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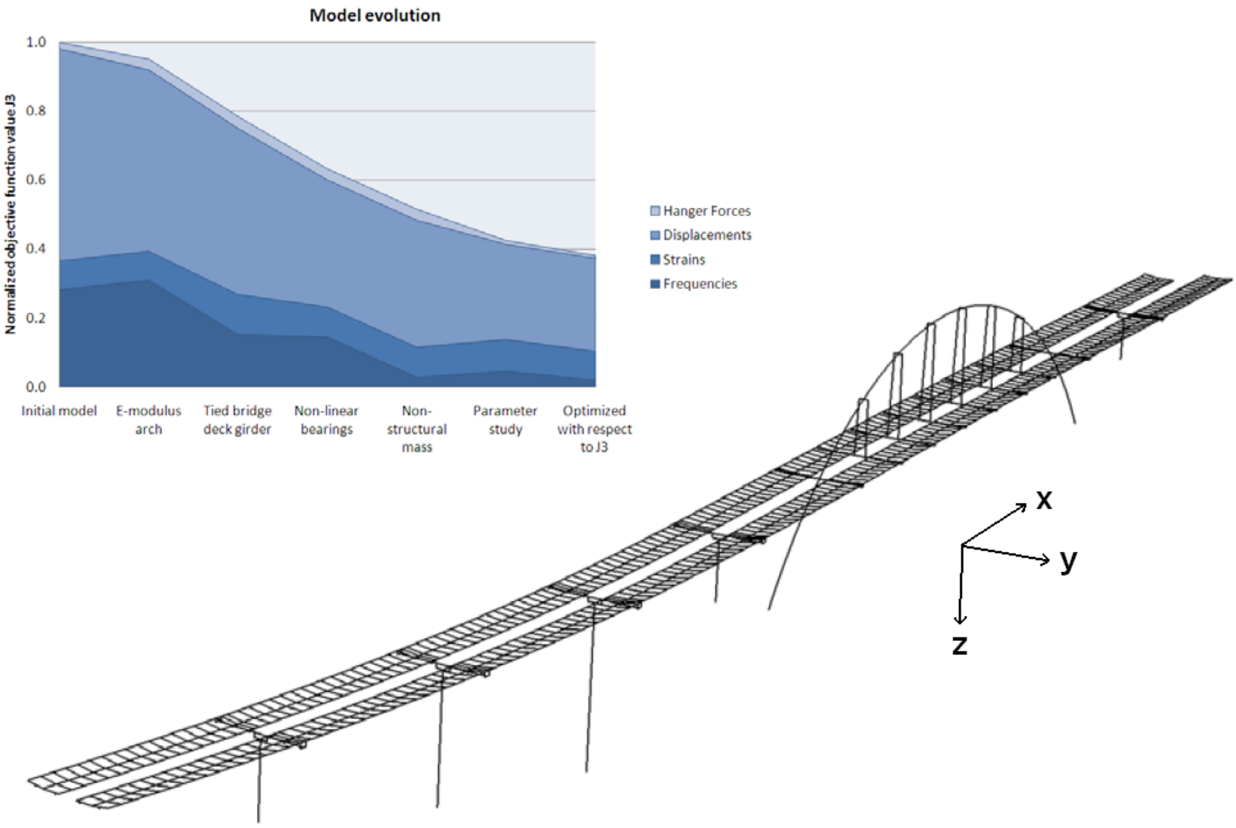

Bridge Assessment and Maintenance based on Finite Element Structural Models and Field Measurements

State-of-the-art review

Hendrik Schlune and Mario Plos

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Cover:
Upper left: Develoment of the objective function J_3 for the introduced model changes
Lower right: Finite element model of the New Svinesund Bridge

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ABSTRACT

Maintenance, upgrading, repair and replacement of existing bridges lead to high cost and considerable disruption of traffic. In current practice, condition assessment and the evaluation of existing bridges are mainly based on visual inspection. Since this practice can only disclose faults limited to the surface of the structure, conclusions concerning the underlying structural health are difficult, if not impossible to derive. For bridge evaluations through finite element (FE) analysis a sound numerical model is needed to guarantee the reliability and safety of existing structures. However, in a FE model, used for the design of structural systems, uncertain parameters, such as support conditions and interactions between structural members can have a significant effect on the results. In existing structures, deterioration, damaging events and incomplete blueprints lead to less certain assumptions. To determine the residual load-carrying capacity and the need for strengthening, repair or replacement, a good approximation of the real behaviour of the structure is required.

In this report a strategy for improved bridge management by means of advanced structural modelling in combination with on-site measurements is presented. Important aspects of finite element modelling, field testing and monitoring, and FE model updating are presented. Through FE model updating, on-site measurements are combined with an initial FE model to obtain new information about the structural behaviour. This make it possible to benefit from the on-site measurements in an optimal way. It leads to a more accurate FE model and allows determining uncertain structural parameters which can not be measured directly.

The study was financed by the Swedish road Administration (Vägverket) and the Swedish railway Administration (Banverket).

Key words: Finite element model updating, bridge assessment, measurements,

Utvärdering och förvaltning av broar med hjälp av finit elementmodellering och fältmätningar
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SAMMANFATTNING

Underhåll, reparation, förstärkning och utbyte av broar innebär höga kostnader och stora störningar för trafiken. Idag baseras utvärderingar av befintliga broars tillstånd och bärförmåga till stor del på visuell inspektion. Eftersom en visuell inspektion endast kan avslöja brister som visar sig på ytan, är det dock svårt eller rentav omöjligt att dra korrekta slutsatser om konstruktionens inre tillstånd. Vid utvärdering av en befintlig bro med hjälp av finit elementanalys (FE-analys) krävs att den numeriska modellen är väl underbyggd för att responsen eller bärförmågan skall anses vara påvisad med erforderlig säkerhet. Då en FE-modell används för brokonstruktion kan osäkra parametrar såsom upplagsförhållanden och samverkan mellan olika delar av konstruktionen ha en avgörande effekt på resultaten. Vid utvärdering av befintliga konstruktioner kan även nedbrytning, skador och brister i dokumentationen leda till osäkra antaganden. För att kunna göra korrekt bestämning av återstående bärförmåga och behov av förstärkning, reparation eller ersättning av en bro behöver dock brons verkliga beteende återspeglas i modellen.

I denna rapport presenteras en strategi för förbättrad broförvaltning genom att kombinera avancerad modellering med mätningar på broar. Viktiga aspekter rörande FE-modellering, fältmätningar och övervakning, och uppdatering av FE-modeller presenteras. Genom uppdatering kombineras fältmätningar med en ursprunglig FE-modell för kunna erhålla säkrare information om en konstruktions respons. Detta gör det också möjligt att kunna utnyttja fältmätningarna på ett optimalt sätt. Resultatet blir en mer korrekt FE-modell för vilken osäkra strukturparametrar som inte kan mätas direkt har kunnat bestämmas.

Arbetet har finansierats av Vägverket och Banverket.

Nyckelord: Finit elementmetod, FEM, FE-modell, uppdatering, utvärdering av broar, mätningar

Contents

Preface

The work presented in this report was part of the research project “Bridge Assessment and Maintenance based on Finite Element Structural Models and Field Measurements” (Utvärdering och förvaltning av broar med hjälp av finit elementmodellering och fältmätningar). The project was financed by the Swedish National Road Administration (Vägverket) and the Swedish National Railroad Administration (Banverket). The project was carried out at Chalmers University of Technology.

In the project a literature study was carried out on how to combine finite element analysis with field measurements to improve the evaluation of existing bridges. After evaluation of the existing methods, a case study on the New Svinesund Bridge was carried out, see Schlune *et al.* (2008a) , Schlune *et al.* (2008b) and Jonsson and Johnson (2007). Based on the case study General conclusions and recommendations have been drawn.

The project group consisted of M.Sc. Ebbe Rosell, M.Sc. Lars Lundh and Tekn. lic. Peter Harryson, from the Swedish Road Administration, Ph.D. Frank Axhag from Banverket, Ph.D. Raid Karoumi from the Royal Institute of Technology Stockholm, M.Sc. Anja Bäurich and Ph.D. Michael Blaschko from Bilfinger Berger, Ph.D. Jan Teigen from the Norwegian Road Administration and professor Kent Gylltoft from Chalmers University of Technlogy. All project group members and co-workers at Chalmers participating in the project are thankfully acknowledged.

Göteborg April 2008

Hendrik Schlune and Mario Plos

1 Background

In order to reach a sustainable development of the society, it is of great importance that the huge investments that are made in the infrastructure can be utilized during the entire lifetime of the structures. Today, many existing structures are replaced or strengthened because their reliability and functionality cannot be guaranteed based on the structural assessments made. This leads to great environmental stresses and a bad usage of the society's resources.

Structural assessments for bridges are normally made using simplified structural models, based on information from drawings. This information is sometimes complemented through material tests from the existing structure and by studying of the original design and construction documentation. Information from field tests and measurements of the real response of the bridge is generally not used, and improvements of the structural models through testing or monitoring have rarely been utilised.

By using modern analysis methods for structural assessment, the intrinsic load carrying capacity can be utilized during the entire lifetime of the structure. By structural verification through field tests and measurements, a better knowledge of the structural response and performance will be achieved, resulting in an improved base also for inspection and maintenance. By extending the lifetime of the structures and by optimising the maintenance, great environmental benefits will be achieved, with less raw-material consumption, reduction of transportation and energy consumption, decreased pollution and less deposit. At the same time substantial costs are avoided, both for the society and for the owner or administrator of the structure.

2 Bridge Assessment and Management

Assessment of concrete structures is treated generally in a state-of-the-art report by CEB (1989). Strategies and procedures for assessments are treated, together with safety policy, decision models and reliability models. Assessment of highway structures are treated by COST (2001). A state-of-the-art review of structural assessment of bridges, with special focus on the possibilities with finite element method (FEM), was made in Plos and Gylltoft (2002). In Plos *et al.* (2004) a strategy for enhanced evaluation of the load carrying capacity of existing bridges, using advanced methods of analysis, was proposed. The work with improved structural assessment of bridges is continued within the EU project Sustainable Bridges, focused on railway bridges, see www.sustainablebridges.net. SAMCO (Structural Assessment, Monitoring and Control) provides free access to numerous publications in the field of assessment, monitoring and control of structures on their homepage, see www.samco.org.

2.1 Bridge management

After the completion of a bridge, it is managed by a bridge administrator during its lifetime. Three main types of measures taken during the bridge management phase can be distinguished:

- **Inspection.** The inspections are planned and are repeated with predicted intervals. Normally, they include visual inspection, but they can also include testing and measurements. In some cases, continuous monitoring using built-in or permanently installed gauges on the bridge is used.

- **Assessment.** An assessment is only made when called for. It can be a structural assessment with respect to the safety or the function of the bridge. It can also be an assessment of the condition of the bridge.

- **Maintenance and repair.** This can either be periodical maintenance or consist of measures called for by an assessment.

A structural model of a bridge is made as a part of the design process. However, this model is often simplified and based on *a priori* assumptions of the bridge. For the management phase an improved structural model is often required. After the bridge is constructed there is a possibility to improve the structural model and to update it through testing and measurements. Such an improved structural model is usually made as a part of a structural assessment. It is not needed only to determine the safety or function of the bridge, but also if the bridge needs to be repaired or reconstructed. It can also be used for improved planning and for decisions regarding inspection and maintenance.

2.2 Structural assessment

The reasons to perform a structural assessment of a bridge can be subdivided in four main categories:

- **Changed requirements.** Requirements for increased traffic loads are the dominating reason for structural assessments in Sweden. Other examples in this category can be changes in codes and regulations, or changed requirements due to a change in use.

- **Planned reconstruction.** A reconstruction often involves interventions into the load carrying structure, which requires a structural evaluation of the bridge.

- **Damage.** A bridge may become damaged due to extreme events like floods, storms and earthquakes. Scour is the main cause for bridge damage in many parts of the world. Damage can also occur due to events that the bridge was not designed for, such as overloading, traffic or ship impact, fire and explosions.

- **Deterioration.** Deterioration can be caused by external environmental loading, e.g. chloride penetration, corrosion, frost, carbonation or fatigue. It can also be caused by reactions inside the material.

A structural assessment is made with respect one or more of the following aspects:

- **Safety.** The load carrying capacity is evaluated with respect to the risk for failure or collapse. It is normally expressed as the load carrying capacity for traffic loads, but can also be expressed by a safety index for given design

- **Function.** An evaluation of the function can be made with respect to e.g. deformations or vibrations.

- **Condition.** An assessment of the condition of a bridge can be made with respect to e.g. cracking in concrete bridges, or the state and development of the deterioration.

The measures or activities included in a structural assessment vary from case to case and may consist of one or more of the following parts:

- **Structural modelling and analyses.** To be able to evaluate safety and function, or to be able to do a more close evaluation of the condition, structural analyses and calculations are needed. A structural assessment of the load carrying capacity includes traditionally this part only.

- **More accurate inspections.** The regularly inspections made may need to be complemented, e.g. for a more careful survey of the extension and cause for damage or deterioration.

- **Testing and measurements.** To better determine the properties of the bridge, testing and measurements can be conducted. These can include determination of material properties, real geometry, bridge condition, damage extensions, traffic and other loads etc. Testing and measurements can also be made in order to verify, calibrate and improve structural models of the bridge. This way, also properties that are hard to measure directly can be evaluated, e.g. boundary conditions, and stiffness of internal connections and of damaged structural elements.

- **Evaluation of safety.** With probabilistic methods a more detailed evaluation of the safety can be made. Probabilistic methods also open for taking into account object specific data, e.g. the traffic situation on the specific bridge.

Depending on the outcome of the assessment, it will result in one of the following actions:

- **Continued use of the bridge as it is,**
  - **without further measures**. If the assessment shows high enough load carrying capacity or safety, and satisfactory function and condition, no specific measures need to be taken. Instead, the assessment is a verification that the bridge fulfils the requirements.
  - **combined with intensified inspection and / or monitoring.** If the original requirements, e.g. regarding function or lifetime, are not fulfilled, the bridge may sometimes still be used, provided that the state and development of the condition are continuously checked through extended inspections or by monitoring.
- **Limitation of requirements.** If the original requirements, e.g. regarding traffic load capacity or lifetime, are not fulfilled, the requirements may sometimes be limited, e.g. by reducing the allowed traffic loads or by allowing continued use for a limited time only.
- **Strengthening or repair.** If the requirements are not fulfilled, strengthening or repair can often improve the bridge performance so that it meets the demands.
- **Replacement.** If strengthening or repair is not sufficient, the bridge superstructure or the entire bridge may need to be replaced.

2.3 A strategy for structural assessment

A strategy for structural assessment of existing bridges was proposed in a joint project between three Swedish universities and the Swedish Road Authorities (Vägverket), see Plos *et al.* (2004). Normally, the assessment starts with an *initial assessment* according to regulations given by the bridge administrators, for road and railway bridges in Sweden, see Vägverket (1998) and Banverket (2000). When the initial assessment does not show a sufficient capacity a more profound assessment is often motivated. In this case, the initial assessment is a very important basis for the further assessment. Therefore, it is important that the documentation is clear and complete, and higher demands on the detailed documentation are needed compared to current practice.

A continued evaluation should be made step by step, and be performed as an integrated part of the decision process. The detailed assessment involves more advanced analysis and judgement, that are not ruled by the detailed regulations and code of practice used in common structural assessment. However, it should have the same aim, intention and safety requirements. It can be based on research results and more enhanced methods for determination of the capacity, and more advanced analysis models of the bridge can be used together with bridge-specific load and material data. Calculations and analyses are made in a continuous interaction with physical investigations of the condition of the bridge, and decisions whether to proceed with the assessment are made successively.

The strategy for a more detailed assessment is illustrated in Figure 2.1. The result of the structural assessments depends on:

- The resistance or capacity of the bridge
- The actions or loads on the specific bridge
- The evaluation methods and analysis models used

As pointed out above, such an assessment should be made step by step and collection of information from testing, measurements and observations on the current bridge should be included as an integrated part of the process. The assessment should be a part of the decision process and can be seen as a project with the aim to reach a well-founded decision regarding the status of the bridge and which possible actions that is needed.

The assessment should be led by a person or group with a clear overall view. The leader or the steering group should have the ability to prioritize between the possible activities with respect to what is most important for the decisions that need to be made and the results that should be achieved. The decision to perform measurements and investigations should be an integrated part of the assessment process, and should not be performed isolated from the analysis and evaluation work.

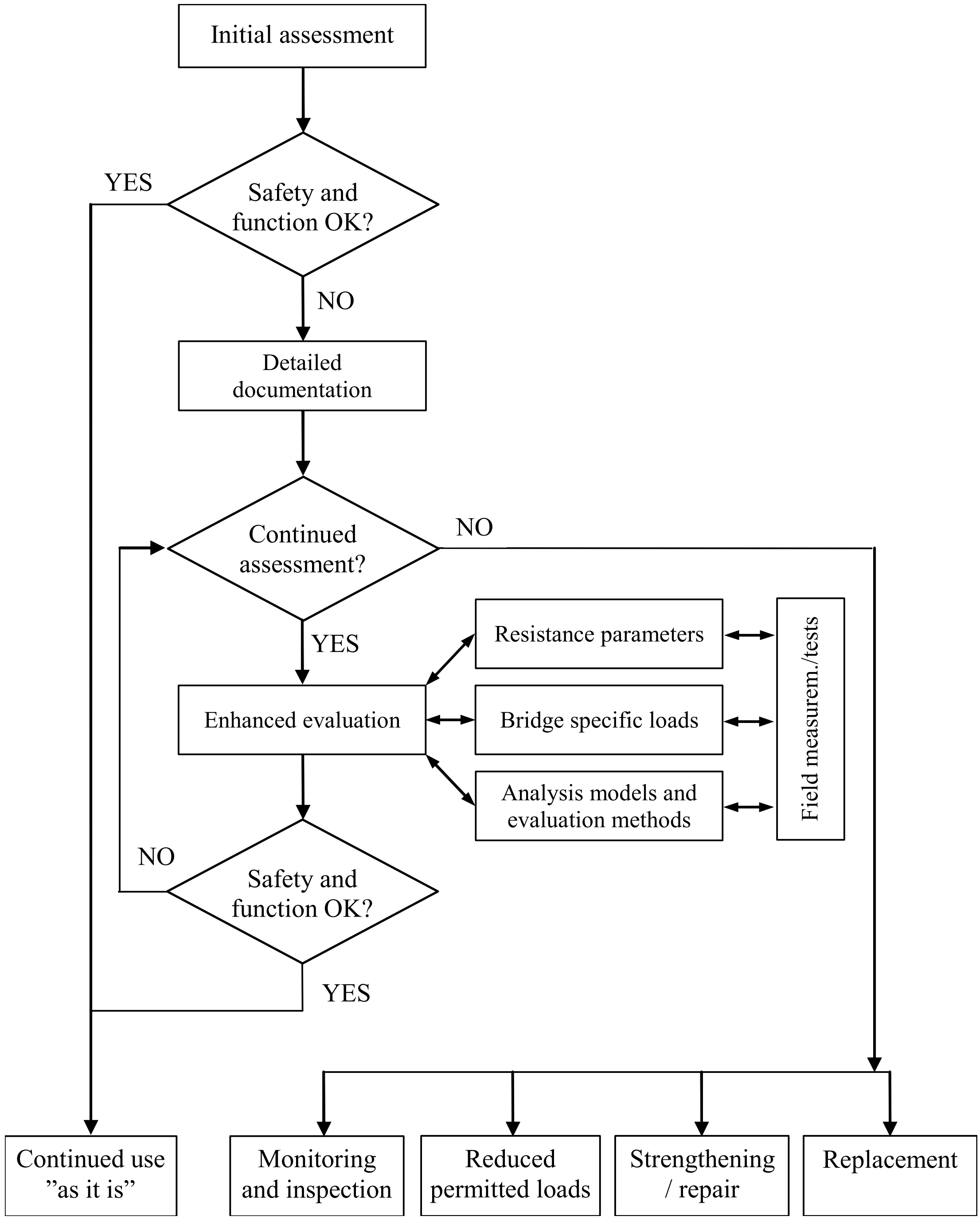

Figure 2.1 Illustration of the strategy for enhanced bridge assessment. Adapted from (Plos, Gylltoft et al. 2004).

2.4 Bridge database

When a bridge is being designed and constructed the information of the bridge is stored in an object database of the bridge. This consists traditionally of drawings and other documents from the design and construction. During the lifetime of the bridge, the bridge database is complemented with information from the management phase, i.e. from inspections, assessments and from maintenance, repair and possible reconstructions. In Figure 2.2, the different phases in the bridge history are shown.

Today the database information is being incorporated in a computerised database system, from which information can be extracted not only regarding the individual bridge, but also about groups of bridges or an entire bridge population. One example is the Swedish management system BaTMan, developed by the Swedish Road Administration (Vägverket) in cooperation with the Swedish National Rail Administration (Banverket) and several other partners. Such systems open the possibility to include not only documents like drawing, protocols, reports and photos in the object database, but also computerised models of the structure.

A computerised model of the structure can contain information about geometry, material, boundary conditions, internal connections, construction history etc. Today it is typically a model for structural analysis of the bridge in a finite element (FE) program. In the future it can be a full three-dimensional (3D) geometrical model including all relevant information about the bridge. Such a model is often referred to as a product model. The product model can include not only data about geometry, material and construction, but also about the condition of the bridge, with links to inspections performed. It can also include all relevant information for structural analysis, with possibilities to represent the structure to different levels of detailing in e.g. FE models coupled to the product model.

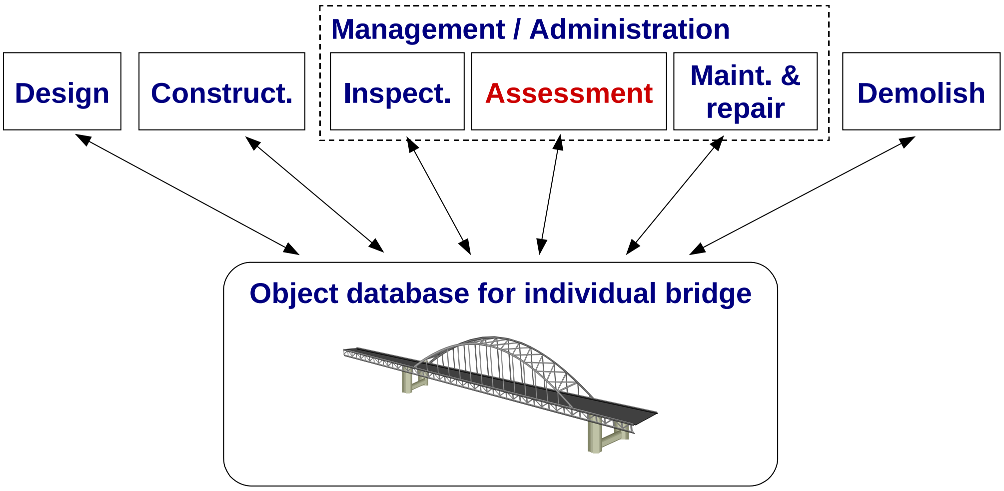

Figure 2.2 Different phases in the history of a bridge. Adapted from (Plos and Gylltoft 2002).

For bridge management, maintenance and assessment it is important that the product model is kept up-to-date and that the structural properties of the bridge are represented correctly. This means that the product model developed for the construction must be updated for the as-built bridge, and thereafter continuously updated with respect to changes of the condition. Updating of the model can be based on documentation from construction and inspections. However, there are often large uncertainties regarding the structural properties. To overcome this, field measurements from testing and monitoring can be used for a more accurate evaluation, both of locally determined parameters, such as material properties, and of parameters that need to be determined from the global response, such as boundary conditions. In order to evaluate the global parameters, structural models need to be used and updated with respect to the measurements made.

2.5 Life cycle cost analysis

Life Cycle Analysis or **Life Cycle Assessment (LCA)** is a process of evaluating the effect of a product on the environment over its entire lifetime. LCA can be used to increase the resource-use efficiency and decrease the environmental impact of e.g. a planned bridge.

The **Life Cycle Cost (LCC)** of a bridge is the total cost of designing, building, operating, maintaining, repairing, replacing and disposing of the bridge, with all costs discounted to a common point in time, i.e. making them time-equivalent. The life cycle cost is the sum of initial and future costs associated with the construction and operation of the bridge over a period of time. Often, only the direct cost for the bridge owner or administrator is included. However, the total life cycle cost of the bridge includes also user and society costs, e.g. from traffic interference in connection with maintenance and repair work.

Life Cycle Cost Analysis (LCCA) is an economical evaluation technique that determines the total cost of owning and operating the bridge over a period of time. The purpose of an LCCA is to compare the cost of different project alternatives, and to determine which alternative that provides the best value for the money spent.

A lot of research and development work has been done internationally within the area of Bridge LCCA, predominantly in the USA. Information is provided on the internet by a number of organisations and projects such as:

- National Cooperative Highway Research Program (NCHPR):
  http://www4.trb.org/trb/crp.nsf/All+Projects/NCHRP+12-43
- National institute of Standard and Technology (NIST):
  http://www.bfrl.nist.gov/bridgelcc
  (for general information about LCCA, see also:
  http://www.wbdg.org/design/lcca.php)
- Federal highway administration:
  http://www.fhwa.dot.gov/infrastructure/asstmgmt/primer04.htm

In Sweden, LCCA has been treated by e.g. Troive (1998) and Karoumi and Sundquist (2001).

It is concluded that LCCA is in general not useful for the determination of the total cost of a project alternative. A LCCA can be made over the entire desired lifetime of a bridge, but can also be used to evaluate different alternatives for existing bridges, e.g. if a bridge should be replaced, repaired, or kept in used for another period of time before repairing or replacing it, maybe with an improved inspection programme or monitoring of the response. The time period has to be the same for all alternatives studied.

Critical for a good LCCA is the indata for the analysis. This includes the possibility to do reliable cost estimates for investments, maintenance and repair measures, but also the possibility to do performance predictions throughout the lifetime of the bridge. It is important to be able to predict how different designs alternatives will meet the requirements for the structure over time, and hence, when and how extensive maintenance and repair measures that will be needed. Consequently, reliable modelling of the structural response as well as deterioration models are needed, as well as good interactive bridge databases.

3 Structural Models for Bridges

3.1 General

The purpose of a structural analysis of a bridge is to determine the cross-sectional forces and moments, or directly the stresses, strains and deformations. Cross-sectional forces and moments are used for design or check of capacity or response in a local analysis. Stresses and strains are used to determine the capacity or the response directly, using the material resistance.

A structural analysis can be made on different levels with respect to the idealisations made. Different levels of idealisations can be made with respect to the geometrical representation of the structure and with respect to the representation of the material response.

3.2 Levels of geometrical representation

Structural models, used for bridge design and assessment, are often simplified. Often, **two-dimensional (2D) models based on beam or frame theory** are used. In normal design and assessment, these are analysed with linear response to determine cross-sectional forces and moments. The structural analysis is often made using a beam or frame design software, but also a general FE program with beam finite elements can be used.

For more complicated geometries, **general three-dimensional (3D) structural element models** are used based on beam, frame and shell theory. Beam or frame design software is sometimes sufficient for the structural analysis, but generally a FE program with structural finite elements, such as beam and shell elements, is preferably used. Here, the geometrical representation of a complicated structure becomes more correct than with 2D beam or frame analysis. Normally, the analysis is linear, and cross-sectional forces and moments are determined. However, this level is also suitable for non-linear analyses of complete bridge structures, where the response or capacity is checked directly in the global analysis. They are also suitable for detailed analysis of e.g. connections in steel bridges.

Continuum element models can be used for detailed analysis of parts where beam, frame or shell theory is not valid, e.g. connections or disturbed regions. It can also be used to be able to include responses that cannot be analysed with simplified models, e.g. bond between reinforcement and concrete. Continuum models are usually not practical for determination of cross-sectional forces and moments. To be valuable they must reasonably well reflect the real material response, which means that non-linear analysis is often required.

3.3 Linear and non-linear analysis

According to the Eurocodes the idealisations with respect to material response can be subdivided into four levels, see CEN (2004a) and CEN (2004b):

Linear elastic analysis. This is the common or standard level for structural analysis of bridges, regardless of the bridge type. It can be used for serviceability limit state (SLS) and for ultimate limit state (ULS). Normally it is used for determination of sectional forces and moments but, in cases where the response really is linear, it can be used for direct stress or deformation analysis as well. Since superposition of results is possible, it is very rational for the determination of critical values for the cross-sectional forces and moments for a large number of load combinations with varying load positions and directions.

Linear analysis with limited re-distribution. This level includes re-distribution of sectional moments and forces obtained through linear elastic analysis, with respect to the non-linear response of the bridge. It is typically used for reinforced concrete bridges in the ULS. Re-distributions of cross-sectional forces and moments between different cross-sections of a bridge are generally not permitted in Sweden. However, it is often needed for redistributions within cross-sections for 3D bridge models with shell elements. It is used for determination of sectional forces and moments.

