Power Scaling Laws for Massive MIMO Relay Systems with Linear Transceivers

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***Abstract*—In this paper, we study the uplink of a relay-assisted multiuser massive MIMO system. The relay station (RS) equipped with M_1 antennas processes the received signal from K single-antenna users, using maximum ratio combining (MRC)/zero-forcing (ZF) detection, and then forwards the post-processed signal to the base station (BS) with M_2 antennas, according to the all-pass relay protocol. In particular, we derive closed-form approximations of the ergodic achievable rate lower bounds, assuming that MRC/ZF combining is performed at the BS. Furthermore, we examine the power scaling law of the proposed system, and find that the transmit powers of relay or users can be reduced in proportion to M_1 or M_2, respectively, without any loss in the achievable rate.**

I. Introduction

In recent years, massive multiple-input multiple-output (MIMO) technology has received unprecedented attention in the wireless communication society. By deploying unconventionally large number of antennas at the base station (BS), massive MIMO systems are capable of combating against undesirable randomness (e.g., small-scale fading, uncorrelated inter-user interference), thus providing significant gains in both spectral efficiency and energy efficiency [1], [2]. More importantly, massive MIMO is viable due to its easy implementation. For instance, simple linear receivers, such as maximum ratio combining (MRC) and zero-forcing (ZF) receivers, have been proved to be near-optimal.

On the other hand, relaying has been extensively explored to provide enhanced coverage and high throughput, especially when the propagation environment experiences significant shadowing. Naturally, massive MIMO combined with relay networks becomes a competitive candidate in future cellular systems. There has already been a number of existing work in this field. In [4], the authors consider a one-way amplify-and-forward (AF) relay station (RS) connecting multiple sources and destinations, they show that the transmit power of each source or RS can be made inversely proportional to the number of relay antennas while maintaining a given quality-of-service. (+comments for ref [5]). Two-way relaying has also engaged huge interest, in [6] and [7], a two-way AF relaying with distributed and centralized antennas are studied, respectively. Their results show that two-way relaying requires more sophisticated techniques to eliminate interferences, while massive MIMO can be well-applied in such a scenario and achieves considerable capacity.

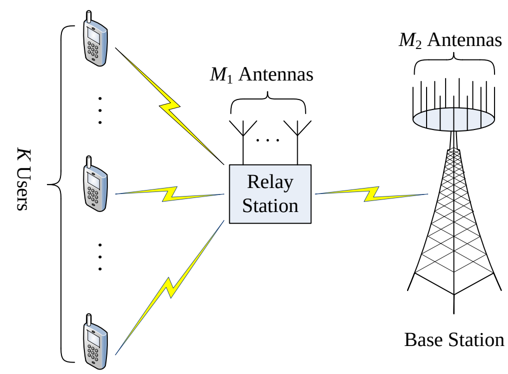

Fig. 1. The scenario of relay system in cellular with large antenna arrays.

The work of this paper based on the scenario where multiuser communicate with the base station via a single relay, which is different from the above multipair-based model, hence the specific signal processing method and the particular system performance of the scenario which both the relay and the base station are equipped with large antenna arrays are still unknown and need further study.

In this paper, we derive the approximations of ergodic achievable rate in closed-form for finite M_1 and M_2. On the other hand, our study shows that the average transmit powers of each user (relay) can be cut down proportionally to $1/M_1\,(1/M_2)$ without ergodic rates degradation when M_1 and M_2 are large enough.

Notation: Throughout this paper, we use capital boldface letters to denote matrices while $\mathrm{Tr}(\,\cdot\,)$ and $\dagger$ denote the trace and conjugate transposition operation respectively. $\mathbf{I}_K$ is the identity matrix of size K. $\overset{a.s.}{\to}$ denote the almost sure convergence. $\mathrm{E}\{x\}$ stands for expectation of variable x.

II. System Model

In this paper, we focus on the uplink transmission. Consider the system model as shown in Fig. 1, in which K users communicate with the base station BS through a relay station RS. Each user is equipped with single antenna while RS and BS are equipped with M_1 and M_2 antennas, respectively. We assume that $M_1 \gg K$, $M_2 \gg K$ and also assume that there are no direct links between users and base station.

