about 1000 Hz, the current transformers should most often be within 5% and 5°.

All transformer type devices suffer more or less from the non-linear behaviour of the core material. This non-linearity causes a "spill-over" or leakage from the large amplitude harmonics, mainly the fundamental, to other harmonics. This leakage, the harmonic distortion and the intermodulation distortion, may be seriously detrimental to the accuracy of the measurement of the low-amplitude harmonics. See further Figure 9 and chapter 5 of this thesis.

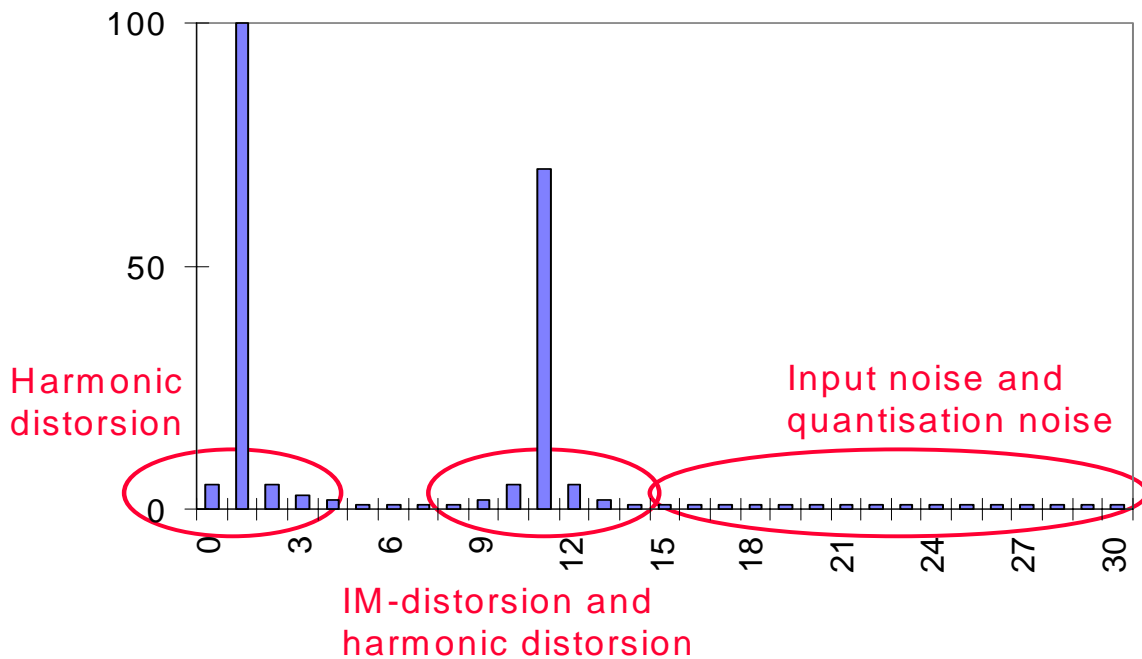


Figure 9. Effects on a measurement of fundamental and eleventh harmonic by input non-linearity, input noise and quantisation.

The transducers of the current clamp type, if used, are part of the power analyser. However, the large variation of transducer accuracy and perhaps also their major contribution to the uncertainty have caused most of the instrument manufacturers to exclude the current transducers from the specification of the instruments.

4.2.4.4 Uncertainty due to definitions and algorithms

An additional source of uncertainty is that quantities such as reactive power, apparent power and power factor have standardised definitions only for the sinusoidal case and, in case of three-phase systems, only for the symmetrical case. Unfortunately, these instruments are most often used to solve problems related to lack of symmetry and lack of sinusoidal signals. Therefore, the result of measurements will differ as these quantities may be calculated by different algorithms and according to different definitions. Further, total (harmonic) distortion is defined in at least three different ways in different

standards, and the phase angle of the harmonics can (and are) displayed with different references.

4.2.4.4.1 Distortion definition

The total harmonic distortion, often abbreviated THD, is normally calculated in one of two different ways:

$$\text{THD} = \sqrt{\frac{\sum_{n>1} U_n^2}{U_{\text{rms}}^2}} \quad \text{or} \quad \text{THD} = \sqrt{\frac{\sum_{n>1} U_n^2}{U_1^2}} \quad (97)$$

In some IEC-standards, the THD is based on comparing rms values while the ANSI-standards compare the rms-value of the harmonics to the amplitude of the fundamental harmonic. Therefore, they are sometimes referred to as the "European" and the "American" THD. The difference will be negligible in most situations, but for highly distorted signals, with a THD above 20 – 30 %, it may be significant. Further, the new European standard for electricity supplied by public distribution systems, EN 50 160, compares the harmonics with the nominal voltage (U_{nom}) for 0.4kV systems or declared voltage (U_{decl}) for medium voltage systems, and limits the number of harmonics to 40:

$$\text{THD} = \sqrt{\frac{\sum_{n=2}^{40} U_n^2}{U_{\text{nom}}^2}} \quad \text{or,} \quad \text{THD} = \sqrt{\frac{\sum_{n=2}^{40} U_n^2}{U_{\text{decl}}^2}} \quad (98)$$

4.2.4.4.2 Reactive power definition

As discussed in an earlier part of this chapter, large errors may occur if the reactive power definition used is not the expected one. There are two major suggestions of how to define reactive power in non-sinusoidal situations. The difference can be defined as

$$D_B = \sqrt{S^2 - P^2 - Q_B^2} = \sqrt{Q_F^2 - Q_B^2} \quad (99)$$

and the size of this quantity is mainly due to the distortion level. Further, many reactive power calculation algorithms, especially in analogue meters, are based on the active power measuring algorithm and some sort of 90 ° phase shift. They will then not quite comply with any of these definitions, although the result most often will be rather close to the Budeanu reactive power. The difference between these and Q_F will be approximately equal to D_B above and may be further studied in Table 2.

4.2.4.4.3 Phase angle reference

The phase angle presentation may also differ. Basically, it is used to indicate a time difference and one must know the time zero-reference, and the frequency related to the phase angle, to know the meaning of a presented value. The time-zero reference can be the voltage zero-crossing, but it can also be chosen as the current zero-crossing, the voltage or the current fundamental-harmonic zero-crossing, and so on. The value of the phase angles will change depending on the choice of time zero-reference. As an example of the reference problem, various possible values of a third order current harmonic phase angle (β_3) are demonstrated. It shows how the value of the phase angle depends on which point in time is chosen as the zero reference and whether a sine or cosine Fourier series representation is assumed. The assumed signal to analyse is a sinusoidal voltage and a current with a 30% third order harmonic. If a sine Fourier series representation is used the analysis will result in

$$u(t) = \hat{U}_1 \sin(\omega_1 t + \alpha_1), \quad i(t) = \hat{I}_1 \sin(\omega_1 t + \beta_1) + \hat{I}_n \sin(3\omega_1 t + \beta_3)$$

and if a cosine representation is used the analysis will result in.

$$u(t) = \hat{U}_1 \cos(\omega_1 t + \alpha_1), \quad i(t) = \hat{I}_1 \cos(\omega_1 t + \beta_1) + \hat{I}_n \cos(3\omega_1 t + \beta_3)$$

It is possible to choose a zero time reference according to:

- 1) the (total) current zero crossing
- 2) the (total) voltage zero crossing
- 3) the zero crossing of the fundamental voltage harmonic ($\alpha_1=0$)
- 4) the zero crossing of the fundamental current harmonic ($\beta_1=0$)

Further, zero-crossing references are normally used when dealing with sine Fourier series representations, since a sinusoidal signal have a zero-crossing at zero phase angle. If a cosine Fourier series representation is used a zero phase angle is associated with the peak value and two more zero time references are possible:

- 5) the fundamental voltage harmonic peak value ($\alpha_1=0$)
- 6) the fundamental current harmonic peak value ($\beta_1=0$)

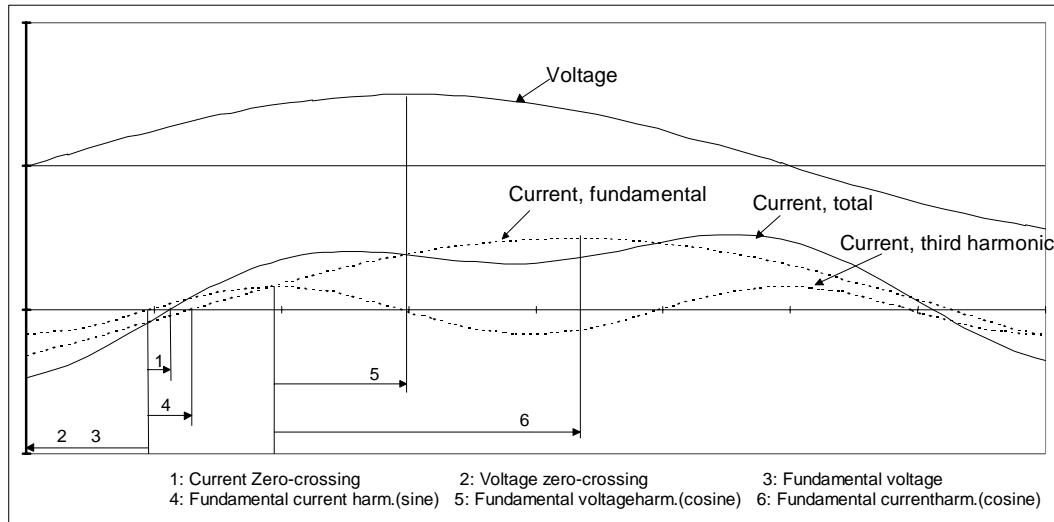


Figure 10. Phase angle of third harmonic of current for different phase angle references.

All the above choices of zero time reference may result in different values of β_3 . The different values of β_3 due to the time reference choices 1) - 6) is demonstrated in Figure 10. Since the voltage is sinusoidal, the zero-crossing of the fundamental harmonic and the total signal coincides and 2) and 3) result in the same value of β_3 . With real life quasi-sinusoidal voltages, there will be a small difference between 2) and 3) which is very hard to determine if the zero time reference is not specified.

The demonstrated multitude of time reference possibilities will add to the uncertainty if the zero-time reference (and the Fourier series representation model) is not specified. In a calibration situation, harmonic phase angles settings that make it possible to determine the time reference must be used

Further, while the time difference in the diagram above is normally expressed as the phase angle related to the frequency of the harmonic of interest, the time difference may also be expressed as a phase angle related to the fundamental frequency. That is, the above phase differences will then be divided by a factor of n .

4.2.4.4.4 Interharmonics

Converters for rail-road electric power are a frequent type of measuring object in Sweden. The power analysers can generally not be expected to handle this situation, neither the measurement of fundamental of the traction power system, 16,7 Hz, nor most of its harmonics. This is because the measuring and calculation techniques often presupposes that no other harmonics than a 50 Hz fundamental harmonic and its integer multiples are present. For example, the frequency of the fifth harmonic of a 16,7 Hz signal,

which is 83.3 Hz, is not an integer multiple of 50 Hz. It will therefore either contribute to the surrounding harmonics of 50 Hz, like the 50, 100 or 150 Hz harmonic, or not show at all, depending on the sampling and calculation algorithms (see the clause about window length and window functions above and chapter 5).

4.2.5 Calibrating a power analyser

Calibrating a power analyser involves at least four steps.

- 1) Determine the calculation algorithms and presentation methods used. These should be stated in the specification of the instrument but are most often omitted. However, if not known, the calibration could show errors that are due to an odd, but not incorrect, choice of calculation algorithm or presentation method and does not constitute a real error. This check should be made for each type of instrument, and does not have to be repeated for each calibrated object.
- 2) Check for errors due to transformer core non-linearity for each type of current probe and voltage transformer included in the system. These errors might not show up in a single calibration. The check should be made more thoroughly, once for each type of probe, and a minor check should be made for each calibration.
- 3) Calibrate the system, for each voltage and current probe, at the fundamental harmonic plus a few harmonics using the harmonic display of the instrument.
- 4) Calibrate the other quantities of interest with one probe set.

For high current probes or high voltage probes it may be necessary to calibrate the probes separately. This should, however, be avoided when possible. See chapter 8 of this work describing a power analyser calibration method.

4.2.6 Conclusions

The main sources of uncertainty for power analysers are:

- For all measurements involving current: the current clamp probe, if used. It is most often not included in the instrument specification.
- For reactive power and power factor: the calculation algorithm, if not known.
- For individual harmonics: the linearity of the transformers and/or current clamps and the frequency range of these, and the phase angle presentation method, if not known.
- For (active) power harmonics, total active power and fundamental active power at low power-factors and reactive power at high power factors: the

phase angle error of the current clamp probe and other phase angle errors of the system.

Calibration of a power analyser should be made for each probe set using the harmonic display and multi-frequency signals. Other quantities can be calibrated with one of the probe sets. If not known, the calculation algorithms and presentation methods must first be validated as well as the linearity of the magnetic type of probes.

4.3 Instrument transformers and harmonics

It is well known that many instrument transformers may have a limited frequency response. In the IEC "General guide on harmonics and interharmonics measurements and instrumentation..."[1], some statements are made on what can be expected of voltage and current (instrument) transformers. This guide is the work of IEC SC77A. The CIGRE working group 05 of study committee 36 (interference) and the IEEE Power system working group also summarised what was known in the first half of the 1980's [24, 26]. References are frequently made to [27] and [28].

To explain the frequency response of ordinary instrument transformers the equivalent circuit diagram of Figure 11 or a similar simplified circuit of Figure 12 are most often used.

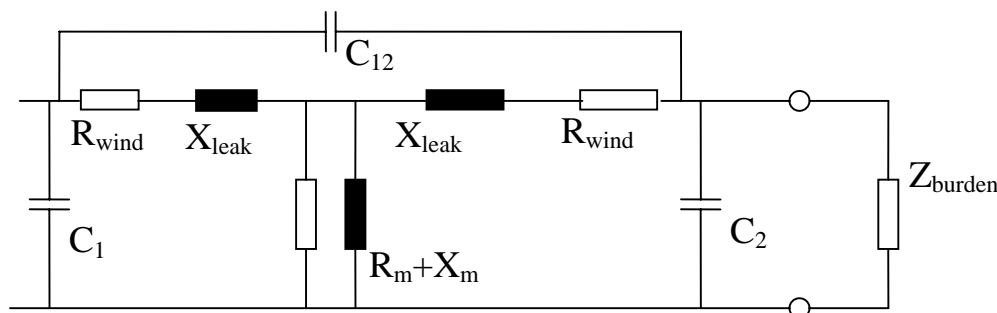


Figure 11 Equivalent circuit diagram for a transformer at medium frequencies.

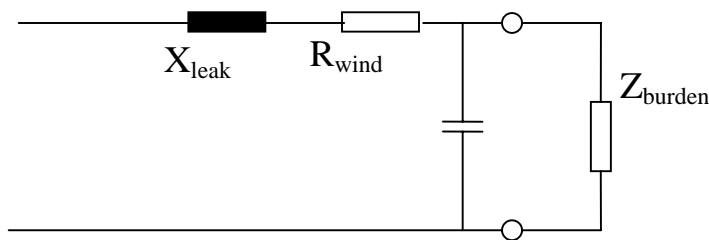


Figure 12. Simplified equivalent circuit diagram.

These are common equivalent diagrams of a transformer except for the capacitors. The capacitors C_1 and C_2 are the lumped stray capacitance of the primary and secondary winding, respectively, and C_{12} is the stray capacitance between the windings. At low frequencies such as 50 Hz they may be negligible but for higher frequencies they may form several resonance circuits, together with the leakage and burden reactance, at various frequencies. From Figure 11, it can further be deduced that grounding (that affects the voltage across C_{12}) as well as the loading (including long cables), especially inductive or capacitive loading, may well affect the frequency response. In some situations, the equivalent circuit diagram of Figure 11 may be reduced to the circuit diagram according to Figure 12.

For high voltages, the increased insulation demand may increase the stray capacitance and the leakage inductance, which will cause resonances to move towards lower frequencies. Further, voltage transformers might be of special designs with inferior frequency response. Cascaded transformers and capacitive voltage transformers (CVT) might be used at higher voltages. In the Swedish 400kV transmission system, CVTs are used as a rule and cascaded transformers are common in high voltage laboratories.

The above equivalent circuit diagrams are not useful for either CVTs or cascaded transformers. An equivalent circuit diagram for cascaded transformers is given by [23] and [29], and for CVTs an equivalent circuit is given by [22]. The first resonance frequency of a CVT most often is in the region 300-600 Hz and cascaded transformers may have their first resonance in the same region.

Two basic evaluation methods are used. The most usual is to test the transformer with a sinusoidal signal, which frequency is increased stepwise. This is most often done at a reduced voltage level, for practical reasons. It is also often more practical to energise the transformer from the low voltage side, but this can give quite different results, according to [24]. A reduced

voltage level can also affect the frequency response due to the change in magnetisation and possible saturation, at least for CVTs according to [28]. It may be possible to use the power system and its harmonic content as the signal source[30], but the natural fluctuation of these harmonics and the high background noise may cause an unreasonable high measurement uncertainty[28]. The second method can be performed at the ordinary voltage level, and is based on an evaluation of the impulse response of the transformer[31, 32].

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5 Measurement of AC-quantities - the digital sampling approach

What is a digital meter? Many "digital" meters use analogue circuits for the entire measuring sequence and then make an analogue-to-digital conversion on the output signal to enhance the readability or the further application of the output. The speed demand of the conversion (the update rate) is often very low and an accurate conversion with a high resolution can easily be achieved. This approach works excellent for many situations, especially when the measured quantity can be directly measured with one analogue circuit, without any kind of further calculation other than simple operations such as low pass filtering. However, power measurements and rms measurement of voltage and current also involves an instantaneous multiplication of voltage and current. (rms measurements can be seen as the special case of power measurement where both signals u and i are identical, that is when the load is a 1Ω resistor.) An analogue multiplication circuit can be made in several ways: mechanically by current through two coils, by electronic multiplication circuits, by the Hall effect, by thermal converters[1], or by pulse-width modulation[2, 3]. The analogue multiplication is always subject to errors of various kind; the most difficult error to handle is often the high demand of linearity that is required of an analogue multiplication circuit. Digital multiplication on the other hand is not subjected to other errors than truncation errors which can easily be kept at a negligible level.

5.1 Introduction to digital sampling measurement techniques

Meters based on digital sampling are most often called "sampling meters" and this terminology is adopted in this work. Still, this might not be quite appropriate as all meters with a digital output have a sampling function. More appropriate would be to make a distinction between these approaches as "analogue signal processing meters" and "digital signal processing meters". Digital multiplication of instantaneous values requires discretisation of both the time and the level. The difficulty of the digital multiplication approach is then often due to the combined demands on the conversion rate (time discretisation) and the resolution (level discretisation).

5.1.1 Numerical or discrete integration

Computers and A-D converters became available in the late 1960's and the beginning of the 1970's. The idea of digital signal processing followed quite rapidly. The major limitations were A-D converter conversion times and slow processor speed. Therefore only the most efficient algorithms were of

interest. As early as 1967 Davis et al [4] showed that for a sinusoidal signal and with phaselocked sampling, the rms value of the signal and the active power of a linear load connected to this signal could be calculated by a numerical integration of as few as three samples taken during precisely one period. (This is analogue to the fact that the active power of a symmetrical three-phase load fed by a symmetrical source is constant.) A sampling wattmeter based on these principles was suggested and a prototype was built by Sacerdoti et al [5] in 1966. The first sampling wattmeter in practical use, as far as the author knows, was built in 1974 in USA by the National Bureau of Standards, NBS [6] The sampling wattmeter described in this work can also use this calculation method and it will be called "discrete integration" in the following.

Real signals are, however, never quite sinusoidal, and it was suggested that if more samples were taken without increasing the sampling rate much, a signal containing some lower harmonics could still be measured without systematic errors due the sampling rate. Several wattmeters were built in UK by the National Physical Laboratory, NPL, based on these principles. It was shown by Clark and Stockton [7] that if neither the voltage nor the current contained harmonics higher than an integer number n , it would still be sufficient with a sampling rate of about 3 samples per period if the number of samples in each sequence, N , was higher than $2*n$. The important thing is then to take these (equal-spaced) samples during exactly M periods and making sure that M and N have no common factors. It was shown that this is equal to taking N samples in one period if the signal is stable, this is further explained in a later section of this chapter.

Whatever measurement algorithm is used, an exact measurement of the (mean) active power of a stationary periodic signal requires that the measurement is made over an (exact) integer number of periods. This can be achieved by synchronising the measuring circuit to the fundamental frequency of the signal. In practical measurements a phase-locking circuit is used, which usually phase-locks to one of the voltages since the voltage tends to be less distorted and easier to phase-lock to. In laboratories the synchronisation can also be made by phase-locking the signal generator to the wattmeter or by phase-locking both the signal generator and the wattmeter to an external frequency, this has been made from time to time in the work presented in this thesis and is used in the latest standard wattmeter presented by The Physichalische Technische Bundesanstalt [8]. In [9] a combined software-hardware phaselocking algorithm is used.

If the samples are taken asynchronously a "truncation" error will occur. This leads to an error, changing with the frequency of the measured signal. However, it is possible to make an analysis of the sampled sequence, either to minimise the error or to compensate for it, or both. If the sampling frequency is much higher than the frequency of the signal, the number of samples can be adjusted such that the measurement is quasi-synchronised. If the voltage and the current are given by $\hat{U}\sin(2\pi f_{ac}t+\alpha)$ and $\hat{I}\sin(2\pi f_{ac}t+\beta)$ the error can then be shown [10] to be equal to:

$$e = UI \left(\cos\phi - \frac{1}{N} \sum_{n=0}^{N-1} \cos\left(2 \cdot \frac{2\pi n f_{ac}}{f_s} + 2\alpha + \beta\right) \right) = \\ = \frac{UI}{N} \cdot \frac{\sin\delta}{\sin 2\pi f_{ac}/f_s} \cos(2\alpha + \beta - 2\pi f_{ac}/f_s)$$

where δ is the missing or extra phase angle due to the truncation and then

$$|e| \leq \frac{UI}{N}.$$

However, if the signal frequency (or one of the harmonics) is close to $f_s/2$, f_s , or a multiple of f_s large errors can occur. The risk for this can be decreased if a high number of samples are taken[10], but not totally avoided. Various designs based on the asynchronous or quasi-synchronous sampling strategy has been built, mainly wide-band wattmeters with a frequency range up to about 100 kHz [10-12].

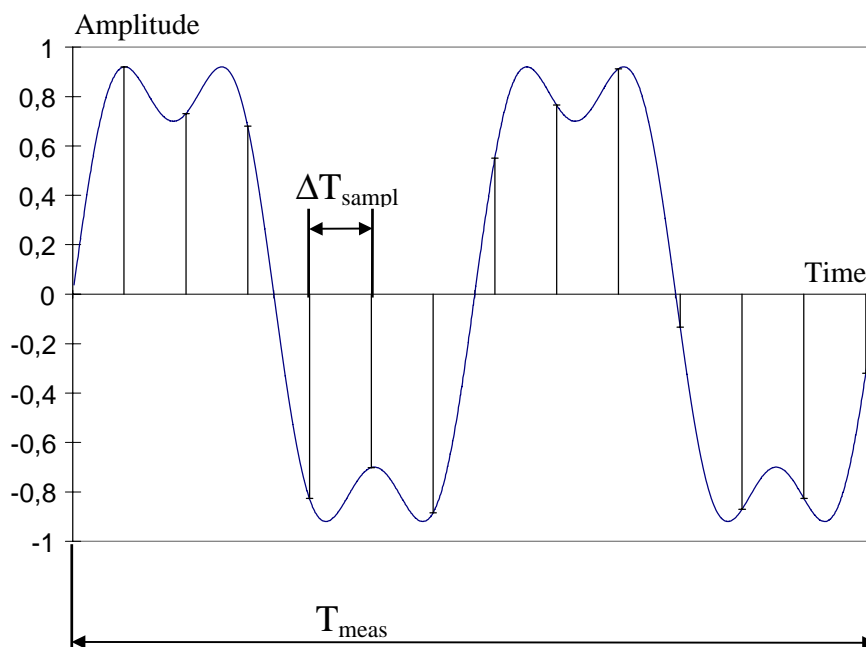
If the signals are taken randomly and the measurement time is much longer than the period time of the lowest frequency contained in the signal the above problem for signals close to $f_s/2$ and its multiples can be avoided. The error will decrease with the number of samples taken, however just by a factor $\sim 1/\sqrt{n}$. With shorter conversion times and higher computer efficiency of modern technology it is possible to get a reasonable accuracy with this method, even if it takes more than one million samples to get an uncertainty of less than 0.1%. This scheme is mainly used for higher frequencies when a measurement strategy based on synchronous sampling would put even higher demand on the sampling rate[13, 14]. The main advantage of this strategy is that the sampling frequency can be lower than the frequency of the signal.