Plastic analysis. Plastic analysis can be based on the lower or upper bound theory of plasticity (static equilibrium or kinematic compatibility). Examples of analysis methods are frame analysis with plastic hinges, yield line theory and strip method for slabs, and strut- and tie models. It can be used to determine cross-sectional forces and moments for all types of bridges in ULS. With these methods, the capacity for plastic deformations needs to be controlled. In Sweden, these methods are generally not permitted for determination of sectional forces and moments in structural analysis of bridges. However, many models used for local (cross-sectional) analysis and design are based on plasticity, e.g. models for shear and torsion capacity, and for analysis of disturbed regions.

Non-linear analysis. Here, the non-linear responses of the materials, as well as non-linear geometrical effects are taken into account directly in the structural analysis. It can be used for all bridge types in the SLS as well as the ULS. It may be used for determination of sectional forces and moments, but also for direct study of the response and analysis of failures and load carrying capacity.

3.4 Modelling levels for design and assessment of bridges

Normally, structural design and assessment of bridges are made in two steps. Through a linear analysis of a structural model, cross-sectional forces and moments are determined for a large number of load combinations. These are then used to design or analyse cross-sections, structural elements and connections of the bridge in a local analysis.

An overview of different levels for structural analysis in bridge assessment in Sweden is given in Plos and Gylltoft (2002). Structural analysis on the common or **“standard**

level" are made using simplified linear 2D beam or frame analysis. Often, separate analyses are made in the two main directions and the structure is designed or analysed separately in these directions.

For more complicated geometries like curved bridges, 3D beam or frame analyses can be used. For slab types of bridges, 3D analyses with shell finite elements are sometimes used. With such a **geometrically improved structural analysis** the geometrical modelling is of course more correct, but the analysis is still linear. Analysis on this level, as well as on the standard level, is used both for design and assessment.

Assessment on this level, in particularly with shell FE analysis, has in practise shown to be unfavourable compared to standard level assessment. Linear analysis leads to high stress concentrations, that need to be re-distributed, and different effects that give increased cross-sectional forces and moments occur in 3D analysis. This is mainly a problem for concrete bridges, and in particularly for analysis of slab bridges or bridges where beams and slabs are interacting.

Non-linear finite element (FE) analysis has proven a great potential for improved bridge assessment, with increased load carrying capacity in many cases. On this level of accuracy, the real response of the bridge is traced in the analysis. The non-linear material response is modelled and effects like cracking, yielding and failures are reflected in the analysis. A non-linear FE analysis can be made either using structural or continuum finite elements. Depending on the level of detailing, conventional local analysis are sometimes needed to check failure risks that are not reflected in the non-linear analysis.

This type of simulations are normally much more demanding and time consuming, and can in practical bridge assessment only be performed for critical load combinations determined through simplified analysis. On the other hand, in addition to a higher capacity, they can give a much deeper insight into the real structural response and hence are very valuable for further decisions regarding further assessment, maintenance or strengthening.

In Sweden, non-linear FE analysis has been used for assessment of a number of concrete bridges, see Plos (2002), Plos and Gylltoft (2002), Plos (2004) and Plos *et al.* (1997). Common bridge types have been arch bridges and slab and slab frame bridges. The bridges have usually been modelled with structural finite elements, such as shell and beam elements, in order to reflect the overall structural behaviour. Failure types not reflected directly in the analyses, like in most cases shear and torsion failures, have been checked with conventional design models. The reinforcement has been modelled with full interaction to the concrete, usually as embedded reinforcement. Even though conservative failure criteria have been used, a higher load carrying capacity, compared to conventional analysis has been shown. Consequently, costly reconstruction or strengthening has been avoided.

From the mid 1990s there are a number of references treating slab bridges of a type common in the USA. Huria *et al.* (1993), Huria *et al.* (1994) compare assessment of such a bridge according to American codes with non-linear FE-analysis. Shahrooz *et al.* (1994) and Ho and Shahrooz (1998) studied the use of non-linear FE analysis for assessment of a skewed bridge of the same type. This bridge was also tested by loading to failure. Song *et al.* (2002) present full-scale tests to failure and non-linear FE analyses of two T-beam bridges. For all bridges in the references, a great capacity reserve was reported compared to the evaluations with conventional methods. The principles for the FE analyses found in the international literature are in accordance with the analyses performed in Sweden.

4 Field Testing and Monitoring

4.1 General

Monitoring, field testing, non destructive testing and safety evaluation of existing concrete structures are comprehensively treated in fib (2003). Field testing and monitoring for improvement of structural models for bridge assessment is also treated by Cruz and Casas (2006). In this chapter, important aspects and principles for field testing and monitoring for improvement of structural models are treated.

The term field test is normally used for a measurement activity that is limited in time and associated with loading of the structure. The term monitoring is used for continuous measurements by permanently installed sensors over a longer period of time. Depending on the type of loading, field tests are often subdivided in static and dynamic field tests.

Typical parameters that can be measured in field tests and through monitoring include:

- Geometry and dimensions
- Deformations/ rotations
- Strains
- Forces
- Accelerations
- Temperature
- Wind speed

In addition to the parameters mentioned above, material parameters usually need to be determined. This is usually made by laboratory tests on samples from the bridge, but can also include non-destructive field tests. Material tests can include mechanical properties like strength, stiffness and fracture energy, but may also include durability parameters like chloride contamination or degree of carbonation. Certain durability parameters, like corrosion rate, may also be monitored on site.

4.2 Field tests

There are basically two types of field tests: diagnostic tests and proof tests. Proof tests are used to verify the performance and load carrying capacity as an alternative to structural analysis with analytical or numerical models. The bridge is exposed to high loading rates and probabilistic methods are used to determine the probability of failure, see Figure 4.1. The test involves a risk to cause damage to the structure. It is not used in Sweden and is not further treated here.

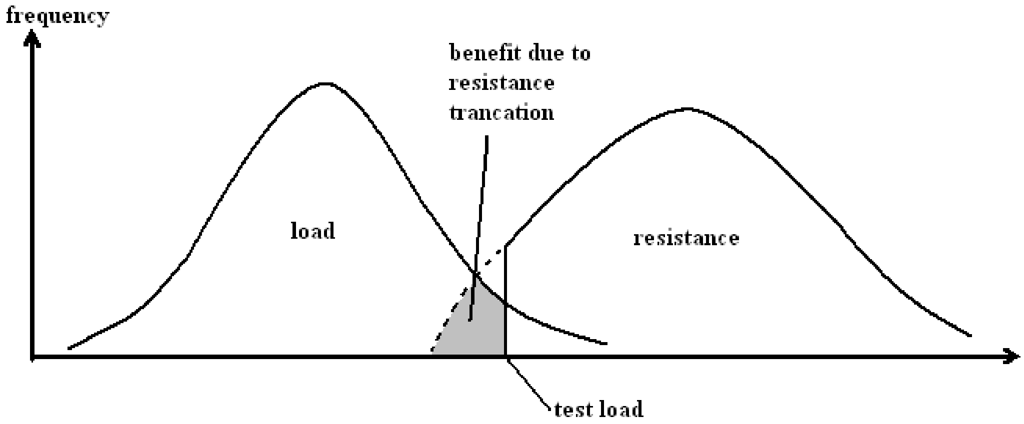

Figure 4.1 Benefit through proof testing

Diagnostic tests are performed to check and improve analytical methods and structural models used. The rate of the applied load is on the level of service conditions. Consequently, the structural response evaluated corresponds to SLS and cannot be extrapolated to ULS without including the non-linear behaviour of the materials, interconnections and boundary conditions.

4.2.1 Static tests

In a static field test the load is applied in fixed positions, or moved at crawl speed along the bridge (pseudo-static). The load from the vehicles or weights used must be measured, as well as loads introduced by jacking. For the load to be representative for serviceability conditions, Cruz and Casas (2006) recommends a load level corresponding to a 5 year return period. It corresponds approximately to 60% of the characteristic live load in bridge codes. They recommend to go never higher than 70% of the design load. In Switzerland, a load level of 80-85% of the code-specified serviceability load is commonly used to test bridges. This is significantly above the expected lifetime maximum load and is a compromise between a diagnostic test and a proof test, see Moses *et al.* (1994).

Static tests are often used to determine the load distribution between main load-carrying members and the support restrain.

4.2.2 Dynamic tests

Also dynamic tests are suitable for improvement of structural models. The tests are used to determine the natural frequencies, mode shapes, damping ratios and the dynamic amplification factor (DAF). Different excitation sources are used:

In a **forced vibration** test, the structure is excited by impulse hammers, drop weights and electro-dynamic shakers. Correlated input and output measurements are taken. In case of large structures like bridges, heavy and costly equipment is needed to provide an excitation on a high level. Forced vibration is therefore more suitable to analyse smaller structures. The accuracy of the measurements of natural frequencies, damping and higher modes is usually higher for forced vibrations than for ambient vibrations.

In an **ambient vibration** test, the freely available excitation sources like wind and traffic are used. The main advantage is that the no expensive artificial excitation devices are needed. In addition, it is not required to shut down the traffic. On the other hand, the signal levels are usually low and the duration of the records has to be longer.

To evaluate the dynamic amplification factor, a vehicle with known weight and axial loads moves at normal to full speed over the bridge. Strains and displacements are measured and compared with results from the same vehicle passing with craw speed over the bridge. From this, the dynamic amplification factor can directly be evaluated for the actual traffic load. This can particularly be useful for railway bridges. As the DAF factor depends for on the condition of the track (road holes, surface roughness) it might change over time when the track changes.

4.3 Monitoring

With monitoring systems, the changes over time of the parameters measured can be followed. If testing is suitable for obtaining structural models that reflect the actual state of a bridge, monitoring is suitable to follow the evolution and changes of the structure in a systematic way. Monitoring can be used to follow the structural response during the construction, but also during the entire service life.

In addition to the parameters that can be measured during a field test, the deterioration processes can be estimated. To identify damage, model based or non-model based approaches can be taken. In model based approaches, a FE model is updated regularly to interpret the changes of measured properties into structural changes. A problem of structural damage identification though FE model updating is that environmental effects, like temperature changes, often have a higher influence on measured properties than local damage, Moon and Aktan (2006). As a high uncertainty is associated with the structural changes due to environmental effects, like temperature dependent material parameters or geometrical changes, it is difficult to interpret the changes of measured properties into damage. Teughels and De Roeck (2004) applied structural damage identification on a highway bridge.

In the future, it will be possible to predict not only the present performance of the bridge, but also the future response, including coming deterioration.

For long time monitoring, sensors and equipment with a high environmental resistance and stability over time are needed. There is at present a rapid development of fibre optic sensors, which can fulfil these requirements. In addition to strain and displacement gauges, they can serve for e.g. crack detection.

4.4 Non-destructive testing

Non-destructive testing methods are available to determine the condition of a bridge. According to fib (2003) typical techniques include:

- **Visual testing** determines the condition of accessible surfaces. Visual signs of damage like cracks, spalls and corrosion products are used to estimate the condition of a bridge.

- **Penetration testing** can be used to detect surface cracks in steel structures. The surface is coated with a visible or fluorescent penetrant that penetrates by capillary action into cracks. After cleaning the surface of the sample from the excess penetrant, a developer is applied to lift the penetrant out of the defects to make the defects visible.

- **Magnetic particle testing** can be used to locate cracks in ferromagnetic materials. Magnetic particles are used to highlight variations in magnetic flux that can indicate a surface crack.

- In **radiographic testing** x-rays or gamma rays pass through the test body and create an image on photosensitive film. The method can detect broken wires and subsurface defects. Radiographic testing is limited to areas that are accessible from both sides.

- **Ultrasonic testing** can be used to detect defects in steel structures or to estimate the compressive strength of concrete. The first case is based on the observation that defects reflect the ultrasonic waves. In the latter case, the velocity of an ultrasonic signal in concrete is measured. From the velocity it is possible to estimate the concrete strength.

- In **impact echo testing** the surface of the structure is excited with a steel ball or a hammer. A receiver is used to determine the wave propagation in the structure. Typical applications of impact echo testing are the location of voids and cracks in concrete, grouted tendon ducts and grouted masonry.

- **Acoustic emission** can be used to monitor the activity of crack propagation in steel and concrete structures by scanning signals in the 300 KHz range. Acoustic emissions are caused by a rapid release of energy in the material that can be caused by plastic deformations or the extension of cracks in a structure under stresses.

- In **eddy current testing** an electromagnetic field creates eddy current in a conductive specimen. The signal is sensed by a coil and allows detecting cracks in steel structures.

- **Rebar locators** are based on electro-magnetic field, radioactive radiation or radar. The effect of magnetic induction is predominantly used in commercial devices and allows to determine the concrete cover, the bar diameter and the position of the rebar.

4.5 Design of field tests for model updating

Designing of field tests includes the selection of sensors, their location in the bridge, and the positions, configurations and magnitudes of the test loads. The structural model should be developed in good time before the test so that it can be used for preliminary calculations. It needs also to be determined if the bridge should be closed to traffic or under normal operation during the test. For a dynamic test, ambient vibration or forced excitation needs to be selected.

It is important that the field test is designed with respect to what model parameters should be determined. Consequently, the uncertainties in the model need to be identified and their importance estimated. The sensor locations and the loading scheme should then be designed so that the influence of each major model parameter can be evaluated based on the measurement results.

5 Finite Element Model Updating

5.1 General

There are different notations used in the context of modelling methods based on experimental observations.

- In cases where dynamic measurements are used to adjust a FE model most often **FE model updating** is used. The objective of model updating is to correct an inaccurate a-priori model to agree with test results.

- **System identification** is a general term to describe mathematical tools and algorithms that build models from measured data. Hence it is often used in mathematical literature.

- When a certain model (for example a FE model) with some unknown parameters is already available, system identification reduces to **parameter estimation**. Parameter estimation deals with estimating parameters in a model based on measured data. It is a branch of statistics and signal processing.

- **Structural identification** is used by Aktan *et al.* (1997) and describes the integration of analytical modelling, followed by experiments for the calibration and verification of the analytical model.

- **Condition assessment, bridge rating and bridge evaluation** are other terms, used in the context of bridge evaluation through load tests and FE analysis.

A process for structural model updating is described in fib (2003). Structural model updating from dynamic tests is treated by e.g. Sanayei *et al.* (1999) and Mottershead and Friswell (1993). Wang *et al.* (2005) give an overview of a procedure for model updating and assessment of bridges based on dynamic and static measurements.

A complicated structure like a bridge has generally many more model parameters that can be varied than test results to match the response with. This means that there are many different ways to modify the model parameters in order to obtain agreement with the measurement results. Consequently, the model updating must also include identification and valuation of the uncertainty of the model parameters. Furthermore, the effect of their variation on the structural response needs to be taken into account.

An updated structural model will provide a good representation of the bridge response in the SLS. However, the model cannot be used directly for analysis of the ULS. For the ULS, the model must also include the non-linear response of the material, the boundaries and the continuity conditions (interaction between different materials and different parts of the structure). However, the updated structural model still provides the best possible starting point for non-linear FE analysis.

5.2 Parameters for updating

Structural parameters that can be updated through FE model updating include:

- **Geometrical parameters.** This includes dimensions like cross-sectional shapes and dimensions, span lengths etc., often represented as cross-sectional areas and inertias in the structural model. It also includes imperfections.

- **Material parameters**, such as the modulus of elasticity (Young's modulus) and the shear modulus. In case of concrete structures, the influence of cracking may need to be included already for service conditions.

- **Stiffness parameters for the boundary conditions**.

- **Stiffness parameters for the continuity conditions**. This includes the interaction between different parts of the structure and between different materials, e.g. joints and connections between structural members, and the interface between materials in composite structures.

- **Dynamic parameters**, such as damping and structural mass.

- **Durability parameters**, associated with deterioration modelling, e.g. corrosion rate, resistance for chloride penetration or carbonation rate.

- **Forces in hangers**

In FE model updating new information is based on prior knowledge combined with measurements. Therefore, the new information depends strongly on the quality of this prior knowledge. To increase the reliability and the accuracy of the updating procedure it can be favourable to determine parameters through local testing or measurements first and to exclude them from the updating procedure. These can typically be material parameters and geometrical parameters including imperfections, as well as durability parameters in case of deterioration modelling. A first improvement of the structural model based on local tests and measurements will increase the possibility to determine correctly the parameters that cannot be tested locally. Parameters that typically need to be determined from tests and measurements of the entire structure are the stiffness parameters for the boundary and continuity conditions, and the effect on the element parameters from damage and deterioration.

Among the uncertain parameters, only those which show a significant effect on the structural response should be chosen as updating parameter. A sensitivity analysis can be performed to identify the most sensitive parameters. The uncertain parameters are usually varied in the a-priori model and the change of the response is studied. That permits the reversal conclusion that the chosen measurements must be designed to be sensitive to the chosen updating parameters.

Usually updating parameters are chosen according to engineering judgement combined with trial-and-error. A very helpful tool to choose the right updating parameter can be a plot of the sensitivity matrix. The sensitivity matrix (or jacobian matrix) shows how much a measured property changes when a possible updating parameter is varied, see Chapter 8.4.1.

5.3 Methodology for model updating

A methodology for structural model updating is given in fib (2003), and is summarised below.

1. **An a-priori structural model** is made to best represent the bridge, based on available knowledge. The level of detail of the model depends on the final purpose. Since the goal is to refine an initial and often incomplete model, it need to be adaptable.

2. **The field test is designed** based on preliminary analysis and experimental studies. This includes sensitivity analyses to determine the optimal excitation or loading schemes, as well as sensor types, numbers and locations.

3. **The field test is performed**. Dynamic tests are performed for verification of global system behaviour and dynamic properties, while static or semi-static tests can provide data also for refined modelling of local behaviour. Detailed visual inspections and material testing can provide improved information of the material response and state of deterioration and damage.

4. **Processing of the measurement data** is made in order to detect and correct errors and to obtain data that is conditioned for structural parameter identification. Data with a higher level of confidence is identified for higher weighing.

5. **Model calibration is performed.** The mechanical properties and the boundary and continuity conditions of the structural model are adjusted so that the model configuration agrees with the physical insight observed during the test and from the processing of the measured data. The numerical values of the set of parameters that should be updated are then determined through a structural parameter identification process. The resulting "calibrated" model is then validated using a set of data not used for calibration.

6. **The calibrated model is used** for decision making and management. The calibrated model serves as the best representation of the bridge structure, and can be used for any structural evaluation under service conditions. It is also the best possible starting point for non-linear FE analysis.

Wang *et al.* (2005) distinguished two steps for model calibration. In the first step, the geometry, material and section properties are adjusted in the model based on measurements and material tests. In the second step, the boundary and continuity conditions are updated based in structural parameter identification methods.

5.4 Techniques for model calibration

Different methods to calibrate a FE model are treated in the following. A comprehensive overview for model updating in structural dynamics is given in

Friswell and Mottershead (1995). The calibration of a FE model is generally an inverse problem, see Figure 5.1. In an inverse problem some values of some model parameters must be obtained from the oberservations.

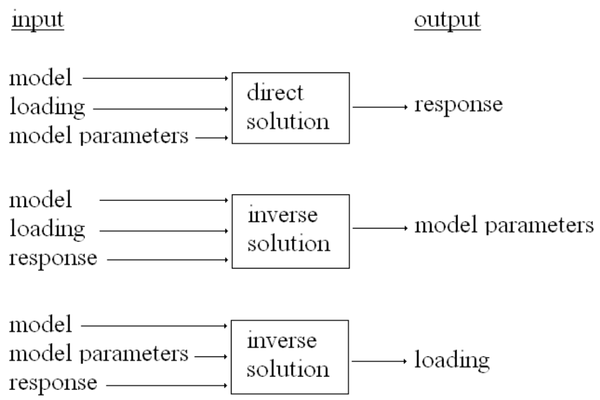

Figure 5.1 Direct and inverse problems, adapted from Johansson (2007)

5.4.1 Manual tuning

5.4.1.1 Method

In manual tuning the FE model is changed manually to increase the agreement between the predictions of the FE model and the measured properties.

Sometimes, measured properties can directly be changed in the FE model. This holds e.g. for material parameters or geometrical dimensions.

In other cases, it is possible to calculate some uncertain parameters in the FE model directly through the measurements. This is e.g. presented Chajes *et al.* (1997), and Huang (2004) where measured strains were used to calculate the effective concrete area in composite bridges.

When there are no reverse procedures to compute parameters in the FE model based on measurements directly, some structural parameters have to be successively changed with engineering judgement and trial and error till sufficient agreement between the FE model and the measurements is obtained, see Enevoldsen *et al.* (2002) and Daniell and Macdonald (2007).

In addition to adjusting some structural parameters of the FE model, manual tuning can also include remodelling of some parts of the structure, adding non structural members and justifying the choice between a consistent or lumped mass matrix. Often this is necessary before a automatic model calibration procedure can be successful, see Brownjohn *et al.* (2001) and Živanović *et al.* (2007). As the updating of specific parameters can be justified by engineering judgement, more confidence in the final model can be obtained, see Daniell and Macdonald (2007). A disadvantage of manual tuning is that the procedures are often limited to a small number of updating parameters. When there are no reverse procedures to compute parameters in the FE model based on measurements directly the variation of the parameters cannot be done very refined as it would lead to an impractical number of model variations.

5.4.1.2 Applications

In the following applications of manual tuning are summarized.

In Chajes *et al.* (1997), strains in a noncomposite steel bridge under passages of a loading vehicle were measured. The measurements showed an unintended composite action between the roadway and the steel girder and a partial support restrain that was not considered in design. The FE model that was based on these observations, indicated that the bridge's load carrying capacity might be substantially higher than initially expected. For the final load rating of the bridge the unintended support restrains were neglected as it was uncertain if it is possible to rely on the restrain for higher load levels.

Based on measured strains in a composite bridge under truck loading Huang (2004) modified the effective concrete section and included the effect of the barriers in the initial FE model. Besides that a core test showed a higher concrete strength than assumed in the design. The so obtained load ratings were approximately 17% to 27 % higher than those based on conventional methods.

Phares (2003) tested a steel girder bridge with commercial equipment and analytical tools. The moment of inertia of the steel girders, the modulus of elasticity and the rotational stiffness at the end support were chosen as updating parameters. After updating, a load rating factor 42% percent higher than initially was found.

Enevoldsen *et al.* (2002) described the manual updating procedure of a steel-truss-arch railway bridge. Joint stiffness were manually changed till the agreement between the measured and calculated strains during a train passage was significantly improved. It was shown that the load distribution and stresses between the initial and the updated model changed in several members with more than 100%.

The FE model of a cable-stayed bridge was tuned by Daniell and Macdonald (2007). Eigenfrequencies and the corresponding modal assurance criterion (MAC) were used as target response. Not only changes of the material parameters but also configurations of the FE model led to an updated model that showed an increased risk for classical flutter.

5.4.1.3 Conclusion

Manual tuning with a small number of updating parameters is an extremely powerful tool to update FE model for bridges. Especially, when it is possible to calculate some uncertain parameters in the FE model directly through the measurements and for a small number of updating parameters, manual tuning has often shown is strength. For a large amount of measured data and updating parameter manual tuning gets impractical.

5.4.2 Direct methods

5.4.2.1 Method

In direct methods in structural dynamics, the entire stiffness and mass matrices are updated in a single step. The user has not the opportunity to select specific updating parameters. The matrix terms get changed without any regards of the shape functions. The physical meaning of the initial finite element is therefore lost. Direct methods reproduce the measured data, including measurement noise, exactly. Thus, there is a requirement for accurate modelling and very high quality measurements including the elimination of faulty sensors. As direct methods are non-iterative, there is no possibility of divergence. The direct methods require less computational time than iterative methods. Friswell and Mottershead (1995) distinguished between three different groups of direct methods:

- In **Lagrange Multiplier Methods** the constrained optimization problem is transformed into an unconstrained optimization problem.

- **Matrix Mixing** is based on the fact that the mass and stiffness matrixes could be constructed directly if all the modes of a structure were measured at all the modelled degrees of freedom. In practical cases the number of measured modes is far fewer than the number of analytical degrees of freedom. The matrix mixing method uses the data from the FE model to complement the missing data.

- **Methods from Control Theory**

5.4.2.2 Applications

In Section 3 of Mottershead and Friswell (1993) numerous references to articles treating direct methods are given.

5.4.2.3 Conclusion

The mentioned disadvantages lead to the conclusion that direct methods are not very useful to update FE models of bridges. Nowadays, only manual tuning or iterative methods are used. Direct methods will not be treated further in this report.

5.4.3 Iterative methods

5.4.3.1 Method

Iterative methods have the advantage over direct methods that the updated model and parameters keep their physical meaning. Therefore, iterative methods are almost exclusively used in today's applications in structural dynamics. Although, the application of iterative updating methods on bridges are nearly exclusively reported in the context of dynamic measurements there is no reason to exclude static measurements like strains and deformations.

In iterative methods, the model updating problem is posed as an optimization problem that is solved iteratively. The differences between the analytical and experimental results are expressed in an objective function (also called penalty function or cost function). The objective function is then minimized by changes to some pre-selected updating parameters. The analytical and experimental results that are compared and brought closer together by the optimization algorithm are often called target responses. As the most common optimization algorithms can only find local minima, a good starting point is essential. Consequently, the updating parameters are usually tuned manually before a computed updating procedure is applied.

The objective function can have contributions from various measured parameters like eigenfrequencies, mode shapes, strains and deflections, see Wang (2005). Objective functions based on the modal assurance criterion (MAC) are very popular in dynamics. Weighting of the target responses can be easily introduced to consider different accuracies and confidence in measured and calculated responses. In addition, it is possible to weight the initial estimate of the updating parameter. This allows considering that some updating parameters are better known than others. Information about the variance of structural parameter are published in JCSS (2001), Ditlevsen and Madsen (1996) or Nowak and Collins (2000).

The objective function can be minimized by gradient or non-gradient (direct) methods. In gradient methods the search direction is determined using the first or second derivative (first order sensitivity matrix or second order sensitivity matrix) of the objective function. For most cases the derivatives have to be calculated numerically, for example with the finite forward approach. This leads to high computational costs for each iteration step. The methods are based on the Taylor's expansion and need usually less iterations than non-gradient methods. When the sensitivity matrix is not known, non-gradient methods can be applied. The Nelder-Mead Simplex-Method and the Rosenbrock-Method are simple to implement and robust. Stochastic methods can reduce the risk to get trapped into a local minimum but need a very high number of iterations. For more information about optimization algorithms see Belegundu and Chandrupatla (1999).

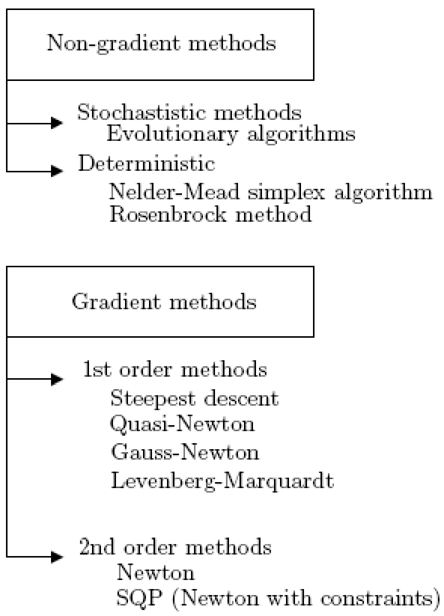

Figure 5.2 Classification of optimization methods, from Runesson et al. (2005)

5.4.3.2 Applications

Brownjohn *et al.* (2003) used eigenfrequencies and mode shapes of a concrete bridge to evaluate the success of an upgrading procedure. The data that were collected before upgrading the bridge were used to update the stiffness of T-beams and diaphragms of the bridge. After converting the simply supported system of the bridge to a jointless structure and adding guard rails the measurements were repeated. The new obtained data were used to update the rotational spring stiffness at the support to provide quantitative evidence that the upgrading work has been successfully performed.

Robert-Nicoud *et al.* (2005) used measurements of deflections, rotations and strains to update and evaluate 5 different FE models of a cantilever bridge. It was found that several models were not capable to reproduce all sets of measurments.

Teughels and De Roeck (2003) described how mode shapes and eigenfrequencies were used to update a FE model of a prestressed concrete bridge. By comparing measured data before and after applying damage to the bridge it was possible to locate the damaged regions.

In Chang *et al.* (2000) an updating procedure based on a weighted objective function is applied on a small scale suspension bridge. Constrained optimization was chosen.

Gentile (2006) used the Rosenbrock-Methods to minimize the weighted difference of eigenfrequencies of a concrete arch bridge. The modulus of elasticity of the concrete in different parts of the bridge were chosen as updating parameters.

Jaishi *et al.* (2007) described the FE model updating method of a concrete-filled steel tubular arch bridge. An objective function based on modal flexibility was chosen. Several feasible starting designs were used and then optimized. This was done to avoid being trapped in a local minimum of the objective function.

5.4.3.3 Conclusion

In all reviewed studies it was shown, that the agreement between the measurements and the FE model could be significantly improved.

It is common practice to conclude that the updating procedure has been successfully performed when the agreement between the target responses could significantly be improved under the condition of updating parameters that stayed in reasonable ranges. These are however only an necessary conditions and not sufficient condition.