During the first phase of the relaying, all users simultaneously transmit their symbols to RS which uses M_1 antennas for receiving. The channel matrix between K users and RS can be expressed as $\mathbf{G}_1 = \mathbf{H}_1\mathbf{D}_1^{1/2}$, in which $\mathbf{H}_1 \in \mathbb{C}^{M_1\times K}$ contains independent and identically distributed (i.i.d.) normal complex Gaussian variable entries while $\mathbf{D}_1 \in \mathbb{R}^{K\times K}$ is slow fading coefficient matrix with $[\mathbf{D}_1]_{kk} = \eta_{1k}$. From the perspective of physics, $\mathbf{H}_1$ represents independent fast fading coefficient while η_{1k} represents slow fading coefficient between kth user and RS which contains pass loss and shadow effect. The received signal at RS can be expressed as

$$\mathbf{y}_R = \mathbf{G}_1\mathbf{x} + \mathbf{n}_R, \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_K]^T$ are transmit symbols of K users with $\mathrm{E}\{\mathbf{x}\mathbf{x}^\dagger\} = \mathbf{P} = \mathrm{diag}(P_1, P_2, \cdots, P_K)$, where P_j are the average transmit powers of jth user for $j = 1, 2, \cdots, K$, $\mathbf{n}_R$ is additive white Gaussian noise (AWGN) vector at R with $\mathrm{E}\{\mathbf{n}_R\mathbf{n}_R^\dagger\} = \sigma_R^2\mathbf{I}_{M_1}$.

During the second phase of the relaying, RS transmits signal $\tilde{\mathbf{y}}_R = \mathbf{W}_1\mathbf{y}_R$ to BS where $\mathbf{W}_1 \in \mathbb{C}^{K\times M_1}$ is precoding matrix. P_r are transmit powers of RS and $\mathbf{W}_1$ is normalized to satisfy the total power constraint of $\mathrm{Tr}(\mathrm{E}\{\tilde{\mathbf{y}}_R\tilde{\mathbf{y}}_R^\dagger\}) = P_r$. Note that RS only uses K of M_1 antennas for transmission while BS uses M_2 antennas for receiving. The channel matrix between RS and BS can be modeled as $\mathbf{G}_2 = \eta_2^{1/2}\mathbf{H}_2$ where $\mathbf{H}_2 \in \mathbb{C}^{M_2\times K}$ contains i.i.d. normal complex Gaussian variable entries and η_2 is slow fading coefficient between RS and BS. The received signal at BS can be expressed as

$$\begin{aligned}\mathbf{y}_{BS} &= \mathbf{G}_2\tilde{\mathbf{y}}_R + \mathbf{n}_{BS}\\ &= \mathbf{G}_2\mathbf{W}_1\mathbf{G}_1\mathbf{x} + \mathbf{G}_2\mathbf{W}_1\mathbf{n}_R + \mathbf{n}_{BS},\end{aligned} \tag{2}$$

where $\mathbf{n}_{BS}$ is AWGN vector at BS with $\mathrm{E}\{\mathbf{n}_{BS}\mathbf{n}_{BS}^\dagger\} = \sigma_{BS}^2\mathbf{I}_{M_2}$. BS uses receiving matrix $\mathbf{W}_2 \in \mathbb{C}^{K\times M_2}$ to recover the symbols of K users

$$\begin{aligned}\tilde{\mathbf{y}}_{BS} =& \mathbf{W}_2\mathbf{y}_{BS}\\ =& \mathbf{W}_2\mathbf{G}_2\mathbf{W}_1\mathbf{G}_1\mathbf{x} + \mathbf{W}_2\mathbf{G}_2\mathbf{W}_1\mathbf{n}_R + \mathbf{W}_2\mathbf{n}_{BS}.\end{aligned} \tag{3}$$

In massive MIMO systems, the acquirement of channel state information (CSI) is important. We assume both RS and BS have local CSI, which means that RS and BS knows $\mathbf{G}_1$ and $\mathbf{G}_2$ respectively. Actually, RS can obtain local CSI with the help of pilot signals in the uplink between users and RS, and BS as well. Hence, our system avoids the overhead of acquirement of global CSI.