All digital sampling wattmeters are built on the basic method of sampling sequences of voltage and current. In order to compute power from these digital sequences, the time relation of these sequences must also be known.

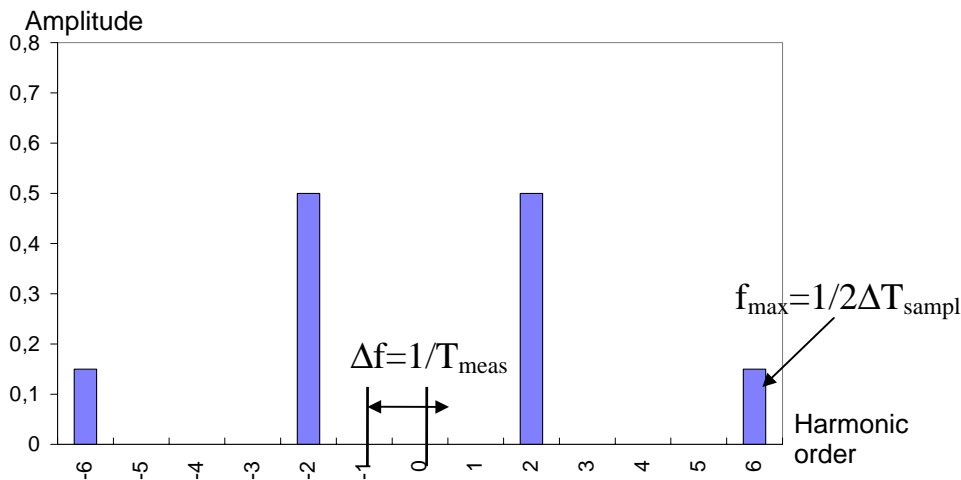
Most systems use simultaneous sampling of voltage and current but that requires two A-D converters. If the same A-D converter (and sample-and-hold) is used for both voltage and current measurements a time shift will occur; this time shift must be compensated to achieve a high accuracy. This design is mainly used for cost reasons, but in a laboratory environment, with stable signals, it can be used to minimize the phase angle uncertainty due to the difference in delay-time of the two A-D converters.

5.1.2 Spectral analysis

All the sampling wattmeters described above uses the same numerical integration principle. This algorithm can be used to calculate the (total) active power but also the rms values of the voltage and the current. It works also for non-sinusoidal signals, at least if the number of harmonics is limited. No information on the spectral composition of the signal can be derived. However, it is well known that a Fourier transform can be made on all stationary periodic signals. According to the Nyquist theorem the sampling rate must then be kept higher than twice the highest harmonic contained in the periodic signal, if a correct result from such a frequency analysis should be obtained. The sampling rate f_s will determine the highest frequency that can be measured, $f_s/2$, while the total measurement time T will determine the lowest frequency that can be resolved and the frequency resolution will be $1/T$.



a) sampled waveform., measurement time T_{meas} and sampling time ΔT_{sampl} indicated



b) discrete spectra of a), frequency resolution Δf and maximum frequency f_{max} indicated

Figure 13. Discrete Fourier transform of signal with 30 % third harmonic, the relations between $T_{meas} - \Delta f$ and between $\Delta T_{sampl} - f_{max}$.

From a mathematical point of view the choice not to measure during an infinite time but during a limited time interval is equal to a multiplication of the signal with a rectangular time window function. The multiplication in the time domain is equal to a convolution in the frequency domain with the Fourier transform of the rectangular window and the Fourier transform of the signal. The Fourier transform of the rectangular window is $\sin(\pi f T) / \pi f T$, where f is the signal frequency and T is the measuring time. As can be seen in Figure 14 below, the convolution with this function has no effect on the measurement of signals with frequencies such that $\sin(\pi f T) / \pi f T = 0$, that is $f_{signal} = n \cdot 1 / T_{meas}$.

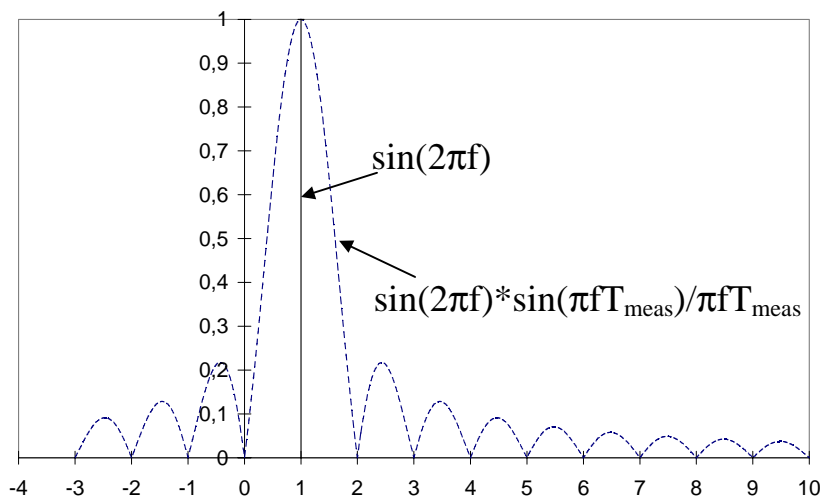


Figure 14. Measurement (harmonic amplitudes) of a sinusoidal signal with a one period rectangular window ($T_{meas} = T_{period}$), showing no effect of the window function if the timing is perfect.

However, if the measurement time is not perfectly synchronised with the signal period, the convolution will cause measurement errors as in Figure 15 below, where signal components of several harmonic orders are indicated despite that the signal is (purely) sinusoidal.

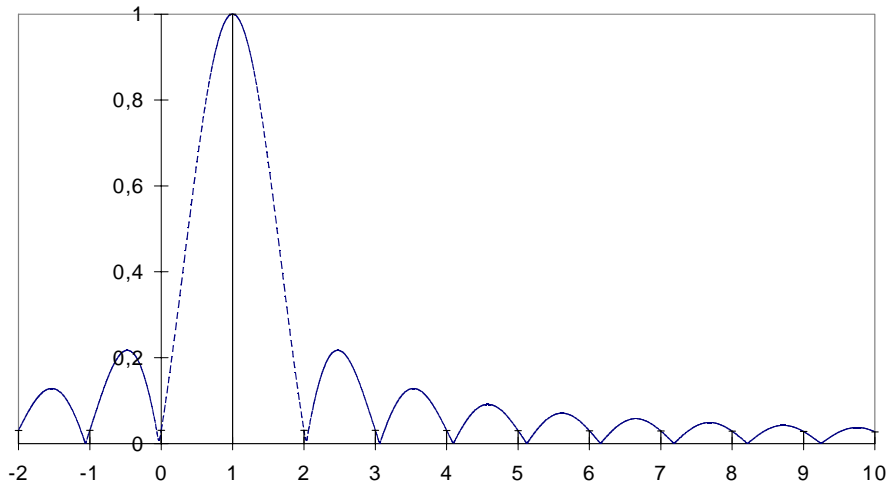


Figure 15. Measurement of a sinusoidal signal with $T_{\text{meas}}=0.97 T_{\text{period}}$ and the resulting harmonic amplitudes due to spectral leakage.

This is called spectral leakage. As can be seen in Figure 15, large errors due to this lack of synchronisation can occur at a distance of several harmonic orders from the signal that is the source of the leakage. Therefore this phenomenon is often called long range spectral leakage.

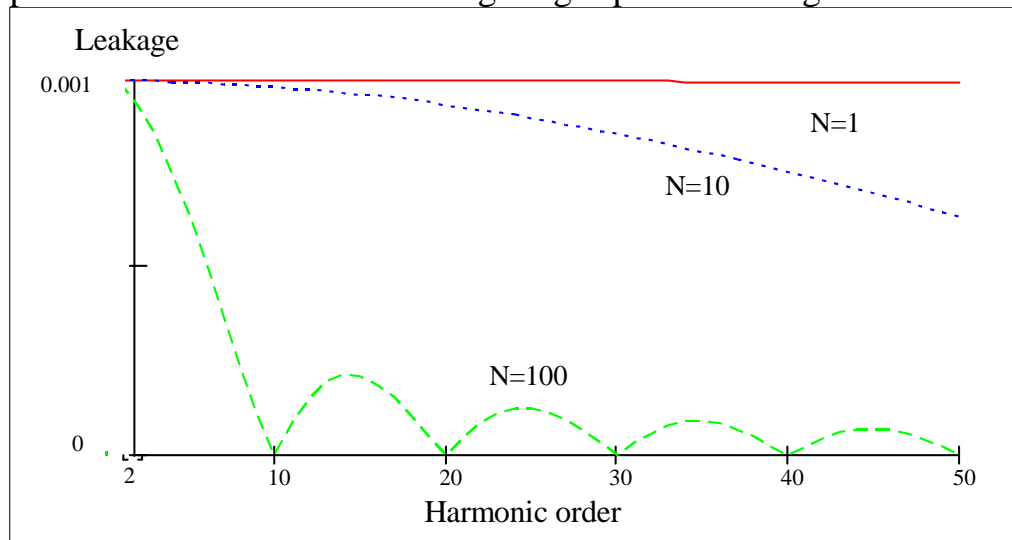


Figure 16. The relative spectral leakage from a large fundamental due to a timing error of 0.001 pu as a function of harmonic order for some different window lengths, N = window in number of periods.

If the timing error is small, the long range leakage from a large fundamental harmonic is more or less directly dependent on the relative timing error and

can only be decreased by a rather large window length, much more than 10 periods. The leakage from a large fundamental harmonic due to a relative timing error of 0.001 pu is shown in Figure 16, for some different window length expressed by the number of periods.

The long range leakage can be very much reduced by the use of another time window function, that has a Fourier transform that decreases more rapidly with the frequency, as the commonly used Hanning window. The Fourier transform of the Hanning window can be seen below where it is compared to the Fourier transform of the rectangular window. However, all such window functions will cause a short-range leakage instead of the long-range leakage, especially to the harmonic nearest a large amplitude harmonic. This may further occur even if the sampling is perfectly synchronised, as can be seen in Figure 17.

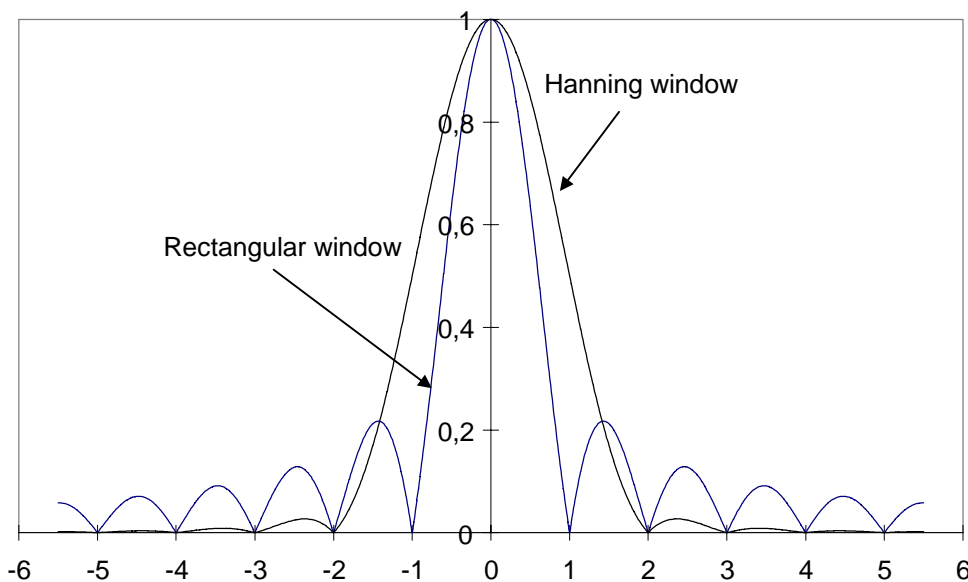


Figure 17. The Hanning window convolution function compared to that of a rectangular window.

Therefore, If the signal is not very stable and not completely periodical it can be advisable to use some kind of window function to avoid the long range leakage, sacrificing some of the accuracy of nearby harmonics. It is then best to chose a window time of several periods so that most of the short range leakage will affect the interharmonic frequencies rather than harmonic frequencies.

In a laboratory environment, however, where the signals can be kept stable, the long range spectral leakage can be kept on a negligible level even when using a rectangular window. The use of another time window would then just introduce a unnecessary short range leakage. A rectangular window is

therefore used in all measurements with the digital sampling wattmeter presented in this work (DSWM). As can be seen by Figure 13, there is an upper limit of the frequency caused by the sampling frequency, or, in other words, the number of samples taken in each period of the fundamental frequency. Harmonic components of the signal that is of higher frequency than this upper limit will influence the measurement in such a way that their effects can not be separated from the data by any post-DFT calculation. Therefore, a so called anti-aliasing (low pass) filter is commonly used to avoid that signal components of higher frequencies will affect the measurements in an uncontrolled way. However, such a filter will always have some influence also on signal components within the frequency of interest. Once again, in a laboratory environment, where stable and low-noise signal generators are used, it is most often better to do without the anti-aliasing filter. No filters but the natural upper bandwidth and the integration time of the sampling instruments have therefore been used for the measurements with the DSWM.

The calculation of amplitudes and (often) the phase angles of each harmonic of voltages and current are frequently used in power quality measurements. This can be made by various algorithms which often is discrete versions of transforms from time-domain to frequency-domain. The most frequently used is the fast Fourier transform, FFT, which is a fast algorithm for DFT. Other algorithms are the Hartley transform, the Walsh transform, the Gabor transform and wavelets. These are often used for special applications, often when the assumption of a stationary signal is not valid.

If just one or a few harmonics are of interest it may be advantageous to extract these harmonics by a best-fit time-domain algorithm instead of making a spectral analysis.

The two basic calculation methods that are used in this work to transform the sampled sequences of the signals into values of the desired quantities is the discrete integration and discrete Fourier transform. By means of discrete integration methods accurate values can also be obtained of other parameters such as apparent power, rms-values of voltage and current with or without DC components and the phase angle between sinusoidal voltage and current.

By means of the second method, Discrete Fourier Transform(DFT) of the digitised signals, the frequency spectra of power, voltage and current can be obtained from the same set of samples as for the first method. From these spectra the total power, voltage and current and most other quantities for the total signal can also be calculated.

The power measurement capability of these methods is investigated for signals that are close to sinusoidal. The result can then rather easily be adapted to analyse the measurement capability for other measured quantities.

5.2 Discrete integration method

The definition of mean power for a periodic signal is

$$P = \frac{1}{T} \int_T u(t) i(t) dt \quad (100)$$

where T should be equal to the period or an integer number of periods. The discrete form of this equation is

$$P = \frac{1}{N} \sum_{j=0}^{N-1} u(j\Delta t) i(j\Delta t) \quad ; \quad N * \Delta t = \text{integer number of } T \quad (101)$$

The question is which conditions have to be fulfilled for this formula to give a correct value of the mean power. To investigate this a Fourier-series error analysis is used.

5.2.1 Errors of the discrete integration method

If $u(t)$ and $i(t)$ are repetitive with a period of T , they can be resolved into Fourier series according to

$$u(t) = \sum_{q=0}^{\infty} \sqrt{2} U_q \sin\left(\frac{2\pi q t}{T} + A_q\right) \quad (102)$$

$$i(t) = \sum_{r=0}^{\infty} \sqrt{2} I_r \sin\left(\frac{2\pi r t}{T} + B_r\right) \quad (103)$$

U_q and I_r are the rms voltage and current harmonic amplitudes. If Equation (102) and Equation (103) are multiplied they will produce the instantaneous power $p(t)$, which then can be expressed by the equation

$$p(t) = u(t) i(t) = P_0 + \sum_{k=1}^{\infty} P_k \cos\left(\frac{2\pi k t}{T} + \Phi_k\right) \quad (104)$$

where each P_k and Φ_k are depending on all voltage and current harmonic amplitudes and phase angles. P_k is called the product harmonics and can be calculated by

$$P_k = \sum_{\substack{|q-r|=k, \\ q+r=k}} U_q I_r \quad (105)$$

That is, voltage harmonics of order q and current harmonics of order r will, together, contribute to the product harmonics of order $(q+r)$ and $(|q-r|)$. Additionally:

$$P_0 = U_0 I_0 + \sum_{q=r} U_q I_r \cos(\Phi_{qr}) \quad (106)$$

where $\Phi_{qr} = A_q - B_r$. P_0 can be identified as the mean power by substituting Equation (104) into Equation (100).

If the average power W is calculated by discrete integration, with N samples made during a time corresponding to M periods, Equation (101) will turn into

$$W = \frac{1}{N} \sum_{j=0}^{N-1} u\left(\frac{jMT}{N}\right) i\left(\frac{jMT}{N}\right) = \frac{1}{N} \sum_{j=0}^{N-1} p\left(\frac{jMT}{N}\right) \quad (107)$$

and the error can then be determined by finding the value of $W - P_0$. By substituting the Fourier series expression of the instantaneous power according to Equation (104) into Equation (107), subtracting P_0 and then switching the summing order, the sampling error can be calculated as

$$W - P_0 = \frac{1}{N} \sum_{k=1}^{\infty} P_k \sum_{j=0}^{N-1} \sin\left(\frac{2\pi jkM}{N} + \Phi_k\right) \quad (108)$$

According to Clark and Stockton [7] this equation can be reduced to

$$W - P_0 = \sum_{k>0} P_k \sin\Phi_k \quad ; \quad \frac{kM}{N} = \text{integer} \quad (109)$$

That is, only some of the product harmonics contribute to the error, the order of which depends of the choice of M and N . Most often the phase angle Φ_k cannot easily be calculated, but a maximum evaluation can be made by:

$$|W - P_0| \leq \sum_{k>0} P_k \quad ; \quad \frac{kM}{N} = \text{integer} \quad (110)$$

The condition of N samples in M periods gives the relation:

$$\frac{kM}{N} = \frac{kf_1}{f_s} = \text{Integer} \quad (111)$$

where f_1 is the fundamental frequency of the measured voltage and current and f_s is the sampling frequency.

This means that the maximum error inherent in the discrete integration method is the sum of all the product harmonics which frequency ($=kf_1$) is equal to an integer multiple of the sampling frequency (f_s). It also means that if M and N are selected such that they have no common factor, the lowest-order product harmonics that will give any contribution to the error is the product harmonic P_N . This in its turn will have the following results:

- If both the voltage and the current are non-sinusoidal, a voltage harmonic of order q will contribute to P_N in combination with current harmonics of the orders $r=N-q$ or $r=N+q$, if they exist. If they do not exist for any q , that is if $N > (q_{\max} + r_{\max})$, then there will be no contribution to the error from the measuring method.
- If one of the signals is truly sinusoidal then the lowest-order voltage or current harmonics that will contribute to P_N will be the $(N-1)$ th. That is, if $N > (q_{\max} + 1)$ or $N > (r_{\max} + 1)$, then there will be no contribution to the error from the measuring method.

For example: if M and N are selected so as to have no common factor and if there are voltage and current harmonics up to the 30th order, but not higher, selecting $N=64$ will be enough to ensure that there will be no error contribution from the discrete integration method.

The result above will directly influence the sampling frequency. The frequency f_s is derived from M and N and the frequency of the first harmonic f_1 as $f_s = f_1 N / M$. This means that the only restriction on f_s , with respect to this error, is that M and N should not have any common factor and e.g. the Nyquist criterion $f_s > 2f_{\max}$ does not have to be fulfilled.

5.2.2 Errors due to quantisation

The finite number of amplitude steps in the ADCs causes a quantisation error which size depends on the quantisation step size Δ . For a n -bit ADC with dynamic range $2D$ the maximum size of the error could be calculated as one step

$$e_{\max} = \Delta = \frac{2D}{2^n} \quad (112)$$

If the error e is modelled as a random variable with an amplitude uniformly distributed between $\pm \Delta/2$, the noise power, or variance, is

$$\sigma_e^2 = \frac{\Delta^2}{12} \quad (113)$$

If P is calculated according to equation (101) and the noise of each channel is according to equation (113) the variance of $\Delta P/P$ will be

$$\sigma_p^2 = \frac{1}{N^2} \sum_{j=0}^{N-1} \left(\frac{u_j \sigma_i}{u_j i_j} \right)^2 + \left(\frac{i_j \sigma_u}{u_j i_j} \right)^2 = \frac{\sigma_e^2}{I_{\text{rms}}^2 N} + \frac{\sigma_e^2}{U_{\text{rms}}^2 N} \quad (114)$$

If the full dynamic range of both the (identical) ADCs is used and the signals are close to sinusoidal then $U_{\text{rms}}^2 = I_{\text{rms}}^2 = D^2/2$ and

$$\sigma_p^2 = 2 \frac{\left(\frac{2D}{2^n} \right)^2}{12N \frac{D^2}{2}} = \frac{1}{3N 2^{2(n-1)}} \quad (115)$$

5.2.3 Errors due to sampling time-jitter

If there is sample timing error Δt when a signal is sampled, and the signal is close to sinusoidal and uses the full dynamic range $2D$ of the ADC, the resulting amplitude error of a sample will be

$$e_j \approx \Delta t \frac{du}{dt} = D\omega\Delta t \cos(\omega t) \leq D\omega\Delta t \quad (116)$$

Therefore a sampling time-jitter will cause amplitude noise which rms-value depends on the derivative of the measured signal. If the jitter is uniformly distributed in the range $\pm\Delta t/2$, then e_j can be approximated as being uniformly distributed in the range $\pm D\omega\Delta t/2$ and the variance σ_j^2 will be

$$\sigma_j^2 < \frac{(D\omega\Delta t)^2}{12} \quad (117)$$

and according to Equation (115115) the variance of $\Delta P/P$ will be

$$\sigma_p^2 < 2 \frac{(D\omega\Delta t)^2}{12N \frac{D^2}{2}} = \frac{(\omega\Delta t)^2}{3N} \quad (118)$$

5.3 Discrete Fourier transform method

The discrete Fourier transform of a sequence $x(n\Delta T)$ can be written as

$$X(k\Delta f) = \frac{1}{N} \sum_{n=0}^{N-1} x(n\Delta T) e^{-j2\pi kn/N} \quad k=0,1,2,\dots,N-1 \quad (119)$$

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta T} = \frac{f_s}{N} \quad (120)$$

where Δf is the frequency resolution and f_s is the sampling frequency.

The value of each $X(k\Delta f)$ is a complex number and the absolute value $|X(k)|$ and phase angle $\Phi(k)$ for each component is given by the formula (Δf is excluded for clarity)

$$|X(k)| = \sqrt{(\text{Re}\{X(k)\})^2 + (\text{Im}\{X(k)\})^2} \quad (121)$$

$$\Phi(k) = \arctan\left(\frac{\text{Im}\{X(k)\}}{\text{Re}\{X(k)\}}\right) \text{ if } \text{Re}\{X(k)\} \geq 0 \text{ else } \arctan\left(\frac{\text{Im}\{X(k)\}}{\text{Re}\{X(k)\}}\right) + \frac{\pi}{2} \quad (122)$$

and the amplitude of the components is given by the formula

$$A(k) = 2 |X(k)| \quad k = 1,2,\dots,N-1 \quad (123)$$

$$A(0) = \operatorname{Re}\{X(0)\} \quad (124)$$

5.3.1 Errors of discrete Fourier transform method

It has been shown, for example by Brigham [15], that $A(k)$ and $\Phi(k)$ are equal to the amplitude and the phase coefficients of the Fourier series representation of $x(t)$,

$$x(t) = A(0) + \sum_{k=1}^{N-1} A(k)\cos(2\pi kt / T + \Phi(k)) \quad (125)$$

if the following conditions are fulfilled:

- 1) $x(t)$ is periodic.
- 2) $x(t)$ does not contain frequency components above $f_s/2$ (the Nyquist criterion).
- 3) The sampling interval corresponds exactly to an integer multiple of periods.

This means that if the above conditions are fulfilled no errors from the discrete Fourier transform will occur. There will, however, appear errors from the ADC quantisation and from sampling time-jitter.

5.3.2 Errors due to quantisation

With a quantisation noise model of an amplitude uniformly distributed in the range $\pm \Delta/2$, Wagdy [16] has shown that the variances of the amplitude, phase and DC-component evaluated by DFT are as follows.

The amplitude variance is

$$\sigma_a^2 = \frac{\Delta^2}{6N} \quad (126)$$

$$\Delta = \frac{D}{2^n} \quad (127)$$

where D is the dynamic range and n is the number of quantisation bits.