Updating parameters are always chosen according to engineering judgement. One unanswered question is if it would have also been possible to update another set of updating parameter. Would the other set of updating parameters also allow improving the agreement between the target responses under the condition of reasonable updating parameter? What set of updating parameter is the correct one?

Another problem that is often neglected is that parameters that are not assigned as updating parameter are also uncertain. It is unanswered how the updating parameter would change if the assumptions of the residual parameters are not exact. In the updating procedure information about the updating parameters is gained based on prior assumptions. It is uncertain how reliable the gained information is as it is based on inaccurate prior information.

Expected errors and inaccuracies in the measurements and the results of the FE model are also often neglected when FE models are updated. A final problem is the optimization algorithm. Can the optimization algorithm find the best values of the updating parameters or is it possible that there are other values of the updating parameters that manage to reproduce the measured data better. What happens if different values of updating parameter can reproduce the measurements with approximately the same agreement?

To assure a successful iterative updating procedure it is therefore recommended to simulate the updating procedure in beforehand. This can be done by simulated measurements. Noise in the measurements, an inaccurate FE model and a wrong choice of updating parameter has to be considered. If under these conditions the updating procedure is always successful it can be applied in practice.

To illustrate the effect of different objective functions and optimization algorithms a simulated example is attached in Appendix A.

5.4.4 FE model updating of structures besides bridges

In Brownjohn *et al.* (2000) eigenfrequencies of different FE models of a high-rise building were compared with test results. Conclusions concerning the necessary degree of detailing of the FE model were drawn. Daniell and Taylor (1999) used ambient vibration test to verify the FE model of a dam. The FE model of a joint was updated by Mottershead *et al.* (1996).

6 Conclusions and Recommendations

6.1 Conclusions

Structural models for bridges are needed for structural assessment and for planning of maintenance and repair. To give a correct base for decision-making, the response of the structural model must agree with the real response of the bridge. To obtain a structural model that well reflects the bridge response, it needs to be checked and updated with respect to the actual properties and the real response of the bridge. Based on field tests, the structural model can be updated using the measurements from the test.

Any structural model developed for a bridge should be a part of the bridge database, so that it can be re-used throughout the lifetime of a bridge. In the future, the structural model will be linked to the product model of the bridge, the core of the object database, containing all relevant information of the bridge. Through measurements from bridge monitoring, the previously calibrated structural model can be kept up-to-date with respect to e.g. damage or deterioration, performing renewed structural updating. In the future, deterioration models used in the prediction of the long-time behaviour can be calibrated using monitoring data.

A structural model of a bridge needs to be updated based on field tests and measurements to really reflect the bridge’s response. On the other hand, field tests need to be evaluated using appropriate structural models in order to provide any valuable information. In the same way, bridge monitoring need to be coupled to structural analysis to make it possible to interpret the changes in measurement result.

For structural model updating, static and dynamic diagnostic tests can be used. Dynamic testing is primarily used to determine dynamic model parameters and parameters that influence the global structural behaviour, e.g. boundary conditions. Static load tests can be used to determine parameters that influence both global and local structural behaviour, such as parameters for element stiffness, or damage distribution. Geometrical properties and material parameters are normally most accurately determined through separate measurements or sample tests. It is important that a field test is designed so that it provides the best possible basis for the model updating.

A draft methodology for updating of FE bridge models through field tests has been developed, based on the literature study and experience gained so far, see also Figure 6.1:

1. Determine the **purpose of the model** and the analysis that will be performed. What kind of response need to be captured? What is the outcome of the analysis?

2. **Develop an initial FE model** that corresponds to the purpose. This includes the choice of an appropriate level of detailing for the model.

3. **Evaluate/ judge if the FE model is likely to fulfil its purpose** with respect to uncertain parameters:

- Identify and value the uncertainties in the input parameters
- Estimate or calculate the influence on the analysis results of the uncertainties in the parameters. If necessary, a sensitivity analysis can be made.
- Identify critical parameters that need to be updated through measurements

4. Decide whether the important uncertain parameters can be best **measured directly** (5) **or indirectly** (6). If parameters can be determined directly, this is normally preferable and limits the number of updating parameters in the updating procedure.

5. **Parameters that can be determined**, or having its uncertainty reduced, **directly** through geometrical measurements, local samples or non-destructive tests **can be used to update the FE model directly**. If the FE model is judged to fulfil its purpose after this improvement, points 6-9 can be omitted

6. If parameters should be determined indirectly, the **updating method has to be chosen**. For manual updating this includes the field test and loading; for programmed iterative updating, it also covers the objective function and the optimization algorithm. The field test should be made so that variations of the uncertain parameters give clear variations of the measured data.

7. Before the field tests are performed it should be **assured that the FE model updating procedure can provide the required information**. This can be done by testing the chosen updating procedure with simulated measurements. The expected variation of the real measurements and inaccuracies in the FE model have to be included when testing the updating procedure. In addition to the demands formulated under point 1, the FE model should now also estimate the target responses as good as possible. Therefore, it can be necessary to modify the FE model.

8. When the updating procedure has shown its validity, the **field test can be performed.** Uncertainties and errors in the measurements are identified. Data is corrected for errors and the corrected measurement data is graded with respect to the level of confidence. The discrepancies between the real measurements and the FE model can be used to assess the validity of the assumptions of measurement distribution and inaccuracies under point 7.

9. The **FE model is updated** and it is checked whether the updating parameters stay in a realistic range.

10. Finally, a **model is obtained that can fulfil its purpose**.

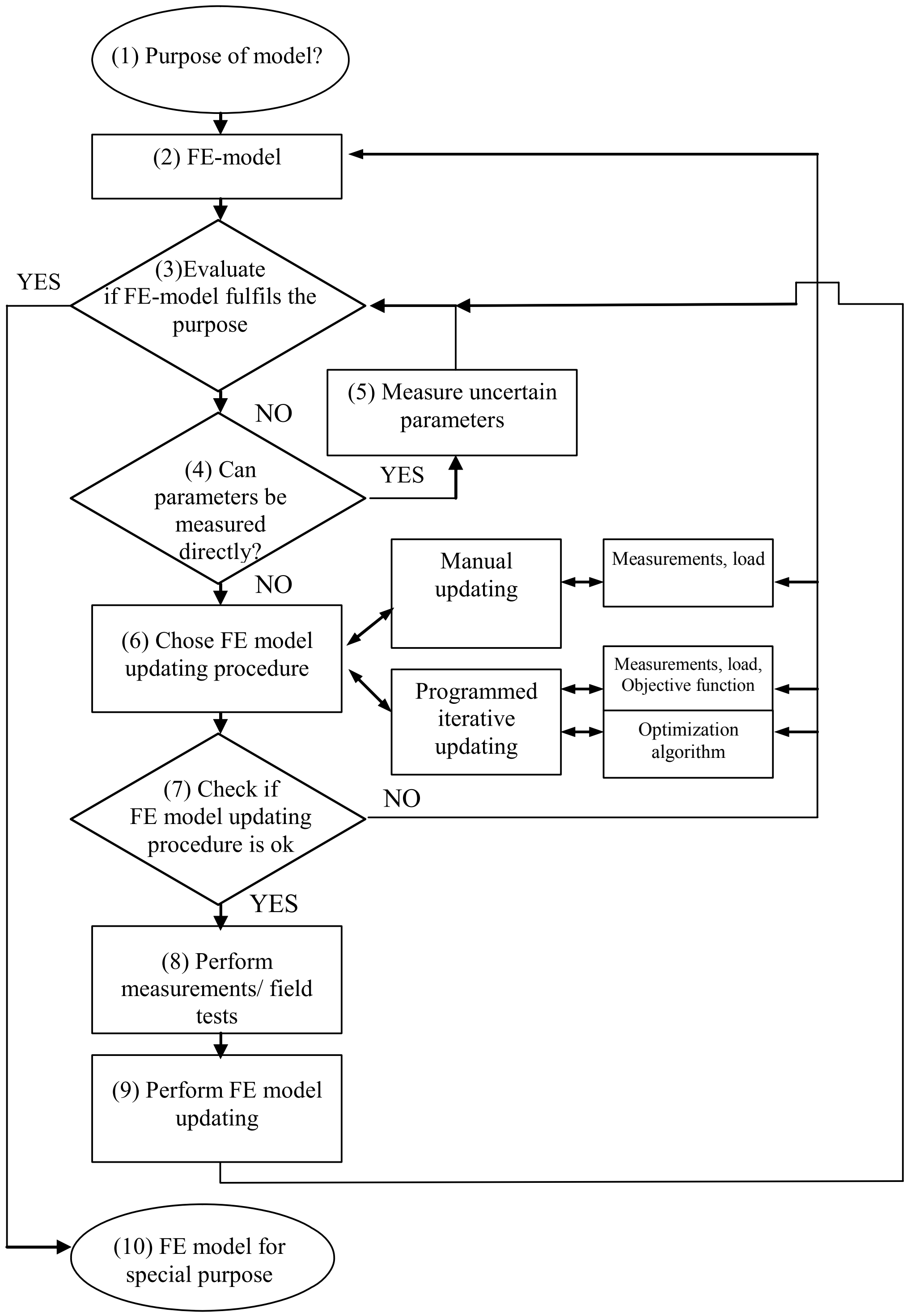

Figure 6.1 Flow chart of improved FE modelling through measurements

The updated model serves as the best representation of the bridge structure for the purpose of it. Since the bridge only can be tested under service conditions, the model represents the response under service conditions only. For analysis in ultimate limit state, the model serves as the best possible starting point. Tests on samples from the bridge can provide with necessary non-linear material or interface properties. Detailed non-linear analyses of critical regions, or even model tests, can provide improved information on the non-linear behaviour of boundary and continuity conditions.

6.2 Recommendations

For the design and the assessment of bridges lower bond assumptions are appropriate. However, the case study on the Svinesund Bridge disclosed the need for an FE model which is as accurate as possible. When FE model updating is intended to be used to estimate structural parameters it must capture all important physical phenomena which influence the measurements. This might include effect like non-linear behaviour of the bearings, temperature effects, construction process, hardening of the concrete with time, non-linear material behaviour and temperature dependent material parameters. This can lead to very complex FE models. On the other hand, it is possible to combine the measurements and the FE model to identify the important physical phenomena which have to be included to capture the real response of the bridge. Based on these observations and the available literature, it is recommended to initially apply FE model updating on bridge types were all important physical phenomena are rather well understood and easy to model.

FE model updating is most beneficial when unavoidable uncertainties concerning the actual load path have a high influence on the results from the a-priori model. This holds for steel-truss bridges, see e.g. Enevoldsen *et al.* (2002), and T-beam bridges with several parallel members. For composite bridges it can be beneficial to substitute the effective width according to codes with more realistic estimates which are based on measurements, see (1998).

6.3 Further Research

To utilize the potential of FE model updating in an optimal way further research is needed. One point that needs to be improved is the accuracy of initial FE models which have to capture all important physical phenomena in an accurate way. It might be necessary to include the building process, temperature dependent material parameters, temperature caused geometrical changes, creep, shrinkage and non-linear behaviour to keep the modelling error small. Through experience, the combination of on-site measurements with FE analysis will lead to a better understanding of important physical phenomena which have to be included when modelling common bridge types.

Furthermore, bridge types which benefit most from on-site measurements and FE model updating should be studied more closely. For these bridge types an appropriate measurement programme including the loading should be found which allows to determine the most important uncertainties.

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8 Appendix A: FE model updating of a simply supported beam

This chapter is heavily drawn on Chapter 8 of Friswell and Mottershead (1995) where more information about the derivations of the used algorithms is given. Both sources can be considered to complement each other.

To illustrate and to clarify important aspect of FE model updating algorithms, a simulated example is used. The effect of different optimization methods, updating parameter and different sets of target responses can be studied.

The analytical model consists of 8 beam elements with equal length, shown in Figure 8.1, containing 18 degrees of freedom. The total span auf the beam is $l = 8m$. Only displacements in the plane are considered.

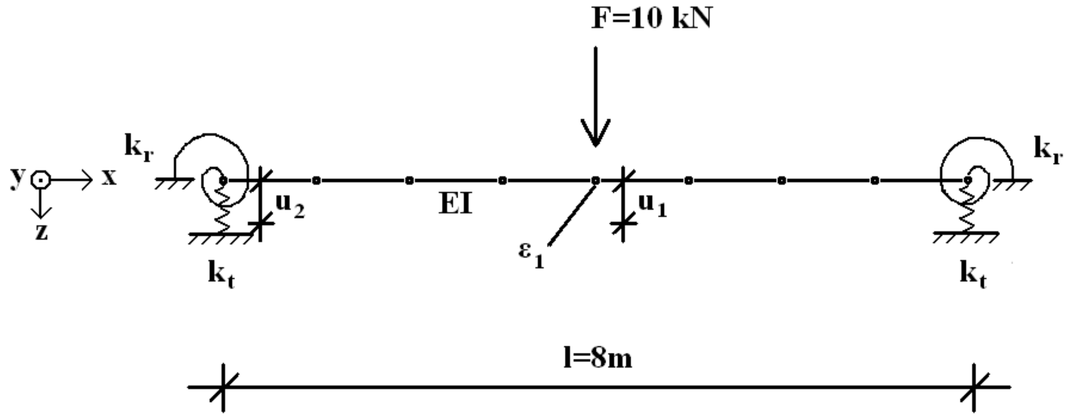

Figure 8.1 Finite element model of the simulated example

8.1 Updating parameters

Updating parameters are the translational spring stiffness, k_t, in z-direction, the rotational spring stiffness, k_r, and the flexural rigidity EI of the beam. The initial estimates and are shown in Table 8.1. The beam is chosen to be a IPE 180 steel beam with the corresponding cross sectional constants.

8.2 Target response

To update the updating parameter the difference in the target responses is minimized. It is assumed that different combinations of the following target responses were chosen:

- the first four eigenfrequencies $f_1 - f_4$

- the displacements in z-direction in the midspan, u_1 ,under load $F = 10kN$

- the displacement in z-direction at the left support, u_2, under load $F = 10kN$

- the strain of the lower flange, ε_1, in the midspan under load $F = 10kN$

8.3 Measurements

The measurements were simulated with the same model as shown in Table 8.1. Instead of using the initial estimate of the updating parameters, now the “correct” parameters were used to simulate the “measured value”.

Table 8.1 Initial and measured parameters and target response

	inital estimate	"measured value"	Units
EI	2.77E+07	2.85E+07	N/m^2
k_t	5.00E+05	1.00E+06	N/m^2
k_r	2.50E+06	5.00E+06	Nm/rad
f_1	10.389	13.319	Hz
f_2	20.379	26.491	Hz
f_3	39.157	46.408	Hz
f_4	77.023	84.882	Hz
u_1	0.026	0.018	m
u_2	0.010	0.005	m
ε_1	0.00040	0.00035	-

8.4 Updating procedures

Different updating procedures are presented in the following.

8.4.1 Unweighted pseudo inverse algorithm – more target responses than updating parameter

The Gauss-Newton algorithm is used to solve the nonlinear least squares problem in an iterative procedure. To improve the conditioning of the problem, normalized updating parameters are used so that their initial values are ones. The sensitivity matrix is a rectangular matrix of order $m \times n$, where m and n are the number of target responses and updating parameter, respectively. The sensitivity matrix is calculated numerically using the forward finite approach:

$$S = \left[S(i,j)\right] \tag{8.1}$$

$$S(i,j) = \frac{zi(\theta j + \Delta\theta j) - zi(j)}{(\theta j + \Delta\theta j) - \theta j} \tag{8.2}$$

For more target responses than updating parameter the **columns of S** are usually **linear independent**. The pseudo inverse can then be calculated according to equation (8.3). After an initial guess of the updating parameters θ_0 the subsequent guesses θ_{k+1} for the normalized updating parameter vector is then calculated according to equation (8.4).

$$S^{+} = \left[S^T * S\right]^{-1} * S^T \tag{8.3}$$

$$\theta_{j+1} = \theta_j + \left[S^T S\right]^{-1} S^T (z_m - z_j) \tag{8.4}$$

where

θ_j, θ_{j+1} — are vectors with the normalized updating parameter in iteration j and $j+1$ respectively; $\theta_j = \left[\theta 1_j \; \theta 2_j \; \theta 3_j\right]^T$

S — is the sensitivity matrix that contains the first derivative of the target responses with respect to the updating parameter (also called Jacobian matrix); in this case $S = \begin{bmatrix} \frac{\partial f_1}{\partial \theta 1} & \cdots & \frac{\partial f_1}{\partial \theta 3} \\ \vdots & \ddots & \vdots \\ \frac{\partial \varepsilon_1}{\partial \theta 1} & \cdots & \frac{\partial \varepsilon_1}{\partial \theta 3} \end{bmatrix}$

z_m — is a vector containing the measured target response $z_m = \left[z1_m \; z2_m \ldots z7_m\right]^T = \left[f_{1m} \; f_{2m} \; f_{3m} \; f_{4m} \; u_{1m} \; u_{2m} \; \varepsilon_{1m}\right]^T$

z_j — is a vector containing the calculated target response in iteration j;

$$z_j = \left[z1_j \; z2_j \; ... z7_j \right]^T = \left[f_{1j} \; f_{2j} \; f_{3j} \; f_{4j} \; u_{1j} \; u_{2j} \; \varepsilon_{1j} \right]^T$$

The results for the first five iterations are summarized in Table 8.2. It can be seen that already after the third iteration all updating parameters are updated correctly. Since all target responses are weighted equally in absolute terms, the algorithm weights effectively target responses with a higher value more than other target responses. Therefore, higher target responses are reproduced faster than lower target responses. This becomes obvious when comparing the relative error of the first and 4th eigenfrequencies after the first iteration. The first eigenfrequency with a lower value has a relative error of 4.3% after the first iteration. The 4th eigenfrequency has a relative error of 1.1% at the same time. The absolute error of both eigenfrequencies is around 0.9 Hz after the first iteration.

Table 8.2 Results for the unweighted Gauss-Newton algorithm

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.060	1.026	1.029	1.029	1.029	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.784	1.992	2.000	2.000	2.000	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.559	1.943	2.000	2.000	2.000	2.000
$\left\Vert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\Vert$	0.708	0.248	0.029	0.000	0.000	0.000	0.000
target respones							
f_1	10.389	12.745	13.269	13.319	13.319	13.319	13.319
f_2	20.379	25.487	26.441	26.491	26.491	26.491	26.491
f_3	39.157	45.343	46.311	46.408	46.408	46.408	46.408
f_4	77.023	83.954	84.650	84.881	84.882	84.882	84.882
u_1	0.026	0.019	0.018	0.018	0.018	0.018	0.018
u_2	0.010	0.006	0.005	0.005	0.005	0.005	0.005
ε_1	0.00040	0.00035	0.00036	0.00035	0.00035	0.00035	0.00035
$\left\Vert \frac{z_m - z_j}{z_m} \right\Vert$	1.162	0.151	0.011	0.000	0.000	0.000	0.000

In Chapter 5.2 it was mentioned that a plot of the sensitivity matrix can be a helpful tool to decide which updating parameter can be estimated with what kind of measurements. In Table 8.3 the normalized sensitivity matrix is plotted. It is calculated according to equation (8.5).

$$S_{normalized} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta 1}\frac{1}{f_1} & \cdots & \frac{\partial f_1}{\partial \theta 3}\frac{1}{f_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial \varepsilon_1}{\partial \theta 1}\frac{1}{\varepsilon_1} & \cdots & \frac{\partial \varepsilon_1}{\partial \theta 3}\frac{1}{\varepsilon_1} \end{bmatrix} \tag{8.5}$$

It shows how much a target response changes relatively when one of the updating parameter is increased. For example, does the strain at midspan decreases with 94.9% percent when the flexural rigidity increases with 100% (see bold entry in Table 8.3). The higher the absolute value in the normalized sensitivity matrix is the more effect does a change of the updating parameter show on measured properties.

Table 8.3 Normalized sensitivity matrix at first iteration

	EI	k_t	k_r
f_1	9.4%	23.3%	6.0%
f_2	9.3%	29.0%	0.0%
f_3	22.9%	14.9%	4.2%
f_4	34.1%	4.6%	6.6%
u_1	-61.0%	-55.5%	-27.3%
u_2	0.0%	-198.0%	0.0%
ε_1	**-94.9%**	0.0%	-15.4%

To account for the different uncertainties of the updating parameters it can be useful to multiply each column of the normalized sensitivity matrix with the expected coefficients of variations of the updating parameters, see Table 8.4. The Table indicates that it is more difficult to decrease the relative error of the bending stiffness than the errors of the spring properties.

Table 8.4 Normalized sensitivity matrix multiplied with the coefficients of variation of the updating parameters

	EI	k_t	k_r
f_1	0.3%	23.3%	6.0%
f_2	0.3%	29.0%	0.0%
f_3	0.7%	14.9%	4.2%
f_4	1.0%	4.6%	6.6%
u_1	-1.8%	-55.5%	-27.3%
u_2	0.0%	-198.0%	0.0%
ε_1	-2.8%	0.0%	-15.4%

Further it can also be valuable to consider the expected accuracy of the measurements to chose the right test configurations. This can be done by dividing each row of the normalized sensitivity by the coefficient of variation of the measurements.

The same can be done to account for the costs of different measurements.

8.4.2 Unweighted pseudo inverse algorithm – more updating parameter than target response or noise data

For more updating parameter than target responses the nonlinear least spare problem can be solved using the More-Penrose pseudoinverse of the sensitivity matrix, S^{+}.

For less target responses than updating parameter the **rows of *S* are** usually **linear independent** and the pseudoinverse can be calculated according to equation (8.5). The normalized updating parameters of the next iteration step are then obtained according to equation (8.6).

$$S^{+} = S^{T}\left[SS^{T}\right]^{-1} \tag{8.6}$$

$$\theta_{j+1} = \theta_{j} + S^{T}\left[SS^{T}\right]^{-1}(z_{m} - z_{j}) \tag{8.7}$$

To illustrate the effect of more updating parameters than target responses, the same example as in Chapter 8.4.1 will be used. Now, it is assumed that only the first two eigenfrequencies were measured. In this case we have two target responses and three updating parameter. The set of equations is therefore underdetermined and there exist an infinite number of solutions. The pseudoinverse searches for the set of updating parameters that needs the smallest parameter changes.

The result of the first 5 iterations is summarized in Table 8.4. The target responses are well reproduced after 3 iterations. Unfortunately the error in the updating parameters does almost not decrease. It can be concluded that this algorithm does not necessarily led to better estimates of updating parameter even if it is possible to perfectly reproduce the measured data.

Table 8.5 Results for unweighted pseudo inverse algorithm, more updating parameter than target response

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.358	1.441	1.441	1.441	1.441	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.681	1.779	1.780	1.780	1.780	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.456	1.529	1.529	1.529	1.529	2.000
$\left\| \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\|$	0.708	0.449	0.477	0.478	0.478	0.478	
target respones							
f_1	10.389	10.389	12.947	13.317	13.319	13.319	13.319
f_2	20.379	20.379	25.734	26.483	26.491	26.491	26.491
f_3	not considered						46.408
f_4							84.882
u_1							0.018
u_2							0.005
ε_1							0.00035
$\left\| \frac{z_m - z_j}{z_m} \right\|$	0.319	0.319	0.040	0.000	0.000	0.000	0.000

8.4.3 Unweighted pseudo inverse algorithm – equal amount of target responses and updating parameters

For an equal amount of target responses and updating parameter all **rows and columns of S are** usually **linear independent.** In this case the inverse of *S* exists and can be used in the algorithm, see equation (8.8).

$$\theta_{j+1} = \theta_j + S^{-1}(z_m - z_j) \tag{8.8}$$

The results for the first five iterations are shown in Table 8.5. It can be seen that the updating parameter and target responses obtain good agreement after a few iterations. Even though it is usually possible to calculate the inverse of *S*, it might be that *S* is ill-conditioned. In the example, no noise in the measurements and a FE model that perfectly represents the real behavior is assumed. In practical cases the ill-conditioning can lead to completely wrong estimates of the updating parameter.

Table 8.6 Results for unweighted pseudoinverse algorithm – equal amount of target response and updating parameter

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta1_j = EI_j / EI_0$	1.000	1.081	1.023	1.029	1.029	1.029	1.029
$\theta2_j = kt_j / kt_0$	1.000	1.770	1.993	2.000	2.000	2.000	2.000
$\theta3_j = kr_j / kr_0$	1.000	1.540	1.926	1.999	2.000	2.000	2.000
$\left\lVert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\rVert$	0.708	0.262	0.037	0.001	0.000	0.000	
target respones							
f_1	10.389	12.747	13.254	13.319	13.319	13.319	13.319
f_2	20.379	25.479	26.436	26.491	26.491	26.491	26.491
f_3	39.157	45.478	46.276	46.408	46.408	46.408	46.408
f_4	not considered						84.882
u_1							0.018
u_2							0.005
ε_1							0.00035
$\left\lVert \frac{z_m - z_j}{z_m} \right\rVert$	0.355	0.061	0.006	0.000	0.000	0.000	0.000

8.4.4 Pseudo inverse algorithm – more target responses than updating parameter – weighting of target responses

As mentioned earlier, all target responses are weighted equally in absolute terms. Therefore, the algorithm weights effectively target responses with a higher value more than others. To avoid this, weighting of the target responses can be introduced. In addition, it is possible to consider that some target responses are more reliable and accurate than others.

The same case as in Chapter 8.4.1 is considered, i.e. all target responses and all updating parameter are included. In addition, it is now assumed that the eigenfrequencies have a coefficient of variation of $V_1 - V_4 = 1\%$, the deflections u_1 and u_2 have a coefficient of variation of $V_{u1} = V_{u2} = 5\%$ and the strain is assumed to have a coefficient of variation of $V_\varepsilon = 10\%$. The chosen variation has to take into account the variation that is introduced by the measurements and the inaccuracies of the FE model. A weighting matrix, W_z, can than be calculated with diagonal elements equal to the reciprocals of the measured variances, according to equation (8.8).

$$W_z = diag\left[\frac{1}{(V_1 f_{1m})^2} \frac{1}{(V_2 f_{2m})^2} \frac{1}{(V_3 f_{3m})^2} \frac{1}{(V_4 f_{4m})^2} \frac{1}{(V_{u1} u_{1m})^2} \cdots \right] \quad (8.9)$$

The normalized updating parameter can then be calculated according to equation (8.9).

$$\theta_{j+1} = \theta_j + \left[S^T W_z S\right]^{-1} S^T W_z (z_m - z_j) \tag{8.10}$$

In Table 8.7 the results are summarized. Compared to the unweighted Gauss-Newton algorithm, the relative error decreases now more evenly for all target responses.

Table 8.7 Results for weighting of the target responses – more target responses than updating parameter

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.140	1.034	1.029	1.029	1.029	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.616	1.954	2.000	2.000	2.000	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.502	1.860	1.995	2.000	2.000	2.000
$\left\lVert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\rVert$	0.708	0.332	0.074	0.003	0.000	0.000	
target respones							
f_1	10.389	12.527	13.177	13.316	13.319	13.319	13.319
f_2	20.379	24.802	26.279	26.489	26.491	26.491	26.491
f_3	39.157	45.199	46.160	46.403	46.408	46.408	46.408
f_4	77.023	85.486	84.569	84.865	84.882	84.882	84.882
u_1	0.026	0.019	0.018	0.018	0.018	0.018	0.018
u_2	0.010	0.006	0.005	0.005	0.005	0.005	0.005
ε_1	0.00040	0.00033	0.00036	0.00036	0.00035	0.00035	0.00035
$\left\lVert \frac{z_m - z_j}{z_m} \right\rVert$	1.162	0.269	0.032	0.001	0.000	0.000	0.000

8.4.5 Pseudo inverse algorithm – more updating parameter than target response or noise data – weighting of target responses

When more updating parameters than target responses are chosen, the measured properties will be reproduced exactly. Therefore weighting of the target responses is not helpful.

8.4.6 Pseudo inverse algorithm – more updating parameter than target responses or noise data – weighting of updating parameter change

To take into account that some updating parameters are better known than others, weighting of the of the updating parameter change can be introduced. The weighting matrix is usually chosen to be a diagonal matrix with the reciprocals of the estimated variances of the corresponding updating parameter. The updating procedure can then be executed according to equation (8.10).

$$\theta_{j+1} = \theta_j + W_\theta^{-1} S^T \left[S W_\theta^{-1} S^T \right]^{-1} (z_m - z_j) \tag{8.11}$$

where

W_θ is the weighting matrix of the updating parameters $W_\theta = diag\left[\frac{1}{(V_{EI} EI_1)^2} \frac{1}{(V_{kt} k_t)^2} \frac{1}{(V_{kr} k_r)^2} \right]$; for normalized updating parameter it turns into $W_e = diag\left[\frac{1}{V_{\theta 1}^2} \frac{1}{V_{\theta 2}^2} \frac{1}{V_{\theta 2}^2} \right]$

In the example, it is assumed that the initial estimate of the bending stiffness of the steel beam is much better than the spring stiffness of the supports. Therefore the bending stiffness is weighted higher than the spring stiffness. A coefficient of variation of $V_{\theta 1} = 3\%$ is assumed for the bending stiffness. The coefficients of variation for the two supports are assumed to be $V_{\theta 2} = V_{\theta 3} = 100\%$. Only the first two eigenfrequencies like in Chapter 8.4.2 are used as target responses.