As for precoding matrix $\mathbf{W}_1$ and $\mathbf{W}_2$, we discuss two linear signal receiving method: MRC and ZF. The precoding matrix $\mathbf{W}_1$ and $\mathbf{W}_2$ can be defined as

$$\mathbf{W}_1 = a_*\mathbf{R}_{1,*}, \mathbf{W}_2 = \mathbf{R}_{2,*}, \tag{4}$$

where a_* is instantaneous power coefficient and $* \in \{\mathrm{mrc}, \mathrm{zf}\}$ represents MRC or ZF method. To be specific, $\mathbf{R}_{1,*} \in \mathbb{C}^{K\times M_1}$ and $\mathbf{R}_{2,*} \in \mathbb{C}^{K\times M_2}$ are signal receiving matrix which use MRC or ZF method.

To begin with, we denote $[\Phi]_{ij} = \varphi_{ij}, [\Theta]_{ij} = \theta_{ij}$ for $i, j = 1, 2, \cdots, K$.

For the MRC method we have

$$\mathbf{R}_{1,\mathrm{mrc}} = \mathbf{G}_1^\dagger, \mathbf{R}_{2,\mathrm{mrc}} = \mathbf{G}_2^\dagger. \tag{5}$$

Base on the total power constraint we have

$$a_{\mathrm{mrc}} = \sqrt{\frac{P_r}{\mathrm{Tr}\left(\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1\mathbf{P}\Phi\mathbf{D}_1^{1/2} + \sigma_R^2\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1^{1/2}\right)}} \tag{6}$$

Hence the processed signal at BS for MRC can be written as

$$\begin{aligned}\tilde{\mathbf{y}}_{BS} =& a_{\mathrm{mrc}}\eta_2\Theta\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1^{1/2}\mathbf{x}\\ &+ a_{\mathrm{mrc}}\eta_2\Theta\mathbf{D}_1^{1/2}\mathbf{H}_1^\dagger\mathbf{n}_R + \eta_2^{1/2}\mathbf{H}_2^\dagger\mathbf{n}_{BS},\end{aligned} \tag{7}$$

where a_{mrc} is defined as (6).

For the ZF method we have

$$\mathbf{R}_{1,\mathrm{zf}} = \left(\mathbf{G}_1^\dagger\mathbf{G}_1\right)^{-1}\mathbf{G}_1^\dagger, \mathbf{R}_{2,\mathrm{zf}} = \left(\mathbf{G}_2^\dagger\mathbf{G}_2\right)^{-1}\mathbf{G}_2^\dagger. \tag{8}$$

In the similar way, the processed signal at BS for ZF can be written as

$$\begin{aligned}\tilde{\mathbf{y}}_{BS} =& a_{\mathrm{zf}}\sqrt{P_t}\mathbf{x} + a_{\mathrm{zf}}\mathbf{D}_1^{-1/2}\Phi^{-1}\mathbf{H}_1^\dagger\mathbf{n}_R\\ &+ \eta_2^{-1/2}\Theta^{-1}\mathbf{H}_2^\dagger\mathbf{n}_{BS},\end{aligned} \tag{9}$$

where

$$a_{\mathrm{zf}} = \sqrt{\frac{P_r}{\mathrm{Tr}\left[\mathbf{P} + \sigma_R^2\mathbf{D}_1^{-1/2}\Phi^{-1}\mathbf{D}_1^{-1/2}\right]}}. \tag{10}$$

From (7) and (9) we note that there are interference in MRC while interference do not exist in ZF, which brings out the main difference between MRC and ZF method. In fact, the interference impair the ergodic rates of MRC in some conditions. But in other cases, we can compensate the impairment of interference to increase the ergodic rates of MRC at the price of the sufficient antennas.

III. Ergodic Rate Analysis

In this section, we will analyze the ergodic rates of MRC and ZF method in detail. However, to get the exact ergodic rates in closed form is extremely difficult. Hence, we will derive the expression of approximate ergodic rates of MRC and ZF rather than exact ergodic rates. Firstly, the ergodic rates satisfy the follow expression

$$R_j \geq \bar{R}_j \equiv \frac{1}{2}\log_2\left(1 + [L_j]^{-1}\right), L_j = \mathrm{E}\left\{\frac{1}{\mathrm{SINR}_j}\right\}, \tag{11}$$

where SINR_j is signal to interference noise ratio (SINR) of jth user, R_j is ergodic rates of jth user, and $\bar{R}_j$ is approximate ergodic rates of jth user. Based on the (11), the approximate ergodic rates of MRC and ZF can be derived in closed form, then we can use $\bar{R}_j$ as an approximation of R_j.