The phase-angle variance is

$$\sigma_\phi^2 = \sigma_a^2 \left(\frac{1}{A_1^2} + \frac{1}{A_2^2} \right) = \frac{\Delta^2}{6N} \left(\frac{1}{A_1^2} + \frac{1}{A_2^2} \right) \quad (128)$$

where A is the amplitude of the two signals between which the phase angle is measured.

The DC component variance is

$$\sigma_{dc}^2 = \frac{\Delta^2}{12N} \quad (129)$$

When the mean power P for each harmonics is calculated from the DFT of U and I then

$$P = UI\cos\Phi \quad (130)$$

and then the variance of $\Delta P/P$ from the quantisation noise will be

$$\sigma_p^2 = \frac{1}{(UI\cos\Phi)^2} \left((I\cos\Phi \sigma_a)^2 + (U\cos\Phi \sigma_a)^2 + (UI\sin^2\Phi \cdot \sigma_\Phi)^2 \right) \quad (131)$$

Which equals

$$\sigma_p^2 = \frac{\sigma_a^2}{U^2} + \frac{\sigma_a^2}{I^2} + \tan^2\Phi \cdot \sigma_\Phi^2 \quad (132)$$

If the full dynamic range is used and U and I are close to sinusoidal then $U^2 = I^2 = D^2/2$ and

$$\sigma_p^2 = 2 \frac{D^2}{2^{2n} 6N \frac{D^2}{2}} + \frac{\tan^2\Phi \cdot D^2}{2^{2n} 6N} \left(\frac{2}{D^2} + \frac{2}{D^2} \right) \quad (133)$$

Which equals

$$\sigma_p^2 = \frac{1}{2^{2n}} \left(\frac{4}{6N} + \frac{4\tan^2\Phi}{6N} \right) \quad (134)$$

5.3.3 Errors due to sampling time-jitter

The jitter causes an amplitude noise variance, which is estimated in 5.2.3 as being:

$$\sigma_j^2 < \frac{(D\omega\Delta t)^2}{12} \quad (135)$$

Combined with Equation (132) this will cause a variance of $\Delta P/P$ from sample time jitter as in

$$\begin{aligned} \sigma_p^2 &< 2 \frac{(D\omega\Delta t)^2}{12N \frac{D^2}{2}} + \frac{\tan^2\Phi(D\omega\Delta t)^2}{12N} \left(\frac{2}{D^2} + \frac{2}{D^2} \right) = \\ &= \frac{(\omega\Delta t)^2}{3N} + \frac{\tan^2\Phi(\omega\Delta t)^2}{3N} \end{aligned} \quad (136)$$

5.4 References

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6 The Digital Sampling Wattmeter

6.1 Wattmeter Design

The Digital Sampling Wattmeter (DSWM) is, basically, a computer that processes data from two synchronised Analogue-to-Digital Converters which measures the voltage and current signals.

The DSWM is organised as shown in Figure 18 below. The most important part of the system is the Analogue-to-Digital Converters(ADCs). They are referred to as DMM-U and DMM-I since they are digital multimeters with ADC-capability. An 5-decade Inductive Voltage Divider, referred to as the IVD, can be used to divide the input voltage down to the 10 V range of the DMM-U. Different shunt resistors are used to convert the input current to the 1 V range of the DMM-I.

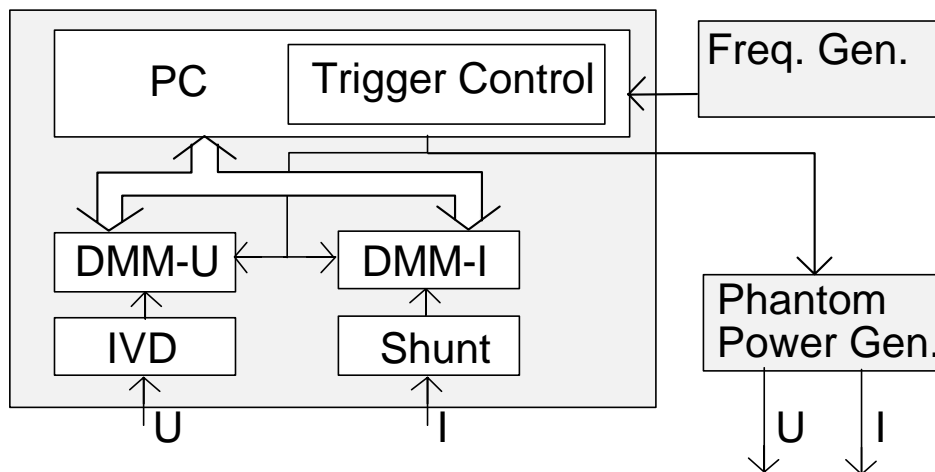


Figure 18 Block diagram of the DSWM-system in "frequency control mode".

The frequency generator is used to create a pulse-train which is divided by the M/N-divider, mounted in the personal computer. The outputs of the M/N-divider is then used to phase-lock the power generator and the DMM-U and DMM-I, in order to get exactly N samples during M periods of the voltage and current. The personal computer is used for instrument control by a standard IEEE 488-bus, and to process the samples taken by the DMM-U and DMM-I.

6.2 Analogue-to-Digital converters

As Analogue-to-Digital converters(ADCs), two high precision digital multimeters with ADC-capability, HP3458, are used. The main features of the HP3458 multimeters are a maximum sampling rate of 100 kHz, a resolution of 18 digits at sampling rates less than 50 kHz and the possibility of being externally triggered. Additionally, when they are operated in the "DCV mode" it is also possible to control the sampling time aperture, which can be used to increase the noise immunity. Each sample is then the result of an integration of the signal during a short time. The multimeters are also equipped with an IEEE488 bus interface and, as an option, a 148 Kbyte reading memory that can contain 37 000 readings at a resolution of 18 bits. The internal memory is necessary for keeping up the sampling speed, due to speed limitations of the bus interface. The sampled data can then be transferred at the end of each sampling sequence instead of after each sample.

However, in the "DCV-mode", the input channel has an upper frequency limit of about 100 kHz according to the specification. Therefore, when good precision at higher frequencies is needed, the "DSDC mode" with an input channel bandwidth of 12 MHz, according to the specification, should be used. The noise immunity will then be lower and a larger amount of samples will be needed to achieve the same stability of the result of the measurements. Therefore, both modes must be investigated to determine which mode has the best performance for a given situation.

Both multimeters are used in *voltage* mode (DCV or DSDC) for best accuracy and therefore shunt resistors are needed to convert current into voltage. Further, the internal divider of the voltage measuring multimeter can be replaced by an external divider, as shown in the block diagram.

6.3 Shunt resistors

Currently, two coaxial shunts, 5A/0.16 Ω and 1A/0.8 Ω , are used. These shunts are built by SP using a design developed by the Mendelejev Institute, St Peterburg. As can be seen from Figure 19 they are of a squirrel-cage design. Their most important properties are an extremely symmetrical design and the use of discrete resistors.

The purpose of the design is to minimise the mutual inductance between the output of the shunt and the current path leading to the resistors and also other, external, current loops. The entire inner structure including the current paths, see Figure 19, is made of double-sided copper-clad boards, normally used for printed circuits boards. They are arranged so that the currents on each side of

the boards are equal in magnitude and of opposite direction. As the boards are thin the current loop areas are small and thereby the magnetic fields caused by these loops inside the shunt also are small. The cylindrical symmetry further minimises these magnetic fields. The influence from magnetic fields caused by external current loops is minimised by the cylindrical symmetric design and the sheet-metal housing. The use of discrete resistors minimises the problems associated with distributed resistances such as skin effects. It also enables the type of current paths used in this design by which the resistors are placed far from the possible unsymmetries at the current input, thereby decreasing the mutual inductance even more. The drawback of the use of discrete resistors is a lower pulse rating. This is, however, of less importance as the system is intended for use in steady state.

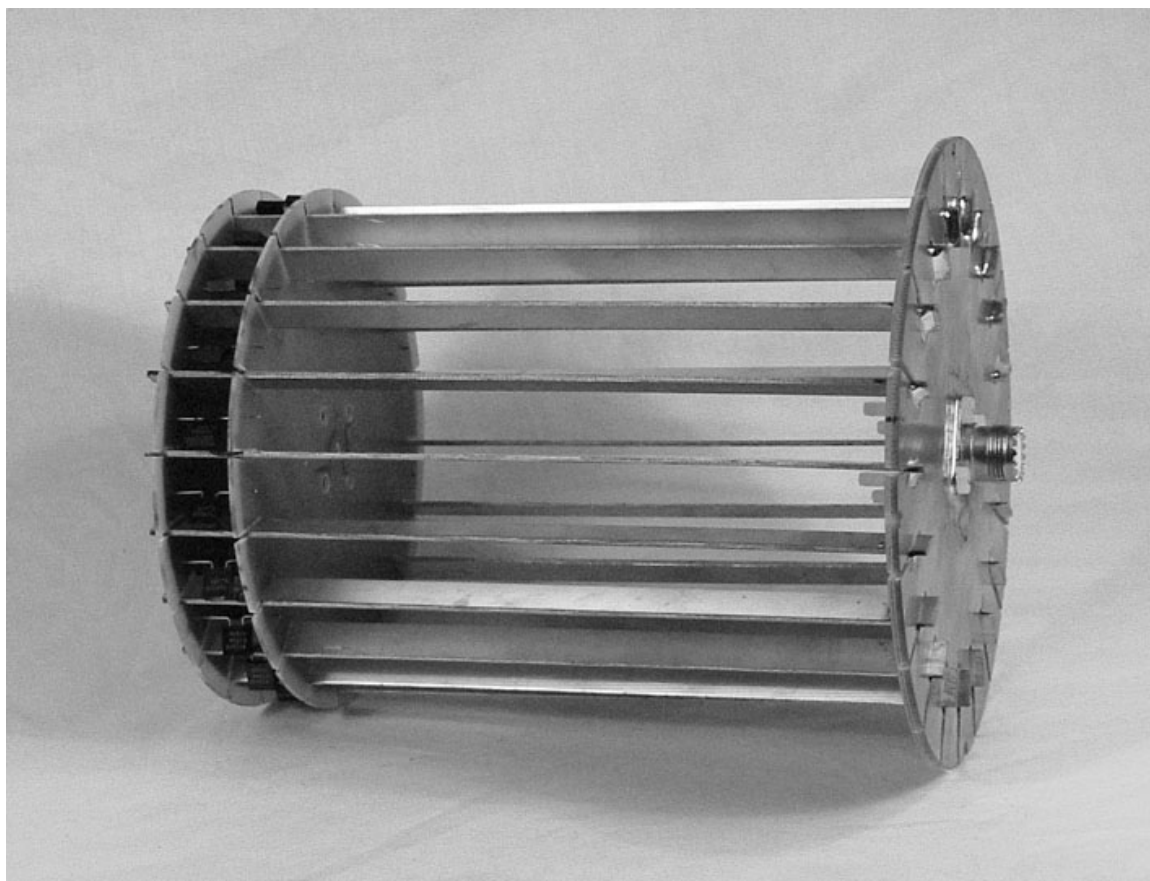


Figure 19 Shunt resistor design

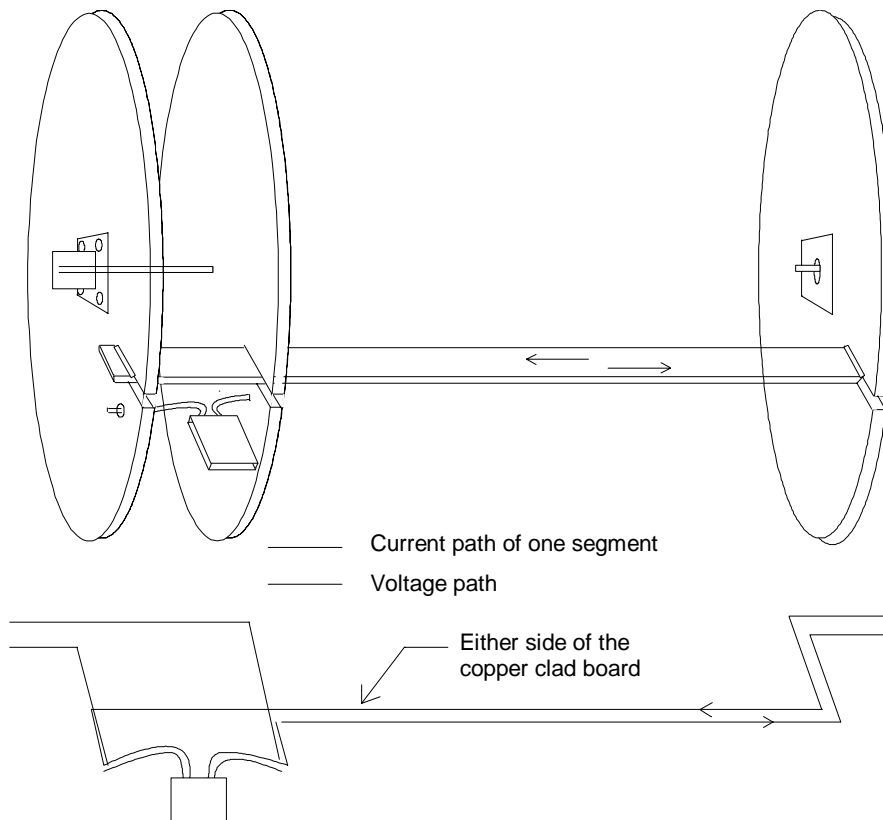


Figure 20 One segment of the shunt resistor and current and voltage paths.

6.4 Inductive voltage divider

The two multimeters have considerably better input channel characteristics in the 10V range, than in the 100V and the 1000V ranges. Therefore, after some tests, an Inductive Voltage Divider (IVD) was added to the system to enable the 10V range to be used for all input voltages. The IVD used is a Siemens “präzisions-spannungsteiler”, $U_{inmax}=6 \cdot f$, which means that the maximum input voltage is $6 \cdot 50 = 300V$ at power line frequency. The IVD is of a 5-decade type and built in the two-stage technique, which minimises the errors due to magnetising current. According to the calibration documents of the Siemens IVD the error is less than 0.5 ppm of input, for both ratio and quadrature.

6.5 Instrument control and data processing

As controller and calculator, a PC with at least two free ISA-board slots is needed. The PC should be equipped with a math processor to reduce the time used for measurement evaluation, especially if the DFT-method is used. At present a -486DX2/66MHz PC is used. An IOtech Personal488 Controller is fitted into one ISA-slot and is used for instrument control by IEEE 488-bus. The other ISA-slot is used for a board specially developed for this system, the M/N-divider.

6.5.1 M/N-divider

The M/N-Divider is designed and built into the computer to synchronise the sampling rate and the power source frequency. This divider can operate in three different modes, and is built up much by much the same principles as in [1, 2]. By means of the M/N-divider the system clock frequency from the frequency generator is divided by two integer numbers M and N, see Figure 21. M and N can be selected by the system and thereby the frequency of the power source and the sampling rate can be controlled and synchronised, in order to get exactly N samples in M periods. This concept for synchronisation is called "frequency control mode". The gate is used to get a controlled and synchronised start of each sampling sequence.

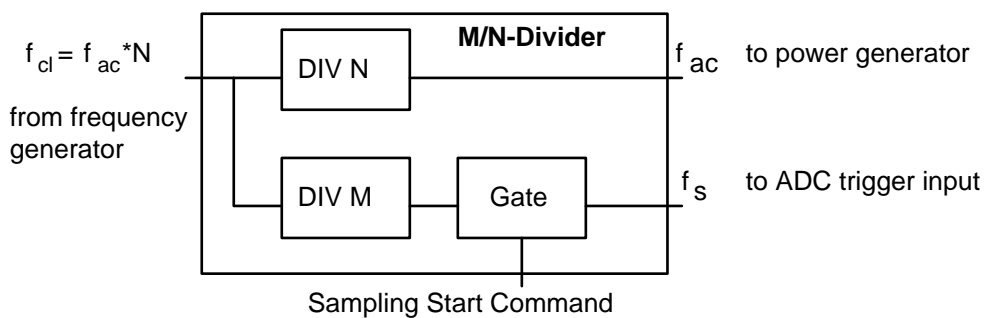


Figure 21 Original M/N-Divider

However, the frequency control input of the ZERA power generator (discussed later) used in the tests requires a frequency of $f_{ac} * 3600$ because the waveforms is synthesised and are built up by 3600 discrete steps that are continuously fed to a digital-to-analogue converter. Thus the M/N-Divider had to be slightly modified according to Figure 22. This type of configuration can be used to synchronise any arbitrary waveform generator that has an external trigger function.

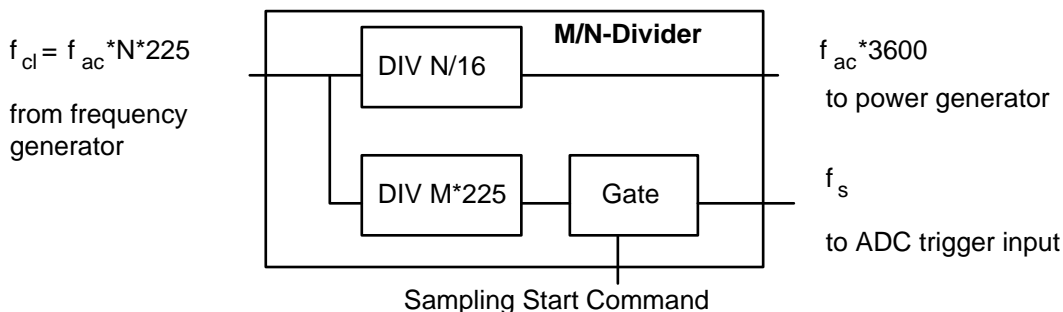


Figure 22 Modified M/N-Divider for the ZERA Power Generator.

The system must also be able to work together with power sources that can not be frequency controlled. In that case the power frequency, f_{ac} , must be

used as reference and a Phase-Locked-Loop(PLL) circuit is inserted and a slightly different configuration of the control circuit is needed. This is shown in Figure 23 below. In this case no system clock frequency generator is needed. This mode of operation is called "phase-locked mode".

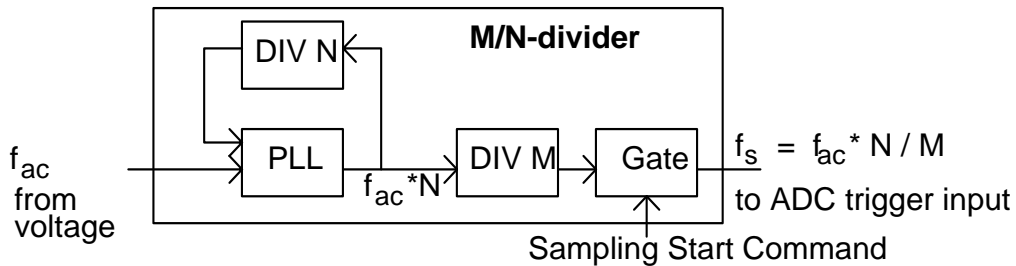


Figure 23 Modified, phase-locking, M/N-divider.

6.5.2 Connections

All instruments are connected to and controlled by the computer through the IEEE 488 bus interface. If the system operates in the "frequency control mode" the frequency generator is also connected to the M/N-Divider by means of a BNC-cable (at TTL level) as shown in Figure 18. The divider output is then connected to the two DMMs (EXT TRIG input) to control the sampling rate (f_s) and to the power source (EXT SYNCH input) to control the frequency (f_{ac}) of the power source. If the system operates in the "phase-locked mode" the voltage is connected to the phase-lock input of the M/N-divider, and the frequency source is omitted.

6.5.3 Software

A DOS software has been developed in the Borland Pascal programming environment, by the author and by Valter Tarasso, SP. A later version for MS Windows is built in the Borland Delphi environment. The software controls different parameters such as the number of samples (N) and the number of periods (M) over which the sampling is to be made. It is also possible to perform multiple measurements and obtain statistics of all results in order to increase the accuracy and to calculate a value of the standard deviation, see Figure 24.

Most of the measurement presented in this work has been done in the DOS environment version; its input and output screen can be studied in Figure 24 and Figure 25. However, a windows version has been developed that enhances man-machine interface, and the output/input screen of this program when making DFT analysis is also shown.

```

POWERMEASURING PROGRAM BY SAMPLING PhaseLock: █
Method Choice=
Presentation Mode TextMo
Calculation Method Spectr

Special Input=
Voltage Div Factor 1
Current Div Factor 1
DMM Mode DCU
Sample Time Aperture (µs) 6
PhaseLock to Voltage YES
ZERA System, EXT SYNCH NO

Input=
Fundamental Frequency 50
Sampling Periods (M) 3
Number of Samples (N) 1024
Voltage DMM Range 100U
Current DMM Range 100U

Output=
Nbr Of StatMeasures 20
Log File on Drive C: OFF
Printout on LPT1 OFF

Graphics params=
Show U/I/P ShowU
Y-Axis max 100
Logarithmic Y-Axis FALSE
Show Spot Frequency 50

Press space bar, "+" or "-" to select
STATUS: N=1024 M= 3 Apertur= 6us TextMode SpectrMethod LinScale DMMMode=DCU
Urange=100 Irange=100 StatON
←↑↓ Move cursor <enter> Accept <ctrl-enter> Accept & Quit ESC Quit

```

Figure 24 Setting options of DSWM

The results of the measurements are shown on the PC screen but can also be saved automatically on disc and/or paper.

```

POWERMEASURING PROGRAM BY SAMPLING PhaseLock: █

P : 5054.5714168 S : 8714.2125290 Pfak : 0.5800377
Urms : 71.1512474 Irms : 122.4744871 Fi : 54.5468087
Udc : -0.0000000 Idc : -0.0000000

PhaseLockbyte=0port=1
STATUS: N=1024 M= 3 Apertur= 6us TextMode MultMethod DMMMode=DCU
Urange=100 Irange=100 StatON
F1-Help F2-New_Logf F3-Prog_par F4-Show_par F6-MeasSI/SP F9-Quit F10-Save_Cnf

```

Figure 25 "Discrete integration method" result window

By use of the "discrete integration method"(DI), described in detail in chapter 5 the total (mean) power can be calculated. By similar calculations of the same samples the RMS-values and the DC-values of the current and the

voltage can also be obtained. Then the apparent power, the reactive power according to Fryze, the power factor and the phase angle (only for sinusoidal signals) can be determined, see Figure 25.

By use of DFT, the amplitudes and phases of 50 harmonics can be calculated, see Figure 26 below. From these values the power of each harmonic can be determined as well as reactive power according to Budeanu as well as most other definitions of "non-active" powers. If the DFT-method is used it is also possible to show the amplitude spectrum graphically.

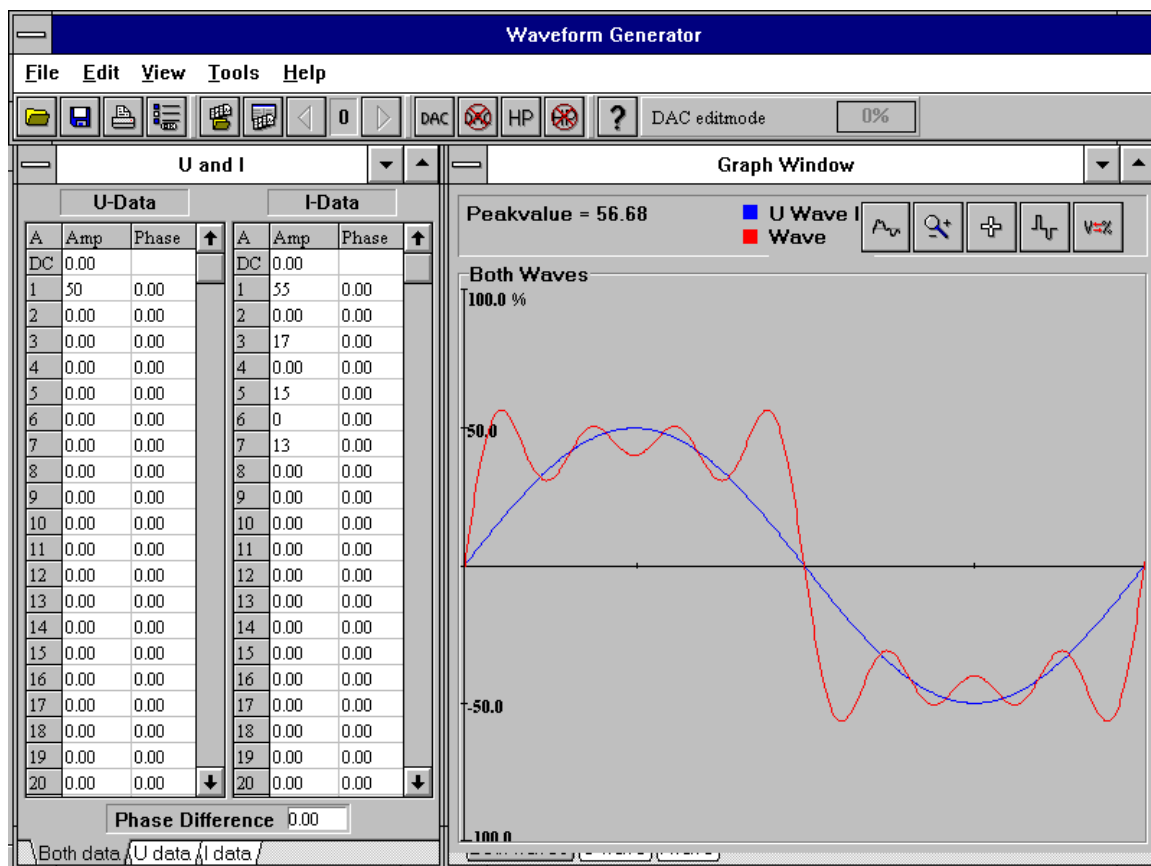


Figure 26 DFT method input and output window for the Windows version of the DSWM.