The results are summarized in Table 8.8. When comparing this results with the results that were obtained when no weighting was introduced (Table 8.4), it can be seen, that the updating parameter are estimated much better. For an underdetermined problem it is possible to obtain better estimates of the updating parameters when the right variances are chosen and the noise level is small.

Table 8.8 Results for weighting of the change of the updating parameter – more updating parameter than target responses

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.000	1.001	1.001	1.001	1.001	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.796	2.013	2.019	2.019	2.019	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.564	1.992	2.065	2.066	2.066	2.000
$\left\| \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\|$	0.708	0.242	0.028	0.043	0.044	0.044	
target respones							
f_1	10.389	12.648	13.269	13.319	13.319	13.319	13.319
f_2	20.379	25.357	26.455	26.491	26.491	26.491	26.491
f_3	not considered						46.408
f_4							84.882
u_1							0.018
u_2							0.005
ε_1							0.00035
$\left\| \frac{z_m - z_j}{z_m} \right\|$	0.319	0.066	0.004	0.000	0.000	0.000	0.000

8.4.7 Weighting of the updating parameter changes and weighting of the target responses – more target responses than updating parameter

As updating parameters and target responses are estimated with certain accuracies it is reasonable to introduce weighting for both sets. This can improve the conditioning of the problem. In this case, the normalized updating parameter can be calculated according to equation (8.12).

$$\theta_{j+1} = \theta_j + \left[S^T W_z S + W_\theta\right]^{-1} S^T W_z (z_m - z_j) \tag{8.12}$$

The results for the example are summarized in Table 8.9. Compared to previous algorithm the convergence is slower. Although weighting of the updating parameter change is introduced, the total updating parameter change is not restricted. Therefore, a high number of iterations can lead to a significant change of the updating parameters.

Table 8.9 *Results of the algorithm with weighting of the updating parameter changes and weighting of the target responses*

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.044	1.044	1.038	1.034	1.032	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.619	1.956	1.995	1.997	1.998	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.829	1.903	1.944	1.967	1.981	2.000
$\left\Vert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\Vert$	0.708	0.209	0.055	0.029	0.017	0.010	
target respones							
f_1	10.389	12.566	13.228	13.301	13.309	13.313	13.319
f_2	20.379	24.548	26.324	26.494	26.493	26.492	26.491
f_3	39.157	44.429	46.306	46.459	46.439	46.426	46.408
f_4	77.023	83.690	84.982	85.017	84.963	84.930	84.882
u_1	0.026	0.019	0.018	0.018	0.018	0.018	0.018
u_2	0.010	0.006	0.005	0.005	0.005	0.005	0.005
ε_1	0.00040	0.00035	0.00035	0.00035	0.00035	0.00035	0.00035
$\left\Vert \frac{z_m - z_j}{z_m} \right\Vert$	1.162	0.267	0.026	0.006	0.004	0.002	0.000

8.4.8 Weighting of the total updating parameter change and weighting of the target responses – more target responses than updating parameter or noise data

In Chapter 8.4.7 the updating parameter change in every iteration step was weighted. Now the updating parameter change from the initial value will be considered. The iteration can than be performed according to equation (8.12).

$$\theta_{j+1} = \theta_j + \left[S^T W_z S + W_\theta \right]^{-1} \left[S^T W_z \left(z_m - z_j \right) - W_\theta \left(\theta_j - \theta_0 \right) \right] \tag{8.13}$$

The results of the first five iterations are summarized in Table 8.10. The measured values and target responses are not reproduced exactly. That is natural as the consideration of the total updating parameter change in the objective function prevents this. Instead a compromise between reproducing the target responses and changing of the updating parameter is found. The convergence is faster than for the algorithm in Chapter 8.4.7.

Table 8.10 Results of the algorithm with weighting of the total updating parameter changes and weighting of the target responses

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.044	1.020	1.016	1.016	1.016	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.619	1.966	2.011	2.012	2.012	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.829	2.011	2.030	2.028	2.028	2.000
$\left\lVert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\rVert$	0.708	0.209	0.020	0.020	0.020	0.020	
target respones							
f_1	10.389	12.566	13.251	13.323	13.323	13.323	13.319
f_2	20.379	24.548	26.298	26.502	26.505	26.505	26.491
f_3	39.157	44.429	46.136	46.327	46.330	46.330	46.408
f_4	77.023	83.690	84.533	84.578	84.578	84.578	84.882
u_1	0.026	0.019	0.018	0.018	0.018	0.018	0.018
u_2	0.010	0.006	0.005	0.005	0.005	0.005	0.005
ε_1	0.00040	0.00035	0.00036	0.00036	0.00036	0.00036	0.00035
$\left\lVert \frac{z_m - z_j}{z_m} \right\rVert$	1.162	0.267	0.023	0.013	0.013	0.013	0.000

8.4.9 Minimum variance method

The minimum variance method assumes that the target responses and the initial estimates of the updating parameter are not correct. An expected variance is assigned to both sets of parameters. Than the iteration algorithm seeks for the updating parameter that have minimum variance. The here applied method is in more detail described in Friswell and Mottershead (1995). One important advantage of the minimum variance method is that it gives information about the variance of the updated parameters, see Figure 8.2. This can be used to assess the quality of the obtained results.

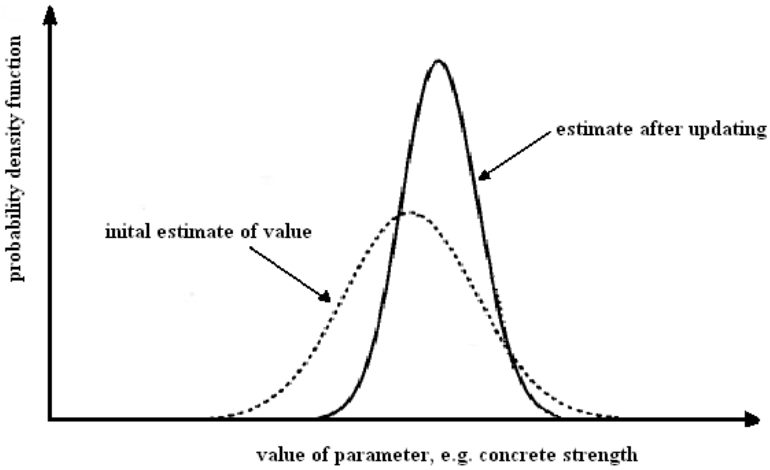

Figure 8.2 Effect of applying the minimum variance method to an updating parameter

The results are summarized in Table 8.10. The major change of the updating parameters happens in the first two iterations. The error in target responses and the second and third updating parameters decreases significantly. The first updating parameter is not updated very well. Under Table 8.10 the initial variance matrix and the variance matrix after 5 iterations are plotted. It becomes obvious, that the variances of all updating parameters decrease. The updating parameters can therefore be considered to be more certain. The chosen measurements decrease the variance of the support stiffness more than the variance of the bending stiffness.

Table 8.11 Results for the minimum variance method

updating parameter	inital value	iteration number					measured values
		1	2	3	4	5	
$\theta 1_j = EI_j / EI_0$	1.000	1.044	1.106	1.106	1.112	1.110	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.619	1.697	1.706	1.708	1.707	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.829	1.592	1.666	1.643	1.655	2.000
$\left\Vert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\Vert$	0.708	0.145	0.156	0.162	0.163	0.163	
target respones							
f_1	10.389	12.566	12.695	12.761	12.759	12.763	13.319
f_2	20.379	24.548	25.164	25.214	25.237	25.230	26.491
f_3	39.157	44.429	45.385	45.496	45.542	45.532	46.408
f_4	77.023	83.690	85.106	85.376	85.460	85.449	84.882
u_1	0.026	0.019	0.019	0.019	0.019	0.019	0.018
u_2	0.010	0.006	0.006	0.006	0.006	0.006	0.005
ε_1	0.00040	0.00035	0.00034	0.00034	0.00034	0.00034	0.00035
$\left\Vert \frac{z_m - z_j}{z_m} \right\Vert$	1.162	0.156	0.063	0.051	0.050	0.050	0.000

$$V_0 = \begin{bmatrix} 0.0009 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; V_5 = \begin{bmatrix} 0.00045 & -0.00005 & -0.00182 \\ -0.00004 & 0.00029 & -0.00052 \\ -0.00182 & -0.00052 & 0.0148 \end{bmatrix}$$

As the updating algorithm did not lead to an improved estimate of the flexural rigidity a new attempt was made. A higher variance of the target responses was assumed. When the initial coefficients of variations of the target responses were multiplied with a factor of 2, the updating procedure led to much better estimates of the updating parameters. After five iterations the following updating parameters were obtained:

$$\theta_5 = \begin{bmatrix} 1.052 \\ 1.718 \\ 1.827 \end{bmatrix}$$

8.4.10 Nelder-Mead Simplex Method

The simplex method is a non gradient method to minimize an objective function. The computation of the sensitivity matrix can therefore be omitted. Depending on the choice of the objective functions weighting of the updating parameter and weighting of the target responses can be introduced. The simplex method is considered to be a robust optimization. For more information see Belegundu and Chandrupatla (1999).

Two different objective functions will be used for this example. The first objective function leads to an unweighted optimization as in Chapter 8.4.1. It is the Euclidean norm of the target responses, see equation (8.14).

$$J = \|z_m - z\| = \sqrt{(f_1 - f_{1m})^2 + (f_2 - f_{2m})^2 + ...(\varepsilon_1 - \varepsilon_{1m})^2} \tag{8.14}$$

The results are shown in Table 8.12. Now, much more iterations are needed to find the right updating parameters. This is to some extend compensated by the fact that each iteration requires less computational time.

Table 8.12 Simplex method – Euclidean norm of target responses in objective function

updating parameter	inital value	iteration number					measured values
		1	2	**5**	**20**	**100**	
$\theta 1_j = EI_j / EI_0$	1.000	1.050	1.050	1.250	1.208	1.029	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.000	1.050	1.200	1.964	2.000	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.000	1.050	1.000	0.734	2.001	2.000
$\left\| \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\|$	0.708	0.707	0.672	0.675	0.657	0.000	
target respones							
f_1	10.389	10.449	10.645	11.270	12.296	13.319	13.319
f_2	20.379	20.500	20.882	22.432	26.817	26.492	26.491
f_3	39.157	39.682	40.124	42.956	46.863	46.409	46.408
f_4	77.023	78.453	78.925	84.534	84.668	84.884	84.882
u_1	0.026	0.025	0.025	0.022	0.021	0.018	0.018
u_2	0.010	0.010	0.010	0.008	0.005	0.005	0.005
ε_1	0.00040	0.00038	0.00038	0.00033	0.00035	0.00035	0.00035
$\left\| \frac{z_m - z_j}{z_m} \right\|$	1.162	1.143	1.037	0.746	0.171	0.000	0.000

The second objective function that was used includes weighting of the target responses and weighting of the total updating parameter change, see equation 8.15. The algorithm used in Chapter 8.4.8 is based on the same objective function.

$$J = (z_m - z)^T W_z (z_m - z) + (\theta - \theta_0)^T W_\theta (\theta - \theta_0) \tag{8.15}$$

Again, much more iterations are needed than in the algorithm that use the sensitivity matrix.

Table 8.13 Simplex method – weighting of the target responses and weighting of the total updating parameter change

updating parameter	inital value	iteration number					measured values
		1	2	**5**	**20**	**100**	
$\theta 1_j = EI_j / EI_0$	1.000	1.000	1.050	1.200	1.021	1.016	1.029
$\theta 2_j = kt_j / kt_0$	1.000	1.050	1.050	1.250	2.057	2.012	2.000
$\theta 3_j = kr_j / kr_0$	1.000	1.000	1.050	1.000	1.208	2.028	2.000
$\left\Vert \frac{\theta_{ref} - \theta_j}{\theta_{ref}} \right\Vert$	0.708	0.690	0.672	0.647	0.397	0.020	
target respones							
f_1	10.389	10.541	10.645	11.103	12.759	13.323	13.319
f_2	20.379	20.758	20.882	22.219	26.670	26.505	26.491
f_3	39.157	39.502	40.124	41.059	46.184	46.330	46.408
f_4	77.023	77.217	78.925	78.567	82.601	84.579	84.882
u_1	0.026	0.025	0.025	0.024	0.020	0.018	0.018
u_2	0.010	0.010	0.010	0.008	0.005	0.005	0.005
ε_1	0.00040	0.00040	0.00038	0.00039	0.00038	0.00036	0.00035
$\left\Vert \frac{z_m - z_j}{z_m} \right\Vert$	1.162	1.064	1.037	0.740	0.134	0.013	0.000

8.4.11 Conclusions

There exists a wide range of algorithm to update a FE model and all managed to reduce the difference in the target responses. It was shown that a better match of the target responses does not necessarily lead to better estimates of the updating parameter. Weighting turned out to be a powerful tool to improve the conditioning of the problem.

9 Appendix B: Model updating of a simplified FE model of the Svinesund Bridge

The methods that performed well on the simply supported beam were applied on a simplified model of the Svinesund Bridge. This should help to determine possible sets of updating parameters and target responses. In addition, it was studied how the estimates of the updating parameters change, when the effect of the actual distribution of the measurements was considered. Finally, it was studied how an inaccurate FE model effects the updating procedures .

9.1 The simplified FE model

The FE model of the bridge, including the element numbers, is shown in Figure 9.1. The model consists of 38 elements and has 216 degrees of freedom.

9.1.1 Coordinate System

The FE model is defined in a Cartesian coordinate system. The x-axis is pointing in the longitudinal direction of the bridge, in approximately northern direction. The y-axis is pointing approximately towards east. The z-axis is pointing downwards. The origin of the coordinate system lies at the midpoint of the arch.

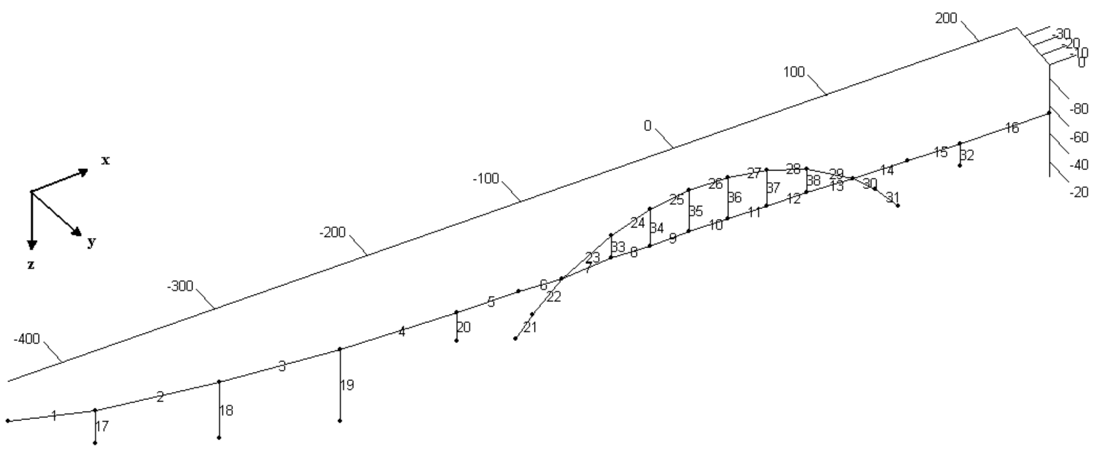

Figure 9.1 Element numbers of the simplified FE model of the Svinesund Bridge

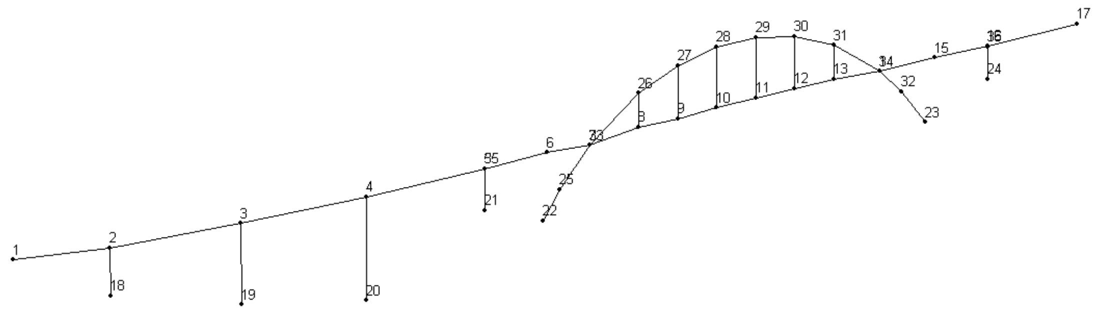

Figure 9.2 Node numbers of the simplified FE model of the Svinesund Bridge

9.1.2 Element type and software

The CALFEM toolbox in MATLAB was used to model the bridge, see Austrell *et al.* (2004). Three dimensional beam elements (ebbeam3e) with 2 nodes and a constant cross-section were chosen. All piles, hangers, carriage-way elements and all arch elements were modelled with the same cross sectional constants, see Table 9.1.

Table 9.1 Cross sectional constants

	Carriage way	Piles	Arch	Hangers
E [GPa]	210.0	37.5	37.5	195.0
G [GPa]	80.70	15.60	15.60	80.70
A [m^2]	0.920	8.120	7.630	0.003
I_y [m^4]	1.17	10.50	9.31	0.01
I_z [m^4]	11.00	40.90	35.00	0.01
K_v [m^4]	2.48	15.00	22.50	0.01
ρ[kg/m^3]	7700	2500	2500	7700

9.1.3 Boundary conditions

At the end abutments, on both sides of the bridge, the carriage way translation were restricted in x-direction and y-direction. All piles were assumed to be fully fixed and all degrees of freedoms were restrained. At the arch supports, the translation in y-direction was assumed to be fixed. The other support conditions of the arch are modelled with springs. The properties of the support springs are shown in Table 9.2.

9.1.4 Internal connections

To be able to study the effect of the stiffness properties of the arch-carriage way connection additional nodes were introduced. One node is part of the carriage way, the other node is part of the arch. Both nodes were then connected with spring elements. The initial properties of the springs are shown in Table 9.2.

The pylon-carriage way connections of the piles closest to the arch are also modelled with spring elements, see Table 9.2. All other connections are assumed to be fully fixed.

Table 9.2 Initial spring stiffness of supports and connections (t=translational; r=rotational)

	arch support	arch-carriage way	pylon-carriage way
kt_x [N*10^9/m]	150.0	93.8	37.5
kt_y [N*10^9/m]	-	562.5	37.5
kt_z [N*10^9/m]	150.0	93.8	37.5
kr_x [Nm*10^9/rad]	3970.0	351.0	0.0
kr_y [Nm*10^9/rad]	908.0	73.5	0.0
kr_z [Nm*10^9/rad]	3970.0	351.0	0.0

9.1.5 Verification of the FE model

To check the overall behaviour of the simplified FE model, the eigenfrequencies and mode shapes were compared with the model developed by Plos and Movaffaghi (2004) and revised by Ülker-Kaustell and Karoumi (2006). As can be seen in Table 9.3 the eigenfrequencies match reasonably well when considering the gross simplifications made. The order of the first four modes was identical.

Table 9.3 Comparison of eigenfrequencies between the simplified model and the model by Ülker-Kaustell and Karoumi (2006)

Mode type	Eigenfrequncies of the simplified FE model [Hz]	Eigenfrequencies of the FE model according to Ülker-Kaustell and Karoumi (2006) [Hz]
1st transversal	0.33	0.40
1st longitudinal	0.76	0.84
2nd transversal	0.95	0.96
3rd transversal	1.17	1.02

The static behaviour of the model was verified by comparing the deflections under load case A with the results obtained from the model developed by Plos and Movaffaghi (2004). The deflections at the midpoint of the arch and at hanger 1 were compared, see Table 9.4. A difference of more than 50% between the models was obtained. This is still in acceptable limits.

The total behaviour of the bridge under the other load cases was checked through the deformed shapes of the FE model.

Table 9.4 Comparison of displacements between the simplified model, the model from Plos and Movaffaghi (2004)

		displacements in z-direction [mm]	
		Simplified model	Model from Plos and Movaffaghi (2004)
arch midpoint	Load case A	8.5	14.8
arch hanger 1	Load case A	36.3	21.1

9.2 Measurements/Target responses

Due to the gross simplifications of the FE model it is not tried to update the model using the actually conducted measurements. Instead, simulated measurements are used. The simulated measurement type and location were chosen to approximately correspond to the actual measurements and loading positions.

Normal forces and moments around the y-axis were assumed to be measured in the southern nodes of elements 22, 26 and 31. The measurements were assumed to be taken under three different load cases. All loads act in z-direction. Load case A corresponds to a force of 1MN in nodes 10 and 11. A load of 2 MN in node 6 was assumed for load case D. For load case E a load of 2 MN in node 8 was applied, see Figure 9.3. That leads to a total of 18 measured cross sectional forces.

In addition, it is assumed, that the first 6 eigenfrequencies were measured. The eigenfrequencies were calculated using a consistent mass matrix.

Finally, the deflections of the bridge in z-direction in nodes 6, 8, 9, 10, 11, 12, 13 and 28 under load cases A, D and E are assumed to be measured, see Figure 9.3. This led to 24 deflection measurements.

In total 48 measurements were assumed to be taken. All measurements were used in the updating procedures.

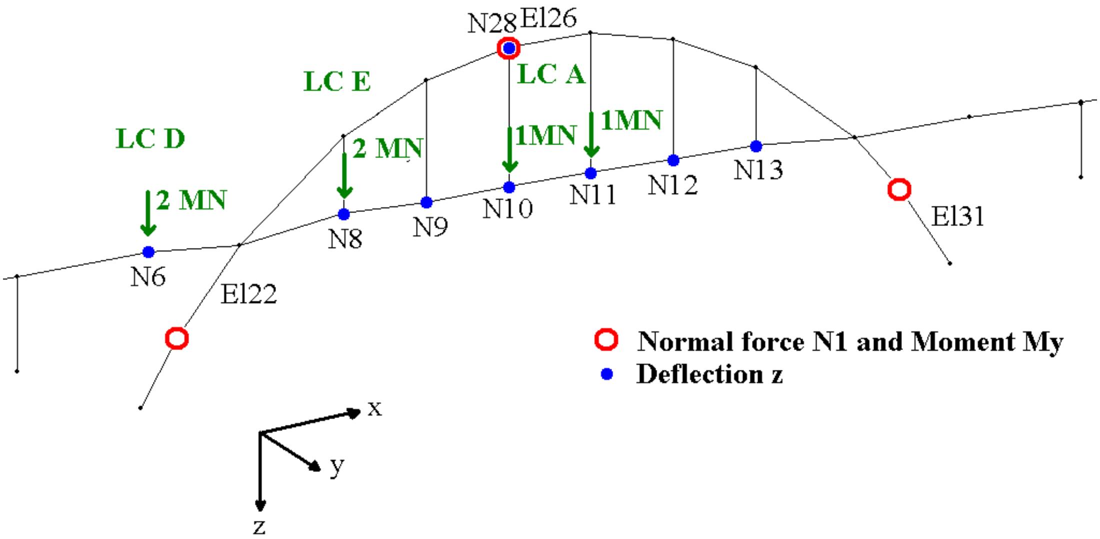

Figure 9.3 Location of measured normal forces, moments and deflections; Location of loads for load cases A, D and E

9.3 Sensitivity study and choice of possible updating parameters

To study the effect of parameter variations a sensitivity study was carried out. This can help to find the right measurements to identify and update the uncertain parameters. In this case, the kind and location of the measurements were already determined by the actually performed measurements. Therefore, the sensitivity study was performed to identify the parameters that could be updated with the already performed measurements. The simplest way of doing this is to plot the normalized sensitivity matrix, see Chapter 5.2 and Chapter 8.4.1. The normalized sensitivity matrix is shown in

Table 9.5.

Each entry of the normalized sensitivity matrix can be calculated according to Equation (9.1) for normalized updating parameters.

$$S_{ij,normalized} = \frac{z_i(\theta_j + h) - z_i(\theta_j)}{z_i(\theta_j)h} \tag{9.1}$$

where

$S_{ij,normalized}$ indicates the dimensionless sensitivity of the *i*th target response z_i with respect to the *j*th updating parameter θ_j

θ_j is the normalized updating parameter

z_i is the *i*th target response

h is the perturbation of the updating parameter that was used to approximate the sensitivity matrix, $h \to 0$ (here 0.01* θ_j was used)

Table 9.5 Normalized sensitivity matrix

					Carriage way							Piles						
				z_m	E	G	A	Iy	Iz	Kv	m	E	G	A	Iy	Iz	Kv	m
				[SI]	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Cross sectinal forces in first node (south) of elements	Loadcase A	N_{22}	-1.41E+06	0.048	0.000	-0.020	0.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2			M_{y22}	-7.23E+05	0.568	0.000	0.167	0.401	0.000	0.000	0.000	-0.007	0.000	0.000	-0.009	0.001	0.000	0.000
3			N_{26}	-2.27E+06	-0.070	0.000	0.016	-0.086	0.000	0.000	0.000	-0.002	0.000	0.000	-0.002	0.000	0.000	0.000
4			M_{y26}	3.69E+06	-0.459	0.000	-0.045	-0.414	0.000	0.000	0.000	-0.010	0.000	0.000	-0.010	0.000	0.000	0.000
5			N_{31}	-1.42E+06	0.051	0.000	-0.016	0.068	0.000	0.000	0.000	-0.011	0.000	0.000	-0.011	0.000	0.000	0.000
6			M_{y31}	-9.24E+05	0.793	0.000	-0.061	0.854	0.000	0.000	0.000	-0.020	0.000	0.000	-0.019	0.000	0.000	0.000
7		Loadcase D	N_{22}	-1.45E+06	0.054	0.000	0.015	0.039	0.000	0.000	0.000	0.021	0.000	0.000	0.021	0.000	0.000	0.000
8			M_{y22}	-1.08E+06	0.086	0.000	-0.101	0.187	0.000	0.000	0.000	-0.221	0.000	0.000	-0.220	-0.001	0.000	0.000
9			N_{26}	1.89E+05	0.103	0.000	0.120	-0.017	0.000	0.000	0.000	0.000	0.000	-0.001	0.001	0.000	0.000	0.000
10			M_{y26}	1.95E+05	-0.677	0.000	0.493	-1.168	0.000	0.000	0.000	-0.225	0.000	-0.001	-0.224	0.000	0.000	0.000
11			N_{31}	-3.79E+04	-0.100	0.000	0.230	-0.329	-0.001	0.000	0.000	-1.012	0.000	0.000	-1.010	-0.002	0.000	0.000
12			M_{y31}	-1.48E+05	0.161	0.000	0.271	-0.111	0.001	0.000	0.000	0.241	0.000	-0.001	0.240	0.002	0.000	0.000
13		Loadcase E	N_{22}	-2.90E+06	-0.008	0.000	0.011	-0.019	0.000	0.000	0.000	0.036	0.000	0.000	0.036	0.000	0.000	0.000
14			M_{y22}	-4.55E+06	-0.169	0.000	-0.043	-0.126	0.000	0.000	0.000	-0.165	0.000	0.000	-0.165	-0.001	0.000	0.000
15			N_{26}	-8.56E+05	-0.045	0.000	-0.021	-0.024	0.000	0.000	0.000	-0.003	0.000	0.000	-0.004	0.000	0.000	0.000
16			M_{y26}	9.53E+05	0.086	0.000	0.070	0.016	0.000	0.000	0.000	-0.151	0.000	0.000	-0.151	0.000	0.000	0.000
17			N_{31}	-3.05E+05	0.110	0.000	-0.022	0.133	0.000	0.000	0.000	-0.407	0.000	0.000	-0.407	-0.001	0.000	0.000
18			M_{y31}	1.57E+06	-0.642	0.000	-0.034	-0.608	0.000	0.000	0.000	-0.072	0.000	0.000	-0.071	-0.001	0.000	0.000
19	Eigenfrequencies [Hz]		1st	0.325	0.087	0.009	-0.048	0.000	0.087	0.009	-0.048	0.004	0.000	0.000	0.000	0.004	0.000	0.000
20			2nd	0.759	0.137	0.000	-0.076	0.119	0.000	0.000	-0.093	0.123	0.000	-0.004	0.122	0.001	0.000	-0.004
21			3rd	0.952	0.121	0.015	-0.070	0.000	0.121	0.015	-0.070	0.009	0.000	-0.001	0.000	0.009	0.000	-0.001
22			4th	1.169	0.166	0.006	-0.355	0.000	0.166	0.006	-0.355	0.001	0.000	0.000	0.000	0.001	0.000	0.000
23			5th	1.464	0.101	0.000	-0.241	0.065	0.002	0.000	-0.275	0.016	0.000	-0.005	0.011	0.006	0.000	-0.005
24			6th	1.531	0.123	0.010	-0.334	0.005	0.117	0.010	-0.335	0.331	0.005	-0.141	0.011	0.319	0.005	-0.141
25	Deflections (in z-direction) [	Loadcase A	Node 6	-0.001	0.921	0.000	-0.572	1.491	-0.001	0.000	0.000	0.111	0.000	-0.001	0.113	-0.002	0.000	0.000
26			Node 8	0.005	0.700	0.000	0.056	0.645	0.000	0.000	0.000	-0.049	0.000	0.000	-0.049	0.001	0.000	0.000
27			Node 9	0.019	0.007	0.000	-0.011	0.017	0.000	0.000	0.000	-0.013	0.000	0.000	-0.013	0.000	0.000	0.000
28			Node 10	0.036	-0.236	0.000	-0.014	-0.222	0.000	0.000	0.000	-0.006	0.000	0.000	-0.006	0.000	0.000	0.000
29			Node 11	0.036	-0.238	0.000	-0.014	-0.224	0.000	0.000	0.000	-0.004	0.000	0.000	-0.004	0.000	0.000	0.000
30			Node 12	0.019	-0.005	0.000	-0.012	0.007	0.000	0.000	0.000	-0.003	0.000	0.000	-0.003	0.000	0.000	0.000
31			Node 13	0.004	0.778	0.000	0.043	0.735	0.000	0.000	0.000	0.015	0.000	0.000	0.015	-0.001	0.000	0.000
32			Node 28	0.008	-0.347	0.000	-0.078	-0.269	0.000	0.000	0.000	-0.022	0.000	0.000	-0.022	0.000	0.000	0.000
33		Loadcase D	Node 6	0.022	-0.801	0.000	-0.010	-0.791	0.000	0.000	0.000	-0.018	0.000	-0.001	-0.016	0.000	0.000	0.000
34			Node 8	-0.004	-0.270	0.000	0.081	-0.351	0.000	0.000	0.000	0.244	0.000	-0.001	0.244	0.001	0.000	0.000
35			Node 9	-0.003	-0.053	0.000	-0.007	-0.047	0.000	0.000	0.000	0.251	0.000	-0.001	0.251	0.001	0.000	0.000
36			Node 10	-0.001	0.381	0.000	-0.246	0.626	0.000	0.000	0.000	0.278	0.000	-0.001	0.279	0.000	0.000	0.000
37			Node 11	0.000	-38.493	-0.004	23.203	-61.666	0.039	-0.004	0.000	12.270	0.001	-0.020	12.177	0.114	0.001	0.000
38			Node 12	0.000	149.853	0.048	-101.969	252.242	-0.460	0.048	0.000	-274.651	-0.017	0.265	-273.572	-1.351	-0.017	0.000
39			Node 13	-0.001	-0.114	0.000	-0.014	-0.097	-0.002	0.000	0.000	-1.112	0.000	0.000	-1.107	-0.004	0.000	0.000
40			Node 28	0.000	-0.867	0.000	-1.734	0.863	0.000	0.000	0.000	1.346	0.000	-0.002	1.346	0.001	0.000	0.000
41		Loadcase E	Node 6	-0.004	-0.270	0.000	0.081	-0.351	0.000	0.000	0.000	0.244	0.000	-0.001	0.244	0.001	0.000	0.000
42			Node 8	0.032	-0.460	0.000	-0.019	-0.441	0.000	0.000	0.000	-0.093	0.000	0.000	-0.092	0.000	0.000	0.000
43			Node 9	0.029	-0.329	0.000	-0.009	-0.321	0.000	0.000	0.000	-0.080	0.000	0.000	-0.080	0.000	0.000	0.000
44			Node 10	0.012	0.029	0.000	0.011	0.018	0.000	0.000	0.000	-0.082	0.000	0.000	-0.082	0.000	0.000	0.000
45			Node 11	-0.003	-2.271	0.000	-0.143	-2.128	-0.001	0.000	0.000	-0.197	0.000	0.000	-0.195	-0.002	0.000	0.000
46			Node 12	-0.011	-0.727	0.000	-0.044	-0.683	0.000	0.000	0.000	-0.180	0.000	0.000	-0.180	-0.001	0.000	0.000
47			Node 13	-0.011	-0.537	0.000	-0.036	-0.501	0.000	0.000	0.000	-0.256	0.000	0.000	-0.255	-0.001	0.000	0.000
48			Node 28	0.006	-0.226	0.000	0.035	-0.261	0.000	0.000	0.000	-0.155	0.000	0.000	-0.155	0.000	0.000	0.000