A. Ergodic rate analysis for MRC

As we know, Φ and Θ are Wishart matrix which satisfy the conjugate symmetry. From the definition of Φ and Θ we obtain

$$\mathrm{E}\left(\varphi_{jj}\right)=M_1, \mathrm{E}\left(\theta_{jj}\right)=M_2, \tag{12}$$

where $j=1,2,\cdots,K$. Then we have

$$\mathrm{E}\left\{\varphi_{jm}\varphi_{jn}^*\right\}=\begin{cases} 0 & ,m\neq n \\ M_1 & ,m=n\neq j \\ M_1^2+M_1 & ,m=n=j \end{cases},$$
$$\mathrm{E}\left\{\theta_{jm}\theta_{jn}^*\right\}=\begin{cases} 0 & ,m\neq n \\ M_2 & ,m=n\neq j \\ M_2^2+M_2 & ,m=n=j \end{cases}, \tag{13}$$

where $j=1,2,\cdots,K$.

From (7) we obtain the SINR of jth user as

$$\mathrm{SINR}_j=\frac{\eta_2^2 P_t S_{\mathrm{I},j}}{\eta_2^2\sum\limits_{i\neq j}P_i S_{\mathrm{I},i}+\eta_2^2\sigma_R^2 S_{\mathrm{N},1}+\eta_2\sigma_{BS}^2 S_{\mathrm{N},2}}, \tag{14}$$

where $S_{\mathrm{I},i}$, $S_{\mathrm{N},1}$ and $S_{\mathrm{N},2}$ are defined as

$$S_{\mathrm{I},i}=\left|\sum_m \theta_{jm}\varphi_{mi}\eta_{1m}^{1/2}\eta_{1i}^{1/2}\right|^2, \tag{15}$$

$$S_{\mathrm{N},1}=\left(\Theta\mathbf{D}_1^{1/2}\Phi\mathbf{D}_1^{1/2}\Theta^\dagger\right)_{jj}, \tag{16}$$

$$S_{\mathrm{N},2}=\frac{1}{a_{mrc}^2}\Theta_{jj}. \tag{17}$$

According to (11), we need $\mathrm{E}\{1/\mathrm{SINR}_j\}$ to obtain approximate ergodic rates $\bar{R}_j$. However, to get exact $\mathrm{E}\{1/\mathrm{SINR}_j\}$ is extremely difficult. Here, we use some technique to simplify the derivation of $\mathrm{E}\{1/\mathrm{SINR}_j\}$. From the law of large numbers [8] we have

$$\frac{\varphi_{mj}}{M_1}\underset{M_1\to\infty}{\overset{a.s.}{\to}}\begin{cases}1, m=j\\ 0, m\neq j\end{cases}, \frac{\theta_{jm}}{M_2}\underset{M_2\to\infty}{\overset{a.s.}{\to}}\begin{cases}1, m=j\\ 0, m\neq j\end{cases}. \tag{18}$$

From (18), (12) and (13) we have

$$\begin{aligned}&\lim_{\substack{M_1\to\infty\\ M_2\to\infty}}\frac{S_{\mathrm{I},j}}{M_1^2 M_2^2}\\ &=\lim_{\substack{M_1\to\infty\\ M_2\to\infty}}\left(\sum_m\sum_n\frac{\theta_{jm}}{M_1}\frac{\theta_{jn}^*}{M_1}\frac{\varphi_{mj}}{M_2}\frac{\varphi_{nj}^*}{M_2}\eta_{1m}^{1/2}\eta_{1n}^{1/2}\eta_{1j}\right)=\eta_{1j}^2.\end{aligned} \tag{19}$$

Therefore the $S_{\mathrm{I},j}$ can be written as

$$S_{\mathrm{I},j}\approx M_1^2M_2^2\eta_{1j}^2, \tag{20}$$

when M_1 and M_2 are sufficiently large.