6.6 System clock

If the system is used in the "frequency control mode" a system clock (f_{cl}) frequency generator is needed. This should be an IEEE-interfaced pulse or function generator with low frequency jitter and a frequency range of at least 1kHz - 1MHz. At present a HP8116A Programmable Pulse/Function Generator is used. Its main characteristic for this application is a 50MHz upper frequency limit.

6.7 Power source

Although not a part of the measuring system, a power source is required to provide the requested test conditions. This power source should generate separate voltages and currents to simulate a power flow. A three-phase power generator, a ZERA Meter Test Board ED 7441, has been used in most measurements involving non-sinusoidal signals. It has a current range of 0 - 100 A and a voltage range of 0 - 380 V. It can generate current and voltage harmonics in the range of f_1 to f_{30} . The fundamental frequency can be set between 40 - 70 Hz and between 15 - 20 Hz. The frequency of the voltage and current can also be phase-locked to a TTL-level signal with the frequency $f_1 * 3600$. In the last part of the project, for instance for the flicker calibrations, a high resolution Digital-to Analogue converter and amplifiers have been used to form a power source together with the windows version software.

6.8 Design goal

The DC specifications of the multimeter and the shunts are within a few ppm. The main errors will therefore be due to AC errors as phase angle errors and frequency dependence. The system is mainly intended for calibration of wide-band wattmeters and power analysers. These instruments have a maximum accuracy of about 0.1% or 1000 ppm. Based on this, an accuracy design goal of 100 ppm of the apparent power was set at 50 Hz.

6.9 References

- [1] J. R. Stockton, "On improved methods for timing the digitisation of AC waveforms," *J. Phys. E. Sci. Instrum.*, Vol. 20, pp 1365-1367, 1987.
- [2] F. J. J. Clark and J. R. Stockton, "Principles and theory of wattmeters operating on basis of regularly spaced sample pairs," *J. Phys. E. Sci. Instrum.*, Vol. 15, pp 645-652, 1982.

7 Verification of the DSWM

The verification of the DSWM system has been made in four steps: 1) A general analysis of the different error contributions of the DSWM in power and rms voltage and current measurements of sinusoidal signals. 2) An analysis of the error contributions from the DSWM for rms voltage and current but especially power measurements for lower frequencies in the range 10 Hz - 1000 Hz. These results have been previously published in [1]. 3) An analysis of the DSWM as a wide-band wattmeter for the audio frequency range 10 Hz - 20 000 Hz. These finding has been published in [2, 3]. 4) An analysis of the DSWM as a harmonic analyser for 50 Hz and harmonics. These finding have also been reported in[4].

7.1 General error analysis

The uncertainty is only analysed for active power in sinusoidal situations, to keep the amount of calculation down to a reasonable level. The evaluation is, however, made in such a way that the uncertainty can be calculated for any non-sinusoidal situation without any further error analysis. The error contributions inherent to the two calculation methods for active power have already been analysed, see chapter 5. The calculation of rms-values, as defined for current by

$$I = \frac{1}{T} \int_T i^2 dt \quad (137)$$

can be considered as a special case of the active power calculation where $u=i$, and therefore the evaluation of the method inherent errors is valid also for rms value measurements.

The DMMs, shunts and dividers will also contribute to the uncertainty. The magnitude of several of these errors depends on conditions as frequency and power-factor. Therefore the total uncertainty of the DSWM in a non-sinusoidal situation will vary with the harmonic content, and the total uncertainty will have to be estimated for each measurement situation, if minimum uncertainty is required. The error analysis is therefore made for the general case, and then the total uncertainty is estimated for some interesting frequencies and power factors.

7.1.1 Error contributions of the instrumentation

The errors can be divided into two main groups, phase-angle errors and amplitude errors. The power-measurement error contribution can be derived from

$$P = UI\cos\Phi \quad (138)$$

which can be differentiated with respect to the amplitudes and the phase angle difference as

$$\Delta P = I\cos(\Phi)\Delta U + U\cos(\Phi)\Delta I - UI\sin(\Phi)\Delta\Phi \quad (139)$$

This expression can then be normalised to express the relative errors of the active power due to the voltage amplitude error, the current amplitude error and the error of the phase angle difference:

$$\frac{\Delta P}{P} = \frac{\Delta U}{U} + \frac{\Delta I}{I} - \tan(\Phi)\Delta\Phi \quad (140)$$

This is not a linear function as $\tan\Phi = 0$ at $\Phi=0$, 1 at $\pi/4$ and approaches infinity as Φ approaches $\pi/2$ (The value of $\Delta\Phi$ must be in radians). This means that a phase angle error will have an insignificant impact on the relative uncertainty of active power at PF=1, but will be the most important error source at low PF values.

In some cases it is more convenient to express the uncertainty with respect to the apparent power instead of the active power, especially at a low power factor to avoid relative uncertainties that approaches infinity. Then the uncertainties calculated with respect to active power should be multiplied by $\cos\Phi$ to obtain the uncertainty with respect to apparent power.

7.1.2 Power errors due to phase-angle errors

The error in Φ will not be directly dependent on the phase angle errors of the voltage channel and the current channel, but on the *difference* between these errors. Therefore, if the phase angle errors of both channels are exactly equal, no power error will occur. As long as the phase angle error signs are not known they will, however, be added as usual in uncertainty analysis.

The phase angle errors are not directly specified for HP3458A. However, when the two instruments are operated on different ranges there will be an

phase angle error $\Delta\Phi$ since the ranges have different bandwidths. The frequency response is stated as to have the roll-off of a one-pole low-pass filter, and the phase angle difference can then be calculated as

$$\Delta\Phi = \arctan\left(\frac{f}{f_{u1}}\right) - \arctan\left(\frac{f}{f_{u2}}\right) \approx \frac{f}{f_{u1}} - \frac{f}{f_{u2}} \quad (141)$$

where f_{u1}, f_{u2} are the different upper band-width limits and $f \ll f_{u1}, f_{u2}$.

According to the HP3458A specification, the 1V range and the 10V range have an upper bandwidth limit of 150kHz and the 100V and the 1000V ranges have a upper limit of 30kHz. The use of a combination of one low voltage range and one high voltage range would result in an active power error of 770ppm of P at 50Hz and $\Phi=30^\circ$, according to Equation (141) and Equation (140), and at 1000Hz the error would be approximately 20 times larger. Such large errors are not acceptable, even if they partly can be compensated for. A separate voltage divider must be added to the voltage channel of the DSWM, and an Inductive Voltage Divider (IVD) which has an extremely low phase-angle error is preferred. Then the number of multimeter ranges used can be limited to the 1V and the 10V range which have the same specified upper band-width limit of 150kHz. Therefore the error analysis is based on the use of an external IVD. The bandwidth phase-angle error will then mainly be caused by the instrument-to-instrument difference of the band-width. This difference is not specified but is estimated to be in the range of 140kHz - 160kHz, which will cause a power uncertainty of

$$\frac{\Delta P}{P} = -\tan(\Phi)\Delta\Phi \approx \pm \tan(\Phi) \left(\frac{f}{140\text{kHz}} - \frac{f}{160\text{kHz}} \right) \approx \pm \tan(\Phi) \cdot 0.9 \cdot 10^{-6} f \quad (142)$$

The IVD phase-angle error is less than $0.5 \mu\text{rad}/D$, where $D = U_2/U_1$, which will cause a power error of

$$\frac{\Delta P}{P} = -\tan(\Phi)\Delta\Phi = \pm \frac{\tan(\Phi) \cdot 0,5\mu\text{rad}}{D} \quad (143)$$

The phase-angle error of the shunts can be estimated from the shunt inductance, which is measured and found to be less than 1 nH for both shunts. The phase-angle errors $\Delta\Phi$ are then approximately equal to $\omega L_{\text{sh}}/R_{\text{sh}}$ and the power uncertainty can then be estimated to be:

$$\frac{\Delta P}{P} = -\tan(\Phi)\Delta\Phi = -\tan(\Phi) \cdot \frac{2\pi f \cdot L_{sh}}{R_{sh}} \quad (144)$$

7.1.2.1 Errors due to delay-time

Delay-time errors are mainly caused by instrument-to-instrument delay-time differences between the EXT TRIG input channels. This difference (Δt) will cause a phase-angle error ($\Delta\Phi = \omega\Delta t$), as in Equation (140), which varies with the frequency such that

$$\frac{\Delta P}{P} = -\tan(\Phi)\omega\Delta t = -2\pi f \tan(\Phi)\Delta t \quad (145)$$

For the HP3458A there is a 75 ns instrument-to-instrument delay-time variability specified which will cause an uncertainty of

$$\frac{\Delta P}{P} = \pm 2\pi f \tan(\Phi) * 75 \cdot 10^{-9} \quad (146)$$

7.1.3 Amplitude Errors

The contribution to the relative active power error from the amplitude error of U and I can be derived directly from (140) as

$$\frac{\Delta P}{P} = \frac{\Delta U}{U} + \frac{\Delta I}{I} \quad (147)$$

If the uncertainty should be stated with respect to apparent power instead of the active power e.g. at low power factors, the uncertainties calculated with respect to active power should be multiplied by $\cos\Phi$ to obtain the uncertainty with respect to apparent power.

The amplitude error is a combination of the DC errors of the digitiser, and the AC errors due to imperfections in the input channels. When the 10V range is used for the DMM-U and the 1V range for the DMM-I, the error contribution from DC-errors can be derived by combining the 3458A DC calibration uncertainty and the specified 1-year-drift, which results in an uncertainty of 4 ppm for each multimeter:

$$\frac{\Delta P}{P} = \frac{\Delta U}{U} + \frac{\Delta I}{I} = 4\text{ppm} + 4\text{ppm} \quad (148)$$

The AC errors of the DMMs are mainly due to the limited bandwidths, which are described as one-pole low-pass filter. Thus, the AC amplitude error can then be estimated according to Equation (147) as

$$\frac{\Delta P}{P} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{gu}}\right)^2}} - 1 + \frac{1}{\sqrt{1 + \left(\frac{f}{f_{gi}}\right)^2}} - 1 \quad (149)$$

where f is the power source frequency and f_{gu} and f_{gi} are the upper bandwidth limits of the voltage channel and the current channel respectively. Thus, when the DMM-U is set to the 10V range, and the DMM-I is set to the 1V range, the band-width limits are 150kHz and the power error will be:

$$\frac{\Delta P}{P} \approx -0.5 \left(\frac{f}{150\text{kHz}} \right)^2 - 0.5 \left(\frac{f}{150\text{kHz}} \right)^2 \approx -2 \cdot f^2 \cdot 22 \cdot 10^{-12} \quad (150)$$

The shunts and the IVD are calibrated separately and the frequency dependence of the corrections is estimated to be negligible. The calibration uncertainty will be their only contribution to the system uncertainty. The amplitude uncertainty of the IVD due to the calibration is 0.5 ppm/D, where $D=U_2/U_1$.

The amplitude uncertainty of the shunt according to the calibration is 2 ppm. However, the temperature-dependence of the resistors will result in a resistance change, which varies among the shunts but is within 0 - 30 ppm at nominal load compared to the no-load situation. The uncertainty due to this is estimated to be ± 10 ppm.

7.1.3.1 Errors due to the sample time-aperture

In the DCV-mode of the multimeters a time-aperture is used to increase the noise immunity. That is, for each sample the signal is integrated during a short time T_a . The signal spectra will then be multiplied by the following aperture factor as

$$f_a(\omega) = f(\omega) \frac{\sin(\pi f T_a)}{\pi f T_a} \quad (151)$$

This will have much the same effect as a one-pole low-pass filter and can be approximated by

$$\frac{\Delta P}{P} = 2 \left(\frac{\sin(\pi f T_a)}{\pi f T_a} - 1 \right) \approx -2 \frac{(\pi f T_a)^2}{6} \approx -2 \cdot 1.65 \cdot f^2 \cdot T_a^2 \quad (152)$$

The value of T_a that is mainly used is $6\mu\text{s}$, which enables a sampling speed of 50kS/s and a resolution of 18 bits. The time aperture is estimated not to cause any power error due to phase angle errors if the same time-aperture is used in both multimeters, because of the high accuracy of the built-in time bases.

7.1.4 Total power-measuring uncertainty

The value of several of the errors is frequency dependent, power-factor dependent or dependent on the sampling rate and/or the number of samples. Also, the calculation method may have an influence on the total uncertainty. Therefore the total possible error of the two calculation methods will be estimated for four different conditions.

In the Table 3 below the total uncertainty for the DSWM is calculated for two different values of frequency and power factor, at $f_{ac}=50\text{Hz}$ and $f_{ac}=1000\text{Hz}$ (both sinusoidal), and at power factors $\text{PF} = 1$ and $\text{PF} = 0.5$. The other parameters of the DSWM are set as follows. The DMMs are set to the DCV mode with an aperture time = $6\mu\text{s}$, which enables an 18 bit resolution of the DMMs. The full dynamic range is used for both the 1V range of the DMM-I and the 10V range of the DMM-U. The sample timing circuit is set to 512 samples in 105 periods for the discrete integration method, and 512 samples in 7 periods for the DFT method. The 5 A shunt with $R=0.16\ \Omega$ is used and the IVD division factor is $D=8\text{V}/230\text{V}=0.035$. The sampling time jitter for this mode is 50ns.

The total uncertainty presented in Table 3 is calculated as the root-square-sum of all estimated uncertainties and errors. This is not quite correct as some of the errors are estimated with a determined sign and some of them can be correlated. However, as a first approximation it is sufficient.

As can be seen in the table the method-dependent errors have an insignificant influence on the total uncertainty due to their low values. According to the table the phase angle errors will dominate at power factors lower than 0.5. It can also be seen that the amplitude errors due to a time-aperture of $6\mu\text{s}$ leads to unnecessary high errors at frequencies around 1000 Hz, compared to the use of the DSDC-mode.

Table 3. Preliminary uncertainty analysis for DSWM, for sinusoidal signals, at 50 Hz and 1000 Hz and power factors 1 and 0.5, No bandwidth corrections made.

	Estimated absolute values of errors (ppm of P)			
	50Hz PF=1	50Hz PF=0.5	1000Hz PF=1	1000Hz PF=0.5
Phase Errors:				
DMM-difference	0.002	78	0.8	1557
IVD	<0.1	49	<0.1	49
Shunt	<0.1	3	<0.1	68
Delay-time	<0.1	41	0.2	815
Amplitude Errors:				
DC, DMM-V	4	4	4	4
DC, DMM-I	4	4	4	4
Band-with,DMM-V	0.1	0.1	22	22
Band-with,DMM-I	0.1	0.1	22	22
IVD	14	14	14	14
Shunt, calibration	2	2	2	2
Shunt, temp dep	10	10	10	10
Sampl. Time Aper.	0.3	0.3	120	120
Discr. Int. Method:				
Quant. noise	0.2	0.2	0.2	0.2
Sampl. jitter	0.4	0.4	8	8
DFT Method:				
Quant. noise	<0.1	<0.1	<0.1	<0.1
Sampl. jitter	0.4	0.8	8	16
Total Discr. Int. Uncertainty:	20	103	126	1763
Total DFT uncertainty:	21	103	126	1763

Most of the errors found by this analysis can be measured and compensated, and therefore the above table should be seen as a first approximation. However, a good compensation of frequency-dependent errors when measuring non-sinusoidal signals is a complex process. The frequency spectra of the signals must be determined and the compensation must be applied for each harmonic separately. The discrete Fourier transform method is ideal for such a compensation while for the discrete integration method efficient compensation is only possible if the signals are close to sinusoidal.

The possible degree of error compensation depends on the uncertainty of the error determination and the stability of the compensated instrument. A general rule is that if the error-mechanisms are complex, the improvement can not be expected to be more than a factor 10 due to stability limits.

In a measuring situation, errors due to loading and the length of the cables should also be taken into account. However, the impedance levels of the DSWM are such that these are negligible in for reasonable choices of wiring.

When measuring power at low power factors, the relative error of the power will be large and approach infinity when the power factors approaches zero. Therefore the errors is mainly expressed as the errors relative the apparent power, S , in the following analysis.

7.1.5 Power Measurements

To check the power measurement capabilities of the DSWM, a comparison was made between the DSWM and the active power measurement system of the Physikalische Technische Bundesanstalt, PTB, which is the national metrology institute of Germany. The comparison was made for some different 50 Hz sinusoidal voltages and currents. A HEG Power Converter C1-2, which is a Time-Division-Multiplying(TDM) power meter with a nominal voltage and current of 120V and 5A, was used as a transfer standard. The results for 120V, 5A, the nominal power of the transfer standard, is analysed here; the full result of the comparison is reported in Appendix A.

Before this comparison the DSWM voltage and current channels were calibrated separately and the phase angle error for the DSWM without shunts and voltage dividers was measured. The phase angle error contributions of the shunts and the IVD were measured separately and the results added. The corrections achieved were then used in the measurements. The transfer standard was calibrated with both the PTB power measurement system and the DSWM for different voltages, currents and power-factors and the results were compared. The differences at 120V, 5A can be studied in Figure 17.

The differences of the two systems were all within the combined estimated uncertainty (1σ) which was estimated as 35 ppm at power factor one and 20 ppm at power factor zero for the DSWM and as 11 ppm at power factor one and 8 ppm at power factor zero for the PTB system. It is also clear that the agreement in phase angle (power factor=0) is better than the agreement in amplitude(power factor=1). Part of the difference was found to be caused by

the shunt temperature rise that was larger than expected due to inefficient cooling.

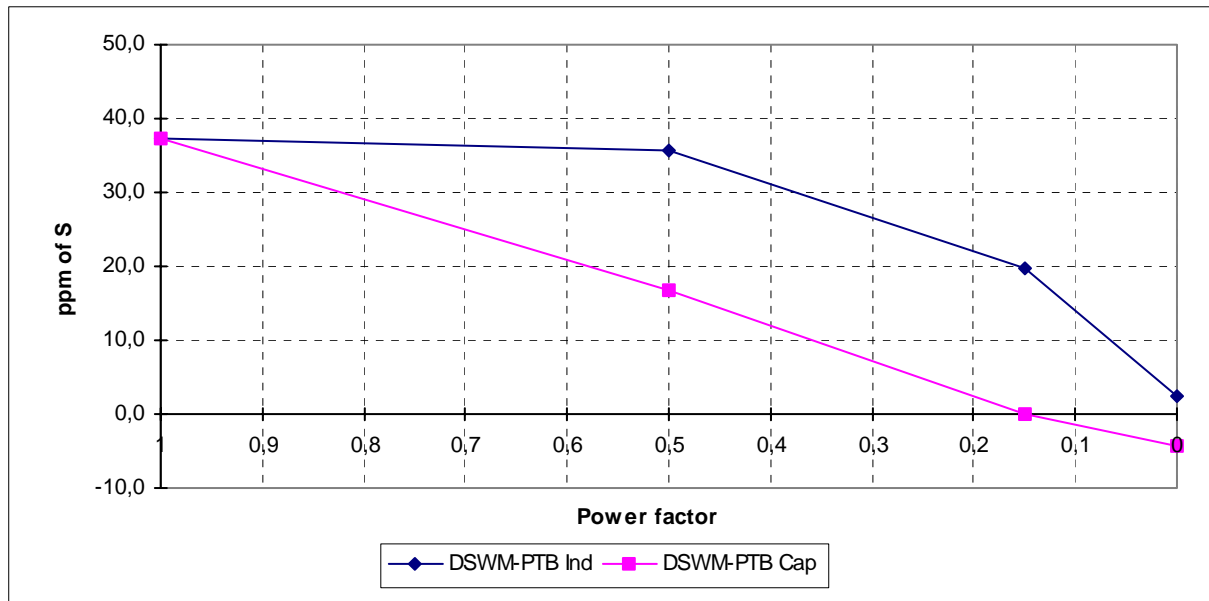


Figure 27. Power measuring differences between the DSWM and the PTB power measuring system, at 120V, 5A.

The DSWM has also been used in two multilateral comparisons at 50Hz, 120 V and 5 A. The first one is a world-wide comparison between national laboratories organised by the BPIM, with among others NIST (USA), NRC (Canada), PTB, (Germany) and NPL (UK). The second one is an European comparison between national laboratories. None of these has yet resulted in official reports but intermediate results are similar to the results of the comparison with PTB above.

7.2 An error analysis for the audio frequency range³

A wattmeter standard, based on digital sampling, has been designed and evaluated for the audio frequency range. The wattmeter is based on two commercially available sampling DMMs. The power spectrum is calculated with DFT analysis. High accuracy is achieved by phase locking and equal-spaced simultaneous sampling.

³ This part of the thesis has been presented at the Conference on Precision Electromagnetic Measurements, July 5-8 1998 in Washington DC, USA, and is accepted for publication in IEEE Transactions on Instrumentation and Measurements.

7.2.1 Introduction

The fast growth of power electronics and higher demands on their efficiency has created a market for high accuracy power meters with a wider frequency range, from a few Hz up to 100 kHz, and even higher. The demands on calibrations increase accordingly. A Digital Sampling Wattmeter (DSWM), mainly composed of commercially available equipment, has been designed at the Swedish National Testing and Research Institute (SP)[1, 5]. The first results showed an uncertainty (1σ) at 50 Hz, 120V and 5A of $\pm 30 \mu\text{W}/\text{W}$ at power factor one and $35 \mu\text{W}/\text{W}$ at a power factor of 0.5. The measuring range of the DSWM has now been extended to cover the audio frequency range 10 Hz - 20 kHz.

7.2.2 Wattmeter Design

The main features of the DSWM are its high accuracy and flexible operation. The basic measuring principle is the equally-spaced, simultaneous sampling of voltage and current during an exact number of periods of the fundamental frequency. The total power is calculated by discrete integration [6] or Discrete Fourier Transform (DFT). The DSWM, described in Fig. 1, is composed of two sampling DMMs, an Inductive Voltage Divider (IVD), coaxial shunt resistors and a PC system used for control and data processing.

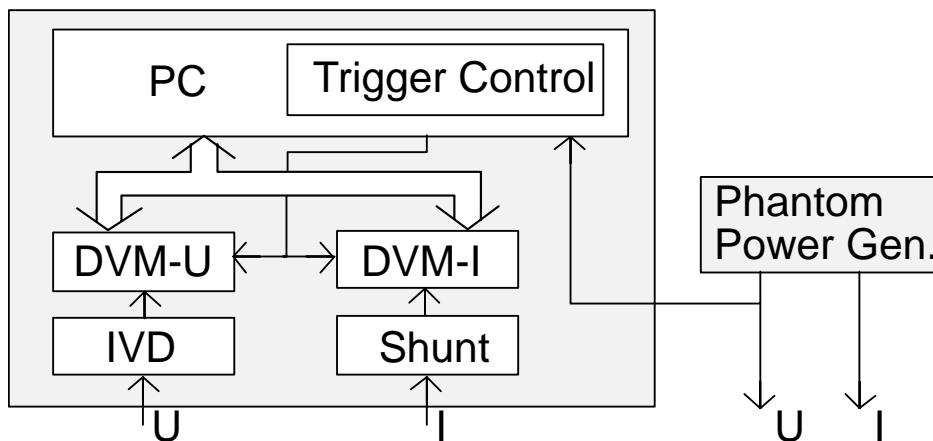


Figure 28. Calibration system, digital sampling wattmeter and power source.