				z_m	Arch							Tendons						
					E	G	A	Iy	Iz	Kv	m	E	G	A	Iy	Iz	Kv	m
				[SI]	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	Cross sectinal forces in first node (south) of elements	Loadcase A	N22	-1.41E+06	0.019	0.000	-0.010	0.029	0.000	0.000	0.000	-0.068	0.000	-0.072	0.004	0.000	0.000	0.000
2			My22	-7.23E+05	-0.406	0.000	-0.133	-0.272	0.000	0.000	0.000	-0.159	0.000	-0.190	0.031	0.000	0.000	0.000
3			N26	-2.27E+06	-0.011	0.000	0.014	-0.026	0.000	0.000	0.000	0.083	0.000	0.080	0.002	0.000	0.000	0.000
4			My26	3.69E+06	0.291	0.000	-0.062	0.354	0.000	0.000	0.000	0.182	0.000	0.237	-0.055	0.000	0.000	0.000
5			N31	-1.42E+06	0.022	0.000	-0.009	0.031	0.000	0.000	0.000	-0.064	0.000	-0.068	0.004	0.000	0.000	0.000
6			My31	-9.24E+05	-0.125	0.000	-0.189	0.066	0.000	0.000	0.000	-0.699	0.000	-0.762	0.064	0.000	0.000	0.000
7		Loadcase D	N22	-1.45E+06	-0.070	0.000	0.006	-0.076	0.000	0.000	0.000	0.007	0.000	0.005	0.002	0.000	0.000	0.000
8			My22	-1.08E+06	0.274	0.000	-0.038	0.313	0.000	0.000	0.000	0.011	0.000	-0.001	0.012	0.000	0.000	0.000
9			N26	1.89E+05	-0.151	0.000	0.029	-0.180	0.000	0.000	0.000	0.065	0.000	0.066	-0.001	0.000	0.000	0.000
10			My26	1.95E+05	0.050	0.000	0.117	-0.068	0.000	0.000	0.000	0.431	0.000	0.533	-0.101	0.000	0.000	0.000
11			N31	-3.79E+04	0.963	0.000	0.031	0.931	0.000	0.000	0.000	0.116	0.000	0.170	-0.054	0.000	0.000	0.000
12			My31	-1.48E+05	-0.666	0.000	0.067	-0.733	0.000	0.000	0.000	0.191	0.000	0.290	-0.098	0.000	0.000	0.000
13		Loadcase E	N22	-2.90E+06	-0.012	0.000	0.002	-0.014	0.000	0.000	0.000	-0.022	0.000	-0.016	-0.006	0.000	0.000	0.000
14			My22	-4.55E+06	0.338	0.000	-0.024	0.362	0.000	0.000	0.000	-0.012	0.000	0.002	-0.014	0.000	0.000	0.000
15			N26	-8.56E+05	-0.039	0.000	0.010	-0.049	0.000	0.000	0.000	0.091	0.000	0.094	-0.002	0.000	0.000	0.000
16			My26	9.53E+05	0.650	0.000	-0.090	0.740	0.000	0.000	0.000	-0.544	0.000	-0.578	0.033	0.000	0.000	0.000
17			N31	-3.05E+05	0.376	0.000	-0.022	0.398	0.000	0.000	0.000	-0.082	0.000	-0.132	0.050	0.000	0.000	0.000
18			My31	1.57E+06	0.615	0.000	0.045	0.569	0.000	0.000	0.000	0.109	0.000	0.185	-0.076	0.000	0.000	0.000
19	Eigenfrequencies		1st	0.325	0.354	0.037	-0.449	0.000	0.354	0.037	-0.449	0.003	0.000	0.000	0.000	0.003	0.000	0.000
20			2nd	0.759	0.207	0.000	-0.398	0.204	0.000	0.000	-0.401	0.029	0.000	0.010	0.019	0.000	0.000	0.000
21			3rd	0.952	0.302	0.047	-0.426	0.000	0.302	0.047	-0.426	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22			4th	1.169	0.266	0.018	-0.143	0.000	0.266	0.018	-0.143	0.033	0.000	0.000	0.000	0.033	0.000	0.000
23			5th	1.464	0.208	0.000	-0.155	0.141	0.000	0.000	-0.221	0.164	0.000	0.153	0.010	0.000	0.000	0.000
24			6th	1.531	0.022	0.001	-0.021	0.016	0.004	0.001	-0.022	0.005	0.000	0.004	0.001	0.000	0.000	0.000
25	Deflections (in z-direction)	Loadcase A	Node 6	-0.001	0.333	0.000	-0.014	0.346	0.000	0.000	0.000	-2.022	0.000	-2.098	0.076	0.000	0.000	0.000
26			Node 8	0.005	-0.150	0.000	-0.241	0.091	0.000	0.000	0.000	-1.464	0.000	-1.473	0.009	0.000	0.000	0.000
27			Node 9	0.019	-0.125	0.000	-0.100	-0.025	0.000	0.000	0.000	-0.855	0.000	-0.845	-0.011	0.000	0.000	0.000
28			Node 10	0.036	-0.113	0.000	-0.065	-0.049	0.000	0.000	0.000	-0.635	0.000	-0.622	-0.013	0.000	0.000	0.000
29			Node 11	0.036	-0.113	0.000	-0.065	-0.049	0.000	0.000	0.000	-0.634	0.000	-0.622	-0.012	0.000	0.000	0.000
30			Node 12	0.019	-0.126	0.000	-0.101	-0.026	0.000	0.000	0.000	-0.852	0.000	-0.843	-0.009	0.000	0.000	0.000
31			Node 13	0.004	-0.166	0.000	-0.269	0.103	0.000	0.000	0.000	-1.588	0.000	-1.604	0.016	0.000	0.000	0.000
32			Node 28	0.008	-0.696	0.000	-0.293	-0.403	0.000	0.000	0.000	0.082	0.000	0.118	-0.036	0.000	0.000	0.000
33		Loadcase D	Node 6	0.022	-0.108	0.000	-0.008	-0.100	0.000	0.000	0.000	-0.006	0.000	-0.005	-0.001	0.000	0.000	0.000
34			Node 8	-0.004	-0.539	0.000	0.070	-0.608	0.000	0.000	0.000	-0.248	0.000	-0.228	-0.020	0.000	0.000	0.000
35			Node 9	-0.003	-0.598	0.000	0.051	-0.649	0.000	0.000	0.000	-0.459	0.000	-0.438	-0.020	0.000	0.000	0.000
36			Node 10	-0.001	-0.362	0.000	0.025	-0.388	0.000	0.000	0.000	-1.101	0.000	-1.109	0.008	0.000	0.000	0.000
37			Node 11	0.000	-50.434	0.000	2.889	-53.295	0.000	0.000	0.000	65.262	0.000	70.179	-4.916	0.000	0.000	0.000
38			Node 12	0.000	489.809	0.000	-24.500	514.128	-0.001	0.000	0.000	-306.702	0.000	-348.828	42.095	0.000	0.000	0.000
39			Node 13	-0.001	0.771	0.000	-0.028	0.798	0.000	0.000	0.000	-0.411	0.000	-0.496	0.085	0.000	0.000	0.000
40			Node 28	0.000	0.028	0.000	0.016	0.010	0.000	0.000	0.000	-0.649	0.000	-0.601	-0.049	0.000	0.000	0.000
41		Loadcase E	Node 6	-0.004	-0.539	0.000	0.070	-0.608	0.000	0.000	0.000	-0.248	0.000	-0.228	-0.020	0.000	0.000	0.000
42			Node 8	0.032	-0.214	0.000	-0.030	-0.185	0.000	0.000	0.000	-0.212	0.000	-0.195	-0.017	0.000	0.000	0.000
43			Node 9	0.029	-0.238	0.000	-0.040	-0.198	0.000	0.000	0.000	-0.337	0.000	-0.315	-0.021	0.000	0.000	0.000
44			Node 10	0.012	-0.249	0.000	-0.099	-0.150	0.000	0.000	0.000	-0.678	0.000	-0.661	-0.018	0.000	0.000	0.000
45			Node 11	-0.003	-0.589	0.000	0.389	-0.979	0.000	0.000	0.000	2.017	0.000	2.124	-0.107	0.000	0.000	0.000
46			Node 12	-0.011	-0.390	0.000	0.074	-0.464	0.000	0.000	0.000	0.292	0.000	0.346	-0.053	0.000	0.000	0.000
47			Node 13	-0.011	-0.320	0.000	0.041	-0.361	0.000	0.000	0.000	0.108	0.000	0.152	-0.044	0.000	0.000	0.000
48			Node 28	0.006	-0.473	0.000	-0.201	-0.272	0.000	0.000	0.000	-0.119	0.000	-0.076	-0.043	0.000	0.000	0.000

					Connection arch-carriage way south						Connection arch-carriage way north					
				z_m	ktx	kty	ktz	krx	kry	krz	ktx	kty	ktz	krx	kry	krz
				[SI]	29	30	31	32	33	34	35	36	37	38	39	40
1	Cross sectinal forces in first node (south) of elements	Loadcase A	N22	-1.41E+06	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2			My22	-7.23E+05	0.005	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	-0.004	0.000
3			N26	-2.27E+06	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4			My26	3.69E+06	-0.001	0.000	0.000	0.000	-0.003	0.000	-0.001	0.000	0.000	0.000	-0.001	0.000
5			N31	-1.42E+06	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
6			My31	-9.24E+05	-0.001	0.000	0.000	0.000	-0.002	0.000	-0.001	0.000	0.000	0.000	0.074	0.000
7		Loadcase D	N22	-1.45E+06	0.000	0.000	0.000	0.000	-0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8			My22	-1.08E+06	-0.002	0.000	0.000	0.000	-0.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9			N26	1.89E+05	0.002	0.000	0.001	0.000	-0.018	0.000	0.001	0.000	0.000	0.000	0.001	0.000
10			My26	1.95E+05	0.008	0.000	0.002	0.000	0.410	0.000	0.003	0.000	0.000	0.000	-0.002	0.000
11			N31	-3.79E+04	0.002	0.000	0.001	0.000	0.003	0.000	0.004	0.000	0.000	0.000	0.007	0.000
12			My31	-1.48E+05	0.006	0.000	0.002	0.000	0.071	0.000	0.001	0.000	0.000	0.000	0.080	0.000
13		Loadcase E	N22	-2.90E+06	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14			My22	-4.55E+06	-0.001	0.000	0.000	0.000	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15			N26	-8.56E+05	-0.001	0.000	0.000	0.000	-0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16			My26	9.53E+05	0.002	0.000	0.000	0.000	-0.045	0.000	-0.001	0.000	0.000	0.000	-0.002	0.000
17			N31	-3.05E+05	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.004	0.000
18			My31	1.57E+06	-0.001	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000	-0.028	0.000
19	Eigenfrequencies		1st	0.325	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001
20			2nd	0.759	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21			3rd	0.952	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22			4th	1.169	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001
23			5th	1.464	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000
24			6th	1.531	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	Deflections (in z-direction)	Loadcase A	Node 6	-0.001	-0.011	0.000	-0.002	0.000	-0.321	0.000	-0.003	0.000	0.000	0.000	0.006	0.000
26			Node 8	0.005	0.002	0.000	0.000	0.000	-0.019	0.000	-0.001	0.000	0.000	0.000	-0.003	0.000
27			Node 9	0.019	0.000	0.000	0.000	0.000	-0.003	0.000	0.000	0.000	0.000	0.000	-0.001	0.000
28			Node 10	0.036	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
29			Node 11	0.036	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	-0.001	0.000
30			Node 12	0.019	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	-0.003	0.000
31			Node 13	0.004	-0.001	0.000	0.000	0.000	-0.003	0.000	0.002	0.000	0.000	0.000	-0.021	0.000
32			Node 28	0.008	-0.001	0.000	0.000	0.000	-0.003	0.000	-0.001	0.000	0.000	0.000	-0.002	0.000
33		Loadcase D	Node 6	0.022	0.000	0.000	0.000	0.000	-0.056	0.000	0.000	0.000	0.000	0.000	0.000	0.000
34			Node 8	-0.004	0.002	0.000	0.002	0.000	-0.186	0.000	0.000	0.000	0.000	0.000	0.001	0.000
35			Node 9	-0.003	0.000	0.000	0.001	0.000	-0.136	0.000	-0.001	0.000	0.000	0.000	0.001	0.000
36			Node 10	-0.001	-0.004	0.000	0.000	0.000	-0.183	0.000	-0.002	0.000	0.000	0.000	0.003	0.000
37			Node 11	0.000	0.501	0.000	0.109	0.000	9.704	0.000	0.094	0.000	0.001	0.000	-0.283	0.000
38			Node 12	0.000	-2.879	0.000	-0.667	0.000	-56.730	0.000	0.092	0.000	0.040	0.000	3.877	0.000
39			Node 13	-0.001	-0.004	0.000	-0.001	0.000	-0.132	0.000	0.002	0.000	0.000	0.000	0.020	0.000
40			Node 28	0.000	-0.030	0.000	-0.001	0.000	-0.850	0.000	-0.012	0.000	0.000	0.000	0.016	0.000
41		Loadcase E	Node 6	-0.004	0.002	0.000	0.002	0.000	-0.186	0.000	0.000	0.000	0.000	0.000	0.001	0.000
42			Node 8	0.032	0.000	0.000	0.000	0.000	-0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000
43			Node 9	0.029	0.000	0.000	0.000	0.000	-0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000
44			Node 10	0.012	0.001	0.000	0.000	0.000	-0.009	0.000	0.000	0.000	0.000	0.000	-0.001	0.000
45			Node 11	-0.003	-0.004	0.000	0.000	0.000	0.028	0.000	0.001	0.000	0.000	0.000	0.006	0.000
46			Node 12	-0.011	-0.001	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.003	0.000
47			Node 13	-0.011	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.005	0.000
48			Node 28	0.006	0.001	0.000	0.000	0.000	-0.016	0.000	-0.001	0.000	0.000	0.000	-0.002	0.000

					Connection pile-carrige way south			Connection pile-carrige way north			support arch south					support arch north				
				z_m	ktx	kty	ktz	ktx	kty	ktz	ktx	ktz	krx	kry	krz	ktx	ktz	krx	kry	krz
				[SI]	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
1	Cross sectinal forces in first node (south) of elements	Loadcase A	N22	-1.41E+06	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
2			My22	-7.23E+05	0.0003	0.0000	0.0000	-0.0011	0.0000	0.0000	-0.003	0.000	0.000	-0.036	0.000	0.000	0.000	0.000	-0.001	0.000
3			N26	-2.27E+06	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4			My26	3.69E+06	0.0000	0.0000	0.0000	-0.0001	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5			N31	-1.42E+06	0.0000	0.0000	0.0000	-0.0001	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
6			My31	-9.24E+05	-0.0001	0.0000	0.0000	0.0001	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.016	0.000
7		Loadcase D	N22	-1.45E+06	0.0000	0.0000	-0.0001	0.0001	0.0000	0.0000	0.000	0.000	0.000	-0.003	0.000	0.000	0.000	0.000	0.000	0.000
8			My22	-1.08E+06	-0.0002	0.0000	0.0000	-0.0009	0.0000	0.0000	-0.001	0.000	0.000	0.073	0.000	0.000	0.000	0.000	-0.001	0.000
9			N26	1.89E+05	0.0000	0.0000	-0.0003	0.0001	0.0000	0.0000	0.000	0.000	0.000	-0.005	0.000	0.000	0.000	0.000	0.001	0.000
10			My26	1.95E+05	0.0000	0.0000	-0.0003	-0.0017	0.0000	0.0000	0.000	0.000	0.000	-0.008	0.000	0.000	0.000	0.000	0.004	0.000
11			N31	-3.79E+04	-0.0005	0.0000	-0.0001	-0.0059	0.0000	0.0000	0.000	-0.001	0.000	-0.006	0.000	0.001	0.000	0.000	0.033	0.000
12			My31	-1.48E+05	0.0003	0.0000	-0.0004	0.0004	0.0000	0.0000	0.000	-0.001	0.000	-0.003	0.000	0.000	0.000	0.000	-0.096	0.000
13		Loadcase E	N22	-2.90E+06	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
14			My22	-4.55E+06	-0.0002	0.0000	0.0000	-0.0007	0.0000	0.0000	-0.001	0.000	0.000	-0.016	0.000	0.000	0.000	0.000	0.000	0.000
15			N26	-8.56E+05	0.0000	0.0000	0.0000	-0.0001	0.0000	0.0000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	-0.001	0.000
16			My26	9.53E+05	0.0000	0.0000	0.0000	-0.0012	0.0000	0.0000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000
17			N31	-3.05E+05	-0.0002	0.0000	0.0000	-0.0025	0.0000	0.0000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.007	0.000
18			My31	1.57E+06	-0.0001	0.0000	0.0000	-0.0001	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.015	0.000
19	Eigenfrequencies		1st	0.325	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001
20			2nd	0.759	0.00011	0.00000	0.00000	0.00051	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21			3rd	0.952	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001
22			4th	1.169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001
23			5th	1.464	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24			6th	1.531	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	Deflections (in z-direction)	Loadcase A	Node 6	-0.001	-0.0003	0.0000	-0.0003	0.0022	0.0000	0.0000	0.005	0.008	0.000	-0.012	0.000	0.001	0.000	0.000	0.003	0.000
26			Node 8	0.005	0.0001	0.0000	0.0000	-0.0008	0.0000	0.0000	-0.002	-0.001	0.000	-0.001	0.000	0.000	0.000	0.000	-0.001	0.000
27			Node 9	0.019	0.0000	0.0000	0.0000	-0.0002	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28			Node 10	0.036	0.0000	0.0000	0.0000	-0.0001	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
29			Node 11	0.036	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30			Node 12	0.019	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
31			Node 13	0.004	-0.0002	0.0000	0.0000	0.0007	0.0000	0.0000	0.000	0.000	0.000	-0.001	0.000	-0.002	-0.002	0.000	-0.001	0.000
32			Node 28	0.008	0.0000	0.0000	0.0000	-0.0002	0.0000	0.0000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
33		Loadcase D	Node 6	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000
34			Node 8	####	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.002	0.000	-0.005	0.000	0.000	0.000	0.000	0.001	0.000
35			Node 9	####	0.000	0.000	0.000	0.001	0.000	0.000	0.002	0.002	0.000	-0.005	0.000	0.000	0.000	0.000	0.001	0.000
36			Node 10	-0.001	0.000	0.000	0.000	0.002	0.000	0.000	0.003	0.005	0.000	-0.008	0.000	0.000	0.000	0.000	0.001	0.000
37			Node 11	0.000	0.023	0.000	-0.005	-0.001	0.000	0.000	-0.123	-0.244	0.000	0.267	0.000	-0.025	0.006	0.000	-0.156	0.000
38			Node 12	0.000	-0.272	-0.002	0.068	-0.991	0.000	0.000	0.420	0.943	0.000	-1.823	0.000	0.242	-0.056	0.000	1.709	0.000
39			Node 13	-0.001	-0.001	0.000	0.000	-0.005	0.000	0.000	0.001	0.001	0.000	-0.009	0.000	0.001	0.000	0.000	0.005	0.000
40			Node 28	0.000	0.000	0.000	-0.001	0.010	0.000	0.000	0.013	0.024	0.000	-0.020	0.000	0.001	0.000	0.000	0.000	0.000
41		Loadcase E	Node 6	####	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.002	0.000	-0.005	0.000	0.000	0.000	0.000	0.001	0.000
42			Node 8	0.032	0.000	0.000	0.000	-0.001	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
43			Node 9	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
44			Node 10	0.012	0.000	0.000	0.000	-0.001	0.000	0.000	-0.001	-0.001	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000
45			Node 11	####	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000
46			Node 12	-0.011	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
47			Node 13	-0.011	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000
48			Node 28	0.006	0.000	0.000	0.000	-0.001	0.000	0.000	-0.001	-0.001	0.000	-0.001	0.000	0.000	0.000	0.000	0.000	0.000

Problems can occur when the value of $z_i(\theta_j)$ is close to zero, see rows 37, 38 and 40. In these cases the entries of the sensitivity matrix become very high, indicating sensitive parameters that are well suited to update the model. In fact, the response is close to zero, leading to numerical and measurement problems. A target response must not only show a significant relative change due to the updating parameters but also show an absolute change that is measurable and does not cause any numerical problems.

Almost all objective functions used in dynamics are based on the relative error in the target responses. When including target responses that show almost no response, they have to be waited differently as the measurement error and the numerical problems increased. A simpler way to avoid getting high contributions from measurements that show a very small response is to exclude them completely from the objective function.

9.3.1 Updating parameters of the carriage way

The entries of the normalized sensitivity matrix representing the carriage way parameters (columns 1-7) indicate that it should be possible to update most of the parameters using the simulated measurements. It can be seen that the shear modulus, *G*, and the Saint-Venant's torsion, constant, K_v, and the moment of inertia around the z-axis, I_z, do not show a significant sensibility. Therefore, they will be excluded from the updating parameters. The mass of the carriage way does only influence the eigenfrequencies. This was expected as the self weight was not considered in the other analysis.

9.3.2 Updating parameters of the piles

The parameters of the piles (columns 8-14) are generally less sensitive than the parameters of the carriage way. Again, the shear modulus, *G*, does not show any significant sensitivity and will therefore be excluded from the updating parameters. The same holds for the Saint-Venant's torsion constant, K_v, of the piles. The mass of the piles is only sensitive to the sixth eigenfrequency and will therefore also be neglected as a possible updating parameter. The area, *A*, of the piles is relative insensitive and it is pretty well known. Therefore, it will also be excluded from the updating parameters. The moment of inertia around the z-axes, I_z, is insensitive in the simplified model. It will also not be considered as an updating parameter. The modulus of elasticity, E, and the moment around the local y-axis, I_y, remain as updating parameters.

9.3.3 Updating parameters of the arch

The shear modulus of the arch, *G*, and the moment of inertia around the z-axis are not sensitive to the chosen measurements. All other parameters are sensitive and can be included as updating parameters.

9.3.4 Updating parameters of the hangers

The shear modulus, *G*, the moment of inertia around the y and z-axis, I_y and I_z, the Saint-Venant's torsion constant, K_v, and the mass, *m*, will be excluded as possible updating parameters. The modulus of elasticity, *E*, and the area, *A*, of the hangers show an effect on the measured properties. At the same time these properties are very well known. Therefore, no parameters of the hangers are chosen as updating parameters.

9.3.5 Updating parameters of the arch-carriage way connection

The connection between the arch and the carriage way has been identified as one of the key sections of the James and Karoumi (2003). This connection is critical for the lateral stability of the arch. The performed measurement program was intended to give more information about this part of the structure.

For the simplified model, only the rotational stiffness around the global y-axis, k_{ry}, at the south side of the bridge showed any considerable sensitivity. Based on the sensitivity study the translational spring stiffness in all directions, k_{tx}, k_{ty} and k_{tz}, are excluded from possible updating parameters. The rotational stiffness around the global x and z-axis, k_{rx} and k_{rz}, will also be neglected. The connection between the arch and the carriage way was designed to be symmetric both sides of the bridge. In addition the sensitivity is low. Therefore it seams reasonable to group the parameters on the north and south side. That means that the same values of spring properties are assigned to the north and south side. Consequently, the number of updating parameters can be reduced and their sensitivity increased.

9.3.6 Updating parameters of the pile-carriage way connection

Based on Ülker-Kaustell and Karoumi (2006) a very high initially stiffness was assumed for the support. They reported that a better agreement between the second eigenfrequency of the theoretical model and the measurements was obtained when the translation was restrained in all directions in the theoretical model. It seems reasonable that the low excitation, used in the ambient vibration tests, did not manage to overcome the static friction. Therefore, the assumption of a fixed connection instead of a connection that is free to move holds for the ambient vibration tests. To check if this behaviour also holds for the load tests the normal force in the top of the pile under self weight (in the designer's model) was multiplied with the coefficient of static friction of the bearing. The result was then compared with the horizontal force obtained by the simplified model at the top of the pile under the different load cases. For load case A and D the obtained force did not manage to overcome the static friction. For load case E the forces were in the same range and it is difficult to predict any behaviour of the bearing. These findings highlight that it is not directly possible to use a model that was calibrated with diagnostic load tests in the ultimate limit state without including the effect of higher loads.

The pile-carriage way connection of the simplified model almost does not show any sensibility. The entries of the sensibility for the second eigenfrequency are 0.00011 and 0.00051 for the translational stiffness in x-direction of the south and north side respectively. As the uncertainty of this parameter is very high (anything between fully fixed $k_{tx} \rightarrow \infty$ and free to move $k_{tx} = 0$) it is kept as a possible updating parameter. As the translational stiffness in y and z-direction is insensitive it is excluded from the updating parameters.

9.3.7 Updating parameters of the arch support

The sensitivity of the parameters that characterize the support of the arch is low. The rotational stiffness around the y-axis, k_{ry}, shows the highest sensitivity. All other properties will not be used as updating parameters.

9.3.8 Conclusions from the sensitivity study

From the 56 updating parameters 17 remain. Two of these 17 can be grouped so that a total of 16 parameters have to be updated in the simplified model.