According to (14) and (20), the $\mathrm{E}\{1/\mathrm{SINR}_j\}$ can be written as

$$\begin{aligned}&\mathrm{E}\left\{\frac{1}{\mathrm{SINR}_j}\right\}\\ &\approx\frac{\eta_2^2\sum_{i\neq j}P_i\mathrm{E}\left\{S_{\mathrm{I},i}\right\}+\eta_2^2\sigma_R^2\mathrm{E}\left\{S_{\mathrm{N},1}\right\}+\eta_2\sigma_{BS}^2\mathrm{E}\left\{S_{\mathrm{N},2}\right\}}{\eta_2^2P_tM_1^2M_2^2\eta_{1j}^2}.\end{aligned} \tag{21}$$

From (12) and (13) we obtain

$$\begin{aligned}\mathrm{E}\left\{S_{\mathrm{I},i}\right\}&=\mathrm{E}\left\{\sum_m\sum_n\theta_{jm}\theta_{jn}^*\varphi_{mi}\varphi_{ni}^*\eta_{1m}^{1/2}\eta_{1n}^{1/2}\eta_{1i}\right\}\\ &=M_1M_2\left[M_1\eta_{1i}^2+M_2\eta_{1j}\eta_{1i}+\eta_{1i}\mathrm{Tr}\left(\mathbf{D}_1\right)\right],\end{aligned} \tag{22}$$

where $i=1,2,\cdots,K, i\neq j$. On the other hand, we have

$$\begin{aligned}\mathrm{E}\left\{S_{\mathrm{N},1}\right\}&=\mathrm{E}\left\{\sum_n\sum_m\theta_{jm}\theta_{jn}^*\varphi_{mn}\eta_{1m}^{1/2}\eta_{1n}^{1/2}\right\}\\ &=M_1M_2\left[M_2\eta_{1j}+\mathrm{Tr}\left(\mathbf{D}_1\right)\right].\end{aligned} \tag{23}$$

Similarly, the $\mathrm{E}\{S_{\mathrm{N},2}\}$ can be written as

$$\begin{aligned}\mathrm{E}\left\{S_{\mathrm{N},2}\right\}=\frac{M_1M_2}{P_r}&\left[\mathrm{Tr}\left(\mathbf{D}_1\right)\mathrm{Tr}\left(\mathbf{D}_1\mathbf{P}\right)+M_1\mathrm{Tr}\left(\mathbf{D}_1^2\mathbf{P}\right)\right.\\ &\left.+\sigma_R^2\mathrm{Tr}\left(\mathbf{D}_1\right)\right]\end{aligned} \tag{24}$$

According to (21)-(24) and (11), the ergodic rates of jth user can be written as

$$R_j\approx\frac{1}{2}\log_2\left(1+\frac{M_1M_2P_jP_r\eta_{1j}\eta_2}{S_{\mathrm{N_{MRC}}}+S_{\mathrm{I}}}\right), \tag{25}$$

where $S_{\mathrm{N_{MRC}}}$ and S_{I} are defined as

$$\begin{aligned}S_{\mathrm{N_{MRC}}}=&\sigma_{BS}^2\left[M_1\frac{\mathrm{Tr}\left(\mathbf{D}_1^2\mathbf{P}\right)}{\eta_{1j}}+\frac{\mathrm{Tr}\left(\mathbf{D}_1\right)\mathrm{Tr}\left(\mathbf{D}_1\mathbf{P}\right)}{\eta_{1j}}\right]\\ &+P_r\sigma_R^2\eta_2\left[M_2+\frac{\mathrm{Tr}\left(\mathbf{D}_1\right)}{\eta_{1j}}\right]+\sigma_R^2\sigma_{BS}^2\frac{\mathrm{Tr}\left(\mathbf{D}_1\right)}{\eta_{1j}},\end{aligned} \tag{26}$$

$$\begin{aligned}S_{\mathrm{I}}=&M_1P_r\eta_2\frac{\sum_{i\neq j}P_i\eta_{1i}^2}{\eta_{1j}}+M_2P_r\eta_2\sum\nolimits_{i\neq j}P_i\eta_{1i}\\ &+P_r\eta_2\mathrm{Tr}\left(\mathbf{D}_1\right)\frac{\sum_{i\neq j}P_i\eta_{1i}}{\eta_{1j}}.\end{aligned} \tag{27}$$

B. Ergodic rate analysis for ZF

Base on the property of wishart matrix [8] we have

$$\mathrm{E}\left\{\left[\Phi^{-1}\right]_{jj}\right\}=\frac{1}{M_1-K}, \mathrm{E}\left\{\left[\Theta^{-1}\right]_{jj}\right\}=\frac{1}{M_2-K}, \tag{28}$$

where $j=1,2,\cdots,K$.