The IVD is used as a better alternative to the internal divider of the DMM. The measured voltage is divided by the IVD to about 8 V and the output of the current shunts is 0.8 V. A special phase-lock and trigger control circuitry, see Fig. 2, has been designed and built into the computer to control of the sampling process in the digital voltmeters. This circuit provides means to take exactly an integer number of samples during a number of periods, either by phase-locking to the voltage as in Fig. 1 or by deriving both the frequency

control signal and the sampling rate control signal from a master clock. The verification described in this paper has been made in the phase-locking mode.

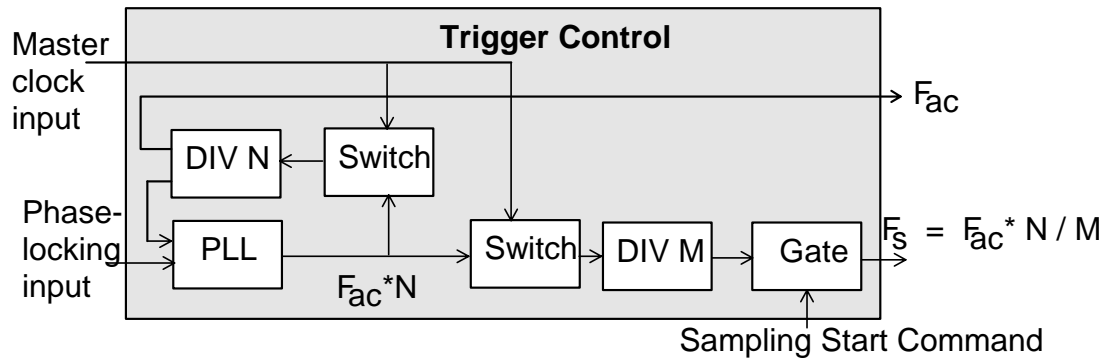


Figure 29. Circuit diagram of the trigger control circuitry which performs the synchronisation between the generated signals and the DSWM

7.2.3 Measurement strategies

DFT is used for the digital signal processing and calculation. It allows correction for frequency-dependent errors even if the signal is non-sinusoidal. Further, DFT makes it possible to measure phase angles with high accuracy at all phase angles. The higher processor workload compared to discrete integration is therefore acceptable.

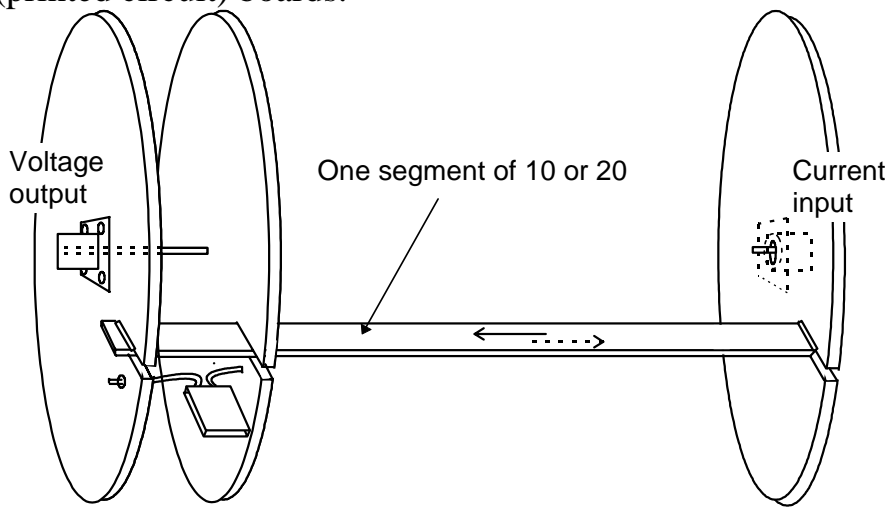
The analogue-to-digital conversion rate is limited to 100 ksample/s, but since exactly N samples are taken during M periods the DSWM can work with a sample rate that is even slower than one sample per period. This would seem to make it impossible to resolve any harmonic other than the fundamental at higher frequencies. For stable signals, however, each set of N samples can easily be rearranged into a virtual one-period sampling sequence, which then can be processed by DFT. This feature has not been fully investigated, but the added uncertainty can be directly derived from the standard deviation of repeated measurements.

At low and medium frequencies the DC measurement mode of the DMM is used, with a heavily reduced integration time. Good AC voltage measurement results at frequencies at or below 50 Hz have been achieved, with a similar setup and with integration times in the order of 100 μ s, by Pogliano [7] and by VTT in Finland [8]. This is also confirmed by measurements using the DSWM. For frequencies above 50 Hz, the integration time should rather be in the order of 10 μ s to avoid large corrections. Even so, at 10 kHz the total correction (for an aperture time of 6 μ s and a 115 kHz bandwidth) is about 9700 μ V/V. A 1% error in the sampling time aperture then represents an error

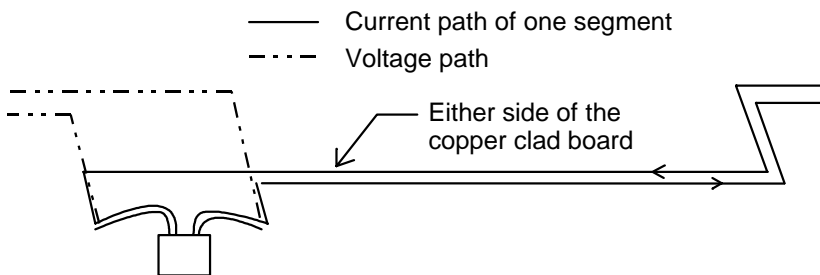
of $120 \mu\text{V/V}$. Therefore the sampling mode of the DMM, incorporating a S/H-circuit with a specified bandwidth of more than 10 MHz is used for frequencies above a few kilohertz. The trade-off is higher noise sensitivity and less stability.

7.2.4 Transducers

Shunt resistors are used to transform the current into a 0.8 V signal. The shunts are of coaxial type with discrete resistors. The design is from the D. I Mendelejev Institute for Metrology but has not previously been published as far as the author knows. The structure of the shunt is shown in Fig. 3. The shunt is of squirrel-cage type where all conductors are made by copper clad (printed circuit) boards.



a) Shunt resistor structure, entirely made of copper clad boards for printed circuits.



b) current and voltage path in one segment of the coaxial shunt

Figure 30. Details of the shunt resistor design

It should be noted that the seemingly large loop in the current path is balanced by an opposite loop from the next segment that is also feeding this resistor. The rather long bifilar current path will keep any inhomogenous field due to the current away from the actual shunt; thus the mutual inductance will be kept down to a minimum. In the new series of shunts that

are under construction the design is slightly changed so that the resistors are placed directly on the segments. This is to facilitate the use of chip resistors and to better define the four-terminal characteristics. The resistors used have low temperature coefficients; the temperature dependence is less than $1 \mu\Omega/\Omega$ per degree. Two shunts of 5 A and 1 A nominal current are evaluated; both have a mutual inductance of $0.5 \text{ nH} \pm 0.5 \text{ nH}$. The input capacitance of the ADC/DMM is $260 \pm 40 \text{ pF}$ in the DC mode. These features and the general AC-DC difference are the main contributors to the uncertainty originating from the shunts.

Two commercially manufactured inductive dividers (IVDs) are used to scale the voltage into the 10 V range. One is used for lower frequencies, 10 - 1000 Hz, and one for higher frequencies, 1 kHz - 20 kHz. For very low frequencies the maximum input voltage of the IVD is limited. However, the built-in divider of the ADC/DVM is then sufficient because the frequency-dependent errors are then low. The main uncertainty contribution from the IVDs is their calibration uncertainty which is $0.5 \mu\text{V}/\text{V}$ relative the input voltage, for both the in-phase and the quadrature component and at 1000 Hz.

7.2.5 Measurement uncertainty

An evaluation of the system accuracy has been made with respect to the following error sources:

- ADC quantisation and sample time jitter errors.
- Phase angle errors of the IVD, shunt resistors, and DMMs, including trigger delay.
- Amplitude errors of the IVD, shunt resistors, and DMMs.
- Loading errors due to the DMM input capacitance.

All parts of the DSWM system were evaluated separately. After corrections, the shunt and IVD uncertainties (including the loading error), were each estimated to be less than $100 \mu\text{V}/\text{V}$ and $100 \mu\text{rad}$.

Special efforts have been made on evaluating the behaviour of the DMMs in the DSWM configuration. The amplitude errors were assessed by comparison with a high accuracy calibrator. The phase angle errors were checked by applying the same 0.8 V to both multimeters and calculating the phase angle difference by DFT. The high resolution and good linearity ($\pm 0.1 \mu\text{V}$ for a 1V signal measured in the 10 V range according to the DC specification, and a measured harmonic distortion of less than 100 ppm at 50 Hz for this system according to [1]) of the multimeters were then required as there is a ten-to-one range difference between the current and the voltage channels.

Amplitude corrections for the integration time aperture and bandwidth were applied as well as a linear best-fit phase-angle correction.

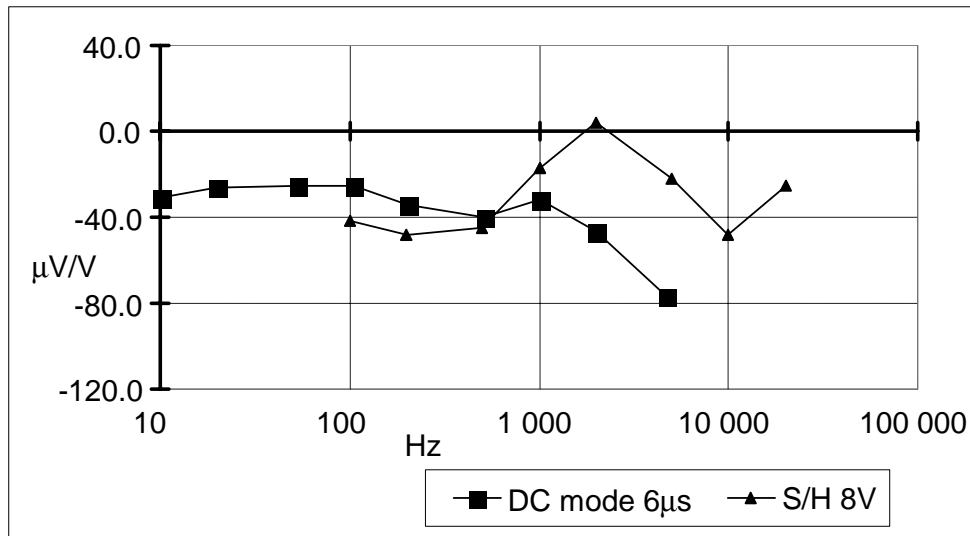


Figure 31. Amplitude frequency response, after applied correction, of one DVM vs frequency.

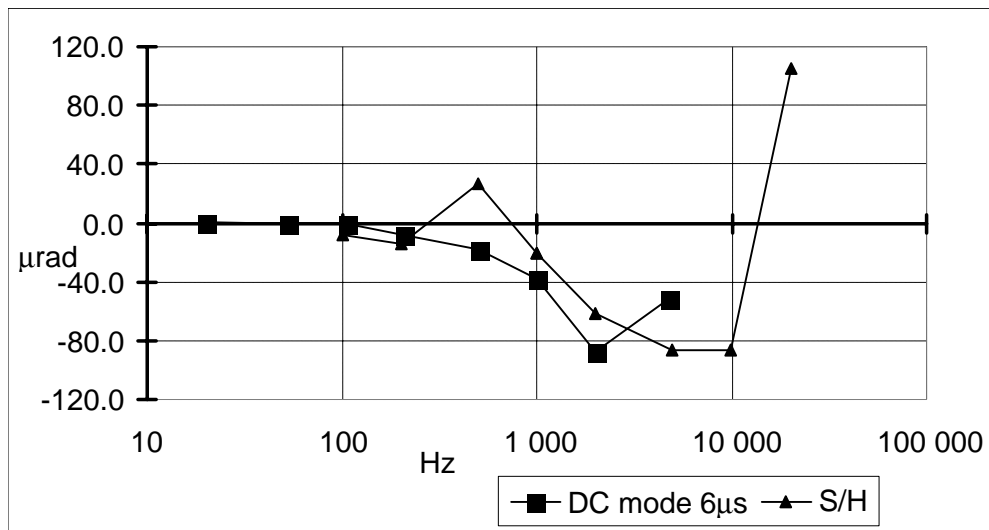


Figure 32. Phase-angle frequency response, after correction, of DSWM without IVD and shunt

The same measurements were made in the sample-and-hold mode. The amplitude frequency responses of this mode were not as consistent as those of the DC mode, but on the other hand the maximum error before corrections was less than 250 $\mu\text{V/V}$. A second-degree best-fit amplitude correction and a linear best-fit phase-angle correction were made. A preliminary frequency

response is shown in Fig. 4 and 5 for the 10 V range. The investigation on the 1 V range gave similar results.

The long-term stability of the DMMs in the DSWM configuration is not yet fully investigated but is estimated to be $\pm 150 \mu\text{V/V}$ for a six month period. The main part of the phase angle correction can be established before each measurement since it only requires a stable 0.8 V AC source.

7.2.6 Conclusion

The amplitude and phase angle corrections and the uncertainties of the separate parts of the Digital Sampling Wattmeter have been evaluated. The total power measurement uncertainty (1σ) at 20 kHz, 120 V and 5 A is estimated to $300 \mu\text{W/W}$ at power factor one, and to $600 \mu\text{W/VA}$ at power factor zero. At 50 Hz the uncertainty (1σ) at 120 V and 5 A is $\pm 30 \mu\text{W/W}$ at power factor one, and $\pm 15 \mu\text{W/VA}$ at power factor zero.

7.3 Verification of a calibration system for power quality instruments⁴

7.3.1 Introduction

The DSWM is a versatile calibration system and is suitable for calibrations of power quality instruments. These instruments are intended for measurements under non-sinusoidal conditions and should be verified for such conditions. Therefore the calibration system uncertainty at measurements of harmonics, especially in the presence of a large fundamental harmonic, is analysed. The analysis is based on common A/D converter characterisation techniques that are adapted for this special application.

⁴ This section was presented at the IEEE Instrumentation and Measurement Technology Conference, May 18 -21 1998 in St Paul, MN, USA, and is accepted for publication in IEEE Trans. on Instr. and Measurements.

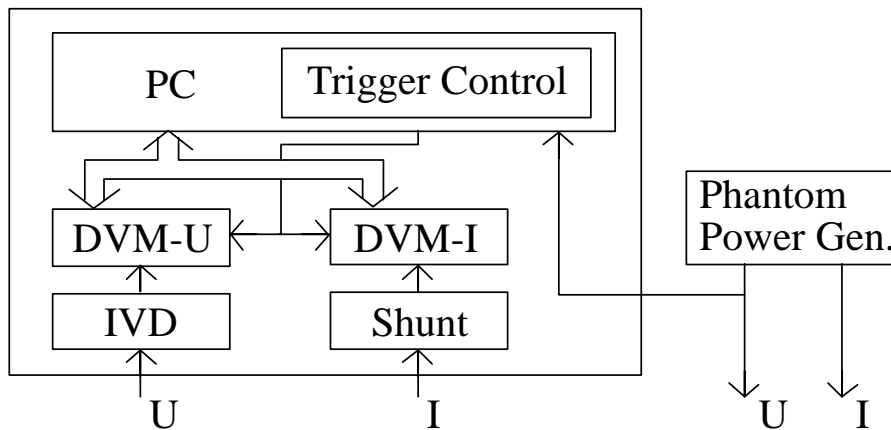


Figure 33. Block diagram of the calibration system

The frequency response is of course an important factor for the accuracy of the system. However, the spectral leakage will also be important for the measurement of harmonics with small amplitudes, especially if these small amplitude harmonics are close to other harmonics with large amplitudes. The IEC guide on harmonics measurements [9] which is valid for power quality meters, states an accuracy for class A instruments of 5 % for measurements of harmonics down to 5% of the range. That means that a calibration system must have an overall harmonic measurement uncertainty, including frequency dependence and spectral leakage, well below 0.25% of the range, preferably about 0.025% of the range.

It is possible to verify the harmonic measurement capability of the system through direct comparison of the amplitudes of the measured harmonics with the RMS-values and the power measured by an AC measurement standard and a power meter. The wattmeter then measures a combination of one multi-frequency and one single-frequency signal. The measurement uncertainty from such an operation will however be poor if the amplitudes of the harmonics are of different magnitudes, which is shown by [10].

This paper describes an alternative verification procedure particularly suited to verify the measurement capability of small amplitude harmonics in the presence of a large fundamental. It is based on frequency response measurements and on dynamic linearity measurements, which are extensively used to characterise A/D converters.

There are some different methods to analyse the dynamic linearity of A/D converters. The histogram method [11, 12] gives the linearity error of each level of a converter for positive slopes as well as negative. The single-tone test and the dual-tone test [13-15] are also two well-known methods that

enable analysis of the non-linearity of an A/D converter in terms of noise, harmonic distortion (HD) and intermodulation distortion (IMD).

The use of single-tone and the dual-tone test, and the HD and IMD calculating methods, are based on the assumption that the non-linear transfer function $H(t)$ of the converter can be approximated with a polynomial function of a reasonably low order,

$$H(t) \approx f(t) + a \cdot f(t)^2 + b \cdot f(t)^3 \dots \quad (153)$$

where the linear transfer function is denoted $f(t)$. If this approximation is valid, the harmonic distortion will be limited to a few harmonics. For some converters such as flash converters and very high speed interleaved converters this approximation might not be valid, if the calculation of the output level $n+1$ might not be correlated to the calculation of the level n , or if the calculation of the output sample with sequence number $n+1$ might not be correlated to the calculation of the sample number n . Then more thorough methods such as the histogram method might be used[16]. For multi-slope-integration type converters, which are used in the calibration system, the polynomial approximation should be valid.

7.3.1.1 Special features of power quality meter characterisation

There are some special features of power quality instrument characterisation in general, and characterisation of a power quality instrument calibration system in particular compared to the usual A/D converter characterisation situation:

- a) The only frequencies of concern are, generally, 50 Hz (or 60 Hz) and its harmonics
- b) Calibrations are made with stationary signals (as opposed to many real life measurements) and therefore the noise is of less importance, as long as it can be reduced by repetitive measurements and averaging.
- c) When a power quality instrument measures voltage harmonics, the fundamental harmonic is normally the only one of such a magnitude that it can cause any considerable spectral leakage. The harmonic distortion values achieved by a single-tone test are then fit to calculate the uncertainty of the measurement of low-amplitude harmonics.
- d) When current harmonics are measured, many harmonics may be of considerable amplitude and the intermodulation distortion may have to be taken into account. The intermodulation distortion may be estimated from the harmonic distortion values or by results from a dual-tone test.
- e) These types of measuring systems consist of several parts that can contribute to the non-linear part of the transfer function: The input voltage

divider or the current transducer, the A/D converter or the sampling control.

- f) The calibration system as well as most power quality instruments phase-locks to the fundamental harmonic of the measured signal. Most phase-locking circuits show an inherent periodic frequency variation of the output (the sampling frequency), with a periodicity that is the same as the fundamental to which it phase-locks. This will also cause spectral leakage but it can not be expressed in terms of a non-linear polynomial transfer function.
- g) When a dual-tone test is made care should be taken that the IMD frequencies are well separated from the HD frequencies. However, if the 50 Hz fundamental and a harmonic are chosen as the two tones, some of the HD and the IMD frequencies will coincide instead of being separated. For a proper dual-tone test the calculating algorithms must be modified to support accurate non-harmonic measurements. This modification will decrease the value of such a test.

7.3.1.2 The single-tone test

Because of considerations described in c) and g) the use of the single-tone test was preferred. When using the single-tone test the input of an A/D converter is subjected to a sinewave that should be as ideal as possible. The single-tone test result SINAD, signal-to-noise-and-distortion, may then be calculated by extracting the fundamental harmonic from the A/D converter output and compare the rms value of the residual with the fundamental. The total harmonic distortion, THD, of an A/D converter may be defined as the rms value of the first five or six harmonics compared to the rms value of the fundamental harmonic. The THD value will be lower than the SINAD because most of the noise will not be included, and all rms values are strictly positive. Each value of the amplitude and the phase angle of the harmonics can be calculated from a measured sequence by sine-fit procedures but DFT-analysis, which is used in our calibration system, have also been shown to be reliable. For our purpose the harmonic distortion of each harmonic is more interesting than the THD or SINAD values.

7.3.1.3 An alternative to filtering

Ideally, when using the single-tone test the source should have much less distortion than the expected distortion on the output of the A/D converter under test. Very often one still will end up being uncertain of the distortion origin. The normal procedure to check if the measured distortion originates from the source or from non-linearity is to add steep filters between the source and the converter. However, at low distortion levels, low frequency and with a source that can not be heavily loaded these filters are not so easily made. Therefore, an alternative solution was investigated:

As soon as any filter is applied between a multifrequency source and an instrument capable of spectrum analysis the relative amplitude of the source harmonics will change, as well as the phase angles of the harmonics, compared to the fundamental. Harmonics that are not originating from the source but from non-linearity acting on the fundamental harmonic will, however, *not* change its relative amplitude or its phase angle compared with the fundamental.

The origin of the harmonic distortion can therefore be determined by comparing the relative amplitude and phase angles before and after the application of any filter that have a calculable effect on the source signal. This method was used to enhance the single-tone test made in this paper.

7.3.1.4 Special considerations when averaging

The strategy of averaging to enhance the accuracy has a dangerous aspect. The amplitudes of the harmonics measured by DFT are slightly biased by the noise, introduced e.g. by quantisation. According to [14] the biased amplitude Y of a harmonic can be approximated by

$$Y = Y_1 + \frac{\sigma_x}{2Y_1} \quad (154)$$

where Y_1 is the real noise-free amplitude and σ_x is the standard deviation of the noise or the noise rms value. The signal-to-noise-ratio can be expressed as

$$S / N = \frac{Y}{\sigma_x} \approx \frac{Y_1}{\sigma_x} \quad (155)$$

If (155) is put into (154) then

$$Y = Y_1 + Y_1 \cdot \frac{1}{2(S / N)^2} = Y_1 \left(1 + \frac{1}{2(S / N)^2} \right). \quad (156)$$

The 1σ -uncertainty of Y due to noise, on the other hand, can be estimated by

$$u_Y \approx \frac{1}{S / N}. \quad (157)$$

By comparing (157) and (156) one can conclude that for $S/N > 2$ or so the bias caused by noise will be negligible compared to the uncertainty due to the same noise. If averaging is applied, however, the uncertainty of Y due to noise can be decreased, almost indefinitely, while the bias will remain unaffected. For example, a sinusoidal signal Y with $S/N=2$ is averaged N times, $N=1000$. The (apparent) relative 1σ -uncertainty due to noise will then be

$$u_{YA} = \frac{u_Y}{\sqrt{n}} \approx \frac{1}{2\sqrt{1000}} \approx 0.016 = 1.6\% \quad (158)$$

but the error of the averaged signal due to the bias will still be

$$\varepsilon_{YA} = \frac{1}{2(S/N)^2} \approx \frac{1}{2(2)^2} = 0.125 = 12.5\%. \quad (159)$$

That is, averaging can only be applied on harmonic amplitudes if S/N is already high. The remedy of this problem is to apply averaging on the complex representation of the harmonics, where no bias is introduced.

7.3.2 Measurements

7.3.2.1 Frequency dependence

The frequency response of the calibration system was measured by comparison with an AC measurement standard and by a theoretical analysis. The result of this investigation has been published[3] and is given in short form here.

Frequency dependent amplitude corrections for both the current and the voltage channel are given by the corrections for the integrating time of the A/D converters and the band-width. The inter-channel phase angle difference is corrected by a linear phase angle correction starting from $30 \mu\text{rad}$ at 50 Hz.

The remaining uncertainties, for sinusoidal signals, after corrections, are estimated to less than $100 \mu\text{V/V}$, $100 \mu\text{A/A}$ and $100 \mu\text{rad}$ respectively for the voltage channel, the current channel and the inter-channel phase angle, for the frequency range 16 - 2 000 Hz.

7.3.2.2 Harmonic distortion

The input of the calibration system voltage-channel A/D was subjected to an 8 V sinewave from a AC voltage calibration standard. The signal was

measured and a complex DFT was calculated based on 1024 samples taken during 3 periods. This measurement was repeated 3*400 times, a trade off between measuring time and precision, and the mean and standard deviation was calculated.

Table 4. Measured harmonic distortion on the 10 V input of the voltage channel A/D converter.