The information gained here can not be directly applied to the more advanced model that will be updated with the real performed measurements. In the simplified model all forces were applied along the centreline of the arch. Therefore, no moments around the global x-axis appeared. The sensitivity of all parameters that predominantly influence the behaviour of the bridge around the x-axis should be checked again using the more advanced model.

The number of updating parameters was kept limited by assuming constant properties of each structural member group. The cross sectional constants are assumed to be the same along the arch, along the carriage way, for all piles and hangers. Gentile (2006), for example, partitioned a concrete arch bridge in different sections. He than updated the modulus of elasticity of different concrete sections along the arch. This procedure can be used to detect defects.

To normalize the sensitivity matrix usually the measured values are used. The normalized sensitivity matrix can contain very high values when the normalization parameter is close to zero. A very high weight is assigned to this target response when the measured value is close to zero. To avoid this, it is recommended to normalize or weight the sensitivity matrix with the values that has the absolute maximum of the measured or calculated values.

9.4 Updating without noise

To simulate the measurements a measurement vector is calculated using the model described in Chapter 9.1. Than the updating parameter are perturbed. Using the updating algorithm it is tried to find the parameters that were used to compute the "measurements".

9.4.1 Perturbation of the updating parameters

In Table 9.6 the parameters that were used to calculate the measurement vector and they perturbed values that are then updated are shown. The perturbation is chosen to give a realistic variation of the parameters.

Table 9.6 Perturbation of cross sectional constants

	Carriage way			Piles			Arch		
	”measured”	factor	perturbed	”measured”	factor	perturbed	”measured”	factor	perturbed
E [GPa]	210	1.08	195.3	37.5	1.11	33.75	37.5	0.91	41.25
G [GPa]	80.70			15.60			15.60		
A [m^2]	0.92	1.11	0.828	8.120			7.63	1.03	7.4011
Iy [m^4]	1.17	1.25	0.948	10.50	0.95	11.025	9.31	1.11	8.379
Iz [m^4]	11.00		11.00	40.90			35.00	1.18	29.75
Kv [m^4]	2.48			15.00			22.50		
[kg/m^3]	7700	1.18	6545	2500			2500	0.91	2750

Table 9.7 Perturbation of stiffness for supports and connections

	”measured”	factor	perturbed
arch-carriage way kr_y [Nm*10^9/rad]	73.5	0.5	147.0
pile-carryage way south kt_x [N*10^9/m]	37.5	0.1	375.0
pile-carryage way north kt_x [N*10^9/m]	37.5	10	3.8
support arch south kr_y [Nm*10^9/rad]	908.0	1.25	681.0
support arch north kr_y [Nm*10^9/rad]	908.0	0.5	1816

9.4.2 First attempt – 16 updating parameters

The pseudo inverse algorithm with weighting of the target responses was applied to update all perturbed parameters. All 48 target responses were used. To calculate the weighting matrix the maximum of the absolute values of the measured parameter and the calculated value was chosen. The weighting matrix was than calculated by the reciprocal of the square root of that value, see Equation (7.2).

$$W_z = diag\left[\frac{1}{(z_1)^2}, \frac{1}{(z_2)^2}, \frac{1}{(z_3)^2}, \frac{1}{(z_4)^2}, \ldots\right] \tag{9.2}$$

where

z_i is the maximum of the absolute values of the *i*th measured and calculated target response; $z_i = \max\left\{\begin{matrix} abs(z_{i,measured}) \\ abs(z_{i,calcualted}) \end{matrix}\right\}$

The normalized updating parameters were then calculated according to equation (5.9).

The updating algorithm did not converge. Especially, the values for the translational spring stiffness of the pile-carriage way connection diverged fast. That shows that it is not possible to update the chosen updating parameters with the available target responses and optimization algorithm.

9.4.3 Second attempt – 14 updating parameters

The translational spring stiffness of the pile-carriage way connection was excluded from the updating procedure. The initial values (without perturbation) were assigned to these properties. Now 14 updating parameters remained. Beside that, the same updating algorithm as in Chapter 9.4.2 was chosen.

As before the updating algorithm did not converge. Especially, the updating parameters of the arch diverged.

9.4.4 Third attempt – 13 updating parameters

9.4.4.1 Weighting of the target responses

It was decided to exclude the rotational stiffness of the arch around the local z-axis, I_z. The algorithm described in section 8.4.4 was applied. The same variance was assumed for target responses. The weighting matrix was calculated according to Equation (7.2). Without the rotational stiffness of the arch around the z-axis as an updating parameter the algorithm converged. In Figure 9.4 the development of the relative error of the target responses and the relative error of the updating parameters is shown. A small number of iterations are needed to reduce the error in the target responses and to find the right updating parameters.

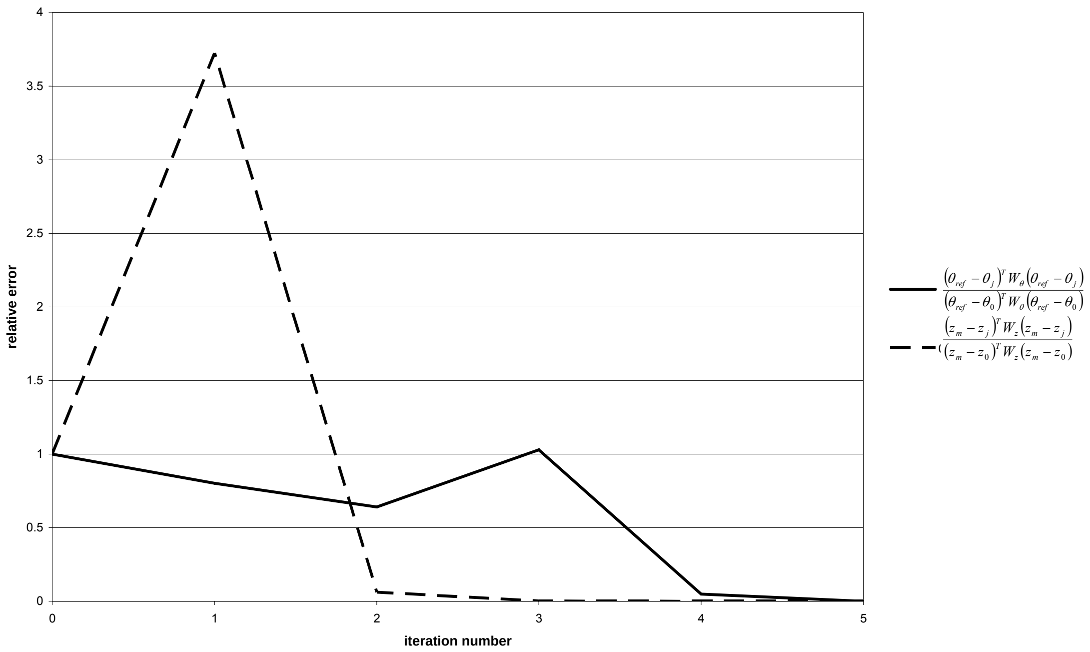

Figure 9.4 Development of the relative error of the target responses and the relative error of the updating parameters (weighting of the target responses)

9.4.4.2 Weighting of the target responses and the total updating parameter change

The algorithm of the previous Chapter is now changed into an algorithm that takes now the total updating parameter change into account. The updating parameters are calculated according to equation (5.12).

The coefficient of variance of each updating parameter is chosen according to Table 9.8. The assumed variance takes also the simplifications of the FE model into account (railings, asphalt layer, stiffeners,…).

A more realistic variation of the target responses is also introduced. In the performed measurement program strains have been measured. As the strains depend on the modulus of elasticity of the concrete their variations are closely related. According to JCSS (2001) the modulus of elasticity of concrete has a coefficient of variation of 15% between different structures. Some additional variation is introduced by the location of the sensors, the sensor accuracy and other factors. A coefficient of variation of 20% is assumed for the cross sectional forces. The eigenfrequencies are assumed to have a coefficient of variation of 1% and the deflection of 10%.

In addition, a factor, *r*, is introduced that allows weighting the confidence of the updating parameters relative to the target responses. The weighting matrix of the target responses and the updating matrix are calculated according to equations (7.3) and (7.4) respectively. To express an equal confidence in the target responses and updating parameters the factor *r=0.5* is chosen.

$$W_z = (1-r) * diag\left[\frac{1}{(V_{z1}z_1)^2}, \frac{1}{(V_{z2}z_2)^2}, \frac{1}{(V_{z3}z_3)^2}, \frac{1}{(V_{z4}z_4)^2}, ...\right] \quad (9.3\ )$$

where

z_i is the maximum of the absolute values of the ith measured and calculated target response; $z_i = \max\begin{Bmatrix} abs(z_{i,measured}) \\ abs(z_{i,calcualted}) \end{Bmatrix}$

$W_\theta = diag\left[\frac{1}{(V_1\theta_1)^2}, \frac{1}{(V_2\theta_2)^2}, \frac{1}{(V_3\theta_3)^2}\right]$ as normalized updating parameter are used it turn into $W_e = r * diag\left[\frac{1}{V_{\theta 1}^2}, \frac{1}{V_{\theta 2}^2}, \frac{1}{V_{\theta 3}^2}, ...\right]$ (9.4)

Table 9.8 Assumed coefficient of variation for updating parameters

		coefficient of variation [%]
carriage way	E	5
	A	10
	I_y	20
	ρ	20
pylons	E	10
	I_y	5
arch	E	10
	A	10
	I_y	10
	ρ	10
arch-carriage way	kr_y	500
support arch south	kr_y	200
support arch north	kr_y	200

The development of the error of the target responses and the updating parameters is shown in Figure 9.5. It can be seen that the error of the updating parameters does not vanish completely. A big part of the residual error is caused by the support stiffness of arch around the global y-axis. This value is not update very precisely. The correct normalized parameter is $\theta_{ref} = 0.83$. The updated value after 5 iterations is $\theta_5 = 0.98$.

The sensitivity of that updating parameter is very low. As the total updating parameter change is considered the value remains pretty unchanged.

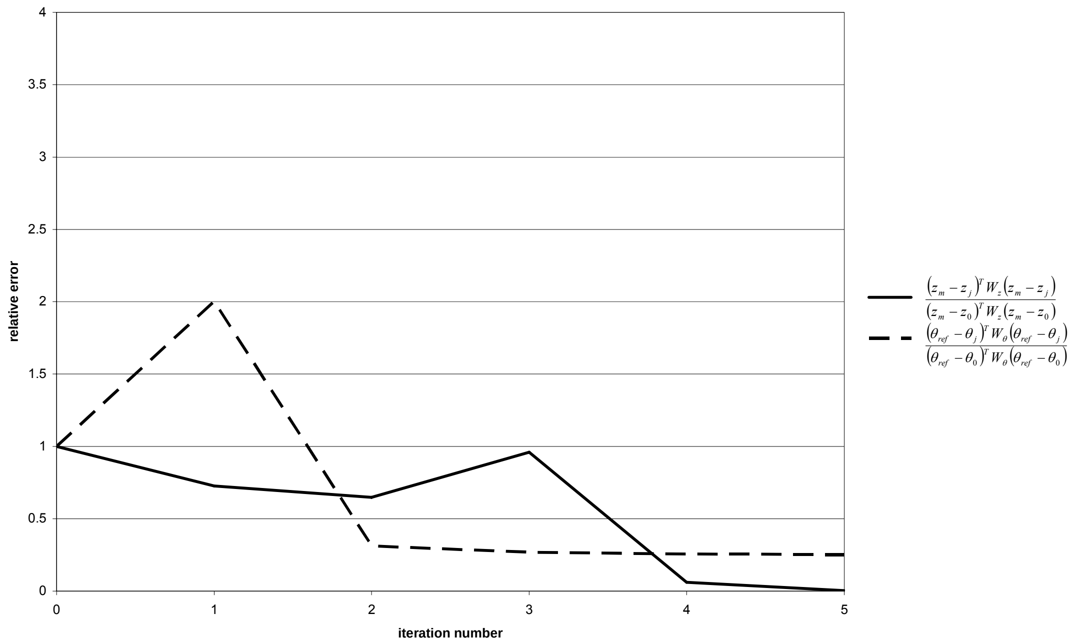

Figure 9.5 Development of the relative error of the target responses and the relative error of the updating parameters (weighting of the target responses and the total updating parameter change)

9.4.4.3 Minimum variance method

The minimum variance method did not converge to the right updating parameters. Only the value for the rotational support stiffness of the arch around the global y-axis at the northern support was upated.

9.4.4.4 Weighting of the target responses – Simplex Method

The Nelder-Mead-Simplex-Method is used to minimize the objective function. The objective function is formulated as the weighted least square of the target responses, see equation 7.6. The weighting matrix was calculated according to equation (7.2).

$$J = (z_m - z)^T W_z (z_m - z) \qquad (9.5)$$

The objective function has the same form as in Chapter 9.4.5.1 but the optimization algorithm is different. The results are summarized in Figure 9.6. Observe that the iteration numbers are different form the previous figures.

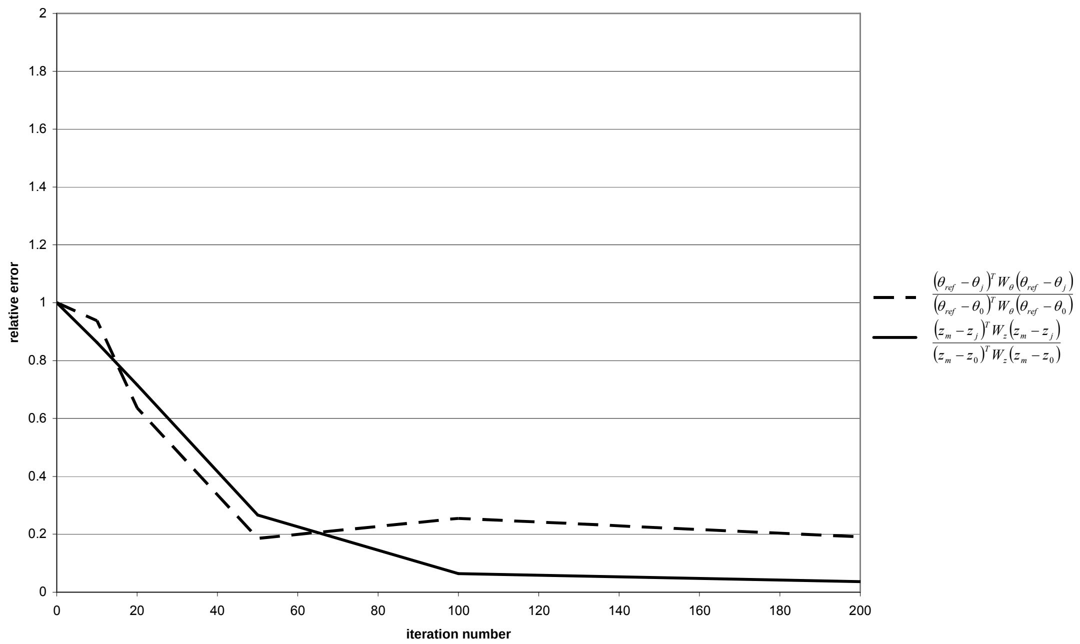

Figure 9.6 Development of the relative error of the target responses and the relative error of the updating parameters (weighting of the target responses – non gradient)

9.4.4.5 Weighting of the target responses - Evolutionary Algorithm

A evolutionary algorithm is used to update the model. An evolutionary algorithm is based on the principles of biological evolution, i.e. natural selection, reproduction, mutation, and survival of the fittest.

1) At first an initial population of N_{pop} chromosomes is generated. That is done by multiplying the initial guesses of the updating parameter with a normal distributed random signal. Each chromosome consists of a vector with all 13 pertubated updated parameters. That leads to a matrix of the size $N_{pop} \times 13$.

2) In the next step the value of the objective function is calculated for all member of the population. The objective function according to Equation 9.5 was chosen.

3) Then the mates are collected. In the here used optimization algorithm the member of the population with the lowest value of the objective function is mated with all other members of the population. Each updating parameter is than chosen with a higher probability from the best chromosome than from the other mate. With a low probability a mutation is performed. That means that the updating parameter neither chosen from the best of the population nor chosen from the other made. Instead, a completely new value of the updating parameter is chosen according to the initially defined gauss distribution. The mating leads to a new generation of genes that are hopefully better than the initial generation.

4) This algorithm continues till the convergence check is passed or the maximum of possible generations is reached.

In Figure 9.7 the results of the application of the evolutionary algorithm to the updating problem is shown. It can be seen that the evolutionary algorithm has an extremely slow convergence rate. Therefore, an impractical number of evaluations of the objective function is needed. It becomes obvious that the chosen evolutionary algorithm has especially problems with the fine tuning. More information about genetic algorithms can be found in Haupt and Haupt (2004).

Figure 9.7 Obtained results when applying an evolutionary algorithm for sizes of the population and different numbers of generations

Size of population Number of generations			5 5	10 10	20 10	10 20	200 10	 measured
carriage way	E	1	1.01	0.99	0.98	0.99	0.98	1.08
	A	1	1.15	0.99	1.04	1.12	1.22	1.11
	Iy	1	1.10	1.34	1.47	1.32	1.66	1.25
		1	1.08	1.10	1.21	1.01	1.04	1.18
piles	E	1	0.99	0.88	1.10	1.01	1.09	1.11
	Iy	1	1.04	1.01	1.01	1.11	1.00	0.95
arch	E	1	1.29	0.98	1.00	0.99	1.05	0.91
	A	1	1.13	1.05	1.00	1.01	0.89	1.03
	Iy	1	1.03	1.14	1.01	1.19	0.83	1.11
		1	0.99	0.97	1.03	1.00	1.10	0.91
arch-carriage way	kry	1	1.00	10.00	1.67	1.44	0.99	0.5
support arch south	kry	1	0.53	0.89	1.05	1.05	1.00	1.25
support arch north	kry	1	1.16	1.27	2.01	0.87	1.00	0.83
$\frac{(\theta_{ref}-\theta_5)^T W_\theta (\theta_{ref}-\theta_5)}{(\theta_{ref}-\theta_0)^T W_\theta (\theta_{ref}-\theta_0)}$		1.00	2.14	1.48	0.98	1.51	2.28	
$\frac{(z_m-z_j)^T W_z (z_m-z_j)}{(z_m-z_0)^T W_z (z_m-z_0)}$		1.00	0.83	0.61	0.37	0.41	0.37	

9.4.5 Probabilistic measurements

In the following, the measurements are assumed to be probabilistic variables. That accounts for the inaccuracies that arise during the measurements. In addition, the calculated target responses can also be inaccurate due to discretization errors in the FE model. The probabilistic values are simulated with a normally distributed random signal. Each entry of the measurement vector, z_m, is multiplied with this distributed random signal. The same coefficients of variations are assumed as in Chapter 9.4.4.2. The standard deviations, σ, are calculated according to equation 9.6.

$$\sigma = \mu V \tag{9.6}$$

where

σ is the standard deviation of the target responses

μ is the mean value of the target responses (is assumed to be equal to z_m)

V is the coefficient of variation of the target responses according to 9.4.4.2 (V=0.01 for eigenfrequencies; V=0.2 for cross sectional forces, V=0.1 for deflections)

9.4.5.1 Weighting of the target responses

The same algorithm as in Chapter 9.4.4.1 was chosen. For the chosen distribution, eight out of ten analysis lead to completely meaningless results. Therefore, the noise was successively decreased till the algorithm converged again. In Table 9.9 the results of the updating procedure with 10%, 20% and 30% of the initial assumed noise are presented. The mean values, μ, and the variances, V, of the normalized updating parameters are shown. The presented values are based on 10 completed updating attempts after the fifth iteration. For each attempt a different distribution of the measurements was assumed. More than 10 updating attempts were needed as some analysis were aborted due to convergence problems, see row 2. In the last row of the table, the mean development of the error norm of the updating parameters is presented. For 10% of the initially assumed noise, the error norm decreased after 5 iterations to 2% percent of the initial error norm. For 20% percent of the initially assumed noise level the error norm decreases to 20% percent of the initial value. For 30% of the initially assumed noise level, the error of the updating parameters is decreased only slightly to 86% of the initial value. That is mainly caused by the wrong updated parameters of the arch support stiffness. The increasing coefficients of variations of some normalized updating parameters indicate that the updated parameters lose their reliability.

Table 9.9 Obtained mean values and variances for normalized updating parameters for noise level reduction factors 0.1, 0.2 and 0.3 for the updating procedure that weights the target responses.

Noise level reduction factor			0.1		0.2		0.3	
Needed attempts to get 10 analysis that were not aborted			11		17		19	
		correct normalized updating parameters	mean value of normalized updating parameters, μ	Coefficient of variation of updating parameters, V [%]	mean value of normalized updating parameters, μ	Coefficient of variation of updating parameters V [%]	mean value of normalized updating parameters, μ	Coefficient of variation of updating parameters, V [%]
carriage way	E	1.08	1.07	1.22	1.10	4.20	1.13	8.55
	A	1.11	1.12	1.78	1.08	4.97	1.06	10.09
	I_y	1.25	1.26	1.11	1.23	4.04	1.20	8.12
	ρ	1.18	1.17	1.86	1.22	5.75	1.26	12.04
piles	E	1.11	1.11	0.35	1.11	1.27	1.09	2.35
	I_y	0.95	0.95	0.73	0.95	1.36	0.97	3.54
arch	E	0.91	0.91	0.71	0.90	1.52	0.90	4.58
	A	1.03	1.03	1.45	1.05	3.48	1.06	7.51
	I_y	1.11	1.11	0.50	1.12	1.64	1.13	3.64
	ρ	0.91	0.91	1.69	0.88	3.41	0.89	8.24
arch-carriage way	kr_y	0.50	0.50	2.67	0.51	5.79	0.51	7.57
support arch south	kr_y	1.25	1.29	21.39	1.56	72.88	1.55	87.48
support arch north	kr_y	0.83	0.54	12.71	0.69	58.50	1.32	210.46
$\frac{(\theta_{ref}-\theta_5)^T W_\theta (\theta_{ref}-\theta_5)}{(\theta_{ref}-\theta_0)^T W_\theta (\theta_{ref}-\theta_0)}$			0.02		0.20		0.86	

9.4.5.2 Weighting of the target responses and weighing of the total updating parameter change

The same algorithm as in Chapter 9.4.4.2 was applied with probabilistic measurements. Several attempts with each noise reduction factor were performed till 10 converged analyses were completed. For the initially assumed noise (noise level reduction factor of 1.0) 17 analyses were submitted to obtain 10 converged analyses, see Table 9.10. Only the arch support stiffness was for some analysis assigned to very unrealistic values. Some mean values of the updating parameters tend towards the correct normalized updating parameters. For some updating parameters the error is increased by the updating procedure. The high coefficients of variation between the different analyses indicate that the obtained updating parameters are not very reliable.

It can be concluded that the updating procedure with the chosen target responses and updating parameters reaches its limits for the assumed measurement accuracy.

In Ülker-Kaustell and Karoumi (2006) it was reported that the measured eigenfreqencies changed with more than 1% depending on the outside temperatures. This leads to a variation of the eigenfrequencies that causes unreliable estimates of the updating parameters. The measured strains in the arch depend on the modulus of elasticity of the concrete. The modulus of elasticity between different structures has a coefficient of variation of V=15% according to JCSS (2001). At least the same variation has to be assumed for the measured strains. That leads to an accuracy of the measured cross sectional forces (measured by strain sensors) which is below the required accuracy that is needed to obtain improved estimates of the updating parameters. The accuracy of the measured deflections is difficult to estimate. The calculated deflections, especially along the carriage way, are probably not very accurate due to the poor representation of the carriage way in the FE model.

To compare this updating procedure with the updating procedure of the previous chapter the results of the same noise level reduction factor are presented in Table 9.10. It can be seen that the algorithm of this chapter is more stable. Fewer attempts are needed to get 10 runs that converged. In addition, it was previously reported that this algorithm also converged for a noise level reduction factor of 1.0. The quality of the obtained updating parameters is not as good as in the previous chapter. The obtained variances are also higher.

Table 9.10 Obtained mean values and variances for normalized updating parameters for noise level reduction factors 0.1, 0.2, 0.3 and 1.0 for the updating procedure that weights the target responses and the total updating parameter change.

Noise level reduction factor			0.1		0.2		0.3		1.0	
Needed attempts to get 10 analysis that were not aborted			10		12		13		17	
		correct normalized updating parameters	mean value of normalized updating parameters, μ	variances of normalized updating parameters, V [%]	mean value of normalized updating parameter, μ	variances of normalized updating parameters, V [%]	mean value of normalized updating parameters, μ	variances of normalized updating parameters, V [%]	mean value of normalize d updating parameter, μ	variances normaliz updatin paramete V [%]
carriage way	E	1.08	1.03	0.00	1.03	0.00	1.03	0.00	1.02	0
	A	1.11	1.10	0.00	1.10	0.01	1.10	0.01	1.08	0
	I_y	1.25	1.29	0.00	1.29	0.01	1.28	0.01	1.28	0
	ρ	1.18	1.17	0.00	1.16	0.01	1.16	0.02	1.18	0
piles	E	1.11	1.09	0.00	1.08	0.02	1.09	0.01	1.09	0
	I_y	0.95	0.99	0.00	0.99	0.01	0.99	0.00	0.98	0
arch	E	0.91	0.94	0.00	0.94	0.01	0.94	0.01	0.96	0
	A	1.03	0.97	0.00	0.97	0.00	0.97	0.00	0.98	0
	I_y	1.11	1.09	0.00	1.08	0.02	1.09	0.01	1.08	0
	ρ	0.91	0.98	0.00	0.98	0.01	0.98	0.01	0.98	0
arch-carriage way	kr_y	0.50	0.56	0.01	0.55	0.15	0.55	0.08	0.55	2
support arch south	kr_y	1.25	1.07	0.73	1.13	2.83	1.12	4.57	1.27	15
support arch north	kr_y	0.83	0.99	0.00	0.99	0.01	0.99	0.02	1.00	0
$\frac{(\theta_{ref}-\theta_5)^T W_\theta (\theta_{ref}-\theta_5)}{(\theta_{ref}-\theta_0)^T W_\theta (\theta_{ref}-\theta_0)}$			0.252		0.27		0.26		0.43	

9.4.5.3 Minimum variance method

The minimum variance method is applied to the same set of updating parameters as in the two previous chapters. The updating algorithm failed to converge.

9.4.5.4 Weighting of the target responses – non gradient method

The Nelder-Simplex-Method is used to minimize the objective function. The objective function is formulated as the weighted least square of the target responses, see equation 9.6. The weighting matrix was calculated according to equation (7.2).

$$J = (z_m - z)^T W_z (z_m - z) \tag{9.7}$$

The objective function has the same form as in Chapter 9.4.5.1 but the optimization algorithm is different.

Noise level reduction factor			0.1		0.2		0.3		1.0	
Needed attempts to get 10 analysis that were not aborted			10		10		10		10	
		correct normalized updating parameters	mean value of normalized updating parameters, μ	variances of normalized updating parameters, V [%]	mean value of normalized updating parameter, μ	variances of normalized updating parameters, V [%]	mean value of normalized updating parameters, μ	variances of normalized updating parameters, V [%]	mean value of normalize d updating parameter, μ	variance normaliz updatin paramete V [%
carriage way	E	1.08	1.02	0.57	0.97	0.14	1.05	0.49	1.03	0.79
	A	1.11	0.99	0.04	1.34	0.32	1.01	0.27	1.01	0.71
	I_y	1.25	1.34	0.33	1.24	0.07	1.30	0.46	1.30	0.86
	ρ	1.18	1.26	0.05	1.06	0.06	1.22	0.07	1.23	0.58
piles	E	1.11	1.06	0.06	1.02	0.06	1.06	0.12	1.05	0.17
	I_y	0.95	1.01	0.05	0.84	0.03	1.03	0.45	1.08	0.63
arch	E	0.91	0.84	0.04	0.92	0.06	0.85	0.04	0.85	0.06
	A	1.03	0.91	0.03	1.12	0.14	0.93	0.04	0.90	0.13
	I_y	1.11	1.10	0.30	0.96	0.03	1.10	0.10	1.14	0.29
	ρ	0.91	0.95	0.06	0.56	0.12	0.96	0.02	1.00	0.38
arch-carriage way	kr_y	0.50	0.58	0.22	0.97	0.07	0.57	0.64	0.63	2.06
support arch south	kr_y	1.25	0.96	0.17	1.06	0.68	0.97	0.12	0.90	2.74
support arch north	kr_y	0.83	1.02	0.26	13.08	1680.49	1.02	0.43	0.95	0.83
$\frac{(\theta_{ref} - \theta_5)^T W_\theta (\theta_{ref} - \theta_5)}{(\theta_{ref} - \theta_0)^T W_\theta (\theta_{ref} - \theta_0)}$					0.89		9.83		0.93	

10 Appendix C: FE model updating of the Svinesund Bridge

The most important results of the FE model updating of the Svinesund Bridge are summarized in Schlune *et al.* (2008a) and Schlune *et al.* (2008b). A detailed description of the manual model refinement steps can be found in Jonsson and Johnson (2007). Here, only some additional information can be found in the following, since it was too space-demanding to be included in the papers

10.1 Model evolution

All responses that have been used for updating are presented in Table 10.1 for the model evolution steps of Schlune *et al.* (2008a). It can be seen that especially an improved agreement for the eigenfrequencies, the displacement and the hanger forces was obtained. The discrepancy for the strains decreased only slightly.