In the same way as Section III-A, we can rewrite SINR of jth user as

$$\mathrm{SINR}_j=\frac{P_j}{\eta_{1j}^{-1}\sigma_R^2[\Phi^{-1}]_{jj}+\eta_2^{-1}\sigma_{BS}^2\frac{1}{a_{zf}^2}[\Theta^{-1}]_{jj}}. \tag{29}$$

Then we have

$$\begin{aligned}&\mathrm{E}\left\{\frac{1}{\mathrm{SINR}_j}\right\}\\ &=\frac{\eta_{1j}^{-1}\sigma_R^2\mathrm{E}\left\{\left[\Phi^{-1}\right]_{jj}\right\}+\eta_2^{-1}\sigma_{BS}^2\mathrm{E}\left\{\frac{1}{a_{zf}^2}\left[\Theta^{-1}\right]_{jj}\right\}}{P_j}.\end{aligned} \tag{30}$$

From (28) we have

$$\mathrm{E}\left\{\frac{1}{a_{zf}^2}\left[\Theta^{-1}\right]_{jj}\right\}=\frac{\left(M_1-K\right)\mathrm{Tr}\left(\mathbf{P}\right)+\sigma_R^2\mathrm{Tr}\left(\mathbf{D}_1^{-1}\right)}{P_r\left(M_1-K\right)\left(M_2-K\right)}. \tag{31}$$

According to the (30) and (31), the ergodic rates of jth user can be written as

$$R_j \approx \frac{1}{2}\log_2\left[1+\frac{(M_1-K)(M_2-K)P_jP_r\eta_{1j}\eta_2}{S_{\mathrm{N_{ZF}}}}\right], \quad (32)$$

where $S_{\mathrm{N_{ZF}}}$ is defined as

$$S_{\mathrm{N_{ZF}}} = (M_1-K)\,\mathrm{Tr}\,(\mathbf{P})\,\sigma_{BS}^2\eta_{1j} + (M_2-K)\,P_r\sigma_R^2\eta_2 + \sigma_R^2\sigma_{BS}^2\eta_{1j}\mathrm{Tr}\left(\mathbf{D}_1^{-1}\right). \quad (33)$$

IV. Power Scaling Law Analysis

In this section, we discuss the power scaling law in the following three cases when both M_1 and M_2 are sufficiently large,

- Case I: $P_j = E_j, P_r = E_r/M_2$
- Case II: $P_j = E_j/M_1, P_r = E_r$
- Case III: $P_j = E_j/M_1, P_r = E_r/M_2$

where where E_j and E_r are fixed for $j = 1, 2, \cdots, K$. In case I, we reduce the transmit powers of relay proportionally to M_2. Obviously, high power efficiency means we can deploy the relay without external power supply fast and conveniently. On the other hand, the average transmit powers of users are cut down proportionally to M_1 in the case(II). In this case, we improve the power efficiency of each user, which is meaningful to the power-saving requirement of mobile devices. Case(III) discusses the situation which combine with the case(I) and case(II).

A. Power scaling law of MRC

For all three cases, the ergodic rates approach to corresponding bounds as M_1 and M_2 grows to infinity. Based on the Section III-A, the upper bounds of case(I) and case(II) can be written as

$$\mathrm{case(I)}: R_j = \frac{1}{2}\log_2\left(1+\frac{E_jE_r\eta_{1j}^2\eta_2}{\sigma_{BS}^2\mathrm{Tr}\,(\mathbf{D}_1^2\mathbf{E})}\right), \quad (34)$$

$$\mathrm{case(II)}: R_j = \frac{1}{2}\log_2\left(1+\frac{E_j\eta_{1j}}{\sigma_R^2}\right), \quad (35)$$

while the upper bound of case(III) can be written as (36), where $\mathbf{E} = \mathrm{diag}(E_1, E_2, \cdots, E_K)$.