Harm. No	Rel. Amplitudes		Phase angle	
	Meas. Value ($\mu\text{V}/\text{V}$)	Stddev ($\mu\text{V}/\text{V}$)	Meas. Value (degree)	Stddev (degree)
2	121	3.9	110	1.3
3	2	0.1	70	83.8
4	3	0.8	-50	20.3
5	9	1.0	165	1.8
6	3	0.6	103	8.4
7	2	0.9	128	31.1
8	1	0.6	-81	97.6
9	4	0.3	160	1.6

The harmonic distortion was measured and can be studied in Table 4. The only harmonic of importance was the second with a relative amplitude of $121\mu\text{V}/\text{V}$ and with a standard deviation of approximately $4\mu\text{V}/\text{V}$. The fifth harmonic was second largest at about $10\mu\text{V}/\text{V}$ and the rest of the first ten harmonics is between 1 and $10\mu\text{V}/\text{V}$. The standard deviations of these harmonics are higher than the rest of harmonics and this suggests that the harmonic distortion fluctuates, with a time constant of the same order as the measurement time.

A simple lowpass RC filter, $R=248\Omega$ and $C=10\mu\text{F}$, was applied between the source and the calibration system. Its effects on the harmonics amplitudes and phase angles, if originating from the source, were calculated and compared with measurements with and without the filter. The comparison can be studied in Table 5.

As one could expect, due to very low amplitudes, no resemblance with the calculated effects of the filter could be seen on the harmonics above the second. On the contrary the relative amplitudes did go up which might indicate that this is mainly a mixture of noise with constant absolute value and harmonic distortion with constant relative value.

Table 5. Relative change of amplitudes and phase angles when the source is lowpass filtered. 8 V A/D input.

Harm. No	Rel. Ampl. Change		Phase angle change	
	Theor. B_T/A_T	Meas. B_M/A_M	Theor. $\Delta\Phi_T$ (degree)	Meas. $\Delta\Phi_M$ (degree)
2	0.71	0.84	18.5	19.1
3	0.53	7.09	46.9	-36.6
4	0.41	4.43	79.4	42.3
5	0.34	1.16	114.0	-248.1
6	0.28	2.38	149.6	-240.8
7	0.25	3.42	185.8	38.3
8	0.22	5.14	222.4	198.7
9	0.19	0.94	259.3	-102.7

The part of the second harmonic distortion that is due to A/D converter non-linearity can now be solved, using the results of Table 5 and the fact that the relative value of the A/D converter distortion does not change. It can be done using a graphical solution as shown in Figure 34 or by trigonometry.

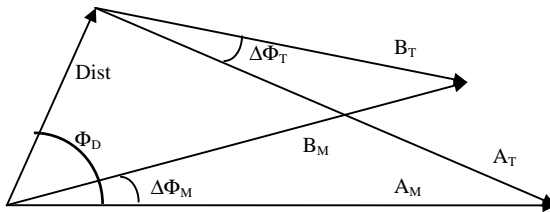


Figure 34. Graphical solution on how to separate A/D converter distortion from source distortion. A_M , B_M and $\Delta\phi_M$ are the measured changes and A_T , B_T and $\Delta\phi_T$ are the theoretical source harmonic changes, due to the RC filter.

Now the amplitude and the phase angle of the second harmonic distortion can be calculated:

$$|Dist| \approx 0.4 \cdot A_M \approx 0.4 \cdot 123 \approx 50 \mu\text{V} / \text{V} \quad (160)$$

$$\phi_{Dist} = \phi_{AM} + \phi_D = 110^\circ + 58^\circ \approx 170^\circ$$

A_M and Φ_{AM} are the second harmonic amplitude and phase angle values from Table 4. The maximum harmonic distortion of the voltage channel A/D converter of the calibration system is therefore less than 50 $\mu\text{V}/\text{V}$ for all harmonics.

The same measurements were repeated at the 1 volt level for the current channel A/D converter, which behaved in a very similar way as the voltage channel converter. The voltage channel completed with the voltage divider was subjected to 230 V and the result was compared to the now known A/D-characteristics and a resistive voltage divider introducing a negligible amount of distortion. The IVD is a magnetic type of divider and as can be seen in Table 6 it does contribute heavily to the harmonic distortion. The major reason for this distortion is probably the non-sinusoidal magnetising current in combination with the input impedance including the source impedance. The current transducers are of coaxial resistive shunt type and are not suspected to contribute to the harmonic distortion. The total result of the investigation can now be studied in Table 6

Table 6. The calculated harmonic distortion of the calibration system.

Harmonic no	Current channel Dist. ($\mu\text{V}/\text{V}$)	Voltage channel ADC Dist. ($\mu\text{V}/\text{V}$)	Voltage channel compl. Dist. ($\mu\text{V}/\text{V}$)
2	40	50	80
3	<10	<10	600
4	<10	<10	80
5	<10	<10	110
>5	<10	<10	<30

It can be concluded that the complete voltage channel harmonic distortion is less than 800 $\mu\text{V}/\text{V}$ with the IVD as the major contributor to the distortion. The current channel harmonic distortion can in the same manner be estimated to be less than 40 $\mu\text{V}/\text{V}$.

7.3.2.3 Intermodulation distortion

In harmonic current measurements, with high amplitude odd harmonics mixed with low amplitude even harmonics, the intermodulation distortion(IMD) can be of interest. As the amplitudes of the different harmonics will vary, and the relation between harmonic distortion(HD) and

IMD is complex, only a very roundabout estimate on how the IMD affects the current harmonic measurement can be done.

Based on the assumption that the non-linearity can be approximated by a polynomial function according to Equation (153) it has been shown, e.g. by [13], that the IMD and the HD are closely connected and that the $IMD(f_1+f_2)$ will be of about the same magnitude as $HD(f_1)+HD(f_2)$, if the amplitudes of the harmonics are the same. Generally, there will be a few combinations of nf_1 and mf_2 , giving $IMD(nf_1 \pm mf_2)$, contributing to the distortion of each harmonic, these can be added as uncorrelated contributions. Further the second harmonic distortion dominates in our calibration system, so the IMD at one particular frequency will have one dominating IMD contribution, with an amplitude of about two times the second harmonic distortion, plus number of small contributions. The total IMD of each harmonic will therefore generally not exceed three times the maximum HD, that is $100\mu A/A$ for the current channel of this calibration system.

7.3.3 Conclusions

The investigation shows that the calibration system harmonic measurement capability is inside the design goal for the calibration system, which is a measurement uncertainty considerably better than $2500\mu V/V$ when measuring low amplitude harmonics in the presence of a large fundamental. However, the major contribution is from the inductive voltage divider, which indicates that more work should be done here in the future, by changing it to a non-inductive device and/or make correction for the distortion. As the distortion probably is mainly due to magnetising current the source impedance is of great importance for the performance of the voltage harmonic measurements. The source impedance should therefore be kept as low as possible.

The current harmonic measurement uncertainty due to harmonic and interharmonic distortion is below $100\mu A/A$.

Further, it is shown that the chosen method is suitable for the verification of the harmonic measurement capability.

It is also shown that using a stable source, averaging and an innovative calculating of the distortion it is possible to measure distortion levels lower than the source distortion level without the use of much filtering.

7.4 References

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8 Specific applications of the DSWM

8.1 Calibration of power analysers

A power analyser is a multifunction instrument and it may have a multitude of output such as:

- rms voltage, current, power, reactive power, power factor.....
- Harmonic analysis: Voltage harmonics, amplitude and phase angle
- Harmonic analysis: Current harmonics, amplitude and phase angle
- Harmonic analysis: Power harmonics
- Harmonic analysis: Total harmonic distortion(THD).
- Detection and recording of transient phenomena
- Flicker

The variation within this kind of instrument is very large, from single phase instrument just capable of measuring a small number of harmonics up to multichannel instrument capable of using all the above functions for a large number of input at the same time. When calibrating a measuring instrument or measuring system that have such a lot of functions the number of calibration points may easily become unmanageable. The number of measuring points can be kept down by a sensible calibration strategy. Some knowledge of the meter and its possible weaknesses is important in order to choose the right strategy.

The most basic quantities for a power analyser is the harmonic analysis of the voltage and current resulting in values for amplitudes and phase angles, of the fundamental harmonic as well as the other harmonics. Most other quantities as active power(P), reactive power(Q), apparent power(S), power factor(PF) and the total harmonic distortion(THD) can all theoretically be directly derived from the harmonic analysis. If the calculation algorithms were completely known it would therefore be possible to calculate corrections for these values directly from the calibration of the harmonic analysis function of the power analysers. However, the software of a modern meter is so complex that it is not possible to be sure of this. Further, some unknown software correction can also be incorporated Therefore all quantities of interest must be calibrated at least in a few points and a magnitude dependence check be made.

Generally the instrument is equipped with three or four current inputs and the same amount of voltage inputs. Sometimes all input are alike and each transducer can be put into any of the inputs. It is possible but not very

efficient to calibrate the instrument in such a way that the corrections for the instrument can be calculated in every possible configuration. Often more than one set of current transducers are used. Most often the most practical method is to calibrate each transducer in one position only.

Some conclusions can be drawn from the earlier chapters about power analyser uncertainties and the section about the DSWM as a calibration system for power analysers: Power analysers may not always use the same algorithms to calculate quantities and it is not always stated in the specification which algorithm is used. This may be the case for such things as THD, reactive power, three-phase apparent power, three-phase power factor, harmonic voltage and current phase angles. The harmonic measuring capacity may be limited for low values due to the effects of input distortion, quantisation noise and phaselocking errors.

Calibrating a power analyser involves at least four steps.

1. Establish which calculation algorithms and presentation methods are used. These should be stated in the specification of the instrument but are most often omitted. However, if not known, the calibration could show errors that are due to an odd, but not incorrect, choice of calculation algorithm or presentation method and not a real error. This check should be made for each type of instrument, and does not have to be repeated for each calibrated object.
2. Check for errors due to transformer core non-linearity for each type of current probe and voltage transformer included in the system. These errors might not show in a single calibration. The check should be made more thoroughly once for each type of probe and a minor check should be made for each calibration.
3. Calibrate the system, for each voltage and current probe, at fundamental harmonic plus a number of harmonics using the harmonic display of the instrument.
4. Calibrate the rest of the quantities of interest with one probe set.

For high current probes or high voltage probes it may be necessary to calibrate the probes separately. This should, however, be avoided when possible. Things like the input impedance may well affect a probe, and an extra uncertainty component must be added. Further the number of calibration points will increase and the user must keep track on a larger number of corrections.

When choosing the harmonic amplitudes and phase angles a few things should be considered: The amplitudes and phase angles should be chosen

such that it can distinguish if an error of some quantity is caused by a real error or is caused by the use of an unexpected algorithm. The phase angles will affect the waveform, and thereby the crest factor, considerably. For example, if it is not known whether the phase reference is the zero-transition of the waveform or the zero-transition of the fundamental harmonic this can be distinguished by choosing appropriate phase angles of the harmonics.

8.1.1 Calibration strategy

Ultimately, it is always the user of the instrument that decides which of the instrument's functions that he wants to be calibrated. However, the harmonic analysis is the most basic function and should always be calibrated. Most often rms voltage and current, active power, reactive power, power factor and total harmonic distortion are also of great interest.

It is not possible, because of the cost, to calibrate all quantities for all possible input values. One measuring point for each quantity and range will be enough if the response to the most important influence factors are checked in one range. The most important influence factors are amplitude and then distortion level, phase angles, waveform. The voltage variation is generally much less than the current variation; the amplitude dependence of the voltage can therefore be checked in a smaller region.

8.1.2 Measurements

If a complete calibration is to be made the following quantities should be included in a calibration: U, I, P, Q, S, PF, dPF, THD, $U_1, U_n(\text{ampl}+\text{phas})$, $I_1, I_n(\text{ampl}+\text{phas})$. The calibration sequence should be as follows:

- Calibration at rated voltage and current. The signal should contain the fundamental harmonic and one more harmonic, for example the fundamental and the seventh harmonic. If the meter is to be used at some special amplitude that are not the rated, this amplitude should be used instead, for example a 600V rated voltage input that is intended for measurement on the 400V system should rather be calibrated at 230 V phase voltage.
- Phase angle or wave-form dependence: The signal should be the fundamental plus one harmonic as above but for another phase angle giving another waveform.

Measurement of U, I, P, $U_1, U_n(\text{ampl}+\text{phase})$, $I_1, I_n(\text{ampl}+\text{phase})$ at the following conditions:

- Amplitude dependence: The fundamental harmonic plus one harmonic; calibration at 20% of rated current for the current and at 50% of the rated voltage.

- Large distortion/many harmonics: The fundamental harmonic plus most harmonics up to the maximum harmonic order; 50% 3rd order current harmonic, then lower by the factor 1/n. 5% 3rd order voltage harmonics then lower by the factor 1/n. The total waveform should resemble a common real-life waveform.
- Low distortion, most harmonics at amplitudes near the harmonic amplitude resolution of the instrument.

The last points should be repeated for every set of current transducers, since the transducers are most prone to cause input distortion and frequency dependence.

8.1.3 Limitations

At the moment the calibration system measurement capability is limited to 10A and 300V. Current clamps up to 100 A can be calibrated by use of a 10 turn primary winding.

8.1.4 Miscellaneous

The setting of the above points should be made such that possible errors in calculation algorithms and possible errors in the use of calibration constants is discovered. This may make it necessary to add some calibration points when first calibrating a certain type of instrument.

In the end it is always the customer who decides the which quantities and which ranges that should be calibrated

The above calibration covers the most usual measuring functions of a power quality meter. Special measuring functions such as flicker measurements must be calibrated separately. Oscilloscope functions are most often not used for measurements, then it does not need a calibration. The above calibration may give some hint of status of the transient recorder function, but do not really cover this function. Today there are no rules on how to measure and characterise transients or voltage dips, therefore there is little need for a calibration of such a function. In the future, however, such a function may also need to be calibrated.

8.2 Calibration of flickermeters

8.2.1 Introduction

The light intensity of a lamp fed by a 50 Hz AC voltage will have a light intensity "flicker", at twice the AC frequency. Fortunately the human eye and brain do not detect intensity variations of this rate. However if the AC

voltage amplitude is changed, or modulated, by a lower frequency, in the range 1-25 Hz, people will tend to notice this and be annoyed by it, even when the variation is a small fraction of the total voltage.

The light intensity is approximately proportional to the square of the voltage. Further, the combination of eye and brain has a certain bandwidth and a certain response function. The response and bandwidth varies from person to person so empirical studies have been performed on a number of test persons. These studies have been aimed at finding first the limit of perception and then the limit where people tend to be annoyed by the flicker. Limits have been set to the level where 50 % of the test persons detect the flicker. This first limit, the perceptibility level, has been established for a number of change rates, both for step-wise changes and for sinusoidal modulation. Other studies have been made that has established the limit for irritation due to stepwise changes [1]. As it takes a certain time for an irritation to build up, these second studies were made in a longer time-frame, while the perceptibility level is essentially based on the instantaneous sensation of the flicker.

Still, in real life, voltage variations may sometimes be step-wise but seldom sinusoidal and they seldom have a constant rate and a constant modulation depth. Establishing the flicker severity from a measured time sequence of a voltage therefore still incorporates an evaluation that partly must be both intuitive and subjective. A flicker meter is an instrument which is designed to quantify, in an accurate and repeatable way, to what degree the light intensity variations caused by a voltage variation will irritate people. These flickermeter generally follow the functional description given in an IEC standard for flickermeters.

Up till a few years ago all flickermeters were based on analogue squaring and filtering functions. Instruments based on such analogue functions are very prone to change with time which results in a low accuracy. Newer instruments are often based on digital signal analysis with much better stability with time. The standard describing the instrument states that a test function should be built into the instrument. This serves only as a indication of the performance, however, and calibrations are needed regularly. This paper analyses what is required of such a calibration; it presents a calibration method for flicker instruments and a calibration system.

8.2.2 Flickermeter design details

A flickermeter with a functionality according the IEC standard [2-5] is designed to simulate the lamp-eye-brain-chain. The lamp is simulated in the meter by a squaring device, the eye-brain function partly by a weighing filter and partly by a further chain of squaring and low pass-functions, see Figure

35. The output of this function is p : "instantaneous flicker sensation". The instantaneous flicker sensation level should be unity (1.00) for a voltage variation equal to the perceptibility level according to [2]. A further statistical function followed by a square-root function completes the instrument. The output of the last function is the "short-time flicker severity level, "Pst", and should be unity for voltage variation equal to the irritation level according to [1].

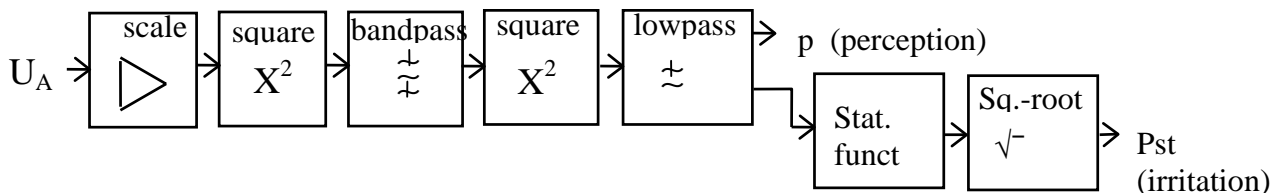


Figure 35. Flicker meter functional block diagram.

The two last block before the output p ("instantaneous flicker sensation level"), is a squaring function followed by a first order low-pass filter with an upper bandwidth $1/(2\pi \cdot 0.3s) \approx 0.5$ Hz. The signal p will therefore approximately be a DC level for voltage change rates well above 0.5 Hz, as can be seen in the 8 Hz instantaneous flicker example in Figure 36. Figure 36 also shows the input of the bandpass filter which essentially is the modulation and the output of the bandpass filter which is close to sinusoidal for a 8 Hz modulation but approximately a 100 ms pulse in the low-frequency rectangular modulation case. For the lower modulation rates the output signal p will have a ripple with a frequency of twice the input change frequency. For very low modulation rates the signal will be a strictly positive pulse train and for single change events such as a step-change the output will be a bandwidth-limited pulse, as seen by the 0.5 Hz flicker example in Figure 36. It is then evident why the instantaneous flicker sensation should be interpreted as the peak value and not the average value for all change rates.

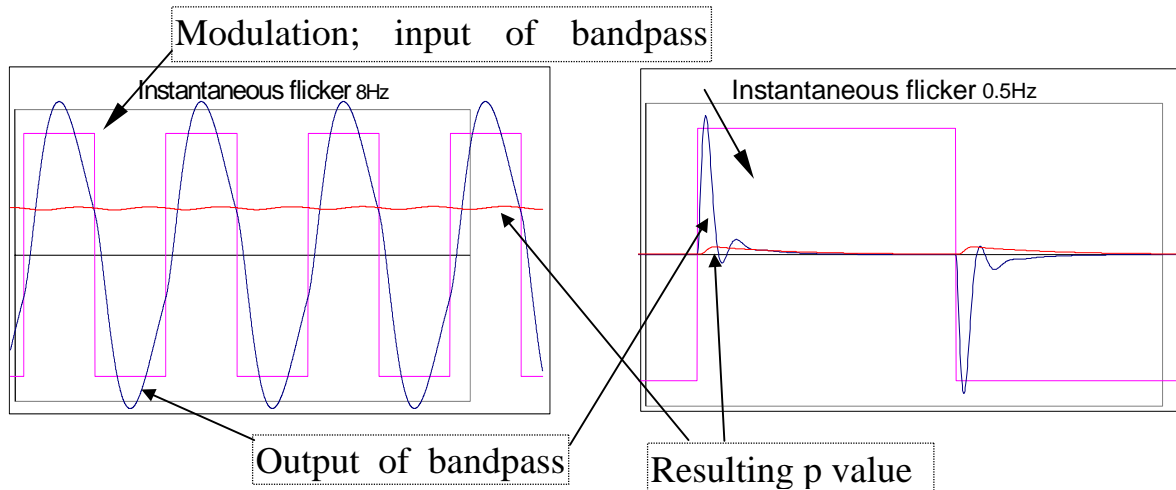


Figure 36 The strictly positive "instantaneous flicker" signal for rectangular modulation, input and output of bandpass filter is also shown.

The blocks with the statistical function and the root function will basically produce an output that is the square root of half the instantaneous flicker level that is not exceeded during about 95 % of the observation time. By these functions the level of irritation can be calculated from the instantaneous flicker signal. For higher modulation rates and steady state, when the instantaneous flicker level is a pure DC, the Pst, or short time flicker severity, output of this block will be equal to:

$$Pst \approx \sqrt{0.501 \cdot p}$$

where p is the instantaneous flicker sensation level. More precisely, for general input signals the short time flicker severity according to [2, 3] shall be:

$$Pst = \sqrt{0,0314P_{0,1} + 0,0525P_{1s} + 0,0657P_{3s} + 0,28P_{10s} + 0,08P_{50s}}$$

where $P_{0,1}$, P_1 , P_3 , P_{10} , P_{50} is the instantaneous flicker level exceeded for 0.1, 1, 3, 10 and 50 % of the observation time and the index s means a smoothed value. For example P_{50s} should be the mean of P_{30} , P_{50} and P_{80} .

8.2.3 Calibration considerations.

The uncertainty requirement of a flicker meter is stated in [2] as $\pm 5\%$. The calibration should therefore aim at a calibration uncertainty less than 1%. There are, basically, two possibilities given in the standard for an accuracy evaluation of a flicker meter. One table, see Table 7, is given that describes some combinations of step-wise change level and frequency that should result in a Pst level of unity. Two other tables, see Table 8, are given that

states some more sinusoidal and rectangular modulations that should result in an instantaneous flicker sensation level of unity for different modulation rates. The former table is given as a performance test table while the other two are called type test tables.

Table 7. Performance test table, settings for Pst = 1 ± 5% [3].

Changes per minutes	Modulation freq. (Hz)	Voltage change (%)
1	0,008333	2,72
2	0,016667	2,21
7	0,58333	1,46
39	0,32500	0,905
110	0,91667	0,725
1620	13,500	0,402

Table 8. Type test tables, rectangular modulation, settings for p=1.00± 5% (Pst about 0.71)[2].

Sinusoidal modulation				Rectangular modulation			
Hz	ΔV/V (%)	Hz	ΔV/V (%)	Hz	ΔV/V (%)	Hz	ΔV/V (%)
0.5	2.340	10.0	0.260	0.5	0.514	10.0	0.205
1.0	1.432	10.5	0.270	1.0	0.471	10.5	0.213
1.5	1.08	11.0	0.282	1.5	0.432	11.0	0.223
2.0	0.882	11.5	0.296	2.0	0.401	11.5	0.234
2.5	0.754	12.0	0.312	2.5	0.374	12.0	0.246
3.0	0.654	13.0	0.348	3.0	0.355	13.0	0.275
3.5	0.568	14.0	0.388	3.5	0.345	14.0	0.308
4.0	0.500	15.0	0.532	4.0	0.333	15.0	0.334
4.5	0.446	16.0	0.480	4.5	0.316	16.0	0.376
5.0	0.398	17.0	0.530	5.0	0.293	17.0	0.413
5.5	0.360	18.0	0.584	5.5	0.269	18.0	0.452
6.0	0.328	19.0	0.640	6.0	0.249	19.0	0.498
6.5	0.300	20.0	0.700	6.5	0.231	20.0	0.546
7.0	0.280	21.0	0.760	7.0	0.217	21.0	0.586
7.5	0.266	22.0	0.824	7.5	0.207	22.0	0.604
8.0	0.256	23.0	0.890	8.0	0.201	23.0	0.680
8.8	0.250	24.0	0.962	8.8	0.199	24.0	0.743
9.5	0.254	25.0	1.042	9.5	0.200		

* N.B. The modulation ΔV/V is defined in the standard as $(U_{max} - U_{min})/U_n$

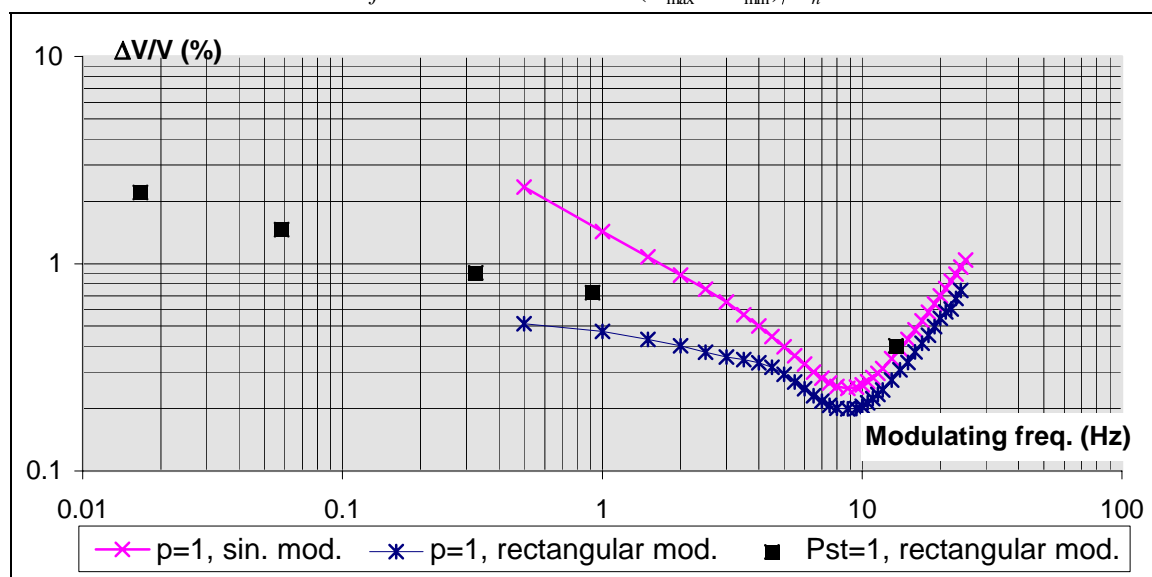


Figure 37. Flicker limit curves for sinusoidal and rectangular modulations. A summary of the tables above.