Table 10.1 Responses for model evolution steps

			Measured value z_m	*Initial model*	*E-modulus increase of concrete arch*	*Tied bearing*	*Non-linear bearing*	*Mass of non structural parts*	*After parameter study*	*Updated with respect to J_3*	*Updated with respect to J_4*	*Updated with respect to J_5*	*Updated with respect to J_6*	*Assumed standard deviation*
Frequency [Hz]			0.425	0.408	0.426	0.430	0.429	0.424	0.433	0.425	0.427	0.425	0.424	0.0016
			0.846	0.460	0.471	0.885	0.884	0.839	0.872	0.864	0.863	0.867	0.862	0.0062
			0.940	0.956	0.995	0.999	0.997	0.932	0.953	0.940	0.940	0.940	0.940	0.0035
			0.999	1.019	1.048	1.060	1.059	0.983	1.003	0.988	0.991	0.987	0.986	0.0033
Strain [µS]	S1	A	-1.3	-1.1	-1.1	-2.5	-2.3	-2.3	-2.2	-2.2	-2.1	-2.2	-2.1	6.0
			-2.3	-2.7	-2.4	-1.2	-1.4	-1.4	-1.4	-1.6	-1.6	-1.6	-1.6	6.0
			-2.2	-2.0	-1.8	-1.9	-1.9	-1.9	-1.8	-1.9	-1.9	-1.9	-1.9	6.0
			-2.0	-1.9	-1.7	-1.8	-1.9	-1.9	-1.8	-1.9	-1.8	-1.9	-1.9	6.0
		B	-2.7	-1.5	-1.5	-2.6	-2.4	-2.4	-2.3	-2.3	-2.3	-2.3	-2.3	6.0
			-1.0	-2.5	-2.3	-1.2	-1.4	-1.4	-1.5	-1.6	-1.6	-1.6	-1.6	6.0
			7.4	6.2	6.1	5.8	6.1	6.1	5.8	5.9	5.9	5.9	5.9	6.0
			-12.0	-10.2	-9.8	-9.6	-9.9	-9.9	-9.5	-9.8	-9.7	-9.8	-9.8	6.0
		C	-3.4	-1.2	-1.2	-2.5	-2.4	-2.4	-2.2	-2.2	-2.2	-2.2	-2.2	6.0
			-0.5	-2.7	-2.4	-1.2	-1.4	-1.4	-1.5	-1.6	-1.6	-1.6	-1.7	6.0
			6.1	5.0	4.9	4.7	4.9	4.9	4.7	4.7	4.7	4.7	4.8	6.0
			-10.3	-8.9	-8.5	-8.4	-8.7	-8.7	-8.3	-8.5	-8.5	-8.6	-8.6	6.0
		D	13.3	8.5	7.8	2.9	3.9	3.9	3.6	3.9	3.9	3.8	4.0	6.0
			-14.8	-11.7	-10.9	-6.0	-7.3	-7.3	-6.9	-7.3	-7.3	-7.3	-7.5	6.0
			-2.7	-1.3	-1.2	-1.3	-1.3	-1.3	-1.3	-1.4	-1.4	-1.4	-1.4	6.0
			-2.0	-1.9	-1.8	-1.9	-2.1	-2.1	-2.0	-2.1	-2.0	-2.1	-2.1	6.0
		E	9.3	16.9	16.0	-2.6	6.3	6.3	5.1	6.6	6.9	6.0	7.3	6.0
			-14.1	-21.9	-20.6	-3.6	-11.8	-11.8	-10.5	-12.1	-12.3	-11.6	-12.8	6.0
			-3.9	-2.7	-2.5	-3.1	-2.8	-2.8	-2.7	-2.8	-2.8	-2.9	-2.8	6.0

		Measured value z_m	Initial model	E-modulus increase of concrete arch	Tied bearing	Non-linear bearing	Mass of non structural parts	After parameter study	Updated with respect to J_3	Updated with respect to J_4	Updated with respect to J_5	Updated with respect to J_6	Assumed standard deviation
		-3.7	-2.3	-2.2	-3.0	-2.7	-2.7	-2.6	-2.7	-2.6	-2.7	-2.7	6.0
S6	A	-7.6	-7.5	-6.9	-6.8	-6.8	-6.8	-6.3	-6.4	-6.4	-6.4	-6.5	6.0
		1.6	2.2	2.0	1.9	1.8	1.8	1.4	1.4	1.4	1.3	1.4	6.0
		-3.1	-2.7	-2.5	-2.5	-2.5	-2.5	-2.4	-2.6	-2.5	-2.6	-2.6	6.0
		-2.2	-2.6	-2.4	-2.4	-2.5	-2.5	-2.4	-2.5	-2.5	-2.5	-2.5	6.0
	B	-4.3	-4.7	-4.3	-4.2	-4.2	-4.2	-3.9	-4.0	-4.0	-4.0	-4.0	6.0
		-2.0	-0.9	-0.8	-0.9	-1.0	-1.0	-1.2	-1.3	-1.3	-1.4	-1.3	6.0
		16.1	14.3	13.1	12.9	13.1	13.1	12.6	13.1	12.9	13.1	13.2	6.0
		-21.6	-19.8	-18.1	-18.1	-18.3	-18.3	-17.7	-18.4	-18.1	-18.5	-18.5	6.0
	C	-6.9	-6.9	-6.3	-6.3	-6.2	-6.2	-5.8	-5.9	-5.9	-5.9	-6.0	6.0
		0.7	1.5	1.4	1.3	1.2	1.2	0.8	0.8	0.8	0.7	0.8	6.0
		13.1	11.7	10.8	10.6	10.8	10.8	10.4	10.8	10.6	10.8	10.8	6.0
		-18.1	-17.1	-15.7	-15.6	-15.8	-15.8	-15.3	-15.9	-15.7	-16.0	-16.0	6.0
	D	-22.3	-21.0	-19.5	-17.1	-18.9	-18.9	-17.9	-18.5	-18.3	-18.5	-18.6	6.0
		18.2	16.8	15.7	13.0	14.5	14.5	13.7	14.1	14.0	14.0	14.2	6.0
		-3.6	-2.0	-1.9	-2.0	-2.1	-2.1	-2.1	-2.1	-2.1	-2.2	-2.2	6.0
		-0.5	-2.1	-1.9	-2.1	-2.2	-2.2	-2.2	-2.3	-2.2	-2.3	-2.3	6.0
	E	17.6	12.9	11.8	14.8	13.5	13.5	12.6	12.5	12.5	12.5	12.5	6.0
		-26.7	-19.8	-18.1	-23.3	-21.1	-21.1	-20.0	-20.0	-19.8	-20.2	-20.0	6.0
		-2.6	-3.6	-3.3	-4.3	-3.8	-3.8	-3.8	-3.9	-3.8	-3.9	-3.9	6.0
		-4.0	-3.3	-3.0	-4.2	-3.7	-3.7	-3.6	-3.7	-3.6	-3.7	-3.6	6.0
S25	A	-32.2	-38.0	-34.0	-33.5	-33.9	-33.9	-31.6	-32.3	-32.2	-32.3	-32.6	6.0
		13.1	16.5	15.7	15.0	15.5	15.5	13.7	13.7	13.8	13.5	13.8	6.0
		-10.7	-11.4	-9.7	-9.8	-9.8	-9.8	-9.5	-9.9	-9.7	-10.0	-10.0	6.0
		-9.5	-11.4	-9.8	-9.9	-9.8	-9.8	-9.5	-9.9	-9.8	-10.0	-10.0	6.0
	B	-21.0	-24.9	-22.2	-21.8	-22.1	-22.1	-20.8	-21.4	-21.2	-21.4	-21.5	6.0
		3.9	5.2	5.4	4.8	5.2	5.2	4.4	4.3	4.4	4.1	4.3	6.0
		-10.1	-10.7	-9.1	-9.2	-9.2	-9.2	-8.8	-9.2	-9.1	-9.3	-9.3	6.0
		-7.6	-9.8	-8.4	-8.5	-8.4	-8.4	-8.2	-8.5	-8.4	-8.6	-8.6	6.0
	C	-29.1	-34.6	-31.0	-30.5	-30.8	-30.8	-28.8	-29.5	-29.4	-29.5	-29.8	6.0
		10.5	13.5	13.0	12.3	12.7	12.7	11.2	11.2	11.3	11.0	11.3	6.0
		-11.2	-11.6	-9.9	-10.0	-10.0	-10.0	-9.6	-10.0	-9.9	-10.1	-10.1	6.0
		-8.3	-10.7	-9.2	-9.3	-9.2	-9.2	-8.9	-9.3	-9.2	-9.4	-9.4	6.0
	D	-1.0	-1.0	-1.0	-0.5	-0.7	-0.7	-0.6	-0.6	-0.6	-0.6	-0.6	6.0
		2.1	2.4	2.1	1.4	1.8	1.8	1.7	1.8	1.8	1.8	1.8	6.0
		0.5	0.6	0.5	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	6.0
		0.4	0.6	0.5	0.5	0.5	0.5	0.5	0.6	0.5	0.6	0.6	6.0
	E	-0.3	-0.8	-0.6	0.8	0.1	0.1	-0.1	-0.3	-0.3	-0.3	-0.3	6.0
		-4.5	-5.5	-4.7	-6.5	-5.6	-5.6	-5.3	-5.3	-5.2	-5.3	-5.2	6.0
		-2.0	-3.0	-2.6	-2.8	-2.7	-2.7	-2.6	-2.7	-2.7	-2.8	-2.7	6.0
		-2.3	-3.1	-2.6	-2.7	-2.7	-2.7	-2.6	-2.7	-2.7	-2.7	-2.7	6.0
N6	A	-10.9	-11.8	-10.7	-12.3	-11.7	-11.7	-10.9	-11.2	-11.1	-11.2	-11.3	6.0
		5.4	6.5	5.9	7.9	7.1	7.1	6.5	6.6	6.5	6.5	6.6	6.0
		-4.6	-2.7	-2.4	-2.2	-2.3	-2.3	-2.2	-2.3	-2.3	-2.3	-2.4	6.0
		-3.2	-2.6	-2.3	-2.1	-2.3	-2.3	-2.2	-2.3	-2.3	-2.3	-2.3	6.0

			Measured value z_m	Initial model	E-modulus increase of concrete arch	Tied bearing	Non-linear bearing	Mass of non structural parts	After parameter study	Updated with respect to J_3	Updated with respect to J_4	Updated with respect to J_5	Updated with respect to J_6	Assumed standard deviation
Displacement [mm]		B	-7.3	-7.9	-7.2	-8.5	-7.9	-7.9	-7.4	-7.6	-7.5	-7.6	-7.7	6.0
			1.0	2.4	2.2	3.8	3.1	3.1	2.7	2.7	2.7	2.6	2.7	6.0
			14.8	14.9	13.6	13.6	13.8	13.8	13.3	13.8	13.6	13.9	13.9	6.0
			-20.9	-20.4	-18.6	-18.3	-18.6	-18.6	-18.0	-18.7	-18.5	-18.8	-18.9	6.0
		C	-10.3	-10.9	-9.9	-11.4	-10.8	-10.8	-10.1	-10.4	-10.3	-10.4	-10.4	6.0
			4.6	5.6	5.1	7.0	6.2	6.2	5.6	5.6	5.6	5.6	5.7	6.0
			11.5	12.3	11.2	11.3	11.4	11.4	11.0	11.4	11.3	11.5	11.5	6.0
			-17.9	-17.6	-16.0	-15.7	-16.0	-16.0	-15.5	-16.1	-15.9	-16.2	-16.3	6.0
		D	3.9	5.4	5.0	2.0	2.9	2.9	2.9	3.1	3.1	3.1	3.2	6.0
			-5.1	-6.4	-5.9	-2.1	-3.2	-3.2	-3.1	-3.4	-3.4	-3.4	-3.5	6.0
			0.2	-0.5	-0.4	0.0	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	6.0
			-1.1	-0.5	-0.5	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	6.0
		E	-5.8	0.1	0.2	-9.5	-4.7	-4.7	-4.6	-3.7	-3.6	-4.0	-3.4	6.0
			2.9	-3.4	-3.2	9.4	3.2	3.2	3.3	2.1	1.9	2.4	1.6	6.0
			-2.6	-1.5	-1.4	0.1	-0.6	-0.6	-0.5	-0.7	-0.7	-0.6	-0.7	6.0
			-1.5	-1.7	-1.6	-0.1	-0.9	-0.9	-0.8	-1.0	-1.0	-0.9	-1.0	6.0
	N1	A	-6.8	-4.5	-4.3	-1.8	-2.7	-2.7	-2.5	-2.6	-2.6	-2.5	-2.6	6.0
			-2.3	-1.9	-1.8	-1.7	-1.6	-1.6	-1.6	-1.7	-1.7	-1.7	-1.7	6.0
			-2.3	-1.8	-1.7	-1.6	-1.8	-1.8	-1.7	-1.8	-1.7	-1.8	-1.8	6.0
		B	-5.6	-4.0	-3.8	-1.8	-2.6	-2.6	-2.3	-2.4	-2.5	-2.4	-2.5	6.0
			7.8	7.6	7.3	7.2	7.5	7.5	7.2	7.4	7.3	7.4	7.4	6.0
			-13.6	-11.6	-11.0	-10.6	-11.1	-11.1	-10.7	-11.0	-10.9	-11.0	-11.1	6.0
		C	-4.4	-4.3	-4.2	-1.8	-2.7	-2.7	-2.5	-2.6	-2.6	-2.5	-2.6	6.0
			6.4	6.2	6.0	5.9	6.2	6.2	5.9	6.0	6.0	6.0	6.1	6.0
			-11.5	-10.0	-9.6	-9.2	-9.6	-9.6	-9.3	-9.6	-9.5	-9.6	-9.6	6.0
		D	-8.8	-6.3	-5.9	-0.7	-1.8	-1.8	-1.7	-2.0	-2.0	-1.9	-2.1	6.0
			-1.0	-0.3	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6.0
			-0.5	-0.3	-0.3	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	6.0
		E	-21.6	-22.2	-20.7	-1.5	-10.9	-10.9	-9.4	-11.3	-11.5	-10.7	-12.1	6.0
			-1.9	-1.1	-1.0	0.1	-0.3	-0.3	-0.3	-0.4	-0.4	-0.4	-0.5	6.0
			-0.1	-1.0	-0.9	-0.1	-0.7	-0.7	-0.6	-0.7	-0.7	-0.7	-0.7	6.0
	Δx of Arch at hanger 1	A	-2.1	-1.7	-1.6	-2.5	-2.3	-2.3	-2.1	-2.1	-2.1	-2.1	-2.1	1.87
		B	-1.5	-0.8	-0.8	-1.6	-1.4	-1.4	-1.2	-1.2	-1.2	-1.2	-1.2	1.87
		C	-2.3	-1.5	-1.4	-2.3	-2.1	-2.1	-1.9	-1.9	-1.9	-1.9	-1.9	1.87
		D	0.6	2.0	2.0	-0.7	-0.3	-0.3	-0.3	-0.2	-0.2	-0.2	-0.1	1.87
		E	11.0	18.2	17.2	6.6	11.7	11.7	10.5	11.5	11.7	11.2	12.0	1.87
	Δy of Arch at hanger 1	A	-0.8	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.87
		B	8.7	10.7	9.9	9.8	10.0	10.0	9.5	9.8	9.7	9.8	9.9	1.87
		C	7.1	9.1	8.5	8.3	8.5	8.5	8.1	8.4	8.3	8.4	8.4	1.87
		D	-0.5	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	1.87
		E	0.6	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.87
	Δz of Arch at hanger 1	A	-0.8	-1.9	-1.8	-2.9	-2.7	-2.7	-2.4	-2.3	-2.4	-2.3	-2.4	1.87
		B	-0.8	-0.5	-0.4	-1.4	-1.1	-1.1	-1.0	-0.9	-1.0	-0.9	-0.9	1.87
		C	-2.1	-1.6	-1.5	-2.6	-2.3	-2.3	-2.0	-2.0	-2.0	-2.0	-2.0	1.87
		D	0.3	1.0	1.1	-1.8	-1.4	-1.4	-1.4	-1.3	-1.3	-1.3	-1.2	1.87
		E	15.8	24.8	23.3	11.3	17.1	17.1	15.5	16.7	16.8	16.3	17.2	1.87

		Measured value z_m	Initial model	E-modulus increase of concrete arch	Tied bearing	Non-linear bearing	Mass of non structural parts	After parameter study	Updated with respect to J_3	Updated with respect to J_4	Updated with respect to J_5	Updated with respect to J_6	Assumed standard deviation
Δx of Arch at midpoint	A	1.1	1.2	1.1	0.2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.87
	B	0.2	1.0	0.9	0.2	0.4	0.4	0.4	0.4	0.4	0.4	0.4	1.87
	C	0.4	1.1	1.1	0.2	0.5	0.5	0.4	0.5	0.5	0.5	0.5	1.87
	D	1.3	2.0	1.9	-0.4	0.0	0.0	0.0	0.1	0.1	0.1	0.1	1.87
	E	8.2	14.8	14.0	5.0	9.4	9.4	8.4	9.2	9.3	8.9	9.6	1.87
Δy of Arch at midpoint	A	0.4	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.87
	B	17.6	17.9	16.5	16.2	16.6	16.6	15.6	16.1	15.9	16.1	16.2	1.87
	C	14.7	15.4	14.2	14.0	14.3	14.3	13.4	13.8	13.7	13.8	14.0	1.87
	D	0.3	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	1.87
	E	0.7	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	1.87
Δz of Arch at midpoint	A	12.0	14.8	13.6	13.0	13.4	13.4	12.4	12.6	12.6	12.6	12.7	1.87
	B	6.6	10.0	9.1	8.7	9.0	9.0	8.4	8.5	8.5	8.5	8.6	1.87
	C	10.5	13.6	12.5	12.0	12.3	12.3	11.4	11.6	11.6	11.6	11.7	1.87
	D	1.0	0.8	0.7	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.6	1.87
	E	-0.8	-1.1	-1.0	-1.9	-1.4	-1.4	-1.2	-1.2	-1.1	-1.2	-1.1	1.87
Δx of Arch at hanger 6	A	3.5	4.7	4.4	3.1	3.6	3.6	3.2	3.3	3.3	3.3	3.4	1.87
	B	1.9	3.2	3.0	2.0	2.4	2.4	2.2	2.2	2.2	2.2	2.3	1.87
	C	3.2	4.3	4.1	2.8	3.3	3.3	3.0	3.1	3.1	3.0	3.1	1.87
	D	1.3	2.9	2.8	-0.2	0.4	0.4	0.4	0.5	0.5	0.4	0.5	1.87
	E	9.2	16.3	15.5	4.3	9.8	9.8	8.7	9.7	9.9	9.4	10.2	1.87
Δy of Arch at hanger 6	A	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.87
	B	8.3	9.9	9.3	9.1	9.4	9.4	8.9	9.2	9.1	9.2	9.3	1.87
	C	6.6	8.5	7.9	7.7	8.0	8.0	7.6	7.8	7.8	7.9	7.9	1.87
	D	-0.4	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	1.87
	E	0.7	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	1.87
Δz of Arch at hanger 6	A	-4.3	-5.7	-5.3	-4.0	-4.4	-4.4	-4.0	-4.1	-4.1	-4.1	-4.2	1.87
	B	-3.0	-3.5	-3.3	-2.3	-2.6	-2.6	-2.4	-2.4	-2.4	-2.4	-2.5	1.87
	C	-4.9	-5.2	-4.8	-3.6	-4.0	-4.0	-3.7	-3.7	-3.7	-3.7	-3.8	1.87
	D	-2.4	-2.7	-2.6	0.5	-0.1	-0.1	-0.1	-0.2	-0.2	-0.1	-0.2	1.87
	E	-12.1	-19.6	-18.6	-6.5	-12.4	-12.4	-11.1	-12.2	-12.4	-11.8	-12.7	1.87
Δz at point 10	A	0.3	0.6	0.5	0.1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.87
	B	-0.8	-2.1	-2.0	-2.1	-2.1	-2.1	-2.1	-2.2	-2.1	-2.2	-2.2	1.87
	C	-0.6	-1.5	-1.4	-1.6	-1.5	-1.5	-1.6	-1.6	-1.6	-1.6	-1.6	1.87
	D	19.3	26.3	26.1	22.5	25.7	25.7	22.9	22.6	22.8	22.2	22.8	1.87
	E	-1.1	0.1	0.1	-1.5	-0.9	-0.9	-0.9	-0.7	-0.7	-0.8	-0.7	1.87
Δz at point 20	A	0.3	0.6	0.5	0.1	0.4	0.4	0.3	0.4	0.4	0.3	0.4	1.87
	B	-0.4	0.2	0.2	-0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0	1.87
	C	0.5	0.4	0.4	0.1	0.3	0.3	0.2	0.2	0.2	0.2	0.2	1.87
	D	19.8	24.2	24.0	20.5	23.6	23.6	21.1	20.8	21.0	20.5	21.0	1.87
	E	-1.1	0.0	0.0	-1.6	-1.0	-1.0	-1.0	-0.8	-0.8	-0.9	-0.8	1.87
Δz at point 30	A	0.3	0.6	0.5	0.2	0.4	0.4	0.3	0.4	0.4	0.4	0.4	1.87
	B	-0.2	0.3	0.3	0.0	0.1	0.1	0.2	0.2	0.2	0.2	0.2	1.87
	C	0.0	0.6	0.5	0.1	0.4	0.4	0.4	0.4	0.4	0.4	0.4	1.87
	D	19.0	22.1	22.0	18.7	21.5	21.5	19.2	19.0	19.2	18.7	19.1	1.87
	E	-1.2	0.1	0.1	-1.6	-1.0	-1.0	-1.0	-0.8	-0.8	-0.9	-0.7	1.87

		Measured value z_m	Initial model	E-modulus increase of concrete arch	Tied bearing	Non-linear bearing	Mass of non structural parts	After parameter study	Updated with respect to J_3	Updated with respect to J_4	Updated with respect to J_5	Updated with respect to J_6	Assumed standard deviation
Δz at point 11	A	-0.7	0.1	0.1	-1.1	-0.8	-0.8	-0.5	-0.4	-0.4	-0.4	-0.4	1.87
	B	-18.3	-20.6	-20.3	-21.2	-21.0	-21.0	-18.8	-18.6	-18.8	-18.3	-18.7	1.87
	C	-17.0	-18.6	-18.3	-19.4	-19.1	-19.1	-17.1	-16.9	-17.0	-16.6	-17.0	1.87
	D	0.0	0.5	0.5	-2.3	-2.0	-2.0	-1.9	-1.8	-1.8	-1.8	-1.7	1.87
	E	22.9	32.8	31.5	19.7	25.4	25.4	23.2	24.3	24.5	23.8	24.8	1.87
Δz at point 21	A	-0.1	-0.6	-0.6	-1.7	-1.4	-1.4	-1.1	-1.0	-1.0	-0.9	-1.0	1.87
	B	-5.3	-3.7	-3.7	-4.6	-4.4	-4.4	-3.7	-3.6	-3.7	-3.5	-3.6	1.87
	C	-5.1	-4.3	-4.2	-5.3	-5.0	-5.0	-4.3	-4.2	-4.3	-4.1	-4.2	1.87
	D	0.3	0.7	0.7	-2.1	-1.8	-1.8	-1.7	-1.6	-1.6	-1.6	-1.6	1.87
	E	19.1	28.9	27.5	15.6	21.4	21.4	19.6	20.7	20.9	20.2	21.2	1.87
Δz at point 31	A	-0.7	-0.6	-0.6	-1.7	-1.4	-1.4	-1.1	-1.0	-1.0	-0.9	-1.0	1.87
	B	6.0	6.1	6.0	5.0	5.3	5.3	5.0	5.1	5.1	5.0	5.1	1.87
	C	3.3	3.9	3.8	2.7	3.0	3.0	3.0	3.0	3.0	3.0	3.1	1.87
	D	-0.4	0.7	0.7	-2.1	-1.8	-1.8	-1.7	-1.6	-1.6	-1.6	-1.5	1.87
	E	19.2	29.1	27.7	15.8	21.6	21.6	19.7	20.8	21.0	20.4	21.4	1.87
Δz at point 22	A	6.4	8.5	8.0	6.8	7.3	7.3	7.2	7.3	7.3	7.3	7.4	1.87
	B	-4.7	0.1	-0.2	-1.2	-0.8	-0.8	-0.1	0.1	0.0	0.2	0.0	1.87
	C	-1.0	1.1	0.8	-0.4	0.1	0.1	0.7	0.9	0.8	1.0	0.9	1.87
	D	2.2	0.7	0.7	-1.6	-1.3	-1.3	-1.2	-1.2	-1.1	-1.2	-1.1	1.87
	E	17.4	23.6	22.3	12.7	17.4	17.4	16.0	16.9	17.1	16.6	17.4	1.87
Δz at point 32	A	7.3	8.5	8.0	6.9	7.3	7.3	7.2	7.3	7.3	7.3	7.4	1.87
	B	15.2	18.2	17.7	16.7	17.1	17.1	16.0	16.0	16.0	15.8	16.1	1.87
	C	14.9	16.4	15.9	14.8	15.3	15.3	14.3	14.4	14.4	14.3	14.5	1.87
	D	0.8	0.7	0.7	-1.6	-1.3	-1.3	-1.2	-1.2	-1.1	-1.2	-1.1	1.87
	E	20.1	24.0	22.7	13.1	17.8	17.8	16.3	17.3	17.4	16.9	17.7	1.87
Δz at point 24	A	23.2	26.1	25.1	24.6	25.0	25.0	23.5	23.6	23.7	23.5	23.8	1.87
	B	4.0	4.7	4.0	3.7	3.9	3.9	4.4	4.7	4.6	4.8	4.7	1.87
	C	8.9	10.7	9.8	9.4	9.7	9.7	9.8	10.0	9.9	10.1	10.1	1.87
	D	2.0	0.6	0.5	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	1.87
	E	-1.2	-1.6	-1.5	-1.9	-1.7	-1.7	-1.4	-1.3	-1.3	-1.3	-1.3	1.87
Δz at point 34	A	23.3	26.1	25.1	24.6	25.0	25.0	23.5	23.6	23.7	23.5	23.8	1.87
	B	24.8	29.1	28.3	27.9	28.2	28.2	26.2	26.2	26.3	25.9	26.3	1.87
	C	31.0	35.7	34.7	34.2	34.6	34.6	32.1	32.0	32.2	31.7	32.2	1.87
	D	1.4	0.6	0.5	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	1.87
	E	-0.2	-1.3	-1.3	-1.7	-1.4	-1.4	-1.2	-1.1	-1.1	-1.1	-1.1	1.87
Δz at point 13	A	23.1	25.6	24.7	23.8	24.3	24.3	22.8	22.9	22.9	22.7	23.0	1.87
	B	-26.3	-32.0	-32.3	-32.9	-32.6	-32.6	-28.3	-27.6	-28.0	-27.0	-27.8	1.87
	C	-21.0	-24.3	-24.8	-25.6	-25.2	-25.2	-21.5	-20.8	-21.2	-20.2	-21.0	1.87
	D	1.6	0.7	0.7	-0.4	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	1.87
	E	4.4	7.1	6.6	2.7	4.7	4.7	4.4	4.8	4.8	4.7	5.0	1.87
Δz at point 23	A	20.1	23.3	22.3	21.5	21.9	21.9	20.7	20.9	20.9	20.8	21.0	1.87
	B	-0.3	4.8	4.2	3.5	3.9	3.9	4.4	4.6	4.5	4.7	4.6	1.87
	C	10.1	10.4	9.6	8.8	9.2	9.2	9.3	9.6	9.5	9.7	9.6	1.87
	D	2.4	0.8	0.7	-0.3	-0.1	-0.1	-0.1	0.0	0.0	-0.1	0.0	1.87
	E	3.1	7.2	6.7	2.8	4.8	4.8	4.5	4.9	4.9	4.8	5.1	1.87