B. Power scaling law of ZF

Like the discussion of the power scaling law of MRC, the upper bounds of case(I) and case(II) can be written as

$$\mathrm{case(I)}: R_j = \frac{1}{2}\log_2\left(1+\frac{E_jE_r\eta_2}{\sigma_{BS}^2\mathrm{Tr}\,(\mathbf{E})}\right), \quad (37)$$

$$\mathrm{case(II)}: R_j = \frac{1}{2}\log_2\left(1+\frac{E_j\eta_{1j}}{\sigma_R^2}\right), \quad (38)$$

while the upper bound of case(III) can be written as (39), where $\mathbf{E} = \mathrm{diag}(E_1, E_2, \cdots, E_K)$.

From (35) and (38) we can find the upper bound of MRC in case(II) and the that of ZF in case(II) are same. Moreover, the slow fading coefficients have important effect on MRC and ZF, and the ergodic rates of jth user not only depends on the η_{1j} but also other slow fading coefficents. As for the distinction of MRC and ZF, we can find that the complex function of $\eta_{1i}, E_i, i = 1, 2, \cdots, K$ make the difference, so we can not decide which method is better by simply decision rules.

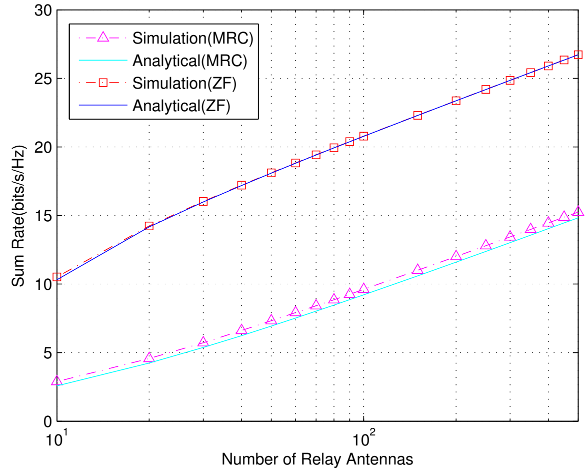

Fig. 2. The transmit powers of relay and the average transmit powers of users are fixed.

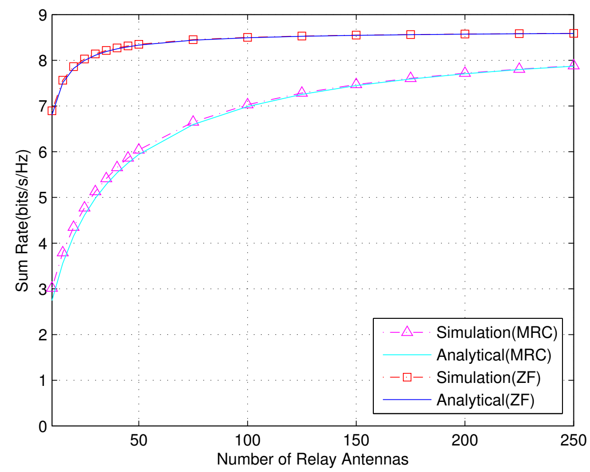

Fig. 3. The transmit powers of relay decrease proportionally to M_2 while the average transmit powers of users are fixed.

V. Numerical Results

The ergodic rates and power scaling laws are verified through Monte Carlo simulations in this section, where we choose $K = 5$, $\mathbf{D}_1 = \mathbf{I}_5$, $\eta_2 = 1$, $\sigma_R^2 = 1$, $\sigma_{BS}^2 = 1$.

Fig. 2 compares the analytical and simulated ergodic rates achieved by MRC and ZF processing, against the number of relay antennas. In the parameter setup, we let $M_1 = M_2$, $P_1 = P_2 = \cdots = P_5 = 5$ and $P_r = 50$. Obviously, our analytical results are very tight for both cases across a wide range.