Normally a performance test table would be the obvious choice for a calibration. However, Pst values are usually given every 10 min. Further, the change rates stated in this table demands even longer measurement times to achieve 1% uncertainty. The performance test table might therefore possibly be feasible for a performance test with a $\pm 5\%$ uncertainty requirement but not for a high accuracy calibration for a reasonable cost. A calibration of the instantaneous flicker sensation output at the somewhat higher voltage change rates given in table 2 is therefore preferred. The performance of the statistical block can be checked by a single Pst measurement after the main calibration.

8.2.3.1 Measuring instantaneous flicker for calibration purposes

The calibration instrument measuring the (output) instantaneous flicker sensation level of the meter under test must have a peak level detection uncertainty of less than 1 % for frequencies below 10 Hz. This is because the output ripple which is twice the frequency of the voltage change will not be negligible for lower frequencies due to the 0.5 Hz low pass filter, see Figure 36. For the sinusoidal modulations it is also possible to measure the AC and the DC rms voltage of the output and calculate the instantaneous flicker value as $V_{DC} + \sqrt{2}V_{AC}$.

8.2.3.2 Measuring the modulation $\Delta V/V$ for calibration purposes

One might consider measuring $\Delta V/V$ by the measurement of the peak(-to-peak) voltage by a data acquisition system. However, measuring about 1 % change of the peak value with an uncertainty of 1 % requires a data acquisition system with a resolution and uncertainty (linearity only) better than an ideal 14-bit DAC. This must be achieved at a sampling rate high enough to find the peak with an uncertainty below 0.01% which equals to a minimum phase angle resolution of $\arccos(1-0.0001)$, which gives a minimum sampling rate of $50\text{Hz} \cdot 2\pi / \arccos(1-0.0001)$ which is about 2 ksample/s. The case with sinusoidal modulation is even worse, it is difficult to find the exact peak voltage of an modulated signal when it is modulated by a sinusoidal signal. This is because the peak of the carrier and the peak of the modulating signal might never coincide and certainly not for both positive and negative peak voltage of the modulated signal in the same period of the modulating signal.

A better alternative for rectangular modulation is to calculate the rms-value of each AC voltage period, or half-period. Then each value will depend on a number of measurements instead of a single value and it is much easier to achieve a low uncertainty. For the same reason spectral analysis is very suitable in the case of the sinusoidal modulation, and may also be used for rectangular modulation.

8.2.3.3 Measuring $\Delta V/V$ with discrete Fourier transform, DFT

When using a reference meter based on spectral analysis by DFT care must be taken not to get errors due to spectral leakage. This can best be achieved if a number of fundamental periods are analysed that also coincides with an integer number of the modulating frequency.

Sinusoidal modulation can be expressed as in the first part of (161) and by substitution by trigonometric identities one can get the expression as in the second part of (161):

$$\begin{aligned} v(t) &= (1 + m \cdot \cos \omega_m t) A \cos \omega t = \\ &= A \cos \omega t + \frac{A \cdot m}{2} (\cos(\omega t + \omega_m t) + \cos(\omega t - \omega_m t)) \end{aligned} \quad (161)$$

where m is the modulation factor, ω_m the modulation frequency and ω the carrier frequency, in this case the 50 Hz power system frequency. Sinusoidal modulation will therefore result in two "sidebands" with an amplitude of $m/2$ relative the power frequency amplitude or the AC amplitude, see Figure 39. Further, in the standard and the tables above the voltage fluctuation ΔV is given as the peak-to-peak value of the fluctuation, while m above is the relative deviation from the nominal rms value, that is:

$$\frac{\Delta V}{V} = 2m = 4 \frac{A_{50\text{Hz}}}{A_{\text{sideband}}} \cdot 100\%$$

If DFT is used to measure rectangular modulation an infinite number of modulation harmonics will occur. Then the first, third, fifth, seventh and preferably the ninth harmonic must be evaluated to achieve a uncertainty of less than 1%.

8.2.3.4 Miscellaneous

The flicker meter must be able to measure correct at a broad range of modulation. This linearity should be checked at some modulation frequency preferably in the central region of the modulation frequency interval. The modulation should then be changed to check that the response to a modulation change is according to the specification. The modulation frequency of 4 Hz might be a good choice.

Both rectangular and sinusoidal modulation should be used in the calibration, but the calibration can concentrate on one of the modulation types and just make check of the other at one or a few modulation frequencies. Sinusoidal modulation is preferred for DFT analysis.

8.2.4 The calibration system

The system is built up from two subsystems, a waveform generator and a waveform analysis system.

8.2.4.1 Waveform generator

The appropriate waveform is calculated by a PC software and sent to a 16-bit digital-to-analogue converter with a maximum output frequency of 100 ksamples/s. The 10V output of the DAC is further amplified by a precision power amplifier to a maximum of 400V. The data sequence of the waveform must be long enough to cover a time which is an integer number of periods of both the modulated and modulating signal. When this waveform is repeated continuously a true amplitude modulated signal is created.

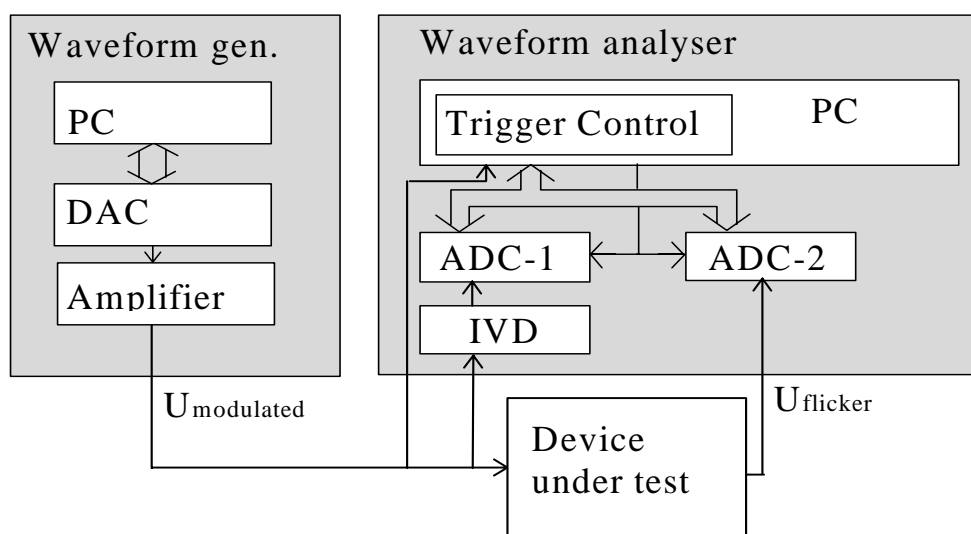


Figure 38. Block diagram of the calibration system

8.2.4.2 The waveform analyser

The waveform analyser is based on two precision voltmeters used in sampling mode. The sampling is phase-locked to the input voltage in such a way that an (exact) integer number of periods are sampled. This sequence is then analysed by a spectral analysis software in the PC. If the number of AC frequency periods are chosen in a way that the time corresponds also to an integer number of periods of the modulating signal this spectral analysis will result in a precise measurement of the modulation. Modulation by 4, 8, 16 and 24 Hz can all be analysed correctly based on a sampled sequence of 25 periods of the line frequency. By a 100-period sequence the 0.5, 1 and 2 Hz modulation can also be measured. Calibration at these modulation frequencies, 0.5, 1, 2, 4, 8, 16, 24 Hz, at sinusoidal modulation is enough to verify the flicker meter when combined with a check of the response to rectangular modulation at both the instantaneous flicker output and at the Pst output.

The harmonic measurement capability of waveform analyser has been established earlier [6, 7] to well within 1000 ppm or 0.1 %. Therefore the measurement of the relative modulation factor m can be performed with an uncertainty of less than 1%, provided that the source is stable and has a low distortion. A typical result from a modulation measurement of a sinusoidal modulation can be seen in graphical form in Figure 39

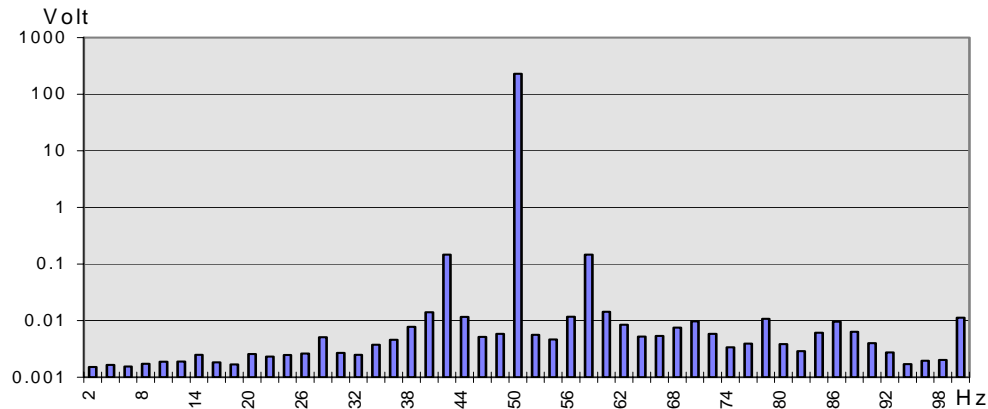


Figure 39. Typical modulation measurement result.

The frequency of the instantaneous flicker ripple will be twice the frequency of the modulation and therefore the instantaneous flicker sensation level, p , can be measured by the second waveform analyser input channel without any windowing errors, at the same time as $\Delta V/V$ is measured.

8.2.5 Conclusion

A flickermeter can be calibrated with a system capable of generating and measuring an AC voltage with sinusoidal or rectangular amplitude modulation. Such a signal is preferably analysed by Discrete Fourier Transform. Provided that a appropriate number of periods are analysed and the waveform source is stable and have a low distortion an uncertainty of less than 1 % can be achieved.

8.3 Measurement errors of energy meters

It is of interest to find out how much energy meters are affected by harmonics. Measurements are one way to estimate these effects. Normally, only harmonics with a frequency that is an integer multiple of 50 Hz are considered. However, in many cases, there are 16.7 Hz and its harmonics present on the Swedish power system, due to the Swedish railway power supply. This is especially true where the input energy of converters for 16.7 Hz power is measured. The 16.7 Hz and its harmonics can be considered as subharmonics and interharmonics from a 50 Hz point of view. Therefore, one

of the measurements are performed with among others an 83.3 Hz component, which is the fifth harmonic of a 16.7 Hz converter.

Measurements of the metering errors of seven different energy meters have been made. Both active and reactive meters have been tested. The metering results of the meters have been compared to the actual powers. The actual powers have been determined by calibrating a three-phase energy meter test system with the DSWM. The measurements have been repeated for different setting of voltage and current harmonics.

8.3.1 Test setup

8.3.1.1 Settings

Three different settings, defined in Table 9 to Table 11 below, were used.

Table 9. First setting, with subharmonics and interharmonics

Harm. No.	Freq. Hz	Phase R			Phase S			Phase T		
		U %	I %	Φ °	U %	I %	Φ °	U %	I %	Φ °
1	16.7	0.3	60	-90	0.3	110	-90	0.3	100	-90
3	50	100	100	0	100	100	0	100	100	0
5	83.3	0.3	20	-90	0.3	10	-90	0.3	6	-90
7	116.7	0.3	23	-90	0.3	10	-90	0.3	6	-90
9	150	0.3	27	-90	0.3	40	-90	0.3	20	-90
11	183	0.3	30	-90	0.3	0	-	0.3	25	-90
13	217	0.3	13	-90	0.3	10	-90	0.3	9	-90

This setting gives: $S=1347\text{VA}$, $P=945\text{W}$, $Q_F=954\text{Var}$, $P_H=0\text{W}$, $S_H=7\text{VA}$, where index H refers to all harmonics but the 50 Hz harmonic.

Table 10. Second setting, odd harmonics only.

Harm. no	Freq. Hz	All phases		
		U %	I %	Φ °
1	50	100	100	0
3	150	5	20	0
5	250	5	20	0
7	350	5	20	0

This setting gives $S=1005,4\text{VA}$, $P=974.9\text{W}$, $Q_F=245.9\text{Var}$, $Q_I=Q_B=0\text{VAr}$, $P_H=30\text{W}$, $S_H=30\text{VA}$.

Table 11. Third setting, even harmonics only.

Harm. no.	Freq. Hz	All phases		
		U %	I %	Φ °
1	50	100	100	0
2	100	5	20	0
4	200	5	20	0
6	400	5	20	0

This setting gives: $S=1005\text{VA}$, $P=974.8\text{W}$, $Q_F=246.3\text{VAr}$, $P_H=30\text{W}$, $Q_I=Q_B=0\text{VAr}$, $S_H=30\text{VA}$

The apparent power is calculated as $S = U_{\text{rms}} \cdot I_{\text{rms}}$ and reactive power is calculated by $Q = \sqrt{S^2 - P^2}$.

8.3.1.2 Test objects

The test objects were:

1. Object A, class 1, static, active and true connected reactive meter.
2. Object B, class 0.5, static, cross-connected, reactive meter.
3. Object C, class 1, static, active meter.
4. Object D, class 1, static, cross-connected, reactive meter.
5. Object E, class 0.2, static, active meter
6. Object F, class 0.2, static, cross-connected, reactive meter.
7. Object G, class 1, Ferraris meter for active energy.

All meters were had a rated voltage and current of $110\text{V}/\sqrt{3}$ and 5A , and were tested at rated voltage and current.

8.3.1.3 The measuring method

A three-phase ZERA meter test equipment was used as voltage and current source. The measuring part of the system was calibrated for all settings with the DSWM, This was done phase-by-phase because the DSWM is a one-phase meter while the measurement was performed in a three phase environment. Then the measurements were performed with the DSWM connected to one phase as an extra precaution.

8.3.2 Measurements

8.3.2.1 Results

The results when using the settings from Table 9 to Table 11 can be studied in Table 12 to Table 14

Table 12. Results when using setting according to Table 9.

Object	Type*	Class	Active energy Errors		Reactive energy Errors	
			%	W	%	VAr
Object A	S,T	1	0.3	3	-96.4	-920
Object B	S,C	0.5			-95.4	-910
Object C	S	1	0.5	5		
Object D	S,C	1			-24.7	-278
Object E**	S	0.2	-2.7	-26		
Object F	S,C	0.2			-74.6	-709
Object G	F	1	-1.9	-18		

* The following denominations are used: S=static, F=Ferraris, C=cross-connected, T=true connected.

** This meter, object E, showed a standard deviation of 0.7%. It may be that the meter can not handle the rather large sub- and interharmonics levels of this test. It also showed a large drift, about 3 %, after connection (with this setting. 1 only)

Table 13. Results when using setting according to Table 10.

Object	Type	Class	Active Errors		Reactive Errors	
			%	W	%	Var
Object A	S,T	1	0.0	0	-93.3	-230
Object B	S,C	0.5			-99.0	-241
Object C	S	1	0.1	1		
Object D	S,C	1			-98.8	-243
Object E	S	0.2	-0.2	-2		
Object F	S,C	0.2			-98.7	-243
Object G	F	1	-0.9	-9		

Table 14. Results when using setting according to Table 11.

Object	Type	Class	Active Errors		Reactive Errors	
			%	W	%	VAr
Object A	S,T	1	-0.1	1	-93.9	-231
Object B	S,C	0.5			-99.1	-245
Object C	S	1	-0.1	-1		
Object D	S,C	1			-98.8	-243
Object E	S	0.2	-0.2	-1.9		
Object F	S,C	0.2			-98.9	-243
Object G	F	1	-0.9	-8.8		

8.3.2.2 Measurement uncertainty

The total energy measurement uncertainty is a combination of the DSWM measurement uncertainty, the stability of the source and the uncertainty due to the measurement resolution that is determined by the measuring time. These uncertainties can be defined by their estimated standard deviations:

Uncertainty source	Standard uncertainty setting 1	Standard uncertainty setting 2-3
DSWM:	0.02%,	0.03%.
Source stability	0.049%,	P: 0.0115%, Q: 0.038%
Resolution	0.072%,	0.087%.

Table 15. The total uncertainty of the measurements

	Active energy	Reactive energy
Setting 1	0.20%	0.20%
Setting 2	0.10%	0.15%
Setting 3	0.10%	0.15%

The uncertainty is calculated according to WECC doc. 19-1990 with a coverage factor 2.

8.3.3 Conclusions

The results from the measurements made at setting one are somewhat erratic, especially the reactive power measurements. This can be due to a number of reasons. Although the meters were operated at their nominal fundamental voltage and current, the rms current may have exceeded the maximum allowed current. Further, the subharmonics are much more prone to saturate

the meters because the saturation limit of an inductive device depends on the inverse of the frequency. Finally, the subharmonics may have upset the meters even if they were not saturated. Because of the risk for saturation, meters subjected to a high degree of subharmonics should be de-rated, and it would have been of interest to redo the measurements of setting one for a lower current. It should be noted that the readings of P and Q of the meters would not have indicated an overload situation.

The measurements of settings 2 and 3 show, as expected, rather low errors of the active power. The Ferraris-type meter shows the highest error, a negative error that is equal to a third of the (positive) active harmonic power of these settings. This kind of error was expected for a meter with a low upper frequency limit. The errors of the reactive energy meters are very high relative the total reactive power (or fictitious or non-active) power Q_F . This must be because they do not measure according the definition of the total reactive or fictitious power Q_F . The size of the errors makes it impossible to apply a correction factor for their measurement error (if they are expected to measure the total or fictitious power Q_F). However, although not directly measured, the errors compared to Q_B or Q_1 of setting 2 and 3 are within a few tenths of a percent relative the apparent power for the cross-connected meters. This should be expected, because according to equation (18) of chapter 4 they should measure $Q=Q_1-P_2+P_4+Q_5-Q_7+P_8$ which is zero for both setting 2 and setting 3, and so is the set values of Q_B and Q_1 . The difference between the true connected reactive meter and Q_B or Q_1 is somewhat higher, this is also expected at these settings if the phase shifting device of the meter does not phase shift all harmonics by $\pi/2$.

8.4 References

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9 Conclusions

It has been shown that accurate measurements of non-sinusoidal electrical signals can be made by taking equally-spaced simultaneous digital samples of voltage and current, during an integer multiple of periods of the fundamental frequency. The digital sampling wattmeter system (DSWM), which is mainly built up of standard laboratory equipment, works by this principle and has been proven to fulfil the accuracy expectations. Two calculation methods have been used to obtain values of the desired quantity: discrete integration (DI), and Discrete Fourier Transform (DFT). A system like the DSWM is versatile and can be used for measurements and calibrations of many quantities. The basic ones are the (total) active power and the amplitude and phase angle of individual harmonics of non-sinusoidal voltages and currents. However, a number of other quantity describing the state of a power system can be calculated by similar and calculations, for example reactive and apparent power, rms and DC voltage and current, phase angle and power factor.

The application of DFT instead of DI, which is used in most other laboratory standard DSWMs, has a few important advantages. In case that currents and voltages are not sinusoidal, a good description of the state of a power system demands that the signal can be separated in its harmonic components. This is true for a number of suggested extensions of the power theory to cover the non-sinusoidal condition. In this case, the DFT is an effective calculation method that allows measurements of the amplitudes and phase angles of individual harmonics of non-sinusoidal voltages and currents. Once these are obtained most quantities can be calculated such as various non-active powers and the (total) harmonic distortion. Further, these calculation methods allows frequency dependent corrections to be used, which enhances also the accuracy of (total) active power measurements of non-sinusoidal signals.

The DSWM was first verified for the 50 Hz sinusoidal situation, and has then taken part in three international comparisons with satisfying results. Final results from the first comparison are presented in Appendix 1. The comparison was made at several power factors and the observed differences were all within 50 ppm. The DSWM has further been verified for sinusoidal signals in the range 10 Hz - 20 kHz, resulting in an estimated uncertainty relative S of 600ppm at 20 kHz and at power factor one and 1200 ppm at power factor zero. The most important additional feature, for the measurement of individual harmonics of distorted signals, is the input distortion of the measuring system, which must be low. This distortion has been verified to be less than 800 ppm for all harmonics and lower than 100

ppm for most harmonics. Today, the DSWM is the Swedish primary reference for all power measurement.

Some AC quantities, such as reactive power are not properly defined for non-sinusoidal situations. Efforts are made in this work to understand and to explain the problems of extending the reactive power definition to cover non-sinusoidal situations. The main conclusion is that reactive power is used to obtain information on more than one property of the power transmission mechanism, e. g. phase angle, transmission efficiency and line voltage drop. No single definition exists that can provide information on all these properties in a non-sinusoidal situation. Moreover, the design of many instruments is such that they do not comply with any of the extended definitions. In a non-sinusoidal situation, these meters will then exhibit extra errors due to this non-compliance.

Some conclusions on future demands on energy meters and other power system metering equipment can be drawn, regarding their measurement capabilities in non-sinusoidal situations. These conclusions are based on an error analysis of these meters and on an analysis on how the responsibility for the harmonic currents and voltages in the power system can be determined and shared. The conclusion is that it is not possible to make a precise determination of the responsibility for harmonics based on any power or energy measurement. The sources of harmonics are not power sources but should rather be modelled as voltage or current sources including source impedances. The harmonic currents, voltages and powers will therefore depend much on the relation between the impedance of the power system and the impedance of the load of interest as well as the impedance and the harmonic level of other nearby loads. Sources of harmonics will therefore interact in a way that may be treated by statistical means but this interaction can not generally be predicted for a single load.

However, a rough estimate of the responsibility for harmonics can be made by a combination of harmonic active power measurements and harmonic current and harmonic voltage measurements. In that case, a high value of harmonic current and/or voltage combined with positive harmonic active power of a reasonable high power factor is a good indication of a disturbing load while a negative active power indicates a disturbed load. High harmonic current and/or voltage combined with a low harmonic active power will then indicate a high degree of interaction and a shared responsibility of the harmonic situation. If the output of an energy meter is to be used for indicating purposes only, the harmonic current is good

indicator, preferably as a relative value, compared to the rated current of the load.

Before the harmonic active power is used to determine the responsibility of current harmonics, more precise estimations of the harmonic impedance of the power system must be made. Further, accurate measurements of harmonics can not be guaranteed today, because of a lack of knowledge of the frequency response of instrument transformers used for metering. More work is needed to determine typical frequency responses and the impact of the burden on the frequency response. In the long run, all instrument transformers should have a verified frequency response to allow for proper harmonic evaluations.

Appendix 1 A bilateral power comparison

As a lead in maintaining the international agreement of the power standard a comparison was made between SP and Physikalisch-Technische Bundesanstalt (PTB), which is the national metrology institute of Germany. The comparison was made for some different 50 Hz sinusoidal voltages and currents. A HEG Power Converter C1-2, which is a Time-Division-Multiplying (TDM) power meter with a nominal voltage and current of 120V and 5A, was used as a transfer standard.

The comparison was made between three power measuring systems at SP and two systems at PTB. SP uses one power measuring system for PF=1, one power measuring system for PF=0, and the DSWM, which is used for all power factors. PTB uses two systems, a system NME 2 for PF=1 and PF=0, and a system PTB-Leistungsnorm for all power factors.

The comparison was made during the time from December 1993 to April 1994. The transfer standard HEG C1-2 was calibrated by each power measuring system at a number of voltages and currents. Each such calibration is presented separately in the following.

