Discreteness Effects in $\Lambda$CDM Simulations

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The effects of particle discreteness in $N$-body simulations are a fundamental point that needs careful thought and analysis, since all such simulations suffer from numerical noise arising from the use of finite-mass particles.
WHAT IS NEW?

1. We assess the actual significance of discreteness effects against statistical scatter.

2. We offer a deeper and wider view of such effects through a thorough wavelet-statistical analysis.

3. We probe particular aspects of the problem, such as the range of scales affected by discreteness and a variety of statistical effects arising from the initial conditions.
Dynamical evolution does not propagate discreteness noise up from the small scales at which it is introduced.

- This is one important aspect of the robustness of cosmological $N$-body simulations, which we can prove rigorously thanks to new and powerful wavelet statistics.

- The point is that the final power spectra, correlation functions and mass variances only show marginal differences, if any, once their scatter is taken into account (see Fig. 1).
• This is also true for diagnostics that are sensitive to the phase of the density fluctuations (see Fig. 2).

• One needs minimum-scatter phase-sensitive statistics, such as our wavelet set, to show that discreteness noise is not propagated upwards (see Fig. 3).
The final correlation functions and mass variances are not shown here, but they look the same.
FIGURE 2
FIGURE 3

$z = 0$

$z = 15$
One should aim to satisfy the condition $\varepsilon \sim 2d$, where $\varepsilon$ is the force resolution and $d$ is the interparticle distance.

- This condition involves several aspects of discreteness, which appear if there is unbalance between force and mass resolution: initial non-Gaussianity from Gaussian initial conditions (see Figs 4 and 5), departure from lognormality and rise of further complexity at low redshifts (see Fig. 6).

- This is a fresh view of the problem, which we discuss together with its implications.
• Concerning the range of scales affected by discreteness, approximately $\varepsilon < s < 2d$, this is a result that again we can prove rigorously thanks to our wavelet statistics.

• Previous attempts to quantify this point neglected the statistical scatter of the diagnostics, which generally dominates over the systematic effects of discreteness, as pointed out above (see first conclusion).
\[ \varepsilon = d/4 \]

\[ \varepsilon = d \]

\[ \varepsilon = 4d \]

\( z = 15 \)

FIGURE 4
\( \varepsilon = \frac{d}{4} \)

\( \varepsilon = d \)

\( \varepsilon = 4d \)

FIGURE 5

\( z = 100 \)
We conclude that discreteness effects can be kept under control by implementing our condition $\varepsilon \sim 2d$ adaptively, not only in AMR codes but also in tree-based codes, and clarify how [See our ApJ paper (arXiv:0804.0294)].