Fig. 3, Fig. 4 and Fig. 5 shows the conclusion of power scaling law in which $E_1 = E_2 = \cdots = E_5 = 5$ and $E_r = 50$.

$$\text{case(III)} : R_j = \frac{1}{2}\log_2\left(1+\frac{E_jE_r\eta_{1j}^2\eta_2}{\sigma_{BS}^2\mathrm{Tr}\left(\mathbf{D}_1^2\mathbf{E}\right)+E_r\sigma_R^2\eta_{1j}\eta_2+\sigma_R^2\sigma_{BS}^2\mathrm{Tr}\left(\mathbf{D}_1\right)}\right) \quad (36)$$

$$\text{case(III)} : R_j = \frac{1}{2}\log_2\left(1+\frac{E_jE_r\eta_{1j}\eta_2}{\sigma_{BS}^2\mathrm{Tr}\left(\mathbf{E}\right)\eta_{1j}+E_r\sigma_R^2\eta_2+\sigma_R^2\sigma_{BS}^2\eta_{1j}\mathrm{Tr}\left(\mathbf{D}_1^{-1}\right)}\right) \quad (39)$$

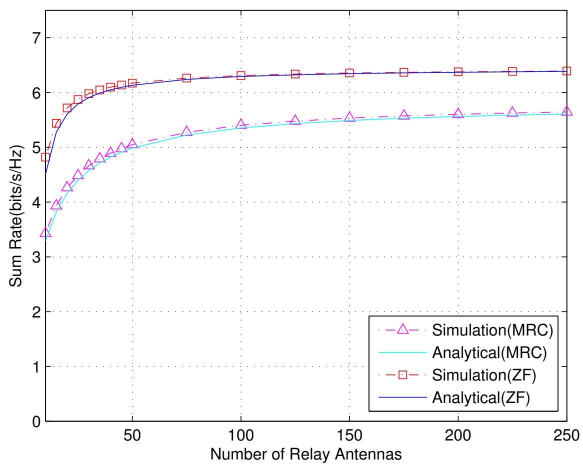

Fig. 4. The average transmit powers of users decrease proportionally to M_1 while the transmit powers of relay are fixed.

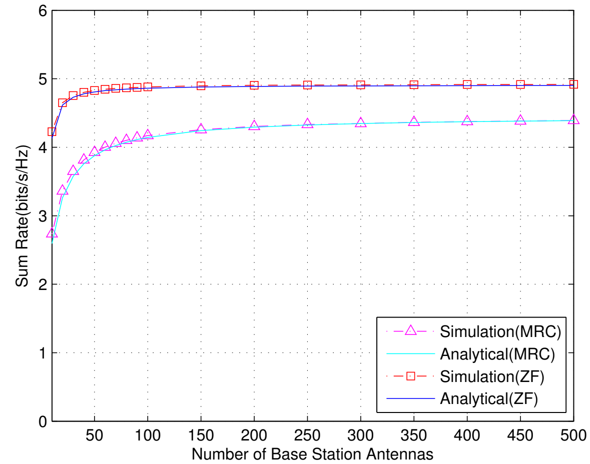

Fig. 5. The average transmit powers of users and the transmit powers of relay decrease proportionally to M_1 and M_2, respectively.

Fig. 3 shows the conclusion of case(I) where $M_2 = 2M_1$. From Fig. 3 we can see that the sum rates of MRC and ZF approach the corresponding upper bounds as the number of antennas increase. In general, the upper bounds of MRC and that of ZF are different, which depends on the slow fading coefficients. Moreover, MRC needs extra antennas to compensate the effect of the interference while ZF approaches the upper bound in a relatively smaller number of antennas.

Fig. 4 shows the conclusion of case(II) where $M_2 = 100$. From (35) and (38) we can see that the upper bounds of MRC and ZF are same, which are different from the case(I). Moreover, unlike the case(I), case(II) reduces the average transmit powers of each user, which is more meaningful to the real situations.

Fig. 5 shows the conclusion of case(III) where $M_1 = 50$. In this case, we not only reduce the average transmit powers of each user, but also reduce the transmit powers of the relay. Like the case(I), the upper bounds of MRC and ZF are different in general. In addition, the upper bound (MRC or ZF) of case(III) is definitely less than that of case(I) and case(II), which is coincide with the common sense, namely, the performance of case(III) is worse than case(I) and case(II) due to the lesser transmit powers.

VI. Conclusions

This paper studied the multiuser relay system where a large number of antennas are equipped at the relay and the base station. We derived the ergodic rates in closed form and discuss three different power cases when MRC or ZF is used. The results has shown that the ergodic rates will keep fixed when the average transmit powers of users is scaled down by the number of the relay antennas, i.e., M_1 or the transmit powers of the relay is scaled down by the number of the base station antennas, i.e. M_2, where M_1 and M_2 grows to infinity.

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