The comparison between the DSWM and the PTB-Leistungsnorm is summarised in diagram form on the last page of this appendix. All differences are well within the combined $k=1$ uncertainty. It can be noticed that one of the points with the largest differences is 120 V, 5 A, which is the 100%-point of the transfer standard. After this comparison, an unexpected loading error of the DSWM were found, due to ineffective cooling of the 5 A current shunt. This loading error accounts for 10 -20 ppm of the difference at 120 V, 5 A. Preliminary results from two more multilateral comparison, which are part of the international co-operation of national laboratories, generally confirm these results.

Calibration of power converter HEG C1-2 no 51 709**Reference: Primary measuring system for power factor one**

Date of measurement: 1994-04-11--14

Input values			Measured output voltage /V	Measure error /ppm	Uncert rel. S. /+ppm
U	I	cos ϕ			
120 V	5 A	1	10.000	+28	20
120 V	1 A	1	2.000 034	+17	20
120 V	0.05 A	1	0.099	-321	100
60 V	5 A	1	5.000 170	+34	20
120 V	0 ²⁾	-	-0.000	-	± 10
0 ¹⁾	5 A	-	0.000 027	-	± 10
0 ¹⁾	0 ²⁾	-	0.000 021	-	± 10

Reference: Primary measuring system for power factor zero, SP no 2

Date of measurement: 1994-04-15--21

Input values			Measured output voltage /V	Measure error /ppm	Uncert rel. S. /+ppm
U	I	cos ϕ			
120 V	5 A	0 i	-0.000	-13.3	5
		0 k	+0.000	+10.8	5
120 V	1 A	0 i	-0.000	-27.4	8
		0 k	-0.000	-5.2	8
60 V	5 A	0 i	-0.000	-17.8	5
		0 k	+0.000	+23.6	5
0 ¹⁾	0 ²⁾	-	+0.000	-	$\pm 10\mu$

Reference: Digital Sampling Watt-Meter

Date of measurement: 1994-04-08--22

Input values			Measured output voltage /V	Measured error rel.S /ppm	Uncert. rel. S, k=1 /±ppm
U	I	cosφ			
120 V	5 A	1	10,000 120	+12	35
		0,5 i	4,999 810	-19	35
		0,5 k	5,000 150	+15	35
		0,15 i	1,499 810	-19	25
		0,15 k	1,500 100	+10	25
		0 i	-0,000 150	-15	20
		0 k	0,000 090	+9	20
		120 V	1 A	1	2,000 032
0,5 i	0,999 956			-22	30
0,5 k	1,000 002			+1	30
0 i	-0,000 038			-19	20
0 k	-0,000 030			-15	20
120 V	0,05 A	1	0,099 969	-310	120
		0,5 i	0,049 965	-350	120
		0,5 k	0,049 968	-320	120
60 V	5 A	1	5,000 080	+16	35
		0,5 i	2,499 970	-6	35
		0,5 k	2,500 085	+17	35
		0 i	-0,000 040	-8	20
		0 k	0,000 075	+15	20
120 V	0 ²⁾	-	-0,000 047	-	±10 μV
0 ¹⁾	5 A	-	0,000 012	-	±10 μV
0 ¹⁾	0 ²⁾	-	0,000 008	-	±10 μV

1) Voltage input short-circuited

2) Current input open

Positive reference voltage: +7,042 213 V

Negative reference voltage: -7,042 276 V

Uncertainty:

±2 ppm

Reference: PTB-NME 2

Date of measurement: 1993-12 - 1994-02

Input values			Measured output voltage /V	Measured error rel.S /ppm	Uncert. rel. S, k=1 /±ppm
U	I	cosφ			
120 V	5 A	1	10,000 536	+54	11
120 V	1 A	1	2,000 091	+46	11
120 V	0,05 A	1	0,099 9711	-289	50
60 V	5 A	1	5,000 215	+43	20
120 V	0 ²⁾	-	-0,000 033	-	±10 μV
0 ¹⁾	5 A	-	0,000 025	-	±10 μV
0 ¹⁾	0 ²⁾	-	0,000 024	-	±10 μV

Input values			Measured output voltage /V	Measured error rel.S /ppm	Uncert. rel. S, k=1 /±ppm
U	I	cosφ			
120 V	5 A	0 i	-0,000090	-9,0	8
		0 k	+0,000082	+8,2	8
120 V	1 A	0 i	-0,000 076	-38,1	8
		0 k	-0,000 019	-9,3	8
60 V	5 A	0 i	-0,000110	-22,1	15
		0 k	+0,000018	+3,6	15
0 ¹⁾	0 ²⁾	-	+0,000 024	-	±10μV

Reference: PTB-Leistungsnorm

Date of measurement: 1993-12 - 1994-02

Input values			Measured output voltage /V	Measured error rel.S /ppm	Uncert. rel. S, k=1 /±ppm
U	I	cosφ			
120 V	5 A	1	10,000 497	+50	30
		0,5 i	5,000 018	-17	30
		0,5 k	5,000314	+31	30
		0,15 i	1,500 004	0	30
		0,15 k	1,500 100	+10	30
		0 i	-0,000 127	-13	30
		0 k	0,000 042	+4	30
120 V	1 A	1	2,000 085	+42	30
		0,5 i	1,000 015	+8	30
		0,5 k	1,000 041	+20	30
		0 i	-0,000 104	-50	30
		0 k	0,000 026	+13	30
120 V	0,05 A	1	0,099 971	-290	120
		0,5 i	0,049 968	-320	120
		0,5 k	0,049 969	-310	120
60 V	5 A	1	5,000 148	+30	35
		0,5 i	2,500 075	+15	35
		0,5 k	2,500 231	+46	35
		0 i	-0,000 126	-25	20
		0 k	0,000 030	+6	20
120 V	0 ²⁾	-	-0,000 033	-	±10 μV
0 ¹⁾	5 A	-	0,000 025	-	±10 μV
0 ¹⁾	0 ²⁾	-	0,000 024	-	±10 μV

1) Voltage input short-circuited

2) Current input open

Positive reference voltage: +7,042 226V

Negative reference voltage: -7,042 312 V

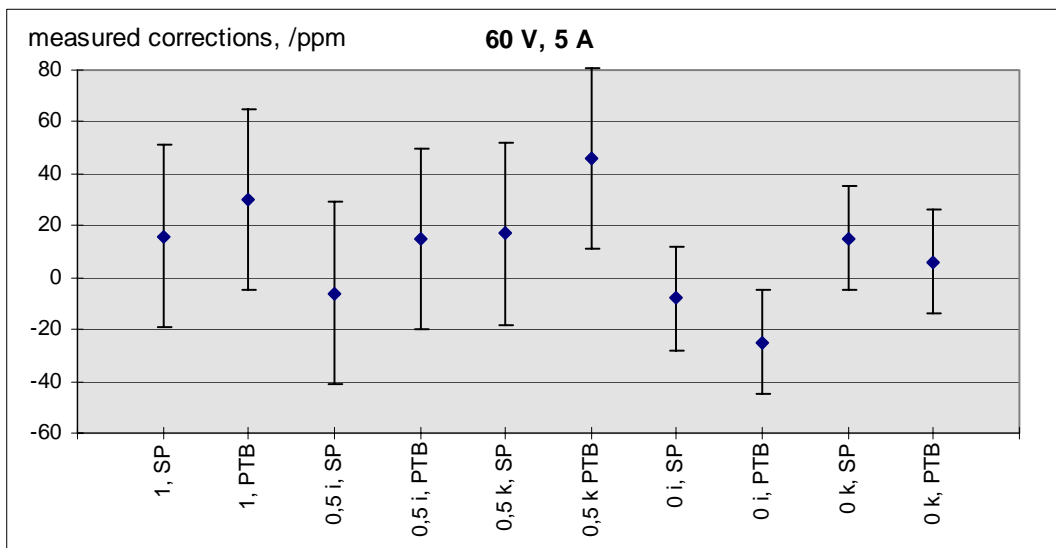
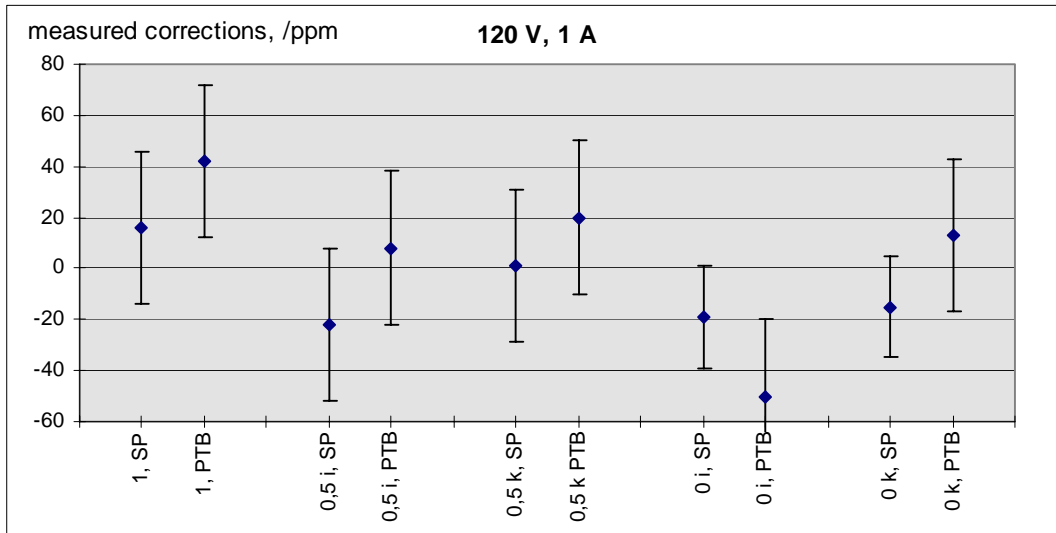
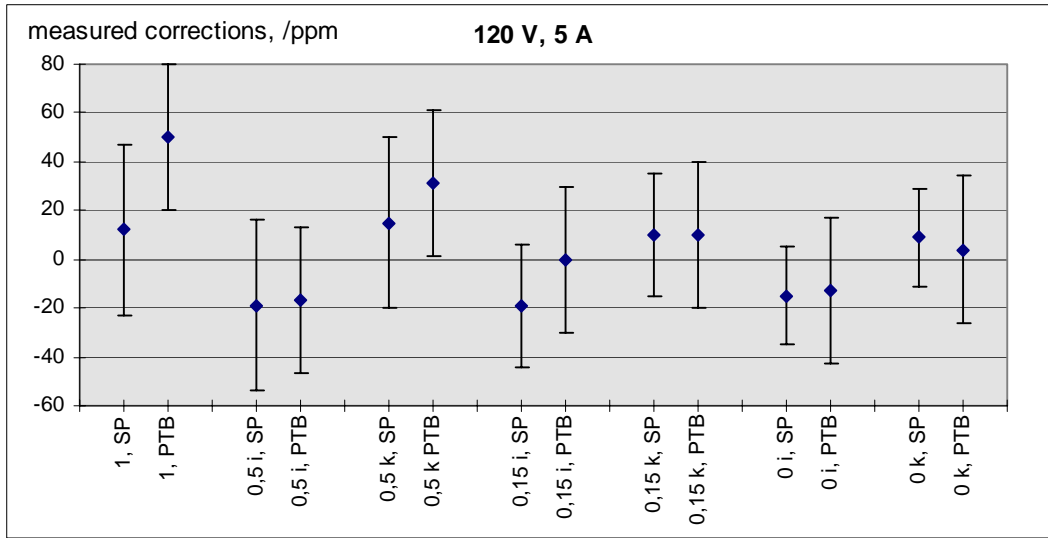


Figure 1. SP-DSWM and PTB-Leistungsnorm comparison in diagram form, for some power factors, at 120V and 5A, 120V and 1A, 60V and 5A,.

Appendix 2 A comparison of power measuring systems⁵

Abstract: An automatic zero power factor standard has been designed and compared to a digital sampling wattmeter standard. The agreement of the two power measuring standards at 120V, 5A and zero power factor is within a few ppm.

Introduction

To improve the traceability of power measurements at zero power factor (PF) at the Swedish National Testing and Research Institute (SP) an automatic zero power factor measuring system (ZPFS) has been designed. The ZPFS is based on the principles described by Inglis et al. [1]. A digital sampling wattmeter (DSWM) which can be used in both sinusoidal and non-sinusoidal conditions is also used at SP as a primary standard for measurement of power [2]. A comparison of the two standards has been made at sinusoidal conditions and evaluated against the estimated uncertainties at zero power factor. The comparison has been made with a power converter as a transfer device. A brief description of the measuring systems and the uncertainties are given together with the results of the comparison.

Automatic zero power factor measuring system

The ZPFS utilises a thermal wattmeter where zero-PF is established when a reversal of the current, or the voltage, results in no change of the output, fig. 1.

⁵ Paper published in IEEE transactions on instrumentation and measurements, Vol. 46, No 2, 1997, by P. Simonsson, S. Svensson and K.-E. Rydler.

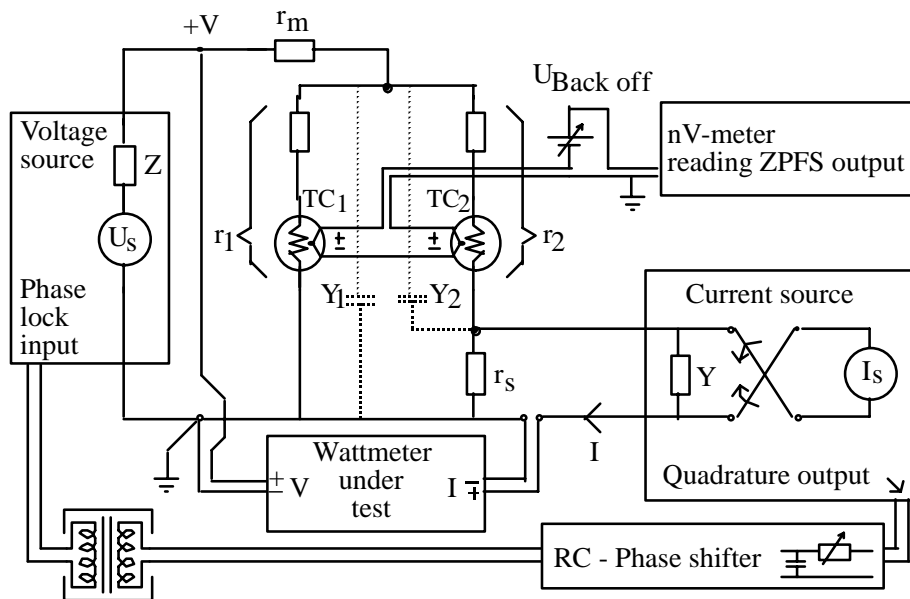


Figure 40 Schematic circuit diagram of the automatic zero power factor measuring system.

The measurement procedure for calibration of a wattmeter under test (UUT) is automated by computer control via the IEEE-488 bus and starts with a warm up period with applied apparent power (S). Then measurements are made at n phase angle settings, approximately equidistantly spaced, covering a small interval around $\pi/2$ rad. The interval is in the order of $100 \mu\text{rad}$ and n is 5 to 7. At each phase setting the ZPFS output (E) and the UUT output (W) are measured at forward (I^+) and reversed current direction (I^-). The measurement sequence is I^+ , I^- and I^+ . The I^+ measurements are averaged to minimise errors due to linear drift. E is approximately a linear function of the applied power. The UUT readings, $W(I^+)$ and $W(I^-)$, are plotted against the change in ZPFS output, $\Delta E = E(I^+) - E(I^-)$. By applying linear regression to the W readings, the UUT errors are determined for $\Delta E = 0$ and then corrected for the phase angle error of the ZPFS, fig. 2.

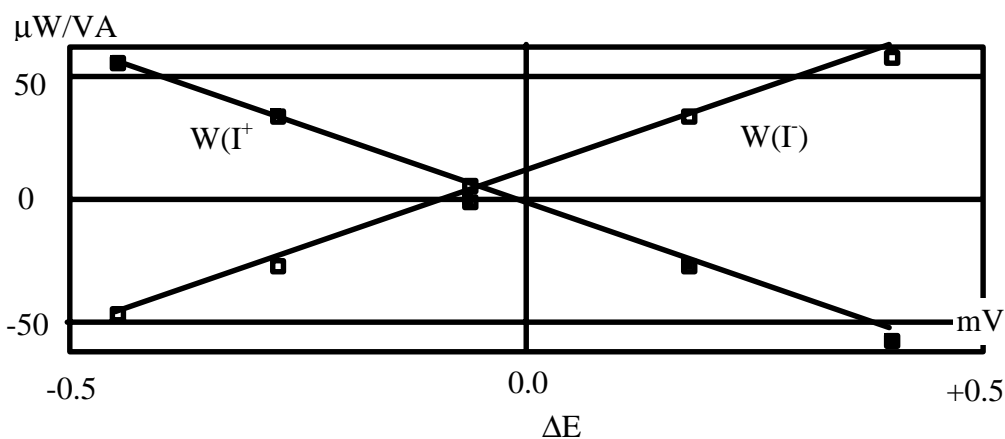


Figure 41 UUT readings, in ppm of S, versus change in ZPFS output.

9.1.1 Uncertainties

At $\Delta E = 0$, the phase difference between V and I is $\phi = \pm(\pi/2 - \alpha_0) + \alpha_1$. The phase angle errors, α_0 and α_1 , are given in [1] as

$$\alpha_0 = \frac{r_s}{2r_m} \left[\frac{|I|}{|V|} \operatorname{Re}(Z) + \frac{|V|}{|I|} \operatorname{Re}(Y) \right] \quad (1)$$

$$\alpha_1 = \phi(r_m) + \phi(r_s) + \frac{1}{2} [r_1 Y_1 + r_2 Y_2 - \phi(r_1) - \phi(r_2)] \quad (2)$$

where $\phi(r_m)$, $\phi(r_s)$, $\phi(r_1)$ and $\phi(r_2)$ are the phase angle of r_m , r_s , r_1 and r_2 respectively. Refer to figure 1 for explanation of V, I, Z, Y, Y_1 , Y_2 , r_m , r_s , r_1 and r_2 . The individual components in (1) and (2) has been measured and the resulting phase angle errors with uncertainties (1σ), at V=120V, I=5A and 53 Hz, are given in below.

$\operatorname{Re}(Z) = 65 \pm 30 \text{ m}\Omega$,	$\operatorname{Re}(Y) < 10 \text{ }\mu\text{S}$
$r_s = 0.16 \text{ }\Omega$,	$\phi(r_s) = +0.8 \pm 1.2 \text{ }\mu\text{rad}$
$r_m = 24 \text{ k}\Omega$,	$\phi(r_m) = -0.8 \pm 0.4 \text{ }\mu\text{rad}$
$r_1 = r_2 = 0.25 \text{ k}\Omega$,	$\phi(r_1) = \phi(r_2) = +0.08 \pm 0.05 \text{ }\mu\text{rad}$
$Y_1 = 1.9 \pm 0.3 \text{ nS}$.	$Y_2 = 0.3 \pm 0.3 \text{ nS}$

Calculation of (1) and (2) gives $\alpha_0 = +0.01 \pm 0.005 \text{ }\mu\text{rad}$ and $\alpha_1 = +0.2 \pm 1.3 \text{ }\mu\text{rad}$. Other errors such as source distortion, imperfect reversal and phase drift during measurement are estimated to be $< 0.3 \text{ }\mu\text{rad}$. The phase angle uncertainty of the ZPFS is $\pm 1.4 \text{ }\mu\text{rad}$ (1σ).

Digital sampling wattmeter

The basic measuring principle of the DSWM is the equally-time-spaced simultaneous sampling of voltage and current during an exact number of periods. From a set of samples the total power can be calculated by discrete integration or the magnitude and phase angle of each harmonic of the power can be calculated using discrete Fourier transform. The DSWM is composed of two sampling DVMs, an inductive voltage divider (IVD), coaxial shunts and a PC for control and data processing, fig. 3. The IVD is used as a better alternative to the internal divider of the DVM. The measured voltage is divided by the IVD to 8V and the output of the current shunts are 0,8V.

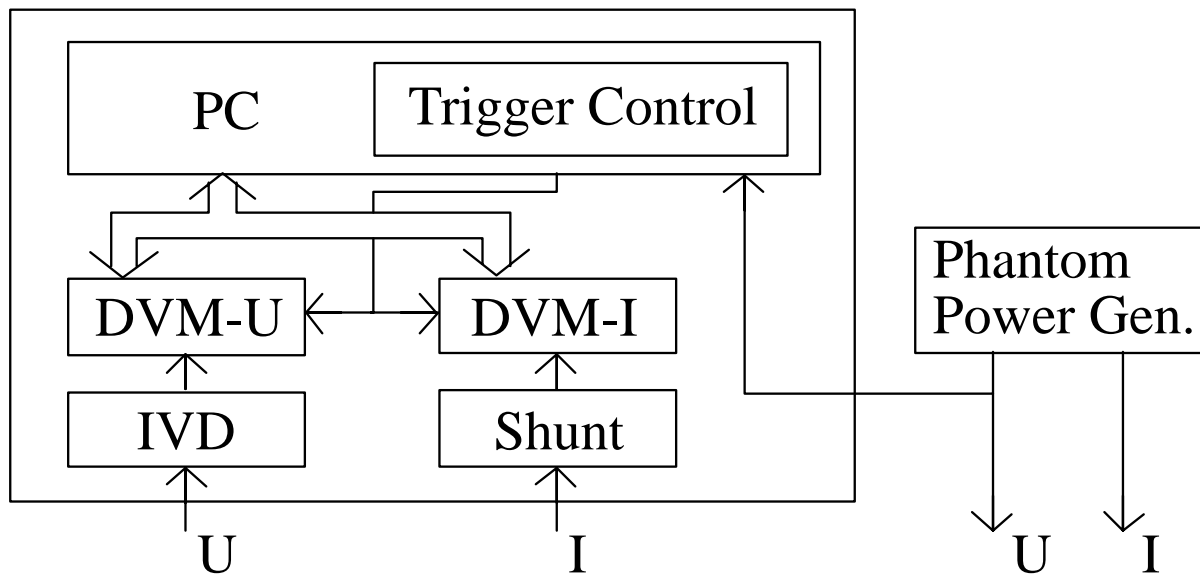


Figure 42 Schematic diagram of the digital sampling wattmeter.

Uncertainties

The accuracy of the DSWM at zero PF will depend on the phase angle errors caused by phase shift differences between the current and the voltage channels and the synchronisation of the sampling.

The phase angle error, ϕ_1 , of the DSWM, without IVD and shunt, is determined by measuring the phase angle between two signals. The measured phase angle will be $\phi_{m1} = \phi_1 + \phi$, where ϕ is the true value. If the two signals are interchanged the measured phase angle will be $\phi_{m2} = \phi_1 - \phi$. Hence the mean of the two measured phase angles is equal to the phase angle error. As the DSWM measures phase angle as $\phi = \arccos(P/S)$, where P is the active power and S is the apparent power, the phase angle between the two signals is chosen to approximately $\pi/2$ rad. in order to get the best resolution.

The phase angle error of the IVD, ϕ_2 , is measured by comparison to a standard IVD. The phase angle error of the shunts, ϕ_3 , are determined by measuring the inductance. At 120V, 5A, zero PF and a measuring frequency of 53Hz the following phase angle errors, with uncertainties (1σ), has been determined:

$$\phi_1 = (30 \pm 9) \mu\text{rad}$$

$$\phi_2 = (0 \pm 4) \mu\text{rad}$$

$$\phi_3 = (0.8 \pm 1.2) \mu\text{rad}$$

After correction for the phase angle errors the uncertainty of the DSWM is $\pm 10 \mu\text{rad}$ (1σ).

Comparison

The comparison of the two power measuring standards has been made at 53Hz with a power converter as a transfer device. In table 1 the errors of the power converter as measured by the ZPFS and the DSWM are given.

Table 16 Errors of the converter as measured by the ZPFS and the DSWM and estimated uncertainties.

Input		Standard	Measured error		Uncertainty rel. S, 1 σ /±ppm
U /V	I /A		rel. S /ppm lag	lead	
120	5	ZPFS	-14.9	+12.3	2
		DSWM	-15	+9	12
120	1	ZPFS	-29	-4	6
		DSWM	-19	-15	13
60	5	ZPFS	-18.8	+24.5	3
		DSWM	-8	+15	12

The uncertainties given includes the above estimated uncertainties of the respective standard and the uncertainties related to the measurements. At 120V and 5A the standard deviation of the mean of the measured values was 0.5 ppm using the ZPFS and 5 ppm using the DSWM. Noise on the output of the converter contributes with an uncertainty of ± 1 ppm of the nominal apparent power.

Conclusion

An automatic zero power factor measuring system has been designed and compared to a digital sampling wattmeter standard. The ZPFS and the DSWM are found to agree within the estimated uncertainties. The ZPFS will be the primary standard of SP for power measurements at zero power factor.

References

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