		Measured value z_m	Initial model	E-modulus increase of concrete arch	Tied bearing	Non-linear bearing	Mass of non structural parts	After parameter study	Updated with respect to J_3	Updated with respect to J_4	Updated with respect to J_5	Updated with respect to J_6	Assumed standard deviation
Δz at point 33	A	20.8	23.3	22.3	21.5	22.0	22.0	20.8	20.9	20.9	20.8	21.0	1.87
	B	27.6	27.8	27.0	26.4	26.7	26.7	24.8	24.8	24.9	24.6	25.0	1.87
	C	31.3	32.7	31.7	30.9	31.3	31.3	29.1	29.1	29.2	28.8	29.3	1.87
	D	0.3	0.8	0.8	-0.3	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	1.87
	E	4.6	7.4	7.0	3.1	5.1	5.1	4.7	5.2	5.2	5.0	5.3	1.87
Δz at point 15	A	21.1	22.7	21.9	21.8	22.0	22.0	20.7	20.8	20.8	20.7	20.9	1.87
	B	-27.3	-33.0	-33.2	-33.2	-33.1	-33.1	-28.8	-28.1	-28.5	-27.5	-28.4	1.87
	C	-23.8	-25.7	-26.1	-26.2	-26.0	-26.0	-22.3	-21.6	-22.0	-21.1	-21.8	1.87
	D	2.5	0.1	0.0	0.6	0.7	0.7	0.6	0.6	0.6	0.6	0.6	1.87
	E	-8.4	-9.4	-8.9	-5.8	-7.3	-7.3	-6.5	-6.7	-6.8	-6.6	-6.9	1.87
Δz at point 25	A	18.6	20.5	19.7	19.6	19.8	19.8	18.8	18.9	18.9	18.8	19.0	1.87
	B	0.9	3.5	2.9	2.9	3.0	3.0	3.6	3.8	3.7	3.9	3.8	1.87
	C	5.9	8.4	7.7	7.6	7.8	7.8	8.0	8.3	8.2	8.3	8.3	1.87
	D	0.9	0.1	0.1	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	1.87
	E	-5.1	-9.3	-8.8	-5.7	-7.2	-7.2	-6.4	-6.6	-6.7	-6.5	-6.8	1.87
Δz at point 35	A	18.7	20.5	19.7	19.6	19.8	19.8	18.8	18.9	18.9	18.8	19.0	1.87
	B	24.3	26.3	25.6	25.5	25.7	25.7	23.9	23.8	23.9	23.6	24.0	1.87
	C	26.9	30.2	29.3	29.2	29.4	29.4	27.3	27.3	27.4	27.1	27.5	1.87
	D	0.1	0.2	0.1	0.7	0.8	0.8	0.7	0.7	0.7	0.7	0.7	1.87
	E	-8.4	-9.1	-8.6	-5.5	-6.9	-6.9	-6.2	-6.4	-6.5	-6.3	-6.6	1.87
Δz at point 26	A	1.7	3.9	3.7	4.4	4.3	4.3	4.4	4.5	4.4	4.5	4.5	1.87
	B	-1.1	-3.1	-3.2	-2.6	-2.7	-2.7	-1.9	-1.7	-1.8	-1.6	-1.8	1.87
	C	-1.0	-2.9	-3.0	-2.3	-2.5	-2.5	-1.7	-1.5	-1.6	-1.4	-1.6	1.87
	D	-1.2	-1.3	-1.3	0.9	0.7	0.7	0.6	0.6	0.5	0.6	0.5	1.87
	E	-12.9	-19.1	-18.0	-8.8	-13.3	-13.3	-11.9	-12.7	-12.9	-12.4	-13.1	1.87
Δz at point 36	A	3.3	3.9	3.7	4.4	4.3	4.3	4.4	4.5	4.5	4.5	4.5	1.87
	B	12.3	14.4	14.1	14.7	14.6	14.6	13.7	13.6	13.7	13.5	13.7	1.87
	C	10.0	11.7	11.5	12.2	12.1	12.1	11.4	11.4	11.4	11.3	11.4	1.87
	D	-0.2	-1.3	-1.3	0.9	0.7	0.7	0.6	0.6	0.5	0.6	0.5	1.87
	E	-13.0	-18.9	-17.9	-8.7	-13.1	-13.1	-11.7	-12.6	-12.7	-12.2	-13.0	1.87
Δz at point 46	A	4.0	4.9	4.7	5.4	5.3	5.3	5.3	5.4	5.4	5.4	5.4	1.87
	B	40.6	47.4	46.9	47.4	47.4	47.4	43.0	42.6	42.9	42.1	42.9	1.87
	C	33.1	39.3	38.8	39.5	39.4	39.4	35.9	35.6	35.8	35.1	35.8	1.87
	D	-0.7	-1.3	-1.3	0.9	0.7	0.7	0.6	0.6	0.5	0.6	0.5	1.87
	E	-12.2	§-18.5	-17.4	-8.3	-12.7	-12.7	-11.4	-12.2	-12.3	-11.9	-12.6	1.87
Δz at point 17	A	-2.4	-3.7	-3.5	-2.2	-2.6	-2.6	-2.2	-2.2	-2.3	-2.2	-2.3	1.87
	B	-19.5	-22.8	-22.4	-21.3	-21.7	-21.7	-19.5	-19.4	-19.5	-19.1	-19.5	1.87
	C	-17.5	-21.4	-20.9	-19.7	-20.1	-20.1	-18.1	-18.0	-18.1	-17.7	-18.1	1.87
	D	-2.4	-2.7	-2.6	0.4	-0.1	-0.1	-0.1	-0.2	-0.3	-0.2	-0.3	1.87
	E	-11.5	-19.2	-18.2	-6.4	-12.1	-12.1	-10.8	-11.9	-12.0	-11.5	-12.4	1.87
Δz at point 27	A	-3.5	-4.3	-4.1	-2.8	-3.2	-3.2	-2.7	-2.8	-2.8	-2.7	-2.8	1.87
	B	-5.1	-6.6	-6.4	-5.3	-5.7	-5.7	-5.0	-5.0	-5.0	-4.9	-5.0	1.87
	C	-6.8	-7.7	-7.4	-6.2	-6.6	-6.6	-5.8	-5.8	-5.9	-5.7	-5.9	1.87
	D	-1.5	-2.7	-2.6	0.4	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.3	1.87
	E	-12.3	-19.3	-18.2	-6.3	-12.1	-12.1	-10.8	-11.9	-12.1	-11.5	-12.4	1.87

			Measured value z_m	*Initial model*	*E-modulus increase of concrete arch*	*Tied bearing*	*Non-linear bearing*	*Mass of non structural parts*	*After parameter study*	*Updated with respect to J_3*	*Updated with respect to J_4*	*Updated with respect to J_5*	*Updated with respect to J_6*	*Assumed standard deviation*
	Δz at point 37	A	-3.4	-4.3	-4.1	-2.8	-3.2	-3.2	-2.7	-2.8	-2.8	-2.7	-2.8	1.87
		B	3.4	2.7	2.8	3.8	3.5	3.5	3.4	3.3	3.3	3.3	3.3	1.87
		C	0.7	0.1	0.3	1.5	1.1	1.1	1.2	1.1	1.1	1.1	1.1	1.87
		D	-2.2	-2.7	-2.6	0.4	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.3	1.87
		E	-12.1	-19.2	-18.2	-6.2	-12.0	-12.0	-10.7	-11.8	-12.0	-11.4	-12.3	1.87
Hanger Load [kN]	Hanger 1E	A	95.9	95.5	86.8	87.9	87.7	87.7	93.3	97.1	95.4	98.6	97.1	11.0
		B	259.5	267.4	263.3	264.1	263.9	263.9	251.9	251.5	252.2	250.1	252.3	11.0
		C	216.5	222.7	216.0	217.0	216.8	216.8	209.4	210.6	210.4	210.1	211.2	11.0
		D	-21.5	-24.3	-25.2	-20.7	-23.4	-23.4	-21.6	-21.2	-21.5	-20.8	-21.3	11.0
		E	288.8	296.4	305.0	313.6	309.3	309.3	295.1	289.2	291.6	286.7	289.4	11.0
	Hanger 1W	A	96.7	95.1	86.4	87.5	87.4	87.4	93.0	96.7	95.1	98.2	96.7	11.0
		B	-21.7	-34.8	-41.7	-40.9	-41.0	-41.0	-21.4	-16.2	-19.0	-12.9	-17.1	11.0
		C	-14.2	-26.0	-35.2	-34.2	-34.3	-34.3	-16.0	-10.2	-13.2	-6.9	-11.0	11.0
		D	-22.4	-24.1	-24.9	-20.5	-23.2	-23.2	-21.4	-21.0	-21.3	-20.6	-21.1	11.0
		E	281.8	288.0	296.4	305.0	300.7	300.7	287.5	281.8	284.1	279.4	281.9	11.0

In Table 10.2 the normalized updating parameters for the model evolution steps are shown. All updating parameters stayed in realistic ranges. The bridge is in general stiffer than assumed.

Table 10.2 Normalized updating parameters for model evolution steps

	Initial model	*After manual refinements (exclusive parameter study)*	*After parameter study*	*Updated with respect to J_3*	*Updated with respect to J_4*	*Updated with respect to J_5*	*Updated with respect to J_6*
E-modulus of concrete at arch base	1.00	1.04	1.07	1.03	1.04	1.02	1.02
E-modulus of concrete at arch crown	1.00	1.17	1.20	1.15	1.17	1.14	1.14
E-modulus of bridge deck girder	1.00	1.00	1.15	1.17	1.16	1.32	1.17
Additional mass of non-structural parts	0.00	1.00	1.00	1.06	1.09	1.07	1.03
Static friction threshold of bearings	-	1.00	1.13	0.91	0.86	0.98	0.81

10.2 Parameter study

After manual model refinements had been introduced into the model, a parameter study was carried out to study the effect of model parameter changes on the objective functions. In the following, the development of the objective function, J_3, according to Schlune *et al.* (2008a) has been plotted for model parameter variations. The contribution from the four different target response types to the objective function are presented separately. In addition, the sum of the contribution is presented. The model parameter that were studied are:

- The E-modulus of the concrete of the arch, see Figure 10.1
- The mass of non-structural parts along the carriageway, see Figure 10.2
- The asphalt stiffness, only for the dynamic analysis, see Figure 10.3
- The rotational spring stiffness of the arch-carriageway connection around the y-axis, see Figure 10.4
- The rotational spring stiffness of the arch foundation around the y-axis, see Figure 10.5
- The static friction coefficients in the bearings, see Figure 10.6
- The E-modulus of the steel of the carriageway, see Figure 10.7

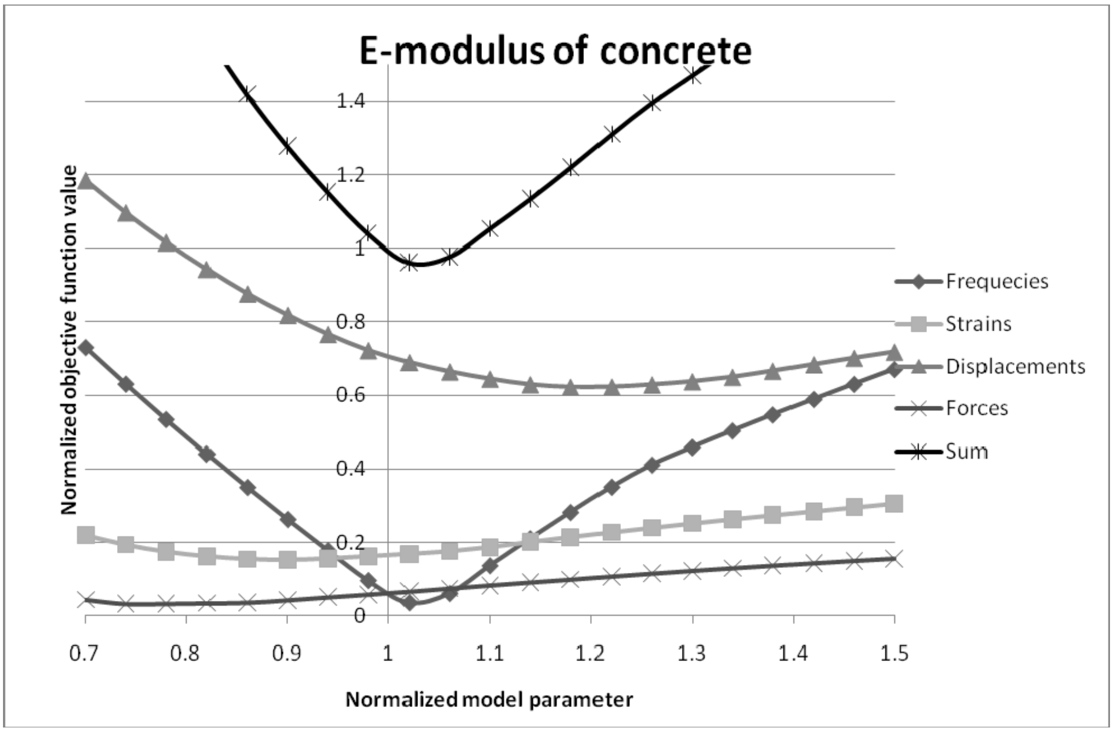

Figure 10.1 Development of the objective function J_3 over changes of the E-modulus of the concrete of the arch

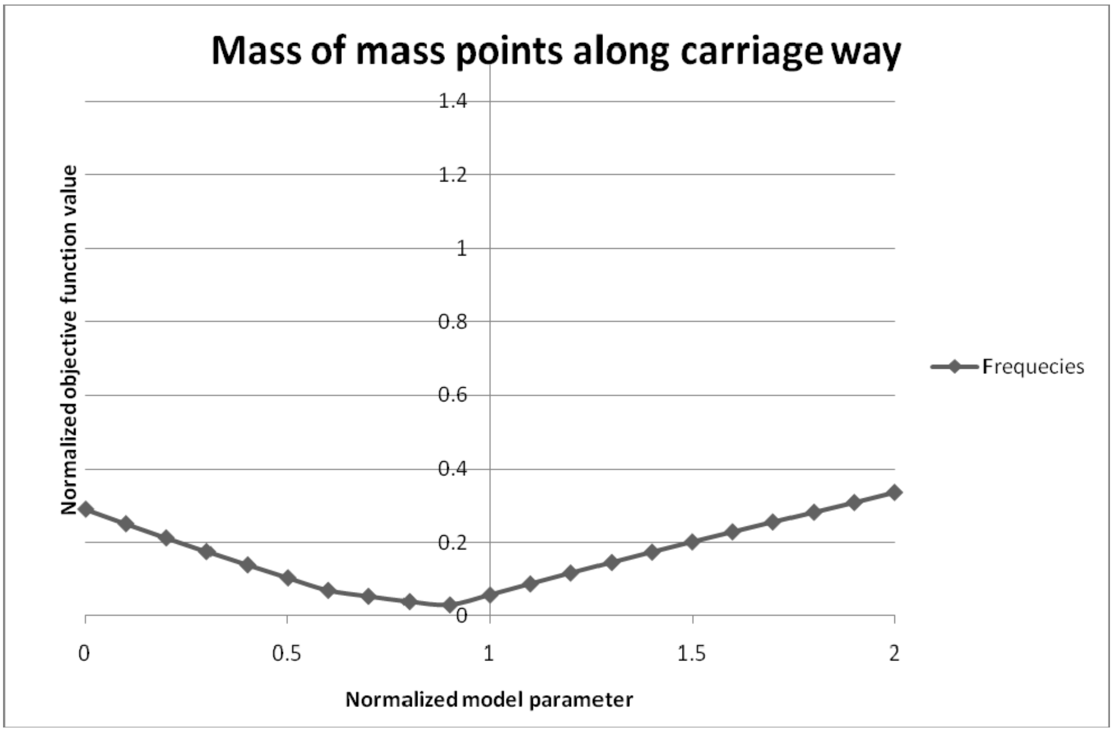

Figure 10.2 *Development of the objective function J_3 over changes of the mass points along the carriage way*

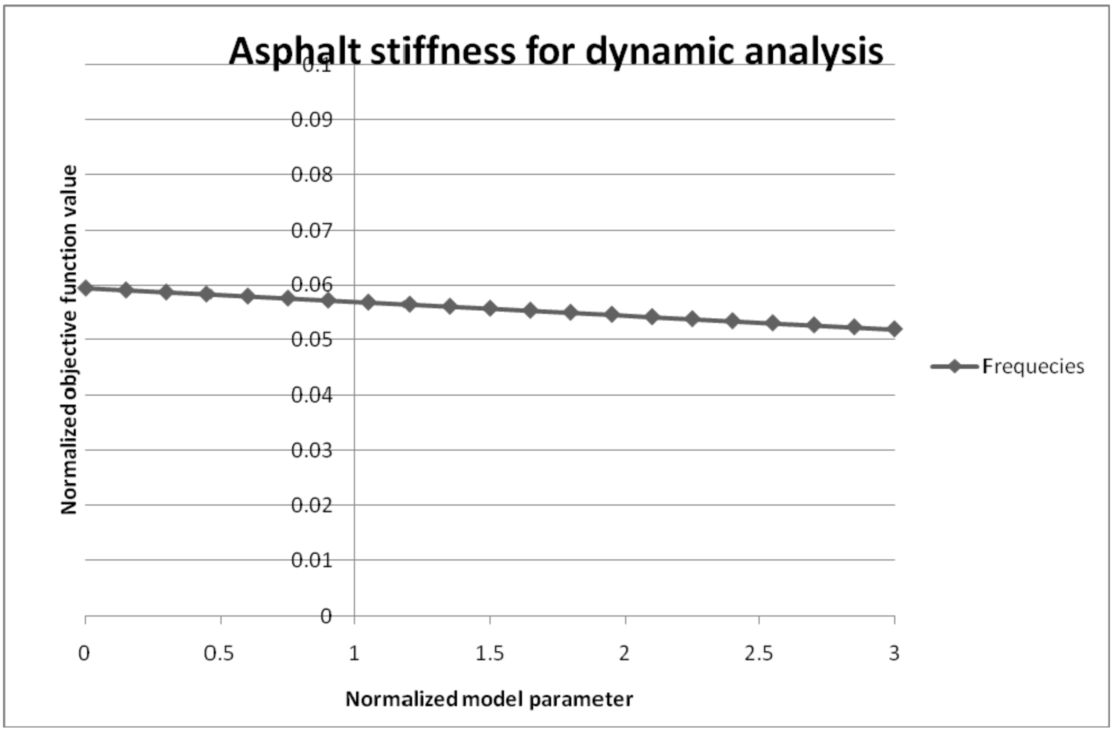

Figure 10.3 *Development of the objective function J_3 over changes of the asphalt stiffness for the dynamic analysis*

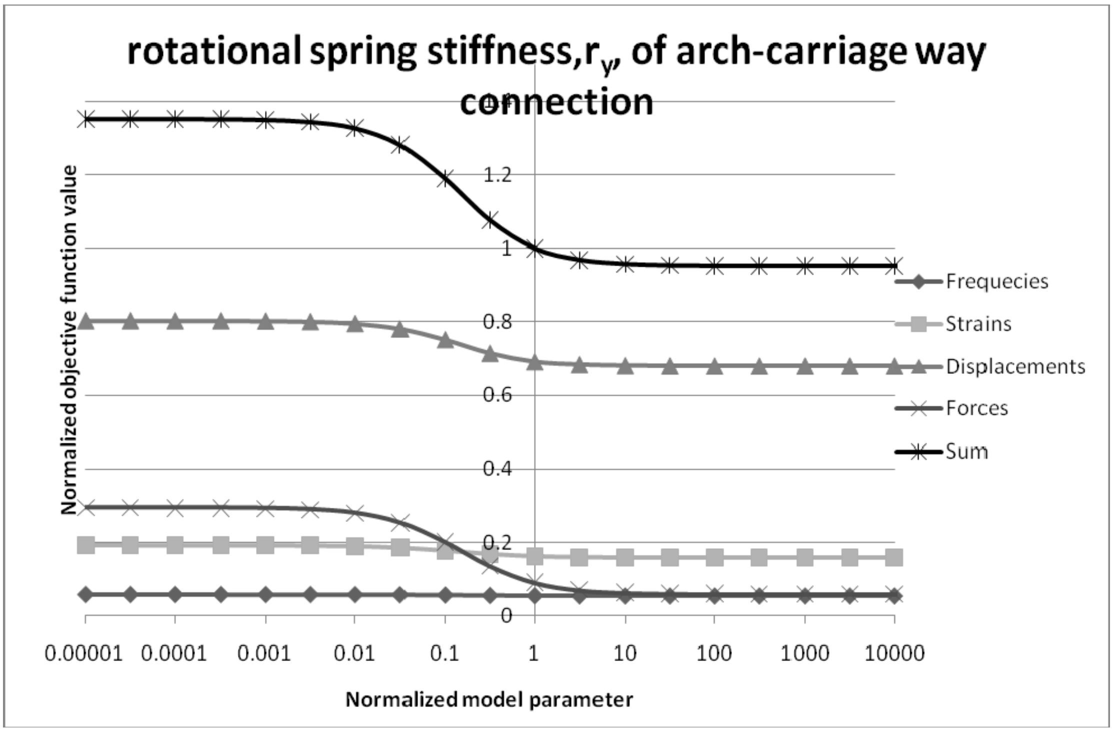

Figure 10.4 Development of the objective function J_3 over changes of the rotational spring stiffness of the arch-carriage way connection

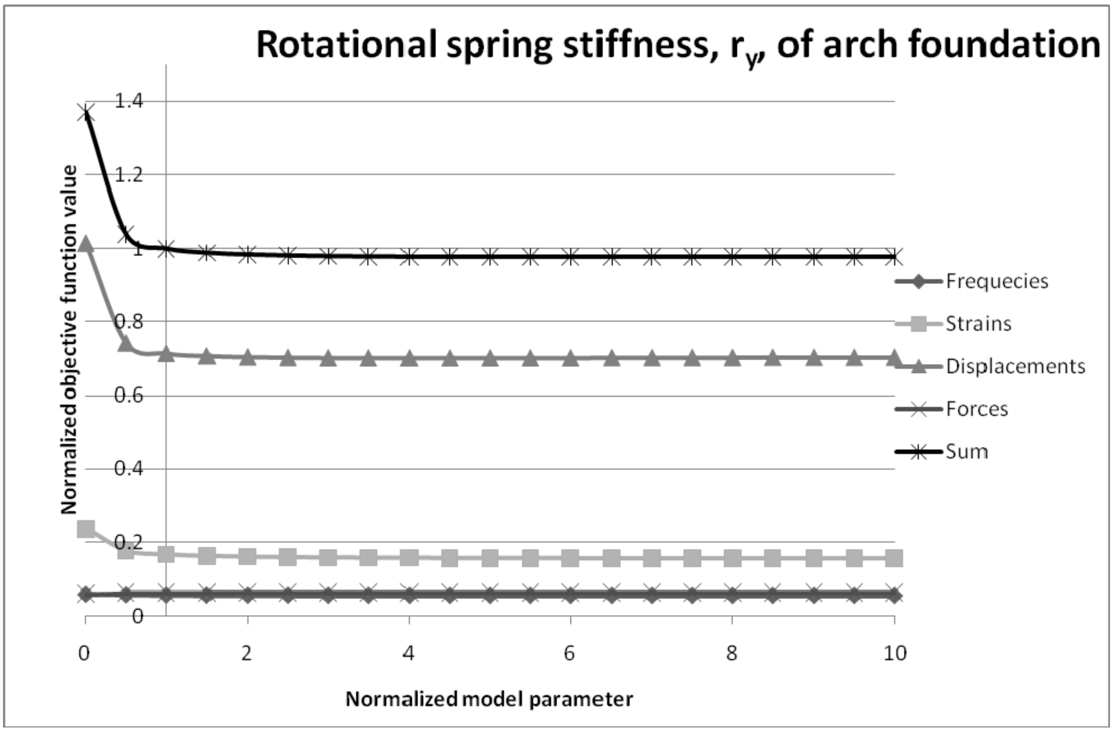

Figure 10.5 Development of the objective function J_3 over changes of the rotational spring stiffness of the arch foundation

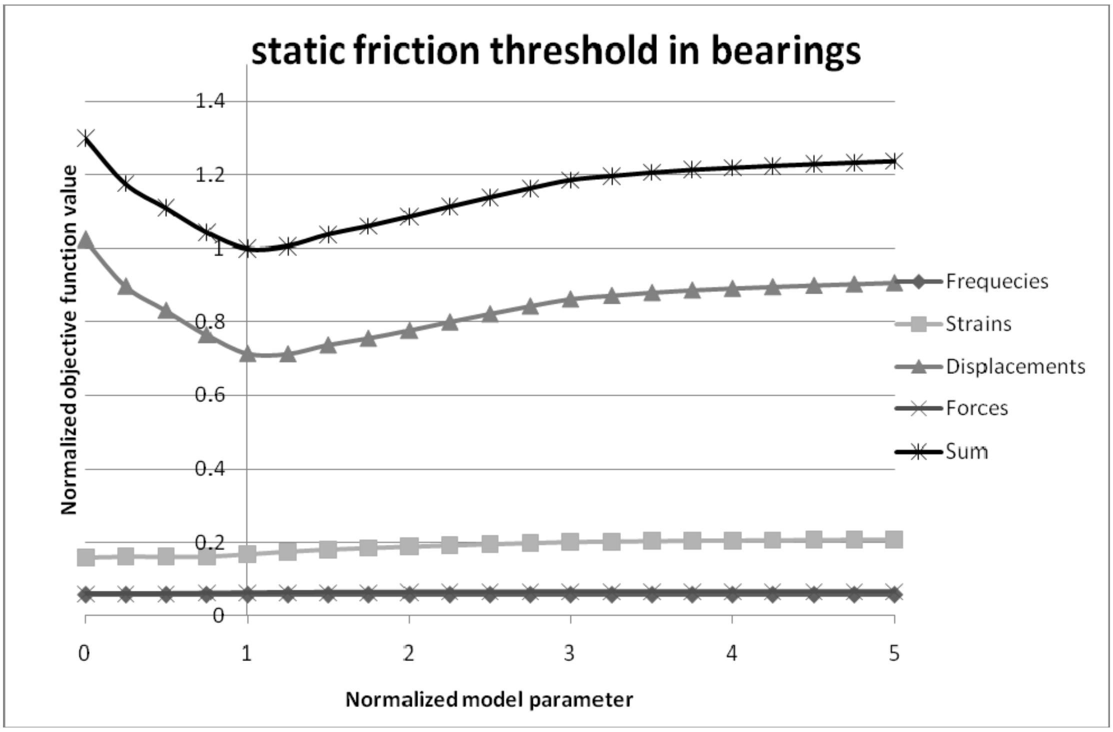

Figure 10.6 Development of the objective function J_3 over changes of the static friction coefficient in the bearings

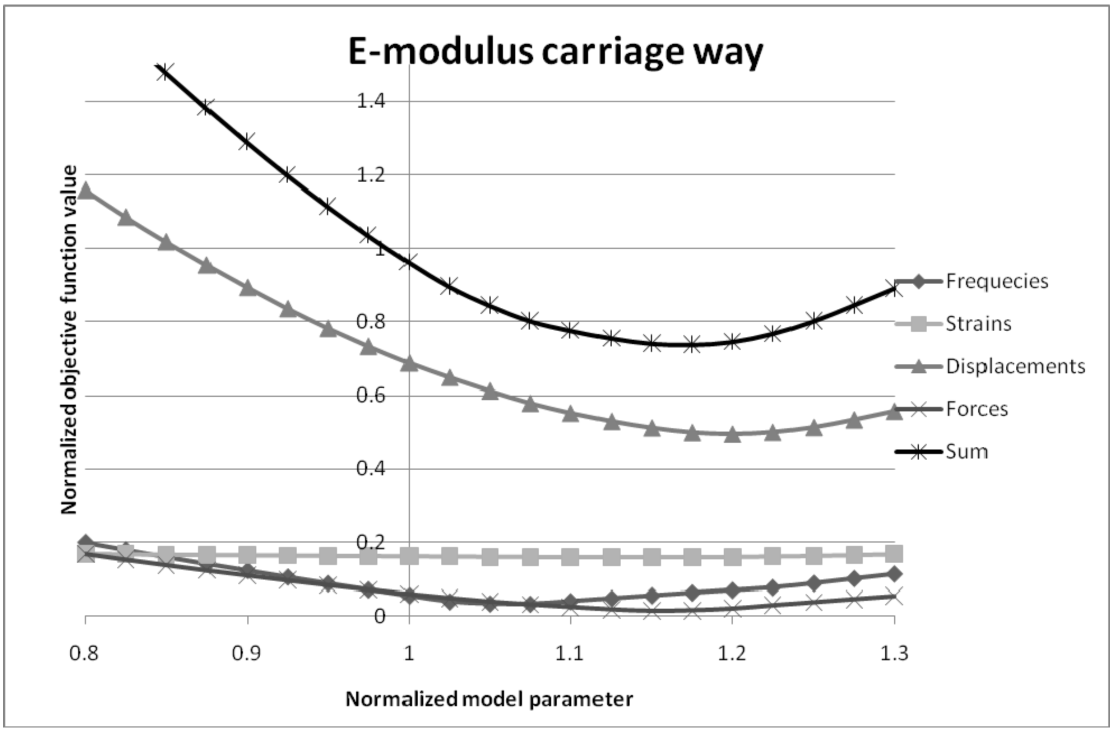

Figure 10.7 Development of the objective function J_3 over changes of the E-modulus of the steel of the carriage way

10.3 Static friction coefficient

To estimate the coefficient of friction, $\mu_{max,T}$,Jonsson and Johnson (2007) assumed a total sliding path of 30,000 m before testing as recommended by Dr. H. Segerer from Maurer Söhne GmbH & Co. KG. In Schlune *et al.* (2008a) a smaller total sliding path is assumed which reduced the initially assumed coefficient of friction $\mu_{max,T}$ to 25% of the values assumed by Jonsson and Johnson (2007).